

Received January 15, 2019, accepted February 23, 2019, date of publication February 28, 2019, date of current version March 20, 2019. *Digital Object Identifier 10.1109/ACCESS.2019.2902271*

Decentralized H_{∞} Filtering for Large-Scaled System Based on T-S Fuzzy Model With the Integrated Event-Triggered Strategy

LIAN[G](https://orcid.org/0000-0001-9841-0961)LIANG ZHANG®, HONGBIN ZHANG®, (Member, IEEE), AND ZHIPENG ZHANG
School of Information and Communication Engineering, University of Electronic Science and Technology of China, Chengdu, China

Corresponding author: Liangliang Zhang (liangliangzhangee@hotmail.com)

ABSTRACT Considering the characteristics of data transmission between the channels, this paper studies the decentralized H_{∞} filter for the large-scaled system with an integrated event-triggered strategy. The Takagi–Sugeno fuzzy model is used to approximate the large-scaled system. Different from the event-triggered control with the single condition, an event-triggered control with an integrated condition is presented to determine when the measurement output should be broadcast and transmitted to its filter and thus leading to a high efficiency of data transmissions. By applying a Lyapunov–Krasovskii functional and linear matrix inequalities, a method of the integrated event-triggered H_{∞} filter performance analysis and designing is developed so that the overall filtering error system concerned to be asymptotically stable with a given disturbance attenuation level. A simulation is finally given to show these effective methods in this paper.

INDEX TERMS Integrated event-triggered strategy, *H*∞ filter, large-scaled system, Takagi-Sugeno fuzzy system.

I. INTRODUCTION

Many of the physical systems are large-scale systems combined by several subsystems, such as the power networks, the urban traffic networks [1] and the digital communications networks [2]. Each subsystem interacts and interrelates through some certain ways. The control method of such class of large-scale systems has attracted many researchers due to its high reliability and low cost. However, owing to the limited bandwidth in the large-scaled system, the conventional control strategy may have some negative effects on the control performance. For example, it wastes limited bandwidth and computing power, even shortens the lifetime of the whole system. Therefore, how to improve transmission efficiency while keeping expected performance has attracted more and more research interests [3].

In conventional control systems, data sampling and transmission are carried out at instant times which are triggered by clocks or timers [4]. For example, the measurements on each sensor node are sampled periodically and sent to the controller or filter at a fixed sampling period or interval.

The associate editor coordinating the review of this manuscript and approving it for publication was Shun-Feng Su.

This type of control strategy is referred to as time-triggered control strategy and is motivated by the sampled data systems theory. Although the time-triggered control strategy is simple to implement and easy to analyse and design, it may be not a good choice for resource-constrained systems as the triggering of samples is independent with the state evolution of the system. The event-triggered solves these problems well and has been favorable for addressing analysis and synthesis of the control and filter problem in the literature [5], [6]. In the event-triggered control strategy, data transmissions are performed only after specified events occur. In this sense, channel resources will only be occupied when ''needed'', and the number of data transmitted can be reduced. Using the above strategies the number of data can be reduced significantly compared with the conventional time-triggered strategy while ensuring the desired target.

Generally, the traditional event-triggered mechanism can be divided into two kinds of schemes: a predesignated state-independent specified event and a predesignated state-dependent specified event. When the system state fluctuates within the allowable range, using a state-independent threshold can reduce the communication burden. However, in addition, since the threshold is a state-independent scalar,

2169-3536 2019 IEEE. Translations and content mining are permitted for academic research only. Personal use is also permitted, but republication/redistribution requires IEEE permission. See http://www.ieee.org/publications_standards/publications/rights/index.html for more information.

it is often difficult to apply the above scheme directly to the stability analysis of the system. Using a predesignated state-dependent specified event-triggered schemes, the number of trigger events can be reduced. However, when large-scaled systems achieve consensus goals, there are still some unnecessary transmission of sampled data. Therefore, in order to reduce unnecessary over-occupation of channel bandwidth by data and reduce the burden of continuous monitoring of hardware devices in pre-specified state-independent specific events, an integrated event triggering control strategy is proposed, in which two integration conditions only depend on discrete measurement output. All the above observations stimulated this study.

It is well-known that the large-scaled system is a strong coupling, non-linear system [7]. In the large-scaled system, subsystems connect to each other and transmit the information for each other. These subsystems have their own control inputs, measurement outputs as well as states. The term 'fuzzy' means it can deal with the concepts which can't be accurate expressed as 'true' or 'false' but rather as 'partially true'. So, the fuzzy logic is better to describe nonlinear systems [8]. In the fuzzy model, T-S fuzzy model as an universal approximator for nonlinear systems has many studies [9]. Zhang *et al.* [10] investigated the non-fragile H_{∞} filter for T-S fuzzy power systems. Reference [11] was concerned with the fault estimation (FE) problem for a class of interconnected nonlinear systems described by Takagi-Sugeno fuzzy models. Wang *et al.* [12] combined adaptive control and the approximation property of large-scaled systems with T-S fuzzy model. Wang *et al.* [13] concerned with the problem of optimization for discrete-time T-S fuzzy systems under an event-triggered strategy. How to optimize the event-triggered strategy motivate this paper.

 H_{∞} filter, which attracted much attention in the past decades [14]. Reference [15] is a good tool for estimating the system states. As we all know that Kalman filter method can only be used to the systems with the external noise which requirements stochastic processes with known statistical properties. In practice, the ideal conditions may not always hold, which limit Kalman filter applied to the practical system [16]. However, *H*∞ filter does not make any assumption about noise characteristics, only requires that the disturbance is energy limited and the energy gain from the disturbance to the estimation error is bounded by a certain level. The theoretical results on H_{∞} filter could be found in the following references. Zhang *et al.* [17] designed H_{∞} filter for the switching system. The problem of event-triggered *H*_∞ filter for networked Markovian jump system was studied in [18]. Reference [19] concerned with the non-fragile distributed H_{∞} filter problem.

Motivated by the above reasons, this paper concerns the large-scaled system. The main contributions of this paper are summarized as follows:

1) We first propose an integrated event-triggered control strategy with nonlinear dynamics for the large-scaled system. Whether the current data of each subsystem

should be transmitted to its filter is simultaneously determined by two threshold condition. Note that the integrated event-triggered strategy is more relaxed since stability requirements are less stringent in an attempt to reduce the transmitted number of sampled data.

