IEEE Access
Multidisciplinary ⋮ Rapid Review ⋮ Open Access Journal

Received January 31, 2019, accepted February 15, 2019, date of publication February 28, 2019, date of current version March 12, 2019.

Digital Object Identifier 10.1109/ACCESS.2019.2900225

Joint Spectrum and Power Allocation for NOMA Enhanced Relaying Networks

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This work was supported in part by the National Natural Science Foundation of China under Grant 61871466 and Grant 61841603, in part by the Guangxi Natural Science Foundation Innovation Research Team Project under Grant 2016GXNSFGA380002, in part by the Projects of Education Department of Guangxi under Grant 2018KY0472, and in part by the Guangxi Natural Science Foundation under Grant 2018JJA170050.

ABSTRACT In this paper, we investigate the joint resource allocation for the non-orthogonal multiple access (NOMA)-enhanced relaying networks involving the subcarrier pair, subcarrier-user assignment, as well as power allocation. In the NOMA-enhanced relaying networks, the relay is capable of communicating with multiple users on one subcarrier using the NOMA technology. To maximize the system throughput, the joint resource allocation problem is formulated as a mixed-integer nonlinear programming problem, which is difficult to tackle in general. Furthermore, there is strong coupling between the subcarrier-user assignment and the power allocation due to the multi-user interference in the NOMA system. To reduce the complexity, we separate the joint resource allocation problem as two subproblems in terms of the subcarrier assignment (subcarrier pair and subcarrier-user assignment) and power allocation, respectively. In particular, we propose a subcarrier assignment scheme based on the simulated annealing algorithm to optimize the subcarrier pair and subcarrier-user assignment with the fixed power allocation. Then, the power allocation problem is transformed as a difference of convex functions programming problem, and the sequence convex programming method is adopted to solve it. The simulation results illustrate that the proposed algorithm can effectively improve the system throughput.

INDEX TERMS NOMA, spectrum allocation, power allocation, relaying system.

I. INTRODUCTION

With the wide application of mobile networks, the number of mobile destinations and the data needed to be processed increase explosively. Meanwhile, mobile internet services have been an unprecedented development. Therefore, the wireless networks face huge challenges involving the network capability, transmission rate, networks delay, and so on. Cooperation relaying is one of the key technologies of LTE-Advanced, which can efficiently improve the network coverage with the assistance of relay. In addition, relaying networks can also enhance the transmission rate since multi-relay system can form a virtual MIMO networks to exploit the diversity.

Reference [1] investigates the resource allocation of the relay-based device-to-device (D2D) networks. In [1], the D2D transmitter can either communicate to the receiver directly or by the relay mode, and the joint optimization problem involving transmission mode selection, subcarrier assignment and power allocation is formulated. Specifically, in the problem solving, the transmission mode selection and resource assignment are formulated as a job assignment problem which can be solved easily. Zainaldin *et al.* [2] study the joint resource allocation and relay selection for bidirectional LTE-Advanced relay networks. In [2], the transmission mode selection is considered, and a three time slot scheme for time division duplexing (TDD) is proposed for LTE-Advanced frame architecture to accommodate a hybrid transmission scheme. Then, a combinatorial optimization problem involving the resource assignment, relay selection, and bidirectional transmission scheme is formulated. In [3], the asymmetric resource allocation is investigated both for decode-and-forward (DF) and amplify-and-forward (AF) relaying system. In the asymmetric resource allocation, the transmission durations between BS to relay and relay to users are not needed to be equal, which improve the degree

The associate editor coordinating the review of this manuscript and approving it for publication was Qilian Liang.

of freedom for transmission. Furthermore, Zhou *et al.* [3] propose an optimal cross-layer based asymmetric resource allocation taking into account the maximum delay threshold, minimum data rate requirement and zero overflow. The energy efficiency of relaying networks is considered by [4]. In [4], the joint resource allocation problem with QoS requirement is formulated as a nonconvex binary mixed-integer nonlinear programming. Then, a concave lower bound on the pricing-based network utility is applied to transform the problem into a convex problem. Reference [5] investigates the resource allocation problem for the full-duplex relay-assisted D2D systems, and proposes a linearly relaxed allocation algorithm to optimize the problem.

Non-orthogonal multiple access (NOMA) is a promising technology to improve the spectrum efficiency [6]–[10]. Due to the superposition coding and successive interference cancellation (SIC), NOMA allows multiple users to share the same spectrum resource, which can significantly improve the spectrum efficiency. In NOMA system, the user with better channel condition will generate co-channel interference to the user with worse channel condition. Therefore, in general, the resource allocation problem of the NOMA system is formulated as a nonconvex optimization problem. In addition, there is strong coupling between the spectrum allocation and power allocation for NOMA system, which makes the optimization problem difficult to solve.

Reference [11] investigates the energy efficient resource allocation for the NOMA system with imperfect CSI. In [11], the energy efficient resource allocation problem is formulated as a probabilistic mixed non-convex optimization problem. Then, Fang *et al.* [11] propose a novel low-complexity suboptimal user scheduling algorithm to solve the problem. Reference [12] investigates the joint resource allocation problem for NOMA enhanced heterogeneous networks (HetNets), and a separate method is considered. Specifically, in [12], the spectrum allocation is modeled as a many-to-one matching game with peer effects, and a low-complexity algorithm based on the swap operations is proposed to enable small base stations and resource blocks to effectively interact with each other. When the spectrum allocation is fixed, the power allocation is still hard to solve because of the nonconvexity. Zhao *et al.* [12] adopt the sequential convex programming to solve the power allocation. Besides the spectrum and power allocation, [13] also considers the rate allocation and SIC decoding policy, and the resource allocation is formulated as a nonconvex optimization problem. Then, the authors recast the nonconvex optimization problem as a linear multiplicative programming problem which is optimized by the branch-and-bound approach. The power allocation problem for NOMA system is considered by [14]. First, a two-user scenario is considered, and two closed-form sub-optimal solutions are derived for the two-user scenario. Then, the sub-optimal solution is extended to the multi-user case by proposing a user-pairing approach as well as a number of power allocation techniques. Energy cooperation is a promising technology for energy harvesting networks in which one system's excessive energy can be shared with the other ones to improve renewable energy efficiency. Reference [15] studies the resource allocation in energy cooperation enabled two-tier HetNets with NOMA technology. In [15], a joint optimization problem is formulated for optimizing the user association, power allocation, transferred energy among BSs as well as grid energy consumption.

