IEEE Access
Multidisciplinary | Rapid Review | Open Access Journal

Received January 27, 2019, accepted February 15, 2019, date of publication February 21, 2019, date of current version March 7, 2019.

Digital Object Identifier 10.1109/ACCESS.2019.2900367

Constructing Chaotic System With Multiple Coexisting Attractors

QIANG LAI[1], (Member, IEEE), CHAOYANG CHEN[2], (Member, IEEE), XIAO-WEN ZHAO[3], JACQUES KENGNE[4], AND CHRISTOS VOLOS[5]

[1]School of Electrical and Automation Engineering, East China Jiaotong University, Nanchang 330013, China
[2]School of Information and Electrical Engineering, Hunan University of Science and Technology, Xiangtan 411201, China
[3]School of Mathematics, Hefei University of Technology, Hefei 230009, China
[4]Laboratory of Automation and Applied Computer, Department of Electrical Engineering, University of Dschang, Dschang P.O. Box 134, Cameroon
[5]Laboratory of Nonlinear Systems, Circuits and Complexity, Department of Physics, Aristotle University of Thessaloniki, Thessaloniki GR-54124, Greece

Corresponding authors: Qiang Lai (laiqiang87@126.com) and Chaoyang Chen (ouzk@163.com)

This work was supported in part by the National Natural Science Foundation of China under Grant 61603137 and Grant 61803139, and in part by the Key Research and Development Program of Jiangxi Province of China under Grant 20181BBE50017.

ABSTRACT This paper reports the method for constructing multiple coexisting attractors from a chaotic system. First, a new four-dimensional chaotic system with only one equilibrium and two coexisting strange attractors is established. By using bifurcation diagrams and Lyapunov exponents, the dynamical evolution of the new system is presented. Second, a feasible and effective method is applied to construct an infinite number of coexisting attractors from the new system. The core of this method is to batch replicate the attractor of the system in phase space via generating multiple invariant sets and the generation of invariant sets depends on the equilibria, which can be extended by using some simple functions with multiple zeros. Finally, we give some numerical results of the appearance of multiple coexisting attractors in the system with sine and sign functions for demonstrating the effectiveness of the method.

INDEX TERMS Chaotic system, coexisting attractors, Lyapunov exponents, equilibrium.

I. INTRODUCTION

As everyone knows, chaos is ubiquitous in nature and human society, and have a great applying potentiality in engineering owing to its unique features including ergodicity, boundedness, self-similarity, initial condition sensitivity, etc. In the past few decades, scholars have conducted extensive and intensive study on chaos, and fruitful research results have been achieved [1]–[4].

With the cognition of the importance of chaos grows, a natural question that refers to how to generate chaos in system has been raised. Chaos generation (or chaotification) has become a research focus with the advent of large quantities of chaotic systems in recent years. Several typical chaotic systems have received much attention, including Lorenz-type systems [5], [6], Sprott family systems [7], no-equilibria systems [8], [9], memristor-based systems [10]–[12] and so on. Most of the previous chaotic systems usually only exist one chaotic attractor for a set of fixed parameters. However, with the deepening of research, more evidence suggests that some simple chaotic systems are likely to generate multiple coexisting attractors from different initial conditions [13]–[18]. Li and Sprott [19], [20] and Li *et al.* [21], [22] gave a comprehensive analysis of the coexisting attractors in Lorenz and Lorenz-type systems and put forward the offset boosting and conditional symmetry method for constructing coexisting attractors in simple systems. Lai and Chen [23] proposed an effective polynomial function method which can produce multiple coexisting butterfly attractors from Sprott B system. Zhang and Chen [24] generated infinitely many chaotic attractors from low-dimensional differential systems by using an effective method. Hens *et al.* [25] applied the concept of partial synchronization to construct infinitely many coexisting attractors from coupled chaotic systems. Kengne *et al.* [26] constructed an extremely simple jerk system with a pair of symmetric strange attractors and analyzed it by using simulation and circuit implementation. Danca [27] studied the coexisting attractors of fractional-order chaotic systems. The phenomenon of coexisting attractors reflects the influence of initial conditions on the diversity of the final evolution state of the system. It is generally believed that the system with coexisting attractors has better

The associate editor coordinating the review of this manuscript and approving it for publication was Ludovico Minati.

flexibility and plasticity in performance. Through reasonable adjustment and control mechanism, the system can switch orderly in multiple states to adapt to different external environments and work requirements. As a matter of fact, the phenomenon of coexisting attractors is common with biological systems [28], [29], electrical systems [30], optical systems [31], neural networks [32] but not limited to these systems. It has a positive effect on the system and promotes the system's diversity.

Both the chaos and coexisting attractors are very important physical phenomena with engineering application values. Hence it is necessary to study them deeply. The previous studies on coexisting attractors are confined to the construction of chaotic systems with finite coexisting attractors, thus it will be very interesting to use some methods for increasing the number of coexisting attractors. In view of these considerations, this letter considers to create a new autonomous chaotic system with coexisting attractors and construct infinitely many coexisting chaotic attractors from this system by using a simple method. Section 2 presents the mathematical model of the new system and analyzes its basic properties. Section 3 introduces the method of generating coexisting attractors and applies it to the proposed system with theoretical and simulation analysis.

