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Mellin Transform-Based Approach for Parameter Estimation of K Distribution Plus Noise

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ABSTRACT Mellin transform is a fast and robust method for parameter estimation. In this paper, the parameter estimator of K plus noise (KpN) model based on the Mellin transform is proposed. First, we start deriving the expression of the second characteristic functions and log-based moments in detail by Mellin transform. And then, we provide the expression of the first fourth order log-culumants which are directly related to the parameters of KpN model. Finally, the numerical computation is applied to realize parameter estimation of KpN model. The Monte Carlo simulations show that the proposed estimator is more efficient than existing methods for clutter to noise ratios under both situations of large and small sample sizes.

INDEX TERMS K plus noise model, parameters estimation, Mellin transform, synthetic aperture radar (SAR).

I. INTRODUCTION

Sea clutter modeling is a basis of image interpretation for maritime environment surveillance by radar systems, such as IPIX and synthetic aperture radar (SAR) [1], [2]. Experimental data from sea-clutter returns have shown that they are often dominated by multipath, shadowing, and ducting mechanisms at low grazing angles and by Bragg scattering from rough surfaces and whitecaps at high grazing angles [3]. The Gaussian clutter model is a reasonable description for sea clutter in a low-resolution maritime radar system [4]. With lower altitude and grazing angles for high-resolution imaging, the Gaussian clutter model is no longer appropriate due to the larger scale structure of the sea surface.

For the sea clutter model in the high-resolution imaging condition, compound K distribution is a good model for sea clutter returns and has received a great deal of attention in the literature e.g., [5] and [6]. Compound K distribution results from a multiplicative texture model for the clutter and fits a wide range of experimental data well; Ward predicted the radar detection performance in sea clutter using the compound K distribution in [7]. However, it was noticed that the observations from some areas were heterogeneous to an extent that even K distributions could not take account of [8].

K distribution plus noise (KpN model) and K distribution plus discrete spikes (KA model) are two extensions to the compound K distribution, experiments carried out in [9] show that the modifications to the compound K distribution have a significant impact on radar target detection performance. Generalized K distribution model is another extension to the compound K distribution, which generalizes the compound K distribution away from the strong-scattering limit, providing a statistical model for weak scattering [10]. KK distribution is a mixture model consisting of the sum of two K components, one associated with whitecap components and one for the sea-spikes. Compared to KA model, the KK model is equally able to model the tail region with the benefit of better matching in the region where the tail extends from the bulk of the distribution and not consider the added thermal noise, so the model has the analytic expression to compute easily [11].

Estimating the parameters of sea clutter model from data samples is a very important application on maritime remote sensing and surveillance. Studies on approaches to estimate parameters of compound K distribution have been deeply investigated. The great majority of the approaches on parameter estimation of K distribution is based on statistic characteristics. The moment estimator is the initial and convenient

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way to solve this problem [12]. However, the disadvantages of the moment estimator are also obvious, the moment estimator is unable to realize equivalent number of looks estimation and ensure sufficient accuracy of estimation. In [13], Iskander *et al.* provided a method based on higher order and fractional moments, that is computationally inexpensive and do not require the solution of nonlinear equations. The Maximum Likelihood (ML) estimator is the most optimal estimator in theory. Unfortunately, the analytic expression is hard to derive due to the complex representation of compound K distribution [14], [15]. Roberts and Furui [16] improved the ML estimator by Expectation Maximum (EM) algorithm at the cost of huge computation burden. Wachowiak *et al.* [17] applied the neural network to deal with the problem, but time-consuming is too large to lead to the method inefficiency. Su and Chen [18] developed an estimation procedure by means of particle swarm optimization (PSO) in. Blacknell and Tough [19] put forward a new estimator for K distribution based on zlog(z), comparing the proposed method with five existing estimators to show the effectiveness in. Hu [20] introduced a method called z^r log(z) which is the extension from zlog(z) and validated the proposed estimator had better performance than the latter when the order is smaller than one. Shi *et al.* [21] raised a fast and robust method of parameter estimation for the K distribution based on Mellin transform, Compared with the moment estimator, the Mellin estimator has an ability to estimate the equivalent apparent number and obtain higher estimation accuracy.

In all, the algorithms aimed at the parameter estimation of K-distributed clutter are various, but the literature on K distribution parameter estimation in the presence of added thermal noise is not sufficient. The analogous problem has been addressed by Fante [22] for log-normally distributed clutter. Watts [23] made further predication on radar detection prediction in K-distributed sea clutter and thermal noise and proposed a K-distributed plus thermal noise estimator which uses higher-order moments estimation (HOME) in. Sutour *et al.* [24] analyzed the K-distributed sea clutter and thermal noise in high range and Doppler resolution radar data and put forward a parameter estimator by calculating the complementary cumulative density function of real data in. Mezache *et al.* [25] introduce two pragmatic methods called curve-fitting estimation and Nelder-Mead algorithm to parameter estimation for KpN model. Sahed *et al.* [26] presented a zlog(z) based closed form approach, replacing the hypergeometric function by the inverse of the harmonic mean of the receive d data. Zhang andYang [27] proposed a method of K distribution shape parameter estimation based on robust statistics. In contrast to moment estimator, the estimator can reduce the interference of the outlier significantly, but the research on other K distribution parameters estimation was not concluded.

In conclusion, the above parameter estimation algorithms for KpN model have either the hard computation burden or low estimation accuracy. Mellin transform has been proved to be effective for K-distributed clutter parameter estimation. In this paper, we intend to generalize the Mellin transform to KpN model and propose a novel estimator based on Mellin transform to achieve parameter estimation for KpN model.

The remainder of this paper organized as follows. Section II gives an overview on the KpN model and existing estimation methods. Section III describes the method that applies the Mellin transform to parameter estimation for KpN model. Section IV presents and compares the experimental results of the proposed algorithm with HOME and zlog(z) method for a given clutter to noise ratio (CNR) in the large or small sample sizes. Finally, summaries and conclusions are given in Section V.