As aforementioned, the relay and NOMA system can bring significant benefits to wireless networks. Accordingly, it is reasonable to investigate the promising application of NOMA in relaying system. Reference [16] investigates the performance improvement method for the cell-edge users of NOMA systems in downlink scenario. In [16], a cell-center user is adopted as a relay between the BS and edge-users to improve the performance of edge-users. In addition, two cooperative relaying schemes (full duplex relaying or half duplex relaying) are proposed, and the performance of two schemes are investigated in terms of outage probability and sum throughput. Reference [17] investigates the power allocation schemes in the NOMA-enabled cooperation relaying system to maximize the energy efficiency. To reduce the complexity, Liu *et al.* [17] decompose the original problem into a bilayer optimization problem. Yang *et al.* [18] propose a collaborative NOMA assisted relaying (CNAR) system, in which the relay node of the CNAR decodes the message for itself from source-relay NOMA signal and transmits the remaining messages to the multiple cell-edge users in relay-destination link. Then, [18] develops a simplified CNRA system to reduce the complexity of the relay. Finally, [18] optimizes the power allocation among NOMA users to minimize the outage probability. Reference [19] studies the resource allocation involving the antenna selection and user scheduling for NOMA-enabled massive MIMO and relaying systems. To decrease the complexity, a novel low-complexity suboptimal user scheduling algorithm is proposed in [19] to maximize the system energy efficiency.

In this paper, we investigate the resource allocation problem in NOMA enhanced relaying networks involving the subcarrier assignment (subcarrier pair and subcarrier-user assignment) as well as power allocation. The resource allocation problem is formulated as a mixed integer nonlinear programming problem. As aforementioned, it is difficult to solve the resource allocation problem due to the strong coupling between the subcarrier assignment and power allocation in NOMA system. To reduce the computational complexity, we first consider the subcarrier assignment with fixed power allocation, and a heuristics method based on simulated annealing algorithm is proposed to solve the subcarrier-user assignment problem. Then, there is only power allocation which is still difficult due to the co-channel interference. The power allocation problem can be transformed into a difference of convex functions (DC) programming problem, and the sequence convex programming method is adopted to solve it.

The contributions of this paper is summarized as follows:

1) In this paper, we consider the resource allocation problem involving the joint subcarrier pair, subcarrier-user assignment as well as power allocation for NOMA enhanced relaying networks, and formulate the problem as a mixed-integer nonlinear programming problem with the aim of maximizing the system overall throughput.
2) The formulated optimization problem is difficult to optimize due to the strong coupling between the subcarrier assignment and power allocation of the NOMA system. To reduce the complexity, we separate the joint resource allocation problem as two subproblems in terms of the subcarrier assignment and power allocation respectively. Then, a heuristics method based on simulated annealing (SA) algorithm is proposed to optimize the subcarrier assignment for NOMA system with fixed power allocation.
3) The power allocation problem is modeled as a nonconvex optimization problem which is difficult to tackle. To solve it, we transform the power allocation problem into a DC programming problem. Then, we approximate the original nonconvex optimization problem as a series of convex problems using first order Taylor expansion. The DC programming algorithm based on Lagrangian dual method is invoked to optimize the approximation problems.

The rest of the paper is organized as follows. Section II introduces the system model of NOMA enhanced relaying networks, and formulates the resource allocation problem as a mixed integer nonlinear programming problem. In section III, a heuristics method based on SA is proposed to optimize the subcarrier assignment problem. The power allocation problem is solved by the DC programming algorithm in IV. In section V, simulation results are provided to evaluate the performance of the proposed algorithm. Finally, we conclude this paper in section VI.

II. SYSTEM MODEL AND PROBLEM FORMULATION

A. SYSTEM MODEL

In this paper, we consider a two-hop cooperation DF relaying system consisting of one BS, one relay as well as K users. The transmission period is divided into two equal time slots, and the half-duplex relay is adopted. Accordingly, the transmission from the BS to users can be regarded as two independent processes. Meanwhile, it is assumed that there are N subcarriers occupied in both the transmission periods of the relaying network. In the considered scenario, the NOMA technology is adopted to improve the spectrum efficiency. That is, each subcarrier can be assigned to multiple users simultaneously from the relay. For the sake of simplification, we assume that each subcarrier can only be assigned to at most two users using NOMA technology. In the first time slot, the BS transmits the superposition signals of two users to the relay. Then, in the second time slot, the relay retransmits the signals to users using NOMA technology. In addition, it is assumed that both the BS and relay have the full knowledge of the channel state information (CSI).