2) The T-S fuzzy model has been proposed to cope with the integrated event-triggered H_{∞} filter. Based on the Lyapunov-Krasovskii functional, a new sufficient condition condition is obtained so that the filtering error system is asymptotically stable and achieves a prescribed filtering performance. A sufficient condition for the integrated event-triggered H_{∞} filter is established by using linear matrix inequalities (LMIs).

II. PROBLEM FORMULATION

In this section, we consider a nonlinear large-scaled system based on T-S fuzzy model [20] which is presented as follows: **Rule** *r*: IF x_1 (*k*) is F_{1i} and . . . and x_g (*k*) is F_{gi} THEN

$$
x_i (k + 1) = A_i x_i (k) + W_i w_i (k) + \sum_{m=1, m \neq i}^{J} C_{mi} x_m (k)
$$

$$
y_i (k) = D_i x_i (k)
$$

$$
z_i (k) = H_i x_i (k),
$$
 (1)

where $x \in R^{n_x}$ is the state vector, $z \in R^{n_z}$ is the signal to be estimated, $y \in R^{n_y}$ is the measurement output vector, $w \in R^{n_w}$ is the process disturbance. A_i , B_i , C_i , D_i and H_i are constant known matrices, $i = 1, 2, ..., r$, where the scalar *r* is the number of IF-THEN rules. The premise variables and the fuzzy sets are denoted as x_g (*k*) and F_{gi} , respectively, where $j = 1, 2, \ldots, g$. The model of dynamic system is inferred as follows:

$$
S_{i}\begin{cases}x_{i}(k+1) = \sum_{i=1}^{r} h_{i}(x(k)) [A_{i}x(k) + W_{i}w_{i}(k) \\ + \sum_{m=1,m \neq i}^{J} C_{mi}x_{m}(k)] \\y_{i}(k) = \sum_{\substack{i=1 \ i=1}}^{r} h_{i}(x(k)) D_{i}x_{i}(k) \\z_{i}(k) = \sum_{i=1}^{r} h_{i}(x(k)) H_{i}x_{i}(k),\end{cases}
$$
(2)

where $h_i(x(k)) = \mu_i(x(k)) \Big/ \sum_{i=1}^r \mu_i(x(k))$ and $\sum_{i=1}^r h_i(x(k))$ $(x (k)) = 1.$

Consequently, we present the decentralized fuzzy filter *Sfi* with undetermined system matrices.

$$
S_{fi}: \begin{cases} x_{fi} (k+1) = \sum_{j=1}^{r} h_{ij} (x (k)) [A_{fji}x_{fi} (k) \\ + B_{fij}y (t_k)] \\ z_{fi} (k) = \sum_{j=1}^{r} h_{ij} (x (k)) C_{fij}x_{fi} (k), \end{cases}
$$
(3)

where $x_f \in R^{n_{xf}}$ is the state vector of the filter, $z_f \in R^{n_{xf}}$ is the estimation of $z(k)$, $y(t_k)$ is the input data received by the filter. A_{fi} , B_{fi} , C_{fi} are the system matrices to be determined.

Considering the following the integrated event-triggered strategy:

tk+¹

$$
= \inf\{k \mid f_1 = e^T(k) \, Qe(k) - \sigma^2 y^T(k) \, Qy(k) \ge 0 \mid f_2 = y^T(k) y(k) - \rho^2 \ge 0 \quad (4)
$$

where $Q = Q^T > 0$ and $e(k) = y(k) - y(t_k)$.

Now, we can rewritten the above event-triggered H_{∞} filter *Sfi*:

$$
S_{\hat{H}}: \begin{cases} x_{\hat{H}}(k+1) = \sum_{j=1}^{r} h_{ij}(x(k)) \left[A_{\hat{H}i} x_{\hat{H}}(k) \right. \\ \left. + B_{\hat{H}j} D_{i} x_{i}(k) - B_{\hat{H}j} e_{i}(k) \right] \\ z_{\hat{H}}(k) = \sum_{j=1}^{r} h_{ij}(x(k)) \, C_{\hat{H}j} x_{\hat{H}}(k), \end{cases} \tag{5}
$$

By defining a new state vector $\xi_i(k) = \left[x_i^T(k) \; x_{fi}^T(k)\right]^T$ and $\tilde{z}(k) = z(k) - z_f(k)$, we can obtain the filtering error system:

$$
\Omega_{i}: \begin{cases} \xi_{i} (k+1) = \sum_{i=1}^{r} [\widetilde{A}_{i} \xi(k) + \widetilde{W}_{i} w(k) + \widetilde{B}_{f i} e(k)] \\ + \sum_{m=1, m \neq i}^{J} \widetilde{C}_{mi} \xi_{m} (k)] \\ \widetilde{z}_{i} (k) = \sum_{i=1}^{r} \widetilde{H}_{i} \xi_{i} (k), \end{cases}
$$
(6)

where

$$
\widetilde{A}_{i}(h) \begin{bmatrix} A_{i}(h) & 0 \\ B_{f\bar{i}}(h) D_{i}(h) & A_{f\bar{i}}(h) \end{bmatrix}
$$
\n
$$
= \sum_{j=1}^{r} \sum_{l=1}^{r} h_{ij}(x(k)) h_{il}(x(k)) \begin{bmatrix} A_{ij} & 0 \\ B_{f\bar{i}l}D_{ij} & A_{f\bar{i}l} \end{bmatrix},
$$
\n
$$
\widetilde{W}_{i}(h) = \begin{bmatrix} w_{i}(h) \\ 0 \end{bmatrix}
$$
\n
$$
= \sum_{j=1}^{r} \sum_{l=1}^{r} h_{ij}(x(k)) h_{il}(x(k)) \begin{bmatrix} w_{ij} \\ 0 \end{bmatrix},
$$
\n
$$
\widetilde{B}_{f\bar{i}}(h) = \begin{bmatrix} 0 \\ -B_{f\bar{i}}(h) \end{bmatrix}
$$
\n
$$
= \sum_{j=1}^{r} \sum_{l=1}^{r} h_{ij}(x(k)) h_{il}(x(k)) \begin{bmatrix} 0 \\ -B_{f\bar{i}l} \end{bmatrix},
$$
\n
$$
\widetilde{H}_{i}(h) = \begin{bmatrix} H_{i}(h) & -C_{f\bar{i}}(h) \end{bmatrix}
$$
\n
$$
= \sum_{j=1}^{r} \sum_{l=1}^{r} h_{ij}(x(k)) h_{il}(x(k)) \begin{bmatrix} H_{ij} & -C_{f\bar{i}l} \end{bmatrix}.
$$

Considering Table 1, it illustrates the integrated relationship in the strategy (4), which determines whether the sampled data is released over the limited communication channel.

