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An Improved Method to Transform Triangular Fuzzy Number Into Basic Belief Assignment in Evidence Theory

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ABSTRACT Dempster-Shafer evidence theory (D-S theory) is a developing theory to solve the uncertain problems and it has an important impact on many fields such as information fusion, expert systems, and machine learning. One of the main points of D-S theory is about how to generate a reliable basic belief assignment. Furthermore, the data collected from the multi-sources may be influenced by noise or other factors which cause conflicts in practical applications. Since the fuzzy number is useful to construct the target model for generating basic belief assignments, in this paper, an improved method to obtain basic belief assignment is proposed based on the triangular fuzzy number and k-means++ algorithm. First, the k-means++ clustering method is used to construct the target model. Then, the difference degree between the target model and sample model is calculated to generate the initial basic belief assignments. After that, the conflicts will be resolved by using the discount coefficient method. Finally, Dempster's combination rule is used to combine initial basic belief assignments to obtain the final result. The applications in recognition problems of the Iris data set and Wine Quality data set illustrate that the proposed method is effective to generate the basic belief assignments and keeps a high recognition rate even under a noisy environment.

INDEX TERMS Basic belief assignment, Dempster-Shafer evidence theory, triangular fuzzy number, white Gaussian noise.

I. INTRODUCTION

Multi-source information fusion technique plays an important role in many fields such as sustainable market valuation [1], health diagnosis of equipment [2], [3], emerging technology commercialization evaluation [4], energy management strategy [5], and decision-making problems [6], [7]. However, the data collected from multi-sources are often affected by environment or other factors that may cause imprecise data in practical application [8], [9]. How to deal with these uncertain problems is still a great challenge [10]–[12]. In order to solve the problems of uncertainty, the researchers put forward many effective methods including the extended fuzzy theory [13]–[15], evidence theory [16], [17], evidential reasoning [18], [19], quantum-based [20], [21], D numbers [22]–[24], R numbers [25], [26], Z numbers [27], [28], entropy [29], [30], and technique for order preferenceby similarity to ideal solution [31]. In among methods, Dempster-Shafer evidence theory (D-S theory) is an efficient math tool in multi-source information fusion technique [32]. This theory was proposed to deal with those uncertain problems by Dempster [33] and improved by Shafer [34] later. D-S theory is the generalization of Bayes' theorem and it is more effective than Bayes' theorem [35]. It extends the space of elementary event to the power set of frame of discernment [36], [37]. This theory has the ability to determine a reliable result by fusing multi-source and uncertain information with the rule of combination [38], [39]. Nowadays, D-S theory is widely applied in many fields such as recommender system [40], decision-making [41], [42], fault diagnosis [43], identification problem [44], [45] and pattern classification [46], [47]. Whereas, the generation of basic belief assignment (BBA) is the main point of D-S theory and it is still an open issue.

Basically, there are two kinds of methods to generate the BBAs [48], [49]. One kind of method is to determine the weight based on the experts' analysis, then generate the BBAs. However, this method often causes conflicts due to

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the subjectivity of experts. The other way to generate the BBAs is based on a target model from sample data. This method has several advantages such as strong objectivity and clear mathematical theory so it has become a popular method in recent years. Specifically, Kang *et al.* [50] proposed a method to transform interval number model into the BBAs. Xu [51] studied the decision making problem and proposed a multi-attribute decision making method based on the similarity degree of fuzzy numbers. In recent years, D-S theory is often combined with fuzzy set theory to construct the target model and handle uncertain problems [52], [53]. As a powerful fuzzy number, triangular fuzzy number is commonly applied in the practice due to its simplicity and practicability [54]. Later on, Jiang *et al.* [55] proposed a method to transform triangular fuzzy number model into the BBAs. In particular, the triangular fuzzy number model (TFN model) in [55] takes the maximum and minimum as its upper and lower bound value, and takes average as its intermediate value. However, in a noisy environment, data collected by multi-sources may be imprecise and even have great difference with each other. When the extremes (i.e., maximum and minimum) are not precise enough to transform into the correct BBAs, the construction of triangular fuzzy number model should be exploited to improve the anti-noise capability.

By studying the existing related works, it is found that k-means++ clustering method uses clustering centers to model the data. For this reason, k-means++ clustering method is used to find three clustering centers to replace the maximum, average and minimum values in TFN model. Compared with Jiang *et al.*'s method, the proposed method considers the weakness of TFN model and introduces k-means++ clustering method to establish triangular fuzzy numbers. This is the main contribution of the proposed method which is the difference between the proposed method and Jiang *et al.*'s method. To some extent, the proposed method can be more flexible and effective than TFN model in [55].