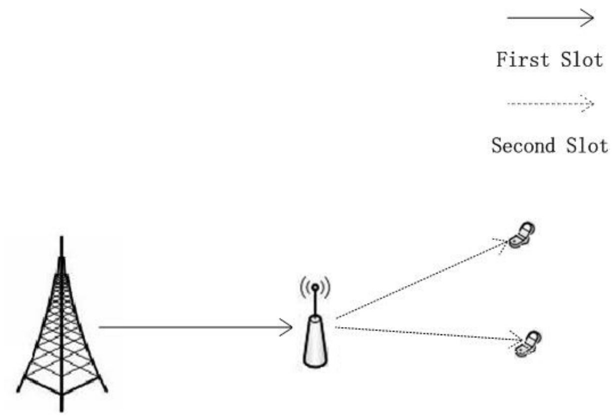

FIGURE 1. Illustration of NOMA-enhanced relaying networks.

B. CHANNEL MODEL

In the considered NOMA enhanced relaying system, it is assumed that the BS transmits the mixed signal of the users sharing the same spectrum to the relay on subcarrier m ($m = 1, 2, \cdots N$) in the first time slot. Then, the relay forwards the received mixed signal to the users using the subcarrier n ($n = 1, 2, \cdots, N$) in the second time slot. It is called that the subcarrier m is paired with subcarrier n. In order to express convenience, we first give the channel model of the second time slot.

In the second time slot, the transmitted signal from the relay on subcarrier n is given as:

$$s_{r,n} = \sum_{k=1}^{K} x_{n,k} \sqrt{p_{r,n,k}} a_k \tag{1}$$

where $x_{n,k}$ is the subcarrier-user assignment indicator variable. $x_{n,k} = 1$, if the subcarrier n is assigned to user k. $p_{r,n,k}$ is the transmit power from the relay to user k on subcarrier n. a_k represents the symbol of the user k.

Then, the received signal of the user k can be given as:

$$y_k = x_{n,k} h_{r,n,k} \sqrt{p_{r,n,k}} a_k + \sum_{j=1, j \neq k}^{K} x_{n,j} h_{r,n,k} \sqrt{p_{r,n,j}} a_j + n_0 \tag{2}$$

where $h_{r,n,k}$ is the channel gain from the relay to user k. n_0 represents the additive white Gaussian noise with variance σ^2.

In the first time slot, the BS transmits the mixed signal of the users sharing subcarrier n to the relay on subcarrier m. The received signal at the relay side can be given as:

$$y_r = \sum_{k=1}^{K} x_{n,k} h_{b,r,m} \sqrt{p_{b,r,m}} a_k + n_0 \tag{3}$$

where $h_{b,r,m}$ is the channel gain from BS to the relay on subcarrier m. $p_{b,r,m}$ means the transmit power from BS to the relay on subcarrier m.

In NOMA system, each subcarrier is adopted to transmit data to multiple users simultaneously. The SIC is applied at the users side to remove the interference from the superposition signals. The mechanism of SIC is that the user with better channel condition can decode the data of the user with weaker channel condition and then proceeds to subtract it from the received signal and decode its own data. The user with weaker channel condition treats the signal of the user with the better channel as noise and decodes its own data from the received signal.

Without loss of generality, we assume that $h_{r,n,1}$, $h_{r,n,2}$, $\cdots$, $h_{r,n,K}$ are in the descending order. In accordance with the NOMA principle, the user with weaker channel condition treats the signal of the users with better channel condition as noise. Therefore, the SINR of the user k can be given by:

$$\gamma_{r,k}^{n} = \frac{|h_{r,n,k}|^2 p_{r,n,k}}{|h_{r,n,k}|^2 \sum_{l=1}^{k-1} x_{n,l} p_{r,n,l} + \sigma^2} \tag{4}$$

C. PROBLEM FORMULATION

In the NOMA cooperation relaying system, the BS transmits the signal to users with the assist of the relay using NOMA technology. We denote a subcarrier pair (m, n). It means that the relay receives the signal from the BS on subcarrier m, and retransmits the signal to the users on subcarrier n. In the following, we analyze the transmission rate on the subcarrier pair (m, n).

As mentioned above, the BS transmits the superposition signal of users on subcarrier m in the first time slot. The SINR of subcarrier m can be expressed as:

$$\gamma_{b,r}^{m} = \frac{|h_{b,r,m}|^2 p_{b,r,m}}{\sigma^2} \tag{5}$$

Then, the achievable rate of the subcarrier m in first time slot can be expressed as:

$$R_{b,r} = \frac{1}{2}\log(1 + \gamma_{b,r}^{m}) \tag{6}$$

where the factor $\frac{1}{2}$ is due to the use of the two time slots frame architecture.

In the second time slot, the relay retransmits the signal to the users on subcarrier n. The achievable rate of the subcarrier n in the second time slot is given as:

$$R_{r,n} = \frac{1}{2}\sum_{k=1}^{K} x_{n,k}\log(1 + \gamma_{r,k}^{n}) \tag{7}$$

Then, the achievable rate of the subcarrier pair (m, n) can be given as:

$$R_{m,n} = \frac{1}{2}u_{m,n}\min\{\log(1 + \gamma_{b,r}^{m}), \sum_{k=1}^{K} x_{n,k}\log(1 + \gamma_{r,k}^{n})\} \tag{8}$$

where $u_{m,n}$ means the subcarrier pair indicator variable. $u_{m,n} = 1$, if the incoming subcarrier m and the outgoing subcarrier n of the relay make a subcarrier pair;

The overall system throughput of the NOMA relaying system can be obtained:

$$R = \frac{1}{2}\sum_{m=1}^{N}\sum_{n=1}^{N} u_{m,n}\min\{\log(1 + \gamma_{b,r}^{m}), \sum_{k=1}^{K} x_{n,k}\log(1 + \gamma_{r,k}^{n})\} \tag{9}$$