II. DESCRIPTION OF THE NEW CHAOTIC SYSTEM

In 2002, Lü and Chen coined a special chaotic system which can be distinguished from other chaotic systems by its linear part $A = (a_{ij})_{3\times 3}$ with $a_{12} \cdot a_{21} = 0$, and the mathematical model of the system is described by [6]

$$\begin{cases} \dot{x} = a(y - x) \\ \dot{y} = by - xz \\ \dot{z} = xy - cz \end{cases} \tag{1}$$

where a, b, c are real numbers. In this section, we present a new chaotic system according to the system (1). An extra variable is introduced to system (1) as a nonlinear input, and then the new system is given as

$$\begin{cases} \dot{x} = a(y - x) \\ \dot{y} = by - xz - w \\ \dot{z} = xy - cz \\ \dot{w} = yz \end{cases} \tag{2}$$

with four state variables x, y, z, w. It is easy to know that the new system (2) is dissipative because its divergence $\nabla V = \partial\dot{x}/\partial x + \partial\dot{y}/\partial y + \partial\dot{z}/\partial z + \partial\dot{w}/\partial w = -(a - b + c) < 0$ with $a + c > b$. We also can get the only equilibrium $O(0, 0, 0, 0)$ of system (2) by assuming $\dot{x} = \dot{y} = \dot{z} = \dot{w} = 0$. The eigenvalues of the characteristic equation at O can be calculated as $\lambda_1 = 0$, $\lambda_2 = -a$, $\lambda_3 = -c$, $\lambda_4 = b$. If $b > 0$, O is unstable. If $b < 0$ and $a > 0$, $c > 0$, the stability of O can be determined by using center manifold theorem since O is a non-hyperbolic equilibrium with $\lambda_1 = 0$. Here we only consider the dynamic analysis of system (2) with $a > 0$, $b > 0$, $c > 0$ and an unstable equilibrium O.

The dynamic behaviors of system (2) versus the parameter b can be illustrated by using the bifurcation diagrams and Lyapunov exponents. The Fig.1 and Fig.2 present the bifrucation diagrams and Lyapunov exponents with regard to $b \in [13, 17]$ and $b \in (17, 30]$ for the fixed parameter values $a = 39$, $c = 3$.

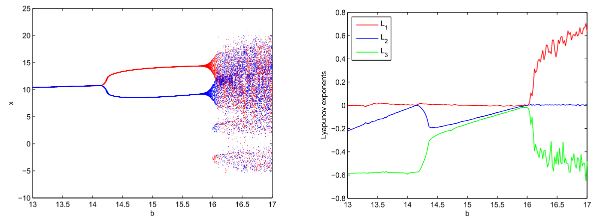


FIGURE 1. **Bifurcation diagrams and Lyapunov exponents of system (2) with $a = 39$, $c = 3$ and $b \in [13, 17]$.**

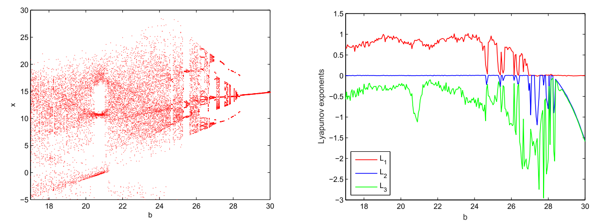


FIGURE 2. **Bifurcation diagrams and Lyapunov exponents of system (2) with $a = 39$, $c = 3$ and $b \in [17, 30]$.**

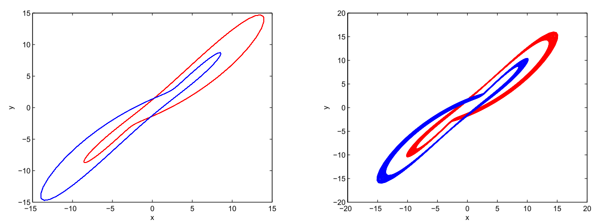


FIGURE 3. **Two coexisting attractors of system (2) with initial values $(\pm1, \pm1, 0, \pm1)$ and: (a) $b = 15$; (b) $b = 16$.**

In Fig.1(a), the independent red and blue branches started from initial values $X_\pm = (\pm1, \pm1, 0, \pm1)$ indicate that system (2) exists coexisting attractors. For $b = 15$, two coexisting limit cycles are observed in system (2) as shown in Fig.3(a). For $b = 16$, two coexisting chaotic attractors are observed in system (2) as shown in Fig.3(b). The Fig.2 shows that system (2) exists chaotic and periodic attractors within $b \in (17, 30]$. By selecting $b = 18, 26, 27, 30$, we can numerically obtain the chaotic and periodic attractors of system (2), as illustrated in Fig.4. The Lyapunov exponents of system (2) with respect to the parameter b shown in Fig.1(b) and Fig.2(b) determines the chaotic and periodic features of system (2), where L_1, L_2, L_3 ($L_1 < L_2 < L_3$) are the first three Lyapunov exponents of system (2), and the minimum Lyapunv exponent L_4 is always less than -5.