For the large-scaled system, there are the following three cases, where the sampled data is not necessary to be released:

$$
\begin{cases}\nCase \ I: f_1 \leq 0, & f_2 \leq 0 \\
Case \ II: f_1 \leq 0, & f_2 \geq 0 \\
Case \ III: f_1 \geq 0, & f_2 \leq 0\n\end{cases} (7)
$$

Note that in the integrated transmission strategy (4), the non-transmitted conditions mentioned in (7) should be fully considered in analysis and design of the system under consideration.

A useful lemma is introduced as following

Lemma 1: Given three matrices $A \in R^{m \times n}$, $B \in R^{m \times n}$ and $M \in R^{n \times n}$, and two positive-definite matrices $A \in R^{m \times m}$ and $Q \in R^{n \times n}$, such that

 $A^T P A - Q + M < 0, \quad B^T P B - Q + M < 0$

then

$$
A^T P A + B^T P B - 2Q + 2M < 0.
$$

III. MAIN RESULTS

A. STABILITY AND INTEGRATED EVENT-TRIGGERED H_{∞} PERFORMANCE ANALYSIS

In this section, we will present a sufficient condition for the integrated event-triggered H_{∞} filter in the form of (5).

Theorem 1 (Stability With H_{∞} *Performance):* For a prescribed real number $\gamma > 0$, the filtering error system with (6) is asymptotically stable and satisfies the H_{∞} performance index if there exist matrices $P_i > 0$ (8), as shown at the top of the next page, where

$$
\Pi_1 = [\hat{C}_i, \hat{C}_{\varepsilon i1}, \hat{C}_{\varepsilon i2}, \hat{C}_{\varepsilon i3}],
$$
\n
$$
\Pi_1 = \frac{1}{2} diag\{\hat{P}_i, \hat{P}_{\varepsilon i1}, \hat{P}_{\varepsilon i2}, \hat{P}_{\varepsilon i3}\},
$$
\n
$$
\hat{C}_i = [\tilde{C}_{i1}P_1, \dots, \tilde{C}_{im,m\neq i}P_m, \dots, \tilde{C}_{iJ}P_J],
$$
\n
$$
\hat{C}_{\varepsilon i1} = [\tilde{C}_{i1}P_1, \dots, \tilde{C}_{im,m\neq i}P_m, \dots, \tilde{C}_{iJ}P_J],
$$
\n
$$
\hat{C}_{\varepsilon i2} = [\tilde{C}_{i1}P_1, \dots, \tilde{C}_{im,m\neq i}P_m, \dots, \tilde{C}_{iJ}P_J],
$$
\n
$$
\hat{C}_{\varepsilon i3} = [\tilde{C}_{i1}P_1, \dots, \tilde{C}_{im,m\neq i}P_m, \dots, \tilde{C}_{iJ}P_J],
$$
\n
$$
\hat{P}_i = diag[P_1, \dots, P_{m,m\neq i}, \dots, P_J],
$$
\n
$$
\hat{P}_{\varepsilon i1} = diag[\varepsilon_{11}P_1, \dots, \varepsilon_{m1,m\neq i}P_m, \dots, \varepsilon_{J1}P_J],
$$
\n
$$
\hat{P}_{\varepsilon i2} = diag[\varepsilon_{12}P_1, \dots, \varepsilon_{m2,m\neq i}P_m, \dots, \varepsilon_{J2}P_J],
$$
\n
$$
\hat{P}_{\varepsilon i3} = diag[\varepsilon_{13}P_1, \dots, \varepsilon_{m3,m\neq i}P_m, \dots, \varepsilon_{J3}P_J],
$$

Proof: Our main research is to guarantee the integrated event-triggered filtering error system is asymptotically stable

with H_{∞} performance under zero initial conditions. First, we construct a Lyapunov function for the filtering error systems as:

$$
V(k) = \sum_{i=1}^{J} V_i(k) = \sum_{i=1}^{J} \xi_i^T(k) P_i \xi_i(k).
$$
(9)
\n
$$
\Delta V(k) = \sum_{i=1}^{J} [V_i(k+1) - V_i(k)]
$$

\n
$$
= \sum_{i=1}^{J} [\xi_i^T(k+1) P_i \xi_i(k+1) - \xi_i^T(k) P_i \xi_i(k)]
$$

\n
$$
= \sum_{i=1}^{J} \{ [\tilde{A}_i(h) \xi_i(k) + \tilde{B}_{fi}(h) e_i(k)
$$

\n
$$
+ \sum_{m=1, m \neq i}^{J} \tilde{C}_{mi} \xi_m(k)]^T
$$

\n
$$
\times P_i [\tilde{A}_i(h) \xi_i(k) + \tilde{B}_{fi}(h) e_i(k)
$$

\n
$$
+ \sum_{m=1, m \neq i}^{J} \tilde{C}_{mi} \xi_m(k)] - \xi_i^T(k) P_i \xi_i(k) \}
$$

\n
$$
= \sum_{i=1}^{J} \{ (\tilde{A}_i(h) \xi_i(k))^T P_i (\tilde{A}_i(h) \xi_i(k))
$$

\n
$$
+ 2 (\tilde{A}_i(h) \xi_i(k))^T P_i (\tilde{B}_{fi}(h) e_i(k))
$$

\n
$$
+ 2 (\tilde{A}_i(h) e_i(k))^T P_i \sum_{m=1, m \neq i}^{J} \tilde{C}_{mi} \xi_m(k)
$$

\n
$$
+ (\tilde{B}_{fi}(h) e_i(k))^T P_i \sum_{m=1, m \neq i}^{J} \tilde{C}_{mi} \xi_m(k)
$$