In this paper, our objective is to optimize the subcarrier pair, subcarrier-user assignment and power allocation to maximize the overall throughput of the NOMA relaying network. The optimization problem can be formulated as:

$$\begin{aligned}
\mathbf{P1}\ \max_{\mathbf{u},\mathbf{x},\mathbf{p}}\ & \frac{1}{2}\sum_{m=1}^{N}\sum_{n=1}^{N} u_{m,n}\min\{\log(1+\gamma_{b,r}^{m}), \\
& \sum_{k=1}^{K} x_{n,k}\log(1+\gamma_{r,k}^{n})\} && (10)\\
s.t.\ & \sum_{m=1}^{N} p_{b,r,m} \le P_b && (11)\\
& \sum_{n=1}^{N} (p_{r,n,k} + p_{r,n,j}) \le P_r && (12)\\
& \sum_{m=1}^{N} u_{m,n} = 1, \quad \forall n && (13)\\
& \sum_{n=1}^{N} u_{m,n} = 1, \quad \forall m && (14)\\
& \sum_{k=1}^{K} x_{n,k} \le 2, \quad \forall n && (15)\\
& u_{m,n} \in \{0, 1\}, \quad \forall m, \forall n && (16)\\
& x_{n,k} \in \{0, 1\}, \quad \forall n, \forall k && (17)\\
& \mathbf{p} \succeq 0 && (18)
\end{aligned}$$

where $\mathbf{u} = [u_{m,n}]$, $\mathbf{x} = [x_{r,n,k}]$ and $\mathbf{p} = [p_{b,r,m}, p_{r,n,k}]$. Constraints (11) and (12) mean the peak power constraints of the BS and relay, respectively. Constraints (13) and (14) are imposed to guarantee that each subcarrier can be paired with at most one subcarrier. Constraint (15) guarantees that each of the N subcarriers can be assigned to at most two users. Constraint (18) is the non-negative constraint of the transmit power.

The formulated optimization problem **P1** is a mixed integer nonlinear programming problem because of the combinatorial nature of the subcarrier pair and subcarrier-user assignment. Comparing with the resource allocation problem of OFDMA system, the proposed optimization problem is more difficult to tackle due to the strong coupling between the subcarrier assignment and power allocation of the NOMA system. Indeed, different power allocation will cause different interference from the user with better channel condition to the user with weaker channel condition, which will influence the subcarrier assignment and vice versa. Therefore, it is hard to find an effective algorithm to obtain the optimal solution of **P1**. To reduce the computation complexity, we separate the problem **P1** as two subproblems in terms of the subcarrier assignment and power allocation, respectively.

III. CHANNEL ASSIGNMENT

A. SUBCARRIER PAIR

In the resource allocation problem of the considered NOMA relaying system, the subcarrier assignment problem includes the subcarrier pair and subcarrier-user assignment. In this subsection, we address the subcarrier pair with fixed subcarrier-user assignment and power allocation.

The subcarrier pair problem with fixed subcarrier-user assignment and power allocation can be formulated as the following combinatorial optimization problem:

$$\max \sum_{m=1}^{N}\sum_{n=1}^{N} u_{m,n}\bar{R}_{m,n} \quad s.t.\ (13),\ (14) \tag{19}$$

where $\bar{R}_{m,n}$ is the achievable rate of the subcarrier pair (m, n) with fixed subcarrier-user assignment and power allocation.

Indeed, problem (19) is known as the assignment problem, which can be optimized by the Hungarian method.

B. SUBCARRIER-USER ASSIGNMENT

As mentioned above, in the considered scenario, each subcarrier is assigned to at most two users using NOMA. In this subsection, we propose a heuristic method based on SA to optimize the subcarrier-user assignment in the second time slot.

1) INITIALIZATION ALGORITHM

In the initialization algorithm, each user and subcarrier will produce their preference lists according to the channel information, respectively. In the matching processing, each user declares its most preferred subcarrier on the basis of its preference list. Then, each subcarrier make a decision to accept the most preferred subcarrier in accordance with the preference list. If the number of the users matched to a certain subcarrier reaches to two, or a certain user has been matched to one subcarrier, the subcarrier and the user will exit the matching processing. Aster a round, all the preference lists of the users and subcarriers will be updated.

2) SIMULATED ANNEALING BASED SUBCARRIER-USER ASSIGNMENT

SA is a classic randomization algorithm and easy to implement, which has excellent performance to tackle the combinatorial optimization problem. In this paper, we adopt the SA to optimize the subcarrier-user assignment with fixed power allocation.

a: ENEGY FUNCTION

In SA, the objective function is also known as energy function. In this paper, the energy function is set as the function of subcarrier-user assignment $\mathbf{x}$.

$$E(\mathbf{x}) = \frac{1}{2}\sum_{m=1}^{N}\sum_{n=1}^{N} \bar{u}_{m,n}\min\{\log(1+\bar{\gamma}_{b,r}^{m}), \sum_{k=1}^{K} x_{n,k}\log(1+\bar{\gamma}_{r,k}^{n})\} \tag{20}$$

b: INITIALIZATION OF THE SOLUTION AND TEMPERATURE

In the proposed mechanism, the subcarrier-user assignment solution obtained by the initialization algorithm is adopted as the initial solution of SA, denoted by $\mathbf{x^0}$. The initial temperature is assumed to be T.

c: UPDATING OF THE SOLUTION AND TEMPERATURE

In the ith iteration, the swap operation is applied as the solution updated scheme. For the swap operation, two certain users exchange their matched subcarrier to generate a new subcarrier assignment solution as the candidate solution, denoted by $\mathbf{x}'$. If the utility of the new solution $\mathbf{x}'$ is better than the old one $\mathbf{x}^i$, $\mathbf{x}'$ will be accepted as the next current solution. Otherwise, Metropolis criterion is performed, i.e., SA will accept the new one with the probability $e^{-\frac{\triangle E}{T}}$, where $\triangle E = E(\mathbf{x}') - E(\mathbf{x}^i)$. The temperature T is updated by: $T(i+1) = t \cdot T(i)$, where t is a constant in the range of (0,1).

