IEEE Access
Multidisciplinary | Rapid Review | Open Access Journal

Received January 18, 2019, accepted February 19, 2019, date of publication March 5, 2019, date of current version March 25, 2019.

Digital Object Identifier 10.1109/ACCESS.2019.2903139

On Total Transmission Power Minimization Approach to Decentralized Beamforming in Single-Carrier Asynchronous Bidirectional Relay-Assisted Communication Networks

SAHAR BASTANIRAD, SHAHRAM SHAHBAZPANAHI, (Senior Member, IEEE), RAZGAR RAHIMI, AND ALI GRAMI, (Senior Member, IEEE)

Department of Electrical, Computer, and Software Engineering, University of Ontario Institute of Technology, Oshawa, ON L1H 7K4, Canada

Corresponding author: Shahram Shahbazpanahi (shahram.shahbazpanahi@uoit.ca)

This project was funded by Canada's Natural Science and Engineering Research Council (NSERC) through its Discovery Grant and Collaborative Research Development grant programs.

ABSTRACT Considered in this paper is a single-carrier *asynchronous* bidirectional cooperative network with amplify-and-forward relays helping two transceivers to exchange information. The network is asynchronous in the sense that the delays of propagation along different relaying paths are significantly different from each other. As a result, the channel between the two transceivers is best modeled with a multi-tap impulse response, which can cause interference between adjacent symbols at the end nodes. In a block-wise communication scheme, a cyclic prefix can be appended to each transmitted block of information symbols to avoid interference between adjacent blocks. To suppress interference within each block, however, the network parameters, namely, the relay complex weights and the transceivers' transmit powers should be judiciously chosen. To do so, we herein minimize, over these parameters, the total transmission power consumption throughout the network, under two constraints which ensure that the transceivers' data rates are above two given thresholds. It is herein proved that solving this total transmission power minimization problem leads to the impulse response of end-to-end channel having only a single non-zero tap. Indeed, our analysis shows that only the relays associated with this non-zero tap have to be selected to participate in the information exchange between the two transceivers. We present a simple 1-D integer search algorithm for optimally determining the index of the non-zero tap of the channel impulse response of the end-to-end channel. We also present computationally simple semi-closed-form expressions for the optimal values of the design parameters. Our numerical results show that under identical rate thresholds, in our power allocation scheme, *for any channel realization*, half of the power budget is allocated to the two transceivers and the remaining half is shared among all the relay nodes.

INDEX TERMS Network beamforming, power control, two-way relay networks, bi-directional cooperative communication, distributed beamforming, power minimization.

I. INTRODUCTION

For the last decade, a considerable volume of research efforts has focused on two-way relay networks (see [2] and references therein). In the simplest form of a two-way (bi-directional) relay network, two transceivers communicate with each other with the help of one or more relay nodes. Indeed, the relay nodes cooperate with each other to establish a bidirectional information exchange between the end-nodes. Associated with each relay node, there exists a path which conveys the signal from one transceiver to the other transceiver or vice versa. Such a path is herein referred to as a relaying path. If the propagation delays of different relaying paths are the same (or approximately the same), the network is said to be synchronous. In a *synchronous* bi-directional amplify-and-forward (AF) relaying scheme, where the transceiver-relay links are frequency-flat, the end-to-end channel (i.e., the channel between the two transceivers) can be described using a frequency-flat channel model. Such a channel model represents the channel between

The associate editor coordinating the review of this manuscript and approving it for publication was Muhamamd Aleem.

the two end-nodes with a complex coefficient which depends on the transceiver-relay channel coefficients as well as on the relays' complex weights.

In real-world applications, however, the signals going through different relays may arrive at each of the two end-nodes with different propagation delays. In other words, different relaying paths could incur different propagation delays. In such *asynchronous* two-way relay networks, a single channel gain (i.e., a frequency-flat model) may not be sufficient to describe the channel between the two transceivers; instead, a multi-tap characterization seems to be a more realistic channel representation. Such a multi-tap channel inevitably produces interference between adjacent symbols (i.e., ISI) at the receiving sides of the two transceivers, specially when the data rates are significantly higher than the inverse of the channel delay spread. Hence, channel equalization becomes inevitable, in order to suppress or eliminate ISI at the receiving sides of the two end-nodes. It is noteworthy that a significant distinction exists between the traditional models that characterize a generic multi-path link and a multi-path characterization of an asynchronous relay channel. This difference is due to the fact that in traditional models of multi-path channels, the end-to-end channel impulse response (E2ECIR) depends solely on the wireless medium. As a result, the equalizer has no control over the channel characteristics and must compensate for the channel as much as possible. In such models, the multi-path channel can be equalized at the receiving sides of the two end-nodes (transceivers), through post-channel equalization, and/or at the transmitting sides of these nodes through pre-channel equalization, otherwise known as pre-coding. But, in an asynchronous bidirectional AF relay (ABAR) network, the E2ECIR depends on the relays' AF complex weights. One can judiciously determine these weights based on some optimality criterion. Hence, in the design process of an ABAR network, besides pre-channel and/or post-channel equalizers, there exist additional degrees of freedom for optimal suppression of ISI. Considering ABAR networks, in [3]–[9], the authors model the channel between the two end-nodes using a multi-tap impulse response and exploit these additional degrees of freedom to the advantage of optimal ISI cancelation. These studies use different optimality criterion: for example, the results of [3] rely on a max-min SNR criterion for multi-carrier systems, the studies in [4] and [5] use the sum-rate as the optimality criterion (for multi-carrier and single-carrier schemes, respectively) and the investigations in [7]–[9] resort to a mean squared error (MSE) metric for single-carrier schemes. These approaches have one aspect in common: *each of them optimizes its corresponding optimality criterion under a constraint on the total transmission power*.

Different from [3]–[9], *we herein aim to design a single-carrier ABAR network, by minimizing the total transmission power consumed throughout the network, under two constraints which require the transceivers' data rates to be above two given thresholds.* Such a design approach allows us to ensure that the smallest possible level of transmit power is consumed throughout the network, for any given thresholds on the data rates at the two end-nodes. The results in [3]–[9] do not address such a design approach. In this paper, we focus on the multiple access broadcast channel (MABC) bidirectional AF relaying technique, which consists of two time slots. In the first time-slot, the two transceivers (end-nodes) simultaneously broadcast their information symbols. Each relay receives a noise-contaminated superposition of the attenuated signals broadcasted by the two end-nodes. In the second time-slot, each relay broadcasts, to the end-nodes, a phase- and amplitude-adjusted version of that relay's received signal. Based on the assumption that both transceivers have the knowledge of the global channel state information (CSI), each transceiver eliminates the self-interference signal from its received signal and uses the remaining signal to detect the symbol of interest. Considering such a communication scheme, we first present a model for the E2ECIR and model the received signals, as well as the effective noise at the two end-nodes, in an MABC-based ABAR network in the absence of *any pre- or post-channel equalizers*. We then present the following contributions:

- By assuming the transmission powers at the two transceivers and the relays' beamforming weights as the design parameters, the paper formulates the problem of minimizing the transmission power consumption throughout the communication scheme, subject to two constraints which ensure that the transceivers' data rates are above two given thresholds.
- We rigorously prove that, solving this total transmission power minimization problem leads to the E2ECIR having only a single non-zero tap. In fact, our analysis shows that only those relays which are associated with this nonzero tap, must be selected to participate in the information exchange between the two end-nodes. We devise a simple 1-dimensional integer search algorithm to find the index of the non-zero tap of the E2ECIR.
- Finally, we show how the optimal values of the relays' complex weights and the transmission powers of the two transceivers can be calculated using computationally efficient expressions.

The problem of total transmission power minimization has been extensively studied in the literature, see [2] and [10]–[14]. The motivation behind minimizing the total transmission power is to ensure the least transmit power is consumed throughout the network, while guaranteeing a certain quality of service at the receiver(s). Indeed, this approach aims to find the most power-efficient design for the network.

It is worth mentioning that the authors of [15]–[20] have also made important contributions to the research on asynchronous relay networks. However, these authors do not study the same bi-directional relay network which we herein consider. In the sequel, we briefly explain the differences between our work and each of these investigations. Wang *et al.* [15] assume that each relay transmits a filtered version of that relay's received signal, while in this paper,

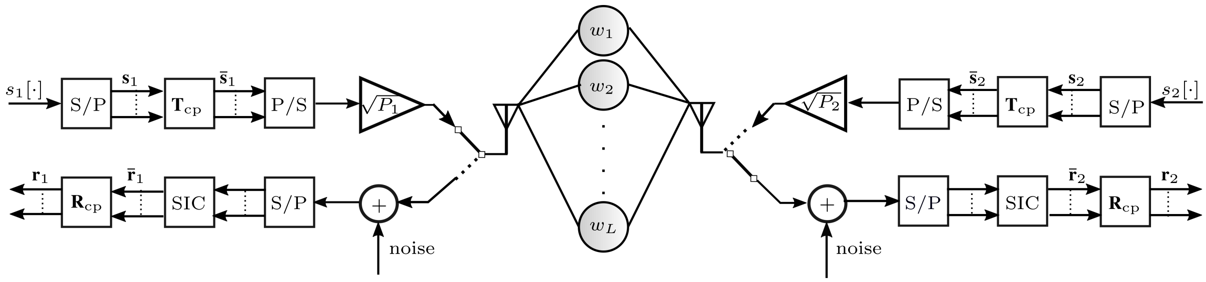

FIGURE 1. System block diagram.

the relays use simple AF relaying protocols. Moreover, considering two linear post-channel equalization techniques, namely minimum mean squared error (MMSE) and zero forcing (ZF) receivers, Wang *et al.* [15] study the diversity of these techniques, while we herein do not assume any type of equalizations and aim to determine the power-optimal values of the transceivers' transmit powers and those of the relays' AF coefficients under two data rate constraints at the two end-nodes. Considering an asynchronous bi-directional relay network of two transceivers and a multi-antenna relay, Fang *et al.* [20] devise a timing offset and channel estimation algorithm and use this algorithm for relay re-synchronization. We do not assume any relay re-synchronization.

The rest of the paper is organized as it follows: Section II presents data modeling and system description. Section III presents the total power minimization problem and the solution thereof. Simulation results are presented in Section IV and conclusions are drawn in Section V.

Notation: We use bold upper- and lower-case letters to represent matrices and vectors, respectively. $tr(\cdot)$ and $E\{\cdot\}$ are used to denote the trace of a matrix and the statistical expectation operator, respectively. The complex conjugate, transpose, and Hermitian transpose of a matrix or a vector are denoted by $(\cdot)^*$, $(\cdot)^T$, and $(\cdot)^H$, respectively. The l_2 norms of a vector is denoted as $\|\cdot\|$. $\boldsymbol{I}_N$ and $\mathbf{0}_{N\times M}$ are used to represent the $N\times N$ identity matrix and the $N\times M$ all-zero matrix, respectively. We use diag($\mathbf{a}$) to denote a diagonal matrix whose diagonal entries are the elements of vector $\mathbf{a}$. The inverse of a matrix is represented by $(\cdot)^{-1}$, while the entry (m, n) and the determinant of matrix $\boldsymbol{\Gamma}$ are given by $\boldsymbol{\Gamma}(m, n)$ and det($\boldsymbol{\Gamma}$), respectively. The magnitude of a complex number is denoted as $|\cdot|$. The smallest integer number greater than the real number z is represented as $\lceil z \rceil$.

II. DATA MODELING

The end-to-end channel model which this paper relies on, was originally developed in [5]. We review this model for the sake of clarity. We consider an ABAR network consisting of two single-antenna transceivers (end-nodes) and L single-antenna relay nodes. The relays use a simple amplify-and-forward relaying protocol to establish a two-way communication link between the two end-nodes. In this network, we assume that the two transceivers can exchange information only through the relays, implying that no direct link exists between the end-nodes. We focus on the AF-based MABC bidirectional relaying technique, where the two end-nodes simultaneously broadcast, in the first time-slot, their information symbols. Each relay receives a noise-contaminated superposition of the faded versions of the signals transmitted by the end-nodes. Each relay adjusts the amplitude and the phase of that relay's received signal and transmits the so-obtained signal to the two end-nodes, in the second time-slot. Based on the assumption that both end-nodes have the knowledge of the global CSI, each end-node eliminates the self-interference signal from that transceiver's received signal and uses the remaining signal to detect the symbols of interest. Due to the fact that different network nodes are distributed at different locations, different relaying paths have different propagation delays. Thus, the end-to-end channel can be modeled using a multi-tap impulse response which causes ISI at the two end-nodes, when the data rates are relatively high. In a block-wise system modeling of such a communication network, the symbols are assumed to be transmitted and received in blocks. As such, the multi-path characteristics of the end-to-end link results in both intra- and inter-block interference. The latter interference can be avoided by inserting CP between consecutively transmitted blocks, as shown in Fig. 1. The effect of intra-block-interference, however, has to be somehow accounted for in the system performance optimization.