III. GENERATION OF COEXISTING ATTRACTORS

The coexisting attractors often have their respective independent basins of attraction in phase space. For the sake of constructing chaotic system with multiple coexisting attractors, a very important thought is to establish multiple invariant sets by extending the number of equilibria of the original system. Taking the system (2) as the original system and we can replicate the attractor of system (2) at different positions in phase space by using some special functions. Here we will apply the method proposed in literature [24] for obtaining coexisting attractors from system (2).

Consider the following nonlinear differential system

$$\dot{X} = F(X), \quad X = (x, y, z, w)^T \in R^4 \tag{3}$$

and assume $\Phi(t, t_0, X_0)$ is the solution of system (3) with respect to initial condition $X_0 = X(t_0)$. Define a compact subset $\Lambda \in R^4$ and the distance between the point X and the set Λ as $d(X, \Lambda) = \inf_{\bar{X}\in A} \|X - \bar{X}\|$. Suppose that $\Lambda_\delta = \{X\,|d(X, \Lambda) < \delta\}$, then $\Lambda \subset \Lambda_\delta$. Based on the literature [33], we can conclude that Λ is an ultimate bound for system (3) if $\lim_{t\to\infty} d(\Phi(t, t_0, X_0), \Lambda) = 0$ for $\forall X_0 \in R^4\backslash\Lambda$ implying that there exists $T > t_0$ such that $\Phi(t, t_0, X_0) \in \Lambda_\delta$ for any $t \geq T, \delta > 0$. Furthermore Λ is considered to be a positively invariant set if $\Phi(t, t_0, X_0) \in \Lambda$ for any $X_0 \in \Lambda, t \geq t_0$. Generally the basin of attraction of the attractor in system (3) is contained in a bounded set if its corresponding invariant set is contained in the bounded set. Thus we just need to construct multiple invariant sets for obtaining multiple coexisting attractors in the system.

Suppose that the system (3) has an attractor with its basin of attraction contained in the following set

$$\Xi = \{(x, y, z, w)\,|\max\{|x|,\ |y|, |z|, |w|\} \leq Q\}$$

where $Q > 0$ is a real number. Both system (3) and the set Ξ can be scaled by applying the transformation $x \to a_1\tilde{x}$, $y \to a_2\tilde{y}$, $z \to a_3\tilde{z}$, $w \to a_4\tilde{w}$ with real numbers $a_i > 0, i = 1, 2, 3, 4$. And then the basin of attraction of the attractor is within the following set

$$\tilde{\Xi} = \{(x, y, z, w)\left||x| \leq \frac{Q}{a_1}, |y| \leq \frac{Q}{a_2}, |z| \leq \frac{Q}{a_3}, |w| \leq \frac{Q}{a_4}\right\}$$

For functions $g_1(x), g_2(y), g_3(z), g_4(w)$ with multiple zeros. By selecting appropriate a_1, a_2, a_3, a_4, we can generate multiple coexisting attractors in the systems $\dot{X} = F(g_1(x), y, z, w)$ along the x-axis, $\dot{X} = F(x, g_2(y), z, w)$ along the y-axis, $\dot{X} = F(x, y, g_3(z), w)$ along the z-axis, $\dot{X} = F(x, y, z, g_4(w))$ along the w-axis and $\dot{X} = F(g_1(x), g_2(y), g_3(z), g_4(w))$ along all the axis. Actually if we apply the functions $g_1(x)$, $g_2(y)$, $g_3(z)$, $g_4(w)$ to the system (3), then the number of the equilibria of system (3) and the attractors around the corresponding equilibria will be extended. To some extent, it means that the number of attractors of system $\dot{X} = F(g_1(x), g_2(y), g_3(z), g_4(w))$ is determined by the number of zeros of functions $g_1(x)$, $g_2(y)$, $g_3(z)$, $g_4(w)$. Next we will illustrate the above results via some numerical examples.

The Fig.4(a) shows the chaotic attractor of system (2) with $a = 39, b = 18, c = 3$ and we can numerically obtain that the attractor is placed in the region

$$\Theta = \{(x, y, z, w)\,||x| \leq 25,\ |y| \leq 25, 0 < z < 40, |w| \leq 80\}$$

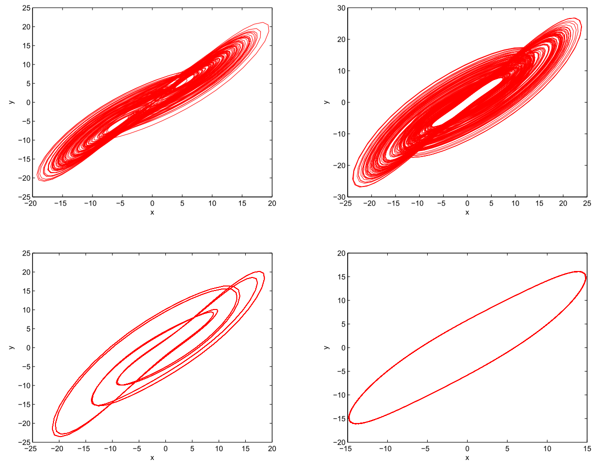


FIGURE 4. Attractors of system (2) with different values of parameter b: (a) $b = 18$; (b) $b = 26$; (c) $b = 27$; (d) $b = 30$.