\n
$$
+ (\sum_{m=1, m \neq i}^{J} \tilde{C}_{mi} \xi_m(k))^T P_i (\tilde{B}_{fi}(h) e_i(k))
$$

\n
$$
+ (\sum_{m=1, m \neq i}^{J} \tilde{C}_{mi} \xi_m(k))^T P_i (\tilde{A}_i(h) \xi_i(k))
$$

\n
$$
+ (\tilde{B}_{fi}(h) e_i(k))^T P_i (\tilde{A}_i(h) \xi_i(k))
$$

+2(
$$
\widetilde{A}_i(h) \xi_i(k)
$$
)^{*T*} $P_i(\widetilde{B}_{fi}(h) e_i(k))$
+ $\varepsilon_{i1}^{-1} e_i^T(k) \widetilde{B}_{fi}^T(h) P_i \widetilde{B}_{fi}(h) e_i(k)$
+ $\varepsilon_{i2}^{-1} \xi_i^T(k) \widetilde{A}_{fi}^T(h) P_i \widetilde{A}_{fi}(h) \xi_i(k)$
+ $\sum_{m=1, m \neq i} \xi_i^T(k) (\widetilde{C}_{im}^T P_i \widetilde{C}_{im} + \varepsilon_{m1} \widetilde{C}_{im}^T P_i \widetilde{C}_{im}$
+ $\varepsilon_{m2} \widetilde{C}_{im}^T P_i \widetilde{C}_{im} \xi_i(k) - \xi_i^T(k) P_i \xi_i(k)$. (10)

Using Schur's complement and *Lemma* 1, we know that LMIs in (11) imply *V* (*k*) for all $\xi_i(k) \neq 0$, and thus, it follows that the large-scaled filtering error system is asymptotically stable.

Now, we present the event-triggered H_{∞} disturbance attenuation performance.

Case I: Under the zero initial condition and by adding and subtracting e^T (*k*) Qe (*k*) and y^T (*k*) y (*k*) in the accumulate operation, one arrives at:

$$
e^{T}(k) e(k) - \gamma^{2} w^{T}(k) w(k)
$$

\n
$$
\leq e^{T}(k) e(k) - \gamma^{2} w^{T}(k) w(k) + V(k+1) - V(k)
$$

\n
$$
+ e^{T}(k) Qe(k) - e^{T}(k) Qe(k) + \gamma^{T}(k) y(k)
$$

\n
$$
- \gamma^{T}(k) y(k)
$$

\n
$$
= \sum_{i=1}^{J} \{[\widetilde{H}_{i}(h) \xi_{i}(k)]^{T} [\widetilde{H}_{i}(h) \xi_{i}(k)] - \gamma^{2} w_{i}^{T}(k) w_{i}(k)
$$

\n
$$
+ e_{i}^{T}(k) Qe_{i}(k) - e_{i}^{T}(k) Qe_{i}(k) + \gamma_{i}^{T}(k) y_{i}(k)
$$

\n
$$
- \gamma_{i}^{T}(k) y_{i}(k) + [\widetilde{A}_{i}(h) \xi_{i}(k) + \widetilde{W}_{i}(h) w_{i}(k)
$$

\n
$$
+ \widetilde{B}_{f i}(h) e_{i}(k) + \sum_{m=1, m \neq 1}^{J} \widetilde{C}_{mi} \xi_{m}(k)]^{T}
$$

\n
$$
\times P_{i}[\widetilde{A}_{i}(h) \xi_{i}(k) + \widetilde{W}_{i}(h) w_{i}(k) + \widetilde{B}_{f i}(h) e_{i}(k)
$$

\n
$$
+ \sum_{m=1, m \neq 1}^{J} \widetilde{C}_{mi} \xi_{m}(k)] - \xi_{i}^{T}(k) P_{i} \xi_{i}(k) \}
$$

\n
$$
+ e_{i}^{T}(k) Qe_{i}(k) - e_{i}^{T}(k) Qe_{i}(k) + \gamma_{i}^{T}(k) y_{i}(k)
$$

\n
$$
+ \widetilde{A}_{i}(h) \xi_{i}(k) + \widetilde{W}_{i}(h) w_{i}(k) + \widetilde{B}_{f i}(h) e_{i}(k)
$$

J

+
$$
\sum_{m=1, m\neq 1} \widetilde{C}_{mi} \xi_m(k) \Gamma P_i [\widetilde{A}_i(h) \xi_i(k) + \widetilde{W}_i(h) w_i(k)
$$

+
$$
\widetilde{B}_{ji}(h) e_i(k) + \sum_{m=1, m\neq 1} \widetilde{C}_{mi} \xi_m(k) \Gamma - \xi_i^T(k) P_i \xi_i(k) \}
$$

$$
\leq \sum_{i=1}^J \{ [\widetilde{A}_i(h) \xi_i(k)]^T P_i [\widetilde{A}_i(h) \xi_i(k)]
$$

+
$$
[\widetilde{W}_i(h) w_i(k)]^T P_i [\widetilde{W}_i(h) w_i(k)]
$$

+
$$
2[\widetilde{A}_i(h) \xi_i(k)]^T P_i [\widetilde{W}_i(h) w_i(k)]
$$

+
$$
2[\widetilde{A}_i(h) \xi_i(k)]^T P_i [\widetilde{W}_i(h) w_i(k)]
$$

+
$$
2[\widetilde{W}_i(h) w_i(k)]^T P_i [\widetilde{B}_{ji}(h) e_i(k)]
$$

+
$$
2[\widetilde{W}_i(h) w_i(k)]^T P_i [\widetilde{B}_{ji}(h) e_i(k)]
$$

+
$$
[\widetilde{B}_{ji}(h) e_i(k)]^T P_i [\sum_{m=1, m\neq 1} \widetilde{C}_{mi} \xi_m(k)]
$$

+
$$
[\widetilde{W}_i(h) w_i(k)]^T P_i [\sum_{m=1, m\neq 1} \widetilde{C}_{mi} \xi_m(k)]
$$

+
$$
[\widetilde{W}_i(h) w_i(k)]^T P_i [\sum_{m=1, m\neq 1} \widetilde{C}_{mi} \xi_m(k)]
$$