II. OVERVIEW ON COMPOUND KpN MODEL AND EXISTING ESTIMATION METHODS

In this section, we will first give an overview on KpN model. Subsequently, in order to validate our proposed method better, we will briefly introduce the other two parameter estimation methods in [23] and [26] as contrast methods; The HOME method is a classical parameter estimation method for KpN model and is often used as a contrast method in many papers [26]–[28]. The zlog(z)-based closed form approach is the recent research production on KpN model parameter estimation, the experiments carried out in [26] indicated that the method had a better performance than the HOME method.

A. COMPOUND K DISTRIBUTION

The compound K distribution is composed of two components. While accounting for the correlation properties of the sea-echoes, this compounding agrees with modulating the square law-detected speckle by the texture [26]. The probability density function (PDF) of the compound K distribution can be expressed by

$$p_X(x) = \int_0^\infty p_{X|Y}(x|\,y)\,p_Y(y)dy \qquad 0 \le x \le \infty \tag{1}$$

The conditional PDF of the speckle component $x|\,y$ can be described by an exponential distribution and the PDF of texture component y can be described by a Gamma distribution, i.e.,

$$p_{X|Y}(x|\,y) = \frac{1}{y}\exp\left(-\frac{x}{y}\right) \tag{2}$$

and

$$p_Y(y) = \frac{b^v y^{v-1}}{\Gamma(v)}\exp(-by) \tag{3}$$

where b is the scale parameter, v is the shape parameter, and Γ is the gamma function. Substituting (2) and (3) into (1), we can derive the analytic expression of the compound K distribution as

$$p_X(x) = \frac{2b^v}{\Gamma(v)}\left(\frac{x}{b}\right)^{\frac{v-1}{2}} K_{v-1}\left(2\sqrt{bx}\right) \tag{4}$$

where $K_{v-1}(\cdot)$ denotes the modified Bessel function of the second kind.

B. KpN MODEL

The influence of thermal noise is ignored in the previous derivation. The PDF of the thermal noise is assumed to be an uncorrelated zero-mean Gaussian distribution in [26]. Based on this assumption, (2) should be modified as

$$p_{X|Y}(x|y) = \frac{1}{p_n + y} \exp\left(-\frac{x}{p_n + y}\right) \tag{5}$$

where $p_n = 2\sigma^2$ is the noise power level.

The above derivation is based on single-look assumption (i.e., the number of looks is equal to 1). When dealing with the multi-look situation (i.e., the number of looks is more than 1), the PDF of the K-distributed clutter plus noise can be represented as same as [26], the detail derivation is as follows.

Considering a noncoherent integration of N pulses of sea clutter data and assuming independent pulse-to-pulse samples, the sum of N pulses is indicated by intensity variable Z_i as

$$Z_i = \sum_{j=1}^{N} X_j \quad i = 1, 2, \cdots M \tag{6}$$

where i is the range cell index and j is the pulse index.

According to the knowledge in probability theory, we can infer the following expression by (5) and (6) as

$$p_{Z_i|Y}(z_i|y) = \frac{z_i^{N-1}}{(p_n + y)^N \,\Gamma(N)} \exp\left(-\frac{z_i}{p_n + y}\right) \tag{7}$$

where $\Gamma(N) = (N-1)!$

C. THE HOME METHOD

On the basis of (7), the Home method [23] suggested that the three parameters of K-distributed plus noise can be estimated by

$$\begin{cases} \hat{v} = \dfrac{18\left(\hat{\mu}_2 - 2\hat{\mu}_1^2\right)^3}{\left(12\hat{\mu}_1^3 - 9\hat{\mu}_1\hat{\mu}_2 + \hat{\mu}_3\right)^2} \\ \hat{p}_n = \hat{\mu}_1 - \left(0.5\hat{v}\left(\hat{\mu}_2 - 2\hat{\mu}_1^2\right)\right)^{1/2} \\ \hat{b} = \dfrac{\hat{v}}{\hat{\mu}_1 - \hat{p}_n} \end{cases} \tag{8}$$

where

$$\begin{cases} \hat{\mu}_1 = \dfrac{\langle Z \rangle}{N} \\ \hat{\mu}_2 = \dfrac{2\langle Z^2 \rangle}{N(N+1)} \\ \hat{\mu}_3 = \dfrac{6\langle Z^3 \rangle}{N(N+1)(N+2)} \end{cases} \tag{9}$$

where $\langle \cdot \rangle$ is the mean operator. Details of the HOME method can be found in [23].

D. THE zlog(z)-BASED CLOSED FORM METHOD

Another method derived in [26] gives the parameter estimation as

$$\begin{cases} \hat{v} = v_{eff} \left(\dfrac{1 - \frac{N-1}{N} \langle Z \rangle \langle Z^{-1} \rangle}{v_{eff}\left(\frac{\langle Z \log(Z) \rangle}{\langle Z \rangle} - \langle \log(Z) \rangle - \frac{1}{N}\right) - \frac{N-1}{N} \langle Z \rangle \langle Z^{-1} \rangle} \right)^2 \\ \hat{p}_n = \frac{\langle Z \rangle}{N} \left(1 - \sqrt{\dfrac{\hat{v}}{v_{eff}}}\right) \\ \hat{b} = \dfrac{N\hat{v}}{\langle Z \rangle - N\hat{p}_n} \end{cases} \tag{10}$$

where v_{eff} is the effective value of the shape parameter, which can be calculated by

$$v_{eff} = v\left(1 + \frac{1}{CNR}\right)^2 = \frac{(N+1)\langle Z \rangle^2}{N\langle Z^2 \rangle - (N+1)\langle Z \rangle^2} \tag{11}$$

III. DERIVATION OF THE NOVEL MELLIN-BASED ESTIMATOR

Mellin transform is an extremely powerful method; it is often yielding closed-form expressions very difficult to come up with other methods or to deduce from the usual tables of integrals. Yet, as opposed to other methods, Mellin transform is very straightforward to apply [29]. The Mellin-estimator in absence of thermal noise is derived in [21]; the experiment results show the estimator has good performance on parameter estimation of compound K distribution. In this section, we generalize the Mellin transform to K-distributed plus noise, and derive a novel estimator based on Mellin transform. Prior to the presentation of the proposed estimation method, we start by introducing the definition of Mellin transform and characteristic function of the second kind.