There are several steps of proposed method. Firstly, the triangular fuzzy number model is constructed by using k-means++ clustering method. After that, the similarity degree between the sample and the target model is calculated. Then the distance between evidence bodies is measured by Jousselme evidence distance. Finally, the Dempster's combination rule is used to obtain the final BBA. Experimental results illustrate that the proposed method has a better performance in transforming TFN model into the BBAs. Meanwhile, the proposed method keeps the high recognition accuracy rate. Furthermore, the proposed method performs better robustness and has the anti-noise capability. To sum up, the proposed method has below contributions.

- The proposed method proposes a reliable method to transform triangular fuzzy number into BBA even in a noisy environment;
- The proposed method improves the construction of triangle fuzzy number with clustering method;
- The proposed method has the high recognition accuracy rate and the capacity of anti-noise.

This paper is organized as follows. In Section 2, some basic concepts are introduced including D-S theory, Jousselme evidence distance, fuzzy set theory and k-means++ clustering method. In Section 3, a new method is presented for transforming TFN model into the BBAs. In Section 4, a numerical example is given to show the procedures of the proposed method. In Section 5, Iris data set and Wine Quality data set are used in the applications of recognition problems to explain and analyze the performance of new method. Finally, conclusions are given in Section 6.

II. PRELIMINARIES

A. DEMPSTER-SHAFER EVIDENCE THEORY

Dempster-Shafer evidence theory, also referred as D-S theory, is an important tool in information fusion and decision-making. It was proposed by Dempster [33] and improved by Shafer [34] later. The D-S theory has the ability to handle imprecise data and uncertain information [56]–[58]. Therefore, it is applied in many fields [59]–[62]. Some preliminaries of D-S theory are given as follows.

1) FRAME OF DISCERNMENT

In D-S theory, a set of hypotheses $\Theta = \{H_1, H_2, \cdots, H_N\}$ called the frame of discernment (FOD). Suppose the FOD includes the exhaustive hypotheses of variable V. A hypothesis is a possible result of V such as H_1. Each element of FOD is mutually exclusive. Let $P(\Theta)$ denotes the power set of Θ:

$$P(\Theta) = \{\emptyset, H_1, \cdots, H_N, \{H_1 \cup H_2\}, \{H_1 \cup H_3\}, \cdots, \Theta\}. \tag{1}$$

The $\emptyset$ denotes the empty set. The $P(\Theta)$ includes 2^N propositions of Θ.

2) BASIC BELIEF ASSIGNMENT

Let A denote a subset on FOD Θ. The BBA is defined as the function $m : P(\Theta) \to [0, 1]$ and

$$m(\emptyset) = 0, \tag{2}$$

$$\sum_{A \subseteq \Theta} m(A) = 1. \tag{3}$$

The function $m(A)$ expresses how strongly the evidence supports A. If $m(A) > 0$, then A is a focal element [63], [64].

3) DEMPSTER'S COMBINATION RULE

Dempster's combination rule, or orthogonal sum noted by $m = m_1 \oplus m_2$, can combine two BBAs m_1 and m_2 to obtain a new BBA. Suppose two BBAs m_1 and m_2 belong to the same FOD and support different propositions. Focal elements of m_1 are denoted as B_i and focal elements of m_2 are denoted as C_j:

$$m(A) = \frac{\sum_{B_i \cap C_j = A} m_1(B_i) m_2(C_j)}{1 - k} \quad (A \neq \emptyset) \tag{4}$$

and

$$k = \sum_{B_i \cap C_j = \emptyset} m_1(B_i) m_2(C_j) \tag{5}$$

where k is a coefficient which illustrates the conflict between two pieces of evidence or BBAs [65], [66]. It should be carefully handled in conflicting management [67], [68]. In addition, to obtain the result of combination between N information sources, the function is defined as

$$m = m_1 \oplus m_2 \oplus \cdots \oplus m_N. \tag{6}$$

4) PIGNISTIC PROBABILITY

Smets [69] proposed a method to transform a BBA to a Pignistic probability function on a FOD. This function is denoted as $BetP$. Let $m(A)$ be a BBA on the FOD Θ, thus its Pignistic probability function $BetP : \Theta \to [0, 1]$ is

$$BetP(A) = \sum_{B \subseteq \Theta} \frac{|A \cap B|}{|B|} \cdot \frac{m(B)}{1 - m(\emptyset)}, \quad \forall A \subseteq \Theta, \tag{7}$$

where $m(\emptyset) \neq 1$ and $|A|$ is the cardinality of subset A.