Fig. 1 illustrates the single-carrier ABAR network we study in this paper. At the transmitter front-end of each transceiver, the information symbols are converted into blocks of N_s symbols, using an "S/P" block, which is a serial-to-parallel conversion block. For $i = \cdots, -2, -1, 0, 1, 2, \cdots$, let the $N_s \times 1$ vector $\mathbf{s}_q(i)$ represent the i^{th} symbol vector broadcasted by Transceiver (TRx) q, that is

$$\mathbf{s}_q(i) = \begin{bmatrix} s_q[iN_s] & s_q[iN_s+1] & \cdots & s_q[iN_s+N_s-1] \end{bmatrix}^T, \quad (1)$$

with $s_q[k]$ being the k^{th} symbol broadcasted by the q^{th} transceiver to all relays, for $q \in \{1, 2\}$. It is assumed that the transmitted symbols are drawn from unit-power constellations, that is $E\{|s_q[k]|^2\} = 1$ and $E\{s_q[k]\} = 0$ hold true, for $q \in \{1, 2\}$ and for any integer k. As shown in Fig. 1, $\mathbf{s}_q(i)$ is appended with the CP through multiplication of $\mathbf{s}_q(i)$ with the matrix $\mathbf{T}_{\text{cp}} \triangleq [\tilde{\mathbf{I}}_{\text{cp}}^T \;\; \mathbf{I}_{N_s}^T]^T$, where $\tilde{\mathbf{I}}_{\text{cp}}$ is a matrix consisting of the last N rows of the $N_s \times N_s$ identity matrix $\mathbf{I}_{N_s}$ and N is the length of the vector of the taps of the E2ECIR. Thus, the $N_t \times 1$ vector of the output of the CP insertion block, denoted as $\bar{\mathbf{s}}_q(i)$, can be written as

$$\begin{aligned}\bar{\mathbf{s}}_q(i) &\triangleq \left[\bar{s}_q[iN_t] \quad \bar{s}_q[iN_t+1] \quad \cdots \quad \bar{s}_q[iN_t+N_t-1]\right]^T \\ &\triangleq \mathbf{T}_{\rm cp}\mathbf{s}_q(i) \\ &= \left[s_q[(i+1)N_s-N] \cdots s_q[(i+1)N_s-1]\right. \\ &\qquad \left. s_q[iN_s] \cdots s_q[(i+1)N_s-1]\right]^T \end{aligned} \tag{2}$$

where $N_t \triangleq N+N_s$ is the length of the transmitted blocks and $\bar{s}_q[iN_t+k]$ is the $k^{\rm th}$ element of $\bar{\mathbf{s}}_q(i)$. The data block $\bar{\mathbf{s}}_q(i)$ is then converted back to serial by a parallel-to-serial convertor block (denoted as "P/S" in Fig. 1). The serial output is first multiplied by $\sqrt{P_q}$, where P_q denotes the transmission power of TRx q, and is then broadcasted to all the relays.

At the receiving front-end of each transceiver, the received signal is organized into blocks of N_t symbols using an S/P block. Self-interference cancellation is then performed on the vectors (blocks) of the received signals using an "SIC" block. Based on the fact that each transceiver has the knowledge of its own transmitted signals and using the assumption that each transceiver knows the channel gains between itself and the relays and that each transceiver calculates the relay beamforming complex weights, each transceiver can cancel, from its received signal, self-interference (which is the signal transmitted by this transceiver after being relayed back to the same transceiver). The CP is then removed from the received signal vectors by multiplying these vectors with the CP removing matrix $\mathbf{R}_{\rm cp} \triangleq [\mathbf{0}_{N_s\times N} \quad \mathbf{I}_{N_s}]$.

A. CHANNEL MODELING

The discrete-time channel model for our system is next presented. The link between TRx q and Relay l is herein assumed to be frequency-flat and reciprocal and is thus represented by a complex coefficient, denoted as g_{lq}, and a propagation delay, denoted as τ'_{lq}. Let w_l stand for the complex weight used by Relay l to amplify-and-forward that relay's received signal. Then one can easily see that the signal going through the $l^{\rm th}$ relaying path is amplified/attenuated by a complex factor, denoted as b_l, which is given as

$$b_l \triangleq w_l g_{l1} g_{l2}. \tag{3}$$

If the $l^{\rm th}$ relaying path delays the signal traveling from one end-node to the other end-node by τ_l seconds, then we can write

$$(\breve{n}_l - 1)T_s < \tau_l \leq \breve{n}_l T_s. \tag{4}$$

Here, $\breve{n}_l = \lceil \frac{\tau_l}{T_s} \rceil$ is the number of discrete-time samples the signal is delayed due to the propagation delay of the $l^{\rm th}$ relaying path and T_s is the symbol period. We emphasize that Relay l, whose relaying delay τ_l satisfies (4), contributes (or subscribes) only to the $\breve{n}_l^{\rm th}$ tap of the E2ECIR. We can now describe the $n^{\rm th}$ tap of the E2ECIR, denoted by $h[n]$, as

$$h[n] = \sum_{l=1}^{L} b_l \delta[n - \breve{n}_l], \quad \text{for } n \in \mathcal{N} \tag{5}$$

where we define $\mathcal{N} \triangleq \{0, 1, \ldots, N-1\}$ and N is the maximum of discrete-time relaying path delays (or the delay spread of $h[\cdot]$) and is given by

$$N = 1 + \max_{1\leq l\leq L} \lceil \frac{\tau_l}{T_s} \rceil = 1 + \max_{1\leq l\leq L} \breve{n}_l \tag{6}$$

In light of (3), the E2ECIR $h[\cdot]$ depends on the vector of the relay weights which is denoted as $\mathbf{w} \triangleq [w_1 \ w_2 \ \ldots \ w_L]^T$. Let $\mathbf{h}(\mathbf{w})$ denote the vector of the end-to-end channel taps, that is

$$\mathbf{h}(\mathbf{w}) \triangleq [h[0] \ \ h[1] \ \ h[2] \ \ \cdots \ \ h[N-1]]^T. \tag{7}$$

To express the contribution of Relay l to $h[n]$, we introduce an $N \times L$ matrix $\mathbf{B}$ whose $(n+1, l)^{\rm th}$ element is given by

$$\mathbf{B}(n+1, l) \triangleq \begin{cases} g_{l1}g_{l2}, & \frac{\tau_l}{T_s} \leq n < \frac{\tau_l}{T_s} + 1 \\ 0, & \text{otherwise,} \end{cases} \tag{8}$$

for $n \in \mathcal{N}$ and $l = 1, \ldots, L$. We now use (3), (5), and (8) to write $\mathbf{h}(\mathbf{w})$ as

$$\mathbf{h}(\mathbf{w}) = \mathbf{B}\mathbf{w} \tag{9}$$

where $\mathbf{h}(\mathbf{w})$ is given as in (7). Note that (8) and (9) imply that $\mathbf{B}$ has only one non-zero entry in each of its columns. This specific structure of $\mathbf{B}$ follows from the fact that each relay subscribes only to one of the taps of the E2ECIR $h[\cdot]$. If row $(n+1)$ of $\mathbf{B}$ is a zero vector, this means that none of the relay subscribes to $h[n]$, and hence, $h[n]$ is zero.

B. RECEIVED NOISE MODELING

The signal passing through each relaying path is contaminated with noise. To model the noise at the receiver front-ends of the transceivers, we recall that τ'_{lq} is defined as the signal delay due to propagation between Relay l and TRx q, while n'_{lq} stands for the corresponding discrete-time equivalent delay (measured in samples) satisfying the condition $\frac{\tau'_{lq}}{T_s} \leq n'_{lq} < \frac{\tau'_{lq}}{Ts} + 1$. Let also $\upsilon_l[n]$ denote the zero-mean spatio-temporally white noise at Relay l which has a variance of σ^2. As the relays employ the AF relaying protocol, the noise received at the $l^{\rm th}$ relay is first multiplied by w_l, and then, arrives at TRx q with attenuation g_{lq} and with a delay of n'_{lq} samples. The superposition of all relay noises received at TRx q is denoted by $\xi_q[n]$ and can be written as

$$\xi_q[n] \triangleq \sum_{l=1}^{L} w_l g_{lq} \upsilon_l[n - n'_{lq}] = \mathbf{v}_{n,q}^T \mathbf{G}_q \mathbf{w} \tag{10}$$

where we defined

$$\begin{aligned}\mathbf{v}_{n,q} &\triangleq [\upsilon_1[n-n'_{1q}] \ \upsilon_2[n-n'_{2q}] \ \cdots \ \upsilon_L[n-n'_{Lq}]]^T \\ \mathbf{G}_q &\triangleq \text{diag}\{g_{1q}, g_{2q}, \cdots, g_{Lq}\}.\end{aligned}$$

The total effective noise at TRx q can then be expressed as

$$\psi_q[n] = \xi_q[n] + \psi'_q[n]$$

where $\psi'_q[n]$ is the corresponding receiver noise. Defining

$$\begin{aligned}\boldsymbol{\psi}_q(i) &\triangleq \left[\psi_q[iN_t] \;\; \psi_q[iN_t+1] \;\; \cdots \;\; \psi_q[iN_t+N_t-1]\right]^T \\ \boldsymbol{\xi}_q(i) &\triangleq [\xi_q[iN_t] \;\; \xi_q[iN_t+1] \;\; \cdots \;\; \xi_q[iN_t+N_t-1]]^T \\ \boldsymbol{\psi}'_q(i) &\triangleq [\psi'_q[iN_t] \;\; \psi'_q[iN_t+1] \;\; \cdots \;\; \psi'_q[iN_t+N_t-1]]^T,\end{aligned}$$

we can now write

$$\boldsymbol{\psi}_q(i) = \boldsymbol{\Upsilon}_q(i)\mathbf{G}_q\mathbf{w} + \boldsymbol{\psi}'_q(i) \tag{11}$$

where $\boldsymbol{\Upsilon}_q(i) \triangleq \left[\mathbf{v}_{iN_t,q} \; \mathbf{v}_{iN_t+1,q} \cdots \mathbf{v}_{(iN_t+N_t-1),q}\right]^T$ is an $N_t \times L$ matrix whose l^{th} column is the noise of Relay l.

C. RECEIVED SIGNAL MODELING

At the output of the SIC block of TRx q, the i^{th} received signal block, denoted as $\bar{\mathbf{r}}_q(i)$ can be written as [21]

$$\bar{\mathbf{r}}_q(i)=\sqrt{P_{\bar{q}}}\mathbf{H}_0(\mathbf{w})\bar{\mathbf{s}}_{\bar{q}}(i) + \sqrt{P_{\bar{q}}}\mathbf{H}_1(\mathbf{w})\bar{\mathbf{s}}_{\bar{q}}(i-1) + \boldsymbol{\psi}_q(i) \tag{12}$$

where $\bar{q} = 1$ for $q = 2$ and $\bar{q} = 2$ for $q = 1$, and we define $\mathbf{H}_0(\mathbf{w})$ and $\mathbf{H}_1(\mathbf{w})$ as

$$\begin{aligned}\mathbf{H}_0(\mathbf{w}) &\triangleq \begin{bmatrix} h[0] & 0 & 0 & \cdots & 0 \\ \vdots & h[0] & 0 & \cdots & 0 \\ h[N-1] & \cdots & \ddots & \cdots & \vdots \\ \vdots & \ddots & \cdots & \ddots & 0 \\ 0 & \cdots & h[N-1] & \cdots & h[0] \end{bmatrix} \\ \mathbf{H}_1(\mathbf{w}) &\triangleq \begin{bmatrix} 0 & \cdots & h[N-1] & \cdots & h[1] \\ \vdots & \ddots & 0 & \ddots & \vdots \\ 0 & \cdots & \ddots & \cdots & h[N-1] \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & \cdots & 0 \end{bmatrix}.\end{aligned} \tag{13}$$

To avoid inter-block-interference, the received signal vector $\bar{\mathbf{r}}_q(i)$ is passed through the CP removal block, and thus, the first N entries of this vector are removed. Denoting the output at the CP removal block as $\mathbf{r}_q(i)$ and based on the fact that $\mathbf{R}_{\text{cp}}\mathbf{H}_1(\mathbf{w}) = \mathbf{0}_{N_s\times N_t}$ holds true, we can write

$$\begin{aligned}\mathbf{r}_q(i) &\triangleq \mathbf{R}_{\text{cp}}\bar{\mathbf{r}}_q(i) = \sqrt{P_{\bar{q}}}\mathbf{R}_{\text{cp}}\mathbf{H}_0(\mathbf{w})\mathbf{T}_{\text{cp}}\mathbf{s}_{\bar{q}}(i) + \mathbf{R}_{\text{cp}}\boldsymbol{\psi}_q(i) \\ &= \sqrt{P_{\bar{q}}}\widetilde{\mathbf{H}}(\mathbf{w})\mathbf{s}_{\bar{q}}(i) + \widetilde{\boldsymbol{\psi}}_q(i).\end{aligned} \tag{14}$$

Here, $\widetilde{\boldsymbol{\psi}}_q(i) \triangleq \mathbf{R}_{\text{cp}}\boldsymbol{\psi}_q(i)$ is an $N_s \times 1$ vector of the effective noise at TRx q after CP removal, the effective channel matrix $\widetilde{\mathbf{H}}(\mathbf{w}) \triangleq \mathbf{R}_{\text{cp}}\mathbf{H}_0(\mathbf{w})\mathbf{T}_{\text{cp}}$ is an $N_s \times N_s$ circulant matrix with element (k, l) given by $\tilde{h}[(k-l) \mod N_s]$, and $\tilde{h}[n]$ is defined as

$$\tilde{h}[n] \triangleq \begin{cases} h[n], & \text{for } 0 \le n < N \\ 0, & \text{for } N \le n < N_s - 1. \end{cases} \tag{15}$$

Note that in (14), the effective channel matrix $\tilde{\mathbf{H}}(\mathbf{w})$ can in general be non-diagonal, and thus can produce interference within each received block.