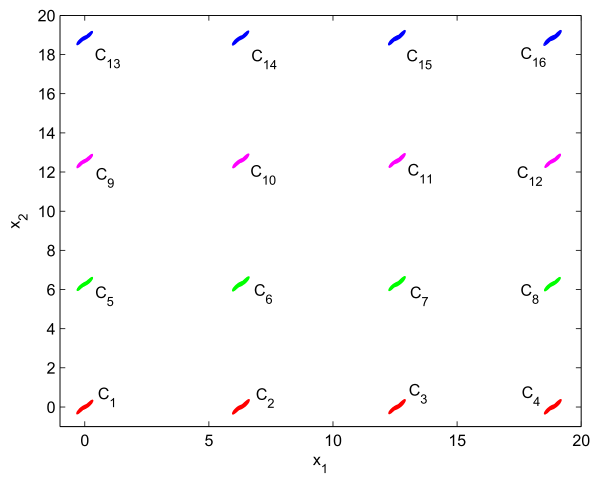


FIGURE 5. Sixteen coexisting chaotic attractors $C_1 - C_{16}$ of system (4) with $a = 39$, $b = 18$, $c = 3$.

Assuming $x = 60x_1, y = 60x_2, z = 60x_3, w = 60x_4$ and applying the function $g_i(\cdot) = \sin(\cdot), i = 1, 2$ to the system (2), we get the following new system

$$\begin{cases} \dot{x}_1 = a(\sin(x_2) - \sin(x_1)) \\ \dot{x}_2 = b\sin(x_2) - 60\sin(x_1)x_3 - x_4 \\ \dot{x}_3 = 60b\sin(x_1)\sin(x_2) - cx_3 \\ \dot{x}_4 = 60\sin(x_2)x_3 \end{cases} \tag{4}$$

Let $a = 39$, $b = 18$, $c = 3$, then the system (4) can yield an infinite number of chaotic attractors from initial values $(1 + 2k\pi, 1 + 2l\pi, 0, 1), k, l = 0, 1, 2, \cdots$ and the attractors are placed along the x_1-axis, x_2-axis. The Fig.5 presents the phase portraits of sixteen coexisting chaotic attractors C_1-C_{16} of system (4) which form along the x_1-axis and x_2-axis in phase space, where C_1-C_4 (red color), C_5-C_8 (green color), C_9-C_{12} (pink color), C_{13}-C_{16} (blue color) are respectively yielded from initial values $(1+2k\pi, 1, 0, 1)$, $(1+2k\pi, 1+2\pi, 0, 1)$, $(1+2k\pi, 1+4\pi, 0, 1)$, $(1+2k\pi, 1+6\pi, 0, 1)$, $k = 0, 2, 4, 6$. All these attractors have same shape. The Fig.6 gives a close look at the attractor C_1.

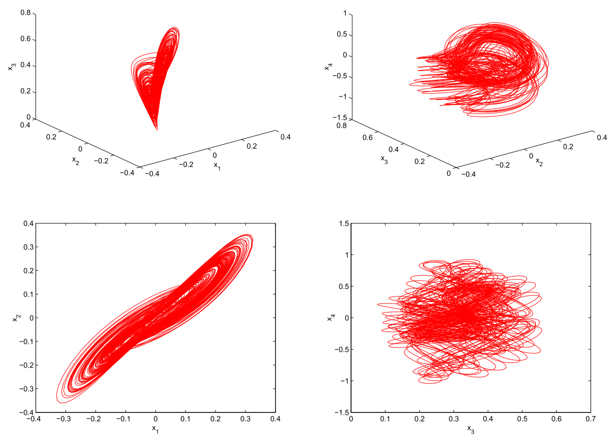


FIGURE 6. Phase portraits of the attractor C_1: (a) x_1-x_2-x_3; (b) x_2-x_3-x_4; (c) x_1-x_2; (d) x_3-x_4.

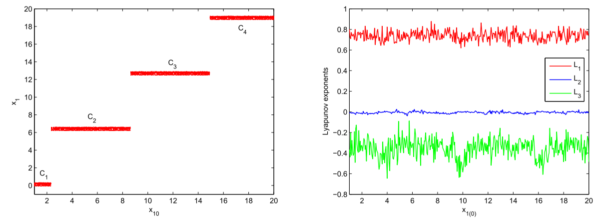


FIGURE 7. Bifurcation diagram and Lyapunov exponents of system (4) with initial values $x_{2(0)} = 1$, $x_{3(0)} = 0$, $x_{4(0)} = 1$ and $x_{1(0)} \in [1, 20]$.