B. JOUSSELME EVIDENCE DISTANCE

A new distance was proposed by Jousselme *et al.* [70] to measure the distance between two bodies of evidence. This method expresses the difference between evidence. Suppose m_1 and m_2 are two BBAs on the same FOD Θ, containing N mutually exclusive and exhaustive hypotheses. The distance between m_1 and m_2 is

$$d_{BBA}(m_1, m_2) = \sqrt{\frac{1}{2}(\vec{m}_1 - \vec{m}_2)^T \underline{\underline{D}}(\vec{m}_1 - \vec{m}_2)}, \tag{8}$$

where $\underline{\underline{D}}$ is a $2^N \times 2^N$ matrix and its elements are

$$D(A, B) = \frac{|A \cap B|}{|A \cup B|}(A, B \in P(\Theta)). \tag{9}$$

The coefficient $\frac{1}{2}$ is required to normalize and to ensure that $0 \leqslant d_{BBA}(m_1, m_2) \leqslant 1$.

C. FUZZY SET THEORY

In classical set theory, an element either belongs or does not belong to the set is certain. However, in real life, some situations are hard to define or describe. The fuzzy set theory was proposed by Zadeh [71] as an extension of the classical concept of set and later applied by many researchers [72], [73]. Fuzzy set theory defines the grade of membership to determine the membership of elements, which is described with a membership function valued in the interval [0, 1]. Furthermore, triangular fuzzy number is one of the simplest and most commonly used fuzzy numbers. Some relative notions on triangular fuzzy number are given as follows.

1) GRADE OF MEMBERSHIP

To explain the relationship between elements and fuzzy set, Zadeh defined the grade of membership [71]. Let U be a space of objects. Let x be a generic element of U, that is, it represents all the elements in U. This relation is denoted as $U = \{x\}$. And let A be a fuzzy set in U. A membership function $\mu_A(x)$ associates each object in U with a real number in the interval [0, 1]. Thus the value of $\mu_A(x)$ at x represents the grade of membership of x in A, denoted as

$$\mu_A : U \to [0, 1]; \quad x \mapsto \mu_A(x) \in [0, 1]. \tag{10}$$

The value of $\mu_A(x)$ has positive correlation with the grade of membership of x in A. In particular, when x does or does not belong to A, the value of $\mu_A(x)$ takes 1 or 0, respectively.

2) TRIANGULAR FUZZY NUMBER

If the membership function of a fuzzy set is defined as follows, it is a triangular fuzzy number, which is denoted as:

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{\omega(x-a)}{b-a}, & 0 \leqslant a \leqslant x \leqslant b; \\ \omega, & x = b; \\ \dfrac{\omega(c-x)}{c-b}, & b \leqslant x \leqslant c; \\ 0, & other. \end{cases} \tag{11}$$

The coefficients a and c are the lower bound and upper bound of fuzzy set. The coefficient b is the value of membership function when the grade of membership equal to 1. In the membership function, ω is the height of triangular fuzzy number. Generally speaking, the coefficient $\omega = 1$, and this fuzzy number is a regular triangular fuzzy number which denotes as $\widetilde{A} = (a, b, c; 1)$. If $\omega \in [0, 1)$, this fuzzy number is a generalized triangular fuzzy number and denotes as $\widetilde{A} = (a, b, c; \omega)$.

D. K-MEANS++ CLUSTERING METHOD

Cluster analysis is an important method in pattern recognition. Clustering divides samples into groups (or clusters) according to the similarity between objects. For example, let S denote a data set with n samples, $S = \{s_1, s_2, \cdots, s_n\}$. After operation, S is divided into c subsets ($2 \leqslant c \leqslant n$). In some sense, samples in the same subset are more similar to each other than to those in other subsets.

There is an important clustering method: k-means++ clustering method [74]. It is often used to model the data and find the local optimums. k-means++ clustering method is a hard clustering method developed from k-means clustering method. Hard clustering method means that each object must be recognized into a certain group (or cluster).

The main steps of k-means++ clustering method are as follows.

Step 1: Randomly select a sample from data set as initial clustering center c_1;

Step 2: Calculate the shortest distance between each sample s_i and known clustering centers, that is, the distance from sample to the nearest clustering center. The shortest distance is denoted by $D(S)$;

Step 3: Calculate the probability of each sample which can be selected as the next clustering center, and the next clustering center is selected based on this probability $P(s)$:

$$P(s) = \frac{D(s)^2}{\sum_{s \in S} D(s)^2}. \tag{12}$$

Step 4: Repeat *Step* 2 and *Step* 3 until K clustering centers are selected;

Step 5: Calculate the distance between each sample s_i of the data set and K clustering centers. According to the shortest distance of samples, they are assigned to corresponding clusters represented by clustering centers;

Step 6: For each cluster, calculate its clustering center c_i again:

$$c_i = \frac{1}{|c_i|} \sum_{s \in c_i} s. \tag{13}$$

Step 7: Repeat *Step* 5 and *Step* 6 until clustering centers c_i no longer change.