D. TOTAL TRANSMISSION POWER FORMULATION

We need to express the total transmission power consumed throughout the network in terms of P_1 and P_2, and $\mathbf{w}$. Based on Fig. 1, the $N_t \times 1$ vector $\bar{\mathbf{x}}_l(i)$ of the i^{th} signal block, transmitted by Relay l, can be represented as

$$\begin{aligned}\bar{\mathbf{x}}_l(i) &\triangleq \left[\bar{x}_l[iN_t] \quad \bar{x}_l[iN_t+1] \quad \cdots \quad \bar{x}_l[iN_t+N_t-1]\right]^T \\ &= w_l\left[\sqrt{P_1}g_{l1}\bar{\mathbf{s}}_1(i) + \sqrt{P_2}g_{l2}\bar{\mathbf{s}}_2(i) + \boldsymbol{\upsilon}_l(i)\right]\end{aligned} \tag{16}$$

where the vector

$$\boldsymbol{\upsilon}_l(i) \triangleq [\upsilon_l[iN_t] \;\; \upsilon_l[iN_t+1] \;\; \cdots \;\; \upsilon_l[iN_t+N_t-1]]^T$$

is the i^{th} noise vector at Relay l and $\bar{x}_l[t]$ is the signal transmitted by this relay at time t. Modeling $\boldsymbol{\upsilon}_l(\cdot)$ as a zero-mean spatio-temporally white stochastic (vector) process whose elements have a variance of σ^2, we can write the total average transmit power of Relay l as

$$\begin{aligned}\widetilde{P}_l &\triangleq \frac{1}{N_t}\mathrm{E}\left\{\bar{\mathbf{x}}_l^H(i)\bar{\mathbf{x}}_l(i)\right\} \\ &= \frac{|w_l|^2}{N_t}\mathrm{E}\Big\{\left[\sqrt{P_1}g_{l1}^*\bar{\mathbf{s}}_1^H(i) + \sqrt{P_2}g_{l2}^*\bar{\mathbf{s}}_2^H(i) + \boldsymbol{\upsilon}_l^H(i)\right] \\ &\quad \times \left[\sqrt{P_1}g_{l1}\bar{\mathbf{s}}_1(i) + \sqrt{P_2}g_{l2}\bar{\mathbf{s}}_2(i) + \boldsymbol{\upsilon}_l(i)\right]\Big\} \\ &= \frac{P_1|g_{l1}|^2|w_l|^2}{N_t}\mathrm{E}\left\{\bar{\mathbf{s}}_1^H(i)\bar{\mathbf{s}}_1(i)\right\} \\ &\quad + \frac{P_2|g_{l2}|^2|w_l|^2}{N_t}\mathrm{E}\left\{\bar{\mathbf{s}}_2^H(i)\bar{\mathbf{s}}_2(i)\right\} + \frac{|w_l|^2}{N_t}\mathrm{E}\left\{\boldsymbol{\upsilon}_l^H(i)\boldsymbol{\upsilon}_l(i)\right\} \\ &= |w_l|^2\left(|g_{l1}|^2P_1 + |g_{l2}|^2P_2 + \sigma^2\right)\end{aligned} \tag{17}$$

where it is assumed that $\bar{\mathbf{s}}_1(\cdot)$, $\bar{\mathbf{s}}_2(\cdot)$, and $\mathbf{v}_l(\cdot)$ are zero-mean mutually independent stationary random vector processes, while the last equality is based on the assumption that the largest time differences between the transceivers' signal arrivals at the relays are negligible, compared to the communication time frame [3]. Using (17), we can represent the total transmission power consumption throughout the communications network as

$$\begin{aligned}P_{\text{total}} &\triangleq P_1 + P_2 + \sum_{l=1}^{L}\widetilde{P}_l \\ &= P_1 + P_2 + \sum_{l=1}^{L}|w_l|^2\left(|g_{l1}|^2P_1 + |g_{l2}|^2P_2 + \sigma^2\right) \\ &= P_1\left(1 + \|\mathbf{G}_1\mathbf{w}\|^2\right) + P_2\left(1 + \|\mathbf{G}_2\mathbf{w}\|^2\right) + \sigma^2\mathbf{w}^H\mathbf{w}.\end{aligned} \tag{18}$$

Note that $\sum_{l=1}^{L}\sigma^2|w_l|^2|g_{lq}|^2 = \mathbf{w}^H\mathbf{D}_q\mathbf{w}$, where the following definition $\mathbf{D}_q \triangleq \sigma^2\mathrm{diag}\{|g_{lq}|^2\}$ is used, for $q \in \{1, 2\}$. Based on this definition, the total transmission power in (18) can be rewritten as

$$P_{\text{total}} = \frac{1}{\sigma^2}\left(\sum_{q=1}^{2} P_q\left(\mathbf{w}^H\mathbf{D}_q\mathbf{w}+\sigma^2\right)+\sigma^4\mathbf{w}^H\mathbf{w}\right). \quad (19)$$

In our design approach, the goal is to minimize P_{total} subject to two constraints on the minimum data rates at the two transceivers.

III. POWER MINIMIZATION

A. PROBLEM DEFINITION

The optimization problem (OP) we consider, can be written as

$$\begin{aligned} &\min_{\mathbf{w},P_1,P_2} \; P_{\text{total}} \\ &\text{s. t. } R_1(P_2,\mathbf{w}) \geq r_1, \quad R_2(P_1,\mathbf{w}) \geq r_2 \\ &\qquad P_1 \geq 0, \quad P_2 \geq 0 \end{aligned} \quad (20)$$

where $R_q(P_{\bar{q}},\mathbf{w})$ is the data rates achieved at TRx q and r_q is the corresponding minimum required data rate, for $q = 1, 2$. We can express the data rate corresponding to the multiple input multiple output (MIMO) data model in (14) as [21]

$$\begin{aligned} &R_q(P_{\bar{q}},\mathbf{w}) \\ &= \frac{1}{2}\log_2\left(\det\left(\mathbf{I}_{N_s}+P_{\bar{q}}\mathbf{C}_q(\mathbf{w})^{-1/2}\tilde{\mathbf{H}}(\mathbf{w})\tilde{\mathbf{H}}^H(\mathbf{w})\mathbf{C}_q(\mathbf{w})^{-1/2}\right)\right) \\ &\times \text{for } q=1,2. \end{aligned} \quad (21)$$

In (21), factor 1/2 accounts for the fact that the information exchange takes two time slots, whereas, for $q = 1, 2$, the matrix $\mathbf{C}_q(\mathbf{w})$ represents the covariance matrix of $\tilde{\boldsymbol{\psi}}_q(k)$ in (14) and is expressed as [7]

$$\begin{aligned} \mathbf{C}_q(\mathbf{w}) &\triangleq \mathrm{E}\{\tilde{\boldsymbol{\psi}}_q(k)\tilde{\boldsymbol{\psi}}_q^H(k)\} \\ &= \sigma^2\left(\mathbf{w}^H\mathbf{G}_q^H\mathbf{G}_q\mathbf{w}+1\right)\mathbf{I}_{N_s} \\ &= \sigma^2\left(\|\mathbf{G}_q\mathbf{w}\|^2+1\right)\mathbf{I}_{N_s} \\ &= \left(\mathbf{w}^H\mathbf{D}_q\mathbf{w}+\sigma^2\right)\mathbf{I}_{N_s} \end{aligned} \quad (22)$$

where last equality follows from the fact that for $q = 1, 2$, $\mathbf{D}_q = \sigma^2\mathbf{G}_q^H\mathbf{G}_q$ holds true.

B. PROBLEM SIMPLIFICATION

To simplify (21), as the $N_s \times N_s$ matrix $\tilde{\mathbf{H}}(\mathbf{w})$ is circulant, we can write

$$\tilde{\mathbf{H}}(\mathbf{w}) = \mathbf{F}^H\mathbf{D}(\mathbf{w})\mathbf{F}, \quad (23)$$

where

$$\mathbf{D}(\mathbf{w}) \triangleq \mathrm{diag}\{H(e^{j0}), H(e^{j\frac{2\pi}{N_s}}), \cdots, H(e^{j\frac{2\pi(N_s-1)}{N_s}})\} \quad (24)$$

and $H(e^{j\omega}) \triangleq \sum_{n=0}^{N-1} h[n]e^{-j\omega n}$ is the frequency response of the channel between the two end-nodes at frequency ω, while $\mathbf{F}$ is the $N_s \times N_s$ DFT matrix with its $(k,k')^{\text{th}}$ element being defined as $\mathbf{F}(k,k') \triangleq N_s^{-\frac{1}{2}}e^{-j2\pi(k'-1)(k-1)/N_s}$, for $k', k \in \{1, 2, \cdots, N_s\}$. By using (22) and (23) in (21), $R_q(P_{\bar{q}},\mathbf{w})$ can be rewritten as

$$\begin{aligned} &R_q(P_{\bar{q}},\mathbf{w}) \\ &= \frac{1}{2}\log_2\left(\det\left(\mathbf{I}_{N_s}+P_{\bar{q}}\mathbf{C}_q^{-1/2}(\mathbf{w})\tilde{\mathbf{H}}(\mathbf{w})\tilde{\mathbf{H}}^H(\mathbf{w})\mathbf{C}_q^{-1/2}(\mathbf{w})\right)\right) \\ &= \frac{1}{2}\log_2\left(\det\left(\mathbf{I}_{N_s}+\frac{P_{\bar{q}}}{\mathbf{w}^H\mathbf{D}_q\mathbf{w}+\sigma^2}\mathbf{F}^H\mathbf{D}(\mathbf{w})\mathbf{F}\mathbf{F}^H\mathbf{D}^H(\mathbf{w})\mathbf{F}\right)\right). \end{aligned} \quad (25)$$