+
$$
[\sum_{m=1, m\neq 1} \widetilde{C}_{mi} \xi_m(k)]^T P_i [\sum_{m=1, m\neq 1} \widetilde{C}_{mi} \xi_m(k)]
$$

-
$$
\xi_i^T(k) P_i \xi_i(k) + \sigma y_i^T(k) Q y_i(k)
$$

-
$$
e_i^T(k) Q e_i(k) + y_i^T(k) y_i(k) - y_i^T(k) y_i(k) \}
$$

(11)

where

$$
\begin{aligned}\n\varsigma_i(k) &= \begin{bmatrix} \xi_i^T & e_i^T(k) & w_i^T(k) \end{bmatrix}^T, \\
\mathcal{Q}_i &= \begin{bmatrix} P_i & 0 \\ 0 & \gamma^2 I \end{bmatrix}, \\
M_i &= \begin{bmatrix} 2 \sum_{m=1, m \neq i}^J \left(\widetilde{C}_{im}^T P_i \widetilde{C}_{im} + \sum_{n=1}^3 \varepsilon_{mn} \widetilde{C}_{im}^T P_i \widetilde{C}_{im} \right) & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \\
\mathcal{A}_{ij} &= \begin{bmatrix} \widetilde{A}_i(h) & \widetilde{B}_{fi}(h) & \widetilde{W}_i(h) \\ \widetilde{B}_i(h) & 0 & 0 \\ \widetilde{A}_i(h) & 0 & 0 \\ 0 & \widetilde{B}_{fi}(h) & 0 \\ 0 & 0 & \widetilde{W}_i(h) \end{bmatrix}\n\end{aligned}
$$

$$
\mathcal{P}_i = \begin{bmatrix} P_i & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & I & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & I & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & Q_i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \varepsilon_{i1}^{-1}P_i & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \varepsilon_{i2}^{-1}P_i & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \varepsilon_{i3}^{-1}P_i \end{bmatrix},
$$
\n(12)

Thus, applying *Lemma* 1, we have

$$
\widetilde{z}^{T}(k)\widetilde{z}(k) - \gamma^{2}w^{T}(k) w(k)
$$
\n
$$
\leq \sum_{i=1}^{J} \left\{ \varsigma_{i}^{T}(k) \left[\mathcal{A}_{ij}^{T} \mathcal{P}_{i} \mathcal{A}_{ij} - \mathcal{Q}_{i} + \mathcal{M}_{i} \right] \varsigma_{i}(k) - y_{i}^{T}(k) y_{i}(k) \right\}.
$$
\n(13)

Using Schur's complement, one knows that LMIs in (11) imply $A_{ij}^T \mathcal{P}_i \mathcal{A}_{ij} - \mathcal{Q}_i + \mathcal{M}_{ij} < 0$. Therefore,

$$
\tilde{z}^T(k)\tilde{z}(k) - \gamma^2 w^T(k) w(k) < 0. \tag{14}
$$

Similarly, in *Case II* and *Case III*,one can obtain that

$$
\widetilde{\zeta}^{T}(k)\widetilde{\zeta}(k) - \gamma^{2}w^{T}(k) w(k)
$$
\n
$$
\leq \sum_{i=1}^{J} \left\{ \varsigma_{i}^{T}(k) \left[\mathcal{A}_{ij}^{T} \mathcal{P}_{i} \mathcal{A}_{ij} - \mathcal{Q}_{i} + \mathcal{M}_{ij} \right] \varsigma_{i}(k) - \rho^{2} \right\}. \quad (15)
$$

It implies that $\|\tilde{z}(k)\|_2 < \gamma \|w(k)\|_2$ for any nonzero $w(k) \in l_2$ [0, ∞). Hence, the filtering error system composed of *J* filter error subsystems as (6) is asymptotically stable and the event-triggered H_{∞} disturbance attenuation performance in (8) is guaranteed for a prescribed γ . The proof is completed.

Remark 1: In our paper, the stability of the system is proved by Lyapunov-Krasovskii's second method. Its basic idea is to construct the Lyapunov-Krasovskii energy function to judge the stability of the system directly. That is to say, Lyapunov-Krasovskii's second method is to analyze the stability from the energetic standpoint (the equation (9) is constructed as a generalized energy function). Consider a function $R^n \to R$ such that $\Delta V(k) \leq 0$ for all values of $k \neq 0$, then the equilibrium is proven to be asymptotically stable.

B. H_{∞} filter design

In this section, the integrated event-triggered H_{∞} filter design problem for the discrete-time interconnected fuzzy system is addressed. Theorem 1 provides a sufficient condition for the H_{∞} filter design, in which the matrix variables P_i are coupled with filter parameter matrices.

Theorem 2: The event-triggered H_{∞} filter design problem is solved if there exist matrices, \widehat{A}_{fij} , \widehat{B}_{fij} , and \widehat{C}_{fij} , and positive constants ε*i*1, ε*i*2 and ε*i*3, which satisfy (17), as shown at the top of the next page, where

