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Bidirectional Quantum Teleportation by Using Six-Qubit Cluster State

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ABSTRACT Quantum teleportation is considered as the most basic process of quantum communication. It is undoubtedly beneficial for the realization of quantum teleportation while transmitting the maximum number of quantum bits with the minimum number of quantum channel resources. An improved bidirectional quantum teleportation protocol based on a quantum channel of six-qubit cluster state is proposed. In this paper, the user Alice can transmit an unknown three-qubit entangled state to the user Bob, and at the same time, the user Bob can transmit an arbitrary single-qubit state to the user Alice by utilizing GHZ-state measurement, bell-state measurement, single-qubit Von Neumann measurement, and unitary operations. Compared with other schemes in quantum resource consumption, necessary operation complexity including operation difficulty and intensity, classical resource consumption, quantum information bits transmitted, and intrinsic efficiency, our scheme has the remarkable advantages in transmitting more quantum information and possesses higher intrinsic efficiency.

INDEX TERMS Quantum teleportation, six-qubit cluster state, bell-state measurement, GHZ-state measurement, Von Neumann measurement, efficiency.

I. INTRODUCTION

Quantum teleportation is a communication protocol that quantum information can be transmitted from one location to another, with the help of classical communication and previously shared quantum entanglement between the sending and receiving location. Since Bennett *et al.* [1] presented the creative quantum teleportation protocol through an entangled quantum channel of Einstein-Podolsky-Rosen (EPR) pair between the sender and the receiver in 1993, several quantum teleportation protocols have been devised by utilizing multi-particle entangled state as quantum channel, such as EPR state [2], GHZ state [3], [4] GHZ-like state [5]–[7] W state [8], [9], W-like state [10], cluster state [11]–[14] *et al.*

Cluster state is a new multi-particle entangled state proposed by Briegel and Raussendorf [15] in 2001. It is known that the n -particle ($n > 3$) cluster state is both robust against decoherence and maximally connected with the better per-

sistency than the GHZ state. Based on this intrinsic attribute the first bidirectional controlled quantum teleportation protocol demonstrated in 2013 was applying cluster state as quantum channel [16]. Soon after, several bidirectional quantum controlled teleportation (BQCT) protocols have been investigated by using multi-particle cluster state as quantum channel. In 2013, A. Yan proposed a protocol by using six-qubit cluster state as quantum channel. In this protocol, two users can simultaneously transmit an arbitrary single-qubit state to each other with the controller Charlie [17]. In 2016, Li *et al.* [18] proposed an asymmetric bidirectional quantum controlled teleportation protocol by using six-qubit cluster state, in which the user Alice can transmit an arbitrary two-qubit entangled state to the user Bob and at the same time the user Bob can transmit an arbitrary single-qubit state to the user Alice. In 2017, B. Choudhury, S. Samanta *et al.* proposed an asymmetric bidirectional quantum controlled teleportation protocol by using nine-qubit cluster state as quantum channel [19]. In this protocol, the user Alice can transmit a three-qubit entangled state to the user Bob and the user Bob can transmit

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a two-qubit entangled state to the user Alice. Although the quantum information bits transmitted have been improved, the intrinsic efficiency has been greatly reduced

When without controller, the feasibility of bidirectional quantum teleportation (BQT) protocols using cluster state as quantum channel has also been proved. For instance, in 2016 Sang *et al.* proposed a bidirectional quantum teleportation protocol by using five-qubit cluster state as quantum channel [20], in which Alice can transmit an arbitrary two-qubit entangled state to Bob, and at the same time Bob can transmit an arbitrary single-qubit state to Alice. Because the qubits carrying information are not transmitted in quantum channel, it is impossible for eavesdropper to intercept the qubits during message transmission process, i.e., intercept of communication. A unique approach for eavesdropper to attack the quantum channel between Alice and Bob is when controller sends message to Alice and Bob. Hence bidirectional quantum teleportation is safer than bidirectional quantum controlled teleportation.

In our work, based on [20], an improved protocol for implementing asymmetric bidirectional quantum teleportation using six-cluster state as a quantum channel is proposed. In our scheme, Alice holds an unknown three-qubit entangled state consisting of three qubits A, B, C and Bob holds an arbitrary single-qubit state comprised of qubit D. The aim of this scheme is to realize the bidirectional transmission of quantum state between Alice and Bob. To achieve this scheme, first of all, particles in quantum channel need to be distributed to two users, i.e. Alice and Bob respectively. Next, two users need to perform proper measurement that is Alice performs GHZ-state measurement, single-qubit Von Neumann measurement on her qubits and Bob performs Bellstate measurement on his qubits. Finally, two users have to perform unitary operations and CNOT operations on their residual states separately to realize the bilateral exchange of the initial quantum states

The outline of this paper is structured as follows: section 2 demonstrates the selection of quantum information bits transmitted and quantum channel. Section 3 details the principle of bidirectional quantum teleportation. Section 4 presents a comparison between this paper and other BQT schemes. Finally, a brief conclusion is drawn in section 5.