Note that in (25), we dropped the factor $1/N_s$ which accounts for number of channel uses. As such, r_1 and r_2 are measured in bits. Using the fact that $\mathbf{F}\mathbf{F}^H = \mathbf{I}_{N_s}$, we can write (25) as

$$\begin{aligned} &R_q(P_{\bar{q}},\mathbf{w}) \\ &= \frac{1}{2}\log_2\left(\det\left(\mathbf{I}_{N_s}+\frac{P_{\bar{q}}}{\mathbf{w}^H\mathbf{D}_q\mathbf{w}+\sigma^2}\mathbf{D}(\mathbf{w})\mathbf{D}^H(\mathbf{w})\right)\right) \end{aligned} \quad (26)$$

where we use the fact that for any two square matrices $\mathbf{X}$ and $\mathbf{Y}$, we can write $\det(\mathbf{XY}) = \det(\mathbf{YX})$. To further simplify (26), we rewrite $\mathbf{D}(\mathbf{w})$ in (24) as

$$\mathbf{D}(\mathbf{w}) = \sqrt{N_s}\mathrm{diag}\{\mathbf{f}_1^H\tilde{\mathbf{h}}(\mathbf{w}), \mathbf{f}_2^H\tilde{\mathbf{h}}(\mathbf{w}), \cdots, \mathbf{f}_{N_s}^H\tilde{\mathbf{h}}(\mathbf{w})\} \quad (27)$$

where the $N_s \times 1$ vector $\mathbf{f}_k$ is defined as

$$\mathbf{f}_k \triangleq \frac{1}{\sqrt{N_s}}\left[1 \quad e^{j\frac{2\pi(k-1)}{N_s}} \quad \cdots \quad e^{j\frac{2(N_s-1)(k-1)\pi}{N_s}}\right]^T, \quad \text{for } k = 1, 2, \ldots, N_s, \quad (28)$$

the $N_s \times 1$ vector $\tilde{\mathbf{h}}(\mathbf{w})$ is obtained by zero-padding $\mathbf{h}(\mathbf{w})$ as

$$\begin{aligned} \tilde{\mathbf{h}}(\mathbf{w}) &\triangleq [\mathbf{h}^T(\mathbf{w}) \;\; \mathbf{0}_{1\times(N_s-N)}]^T \\ &= [h[0] \; h[1] \; \cdots \; h[N-1] \;\; \mathbf{0}_{1\times(N_s-N)}]^T \\ &= \tilde{\mathbf{B}}\mathbf{w}, \end{aligned} \quad (29)$$

and the following definition $\tilde{\mathbf{B}} \triangleq [\mathbf{B}^T \;\; \mathbf{0}^T_{(N_s-N)\times L}]^T$ is used. Using (27), we can rewrite (26) as

$$R_q(P_{\bar{q}},\mathbf{w}) = \frac{1}{2}\log_2\left(\prod_{k=1}^{N_s}\left(1+\frac{P_{\bar{q}}}{\mathbf{w}^H\mathbf{D}_q\mathbf{w}+\sigma^2}|D_{kk}(\mathbf{w})|^2\right)\right) \quad (30)$$

where $D_{kk}(\mathbf{w})$ stands for the k^{th} diagonal element of $\mathbf{D}(\mathbf{w})$. We now use (19) and (30) to write the OP (20) equivalently as

$$\begin{aligned} &\min_{\mathbf{w},P_1,P_2} \frac{1}{\sigma^2}\left(\sum_{q=1}^{2} P_q\left(\mathbf{w}^H\mathbf{D}_q\mathbf{w}+\sigma^2\right)+\sigma^4\mathbf{w}^H\mathbf{w}\right) \\ &\text{s.t. } \frac{1}{2}\log_2\left(\prod_{k=1}^{N_s}\left(1+\frac{P_{\bar{q}}|D_{kk}(\mathbf{w})|^2}{\mathbf{w}^H\mathbf{D}_q\mathbf{w}+\sigma^2}\right)\right) \geq r_q, \text{ for } q=1,2. \\ &\quad P_1 \geq 0 \quad , P_2 \geq 0. \end{aligned} \quad (31)$$

One can assume, without of loss optimality, that the first two constraints in (31) are satisfied with equality. Hence, we rewrite the OP (31) as

$$\min_{\mathbf{w},P_1,P_2} \frac{1}{\sigma^2}\left(\sum_{q=1}^{2} P_q\left(\mathbf{w}^H\mathbf{D}_q\mathbf{w}+\sigma^2\right)+\sigma^4\mathbf{w}^H\mathbf{w}\right)$$

$$\text{s.t.}\ \log_2\left(\prod_{k=1}^{N_s}\left(1+\frac{P_{\bar{q}}|D_{kk}(\mathbf{w})|^2}{\mathbf{w}^H\mathbf{D}_q\mathbf{w}+\sigma^2}\right)\right)=2r_q,\ \text{for } q=1,2.$$
$$P_1\geq 0,\quad P_2\geq 0. \tag{32}$$

We next define $2N_s$ new optimization variables γ_{qk}, for $q=1,2$ and for $k=1,\cdots,N_s$, as

$$\gamma_{qk}\triangleq\frac{P_{\bar{q}}|D_{kk}(\mathbf{w})|^2}{\mathbf{w}^H\mathbf{D}_q\mathbf{w}+\sigma^2}=\frac{P_{\bar{q}}|\mathbf{f}_k^H\tilde{\mathbf{h}}(w)|^2}{\mathbf{w}^H\mathbf{D}_q\mathbf{w}+\sigma^2}=\frac{P_{\bar{q}}|\mathbf{f}_k^H\tilde{\mathbf{B}}\mathbf{w}|^2}{\mathbf{w}^H\mathbf{D}_q\mathbf{w}+\sigma^2}, \tag{33}$$

where the first equality follows from the expression of $\mathbf{D}(\mathbf{w})$ in (27) and the second equality follows from the expression of $\tilde{\mathbf{h}}(\mathbf{w})$ in (29). Based on (33), the first two constraints in (32) can be written as

$$\sum_{k=1}^{N_s}\log_2\left(1+\gamma_{qk}\right)=2r_q,\quad \text{for } q=1,2. \tag{34}$$

Using (33), we can write $P_{\bar{q}}$ as

$$P_{\bar{q}}=\frac{\gamma_{qk}\left(\mathbf{w}^H\mathbf{D}_q\mathbf{w}+\sigma^2\right)}{|\mathbf{f}_k^H\tilde{\mathbf{B}}\mathbf{w}|^2},\quad \text{for } k\in\{1,2,\cdots,N_s\}. \tag{35}$$

Since (35) holds for any $k\in\{1,2,\cdots,N_s\}$, we need to impose the following constraints on $\{\gamma_{qk}\}_{k=1}^{N_s}$:

$$\frac{\gamma_{qk}}{|\mathbf{a}_k^H\mathbf{w}|^2}=\frac{\gamma_{qk'}}{|\mathbf{a}_{k'}^H\mathbf{w}|^2}\quad \text{for } q\in\{1,2\} \tag{36}$$

where we define the $L\times 1$ vector $\mathbf{a}_k$ as

$$\mathbf{a}_k\triangleq\tilde{\mathbf{B}}^H\mathbf{f}_k,\quad \text{for } k\in\{1,2,\cdots,N_s\}. \tag{37}$$

Also, in order to guarantee that $P_q\geq 0$ holds true, we must ensure that $\gamma_{qk}\geq 0$ holds true for $k\in\{1,2,\cdots,N_s\}$ and $q\in\{1,2\}$. Using (36), we rewrite (35) as

$$P_{\bar{q}}=\frac{1}{N_s}\left(\mathbf{w}^H\mathbf{D}_q\mathbf{w}+\sigma^2\right)\sum_{k=1}^{N_s}\frac{\gamma_{qk}}{|\mathbf{a}_k^H\mathbf{w}|^2}. \tag{38}$$

Using (38), we now rewrite the objective function in (32) as

$$P_{\text{total}}=\frac{1}{\sigma^2}\left(\frac{1}{N_s}\prod_{q=1}^{2}\left(\mathbf{w}^H\mathbf{D}_q\mathbf{w}+\sigma^2\right)\sum_{k=1}^{N_s}\frac{\gamma_{1k}+\gamma_{2k}}{|\mathbf{a}_k^H\mathbf{w}|^2}+\sigma^4\mathbf{w}^H\mathbf{w}\right). \tag{39}$$

Let us define two $N_s\times 1$ vectors $\boldsymbol{\gamma}_1$ and $\boldsymbol{\gamma}_2$ as

$$\boldsymbol{\gamma}_1\triangleq[\gamma_{11}\ \gamma_{21}\cdots\ \gamma_{N_s1}]^T \tag{40}$$
$$\boldsymbol{\gamma}_2\triangleq[\gamma_{12}\ \gamma_{22}\cdots\ \gamma_{N_s2}]^T. \tag{41}$$

Using (34), (39), (40), and (41), the OP (32) can be equivalently written as

$$\min_{\boldsymbol{\gamma}_1,\boldsymbol{\gamma}_2,\mathbf{w}}\ \frac{1}{\sigma^2}\left(\frac{1}{N_s}\prod_{q=1}^{2}\left(\mathbf{w}^H\mathbf{D}_q\mathbf{w}+\sigma^2\right)\sum_{k=1}^{N_s}\frac{\gamma_{1k}+\gamma_{2k}}{|\mathbf{a}_k^H\mathbf{w}|^2}+\sigma^4\mathbf{w}^H\mathbf{w}\right)$$
$$\text{s.t.}\ \frac{1}{2}\sum_{k=1}^{N_s}\log_2\left(1+\gamma_{1k}\right)=r_1$$
$$\frac{1}{2}\sum_{k=1}^{N_s}\log_2\left(1+\gamma_{2k}\right)=r_2$$
$$\frac{\gamma_{qk}}{|\mathbf{a}_k^H\mathbf{w}|^2}=\frac{\gamma_{qk'}}{|\mathbf{a}_{k'}^H\mathbf{w}|^2},\ \text{for } q=1,2 \text{ and } k',$$
$$k=1,2,\ldots,N_s\gamma_{qk}\geq 0,$$
$$\text{for } q=1,2 \text{ and } k=1,2,\ldots,N_s. \tag{42}$$

C. SOLVING A RELAXED VERSION OF THE OP (42)

To solve the OP (42), let us consider the following OP:

$$\min_{\boldsymbol{\gamma}_1,\boldsymbol{\gamma}_2,\mathbf{w}}\ \frac{1}{\sigma^2}\left(\frac{1}{N_s}\prod_{q=1}^{2}\left(\mathbf{w}^H\mathbf{D}_q\mathbf{w}+\sigma^2\right)\sum_{k=1}^{N_s}\frac{\gamma_{1k}+\gamma_{2k}}{|\mathbf{a}_k^H\mathbf{w}|^2}+\sigma^4\mathbf{w}^H\mathbf{w}\right)$$
$$\text{s.t.}\ \frac{1}{2}\sum_{k=1}^{N_s}\log_2\left(1+\gamma_{1k}\right)=r_1$$
$$\frac{1}{2}\sum_{k=1}^{N_s}\log_2\left(1+\gamma_{2k}\right)=r_2 \tag{43}$$

where we have considered only the first two constraints in (42). We now aim to solve the relaxed OP (43). Later we show that any solution to the relaxed OP (43) is also optimal for the original OP (42). Let $P_T^{\min}$ denote the minimum value of P_{total} obtained by solving (43), while the corresponding optimal values of the optimization variables be given by $(\boldsymbol{\gamma}_1^\circ,\boldsymbol{\gamma}_2^\circ,\mathbf{w}^\circ)$. Consider the following maximization problem:

$$\max_{\boldsymbol{\gamma}_1,\boldsymbol{\gamma}_2,\mathbf{w}}\ \frac{1}{2}\sum_{k=1}^{N_s}\log_2\left(1+\gamma_{1k}\right)$$
$$\text{s.t.}\ \frac{1}{2}\sum_{k=1}^{N_s}\log_2\left(1+\gamma_{2k}\right)=r_2$$
$$\frac{1}{\sigma^2}\left(\frac{1}{N_s}\sum_{k=1}^{N_s}\frac{\gamma_{1k}+\gamma_{2k}}{|\mathbf{a}_k^H\mathbf{w}|^2}\prod_{q=1}^{2}\left(\mathbf{w}^H\mathbf{D}_q\mathbf{w}+\sigma^2\right)+\sigma^4\mathbf{w}^H\mathbf{w}\right)$$
$$\leq P_T^{\min}. \tag{44}$$

Let $(\hat{\boldsymbol{\gamma}}_1^\circ,\hat{\boldsymbol{\gamma}}_2^\circ,\hat{\mathbf{w}}^\circ)$ be the solution to the OP (44) and $R_1{}^{\max}$ denote the maximum achievable value for the rate of TRx 1 obtained by solving (44), for the given budget $P_T^{\min}$. We now show that $R_1{}^{\max}=r_1$ holds true. To do this, we rely on contradiction: if $R_1{}^{\max}<r_1$, then $(\boldsymbol{\gamma}_1^\circ,\boldsymbol{\gamma}_2^\circ,\mathbf{w}^\circ)$ leads to a higher value for the objective function of (44). Indeed, $(\boldsymbol{\gamma}_1^\circ,\boldsymbol{\gamma}_2^\circ,\mathbf{w}^\circ)$ being a solution to (43), leads to a higher value for $\frac{1}{2}\sum_{k=1}^{N_s}\log_2\left(1+\gamma_{1k}\right)$, while $\frac{1}{2}\sum_{k=1}^{N_s}\log_2\left(1+\gamma_{2k}\right)=r_2$, and at the same time, $P_{\text{total}}=P_T^{\min}$ and this contradicts the optimality of $(\hat{\boldsymbol{\gamma}}_1^\circ,\hat{\boldsymbol{\gamma}}_2^\circ,\hat{\mathbf{w}}^\circ)$ for (44). On the other hand, we can easily show that $R_1{}^{\max}$ cannot be greater than r_1 either. Otherwise, if $R_1{}^{\max}>r_1$, then one can scale down the optimal value $\hat{\boldsymbol{\gamma}}_1^\circ$ by some real $\alpha<1$ such that $\frac{1}{2}\sum_{k=1}^{N_s}\log_2\left(1+\alpha\hat{\gamma}_{1k}^\circ\right)=r_1$ holds true, without violating the constraint in (44), i.e., $P_{\text{total}}<P_T^{\min}$. In other words, the $(\alpha\hat{\boldsymbol{\gamma}}_1^\circ,\hat{\boldsymbol{\gamma}}_2^\circ,\hat{\mathbf{w}}^\circ)$ results in a lower P_{total}, while satisfying

the two constraints in (43). This contradicts the optimality of $(\boldsymbol{\gamma}_1^\circ, \boldsymbol{\gamma}_2^\circ, \mathbf{w}^\circ)$ for (43). Therefore, we can conclude that $R_1{}^{\max} = r_1$ holds true, meaning that any solution to the OP (44) is indeed a solution to the OP (43) and any solution to the OP (43) *inherits all properties of the solutions to* (44).