A. MELLIN TRANFORMS

Let $p(x)$ denotes a complex-valued function of the real, positive variable x, The Mellin transform of $p(x)$ will be denoted by $MT[p(x)](s)$. The definition of the Mellin transform involves an integral

$$MT[p(x)](s) = \int_0^\infty x^{s-1} p(x) dx \tag{12}$$

According to the (12), we can make a further definition of the first characteristic function of the second kind as

$$\phi_Z(s) = MT[p_Z(x)](s) = \int_0^\infty x^{s-1} p_Z(x) dx \tag{13}$$

where $p_Z(x)$ denotes the PDF of K-distributed plus noise in this paper.

The derivation of (13) at $s = 1$ can be used as the definition of log moments:

$$\tilde{m}_k = \left.\frac{d^k \phi_Z(s)}{ds^k}\right|_{s=1} = \int_0^\infty (\ln x)^k p_Z(x) dx \tag{14}$$

Combining (13) and (14), we can acquire the definition of the second characteristic function of the second kind and log-cumulants as follows.

$$\xi_Z(s) = \ln \phi_Z(s) \tag{15}$$

$$\tilde{c}_k = \left.\frac{d^k \xi_Z(s)}{ds^k}\right|_{s=1} \tag{16}$$

Equation (16) is the key to solve the problem on parameter estimation of the K-distributed plus noise.

B. KpN MODEL ESTIMATOR BASED ON MoLC

In section III.A, we have given out the definition of Mellin transform and characteristic function of the second kind. We now provide the exhaustive derivation of our proposed method based on Mellin transform.

Substituting (3) and (5) into (1), we can yield the expression of the K-distributed plus noise as

$$p_X(x) = \int_0^\infty \frac{b^v y^{v-1}}{\Gamma(v)(p_n+y)} \exp\left(-by - \frac{x}{p_n+y}\right) dy \tag{17}$$

When considering the multi-pulse situation [$N \geq 1$, see Eq. (6)], the (17) can be rewritten as

$$p_Z(z) = \int_0^\infty \frac{b^v y^{v-1} z^{N-1}}{\Gamma(v)\Gamma(N)(p_n+y)^N} \exp\left(-by - \frac{z}{p_n+y}\right) dy \tag{18}$$

Substituting (18) into (11), Mellin transform of the intensity variable z is given by

$$\phi_Z(s) = \frac{b^v}{\Gamma(v)\Gamma(N)} \int_0^\infty \frac{y^{v-1}}{(p_n+y)^N} \exp(-by) \times \int_0^\infty z^{s+N-1} \exp\left(-\frac{z}{p_n+y}\right) dz dy \tag{19}$$

The second integral can be used the following formula to calculate [30]:

$$\int_0^\infty z^m \exp\left(-\beta z^n\right) dx = \frac{\Gamma(\gamma)}{n\beta^\gamma} \tag{20}$$

Substituting (19) into (18), double integral can be simplified to

$$\phi_Z(s) = \frac{\Gamma(s+N-1)\, b^v}{\Gamma(v)\Gamma(N)} \int_0^\infty \frac{y^{v-1}\exp(-by)}{(p_n+y)^{1-s}} dy \tag{21}$$

In order to solve the remainder integral, we need use another formula [30, p. 348, eq. 3.383.4]:

$$\int_u^\infty y^{v-1}(y-u)^{\mu-1}\exp(-\beta y)\, dy = \beta^{-\frac{\mu+v}{2}} u^{\frac{\mu+v-2}{2}} \Gamma(\mu) \exp\left(-\frac{\beta u}{2}\right) W_{\frac{v-\mu}{2}, \frac{1-\mu-v}{2}}(\beta u) \tag{22}$$

where $W(\cdot)$ denotes the Whittaker function [30, p. 1025, Secs. 9.22–9.33].

Substituting (22) into (21), we can obtain the expression of Mellin transform of K-distributed plus noise as

$$\begin{aligned}\phi_Z(s) &= \frac{\Gamma(N+s-1)\, b^v}{\Gamma(v)\Gamma(N)} \int_0^\infty y^{v-1}(p_n+y)^{s-1}\exp(-by)\, dy \\ &= \frac{\Gamma(N+s-1)\exp\left(\frac{bp_n}{2}\right)}{\Gamma(N)\, b^{\frac{s-v}{2}}(p_n)^{\frac{2-v-s}{2}}} W_{\frac{s-v}{2}, \frac{1-v-s}{2}}(bp_n)\end{aligned} \tag{23}$$

On the basis of (23) and (15), the second characteristic function of the second kind is given by

$$\begin{aligned}\xi_Z(s) &= \ln\phi_Z(s) \\ &= \ln\Gamma(N+s-1) + \frac{v-s}{2}\ln b + \frac{v+s-2}{2}\ln p_n \\ &\quad + \frac{bp_n}{2} + \ln W_{\frac{s-v}{2}, \frac{1-v-s}{2}}(bp_n) - \ln\Gamma(N)\end{aligned} \tag{24}$$

In (24), the logarithm of the Whittaker function is too difficult to handle, the equivalent expression is given here [30]:

$$W_{k,\mu}(z) = \exp\left(-\frac{1}{2}z\right) z^{\frac{1}{2}+\mu} U\left(\frac{1}{2}+\mu-k, 1+2\mu, z\right) \tag{25}$$

Making use of (25), (24) can be simplified as

$$\begin{aligned}\xi_Z(s) &= \ln\Gamma(s+N-1) + (1-s)\ln b - \ln\Gamma(N) \\ &\quad + \ln\left[U(1-s, 2-v-s, bp_n)\right]\end{aligned} \tag{26}$$

where $U(\cdot)$ is the Tricomi function, and the Tricomi function can be written as the form of definite integral as [31]

$$U(a,c,z) = \frac{\int_A^\infty \exp(-zt) * (t-A)^{a-1}(t+B)^{c-a-1}\, dt}{\exp(-Az)\Gamma(a)} \tag{27}$$

where $A = 1 - B$.