II. THE SELECTION OF QUANTUM INFORMATION BITS TRANSMITTED AND QUANTUM CHANNEL

A. THE SELECTION OF QUANTUM INFORMATION BITS TRANSMITTED

1) IN THEORETICAL APPLICATION

In the idea environment, bidirectional quantum teleportation can perfectly realize the bilateral transmission of quantum states, that is point to point. Since bidirectional quantum teleportation protocol was first proposed in 2013, many bidirectional quantum teleportation protocols have been proposed. However, in these presented protocols, quantum information bits transferred by two communicating parties are generally two single-qubit states [16], [17], [21]–[24] a single-qubit

state and a two-qubit state [20], [25], [26], two two-qubit states [27], [28], the aim of this paper is to increase the number of quantum information bits transmitted by communicating parties. So, it is an innovational theory to use a three-qubit entangled state and a single-qubit state as quantum information bits transmitted.

2) IN EXPERIMENTAL APPLICATION

Noise is the key issue in quantum teleportation process. Unlike traditional communication, weak quantum signal is transmitted by quantum channel and it is easily affected by noise. A noisy quantum channel can be a regular communication channel where the input of a pure quantum state can produce a mixed state as output as the data qubits become entangled with the environment. Different noise environments need different solutions. Our proposed scheme can solve a common noise environment where quantum channel only causes an error on single-qubit [29].

When quantum information bits transmitted is a three-qubit entangled state $|\varphi\rangle_{ABC} = a_0|000\rangle + a_1|111\rangle$, suppose that quantum channel flips a single-qubit, say the first one, which can be presented as $|\varphi\rangle_{ABC'} = a_0|100\rangle + a_1|011\rangle$. In order to correct the false quantum state back into the original quantum state, we should make sure which qubit has been flipped. The method is expressed as follows.

Firstly, comparing the first two qubits, we find they are different which is not allowed for any valid codeword. Therefore, an error is occurred and furthermore, it flips either the first or second qubit. Note that measuring the first and second qubit will destroy the superposition in the codeword, so we just measure the difference between them.

Then comparing the first and third qubit, we find that since the first qubit was flipped, it will disagree with the third; if the second qubit had been flipped, the first and third would have agreed. Therefore, the error is narrowed down to the first qubit and finally we flip it back.

When quantum information bits transmitted is a single-qubit quantum state $|\varphi\rangle_D = b_0|0\rangle + b_1|1\rangle$, suppose that quantum channel flips a single-qubit, which can be elaborated as $|\varphi\rangle_D = b_0|1\rangle + b_1|0\rangle$. We can recover the original single-qubit state by using operation of $\sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ on the false quantum state.

This is a quantum error correction scheme for qubit flipping. In order to deal with the phenomenon of qubit flipping effectively, we need to select an initial state which is only include states $|000\rangle$ and $|111\rangle$. So, an unknown three-qubit entangled state $a_0|000\rangle + a_1|111\rangle$ and a single-qubit state $b_0|0\rangle + b_1|1\rangle$ used as the initial quantum states transmitted can easily handle the phenomenon of qubit flipping in the noise environment.

B. THE SELECTION OF QUANTUM CHANNEL

Efficiency is the key to judge the feasibility of quantum teleportation, the formula is described as $\tau = q_s / (q_u + b_r)$ where q_s indicates the number of qubits consisting of the

quantum information to be transmitted q_u is the number of the qubits used as the quantum channel (except for those chosen for security checking) and b_t represents the number of qubits used as the classical channel. Although b_t has no effect on efficiency which can also be expressed as $\tau = q_s/q_u$ classical communication served as the bridge between two communicating parties in quantum teleportation is regarded as an obvious requirement to avoid the occurrence of faster-than-light (FTL) signaling. Therefore, the formula of efficiency characterized as $\tau = q_s/(q_u + b_t)$ is more rational than $\tau = q_s/q_u$. In this paper quantum information bits transmitted in communicating parties are an unknown three-qubit entangled state and a single-qubit state. When the quantum information bits transmitted are finalized, the measurement basis will determine whether quantum information resource in quantum channel is consumed in an economic way. We have summarized it as follows: when our scheme is in the measurement basis consisting of a GHZ-state measurement, a Bellstate measurement and a single-qubit state measurement, quantum channel will become a six-qubit entangled state; when our scheme is in the measurement basis comprised of three Bell-state measurements and a single-qubit state measurement, quantum channel will become a seven-qubit entangled state; when our scheme is in the measurement basis made up of four Bell-state measurements, quantum channel will become an eight-qubit entangled state. As is clear from the above discussion, selecting a six-qubit entangled state as quantum channel can not only reduce quantum resource consumption (quantum information bits in quantum channel) but also improve intrinsic efficiency. In our scheme, six-qubit cluster state is selected as quantum channel due to cluster state's unique superiority, namely maximum connectedness and a high persistency of entanglement.

III. BIDIRECTIONAL QUANTUM TELEPORTATION

Our scheme can be described as follows. The framework of this scheme is illustrated in Figure 1.