$$
P_1 = \begin{bmatrix} P_{1i} & P_{2i} \\ P_{2i} & P_{2i} \end{bmatrix},
$$

$$
\Phi_{1} = \begin{bmatrix} A_{i}^{T} (h) P_{1i} & D_{i}^{T} (h) \widehat{B}_{j\hat{1}}^{T} (h) \\ \widehat{A}_{j\hat{1}}^{T} (h) & \widehat{A}_{j\hat{1}}^{T} (h) \end{bmatrix},
$$
\n
$$
\Phi_{2} = \begin{bmatrix} -\widehat{B}_{j\hat{1}}^{T} (h) & -\widehat{B}_{j\hat{1}}^{T} (h) \end{bmatrix},
$$
\n
$$
\Pi_{1} = \begin{bmatrix} \widehat{C}_{i}, \widehat{C}_{\varepsilon i1}, \widehat{C}_{\varepsilon i2}, \widehat{C}_{\varepsilon i3} \end{bmatrix},
$$
\n
$$
\Pi_{1} = \frac{1}{2} diag \{ \widehat{P}_{i}, \widehat{P}_{\varepsilon i1}, \widehat{P}_{\varepsilon i2}, \widehat{P}_{\varepsilon i3} \},
$$
\n
$$
\widehat{C}_{i} = \begin{bmatrix} \widetilde{C}_{i1} P_{1}, \dots, \widetilde{C}_{im,m \neq i} P_{m}, \dots, \widetilde{C}_{iJ} P_{J} \end{bmatrix},
$$
\n
$$
\widehat{C}_{\varepsilon i1} = \begin{bmatrix} \widetilde{C}_{i1} P_{1}, \dots, \widetilde{C}_{im,m \neq i} P_{m}, \dots, \widetilde{C}_{iJ} P_{J} \end{bmatrix},
$$
\n
$$
\widehat{C}_{\varepsilon i2} = \begin{bmatrix} \widetilde{C}_{i1} P_{1}, \dots, \widetilde{C}_{im,m \neq i} P_{m}, \dots, \widetilde{C}_{iJ} P_{J} \end{bmatrix},
$$
\n
$$
\widehat{P}_{i} = diag \begin{bmatrix} P_{1}, \dots, \widetilde{C}_{im,m \neq i} P_{m}, \dots, P_{J} \end{bmatrix},
$$
\n
$$
\widehat{P}_{\varepsilon i1} = diag \begin{bmatrix} \varepsilon_{11} P_{1}, \dots, \varepsilon_{m1, m \neq i} P_{m}, \dots, \varepsilon_{J1} P_{J} \end{bmatrix},
$$
\n
$$
\widehat{P}_{\varepsilon i2} = diag \begin{bmatrix} \varepsilon_{12} P_{1}, \
$$

Furthermore, the matrices of the event-triggered H_{∞} filter are given by

$$
A_{fi} (h) = P_{2i}^{-1} \widehat{A}_{fi} (h), B_{fi} (h) = P_{2i}^{-1} \widehat{B}_{fi} (h),
$$

\n
$$
C_{fi} (h) = \widehat{C}_{fi} (h).
$$
\n(18)

IV. NUMERICAL EXAMPLE

In this section, an example presented to illustrate the application of the proposed filter design approach and its performance.

Considering an inverted pendulum system [21] consisting of three subsystems, the T-S fuzzy model of the system is as below

Rule 1: IF *xi*¹ (*k*) is −88◦ THEN

$$
\begin{cases}\n x_i (k+1) = A_{i1} x_i (k) + W_{i1} w_i (k) \\
 + \sum_{m=1, m \neq i}^{3} C_{mi} x_m (k) \\
 y_i (k) = D_{i1} x_i (k).\n\end{cases}
$$
\n(19)

Rule 2: IF x_{i1} (*k*) is 0° THEN ϵ

$$
\begin{cases}\n x_i (k+1) = A_{i2} x_i (k) + W_{i2} w_i (k) \\
 + \sum_{m=1, m \neq i}^{3} C_{mi} x_m (k) \\
 y_i (k) = D_{i2} x_i (k).\n\end{cases}
$$
\n(20)

Rule 3: IF *xi*¹ (*k*) is 88◦ THEN

$$
\begin{cases}\n x_i (k+1) = A_{i3} x_i (k) + W_{i3} w_i (k) \\
 + \sum_{m=1, m \neq i}^{3} C_{mi} x_m (k) \\
 y_i (k) = D_{i3} x_i (k).\n\end{cases}
$$
\n(21)

where

$$
A_{11} = \begin{bmatrix} 1 & 0.00 \times 45 \\ -0.3475 & 0.9 \end{bmatrix};
$$

\n
$$
A_{12} = \begin{bmatrix} 1 & 0.0045 \\ -0.3300 & 0.9 \end{bmatrix};
$$

\n
$$
A_{13} = \begin{bmatrix} 1 & 0.0045 \\ -0.3475 & 0.9 \end{bmatrix};
$$

\n
$$
A_{21} = \begin{bmatrix} 1 & 0.0045 \\ -0.3330 & 0.9 \end{bmatrix};
$$

\n
$$
A_{22} = \begin{bmatrix} 1 & 0.0045 \\ -0.3150 & 0.9 \end{bmatrix};
$$

\n
$$
A_{31} = \begin{bmatrix} 1 & 0.0049 \\ -0.7330 & 0.9 \end{bmatrix};
$$

\n
$$
A_{32} = \begin{bmatrix} 1 & 0.0049 \\ -0.3550 & 0.9 \end{bmatrix};
$$

\n
$$
A_{33} = \begin{bmatrix} 1 & 0.0049 \\ -0.7330 & 0.9 \end{bmatrix};
$$

\n
$$
W_{11} = W_{12} = W_{13} = \begin{bmatrix} 0 \\ 0.0025 \end{bmatrix};
$$

\n
$$
W_{21} = W_{22} = W_{23} = \begin{bmatrix} 0 \\ 0.0019 \end{bmatrix};
$$

\n
$$
W_{31} = W_{32} = W_{33} = \begin{bmatrix} 0 \\ 0.0016 \end{bmatrix};
$$

FIGURE 1. The inverted pendulums connected by three subsystems.

$$
D_{11} = D_{12} = D_{13} = \begin{bmatrix} 1 & 0 \end{bmatrix};
$$

\n
$$
D_{21} = D_{22} = D_{23} = \begin{bmatrix} 1 & 0 \end{bmatrix};
$$

\n
$$
D_{31} = D_{32} = D_{33} = \begin{bmatrix} 1 & 0 \end{bmatrix};
$$

\n
$$
H_{11} = H_{12} = H_{13} = \begin{bmatrix} 0 & 1 \end{bmatrix};
$$

\n
$$
H_{21} = H_{22} = H_{23} = \begin{bmatrix} 0 & 1 \end{bmatrix};
$$

\n
$$
H_{31} = H_{32} = H_{33} = \begin{bmatrix} 0 & 1 \end{bmatrix};
$$

\n
$$
C_{21} = \begin{bmatrix} 0 & 0 \ 0.0050 & 0 \end{bmatrix};
$$

\n
$$
C_{12} = \begin{bmatrix} 0 & 0 \ 0.0040 & 0 \end{bmatrix};
$$

\n
$$
C_{13} = \begin{bmatrix} 0 & 0 \ 0.0000 & 0 \end{bmatrix};
$$

\n
$$
C_{31} = \begin{bmatrix} 0 & 0 \ 0.0030 & 0 \end{bmatrix};
$$

\n
$$
C_{32} = \begin{bmatrix} 0 & 0 \ 0.0025 & 0 \end{bmatrix},
$$

and $w_i = \sin(0.04\pi k)e^{-0.05k}$, $i = 1, 2, 3$, The simplified model of inverted pendulum is described in Fig. 2.