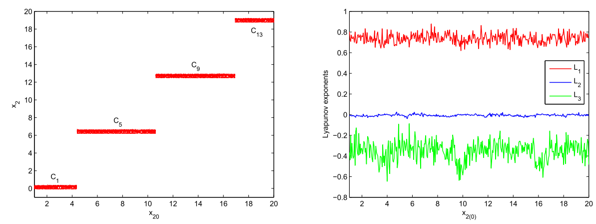


FIGURE 8. Bifurcation diagram and Lyapunov exponents of system (4) with initial values $x_{1(0)} = 1$, $x_{3(0)} = 0$, $x_{4(0)} = 1$ and $x_{2(0)} \in [1, 20]$.

Denote $X_0 = (x_{1(0)}, x_{2(0)}, x_{3(0)}, x_{4(0)})$ as the initial value of system (4) and let $x_{3(0)} = 0$, $x_{4(0)} = 1$. Fix $x_{2(0)} = 1$, we can get the bifurcation diagram and Lyapunov exponents of system (4) with regard to $x_{1(0)} \in [1, 20]$ as illustrated in Fig.7. The Fig.7(a) indicates that system (4) generates four coexisting attractors with the variation of $x_{1(0)} \in [1, 20]$ and these attractors are C_1-C_4 shown in Fig.5. We also get the bifurcation diagram and Lyapunov exponents of system (4) with regard to $x_{2(0)} \in [1, 20]$ and $x_{1(0)} = 1$ as shown in Fig.8. The Fig.8(a) implies that system (4) generates four coexisting attractors with the variation of $x_{2(0)} \in [1, 20]$ and these attractors are C_1, C_5, C_9, C_{13} shown in Fig.5. The first three Lyapunov exponents L_1, L_2, L_3 shown in Fig.7(b) and Fig.8(b) indicate that all these attractors have nearly the same chaotic feature as the change of Lyapunov exponents is very small under different values of $x_{1(0)}, x_{2(0)}$. Thus we can obtain that system (4) has the ability to reproduce attractors in phase space and it is easy to numerically verify that system (4) can generate an infinitely number of coexisting attractors along

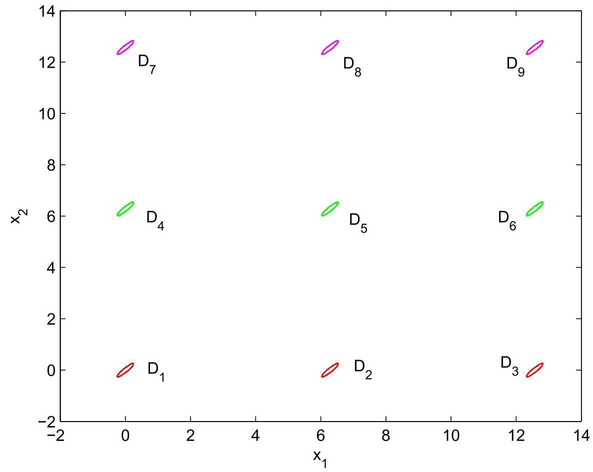


FIGURE 9. Nine coexisting periodic attractors of system (4) with $a = 39$, $b = 30$, $c = 3$.

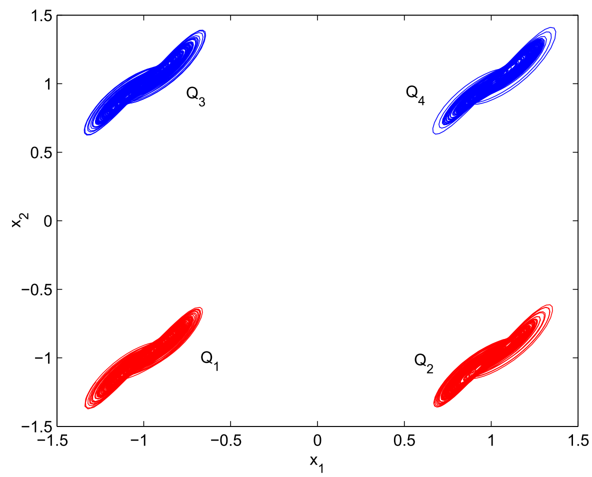


FIGURE 10. Four coexisting chaotic attractors Q_1-Q_4 of system (5) with $a = 39$, $b = 18$, $c = 3$ and the function $x - \mathrm{sgn}(x)$.

x_1-axis, x_2-axis if we choose more different initial values. We also can obtain multiple coexisting periodic attractors from system (4) with parameters $a = 39$, $b = 30$, $c = 3$ and initial values $(1 + 2k\pi, 1 + 2l\pi, 0, 1)$, $k, l = 0, 1, 2, \cdots$ as shown in Fig.9.