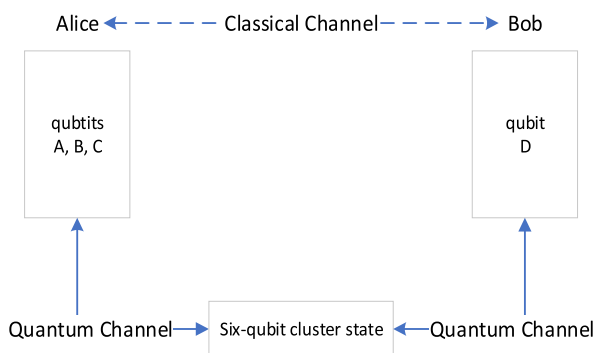


FIGURE 1. The framework of bidirectional quantum teleportation.

Assume that Alice is in the possession of an unknown three-qubit entangled state, which is given by

$$|\varphi\rangle_{ABC} = a_0|000\rangle + a_1|111\rangle \quad (1)$$

where a_0, a_1 are arbitrary complex coefficients with $|a_0|^2 + |a_1|^2 = 1$. At the same time, Bob possesses an arbitrary singlequbit state which can be expressed as

$$|\varphi\rangle_D = b_0|0\rangle + b_1|1\rangle \quad (2)$$

where b_0, b_1 are arbitrary complex coefficients with $|b_0|^2 + |b_1|^2 = 1$. Here, the three qubits A, B, C belong to Alice and the single qubit D belongs to Bob.

Now, Alice wants to transmit the three-qubit entangled state to Bob and Bob wants to transmit the single-qubit state to Alice through a single quantum channel consisting of six-qubit cluster state shared by Alice and Bob. The six-qubit cluster state used as quantum channel has the form

$$|C_6\rangle_{123456} = \frac{1}{2} (|000000\rangle + |000111\rangle + |111000\rangle + |111111\rangle)_{123456} \quad (3)$$

where the qubits 1 and 6 belong to Alice and qubits 2,3,4,5 belong to Bob, respectively. Therefore, the initial state of the total system can be defined as

$$\begin{aligned} |\varphi\rangle_{ABCD123456} &= |\varphi\rangle_{ABC} \otimes |\varphi\rangle_D \otimes |C_6\rangle_{123456} \\ &= \frac{1}{2} (a_0|000\rangle + a_1|111\rangle) \otimes (b_0|0\rangle + b_1|1\rangle) \\ &\quad \otimes (|000000\rangle + |000111\rangle + |111000\rangle \\ &\quad + |111111\rangle) \end{aligned} \quad (4)$$

To implement bidirectional quantum teleportation, the scheme is executed in the following way. Alice firstly performs a GHZ-state measurement on her three qubits A, B, 1 in the measurement basis consisting of $|\xi^\pm\rangle_{AB1}$ and $|\zeta^\pm\rangle_{AB1}$, which can be characterized as

$$\begin{aligned} |\xi^\pm\rangle_{AB1} &= \frac{1}{\sqrt{2}} (|000\rangle \pm |111\rangle)_{AB1} \\ |\zeta^\pm\rangle_{AB1} &= \frac{1}{\sqrt{2}} (|001\rangle \pm |110\rangle)_{AB1} \end{aligned} \quad (5)$$

After Alice tells her measurement results to Bob via classical means Bob performs a Bell-state measurement (BSM) on his two qubits D, 5 in the measurement basis made up of $|\phi^\pm\rangle_{D5}$ and $|\varphi^\pm\rangle_{D5}$, which have the form

$$\begin{aligned} |\phi^\pm\rangle_{D5} &= \frac{1}{\sqrt{2}} (|00\rangle \pm |11\rangle)_{D5} \\ |\varphi^\pm\rangle_{D5} &= \frac{1}{\sqrt{2}} (|01\rangle \pm |10\rangle)_{D5} \end{aligned} \quad (6)$$

Afterwards Bob informs Alice of his measurement result by classical communication Based on Bob's measurement result, Alice takes single-qubit Von Neumann measurement on her qubit C in the measurement basis comprised of the following elements

$$\begin{aligned} |+\rangle_C &= \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle) \\ |-\rangle_C &= \frac{1}{\sqrt{2}} (|0\rangle - |1\rangle) \end{aligned} \quad (7)$$

and her measurement result is then conveyed to Bob through a classical channel. Therefore, the initial state of the total system can be transformed into the state