IV. POWER ALLOCATION

A. POWER ALLOCATION USING DC PROGRAMMING

In the previous section, the subcarrier assignment with fixed power allocation has been optimized by the subcarrier pair method and SA based subcarrier-user assignment. There is only the power allocation variables in the resource allocation problem when the subcarrier assignment is fixed. Assuming the subcarrier n is assigned to users k and j, and the channel condition of user j is better than that of user k, the optimization problem **P1** can be rewritten as:

$$\mathbf{P2} \quad \max_{\mathbf{p}} \frac{1}{2}\sum_{m=1}^{N}\sum_{n=1}^{N}\min\{\log(1+\gamma_{b,r}^{m}), \log(1+\gamma_{r,j}^{n}) + \log(1+\gamma_{r,k}^{n})\} \tag{21}$$
$$s.t.\ (11),\ (12),\ (18)$$

where $\gamma_{r,j}^{n} = \frac{|h_{r,n,j}|^2 p_{r,n,j}}{\sigma^2}$, and $\gamma_{r,k}^{n} = \frac{|h_{r,n,k}|^2 p_{r,n,k}}{|h_{r,n,k}|^2 p_{r,n,j}+\sigma^2}$.

Since (21) is a minimization function of two functions, it is obvious that only when the capacity of two hops are equivalent could the achievable capacity be maximized.

Thus, the following relationship can be obtained:

$$\log(1+\frac{|h_{b,r,m}|^2 p_{b,r,m}}{\sigma^2}) = \log(1+\frac{|h_{r,k,n}|^2 p_{r,n,j}}{\sigma^2}) + \log(1+\frac{|h_{r,n,k}|^2 p_{r,n,k}}{|h_{r,n,k}|^2 p_{r,n,j}+\sigma^2}) \tag{22}$$

We can obtain the relationship of $p_{b,r,m}$, $p_{r,n,k}$ and $p_{r,n,j}$:

$$|h_{b,r,m}|^2 p_{b,r,m} = |h_{r,n,k}|^2 p_{r,n,k} + |h_{r,n,j}|^2 p_{r,n,j} + \frac{|h_{r,n,k}|^2 p_{r,n,k}}{\sigma^2 + p_{r,n,j}} \tag{23}$$

It can be observed from (23) that the relationship among $p_{b,r,m}$, $p_{r,n,k}$ and $p_{r,n,j}$ is nonlinear, which makes the original optimization problem extremely difficult to be solved. Considering the transmit power from the relay to user k and user j on subcarrier n have the same order, we make the following assumption to simplify the problem:

$$\frac{|h_{r,n,k}|^2 p_{r,n,k}}{\sigma^2 + p_{r,n,j}} \approx c \tag{24}$$

where c is a constant.

Then, the transmit power $p_{b,r,m}$ can be represented by $p_{r,n,k}$ and $p_{r,n,j}$:

$$p_{b,r,m} \approx \frac{|h_{r,n,k}|^2 p_{r,n,k}}{|h_{b,r,m}|^2} + \frac{|h_{r,n,j}|^2 p_{r,n,j}}{|h_{b,r,m}|^2} + \frac{c}{|h_{b,r,m}|^2} \tag{25}$$

Therefore, problem **P2** can be simplified as the function of $p_{r,n,k}$ and $p_{r,n,j}$:

$$\mathbf{P3} \quad \max \frac{1}{2}\sum_{n=1}^{N}\{\log(1+\frac{|h_{r,n,j}|^2 p_{r,n,j}}{\sigma^2}) + \log(1+\frac{|h_{r,n,k}|^2 p_{r,n,k}}{|h_{r,n,k}|^2 p_{r,n,j}+\sigma^2})\} \tag{26}$$

$$s.t. \sum_{m=1}^{N}(\frac{|h_{r,n,k}|^2 p_{r,n,k}}{|h_{b,r,m}|^2} + \frac{|h_{r,n,j}|^2 p_{r,n,j}}{|h_{b,r,m}|^2} + \frac{c}{|h_{b,r,m}|^2}) \le P_b \tag{27}$$

$$\sum_{n=1}^{N}(p_{r,n,k}+p_{r,n,j}) \le P_r \tag{28}$$

$$p_{r,n,k} \ge 0, \quad p_{r,n,j} \ge 0, \ \forall n, k, j \tag{29}$$

It is worth nothing that problem **P3** is not a convex optimization problem since the later term of (26) are not concave, which makes it still difficult to be sloved. In this section, we adopt the sequential convex programming approach in [20] to solve problem **P3**. Indeed, the nonconvexity of the objective function is owing to the nonconvexity of the rate function of user k on subcarrier n. The achievable rate of user k in the second time slot can be transformed as a difference of the convex functions in accordance with the property of logarithm function as follows:

$$\mathrm{R}_{r,k}^{n} = \log(|h_{r,n,k}|^2 p_{r,n,j} + |h_{r,n,k}|^2 p_{r,n,k} + \sigma^2) - \log(|h_{r,n,k}|^2 p_{r,n,j} + \sigma^2) \tag{30}$$

Let $H(p_{r,n,j}) = \log(|h_{r,n,k}|^2 p_{r,n,j} + \sigma^2)$. Convex function $H(p_{r,n,j})$ can be approximated by a linear function $H(p_{r,n,j}, p_{r,n,j}^{k})$ using first order Taylor expansion:

$$H(p_{r,n,j}, p_{r,n,j}^{i}) = H(p_{r,n,j}^{i}) + \nabla H(p_{r,n,j}^{i})(p_{r,n,j} - p_{r,n,j}^{i}) \tag{31}$$

where $p_{r,n,j}^{i}$ is the solution of $p_{r,n,j}$ from the ith iteration. $\nabla H(p_{r,n,j}^{i})$ is the gradient at the point $p_{r,n,j}^{i}$, and $\nabla H(p_{r,n,j}^{i}) = \frac{|h_{r,n,k}|^2}{|h_{r,n,k}|^2 p_{r,n,j}^{i}+\sigma^2}$.