Let $g_1(x_1) = x_1 - \mathrm{sgn}(x_1)$, $g_2(x_2) = x_2 - \mathrm{sgn}(x_2)$, then the system (4) can be transformed into the following system

$$\begin{cases} \dot{x}_1 = a(x_2 - \mathrm{sgn}(x_2) - x_1 + \mathrm{sgn}(x_1)) \\ \dot{x}_2 = b(x_2 - \mathrm{sgn}(x_2)) - 60(x_1 - \mathrm{sgn}(x_1))x_3 - x_4 \\ \dot{x}_3 = 60b(x_1 - \mathrm{sgn}(x_1))(x_2 - \mathrm{sgn}(x_2)) - cx_3 \\ \dot{x}_4 = 60(x_2 - \mathrm{sgn}(x_2))x_3 \end{cases} \tag{5}$$

It is easy to verify that system (5) has four different equilibria $O_1(-1, -1, 0, 0)$, $O_2(1, -1, 0, 0)$, $O_3(-1, 1, 0, 0)$ and $O_4(1, 1, 0, 0)$. The numerical results show that system (5) coexists four chaotic attractors Q_1-Q_4 around the equilibria and all these attractors are yielded from initial values $(-1, -1, 0, 1)$, $(1, -1, 0, 1)$, $(-1, 1, 0, 1)$, $(1, 1, 0, 1)$, as shown in Fig.10. If the functions $g_1(x_1), g_2(x_2)$ are replaced

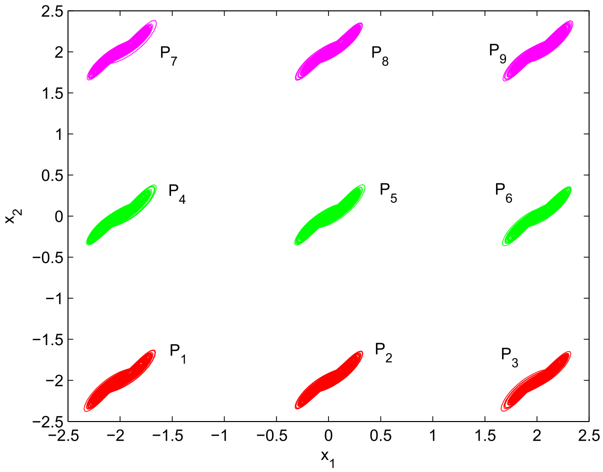


FIGURE 11. Nine coexisting chaotic attractors P_1-P_9 of system (5) with $a = 39$, $b = 18$, $c = 3$ and the function $x - \mathrm{sgn}(x+1) - \mathrm{sgn}(x-1)$.

by $g_1(x_1) = x_1 - \sum_{i=1}^{n} k_i \mathrm{sgn}(x_1 + s_i)$, $g_2(x_2) = x_2 - \sum_{i=1}^{n} k_i \mathrm{sgn}(x_2 + s_i)$ (k_i s_i are real numbers), then an infinite number of coexisting attractors will be generated in system (5). The Fig.11 shows nine coexisting chaotic attractors in system (5) with $g_1(x_1) = x_1 - \mathrm{sgn}(x_1+1) - \mathrm{sgn}(x_1-1)$, $g_2(x_2) = x_2 - \mathrm{sgn}(x_2+1) - \mathrm{sgn}(x_2-1)$. Actually we also can construct multiple coexisting attractors along x_3-axis, x_4-axis by using sine function, sign function and other functions with multiple zeros. Moreover, we can get not only the same type of coexisting attractors, but also different types of coexisting attractors for selecting proper system parameters and initial values.

IV. CONCLUSIONS AND DISCUSSIONS

A novel four-dimensional chaotic system with only one equilibrium with non-hyperbolic feature was created. Based on this novel system, a simple method was used to produce an infinite number of coexisting chaotic attractors, and some numerical examples were given to illustrate the effectiveness of the method. For selecting some initial values, sixteen (or nine) coexisting chaotic attractors are obtained in the system with sine (or sign) function. Actually the appearance of multiple coexisting attractors depends on the number of equilibria which determines the location of domain of attraction to some extent. So if the number of equilibrium points can be increased by some methods, the system will easily generate multiple coexisting attractors. We will continue to forward the research of the coexisting attractors via proposing its generation methods and engineering applications.

REFERENCES

[1] A. Garfinkel, M. L. Spano, W. L. Ditto, and J. N. Weiss, "Controlling cardiac chaos," *Science*, vol. 257, no. 5074, pp. 1230–1235, 1992.

[2] S. Hayes, C. Grebogi, and E. Ott, "Communicating with chaos," *Phys. Rev. Lett.*, vol. 70, no. 20, pp. 3031–3034, May 1993.

[3] L. Bakemeier, A. Alvermann, and H. Fehske, "Route to chaos in optomechanics," *Phys. Rev. Lett.*, vol. 114, no. 1, 2015, Art. no. 013601.

[4] J. Kadmon and H. Sompolinsky, "Transition to chaos in random neuronal networks," *Phys. Rev. X*, vol. 5, no. 4, 2015, Art. no. 041030.

[5] G. Chen and T. Ueta, "Yet another chaotic attractor," *Int. J. Bifurcation Chaos*, vol. 9, no. 7, pp. 1465–1466, 1999.

[6] J. Lü and G. Chen, "A new chaotic attractor coined," *Int. J. Bifurcation Chaos*, vol. 12, no. 3, pp. 659–661, 2002.