$$\begin{aligned}
 |\varphi\rangle_{ABCD123456} &= |\varphi\rangle_{ABC} \otimes |\varphi\rangle_D \otimes |C_6\rangle_{123456} \\
 &= \text{bar}|\xi^+\rangle_{AB1} |\phi^+\rangle_{D5} |+\rangle_C |\eta^1\rangle_{2346} \\
 &\quad + |\xi^+\rangle_{AB1} |\phi^+\rangle_{D5} |-\rangle_C |\eta^2\rangle_{2346} \\
 &\quad + |\xi^+\rangle_{AB1} |\phi^-\rangle_{D5} |+\rangle_C |\eta^3\rangle_{2346} \\
 &\quad + |\xi^+\rangle_{AB1} |\phi^-\rangle_{D5} |-\rangle_C |\eta^4\rangle_{2346} \\
 &\quad + |\xi^+\rangle_{AB1} |\varphi^+\rangle_{D5} |+\rangle_C |\eta^5\rangle_{2346} \\
 &\quad + |\xi^+\rangle_{AB1} |\varphi^+\rangle_{D5} |-\rangle_C |\eta^6\rangle_{2346} \\
 &\quad + |\xi^+\rangle_{AB1} |\varphi^-\rangle_{D5} |+\rangle_C |\eta^7\rangle_{2346} \\
 &\quad + |\xi^+\rangle_{AB1} |\varphi^-\rangle_{D5} |-\rangle_C |\eta^8\rangle_{2346} \\
 &\quad + |\xi^-\rangle_{AB1} |\phi^+\rangle_{D5} |+\rangle_C |\eta^9\rangle_{2346} \\
 &\quad + |\xi^-\rangle_{AB1} |\phi^+\rangle_{D5} |-\rangle_C |\eta^{10}\rangle_{2346} \\
 &\quad + |\xi^-\rangle_{AB1} |\phi^-\rangle_{D5} |+\rangle_C |\eta^{11}\rangle_{2346} \\
 &\quad + |\xi^-\rangle_{AB1} |\phi^-\rangle_{D5} |-\rangle_C |\eta^{12}\rangle_{2346} \\
 &\quad + |\xi^-\rangle_{AB1} |\varphi^+\rangle_{D5} |+\rangle_C |\eta^{13}\rangle_{2346} \\
 &\quad + |\xi^-\rangle_{AB1} |\varphi^+\rangle_{D5} |-\rangle_C |\eta^{14}\rangle_{2346} \\
 &\quad + |\xi^-\rangle_{AB1} |\varphi^-\rangle_{D5} |+\rangle_C |\eta^{15}\rangle_{2346} \\
 &\quad + |\xi^-\rangle_{AB1} |\varphi^-\rangle_{D5} |-\rangle_C |\eta^{16}\rangle_{2346} \\
 &\quad + |\zeta^+\rangle_{AB1} |\phi^+\rangle_{D5} |+\rangle_C |\eta^{17}\rangle_{2346} \\
 &\quad + |\zeta^+\rangle_{AB1} |\phi^+\rangle_{D5} |-\rangle_C |\eta^{18}\rangle_{2346} \\
 &\quad + |\zeta^+\rangle_{AB1} |\phi^-\rangle_{D5} |+\rangle_C |\eta^{19}\rangle_{2346} \\
 &\quad + |\zeta^+\rangle_{AB1} |\phi^-\rangle_{D5} |-\rangle_C |\eta^{20}\rangle_{2346} \\
 &\quad + |\zeta^+\rangle_{AB1} |\varphi^+\rangle_{D5} |+\rangle_C |\eta^{21}\rangle_{2346} \\
 &\quad + |\zeta^+\rangle_{AB1} |\varphi^+\rangle_{D5} |-\rangle_C |\eta^{22}\rangle_{2346} \\
 &\quad + |\zeta^+\rangle_{AB1} |\varphi^-\rangle_{D5} |+\rangle_C |\eta^{23}\rangle_{2346} \\
 &\quad + |\zeta^+\rangle_{AB1} |\varphi^-\rangle_{D5} |-\rangle_C |\eta^{24}\rangle_{2346} \\
 &\quad + |\zeta^-\rangle_{AB1} |\phi^+\rangle_{D5} |+\rangle_C |\eta^{25}\rangle_{2346} \\
 &\quad + |\zeta^-\rangle_{AB1} |\phi^+\rangle_{D5} |-\rangle_C |\eta^{26}\rangle_{2346} \\
 &\quad + |\zeta^-\rangle_{AB1} |\phi^-\rangle_{D5} |+\rangle_C |\eta^{27}\rangle_{2346}
 \end{aligned}$$

TABLE 1. The collapsed states composed of qubit 2,3,4,6 after GHZ-state measurement, bell-state measurement and single-qubit measurement.