Let $A = 0$,

$$U(a,c,z) = \frac{\int_0^\infty \exp(-zt) * (t)^{a-1}(t+1)^{c-a-1}\, dt}{\Gamma(a)} \tag{28}$$

Substituting (28) into (26), the expression of $\xi_Z(s)$ is

$$\begin{aligned}&\xi_Z(s) \\ &= \ln\Gamma(N+s-1) - \ln\Gamma(N) + v\ln b + (v+s-1)\ln(p_n) \\ &\quad + \ln\left(\int_0^\infty \exp(-bp_n t) * t^{v-1}(t+1)^{s-1}\, dt\right) - \ln\Gamma(v)\end{aligned} \tag{29}$$

In order to get concise representation, we give the following notations in advance.

$$\begin{aligned}A &= \int_0^\infty \exp(-bp_n t) * (t)^{v-1}\, dt \\ B &= \int_0^\infty \exp(-bp_n t) * (t)^{v-1}\ln(1+t)\, dt \\ C &= \int_0^\infty \exp(-bp_n t) * (t)^{v-1}\ln(1+t)\ln(1+t)\, dt \\ D &= \int_0^\infty \exp(-bp_n t) * (t)^{v-1}\ln(1+t)\ln(1+t)\ln(1+t)\, dt\end{aligned} \tag{30}$$

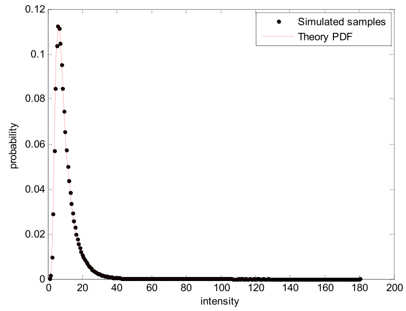


FIGURE 1. Curve-fitting between the simulated samples and theory PDF.

The first, second, third and fourth-order log cumulants in the theory are

$$\left.\frac{d\xi_Z(s)}{ds}\right|_{s=1} = \Phi(N) + \ln(p_n) + \frac{B}{A}$$
$$\left.\frac{d^2\xi_Z(s)}{ds^2}\right|_{s=1} = \Phi(1,N) + \frac{CA - B^2}{A^2}$$
$$\left.\frac{d^3\xi_Z(s)}{ds^3}\right|_{s=1} = \Phi(2,N) + \frac{DA^2 - 3ABC + 2B^3}{A^3} \quad (31)$$

where $\Phi(\cdot)$ denotes the digamma function [32] (i.e., the logarithmic derivative of the gamma function); $\Phi(m,\cdot)$ is the m-th-order polygamma function [32] (i.e., the m-th-order derivative of the digamma function).

On the other hand, the log cumulants calculated by sample data are identical to the situation absence of noise. That is,

$$\begin{cases} \hat{\hat{c}}_1 = \dfrac{1}{N}\displaystyle\sum_{i=1}^{N}[\ln(x_i)] \\ \hat{\hat{c}}_k = \dfrac{1}{N}\displaystyle\sum_{i=1}^{N}\left[\ln\left(x_i - \hat{\hat{c}}_1\right)^k\right], \quad k \geq 2 \end{cases} \quad (32)$$

where x_i is the gray value of pixels in the image.

Combining (31) and (32), the Mellin-transform-based estimator can be represented as

$$\Phi(N) + \ln(p_n) + \frac{B}{A} = \frac{1}{N}\sum_{i=1}^{N}[\ln(x_i)]$$
$$\Phi(1,N) + \frac{CA - B^2}{A^2} = \frac{1}{N}\sum_{i=1}^{N}\left[\ln\left(x_i - \hat{\hat{c}}_1\right)^2\right]$$
$$\Phi(2,N) + \frac{DA^2 - 3ABC + 2B^3}{A^3} = \frac{1}{N}\sum_{i=1}^{N}\left[\ln\left(x_i - \hat{\hat{c}}_1\right)^3\right] \quad (33)$$

In general, the number of pulses, N, is known a priori in the radar processor [11] and the log cumulants in (32) can be calculated directly from the data. That leaves the parameters v, b, and p_n, which can be obtained by (33) through numerical solution.

TABLE 1. Parameters of KpN model simulation corresponding Fig. 1.

Shape Parameter (v)	1
Scale Parameter (b)	2
Noise Power (p_n)	0.5
Sample Sizes (M)	10000
Independent Trials (n)	1000
Integrated Pulses (N)	10