III. THE PROPOSED METHOD

For domain $X = (x_1, x_2, \cdots, x_i, \cdots, x_n)$ with n classes, each class has k attributes, the attributes of class x_i are denoted as $x_{i_1}, x_{i_2}, \cdots, x_{i_k}$. The origin data set is divided into two parts: training data and test data. Training data are used to build TFN model and test data are used to test the method. Let a be a test sample from test data, the attributes of a are denoted as $a_1, a_2, \cdots, a_k$. The steps to generate the BBA are shown in Fig. 1.

Step 1: Construct triangular fuzzy number model.

In this step, the training data is used to establish the target model by applying k-means++ method.

Select m instances from class x_i as training data. The matrix of training data is a $k \times m$ matrix. Each of columns is an instance with k attributes. Each of rows is one attribute. k-means++ method is used to obtain clustering centers for each attribute of class x_i. The training data includes m instances, so each attribute includes m values. After using k-means++ clustering method, m instances are divided into 3 clusters. Every cluster is characterized by a value named clustering center. Here the clustering centers are denoted as c_1, c_2 and c_3. After that, the clustering centers are used to construct the TFN model so the values of TFN are denoted as $a_{ij} = c_1$, $b_{ij} = c_2$ and $c_{ij} = c_3$.

Let $\widetilde{A}_{ij}$ denote the TFN model of class x_i with attribute j. Thus the triangular fuzzy number model is

$$\widetilde{A}_{ij} = (a_{ij}, b_{ij}, c_{ij}; 1). \tag{14}$$

Let T_i denote all TFN models of class x_i,

$$T_i = (\widetilde{A}_{i1}, \widetilde{A}_{i2}, \cdots, \widetilde{A}_{ik}). \tag{15}$$

Whereas, each class of training data has k TFN models, n classes have $n \times k$ TFN models. Thus they can be expressed as a $n \times k$ TFN model matrix T,

$$T = (T_1, T_2, \cdots, T_i, \cdots, T_n)'. \tag{16}$$

In D-S theory, each subset of the power set $P(\Theta)$ associates with a real number in the interval [0, 1]. This kind of mapping relation reflects how strongly the evidence supports different propositions. The power set $P(\Theta)$ includes empty set, singletons and their supersets. However, matrix T only includes individual classes, that is, the singletons of $P(\Theta)$. To produce corresponding TFN model of multi-subset, the intersections of TFN models (i.e., the generalized triangular fuzzy number) are taken in each column. Xiao *et al*. [75] proposed a method to determine generalized triangular fuzzy number, some cases are shown in the Fig. 2. As for the upper and lower bound of generalized triangular fuzzy number, its lower bound is the higher value in lower bounds of two triangular fuzzy numbers, its upper bound is the lower value in upper bounds of two triangular fuzzy numbers. Mostly, the intersection is a smaller triangle which is shown in Fig. 2 (1). In this case, the generalized triangular fuzzy number is taken as the triangular fuzzy number $(a, b, c; \omega_1)$. Whereas, there are other cases like Fig. 2 (2-4) where the intersection is not a triangular. Thus the following measure should be taken. The height of generalized triangular fuzzy number is the greatest value in ω_1, ω_2 and ω_3, which is denoted as ω_h. Suppose the greatest value in ω_1, ω_2 and ω_3 is ω_1 in Fig. 2, that is, $\omega_h = \omega_1$. Therefore, the generalized triangular fuzzy number is $(a, b, c; \omega_1)$.

When there is no intersection, the triangular fuzzy number should take (0, 0, 0; 0) to represent this case. In domain, n classes generate $2^n - n - 1$ generalized triangular fuzzy numbers. Thus there are totally $2^n - 1$ triangular fuzzy numbers in a column. The same operations are taken for each column of the $n \times k$ matrix T and the $(2^n - 1) \times k$ matrix TI of target model is obtained,

$$TI = (T_1, T_2, \cdots, T_{2^n-1})'. \tag{17}$$

Step 2: Construct difference matrix of model and sample.

In this step, an augmented matrix is first obtained by combining the matrix TI with the sample test. Thus the relation between training data and test data is reflected in a matrix. After that, the difference matrix and similarity matrix will be obtained based on features of TFN.