$ \eta^1\rangle_{2346}$	$= \eta^{10}\rangle_{2346} = a_0b_0 0000\rangle + a_0b_1 0011\rangle + a_1b_0 1100\rangle + a_1b_1 1111\rangle$
$ \eta^2\rangle_{2346}$	$= \eta^9\rangle_{2346} = a_0b_0 0000\rangle + a_0b_1 0011\rangle - a_1b_0 1100\rangle - a_1b_1 1111\rangle$
$ \eta^3\rangle_{2346}$	$= \eta^{12}\rangle_{2346} = a_0b_0 0000\rangle - a_0b_1 0011\rangle + a_1b_0 1100\rangle - a_1b_1 1111\rangle$
$ \eta^4\rangle_{2346}$	$= \eta^{11}\rangle_{2346} = a_0b_0 0000\rangle - a_0b_1 0011\rangle - a_1b_0 1100\rangle + a_1b_1 1111\rangle$
$ \eta^5\rangle_{2346}$	$= \eta^{14}\rangle_{2346} = a_0b_0 0011\rangle + a_0b_1 0000\rangle + a_1b_0 1111\rangle + a_1b_1 1100\rangle$
$ \eta^6\rangle_{2346}$	$= \eta^{13}\rangle_{2346} = a_0b_0 0011\rangle + a_0b_1 0000\rangle - a_1b_0 1111\rangle - a_1b_1 1100\rangle$
$ \eta^7\rangle_{2346}$	$= \eta^{16}\rangle_{2346} = a_0b_0 0011\rangle - a_0b_1 0000\rangle + a_1b_0 1111\rangle - a_1b_1 1100\rangle$
$ \eta^8\rangle_{2346}$	$= \eta^{15}\rangle_{2346} = a_0b_0 0011\rangle - a_0b_1 0000\rangle - a_1b_0 1111\rangle + a_1b_1 1100\rangle$
$ \eta^{17}\rangle_{2346}$	$= \eta^{26}\rangle_{2346} = a_0b_0 1100\rangle + a_0b_1 1111\rangle + a_1b_0 0000\rangle + a_1b_1 0011\rangle$
$ \eta^{18}\rangle_{2346}$	$= \eta^{25}\rangle_{2346} = a_0b_0 1100\rangle + a_0b_1 1111\rangle - a_1b_0 0000\rangle - a_1b_1 0011\rangle$
$ \eta^{19}\rangle_{2346}$	$= \eta^{28}\rangle_{2346} = a_0b_0 1100\rangle - a_0b_1 1111\rangle + a_1b_0 0000\rangle - a_1b_1 0011\rangle$
$ \eta^{20}\rangle_{2346}$	$= \eta^{27}\rangle_{2346} = a_0b_0 1100\rangle - a_0b_1 1111\rangle - a_1b_0 0000\rangle + a_1b_1 0011\rangle$
$ \eta^{21}\rangle_{2346}$	$= \eta^{30}\rangle_{2346} = a_0b_0 1111\rangle + a_0b_1 1100\rangle + a_1b_0 0011\rangle + a_1b_1 0000\rangle$
$ \eta^{22}\rangle_{2346}$	$= \eta^{29}\rangle_{2346} = a_0b_0 1111\rangle + a_0b_1 1100\rangle - a_1b_0 0011\rangle - a_1b_1 0000\rangle$
$ \eta^{23}\rangle_{2346}$	$= \eta^{32}\rangle_{2346} = a_0b_0 1111\rangle - a_0b_1 1100\rangle + a_1b_0 0011\rangle - a_1b_1 0000\rangle$
$ \eta^{24}\rangle_{2346}$	$= \eta^{31}\rangle_{2346} = a_0b_0 1111\rangle - a_0b_1 1100\rangle - a_1b_0 0011\rangle + a_1b_1 0000\rangle$

$$\begin{aligned}
 &+ |\zeta^-\rangle_{AB1} |\phi^-\rangle_{D5} |-\rangle_C |\eta^{28}\rangle_{2346} \\
 &+ |\zeta^-\rangle_{AB1} |\varphi^+\rangle_{D5} |+\rangle_C |\eta^{29}\rangle_{2346} \\
 &+ |\zeta^-\rangle_{AB1} |\varphi^+\rangle_{D5} |-\rangle_C |\eta^{30}\rangle_{2346} \\
 &+ |\zeta^-\rangle_{AB1} |\varphi^-\rangle_{D5} |+\rangle_C |\eta^{31}\rangle_{2346} \\
 &+ |\zeta^-\rangle_{AB1} |\varphi^-\rangle_{D5} |-\rangle_C |\eta^{32}\rangle_{2346}
 \end{aligned} \tag{8}$$

where $|\eta^i\rangle_{2346}$ ($i = 1, 2, 3, \dots, 32$) are mutually orthometric four-qubit state shown in Table 1.

It can be seen from the above formula that after three times measurements, the total quantum system can be collapsed into $|\eta^i\rangle_{2346}$. Although $|\eta^i\rangle_{2346}$ has 32 possible quantum states, Table 1 shows that half of the 32 possible quantum states are the same. So broadly speaking, there are only 16 possible collapsed states in this scheme, which can lessen operation complexity. On the basis of above measurement results, Alice and Bob perform appropriate unitary transformation on their own qubits respectively and the different unitary operations used for different measurement results are summarized in Table 2. After performing unitary operations, the collapsed state $|\eta^i\rangle_{2346}$ can be evolved into

$$a_0b_0|0000\rangle + a_0b_1|0011\rangle + a_1b_0|1100\rangle + a_1b_1|1111\rangle \tag{9}$$

Then Alice and Bob carry out CNOT operation in twice. For the first time, Bob performs CNOT operation on

TABLE 2. The possible outcomes of measurement and corresponding unitary operations.