[7] J. C. Sprott, "Simple chaotic systems and circuits," *Amer. J. Phys.*, vol. 68, no. 8, pp. 758–763, 2000.

[8] V.-T. Pham, C. Volos, S. Jafari, Z. Wei, and X. Wang, "Constructing a novel no-equilibrium chaotic system," *Amer. J. Phys.*, vol. 24, no. 5, 2014, Art. no. 1450073.

[9] S. Jafari, J. C. Sprott, and S. M. R. H. Golpayegani, "Elementary quadratic chaotic flows with no equilibria," *Amer. J. Phys.*, vol. 377, no. 9, pp. 699–702, 2013.

[10] C. Li, J. C. T. Wesley, H. C. I. Herbert, and T. Lu, "A memristive chaotic oscillator with increasing amplitude and frequency," *IEEE Access*, vol. 6, pp. 12945–12950, 2018.

[11] J. Kengne, Z. N. Tabekoueng, V. K. Tamba, and A. N. Negou, "Periodicity, chaos, and multiple attractors in a memristor-based Shinriki's circuit," *Chaos*, vol. 25, no. 10, 2015, Art. no. 103126.

[12] J. Kengne, A. N. Negou, and D. Tchiotsop, "Antimonotonicity, chaos and multiple attractors in a novel autonomous memristor-based jerk circuit," *Nonlinear Dyn.*, vol. 88, no. 4, pp. 2589–2608, 2017.

[13] Q. Lai and S. Chen, "Coexisting attractors generated from a new 4D smooth chaotic system," *Int. J. Control, Automat. Syst.*, vol. 14, no. 4, pp. 1124–1131, 2016.

[14] Q. Lai, T. Nestor, J. Kengne, and X.-W. Zhao, "Coexisting attractors and circuit implementation of a new 4D chaotic system with two equilibria," *Chaos, Solitons Fractals*, vol. 107, pp. 92–102, Feb. 2018.

[15] Q. Lai, B. Norouzi, and F. Liu, "Dynamic analysis, circuit realization, control design and image encryption application of an extended Lü system with coexisting attractors," *Chaos, Solitons Fractals*, vol. 114, pp. 230–245, Sep. 2018.

[16] D. de Carvalho Braga and L. F. Mello, "A study of the coexistence of three types of attractors in an autonomous system," *Int. J. Bifurcation Chaos*, vol. 23, no. 12, 2014, Art. no. 1350203.

[17] Q. Li, H. Zeng, and X.-S. Yang, "On hidden twin attractors and bifurcation in the Chua's circuit," *Nonlinear Dyn.*, vol. 77, nos. 1–2, pp. 255–266, 2014.

[18] M. F. Danca, N. Kuznetsov, and G. Chen, "Unusual dynamics and hidden attractors of the Rabinovich–Fabrikant system," *Nonlinear Dyn.*, vol. 88, no. 1, pp. 791–805, 2017.

[19] C. Li and J. C. Sprott, "Multistability in the Lorenz system: A broken butterfly," *Int. J. Bifurcation Chaos*, vol. 24, no. 10, 2014, Art. no. 1450131.

[20] C. Li and J. C. Sprott, "Multistability in a butterfly flow," *Int. J. Bifurcation Chaos*, vol. 23, no. 12, 2013, Art. no. 1350199.

[21] C. Li, W. J.-C. Thio, J. C. Sprott, H. H.-C. Iu, and Y. Xu, "Constructing infinitely many attractors in a programmable chaotic circuit," *IEEE Access*, vol. 6, pp. 29003–29012, 2018.

[22] C. Li, J. C. Sprott, and H. Xing, "Constructing chaotic systems with conditional symmetry," *Nonlinear Dyn.*, vol. 87, no. 2, pp. 1351–1358, 2017.

[23] Q. Lai and S. Chen, "Generating multiple chaotic attractors from Sprott B system," *Int. J. Bifurcation Chaos*, vol. 26, no. 11, 2016, Art. no. 1650177.

[24] X. Zhang and G. Chen, "Constructing an autonomous system with infinitely many chaotic attractors," *Chaos*, vol. 27, no. 7, 2017, Art. no. 071101.

[25] C. R. Hens, R. Banerjee, U. Feudel, and S. K. Dana, "How to obtain extreme multistability in coupled dynamical systems," *Phys. Rev. E, Stat. Phys. Plasmas Fluids Relat. Interdiscip. Top.*, vol. 85, no. 3, 2012, Art. no. 035202.

[26] J. Kengne, Z. T. Njitacke, and H. B. Fotsin, "Dynamical analysis of a simple autonomous jerk system with multiple attractors," *Nonlinear Dyn.*, vol. 83, nos. 1–2, pp. 751–765, 2016.

[27] M.-F. Danca, "Hidden chaotic attractors in fractional-order systems," *Nonlinear Dyn.*, vol. 89, no. 1, pp. 577–586, 2017.

[28] E. M. Ozbudak, M. Thattai, H. N. Lim, B. I. Shraiman, and A. Oudenaarden, "Multistability in the lactose utilization network of *Escherichia coli*," *Nature*, vol. 427, no. 6976, pp. 737–740, 2004.