Then, (30) can be rewritten as:

$$\mathrm{R}_{r,k}^{n} \approx \log(|h_{r,n,k}|^2 p_{r,n,j} + |h_{r,n,k}|^2 p_{r,n,k} + \sigma^2) - H(p_{r,n,j}, p_{r,n,j}^{i}) \tag{32}$$

Obviously, (32) is a concave function.

Substitute (32) into (26), the problem **P3** can be reformulated as follows:

$$\mathbf{P4} \quad \max \frac{1}{2}\sum_{n=1}^{N}\{\log(1+\frac{|h_{r,n,j}|^2 p_{r,n,j}}{\sigma^2}) + \log(|h_{r,n,k}|^2 p_{r,n,j} + |h_{r,n,k}|^2 p_{r,n,k} + \sigma^2) - H(p_{r,n,j}, p_{r,n,j}^{i})\} \tag{33}$$

$$s.t. \ (27), (28), (29)$$

It can be seen that problem **P4** is a convex optimization problem after the linear approximation of $H(p_{r,n,j})$. Next, the Lagrangian dual method is adopted to optimize **P4**.

The Lagrangian function of **P4** can be written as:

$$\begin{aligned} &L(p_{r,n,k}, p_{r,n,j}, \lambda, \mu) \\ &= \frac{1}{2}\sum_{n=1}^{N}\{\log(1+\frac{|h_{r,n,j}|^2 p_{r,n,j}}{\sigma^2}) \\ &\quad + \log(|h_{r,k,n}|^2 p_{r,n,j} + |h_{r,k,n}|^2 p_{r,n,k} + \sigma^2) \\ &\quad - H(p_{r,n,j}^{i}) - \nabla H(p_{r,n,j}^{i})(p_{r,n,j} - p_{r,n,j}^{k})\} \\ &\quad + \lambda[\sum_{m=1}^{N}(\frac{|h_{r,n,k}|^2 p_{r,n,k}}{|h_{b,r,m}|^2} + \frac{|h_{r,n,j}|^2 p_{r,n,j}}{|h_{b,r,m}|^2} \\ &\quad + \frac{c}{|h_{b,r,m}|^2}) - P_b] \\ &\quad + \mu[\sum_{n=1}^{N}(p_{r,n,k}+p_{r,n,j}) - P_r] \end{aligned} \tag{34}$$

where λ and μ are the Lagrangian multipliers with respect to constraint (27) and (28), respectively.

Then, the dual function can be obtained:

$$g(\lambda, \mu) = \max_{p_{r,n,k}, p_{r,n,j}} L(p_{r,n,k}, p_{r,n,j}, \lambda, \mu) \tag{35}$$

Setting the partial derivation of (34) with respect to $p_{r,n,j}$ and $p_{r,n,k}$ to zero, we can obtain:

$$\begin{aligned}\frac{\partial L}{\partial p_{r,n,j}} &= \frac{|h_{r,n,j}|^2}{2(1+\frac{|h_{r,n,j}|^2 p_{r,n,j}}{\sigma^2})} \\ &+\frac{|h_{r,n,k}|^2}{2(|h_{r,n,k}|^2 p_{r,n,k}+|h_{r,n,k}|^2 p_{r,n,j}+\sigma^2)} \\ &-\nabla H(p^i_{r,n,j})+\lambda(\frac{|h_{r,n,j}|^2}{|h_{b,r,m}|^2})+\mu=0 \quad (36)\\ \frac{\partial L}{\partial p_{r,n,k}} &= \frac{|h_{r,n,k}|^2}{2(|h_{r,n,k}|^2 p_{r,n,k}+|h_{r,n,k}|^2 p_{r,n,j}+\sigma^2)} \\ &+\lambda(\frac{|h_{r,n,k}|^2}{|h_{b,r,m}|^2})+\mu=0 \quad (37)\end{aligned}$$

The optimal power allocation of **P4** can be obtained using KKT conditions:

$$\hat{p}_{r,n,j} = [\frac{\sigma^2}{2(\nabla H(p^i_{r,n,j})+\lambda(\frac{|h_{r,n,k}|^2}{|h_{b,r,m}|^2}-\frac{|h_{r,n,j}|^2}{|h_{b,r,m}|^2}))}-\frac{\sigma^2}{|h_{r,n,j}|^2}]^+ \quad (38)$$

$$\hat{p}_{r,n,k} = [\frac{1}{2(-\lambda\frac{|h_{r,n,k}|^2}{|h_{b,r,m}|^2}-\mu)}-\frac{\sigma^2}{|h_{r,n,k}|^2}-p_{r,n,j}]^+ \quad (39)$$

where $[a]^+ = \max\{0, a\}$.

The iterative formula of $p_{r,n,j}$ of the original problem is:

$$p^{i+1}_{r,n,j} = p^i_{r,n,j}+\theta(\hat{p}_{r,n,j}-p^i_{r,n,j}) \quad (40)$$

where θ is the iteration stepsize.