Alice _{AB1}	Bob _{D5}	Alice _C	Measurement results	unitary transformation
$ \xi^+\rangle_{AB1}$	$ \phi^+\rangle_{D5}$	$ +\rangle_C$	$ \eta^1\rangle_{2346}$	$I_2 \otimes I_3 \otimes I_4 \otimes I_6$
$ \xi^+\rangle_{AB1}$	$ \phi^+\rangle_{D5}$	$ -\rangle_C$	$ \eta^2\rangle_{2346}$	$\sigma_2^z \otimes I_3 \otimes I_4 \otimes I_6$
$ \xi^+\rangle_{AB1}$	$ \phi^-\rangle_{D5}$	$ +\rangle_C$	$ \eta^3\rangle_{2346}$	$\sigma_2^z \otimes \sigma_3^z \otimes \sigma_4^z \otimes I_6$
$ \xi^+\rangle_{AB1}$	$ \phi^-\rangle_{D5}$	$ -\rangle_C$	$ \eta^4\rangle_{2346}$	$I_2 \otimes \sigma_3^z \otimes \sigma_4^z \otimes I_6$
$ \xi^+\rangle_{AB1}$	$ \phi^+\rangle_{D5}$	$ +\rangle_C$	$ \eta^5\rangle_{2346}$	$I_2 \otimes I_3 \otimes \sigma_4^z \otimes \sigma_6^z$
$ \xi^+\rangle_{AB1}$	$ \phi^+\rangle_{D5}$	$ -\rangle_C$	$ \eta^6\rangle_{2346}$	$I_2 \otimes \sigma_3^z \otimes \sigma_4^z \otimes \sigma_6^z$
$ \xi^+\rangle_{AB1}$	$ \phi^-\rangle_{D5}$	$ +\rangle_C$	$ \eta^7\rangle_{2346}$	$I_2 \otimes I_3 \otimes \sigma_4^x \otimes (-i)\sigma_6^y$
$ \xi^+\rangle_{AB1}$	$ \phi^-\rangle_{D5}$	$ -\rangle_C$	$ \eta^8\rangle_{2346}$	$I_2 \otimes \sigma_3^z \otimes \sigma_4^x \otimes (-i\sigma_6^y)$
$ \xi^-\rangle_{AB1}$	$ \phi^+\rangle_{D5}$	$ +\rangle_C$	$ \eta^9\rangle_{2346}$	$\sigma_2^z \otimes I_3 \otimes I_4 \otimes I_6$
$ \xi^-\rangle_{AB1}$	$ \phi^+\rangle_{D5}$	$ -\rangle_C$	$ \eta^{10}\rangle_{2346}$	$I_2 \otimes I_3 \otimes I_4 \otimes I_6$
$ \xi^-\rangle_{AB1}$	$ \phi^-\rangle_{D5}$	$ +\rangle_C$	$ \eta^{11}\rangle_{2346}$	$I_2 \otimes \sigma_3^z \otimes \sigma_4^z \otimes I_6$
$ \xi^-\rangle_{AB1}$	$ \phi^-\rangle_{D5}$	$ -\rangle_C$	$ \eta^{12}\rangle_{2346}$	$\sigma_2^z \otimes \sigma_3^z \otimes \sigma_4^z \otimes I_6$
$ \xi^-\rangle_{AB1}$	$ \phi^+\rangle_{D5}$	$ +\rangle_C$	$ \eta^{13}\rangle_{2346}$	$I_2 \otimes \sigma_3^z \otimes \sigma_4^x \otimes \sigma_6^z$
$ \xi^-\rangle_{AB1}$	$ \phi^+\rangle_{D5}$	$ -\rangle_C$	$ \eta^{14}\rangle_{2346}$	$I_2 \otimes I_3 \otimes \sigma_4^z \otimes \sigma_6^z$
$ \xi^-\rangle_{AB1}$	$ \phi^-\rangle_{D5}$	$ +\rangle_C$	$ \eta^{15}\rangle_{2346}$	$I_2 \otimes \sigma_3^z \otimes \sigma_4^x \otimes (-i\sigma_6^y)$
$ \xi^-\rangle_{AB1}$	$ \phi^-\rangle_{D5}$	$ -\rangle_C$	$ \eta^{16}\rangle_{2346}$	$I_2 \otimes I_3 \otimes \sigma_4^x \otimes (-i)\sigma_6^y$
$ \xi^+\rangle_{AB1}$	$ \phi^+\rangle_{D5}$	$ +\rangle_C$	$ \eta^{17}\rangle_{2346}$	$\sigma_2^x \otimes \sigma_3^x \otimes I_4 \otimes I_6$
$ \xi^+\rangle_{AB1}$	$ \phi^+\rangle_{D5}$	$ -\rangle_C$	$ \eta^{18}\rangle_{2346}$	$i\sigma_2^y \otimes \sigma_3^x \otimes \sigma_4^z \otimes \sigma_6^z$
$ \xi^+\rangle_{AB1}$	$ \phi^-\rangle_{D5}$	$ +\rangle_C$	$ \eta^{19}\rangle_{2346}$	$\sigma_2^x \otimes \sigma_3^x \otimes I_4 \otimes \sigma_6^z$

TABLE 2. The possible outcomes of measurement and corresponding unitary operations.