At first, the triangular fuzzy numbers of sample a are constructed. Suppose the attributes of sample a are $a = (a_1, a_2, \cdots, a_k)$. The triangular fuzzy number of attribute a_k is

$$\widetilde{a}_k = (a_k, a_k, a_k; 1). \tag{18}$$

The triangular fuzzy numbers matrix of the sample is

$$T_a = (\widetilde{a}_1, \widetilde{a}_2, \cdots, \widetilde{a}_k). \tag{19}$$

The matrix TI of target model is combined with matrix T_a of the sample. An augmented matrix TA is built:

$$TA = (T_1, T_2, \cdots, T_{2^n-1}, T_a)'. \tag{20}$$

After that, the maximum of each column is taken to normalize the matrix TA. Each column is divided by k times its maximum. Here the coefficient k is an appropriate number

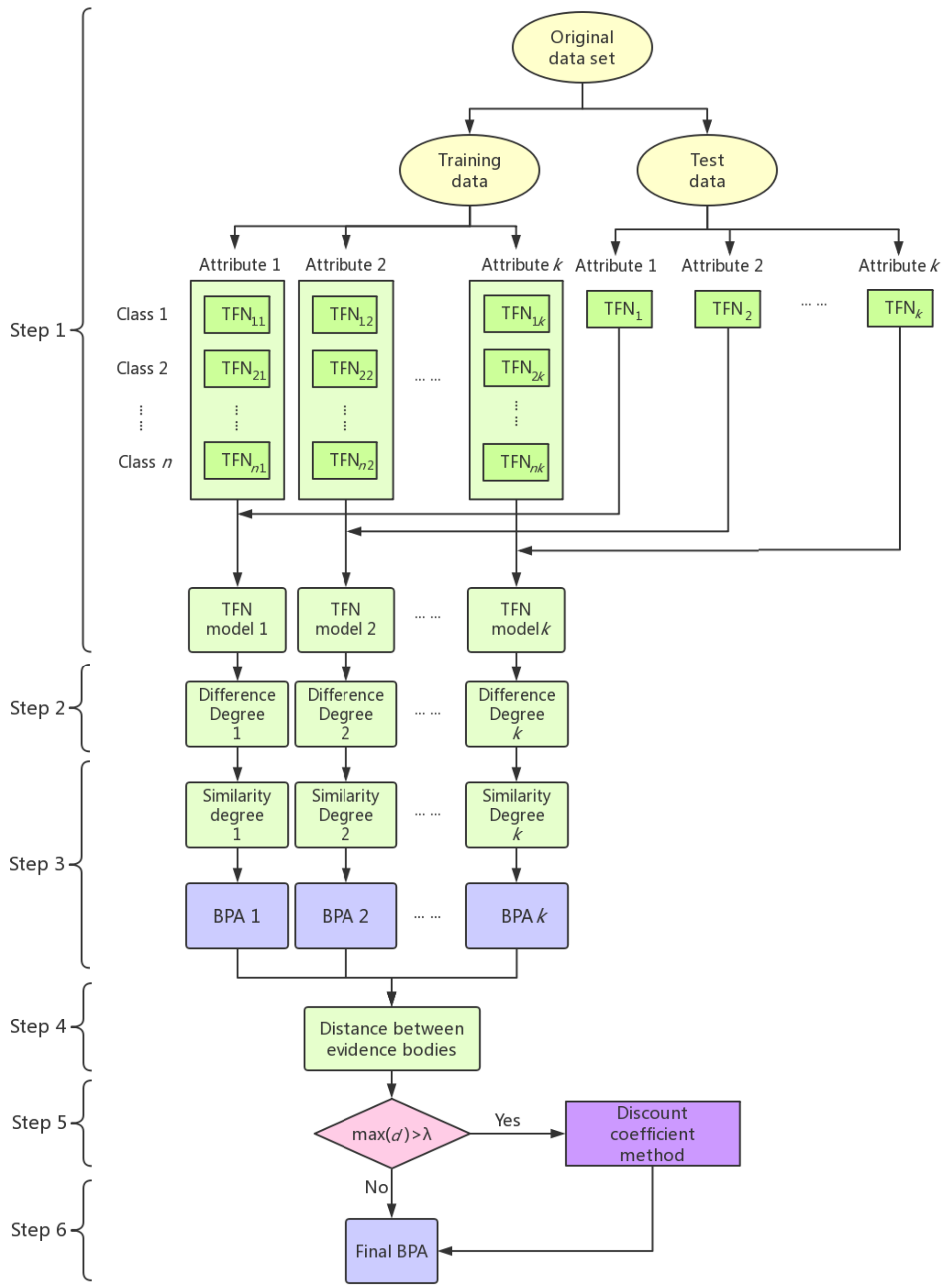

FIGURE 1. **Steps to obtain the final BBA.**

bigger than 1. Each element of matrix TA is normalized into interval [0,1] so that a matrix TN is obtained,

$$TN = (\overline{T}_1, \overline{T}_2, \cdots, \overline{T}_{2^n-1}, \overline{T}_a)'. \tag{21}$$