B. OPTIMIZING THE DUAL PROBLEM

In this subsection, we solve the dual problem with respect to problem **P4**.

$$\begin{aligned}&\min_{\lambda,\mu} \quad g(\lambda,\mu) &(41)\\ &s.t.\ \lambda \geq 0; \quad \mu \geq 0 &(42)\end{aligned}$$

The dual problem is a convex optimization problem. Consequently, the sub-gradient method is adopted to optimize it. The sub-gradient at point λ and μ can be given as:

$$\Delta\lambda = P_b - \sum_{m=1}^{N}(\frac{|h_{r,n,k}|^2\hat{p}_{r,n,k}}{|h_{b,r,m}|^2}+\frac{|h_{r,n,j}|^2\hat{p}_{r,n,j}}{|h_{b,r,m}|^2}+\frac{c}{|h_{b,r,m}|^2}) \quad (43)$$

$$\Delta\mu = P_r - \sum_{n=1}^{N}(\hat{p}_{r,n,k}+\hat{p}_{r,n,j}) \quad (44)$$

The Lagrangian multiplier λ and μ can be updated by:

$$\begin{aligned}\lambda(i+1) &= \lambda(i) - v\Delta\lambda &(45)\\ \mu(i+1) &= \mu(i) - \delta\Delta\mu &(46)\end{aligned}$$

where v and δ are the iteration stepsizes corresponding to λ and μ, respectively.

The details of the proposed power allocation algorithm based on the DC programming is shown in **Algorithm 1**.

Algorithm 1 The Power Allocation Optimization Using DC Programming

```
Initialization
set i = 0.
Initialize the power allocation variables $p^0_{r,n,j}$, ∀ n, j,
the Lagrangian multiplier λ(0), μ(0) and constant c(0).

Set the convergence threshold $\tau_{thr}$.
Updating
while $p^i_{r,n,j}$ - $p^{i-1}_{r,n,j} \geq \tau_{thr}$ do
    i = i + 1;
    Compute the optimal power allocation of P4 $\hat{p}_{r,n,j}$ and
    $\hat{p}_{r,n,k}$ according to (38) and (39).
    Updating power allocation $p^i_{r,n,j}$ according to (40);
    Updating constant c(i) according to (24);
    Solve the dual problem (41) and updating Lagrangian
    multiplier λ and μ according to (45) and (46)
    respectively.
end while
```

V. SIMULATION RESULTS AND DISCUSSIONS

In this section, we present the simulation to evaluate the performance of the proposed joint resource allocation scheme based on DC programming. In the simulation, there are one BS, one relay and multiple users. The Okumura-Hata model is adopted as the large scale fading, and the small scale fading is modeled as Rayleigh fading. In the following, we first introduce some benchmarks to evaluate the performance gain of the proposed algorithm.

- Subcarrier assignment without SA: In this benchmark, we only consider the subcarrier pair and subcarrier-user assignment with fixed power allocation. The subcarrier-user assignment is optimized by the proposed subcarrier-user assignment algorithm without SA.
- Joint spectrum and power allocation without SA (JSPA-2): In this benchmark, the subcarrier pair, subcarrier-user assignment as well as power allocation are considered. The spectrum allocation is solved by the proposed subcarrier assignment algorithm without SA, and the power allocation is optimized by the DC programming.
- Joint spectrum and power allocation without subcarrier pair and SA (JSPA-3): In this benchmark, we only consider the subcarrier-user assignment and power allocation. the subcarrier-user assignment is solved by the proposed subcarrier-user assignment algorithm without SA, and the power allocation is optimized by the DC programming.

Fig.2 compares the performance of the proposed subcarrier assignment scheme with subcarrier assignment scheme without SA and random search. It can be observed from Fig. 2 that the system rate obtained by the proposed subcarrier assignment scheme and the other two benchmarks increase with the increase of the number of users. This result comes

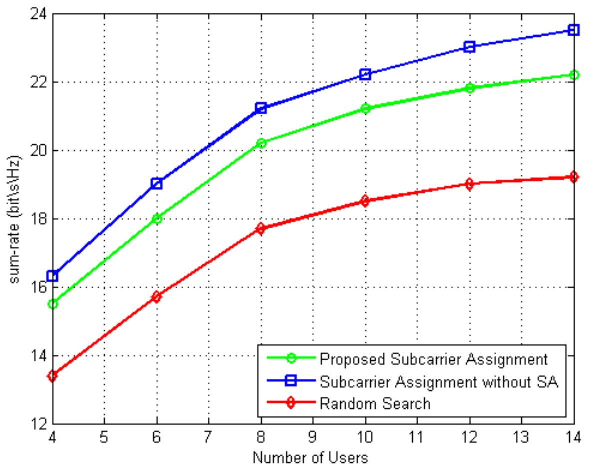

FIGURE 2. System sum rate with fixed power allocation vs different number of users.

from the fact that more users can result in more system throughput due to the multi-user diversity. It can also be observed that the proposed subcarrier assignment scheme outperforms the assignment scheme without SA, which illustrates that SA can effectively improve the search efficiency of the subcarrier-user assignment problem. Indeed, SA is an improvement greedy algorithm which has better global search ability since SA accepts the worse solution with a certain probability.

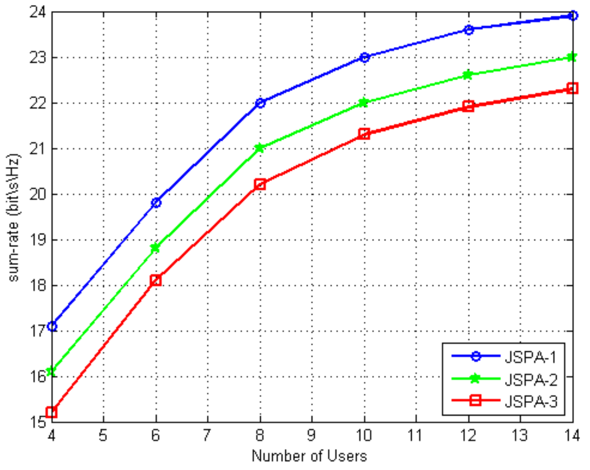

FIGURE 3. System sum rate vs different number of users.