It is shown in [4] that in the OP (44), one can assume, without any loss of optimality, that $\mathbf{w}$ can be restricted to the set

$$\mathcal{A} \triangleq \left\{\mathbf{w} \middle| |\mathbf{a}_k^H \mathbf{w}| = |\mathbf{a}_{k'}^H \mathbf{w}|, \qquad \text{for } k, k' \in \{1, 2, \cdots, N_s\}\right\}. \tag{45}$$

Since the solution to (43) inherits all the properties of the solutions to (44), the value of $\mathbf{w}$, which is optimal for (43), must also belong to the set $\mathcal{A}$, i.e., we can add the constraint

$$\mathbf{w} \in \mathcal{A}. \tag{46}$$

to the OP (43), without loss of optimality, we rewrite the OP (43) equivalently as in (47), as shown at the top of the next page.

D. SOLVING THE ORIGINAL OP (42)

We now prove that any solution to the relaxed OP (47) satisfies the relaxed constraints $\frac{\gamma_{qk}}{|\mathbf{a}_k^H \mathbf{w}|^2} = \frac{\gamma_{qk'}}{|\mathbf{a}_{k'}^H \mathbf{w}|^2}$ and $\gamma_{qk} \geq 0$, for $q \in \{1, 2\}$, and thus, any solution to the relaxed OP (47) is a solution to the original OP in (42). To show this, we note that solving the inner minimization in (47) amounts to solving two separate minimizations, which are given as

$$\min_{\boldsymbol{\gamma}_1} \sum_{k=1}^{N_s} \gamma_{1k}, \quad \text{s.t.} \sum_{k=1}^{N_s} \log_2(1+\gamma_{1k}) = 2r_1 \tag{48}$$

and

$$\min_{\boldsymbol{\gamma}_2} \sum_{k=1}^{N_s} \gamma_{2k}, \quad \text{s.t.} \sum_{k=1}^{N_s} \log_2(1+\gamma_{2k}) = 2r_2. \tag{49}$$

The following lemma is used to simplify the OPs in (48) and (49).

Lemma 1: Consider the following OP:

$$\min_{\mathbf{x}} \sum_{i=1}^{M} x_i \quad \text{s. t.} \sum_{i=1}^{M} \log_2(1+x_i) = c \tag{50}$$

where $\mathbf{x} = [x_1 \ x_2 \ \cdots x_M]^T$ is an $M \times 1$ vector with non-negative entries and c is a constant. At the optimum of (50), all x_i's are equal for all i (i.e., $x_i = x_j$, for $i, j \in \{1, 2, \cdots, M\}$).

Proof: See the appendix.

Using Lemma 1, we conclude that at the optimum of the OPs (48) and (49), for any k and k', $\gamma_{1k} = \gamma_{1k'} \triangleq \beta_1$ and $\gamma_{2k} = \gamma_{2k'} \triangleq \beta_2$ hold true. Thus, the first two constraints in (47) can be rewritten as $\beta_1 = 2^{\frac{2r_1}{N_s}} - 1$ and $\beta_2 = 2^{\frac{2r_2}{N_s}} - 1$, and hence, $\beta_1 = \gamma_{1k} \geq 0$ and $\beta_2 = \gamma_{2k} \geq 0$ hold true because $r_1 \geq 0$ and $r_2 \geq 0$. Using the fact that any $\mathbf{w}$ which is optimal for (47) must belong to $\mathcal{A}$ (where $|\mathbf{a}_k^H \mathbf{w}| = |\mathbf{a}_{k'}^H \mathbf{w}|$ holds true for any k and k'), we conclude that $\frac{\gamma_{qk}}{|\mathbf{a}_k^H \mathbf{w}|^2} = \frac{\gamma_{qk'}}{|\mathbf{a}_{k'}^H \mathbf{w}|^2}$ holds true for any k and k'. Hence, any solution to the relaxed OP (47) is feasible for, and thus, is a solution to the original problem in (42). As a result, solving (47) provides a solution to (42). In order to solve (47), we can write the set $\mathcal{A}$ in (45) as [7]

$$\mathcal{A} = \bigcup_{n=0}^{N-1} \mathcal{B}_n. \tag{51}$$

Here, $\mathcal{B}_n$ is the set of those values of $\mathbf{w}$ which result in only $h[n]$ being non-zero, for some $n \in \mathcal{N}$, while all other taps of $h[\cdot]$ are zero[1,2]. The sets $\{\mathcal{B}_n\}_{n=0}^{N-1}$ are mutually exclusive, that is

$$\mathcal{B}_{n_1} \bigcap \mathcal{B}_{n_2} = \emptyset, \text{ for } n_1 \neq n_2. \tag{52}$$

The reason is that each relay subscribes to only one of the taps of the E2ECIR $h[\cdot]$, For any $\mathbf{w} \in \mathcal{A}$, we can now write

$$\begin{aligned} |\mathbf{a}_k^H \mathbf{w}|^2 &= \frac{1}{N_s} \sum_{k'=1}^{N_s} |\mathbf{a}_{k'}^H \mathbf{w}|^2 = \frac{1}{N_s} \sum_{k'=1}^{N_s} |\mathbf{f}_{k'}^H \tilde{\mathbf{B}} \mathbf{w}|^2 \\ &= \frac{1}{N_s} \|\tilde{\mathbf{B}} \mathbf{w}\|^2 = \mathbf{w}^H \mathbf{B}^H \mathbf{B} \mathbf{w}, \end{aligned} \tag{53}$$

where the first equality relies on the fact that for any $\mathbf{w} \in \mathcal{B}_n$, and for any k and k', $|\mathbf{a}_k^H \mathbf{w}|^2 = |\mathbf{a}_{k'}^H \mathbf{w}|^2$ holds true, the second equality follows from the definition of $\mathbf{a}_k$ in (37), the third equality is based on the Parseval's theorem $\left(\sum_{k=1}^{N_s} |\mathbf{f}_k^H \tilde{\mathbf{B}} \mathbf{w}|^2 = \|\tilde{\mathbf{B}} \mathbf{w}\|^2\right)$, and the fourth equality follows from the fact that the last $N_s - N$ rows of $\tilde{\mathbf{B}} \mathbf{w}$ are zero. Using (53), the OP (47) can be rewritten as in (54), as shown at the top of the next page. As the sets $\{\mathcal{B}_n\}_{n=0}^{N-1}$ are mutually exclusive, we can decompose the OP (54) into a set of maximum[3] N subproblems, and then, separately solve each of these subproblems. Therefore, we will have a maximum of N candidate values for the optimal $\mathbf{w}$, Among these candidates, the one which leads to a minimum total transmission power, is indeed the solution to the OP (54) (or to the OP (47)). To further elaborate on how to solve the OP (54), let us rewrite this OP as in (55), as shown at the top of the next page, where we have used the fact that the sets $\{\mathcal{B}_n\}_{n=0}^{N-1}$ are mutually exclusive. We next define the vector $\mathbf{w}_n$ as the vector of the weights of the relays associated with $h[n]$, when $\mathcal{B}_n$ is not empty. For any $\mathbf{w} \in \mathcal{B}_n$, we have

$$\tilde{\mathbf{h}}^H(\mathbf{w}) \tilde{\mathbf{h}}(\mathbf{w}) = \mathbf{h}^H(\mathbf{w}) \mathbf{h}(\mathbf{w}) = \mathbf{w}^H \mathbf{B}^H \mathbf{B} \mathbf{w} = \mathbf{w}_n^H \mathbf{b}_n \mathbf{b}_n^H \mathbf{w}_n \tag{56}$$

where $\mathbf{b}_n$ is an $L_n \times 1$ vector whose l^{th} element is the same as the l^{th} non-zero element of the $(n+1)^{\text{th}}$ column of $\mathbf{B}^H$

[1]In other words, for $\mathbf{w} \in \mathcal{A}$, the E2ECIR $h[\cdot]$ has only a single non-zero tap and the beamforming weights of those relays which do not subscribe to this non-zero tap will be zero.

[2]Note that if no relay is associated with tap n of the E2ECIR $h[\cdot]$, then $\mathcal{B}_n$ will be empty.

[3]Note that some of $\mathcal{B}_n$'s could be empty.

$$\min_{\mathbf{w}} \min_{\gamma_1,\gamma_2} \frac{1}{\sigma^2}\left(\frac{1}{N_s}\left(\prod_{q=1}^{2}\left(\mathbf{w}^H\mathbf{D}_q\mathbf{w}+\sigma^2\right)\right)\frac{1}{|\mathbf{a}_k^H\mathbf{w}|^2}\left(\sum_{k=1}^{N_s}\gamma_{1k}+\sum_{k=1}^{N_s}\gamma_{2k}\right)+\sigma^4\mathbf{w}^H\mathbf{w}\right)$$
$$\text{s.t.} \sum_{k=1}^{N_s}\log_2(1+\gamma_{1k})=2r_1$$
$$\sum_{k=1}^{N_s}\log_2(1+\gamma_{2k})=2r_2$$
$$\mathbf{w}\in\mathcal{A}. \tag{47}$$

$$\min_{\mathbf{w}} \frac{1}{\sigma^2}\left(N_s\frac{\left(\mathbf{w}^H\mathbf{D}_1\mathbf{w}+\sigma^2\right)\left(\mathbf{w}^H\mathbf{D}_2\mathbf{w}+\sigma^2\right)(\beta_1+\beta_2)}{\mathbf{w}^H\mathbf{B}^H\mathbf{B}\mathbf{w}}+\sigma^4\mathbf{w}^H\mathbf{w}\right), \quad \text{s.t. } \mathbf{w}\in\bigcup_{n=0}^{N-1}\mathcal{B}_n. \tag{54}$$

$$\min_{0\le n\le N-1,\mathcal{B}_n\neq\emptyset} \min_{\mathbf{w}\in\mathcal{B}_n} \frac{1}{\sigma^2}\left(N_s\frac{\left(\mathbf{w}^H\mathbf{D}_1\mathbf{w}+\sigma^2\right)\left(\mathbf{w}^H\mathbf{D}_2\mathbf{w}+\sigma^2\right)(\beta_1+\beta_2)}{\mathbf{w}^H\mathbf{B}^H\mathbf{B}\mathbf{w}}+\sigma^4\mathbf{w}^H\mathbf{w}\right), \tag{55}$$

$$\min_{0\le n\le N-1,\mathcal{B}_n\neq\emptyset} \min_{\mathbf{w}_n} \frac{1}{\sigma^2}\left(\frac{(\beta_1'+\beta_2')\left(\mathbf{w}_n^H\mathbf{D}_1^{(n)}\mathbf{w}_n+\sigma^2\right)\left(\mathbf{w}_n^H\mathbf{D}_2^{(n)}\mathbf{w}_n+\sigma^2\right)}{\mathbf{w}_n^H\mathbf{b}_n\mathbf{b}_n^H\mathbf{w}_n}+\sigma^4\mathbf{w}_n^H\mathbf{w}_n\right) \tag{57}$$

$$\min_{0\le n\le N-1,\mathcal{B}_n\neq\emptyset} \min_{\mathbf{w}_n} \sigma^2\left(\frac{(\beta_1'+\beta_2')\left(1+\mathbf{w}_n^H\mathbf{Q}_1^{(n)}\mathbf{w}_n\right)\left(1+\mathbf{w}_n^H\mathbf{Q}_2^{(n)}\mathbf{w}_n\right)}{\mathbf{w}_n^H\mathbf{b}_n\mathbf{b}_n^H\mathbf{w}_n}+\mathbf{w}_n^H\mathbf{w}_n\right). \tag{58}$$

and L_n counts the number of these non-zero elements. Note that L_n counts also the number of relays which subscribe to $h[n]$. Using (56), we rewrite the OP (55) as in (57), as shown at the top of this page, where $\beta_1' \triangleq N_s\beta_1$ and $\beta_2' \triangleq N_s\beta_2$ and $\mathbf{D}_q^{(n)}$, for $q = 1, 2$, is a diagonal matrix with its diagonal elements being a subset of those diagonal elements of $\mathbf{D}_q$ which correspond to the relays that subscribe to $h[n]$. To arrive from (55) to (57), we rely on the fact that when $\mathbf{w} \in \mathcal{B}_n$, then those entries of $\mathbf{w}$ which do not correspond to the entries of $\mathbf{w}_n$, are all zero. Defining $\mathbf{Q}_q^{(n)} \triangleq \frac{1}{\sigma^2}\mathbf{D}_q^{(n)}$, for $q = 1, 2$, we can further simplify (57) as in (58), as shown at the top of this page. It is well-known that for $\mathcal{B}_n \neq \emptyset$, the inner minimization in (58) has a semi-closed-form solution for the optimal $\mathbf{w}_n$, denoted as $\mathbf{w}_n^{\text{o}}$, which is given by [12]

$$\mathbf{w}_n^{\text{o}} = \sqrt{\frac{(\beta_1'+\beta_2')}{\kappa_n\lambda_n}}((\beta_1'+\beta_2')\mathbf{Q}_2^{(n)}+\lambda_n(z_n\mathbf{Q}_1^{(n)}+\mathbf{I}_{L_n}))^{-1}\mathbf{b}_n \tag{59}$$

where

$$\kappa_n = \mathbf{b}_n^H(z_n\mathbf{Q}_1^{(n)}+\mathbf{I}_{L_n})((\beta_1'+\beta_2')\mathbf{Q}_2^{(n)}+\lambda_n(z_n\mathbf{Q}_1^{(n)}+\mathbf{I}_{L_n}))^{-2}\mathbf{b}_n. \tag{60}$$