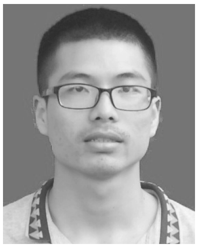
[29] M. Laurent and N. Kellershohn, "Multistability: A major means of differentiation and evolution in biological systems," *Nature*, vol. 24, no. 11, pp. 418–422, 1999.

[30] S. Banerjee, "Coexisting attractors, chaotic saddles, and fractal basins in a power electronic circuit," *IEEE Trans. Circuits Syst. I, Fundam. Theory Appl.*, vol. 44, no. 9, pp. 847–849, Sep. 1997.

[31] H. G. Solari, E. Eschenazi, R. Gilmore, and J. R. Tredicce, ''Influence of coexisting attractors on the dynamics of a laser system,'' *Opt. Commun.*, vol. 64, no. 1, pp. 49–53, 1987.

[32] C.-Y. Cheng, K.-H. Lin, and C.-W. Shih, ''Multistability and convergence in delayed neural networks,'' *Phys. D, Nonlinear Phenomena*, vol. 225, no. 1, pp. 61–74, Jan. 2007.

[33] D. Li, J. Lü, X. Wu, and G. Chen, ''Estimating the ultimate bound and positively invariant set for the Lorenz system and a unified chaotic system,'' *J. Math. Anal. Appl.*, vol. 323, no. 2, pp. 844–853, 2006.



QIANG LAI was born in Jiangxi, China, in 1987. He received the Ph.D. degree in control theory and control engineering from the Huazhong University of Science and Technology, in 2014. He is currently an Associate Professor with the School of Electrical and Automation Engineering, East China Jiaotong University, Nanchang, China. He is also a member of the Key Laboratory of Advanced Control and Optimization of Jiangxi Province. He has authored more than 30 journal papers. His current research interests include nonlinear dynamics, secure communication, complex networks, multi-agent systems, gene regulatory networks, chaos control, and chaotification. He is also a member of the Technical Program Committee of several international conferences. He served as a Reviewer for more than 30 renowned international journals, including the IEEE Transactions on Circuit and Systems I, the IEEE Transactions on Industrial Electronics, the IEEE Access, *Nonlinear Dynamics*, the *International Journal of Bifurcation and Chaos*, and *Robotics and Autonomous Systems*.

CHAOYANG CHEN received the B.Sc. degree in mathematics and applied mathematics from the Hunan University of Science and Technology, in 2007, the M.Sc. degree in applied mathematics from Guangxi Teachers Education University, in 2010, and the Ph.D. degree in control theory and control engineering from the Huazhong University of Science and Technology, in 2014. He was a Postdoctoral Fellow with Central South University, from 2015 to 2017. He is currently an Associate Professor with the School of Information and Electrical Engineering, Hunan University of Science and Technology, Xiangtan, China. His research interests include adaptive control, nonlinear control, neural network control, networked control systems, complex networks, and multi-agent systems.

XIAO-WEN ZHAO received the B.Sc. degree in mathematics and applied mathematics from Anqing Normal University, in 2006, the M.Sc. degree in applied mathematics from Anhui University, in 2009, and the Ph.D. degree in control theory and control engineering from the Huazhong University of Science and Technology, in 2017. He is currently a Lecturer with the School of Mathematics, Hefei University of Technology, Hefei, China. His current research interests include cooperative control of multi-agent systems, nonlinear dynamics, complex networks, fractional-order systems, chaos control, and synchronization.

JACQUES KENGNE was born in Bamougoum, Cameroon, in 1971. He received the M.Sc. and Ph.D. degrees in electronics from the Faculty of Sciences, University of Dschang, in 2007 and 2011, respectively. From 2010 to 2012, he was a Lecturer with the Department of Electrical Engineering, University of Dschang. From 2012 to 2016, he was a Senior Lecturer. In 2016, he was appointed as an Associate Professor with the University of Dschang. He has authored or co-authored more than 50 journal papers. His research interests include nonlinear systems and circuits, chaos, multistability, and chaos synchronization with applications. He serves as a Reviewer for renowned international journals, including *Chaos*, the *International Journal of Bifurcation and Chaos*, *Nonlinear Dynamics*, *Chaos, Solitons and Fractals*, and the IEEE Transactions on Circuits and Systems I.

CHRISTOS VOLOS received the Diploma degree in physics, the M.Sc. degree in electronics, and the Ph.D. degree in chaotic electronics from the Physics Department, Aristotle University of Thessaloniki, Greece, in 1999, 2002, and 2008, respectively, where he is currently an Assistant Professor. He is also a member of the Laboratory of Nonlinear Systems, Circuits and Complexity, Physics Department, Aristotle University of Thessaloniki. His current research interests include the design and study of analog and mixed-signal electronic circuits, chaotic electronics and their applications (secure communication, cryptography, and robotics), experimental chaotic synchronization, chaotic UWB communication, and measurement and instrumentation systems.