After normalizing, each element of the matrix TN is in the interval [0,1]. Based on the matrix TN, the left average area and right average area of each triangular fuzzy number in it are calculated according to the paper [55]. Let $\widetilde{A}_{ij} = (a_{ij}, b_{ij}, c_{ij}; \omega_{ij})$ denote a TFN model of matrix TN, its left average area $S_L(\widetilde{A}_{ij})$ and right average area $S_R(\widetilde{A}_{ij})$ are

$$S_L(\widetilde{A}_{ij}) = \frac{\frac{(a_{ij}+b_{ij})\omega_{ij}}{2} + \frac{(b_{ij}+c_{ij})\omega_{ij}}{2}}{2}, \tag{22}$$

$$S_R(\widetilde{A}_{ij}) = \frac{\frac{(1-b_{ij}+1-c_{ij})\omega_{ij}}{2} + \frac{(1-b_{ij}+1-a_{ij})\omega_{ij}}{2}}{2}. \tag{23}$$

Then the sample difference degrees between the target models and the sample are calculated to get a $(2^n - 1) \times k$

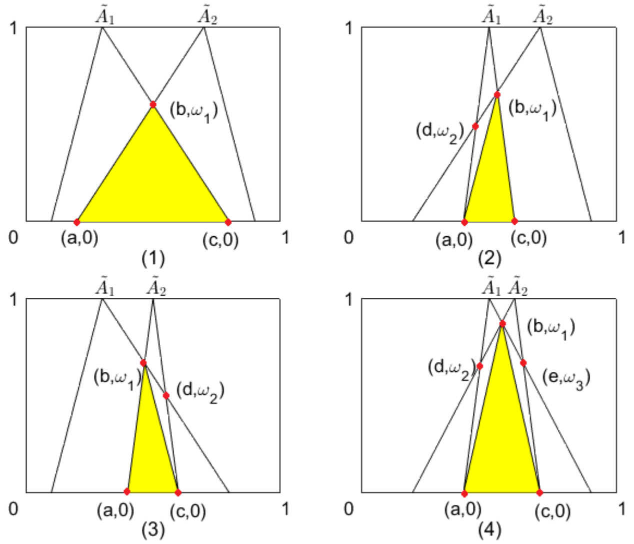

FIGURE 2. The intersection of two triangular fuzzy numbers.

matrix denoted as *difa*.

$$difa = |S_L(\widetilde{A}) - \omega S_L(\widetilde{a}_s)| + |S_R(\widetilde{A}) - \omega S_R(\widetilde{a}_s)|. \quad (24)$$

In Eq. (24), $\widetilde{A}$ and $\widetilde{a}_s$ represent the TFN models of target model and the sample, respectively. $S_L(\widetilde{A})$ and $S_L(\widetilde{a}_s)$ denote the left average area of target model and sample, while $S_R(\widetilde{A})$ and $S_R(\widetilde{a}_s)$ denote the right average area of target model and sample. ω is the height of sample TFN model.

Step 3: Construct similarity matrix.

The sample difference degree reflects the difference of sample and target model by calculating the difference in area via Eq. (24). However, to express how strongly the model supports different samples, the sample difference degree is not an appropriate way. Generally speaking, the difference expresses opposing degree while the similarity expresses supporting degree. Thus in this step, the matrix *difa* should be transformed into similarity matrix. For each element of matrix *difa*, its similarity degree is calculated by

$$sim(i,j) = \omega_{ij} \cdot e^{-r \cdot difa(i,j)}, \quad (25)$$

where the coefficients i and j in Eq. (25) and Eq. (26) are used to denote an element in matrix, that is, the element in the ith row and the jth column. The coefficient r is difference degree coefficient which can adjust the result of Eq. (25). Mostly, r is bigger than 1 and satisfy the equation $0 \leqslant r \cdot min(difa(i,j)) \leqslant 5$, since $e^{-5} = 0.0067$ which is small enough. ω is the height of triangular fuzzy number. If $\omega = 0$, this intersection of two triangular fuzzy number is an empty set, or two triangular fuzzy number do not intersect, so the result is 0. In Eq. (25), the larger the value of $difa(i,j)$, the smaller the value of $sim(i,j)$. There is a negative correlation between the similarity degree and the difference degree. To obtain the initial BBAs, each column of matrix *sim* should be normalized by

$$BBA(i,j) = \frac{sim(i,j)}{\sum sim(:,j)}. \quad (26)$$

Step 4: Calculate the distance between evidence bodies.

Jousselme evidence distance expresses the conflict between evidence. In this step, the Jousselme evidence distance is calculated to estimate the existence of conflicts.