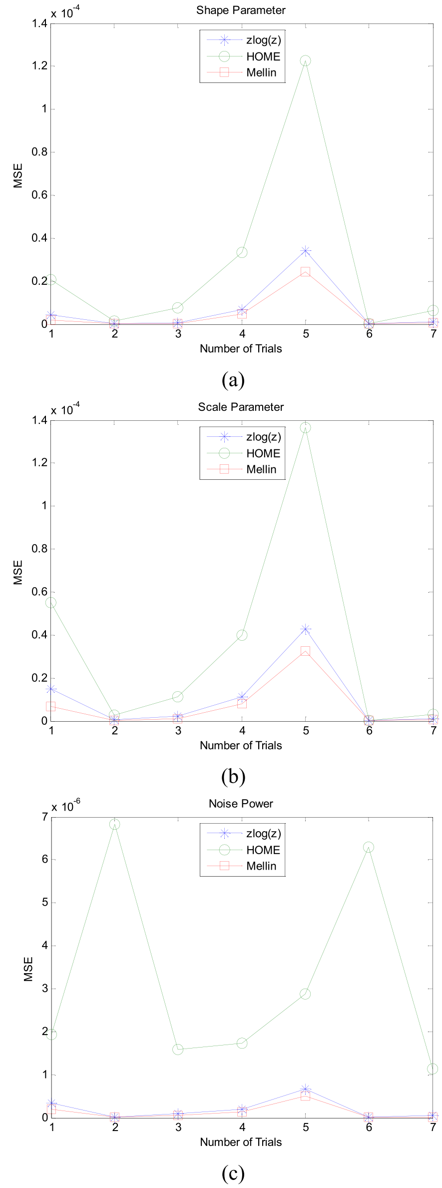


FIGURE 2. Comparison of MSE estimates of shape parameter, scale parameter and noise power of a KpN model for the HOME, the zlog(z)-based closed form, and the proposed estimation methods, the experiment parameters are same as Table 2.

IV. EXPERIMENTAL RESULTS

In this section, we evaluate the performance of our proposed method and compare it with other two popular methods through Monte-Carlo simulations. For convenience,

TABLE 2. Sample mean and sample standard deviation of the three parameter estimates of v, b and p_n at CNR = −3, 0 and 10DB, from home method [23], zlog(z) closed-form method [26] of K-clutter plus noise for M = 10000 samples, N = 10 integrated pulses, and n = 1000 independent trials.

Parameters of Simulated K-clutter Plus Noise (True Values)					Mean and Standard Deviation of the Estimated Parameters					
Methods	v	CNR(dB)	b	p_n	$E[\hat{v}]$	$std(\hat{v})$	$E[b]$	$std(b)$	$E[\hat{p}_n]$	$std(\hat{p}_n)$
zlog(z)	0.5	-3	1.4976	0.6661	0.5001	0.0022	1.4979	0.0039	0.6660	6.0275e-4
HOME					0.5003	0.0047	1.4981	0.0076	0.6660	0.0014
Proposed					0.5001	0.0015	1.4978	0.0027	0.6661	4.4926e-4
zlog(z)	0.1	0	0.2	0.5	0.0999	3.7849e-4	0.1998	7.3877e-4	0.5001	2.6392e-4
HOME					0.0999	0.0013	0.1998	0.0016	0.5000	0.0027
Proposed					0.0999	1.5074e-4	0.1999	3.7154e-4	0.5001	1.0461e-4
zlog(z)	0.5		1	0.5	0.5000	9.2661e-4	0.9999	0.0016	0.5000	3.0219e-4
HOME					0.5005	0.0029	1.0004	0.0034	0.4998	0.0013
Proposed					0.4998	6.2594e-4	0.9996	0.0011	0.5001	2.2877e-4
zlog(z)	1		2	0.5	0.9992	0.0027	1.9988	0.0034	0.5001	4.4987e-4
HOME					0.9999	0.0059	1.9995	0.0065	0.4999	0.0014
Proposed					0.9994	0.0022	1.9991	0.0029	0.5001	3.7051e-4
zlog(z)	1.5		3	0.5	1.5022	0.0060	3.0024	0.0067	0.4997	8.4235e-4
HOME					1.5062	0.0114	3.0064	0.0120	0.4990	0.0017
Proposed					1.5001	0.0051	3.0000	0.0059	0.5000	7.2189e-4
zlog(z)	0.1	10	0.11	0.0909	0.1001	1.7792e-4	0.1101	2.6578e-4	0.0909	6.2507e-5
HOME					0.1003	6.5254e-4	0.1103	4.7033e-4	0.0897	0.0026
Proposed					0.1000	9.4669e-5	0.1100	1.6151e-4	0.0909	2.3341e-5
zlog(z)	1		1.1	0.0909	1.0000	9.8040e-4	1.1000	0.0011	0.0909	2.1178e-4
HOME					0.9997	0.0026	1.0998	0.0017	0.0910	0.0011
Proposed					1.0001	7.1599e-4	1.1001	8.3358e-4	0.0909	1.2114e-4

the power of the received clutter is normalized to unity so that the first-order moment $p_n + v/b \approx 1$, this operation is the same as [26]. The simulations follow the two steps, we provide a method to produce random samples fitting for the KpN model in the first step, and in the second step, we estimate the parameters of KpN model from the samples by the three methods, respectively in different sample sizes, CNR and integrated pulses.

We can use the following Matlab routine to produce the KpN model samples

$$Z = gamrnd\left(N, p_n + gamrnd\left(v, 1/b, n, M\right)\right)$$

Fig. 1 shows the simulated sample data match well with the theory PDF.

Subsequently, we want to show how efficient the proposed Mellin-based estimator is, given by (33), compared to the existing zlog(z)-based closed form method [26] and HOME method [23], which are introduced in section II. The mean and standard deviation (STD), and the mean square error (MSE) criteria [26] are considered to assess the quality of the estimation methods of interest. To this effect, Table 2 summaries the sample mean and the sample standard deviation of the three estimated parameters, namely, the shape parameter v, the scale parameter b and the noise power p_n, for CNR = −3, 0, 10 dB, numbers of samples M = 10000, the number of noncoherent integrated pulses N = 10 and estimation of the unknown parameters are obtained by n = 1000 independent trials. The parameter estimation results of the three methods are displayed in the Table 2, and we make a further investigation on the MSE for the three methods and the results are displayed by Fig. 2 (the parameters are similar to the Table 2).

TABLE 3. Parameters corresponding to Fig.3 (CNR = −3dB).