Here, $z_n \in \left(\frac{\beta_1'+\beta_2'}{\|\mathbf{g}_1^{(n)}\|^2}, +\infty\right)$ is an intermediate parameter obtained by solving (61), as shown at the top of the next page, and λ_n is an implicit function of z_n, which is calculated numerically as the *only* positive solution to the following equation:

$$\sum_{i=1}^{L_n}\frac{z_n(z_n|g_{i1}^{(n)}|^2+1)^{-1}|g_{i1}^{(n)}|^2|g_{i2}^{(n)}|^2}{(\beta_1'+\beta_2')|g_{i2}^{(n)}|^2(z_n|g_{i1}^{(n)}|^2+1)^{-1}+\lambda_n} = 1. \tag{62}$$

Note that here, $g_{iq}^{(n)}$ is the coefficient representing the link between TRx q and the i^{th} relay which subscribes to $h[n]$. In (58), we can now find the optimal tap n, for $n = 0, 1, \ldots, N-1$, by determining which $\{\mathbf{w}_n^{\text{o}}\}_{n=0}^{N-1}$ yields the smallest value for the objective function in (58). Hence, the optimal n, denoted as n^{o}, can be found as in (63), as shown at the top of the next page. Indeed, in the 1-dimensional integer search in (63), we are looking for a subset of the relays which subscribe to one of the taps of the E2ECIR that yields the smallest possible value for the total transmission power among other relay sets. The search for the optimal tap index n is limited only to those taps of $h[\cdot]$ which can be non-zero. If for any n, none of the relays subscribes to $h[n]$, then $h[n] = 0$, which means that row $(n+1)$ of $\mathbf{B}$ is zero.

$$\hbar_n(z_n) \triangleq \frac{1/z_n^2 + \lambda_n \mathbf{b}_n^H \left(\left(\beta_1' + \beta_2'\right) \mathbf{Q}_2^{(n)} - \lambda_n (z_n \mathbf{Q}_1^{(n)} + \mathbf{I}_{L_n}) \right)^{-2} \mathbf{Q}_1^{(n)} \mathbf{b}_n}{\lambda_n^2 \mathbf{b}_n^H \left(\left(\beta_1' + \beta_2'\right) \mathbf{Q}_2^{(n)} + \lambda_n (z_n \mathbf{Q}_1^{(n)} + \mathbf{I}_{L_n}) \right)^{-2} (z_n \mathbf{Q}_1^{(n)} + \mathbf{I}_{L_n}) \mathbf{b}_n} - \frac{1}{\beta_1' + \beta_2'} = 0 \tag{61}$$

$$n^{\mathrm{o}} = \arg \min_{0 \le n \le N-1, \mathcal{B}_n \neq \emptyset} \quad \sigma^2 \left(\frac{\left(\beta_1' + \beta_2'\right) \left(\mathbf{w}_n^{\mathrm{o},H} \mathbf{Q}_1^{(n)} \mathbf{w}_n^{\mathrm{o}} + 1 \right) \left(\mathbf{w}_n^{\mathrm{o},H} \mathbf{Q}_2^{(n)} \mathbf{w}_n^{\mathrm{o}} + 1 \right)}{\mathbf{w}_n^{\mathrm{o},H} \mathbf{b}_n \mathbf{b}_n^H \mathbf{w}_n^{\mathrm{o}}} + \mathbf{w}_n^{\mathrm{o},H} \mathbf{w}_n^{\mathrm{o}} \right) \tag{63}$$

Having n^{o}, we calculate the optimal P_1 and P_2 (denoted by P_1^{o} and P_2^{o}, respectively) as

$$P_1^{\mathrm{o}} = \frac{\beta_2' \left(1 + \mathbf{w}_{n^{\mathrm{o}}}^{\mathrm{o},H} \mathbf{Q}_2^{(n^{\mathrm{o}})} \mathbf{w}_{n^{\mathrm{o}}}^{\mathrm{o}} \right)}{\mathbf{w}_{n^{\mathrm{o}}}^{\mathrm{o},H} \mathbf{b}_{n^{\mathrm{o}}} \mathbf{b}_{n^{\mathrm{o}}}^H \mathbf{w}_{n^{\mathrm{o}}}^{\mathrm{o}}}, \tag{64}$$

$$P_2^{\mathrm{o}} = \frac{\beta_1' \left(1 + \mathbf{w}_{n^{\mathrm{o}}}^{\mathrm{o},H} \mathbf{Q}_1^{(n^{\mathrm{o}})} \mathbf{w}_{n^{\mathrm{o}}}^{\mathrm{o}} \right)}{\mathbf{w}_{n^{\mathrm{o}}}^{\mathrm{o},H} \mathbf{b}_{n^{\mathrm{o}}} \mathbf{b}_{n^{\mathrm{o}}}^H \mathbf{w}_{n^{\mathrm{o}}}^{\mathrm{o}}} \tag{65}$$

Our proposed algorithm is summarized as Algorithm 1.

Remark 1: The proposed scheme enjoys a distributed implementation as explained in the sequel. As the two transceivers are assumed to have the knowledge of global CSI, each transceiver can use (59)-(65) to obtain the design parameters. One of the two end-nodes can then broadcast three parameters, namely $\kappa_{n^{\mathrm{o}}}$, $z_{n^{\mathrm{o}}}$, and $\lambda_{n^{\mathrm{o}}}$ to all the active relays (i.e., those relays contributing to $h[n^{\mathrm{o}}]$. Each of the $L_{n^{\mathrm{o}}}$ active relays can then use (59), with $n = n^{\mathrm{o}}$, to calculate its own beamforming weight using these three parameters along with its own local channel state information. Indeed, in (59), the matrix $\mathbf{Q}_q^{(n^{\mathrm{o}})}$, for $q = 1, 2$, is a diagonal matrix with its i^{th} entry being amplitude squared of the channel coefficient between TRx q and the i^{th} relay which subscribes to tap n^{o} of the E2ECIR. Also, the i^{th} element of the vector $\mathbf{b}_{n^{\mathrm{o}}}$ is the product of the two channel coefficients representing the two links between the two end-nodes and the i^{th} relay which subscribes to $h[n^{\mathrm{o}}]$. As such, the i^{th} active relay can calculate its own beamforming vector using its own local CSI along with the three parameters $\kappa_{n^{\mathrm{o}}}$, $z_{n^{\mathrm{o}}}$, and $\lambda_{n^{\mathrm{o}}}$.

Remark 2: Note that our results in this paper differ from the results of [4] in several aspects: i) The communication scheme studied in [4] is a multi-carrier scheme, while we herein assume a single-carrier network, ii) The design approach of [4] is based on maximizing the sum-rate over all sub-carriers and at the two transceivers under a total transmission power budget, whereas our design approach in this paper relies on minimizing the total transmission power subject to two constraints on the minimum data rates at the two end-nodes, iii) In [4], the design parameters at each transceiver include transmit powers over all sub-carriers at the two transceivers, but in this paper, per transceiver, there is only one design parameter, which is the transmission power of that transceiver. Due to these differences, the multi-carrier sum-rate maximization based method of [4] cannot be used to obtain the design parameters which minimizes the total transmission power in the single-carrier scheme we consider in this paper. Indeed, in [4], the input parameter is the maximum total transmission power consumed throughout the network, while the input parameters in our problem in this paper are the minimum data rates at the two transceivers. Note that while deriving our solution, we relied on the results of [4]. More specifically, to solve the relaxed problem in (43), we introduced the OP (44). The latter OP has been solved in [4], where it has been shown that at the optimum, $\mathbf{w} \in \mathcal{A}$ holds true. We proved that the solution to (43) is the same as the solution to (44). Hence, we could obtain the solution to (44) from the solution to (44), *if and only if* we knew the value of $P_T^{\min}$. Obviously, the value of $P_T^{\min}$ is indeed what we are trying to obtain and it is not known to us. Hence, we cannot use the solution to (44) to find the solution to (43). Nevertheless, we benefited from the fact that the optimal $\mathbf{w}$ for the OP (44) belongs to the set $\mathcal{A}$, and hence, the optimal value for $\mathbf{w}$ in the OP (43) also belongs to the set $\mathcal{A}$. This fact allowed us to write the OP (43) as in (47), thereby enabling us to solve the problem of interest. It is also noteworthy that our technique in this paper and the algorithm of [4] result in a relay selection scheme, but the parameters and the criteria used in the process of relay selection in these techniques are different, so are the algorithms used in the two methods to obtain the corresponding optimal values of the relay beamforming weights and the optimal values of transceivers transmit powers.

IV. NUMERICAL RESULTS

We assume an ABAR network where $L = 60$ single-antenna relays are used to facilitate information exchange between two transceivers. Based on a block transmission scheme, the signals are transmitted in blocks of $N_s = 64$ symbols. In each simulation run, the coefficient g_{lq} is modeled as a zero-mean complex Gaussian random variable whose variance is inversely proportional to the path loss given by $(\tau_{l_q}')^{-3}$, that is the path loss exponent is chosen to be equal to 3. The random propagation delay τ_{l_q}' is uniformly drawn from the interval $[T_s,\ 4T_s]$.

Fig. 2 shows the average minimum total transmission power P_{total}, the average of the corresponding relay transmit powers P_r, and the averages of the corresponding transmit transceiver powers P_1 and P_2 versus $r \triangleq r_1 = r_2$ in dB. In this figure, we compare the performance of our proposed method scheme with two other techniques, 1) the equal power allocation algorithm, denoted as the EPA method and 2) the modified equal power allocation, abbreviated as the MEPA

Algorithm 1 : Proposed Power-Optimal Network Beamforming

1: Set $n = 0$.

2: If none of the relays subscribes to $h[n]$ (i.e., if all entries of row $(n+1)$ of $\mathbf{B}$ are zero), then go to Step 12.