Fig.3 depicts the different system sum rate obtained by the proposed joint spectrum and power allocation algorithm (JSPA-1), JSPA-2 and JSPA-3 with the increase of the number of users. The result of Fig.3 shows that the proposed algorithm achieves better performance than JSPA-2 and JSPA-3 as a result of the improvement of the SA. In addition, Fig.3 illustrates the necessary of the subcarrier pair due to the better performance of JSPA-2 than that of JSPA-3.

In Fig.4, we compare the performance of the NOMA system with that of the OMA system. It can be observed that both the performance of these two systems increase with the increase of the users due to the multi-user diversity. Furthermore, the slope of increase curve decreases with the increase of the number of users since the system sum rate is bounded by the spectrum resource. Meanwhile, the performance of the NOMA obviously outperforms the OMA system. The result illustrates that NOMA technology can effectively improve the system rate by exploiting not only the frequency domain but also the power domain for multiple access.

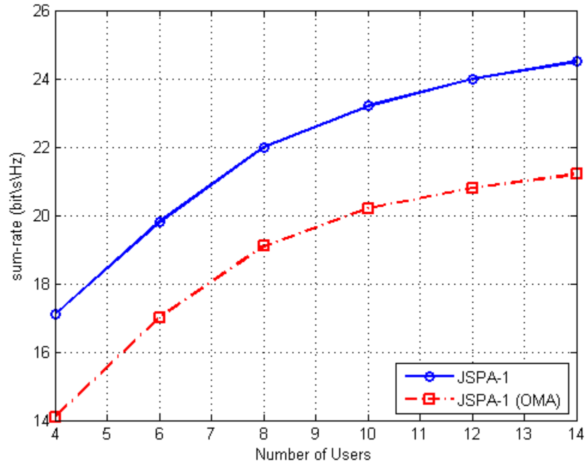

FIGURE 4. System sum rate obtained by the proposed algorithm for NOMA and OMA, respectively.

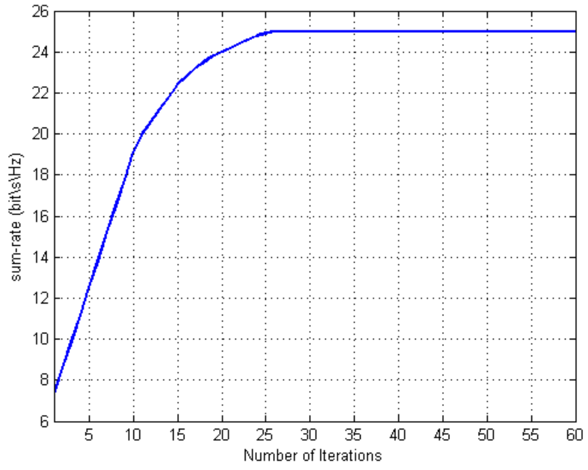

FIGURE 5. Convergence of the proposed algorithm.

Next, we verify the convergence of the proposed algorithm. Fig.5 shows the system sum rate of the proposed joint spectrum and power allocation algorithm with number of iterations. It can be observed that the convergence of the proposed algorithm occurs within 25 iterations, which demonstrates the convergence of the proposed algorithm.

Fig.6 shows the system sum rate obtained by the proposed algorithm with increase of the number of users for different BS transmit power. The result shows that the system performance is improved with the increase of the transmit power.

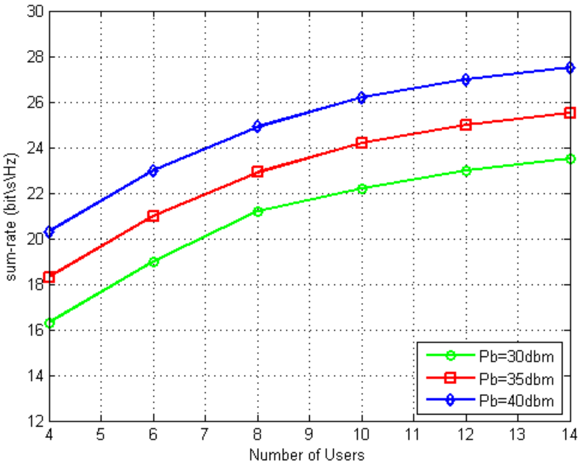

FIGURE 6. System sum rate vs different BS transmit power.

Nevertheless, the performance promotion is limited when the transmit power became more and more enhancement. Therefore, it is not feasible to improve the system performance by increasing transmit power blindly.

VI. CONCLUSION

In this paper, the joint resource allocation problem of NOMA enhanced relaying networks was investigated with the aim of maximizing the system throughput. The resource allocation involving the subcarrier pair, subcarrier-user assignment as well as power allocation was formulated as a mixed integer nonconvex optimization problem which was difficult to tackle. In addition, there was strong coupling between the spectrum allocation and power allocation due to the multi-user interference caused by the NOMA technology, which further improved the difficulty of optimizing the problem. To solve it, we transformed the formulated optimization problem as a DC programming problem. Then, the original problem was approximated as a series of convex optimization problems by the first order Taylor expansion. The sequence convex programming method was applied to solve the approximation problems. Simulation results illustrated that the proposed algorithm can effectively improve the system performance.

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