Shape Parameter (v)	0.2	0.4	0.6	0.8	1	1.2	1.4
Scale Parameter (b)	0.6	1.2	1.8	2.4	3	3.6	4.2
Noise Power (p_n)	0.6661						
Sample Size (M)	10000						
Independent Trials (n)	1000						
Integrated Pulses (N)	10						

The results in the Table 2 and Fig. 2 show that the proposed method has the comparative estimation accuracy with the other two methods, and our method has the smallest MSE and STD in the three methods.

Furthermore, we compare the computational efficiency among the three methods. The corresponding computation

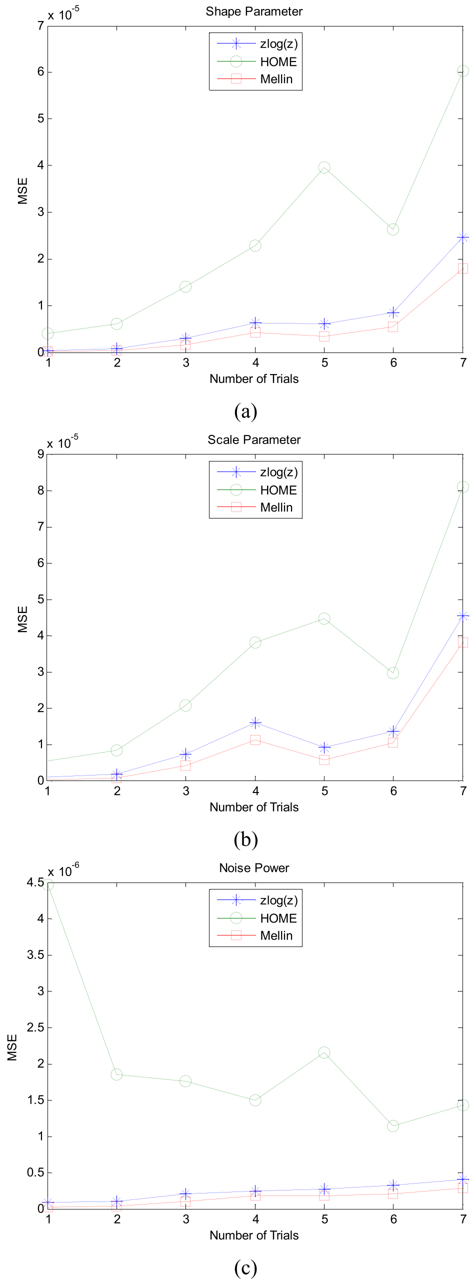


FIGURE 3. Comparison of MSE estimates of shape parameter, scale parameter and noise power of a KpN model for the HOME, the zlog(z)-based closed form, and the proposed estimation methods for CNR = −3dB, the experiment parameters are same as Table 3.

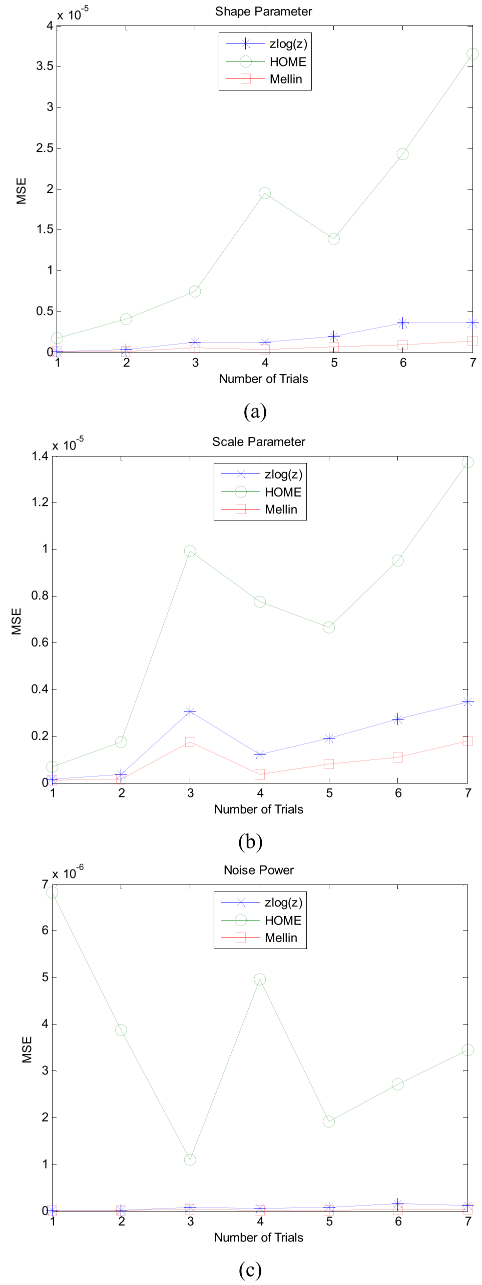
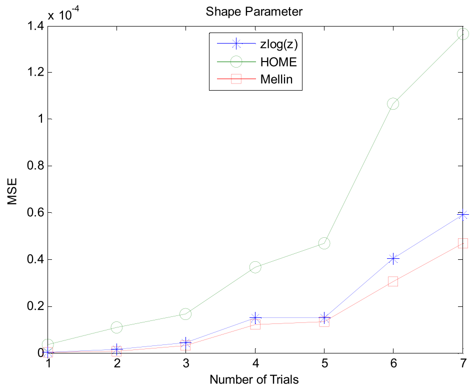


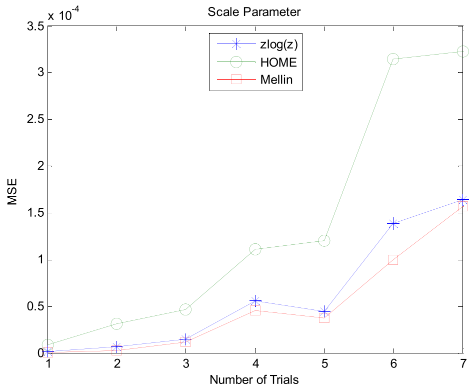
FIGURE 4. Comparison of MSE estimates of shape parameter, scale parameter and noise power of a KpN model for the HOME, the zlog(z)-based closed form, and the proposed estimation methods for CNR = 0dB, the experiment parameters are same as Table 4.