$ \zeta^+\rangle_{AB1}$	$ \phi^-\rangle_{D5}$	$ -\rangle_C$	$ \eta^{20}\rangle_{2346}$	$i\sigma_2^y \otimes \sigma_3^x \otimes \sigma_4^z \otimes I_6$
$ \zeta^+\rangle_{AB1}$	$ \phi^+\rangle_{D5}$	$ +\rangle_C$	$ \eta^{21}\rangle_{2346}$	$\sigma_2^x \otimes \sigma_3^x \otimes \sigma_4^z \otimes \sigma_6^z$
$ \zeta^+\rangle_{AB1}$	$ \phi^+\rangle_{D5}$	$ -\rangle_C$	$ \eta^{22}\rangle_{2346}$	$i\sigma_2^y \otimes \sigma_3^x \otimes \sigma_4^z \otimes \sigma_6^z$
$ \zeta^+\rangle_{AB1}$	$ \phi^-\rangle_{D5}$	$ +\rangle_C$	$ \eta^{23}\rangle_{2346}$	$\sigma_2^x \otimes \sigma_3^x \otimes i\sigma_4^y \otimes \sigma_6^z$
$ \zeta^+\rangle_{AB1}$	$ \phi^-\rangle_{D5}$	$ -\rangle_C$	$ \eta^{24}\rangle_{2346}$	$\sigma_2^x \otimes i\sigma_3^y \otimes \sigma_4^z \otimes \sigma_6^z$
$ \zeta^-\rangle_{AB1}$	$ \phi^+\rangle_{D5}$	$ +\rangle_C$	$ \eta^{25}\rangle_{2346}$	$i\sigma_2^y \otimes \sigma_3^x \otimes \sigma_4^z \otimes \sigma_6^z$
$ \zeta^-\rangle_{AB1}$	$ \phi^+\rangle_{D5}$	$ -\rangle_C$	$ \eta^{26}\rangle_{2346}$	$\sigma_2^x \otimes \sigma_3^x \otimes I_4 \otimes I_6$
$ \zeta^-\rangle_{AB1}$	$ \phi^-\rangle_{D5}$	$ +\rangle_C$	$ \eta^{27}\rangle_{2346}$	$i\sigma_2^y \otimes \sigma_3^x \otimes \sigma_4^z \otimes I_6$
$ \zeta^-\rangle_{AB1}$	$ \phi^-\rangle_{D5}$	$ -\rangle_C$	$ \eta^{28}\rangle_{2346}$	$\sigma_2^x \otimes \sigma_3^z \otimes I_4 \otimes \sigma_6^z$
$ \zeta^-\rangle_{AB1}$	$ \phi^+\rangle_{D5}$	$ +\rangle_C$	$ \eta^{29}\rangle_{2346}$	$i\sigma_2^y \otimes \sigma_3^x \otimes \sigma_4^z \otimes \sigma_6^z$
$ \zeta^-\rangle_{AB1}$	$ \phi^+\rangle_{D5}$	$ -\rangle_C$	$ \eta^{30}\rangle_{2346}$	$\sigma_2^x \otimes \sigma_3^x \otimes \sigma_4^z \otimes \sigma_6^z$
$ \zeta^-\rangle_{AB1}$	$ \phi^-\rangle_{D5}$	$ +\rangle_C$	$ \eta^{31}\rangle_{2346}$	$\sigma_2^x \otimes i\sigma_3^y \otimes i\sigma_4^z \otimes \sigma_6^z$
$ \zeta^-\rangle_{AB1}$	$ \phi^-\rangle_{D5}$	$ -\rangle_C$	$ \eta^{32}\rangle_{2346}$	$\sigma_2^x \otimes \sigma_3^x \otimes i\sigma_4^y \otimes \sigma_6^z$

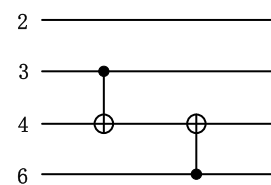


FIGURE 2. CNOT operations on qubits 2,3,4,6.

qubits 3 and 4, where qubit 3 as control qubit and qubit 4 as target qubit. For the second time, Alice performs CNOT operation on qubits 4 and 6, where qubit 6 as control qubit and qubit 4 as target qubit. The CNOT operations on qubits 2,3,4,6 are shown in Figure 2.

Finally, the desired quantum states which are defined as $a_0b_0|0000\rangle + a_0b_1|0001\rangle + a_1b_0|1110\rangle + a_1b_1|1111\rangle$ can be reconstructed successfully. The theoretical quantum circuit of this scheme is shown in Figure 3.

Now, let us take an example to demonstrate the principle of this asymmetric bidirectional quantum teleportation.

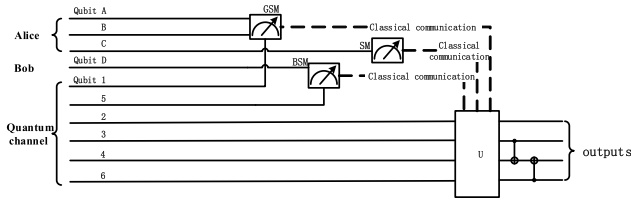


FIGURE 3. Quantum circuit of bidirectional quantum teleportation by using six-qubit cluster state in which Alice has an unknown three-qubit entangled state and Bob has an arbitrary single-qubit state.