According to the Eq. (8) and Eq. (9), the distance between evidence bodies is calculated, the matrix of distance is denoted as d. Each attribute is considered as an information source, thus the matrix d is a $k \times k$ matrix.

Step 5: Resolve conflict if it exists.

Since the conflict is detected, these conflicting problems should be resolved. In this step, the discount coefficient method is used to resolve the conflicts.

A conflict threshold $\lambda (0 \leqslant \lambda \leqslant 1)$ is set to detect the conflict. The higher the value of conflict threshold, the higher the toleration degree to conflict. Mostly, 0.5 is regarded as a dividing line. After obtaining the distance matrix d, the average of each row in the matrix is calculated, then normalized, where the result matrix is $\overline{d}$, and the maximum of $\overline{d}$ is denoted as $max(\overline{d})$. If $max(\overline{d}) \leqslant \lambda$, the conflict is acceptable; if $max(\overline{d}) > \lambda$, some adjustments are needed. The conflicting evidence will be adjusted by using the discount coefficient method [76].

At first, the comparative matrix P is constructed based on the ith and jth element of $\overline{d}$, that is, the normalized average value of the ith row and the jth row in matrix d,

$$P(i,j) = \frac{\overline{d}_i}{\overline{d}_j}. \quad (27)$$

Then the eigenvector e_i corresponding to the maximum eigenvalue of the comparative matrix P is calculated, and its absolute value $|e_i|$ is taken. $|e_i|$ is normalized by dividing its maximum element. After normalization, the discount coefficient $\overline{|e_i|}$ is obtained. Finally, the initial BBAs are discounted by using discount coefficient $\overline{|e_i|}$ and the initial BBAs are obtained.

Step 6: Produce the final BBA.

In this step, the final BBA is combined based on the initial BBAs. According to the Dempster's combination rule, the final BBA is produced by Eq. (4) and Eq. (5).

IV. NUMERICAL EXAMPLE

In this section, the Iris data set is used, where there are 3 classes named *Setosa* (*Se*), *Versicolour* (*Ve*) and *Virginica* (*Vi*). Iris data set has 150 instances and 4 attributes: Sepal length (SL), Sepal width (SW), Petal length (PL) and Petal width (PW). According to the study [77], [78], each attribute can be seen as an independent information source. This assumption is also taken in this experiment.

Firstly, 80% of instances are selected as training data at random used to construct the target model. According to experience and experimental effect in [55], the coefficient of normalization k is set as 1.2; the difference degree coefficient r is set as 16; the conflict threshold λ is set as 0.5 in this experiment. Tables 1–4 show the matrix TI.

Fig. 3 shows the triangular fuzzy number models of *Setosa*, *Versicolour*, *Virginica* and their intersections

TABLE 1. The attribute Sepal length in triangular fuzzy number matrix *TI*.

Hypothesis	Sepal length
$\{Se\}$	(4.7235,5.1563,5.5286;1)
$\{Ve\}$	(4.9500,5.6471,6.4048;1)
$\{Vi\}$	(5.9000,6.5250,7.4750;1)
$\{Se,Ve\}$	(4.9500,5.3271,5.5286;0.5410)
$\{Se,Vi\}$	(0,0,0;0)
$\{Ve,Vi\}$	(5.9000,6.1282,6.4048;0.3651)
$\{Se,Ve,Vi\}$	(0,0,0;0)

TABLE 2. The attribute Sepal width in triangular fuzzy number matrix *TI*.

Hypothesis	Sepal width
$\{Se\}$	(3.1412,3.4938,4.0429;1)
$\{Ve\}$	(2.2000,2.6765,2.9190;1)
$\{Vi\}$	(2.7000,3.0313,3.1250;1)
$\{Se,Ve\}$	(0,0,0;0)
$\{Se,Vi\}$	(0,0,0;0)
$\{Ve,Vi\}$	(2.7000,2.8264,2.9190;0.3817)
$\{Se,Ve,Vi\}$	(0,0,0;0)

TABLE 3. The attribute Petal length in triangular fuzzy number matrix *TI*.

Hypothesis	Petal length
$\{Se\}$	(1.4059,1.4714,1.5188;1)
$\{Ve\}$	(3.4000,4.0118,4.6524;1)
$\{Vi\}$	(4.9750,5.5625,6.3000;1)
$\{Se,Ve\}$	(0,0,0;0)
$\{Se,Vi\}$	(0,0,0;0)
$\{Ve,Vi\}$	(0,0,0;0)
$\{Se,Ve,Vi\}$	(0,0,0;0)

TABLE 4. The attribute Petal width in triangular fuzzy number matrix *TI*.