was performed using optimized MATLAB R2014a codes run on a Windows 7.0 operational systems; the hardware environment was an Inter Core i5-7500 3.4-GHz CPU processor with 8-GB memory. As a result, the run time for each independent trail in average is 0.3119s, 0.4268s, and 0.8984s for the zlog(z), HOME, and the proposed method, respectively. This is reasonable because the proposed method involved a numerical calculation to estimate the three parameters [see (33)] in comparison with the zlog(z) and HOME methods, which leads to a slightly heavier computation burden in the proposed method than in zlog(z)-based method or HOME method. However, the time efficiency of the proposed estimator is sufficiently acceptable because the run time in average is in the same order of magnitude for the proposed estimator and zlog(z)-based or HOME estimator, as Mellin-based estimators have been widely used in radar data processing [33].

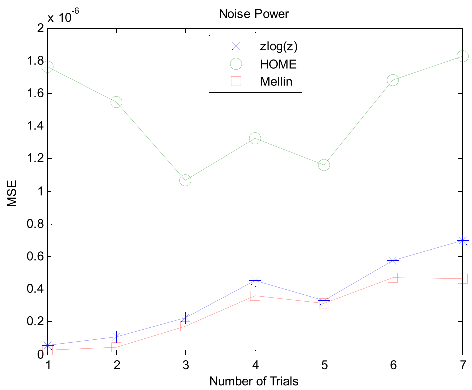
To summarize, the foregoing results indicate that a better estimation accuracy and a slightly expensive computation cost in the proposed estimator than in the zlog(z)-based or HOME estimator.



(a)

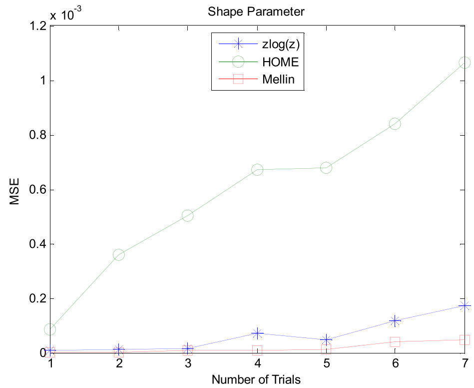


(b)

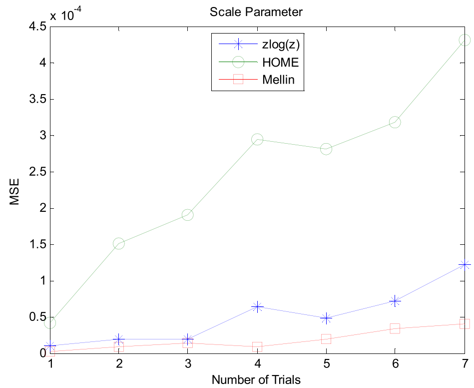


(c)

FIGURE 5. Comparison of MSE estimates of shape parameter, scale parameter and noise power of a KpN model for the HOME, the zlog(z)-based closed form, and the proposed estimation methods for CNR = 10dB, the experiment parameters are same as Table 5.



(a)



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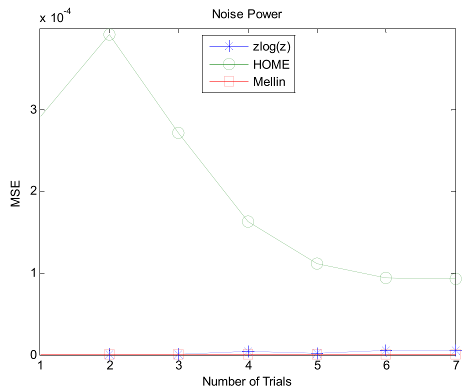


(c)

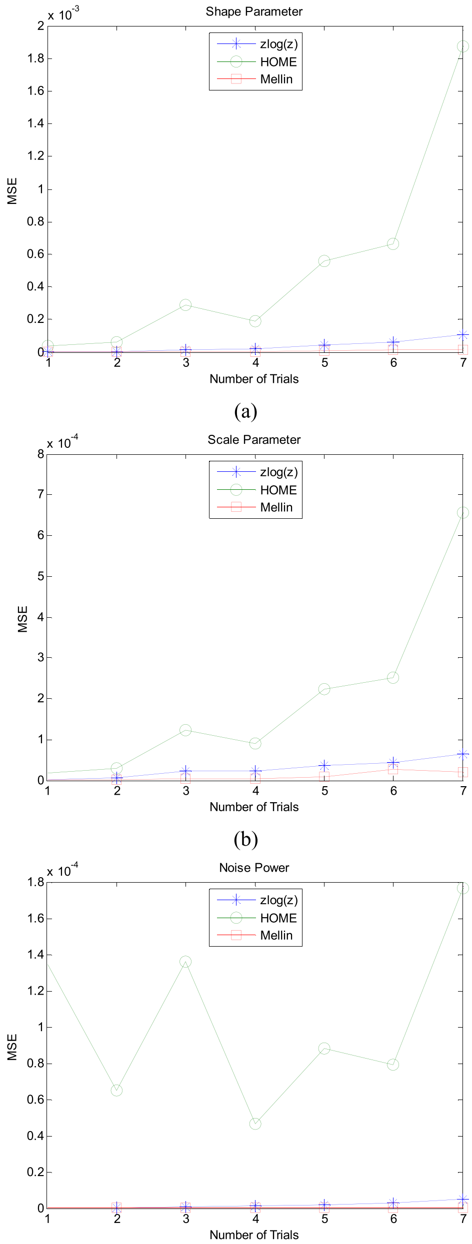
FIGURE 6. Comparison of MSE estimates of shape parameter, scale parameter and noise power of a KpN model for the HOME, the zlog(z)-based closed form, and the proposed estimation methods for CNR = 10dB, the experiment parameters are same as Table 5 except for M = 300.