FIGURE 2. Simulation results for $z_1(k)$ and $z_{f1}(k)$.

In what follows, we design the integrated event-triggered filter under the given transmission strategy (3). Let $\sigma_i = 0.01, i = 1, 2, 3, \varepsilon_{i1} = \varepsilon_{i2} = \varepsilon_{i3} = 20, i = 1, 2, 3$ and $\rho_i = 0.01$. we obtain the following H_{∞} filter parameter matrices:

$$
A_{f11} = \begin{bmatrix} -0.0861 & -0.0203 \\ 3.4978 & 0.8262 \end{bmatrix}; \quad B_{f11} = \begin{bmatrix} -2.2627 \\ 9.7132 \end{bmatrix};
$$

FIGURE 3. Release time and release interval of subsystem 1 under the couple event-triggered strategy.

FIGURE 4. Simulation results for $z_2(k)$ and $z_{f2}(k)$.

FIGURE 5. Release time and release interval of subsystem 2 under the couple event-triggered strategy.

$$
A_{f12} = \begin{bmatrix} -0.0859 & -0.0203 \\ 3.5080 & 0.8284 \end{bmatrix}; \quad B_{f12} = \begin{bmatrix} -2.6577 \\ 4.5223 \end{bmatrix};
$$

\n
$$
A_{f13} = \begin{bmatrix} -0.1817 & -0.0549 \\ 3.3475 & 0.6101 \end{bmatrix}; \quad B_{f13} = \begin{bmatrix} -3.095 \\ 4.9069 \end{bmatrix};
$$

\n
$$
A_{f21} = \begin{bmatrix} -0.3428 & -0.1542 \\ 1.4677 & 0.6602 \end{bmatrix}; \quad B_{f21} = \begin{bmatrix} -0.9773 \\ 4.1471 \end{bmatrix};
$$

FIGURE 6. Simulation results for $z_3(k)$ and $z_{f3}(k)$.

FIGURE 7. Release time and release interval of subsystem 3 under the couple event-triggered strategy.

$$
A_{f22} = \begin{bmatrix} -0.3489 & -0.1534 \\ 1.4936 & 0.6570 \end{bmatrix}; \quad B_{f22} = \begin{bmatrix} -0.9602 \\ 4.0736 \end{bmatrix};
$$

\n
$$
A_{f23} = \begin{bmatrix} -0.5477 & -0.0815 \\ 2.3308 & 1.0090 \end{bmatrix}; \quad B_{f23} = \begin{bmatrix} -3.095 \\ 4.9069 \end{bmatrix};
$$

\n
$$
A_{f31} = \begin{bmatrix} 0.0640 & -0.0612 \\ -0.2724 & 0.2605 \end{bmatrix}; \quad B_{f31} = \begin{bmatrix} -0.0594 \\ 0.2521 \end{bmatrix};
$$

\n
$$
A_{f32} = \begin{bmatrix} 0.0266 & -0.0422 \\ -0.1131 & 0.1795 \end{bmatrix}; \quad B_{f32} = \begin{bmatrix} -0.0557 \\ 0.2369 \end{bmatrix};
$$

\n
$$
A_{f33} = \begin{bmatrix} -0.0186 & -0.0355 \\ -0.0792 & 0.1509 \end{bmatrix}; \quad B_{f33} = \begin{bmatrix} -0.1064 \\ 0.4448 \end{bmatrix};
$$

\n
$$
C_{f11} = \begin{bmatrix} 4.1328 & -1.5617 \end{bmatrix};
$$

\n
$$
C_{f12} = \begin{bmatrix} 2.6248 & -0.0482 \end{bmatrix};
$$

\n
$$
C_{f21} = \begin{bmatrix} 0.0176 & -6896 \end{bmatrix};
$$

\n
$$
C_{f22} = \begin{bmatrix} 0.1458 & -0.6903 \end{bmatrix};
$$

\n
$$
C_{f23} = \begin{bmatrix} 0.1776 & -0.2925 \end{bmatrix};
$$

\n
$$
C_{f31} = \begin{bmatrix} -0.0767 & -0.2589 \end{bmatrix};
$$

\n

Under the integrated event-triggered strategy and the filter parameter with $\rho_i = 0.01$, the state responses of a large-scaled system (2) are depicted in Figs. 2, 4 and 6, where

the initial states are chosen as $x_1^T(0) = [0.3, 0], x_2^T(0) =$ $[0.3, 0], x_2^T(0) = [0.3, 0]$

TABLE 2. Comparison with different schemes.

Through Table 2, it can be seen that under the same conditions, the transmit data by the integrated event-triggered scheme is less than that of the two traditional event-triggered schemes.

V. CONCLUSION

In the above part, the integrated event-triggered transmission strategy has been introduced for the large-scaled system. And the integrated event-triggered H_{∞} filtering performance problem has been investigated. The integrated event-triggered means event-triggered control can determine by two transmission conditions. The advantage of the proposed strategy is to reduce the pressure of channel transmission and improve data communication efficiency. By applying a Lyapunov-Krasovskii functional and linear matrix inequalities(LMIs), a method of the integrated event-triggered H_{∞} distributed filter design is developed for the filtering error system concerned to be asymptotically stable with a given disturbance attenuation level. A simulation is finally given to show the effective methods in this paper.