3: Let $\mathbf{b}_n$ be an $L_n \times 1$ vector whose l^{th} element is the same as the l^{th} non-zero element of column $(n+1)$ of $\mathbf{B}^H$, where L_n counts the number of the non-zero elements in column $(n+1)$ of $\mathbf{B}^H$. Define $\mathbf{Q}_q^{(n)} \triangleq \text{diag}\{|g_{1q}^{(n)}|^2, |g_{2q}^{(n)}|^2, \ldots, |g_{L_n,q}^{(n)}|^2\}$, where $g_{iq}^{(n)}$ is the channel coefficient between TRx q and the i^{th} relay which subscribes to $h[n]$, for $q = 1, 2$. Let $\mathbf{g}_1^{(n)} = [g_{1q}^{(n)}, g_{2q}^{(n)}, \ldots g_{L_n,q}^{(n)}]^T$ be the channel vector between TRx 1 and the relays associated with $h[n]$. Let N_s be the number of symbols in each block and define $\beta_1' \triangleq N_s\left(2^{\frac{2r_1}{N_s}} - 1\right)$ and $\beta_2' \triangleq N_s\left(2^{\frac{2r_2}{N_s}} - 1\right)$. Also, define $\hbar_n(z)$, for $z \in \left(\frac{\beta_1'+\beta_2'}{\|\mathbf{g}_1^{(n)}\|^2}, +\infty\right)$ as

$$\hbar_n(z) \triangleq 1 - (\beta_1' + \beta_2')\frac{1/z^2 - \lambda_n \mathbf{b}_n^H((\beta_1' + \beta_2')\mathbf{Q}_2^{(n)} - \lambda_n(z\mathbf{Q}_1^{(n)} + \mathbf{I}_{L_n}))^{-2}\mathbf{Q}_1^{(n)}\mathbf{b}_n}{\lambda_n^2 \mathbf{b}_n^H((\beta_1' + \beta_2')\mathbf{Q}_2^{(n)} + \lambda_n(z\mathbf{Q}_1^{(n)} + \mathbf{I}_{L_n}))^{-2}(z\mathbf{Q}_1^{(n)} + \mathbf{I}_{L_n})\mathbf{b}_n}, \quad \text{for } n = 0, 1, \ldots, N-1$$

where for any given value of $z \in \left(\frac{\beta_1'+\beta_2'}{\|\mathbf{g}_1^{(n)}\|^2}, +\infty\right)$, λ_n is the only real and positive solution to the following non-linear equation:

$$\sum_{i=1}^{L_n} \frac{z(z|g_{i1}^{(n)}|^2 + 1)^{-1}|g_{i1}^{(n)}|^2|g_{i2}^{(n)}|^2}{(\beta_1' + \beta_2')|g_{i2}^{(n)}|^2(z|g_{i1}^{(n)}|^2 + 1)^{-1} + \lambda_n} = 1, \quad \text{for } n = 0, 1, \ldots, N-1.$$

4: Let $p_l \triangleq \frac{\beta_1' + \beta_2'}{\|\mathbf{g}_1^{(n)}\|^2}$, choose p_u to be a sufficiently large number, and choose the real scalar ϵ arbitrarily small.

5: Let $k = 1$ and set $z_n^{(k)} = (p_l + p_u)/2$.

6: If for $\hbar_n(z_n^{(k)}) > 0$, then set $p_u = z_n^{(k)}$. If $\hbar_n(z_n^{(k)}) < 0$, then set $p_l = z_n^{(k)}$.

7: Calculate $z_n^{(k+1)} = (p_l + p_u)/2$.

8: If $|z_n^{(k+1)} - z_n^{(k)}| < \epsilon$, go to Step 9, otherwise let $k = k + 1$ and go to Step 6.

9: Set $z_n = z_n^{(k+1)}$ and calculate κ_n using

$$\kappa_n = \mathbf{b}_n^H(z_n\mathbf{Q}_1^{(n)} + \mathbf{I}_{L_n}))\lambda_n(z_n\mathbf{Q}_1^{(n)} + \mathbf{I}_{L_n}))^{-2}\mathbf{b}_n.$$

10: Calculate the optimal value of $\mathbf{w}_n$, denoted as $\mathbf{w}_n^{\text{o}}$, from

$$\mathbf{w}_n^{\text{o}} = \sqrt{\frac{(\beta_1' + \beta_2')}{\kappa_n \lambda_n}}((\beta_1' + \beta_2')\mathbf{Q}_2^{(n)} + \lambda_n(z_n\mathbf{Q}_1^{(n)} + \mathbf{I}_{L_n}))^{-1}\mathbf{b}_n.$$

11: Calculate the cost function $f_n(\mathbf{w}_n^{\text{o}})$ as

$$f_n(\mathbf{w}_n^{\text{o}}) = \sigma^2\left(\frac{(\beta_1' + \beta_2')\left(\mathbf{w}_n^{\text{o},H}\mathbf{Q}_1^{(n)}\mathbf{w}_n^{\text{o}} + 1\right)\left(\mathbf{w}_n^{\text{o},H}\mathbf{Q}_2^{(n)}\mathbf{w}_n^{o} + 1\right)}{\mathbf{w}_n^{\text{o},H}\mathbf{b}_n\mathbf{b}_n^H\mathbf{w}_n^{\text{o}}} + \mathbf{w}_n^{\text{o},H}\mathbf{w}_n^{\text{o}}\right).$$

12: Let $n = n + 1$, if $n < N$ go to Step 2, otherwise go to the next step.

13: Find the optimal n, denoted as n^{o}, which results in the smallest value for $f_n(\mathbf{w}_n^{\text{o}})$, i.e., $n^{\text{o}} = \arg \min_{0 \le n \le N-1, \mathcal{B}_n \neq \emptyset} f_n(\mathbf{w}_n^{\text{o}})$.

14: Let $\mathbf{w}_{\text{opt}}$ stand for the optimal $\mathbf{w}$. If Relay l subscribes to tap n^{o} of the E2ECIR, then the l^{th} element of $\mathbf{w}_{\text{opt}}$ is the same as that entry of $\mathbf{w}_{n^{\text{o}}}^{\text{o}}$ which corresponds to Relay l, and if Relay l does not subscribe to tap n^{o} of the E2ECIR, then the l^{th} entry of $\mathbf{w}_{\text{opt}}$ is zero.

15: Calculate the transceiver transmit powers as

$$P_1^{\text{o}} = \frac{\beta_2'\left(1 + \mathbf{w}_{n^{\text{o}}}^{\text{o},H}\mathbf{Q}_2^{(n^{\text{o}})}\mathbf{w}_{n^{\text{o}}}^{\text{o}}\right)}{\mathbf{w}_{n^{\text{o}}}^{\text{o},H}\mathbf{b}_{n^{\text{o}}}\mathbf{b}_{n^{\text{o}}}^H\mathbf{w}_{n^{\text{o}}}^{\text{o}}}, \quad P_2^{\text{o}} = \frac{\beta_1'\left(1 + \mathbf{w}_{n^{\text{o}}}^{\text{o},H}\mathbf{Q}_1^{(n^{\text{o}})}\mathbf{w}_{n^{\text{o}}}^{\text{o}}\right)}{\mathbf{w}_{n^{\text{o}}}^{\text{o},H}\mathbf{b}_{n^{\text{o}}}\mathbf{b}_{n^{\text{o}}}^H\mathbf{w}_{n^{\text{o}}}^{\text{o}}}. \tag{66}$$

algorithm. In the EPA technique, the total transmission power consumption throughout the network is assumed to be evenly distributed among all nodes, and thus, in the EPA technique each node in the system receives $1/(L+2)$ of the total transmission power. In the MEPA technique, half of the total transmission power is assumed to be shared between the two

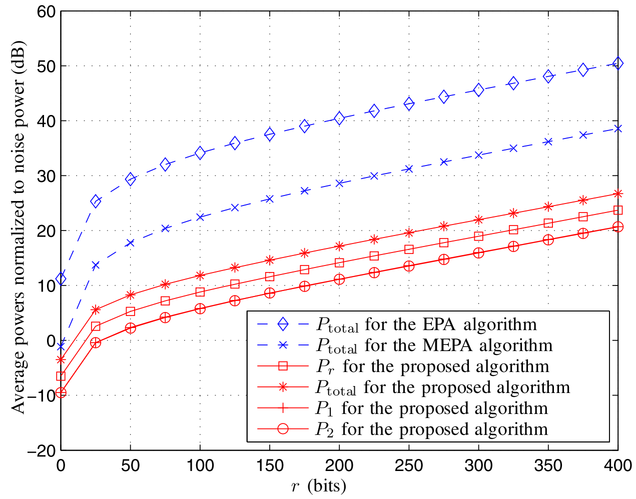

FIGURE 2. The average P_{total}, the corresponding average P_r, the corresponding average P_1 and P_2, for the proposed method, the average total transmission power for the EPA technique and the average total transmission power for the MEPA method versus $r_1 = r_2 = r$.

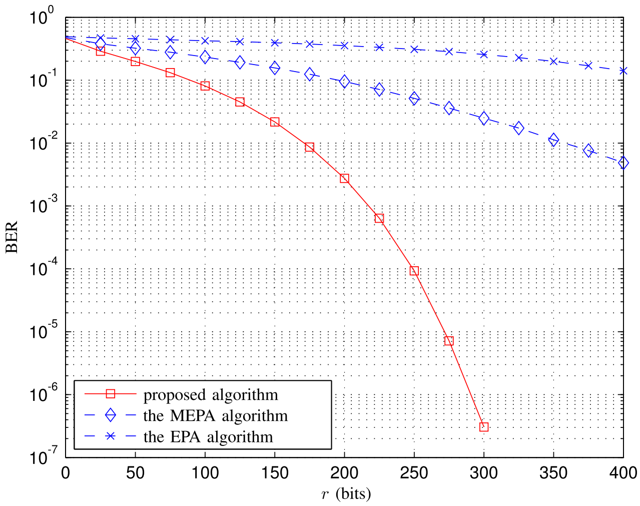

FIGURE 3. Bit error rate versus $r_1 = r_2 = r$ for different methods.

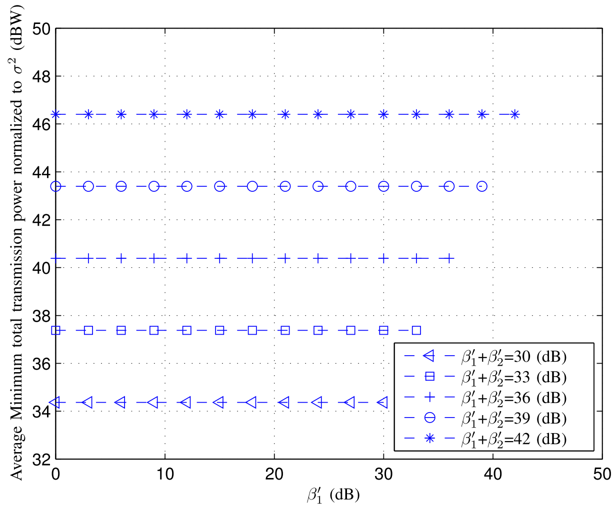

FIGURE 4. The minimum total transmission power versus β_1' for different values of β_2'.

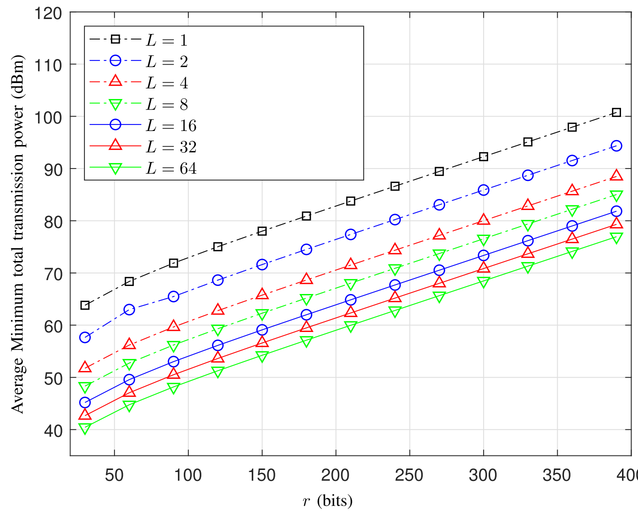

FIGURE 5. The average minimum total transmission power versus r for different number of relays.

end-nodes and the remaining half of the total transmission power is assumed to be evenly distributed among all relay nodes. Fig. 2 shows that our algorithm outperforms the EPA algorithm with a large margin and also consumes less transmission power than the MEPA method does. Indeed, as shown in this figure, our proposed solution outperforms the MEPA method and the EPA algorithm by more that 10 dB and 20 dB, respectively, when r is larger than 50 bits. Moreover, it is shown in Fig. 2 that for our algorithm, the average relay transmit power is always 3 dB below the average minimum total transmission power. Indeed, in our power allocation scheme, *for any channel realization*, half of the power budget is allocated to the two transceivers and the remaining half is shared among all the relay nodes, when the rate thresholds are identical. It can be seen from this figure that the *average* of transceiver powers P_1 and P_2 are identical. Note that P_1 and P_2 may not be identical for a given channel realization.

In Fig. 3, assuming QPSK modulation, we illustrate the end-to-end average bit error rates (BERs) performance for our proposed algorithm which is plotted versus r and is compared with those of the EPA and MEPA schemes. As can be seen from this figure, our total transmission power minimization approach outperforms the EPA and MEPA methods.

Fig. 4 shows the minimum total transmission power P_{total} achieved by our power minimization method for one random channel realization, versus $\beta_1' = N_s(2^{\frac{2r_1}{N_s}} - 1)$ and for different values of $\beta_1' + \beta_2'$. It can be seen from this figure that as long as the sum of β_1' and β_2' remains constant, changing β_1' does not change the minimum total transmission power of the network. This phenomenon is the direct result of the fact that the solution to the problem of total transmission power minimization depends on $\beta_1' + \beta_2'$.