In order to make a further effort to confirm our proposed method, we make detailed researches on the MSE of the three methods in the different situations to pay more attention to check the estimation accuracy of different methods. This is because the results (not provided in order to save space) of time efficiency of different methods are similar with the foregoing results.

Figs. 3-5 display the MSE of the three methods in different CNRs (−3, 0, 10dB) and Figs. 6-7 display the MSE in the small sample sizes ($M = 300, 600$) compared with the previous experiments in $M = 10000$. Fig. 8 displays the MSE in the small number of noncoherent integrated pulse ($N = 2$) compared with the previous experiments in $N = 10$ integrated pulses. The detailed settings of the parameters are contained in the Tables 3-5.

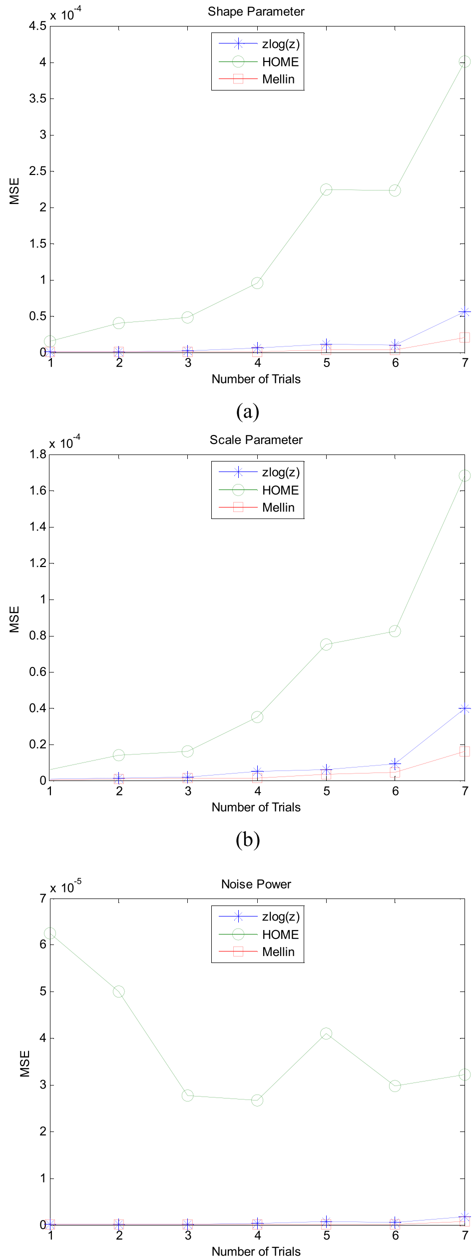
The parameters of Figs. 6-8 are almost identical to Table 5, the differences are the sample sizes $M = 300$ in Fig. 6, $M = 600$ in Fig. 7, and integrated pulses $N = 2$ in Fig. 8.

Observing the Figs. 3-5, the higher CNR is, the higher estimation accuracy of three parameters for three methods is.



(c)

FIGURE 7. Comparison of MSE estimates of shape parameter, scale parameter and noise power of a KpN model for the HOME, the zlog(z)-based closed form, and the proposed estimation methods for CNR = 10dB, the experiment parameters are same as Table 5 except for M = 600.



(c)

FIGURE 8. Comparison of MSE estimates of shape parameter, scale parameter and noise power of a KpN model for the HOME, the zlog(z)-based closed form, and the proposed estimation methods for CNR = 10dB, the experiment parameters are same as Table 5 except for N = 2.

And, the MSE curves of three parameters of proposed Method and zlog(z)-based closed form method are lower than HOME method all the time regardless of the high CNR or low CNR. The performance of our proposed method and zlog(z)-based closed form method is comparative at CNR $=$ 10dB, but in the situations of low CNR, our proposed method has the more remarkable performance than zlog(z)-based closed form method.

Observing the Figs. 6-7, we can easily find that the small sample sizes have a significant impact on the estimation accuracy of the HOME method rather than the others and our proposed method has the smallest MSE in the three methods.

Comparing the Fig. 5 with Fig. 8, we find that the integrated pulse count also has an influence on performance of the three methods. Obviously, the HOME method is at a disadvantage with the others, and our proposed method has a slight preponderance on performance than the zlog(z)-based closed form method.

TABLE 4. Parameters corresponding to Fig.4 (CNR = 0dB).

Shape Parameter (v)	0.2	0.4	0.6	0.8	1	1.2	1.4
Scale Parameter (b)	0.4	0.8	1.2	1.6	2	2.4	2.8
Noise Power (p_n)	0.5						
Sample Size (M)	10000						
Independent Trials (n)	1000						
Integrated Pulses (N)	10						

TABLE 5. Parameters corresponding to Fig.5 (CNR = 10*dB*).

Shape Parameter (v)	0.2	0.4	0.6	0.8	1	1.2	1.4
Scale Parameter (b)	0.22	0.44	0.66	0.88	1.1	1.32	1.54
Noise Power (p_n)	0.0909						
Sample Size (M)	10000						
Independent Trials (n)	1000						
Integrated Pulses (N)	10						

V. CONCLUSION

A novel K-clutter plus thermal noise parameter estimation method based on Mellin transform has been proposed in this paper. At first, we introduce the K distribution and give the generalized expression in the presence of added thermal noise. Next, a useful tool called Mellin transform has been suggested and the novel estimator based on Mellin transform has been proposed by the mathematical derivation. Consequently, the simulation experiments have shown that the new estimator has comparative estimation accuracy and lower STD and MSE than the HOME method and zlog(z)-based closed form method.

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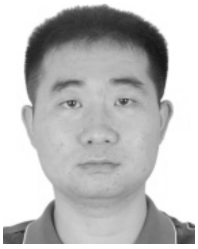

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