REFERENCES

- [1] Y. Zhang, E. Yao, H. Wei, T. Zuo, and S.-S. Liu, ''Constrained multinomial Probit route choice modeling for passengers in large-scaled metro networks in China,'' *Transp. Res. Procedia*, vol. 25, pp. 2385–2395, 2017.
- [2] C. Lin, G. Wu, C. W. Yu, and M. S. Obaidat, "Clustering and splitting charging algorithms for large scaled wireless rechargeable sensor networks,'' *J. Syst. Softw.*, vol. 113, pp. 381–394, Mar. 2016.
- [3] Y. Zou, J. Lam, Y. Niu, and D. Li, ''Constrained predictive control synthesis for quantized systems with Markovian data loss,'' *Automatica*, vol. 55, pp. 217–225, Apr. 2015.
- [4] K. Galloway, K. Sreenath, A. D. Ames, and J. W. Grizzle, ''Torque saturation in bipedal robotic walking through control Lyapunov function-based quadratic programs,'' *IEEE Access*, vol. 3, pp. 323–332, 2015.
- [5] C. Dou, D. Yue, J. M. Guerrero, and Q.-L. Han, ''Multi-agent system-based event-triggered hybrid control scheme for energy Internet,'' *IEEE Access*, vol. 5, pp. 3263–3272, 2017.
- [6] L. Ding, Q.-L. Han, X. Ge, and X.-M. Zhang, ''An overview of recent advances in event-triggered consensus of multiagent systems,'' *IEEE Trans. Cybern.*, vol. 48, no. 4, pp. 1110–1123, Apr. 2018.
- [7] D. D. Siljak, *Large-Scale Dynamic Systems: Stability and Structure*. Amsterdam, The Netherlands: North Holland, 1978.
- [8] G. J. Klir and B. Yuan, *Fuzzy Sets and Fuzzy Logic: Theory and Applications*. Upper Saddle River, NJ, USA: Prentice-Hall, 1995.
- [9] T. Taniguchi, K. Tanaka, H. Ohtake, and H. O. Wang, ''Model construction, rule reduction, and robust compensation for generalized form of Takagi–Sugeno fuzzy systems,'' *IEEE Trans. Fuzzy Syst.*, vol. 9, no. 4, pp. 525–538, Aug. 2001.
- [10] L. Zhang, H. Zhang, and X. Ding, ''Non-fragile *H*∞ filtering for large-scale power systems with sensor networks,'' *IET Gener., Transmiss. Distrib.*, vol. 11, no. 4, pp. 968–977, 2017.
- [11] X.-J. Li, J.-J. Yan, and G.-H. Yang, "Adaptive fault estimation for T-S fuzzy interconnected systems based on persistent excitation condition via reference signals,'' *IEEE Trans. Cybern.*, to be published.
- [12] H. Wang, W. Liu, J. Qiu, and P. X. Liu, "Adaptive fuzzy decentralized control for a class of strong interconnected nonlinear systems with unmodeled dynamics,'' *IEEE Trans. Fuzzy Syst.*, vol. 26, no. 2, pp. 836–846, Apr. 2018.
- [13] X.-L. Wang and G.-H. Yang, ''Event-triggered *H*∞ filtering for discrete-time T-S fuzzy systems via network delay optimization technique,'' *IEEE Trans. Syst., Man, Cybern., Syst.*, to be published.
- [14] L. Zhang, H. Zhang, and H. Gao, ''Decentralized non-fragile event-triggered H_{∞} filtering for large-scaled power system based on T-S fuzzy model,'' *IEEE Access*, vol. 6, pp. 64540–64548, 2018.
- [15] Z. Zhang, Z. Zhang, P. Shi, H. R. Karimi, and H. Zhang, ''Finite-time *H*∞ filtering for T–S fuzzy discrete-time systems with time-varying delay and norm-bounded uncertainties,'' *IEEE Trans. Fuzzy Syst.*, vol. 23, no. 6, pp. 2427–2434, Dec. 2015.
- [16] R. E. Kalman, "A new approach to linear filtering and prediction problems,'' *J. Basic Eng.*, vol. 82, no. 1, pp. 35–45, 1960.
- [17] M. Zhang, P. Shi, Z. Liu, L. Ma, and H. Su, " H_{∞} filtering for discretetime switched fuzzy systems with randomly occurring time-varying delay and packet dropouts,'' *Signal Process.*, vol. 143, no. 6, pp. 320–327, Feb. 2018.
- [18] H. Wang, P. Shi, and R. K. Agarwal, ''Network-based event-triggered filtering for Markovian jump systems,'' *Int. J. Control*, vol. 89, no. 6, pp. 1096–1110, 2016.
- [19] D. Zhang, W. Cai, Q.-G. Wang, and L. Xie, "Nonfragile distributed filtering for T–S fuzzy systems in sensor networks,'' *IEEE Trans. Fuzzy Syst.*, vol. 23, no. 5, pp. 1883–1890, Oct. 2015.
- [20] T. Takagi and M. Sugeno, ''Fuzzy identification of systems and its applications to modeling and control,'' *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. SMC-15, no. 1, pp. 116–132, Jan./Feb. 1985.
- [21] Y.-F. Chen and A.-C. Huang, "Adaptive control of rotary inverted pendulum system with time-varying uncertainties,'' *Nonlinear Dyn.*, vol. 76, no. 1, pp. 95–102, 2014.

LIANGLIANG ZHANG was born in Zhejiang, China, in 1990. She received the B.E. degree from the City College, Zhejiang University, and the M.S. degree in circuits and systems from the University of Electronic Science and Technology of China, Chengdu, Sichuan, in 2012 and 2014, respectively, where she is currently pursuing the Ph.D. degree in circuits and systems. Her key research interests include fuzzy control, event-triggered H_{∞} filtering, and switched systems.

HONGBIN ZHANG received the B.Eng. degree in aerocraft design from Northwestern Polytechnical University, Xi'an, China, in 1999, and the M.Eng. and Ph.D. degrees in circuits and systems from the University of Electronic Science and Technology of China, Chengdu, in 2002 and 2006, respectively. He has been with the School of Electrical Engineering, University of Electronic Science and Technology of China, since 2002, where he is currently a Professor. From 2008 to 2010, he was

a Research Fellow with the Department of Manufacturing Engineering and Engineering Management, City University of Hong Kong, Hong Kong. His current research interests include intelligent control and autonomous cooperative control. and integrated navigation.

ZHIPENG ZHANG was born in Jiangxi, China, in 1994. He received the B.E. degree in electronic science and technology from Nanchang Hangkong University, Nanchang, China, in 2017. He is currently pursuing the M.E. degree in electronics and communication engineering with the University of Electronic Science and Technology of China, Chengdu. His current research interests include computer vision, intelligent control, and target recognition.

 \sim \sim \sim