In our next simulation example, we assume that the two transceivers are located at (5 km, 0 km) and (−5 km, 0 km), and that the relays locations are randomly distributed in a square area, with dimensions 5 km × 5 km, centered around the point (0 km, 0 km). We assume a path-loss exponent of 3.8, while the standard deviation of shadowing is 8 dB. We model the small-scale fading channel coefficients as zero-mean complex Gaussian random variables with unit variance. The relays' and transceivers' noises are modeled as zero-mean spatio-temporally white Gaussian random processes with variance $\sigma^2 = -130$ dBm. Fig. 5 illustrates the average minimum total transmission power versus r for networks with different numbers of relays. As Fig. 5 shows, when number of relays is small, doubling this number results in about 6 dB saving in total transmit power, while for large number of relays, this saving is about 2-3 dB.

V. CONCLUSIONS

Considered is a single-carrier *asynchronous* bidirectional cooperative network, where multiple amplify-and-forward relays rely on the so-called multiple access broadcast channel protocol to establish a two-way information exchange between two transceivers. The network considered herein is assumed to be asynchronous in the sense that the signals traveling between the two end-nodes through different relaying paths undergo different propagation delays. As a result, such a two-way relay channel can be seen as a multi-path link which can cause inter-symbol interference (ISI) at the two end-nodes. In a block-wise communication scheme, ISI results in both inter- and intra-block interference. While inter-block interference can be eliminated by cyclic prefix insertion and deletion, tackling intra-block interference requires the *optimal design* of the underlying bidirectional relay channel. Considering the transceivers' transmission powers and the relay complex beamforming weights as the design parameters, we optiamlly designed this bidirectional relay channel via minimizing the total transmission power consumption throughout the communication scheme, under two constraints on the minimum data rates at the two transceivers. Our rigorous proof showed that, at the optimum, only a subset of the relays will have to participate in the information exchange between the two transceivers and the rest of the relays have to be turned off. More specifically, our power-optimal rate-constrained design of the asynchrnous bidirectional relay channel requires the impulse response of the end-to-end channel be single-tap. Therefore, only the relays associated with that single tap have to be turned on and the rest of the relays will have to be turned off. We devised a simple 1-dimensional integer search algorithm to optimally find the index of the non-zero tap of the impulse response of the end-to-end channel. We also developed a computationally efficient semi-closed-form expression for the optimal values of the transceivers' transmission powers and those of the relays' complex weights. Our numerical results showed that our proposed scheme outperforms an equal power allocation scheme, where all nodes in the network consume the same amount of power, while satisfying the same constraints on the transceivers' data rates as our approach does.

VI. Acknowledgment

This paper was presented in part as a conference paper in [1].

APPENDIX
PROOF OF LEMMA 1

To prove this lemma, we note that the cost function in (50) can be replaced with $1 + \frac{1}{M}\sum_{i=1}^{M} x_i = \frac{1}{M}\sum_{i=1}^{M}(1 + x_i)$ and the constraint can be replaced with $\left(\prod_{i=1}^{M}(1 + x_i)\right)^{1/M} = c$. Hence, solving (50) is equivalent to finding M non-negative numbers $\{1 + x_i\}_{i=1}^{M}$ which have the smallest possible arithmetic mean while their geometric mean is constant. We know that the arithmetic mean of such numbers is always larger than or equal to their geometric mean with the equality occurring when all these numbers are equal. Hence, the cost function in (50) is minimized when all x_i's are equal. The proof is complete. ∎

REFERENCES

[1] S. Bastanirad, S. Shahbazpanahi, and A. Grami, "Jointly optimal distributed beamforming and power control in asynchronous two-way relay networks," in *Proc. 49th Asilomar Conf. Signals, Syst. Comput.*, Nov. 2015, pp. 963–967.
[2] V. Havary-Nassab, S. Shahbazpanahi, and A. Grami, "Optimal distributed beamforming for two-way relay networks," *IEEE Trans. Signal Process.*, vol. 58, no. 3, pp. 1238–1250, Mar. 2010.
[3] R. Vahidnia and S. Shahbazpanahi, "Multi-carrier asynchronous bi-directional relay networks: Joint subcarrier power allocation and network beamforming," *IEEE Trans. Wireless Commun.*, vol. 12, no. 8, pp. 3796–3812, Aug. 2013.
[4] R. AliHemmati and S. Shahbazpanahi, "Sum-rate optimal network beamforming and subcarrier power allocation for multi-carrier asynchronous two-way relay networks," *IEEE Trans. Signal Process.*, vol. 63, no. 15, pp. 4129–4143, Aug. 2015.
[5] M. Askari and S. Shahbazpanahi, "On sum-rate maximization approach to network beamforming and power allocation for asynchronous single-carrier two-way relay networks," *IEEE Access*, vol. 5, pp. 13699–13711, 2017.
[6] J. Mirzaei and S. Shahbazpanahi, "On achievable SNR region for multi-user multi-carrier asynchronous bidirectional relay networks," *IEEE Trans. Wireless Commun.*, vol. 14, no. 6, pp. 3219–3230, Jun. 2015.
[7] R. Vahidnia and S. Shahbazpanahi, "Single-carrier equalization for asynchronous two-way relay networks," *IEEE Trans. Signal Process.*, vol. 62, no. 22, pp. 5793–5808, Nov. 2014.
[8] R. Vahidnia, S. Shahbazpanahi, and A. Minasian, "Pre-channel equalization and distributed beamforming in asynchronous single-carrier bi-directional relay networks," *IEEE Trans. Signal Process.*, vol. 64, no. 15, pp. 3968–3983, Aug. 2016.
[9] F. E. Dorcheh and S. Shahbazpanahi, "Jointly optimal pre-and post-channel equalization and distributed beamforming in asynchronous bidirectional relay networks," *IEEE Trans. Signal Process.*, vol. 65, no. 17, pp. 4593–4608, Sep. 2017.
[10] L. P. Qian, Y. Wu, and Q. Chen, "Transmit power minimization for outage-constrained relay selection over rayleigh-fading channels," *IEEE Commun. Lett.*, vol. 18, no. 8, pp. 1383–1386, Aug. 2014.
[11] T. Wang, Y. Fang, and L. Vandendorpe, "Power minimization for OFDM transmission with subcarrier-pair based opportunistic DF relaying," *IEEE Commun. Lett.*, vol. 17, no. 3, pp. 471–474, Mar. 2013.
[12] S. Shahbazpanahi and M. Dong, "A semi-closed-form solution to optimal distributed beamforming for two-way relay networks," *IEEE Trans. Signal Process.*, vol. 60, no. 3, pp. 1511–1516, Mar. 2012.
[13] H. Chen, A. B. Gershman, and S. Shahbazpanahi, "Filter-and-forward distributed beamforming in relay networks with frequency selective fading," *IEEE Trans. Signal Process.*, vol. 58, no. 3, pp. 1251–1262, Mar. 2010.
[14] S. Talwar, Y. Jing, and S. Shahbazpanahi, "Joint relay selection and power allocation for two-way relay networks," *IEEE Signal Process. Lett.*, vol. 18, no. 2, pp. 91–94, Feb. 2011.

[15] H.-M. Wang, X.-G. Xia, and Q. Yin, "A linear analog network coding for asynchronous two-way relay networks," *IEEE Trans. Wireless Commun.*, vol. 9, no. 12, pp. 3630–3637, Dec. 2010.
[16] H. Wang and X.-G. Xia, "Asynchronous cooperative communication systems: A survey on signal designs," *Sci. China Inf. Sci.*, vol. 54, no. 8, pp. 1547–1561, Aug. 2011.
[17] Z. Zhong, S. Zhu, and G. Lv, "Distributed space-time code for asynchronous two-way wireless relay networks under frequency-selective channels," in *Proc. IEEE Int. Conf. Commun.*, Jun. 2009, pp. 1–5.
[18] W. Zhang, H. Wang, Q. Yin, and W. Wang, "Space frequency code for cooperative communications with both timing errors and carrier frequency offsets," *IEICE Trans. Commun.*, vols. 93–B, pp. 3505–3508, Dec. 2010.
[19] Z. Zhong, S. Zhu, and G. Lv, "Distributed space-time code for asynchronous two-way wireless relay networks under frequency-selective channels," in *Proc. IEEE Int. Conf. Commun.*, Jun. 2009, pp. 1–5.
[20] Z. Fang, F. Liang, Z. Wang, and D. Chen, "Low complexity timing estimation and resynchronization for asynchronous bidirectional communications with multiple antenna relay," *Int. J. Commun. Syst.*, vol. 28, no. 6, pp. 1140–1150, Apr. 2015.
[21] Z. Wang and G. B. Giannakis, "Wireless multicarrier communications," *IEEE Signal Process. Mag.*, vol. 17, no. 3, pp. 29–48, May 2000.

SAHAR BASTANIRAD received the B.Sc. degree in electrical engineering from Shahid Behesthi University, Tehran, Iran, in 2012, and the M.Sc. degree in electrical and computer engineering from the University of Ontario Institute of Technology, Oshawa, ON, Canada, in 2015. Since 2016, she has been an ASIC/Layout Design Engineer with AMD Inc.

SHAHRAM SHAHBAZPANAHI (M'02–SM'10) was born in Sanandaj, Kurdistan, Iran. He received the B.Sc., M.Sc., and Ph.D. degrees from the Sharif University of Technology, Tehran, Iran, in 1992, 1994, and 2001, respectively, all in electrical engineering. From 1994 to 1996, he was a Faculty Member with the Department of Electrical Engineering, Razi University, Kermanshah, Iran. From 2001 to 2003, he was a Postdoctoral Fellow with the Department of Electrical and Computer Engineering, McMaster University, Hamilton, ON, Canada. From 2003 to 2004, he was a Visiting Researcher with the Department of Communication Systems, University of Duisburg-Essen, Duisburg, Germany. From 2004 to 2005, he was a Lecturer and an Adjunct Professor with the Department of Electrical and Computer Engineering, McMaster University. In 2005, he joined the Faculty of Engineering and Applied Science, University of Ontario Institute of Technology, Oshawa, ON, Canada, where he is currently a Full Professor. His research interests include statistical and array signal processing; space-time adaptive processing; detection and estimation; multi-antenna, multi-user, and cooperative communications; spread spectrum techniques; DSP programming; and hardware/real-time software design for telecommunication systems. He also served as an elected member of the Sensor Array and Multichannel (SAM) Technical Committee of the IEEE Signal Processing Society. He was a recipient of several awards, including the Early Researcher Award from Ontario's Ministry of Research and Innovation, the NSERC Discovery Grant (three awards), the Research Excellence Award from the Faculty of Engineering and Applied Science, University of Ontario Institute of Technology, and the Research Excellence Award, Early Stage, from the University of Ontario Institute of Technology. He has served as an Associate Editor for the IEEE TRANSACTIONS ON SIGNAL PROCESSING and the IEEE SIGNAL PROCESSING LETTERS. He served as a Senior Area Editor for the IEEE SIGNAL PROCESSING LETTERS.

RAZGAR RAHIMI was born in Saqqez, Iran. He received the B.Sc. degree in electrical engineering from the Iran University of Science and Technology, in 2005, the M.Sc. degree in electrical engineering from Shahed University, Tehran, Iran, in 2010, and the Ph.D. degree from the University Of Ontario Institute Of Technology, Oshawa, ON, Canada, in 2018. He was a Telecommunication Engineer with the Iran Control and Communications Systems Supply Company, from 2005 to 2014. He is currently a Lecturer with the Faculty of Engineering and Applied Science, University of Ontario Institute of Technology. His current research interests include signal processing, cognitive radio networks, and cooperative wireless communications.

ALI GRAMI (M'86–SM'06) received the B.Sc. degree from the University of Manitoba, the M.Eng. degree from McGill University, and the Ph.D. degree from the University of Toronto, all in electrical engineering. Before joining academia, he was with the high-tech industry for many years, where he was the Lead Researcher and a Principal Designer of the first North-American broadband access satellite system. He has taught at the University of Ottawa and Concordia University, and has been an Adjunct Faculty Member with Ryerson University and York University. He is currently a Founding Faculty Member with the University of Ontario Institute of Technology (UOIT), Canada. At UOIT, he has also led the development of programs toward bachelor's, master's, and doctoral degrees in electrical engineering. He has authored the undergraduate level textbooks *Introduction to Digital Communications* (Elsevier, 2016) and *Probability, Random Variables, Statistics, and Random Processes* (Wiley, 2019).