Suppose that Alice’s measurement outcome is $|\xi^+\rangle_{AB1}$ and at the same time Bob’s measurement outcome is $|\phi^+\rangle_{D5}$ the corresponding collapsed state of qubits C, 2,3,4,6 are given as

$$|\varphi\rangle_{C2346} = (a_0b_0 |00000\rangle + a_0b_1 |00011\rangle + a_1b_0 |11100\rangle + a_1b_1 |11111\rangle)_{C2346} \quad (10)$$

Subsequently, Alice takes singlequbit Von Neumann measurement on his qubit C. If Alice’s result is $|+\rangle_C$, then the remaining qubits 2,3,4,6 are collapsed into the state

$$|\varphi\rangle_{2346} = (a_0b_0 |0000\rangle + a_0b_1 |0011\rangle + a_1b_0 |1100\rangle + a_1b_1 |1111\rangle)_{2346} \quad (11)$$

Then Alice and Bob perform unitary operations which are I_6 and $I_2 \otimes I_3 \otimes I_4$ respectively

And after that Bob needs to carry out a Controlled-NOT operation on his qubits with qubit 3 as control qubit and qubit 4 as target qubit, then the state in (11) can be evolved into

$$|\varphi'\rangle_{2346} = (a_0b_0 |0000\rangle + a_0b_1 |0011\rangle + a_1b_0 |1110\rangle + a_1b_1 |1101\rangle)_{2346} \quad (12)$$

Finally, Alice performs another Controlled-NOT operation on his qubit with qubit 6 as control qubit and qubit 4 as target qubit, then the state becomes the following state

$$\begin{aligned} |\phi\rangle_{2346} &= (a_0b_0 |0000\rangle + a_0b_1 |0001\rangle + a_1b_0 |1110\rangle + a_1b_1 |1111\rangle)_{2346} \\ &= (a_0 |000\rangle + a_1 |111\rangle)_{234} \otimes (b_0 |0\rangle + b_1 |1\rangle)_6 \end{aligned} \quad (13)$$

Thus, the asymmetric bidirectional quantum teleportation is successfully realized and the scheme is fully described for all cases in Table 2.

IV. COMPARISON

In this scheme, without controller a six-qubit cluster state is used as quantum channel for the purpose of bipartite exchange of an unknown three-qubit entangled state and an arbitrary single-qubit state. Now, let us make some comparisons among our scheme and other schemes [20], [21], [28]. Comparisons are made from five aspects, namely the quantum resource consumption, the necessary operation complexity including operation difficulty and intensity, the classical

TABLE 3. Comparisons between four BQT protocols.

Protocol	QRC	NO	CRC	QIBT	τ
V	EPR	4 SM	4	2	1/3
M	Five-Q CS	2 BSM 1 SM	5	3	3/10
S	Eight-Q ES	8 SM 1 BSM	8	4	1/4
Our	Six-Q CS	1 GSM 1 SM	5	4	4/11

QRC quantum resource consumption, Q qubit, CS cluster state, ES entangled state, NO necessary operation, CRC classical resource consumption, QIBT quantum information bits transmitted, SM single-qubit measurement, BSM Bell-state measurement, GSM GHZ-state measurement.

resource consumption, quantum information bits transmitted and the intrinsic efficiency, which is defined as $\tau = q_s / (q_u + b_t)$ in section II. The comparisons are summarized in Table 3.

From Table 3, the quantum information bits transmitted in V, M, S schemes are respectively two single-qubit states, a two-qubit state and a single-qubit state, two two-qubit states. with the number of quantum information bits transmitted increasing, quantum resource consumption and classical resource consumption will both become more complex and efficiencies are corresponding decreasing in V, M, S schemes.

Our scheme is an innovation and expansion on the basis of the M scheme, so comparing the M scheme with our scheme, we can see several differences which are shown as follows: (1) the quantum resource consumption in the M scheme is less than our scheme. (2) the operation complexity in our scheme is more difficult than the M scheme. (3) the remarkable advantages in our scheme are transmitting more quantum information and possessing higher intrinsic efficiency.

V. CONCLUSION

In summary, an ideal of asymmetric bidirectional quantum teleportation by using six-qubit cluster state as quantum channel is proposed. In our scheme, Alice and Bob are not only senders but also receivers. Alice can transmit an unknown three-qubit entangled state to Bob and Bob can transmit an arbitrary single-qubit state to Alice by using quantum measurements and corresponding unitary operations. What’s more, we have not only certified the feasibility of the scheme but also demonstrated the scheme’s innovation by comparing our scheme with other schemes in quantum and classical resource consumption, operation complexities, quantum information bits transmitted and intrinsic efficiencies. Finally as we all know, quantum secure direct communication (QSDC) considered as an elegant means for secret communication can be used in some urgent circumstances. Based on quantum teleportation, many QSDC schemes have been devised [30], [31]. And the bidirectional quantum teleportation regarded as the expansion of quantum teleportation can also be used to implement bidirectional QSDC, this method can make the implementation of QSDC simple and secure. But at present, bidirectional QSDC can only encode one-bit message on single-qubit and transmit one-

bit information every time by using bidirectional quantum teleportation [23], so we hope that more information can be transmitted at a time in bidirectional QSDC by using our proposed bidirectional quantum teleportation in the future.

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