Hypothesis	Petal width
$\{Se\}$	(0.1765,0.2688,0.2857;1)
$\{Ve\}$	(1.0000,1.2412,1.4714;1)
$\{Vi\}$	(1.8000,2.0500,2.0875;1)
$\{Se,Ve\}$	(0,0,0;0)
$\{Se,Vi\}$	(0,0,0;0)
$\{Ve,Vi\}$	(0,0,0;0)
$\{Se,Ve,Vi\}$	(0,0,0;0)

TABLE 5. The triangular fuzzy number models of sample.

Attributes	TFN models
Sepal length	(5.1,5.1,5.1;1)
Sepal width	(3.5,3.5,3.5;1)
Petal length	(1.4,1.4,1.4;1)
Petal width	(0.2,0.2,0.2;1)

for Sepal length. The regular triangular fuzzy numbers are (4.7235,5.1563,5.5286;1), (4.9500,5.6471,6.4048;1) and (5.9000,6.5250,7.4750;1).

After that, a sample is selected from class *Setosa* as test data. The test data is [5.1,3.5,1.4,0.2]. For Sepal length, its triangular fuzzy number is (5.1,5.1,5.1;1). Thus the triangular fuzzy numbers of test data are shown in Table 5:

The matrix of target model is combined with the matrix of sample. An augmented matrix *TA* is built. Then the matrix *TA* should be normalized with coefficient k.

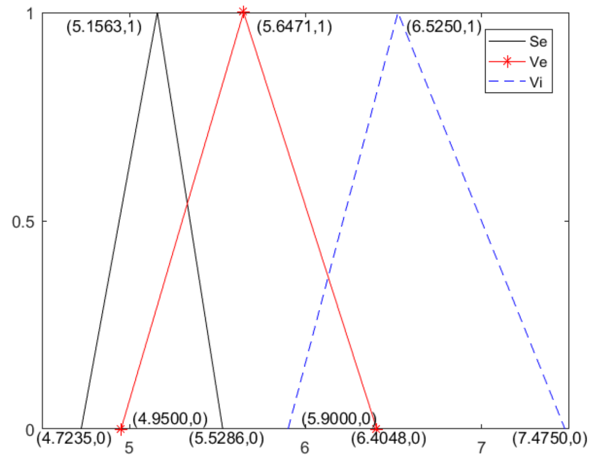

FIGURE 3. An example of Sepal length.

After that, the difference and similarity matrix are calculated. The similarity degree between sample's Sepal length and triangular fuzzy number(*SL*) is shown in Table 6:

TABLE 6. The similarity degree of Sepal length.

Hypothesis	Similarity degree
$\{Se\}$	0.8478
$\{Ve\}$	0.1047
$\{Vi\}$	0.0024
$\{Se,Ve\}$	0.3635
$\{Se,Vi\}$	0
$\{Ve,Vi\}$	0.0795
$\{Se,Ve,Vi\}$	0

The BBAs are obtained after normalizing the similarity degree. The distance between evidences is calculated. After that, the conflicts should be resolved by the discount efficient method. The processed BBAs are shown in Table 7, where $m(\emptyset) = 0$.

According to the function of Pignistic probability, the final BBA can be transformed to probability. Thus the probability of *Setosa*, *Versicolour* and *Virginica* are:

$$BetP(\{Se\}) = 1.0000,$$
$$BetP(\{Ve\}) = 1.2583 \times 10^{-13},$$
$$BetP(\{Vi\}) = 2.4730 \times 10^{-22}.$$

This result illustrates that the test sample is recognized as class *Setosa*, which is consistent with the real class.

V. APPLICATION

A. THE IRIS DATA SET

1) RESULT COMPARISON IN IRIS DATA SET UNDER THE ENVIRONMENT WITHOUT NOISE

In this section, in order to demonstrate the effectiveness and superiority of the proposed method, the proposed method is compared with Jiang *et al.*'s method [55] and Kang *et al.*'s method [50] based on Iris data set.

TABLE 7. The BBAs between target model and sample.

Hypothesis	Sepal length	Sepal width	Petal length	Petal width	Final result
$\{Se\}$	0.6065	0.9047	1.0000	1.0000	1.0000
$\{Ve\}$	0.0749	0.0018	5.2731×10^{-6}	6.7650×10^{-7}	1.2583×10^{-13}
$\{Vi\}$	0.0017	0.0247	2.8340×10^{-9}	1.2492×10^{-11}	2.4730×10^{-22}
$\{Se,Ve\}$	0.2600	0	0	0	0
$\{Se,Vi\}$	0	0	0	0	0
$\{Ve,Vi\}$	0.0569	0.0688	0	0	0
$\{Se,Ve,Vi\}$	0	0	0	0	0

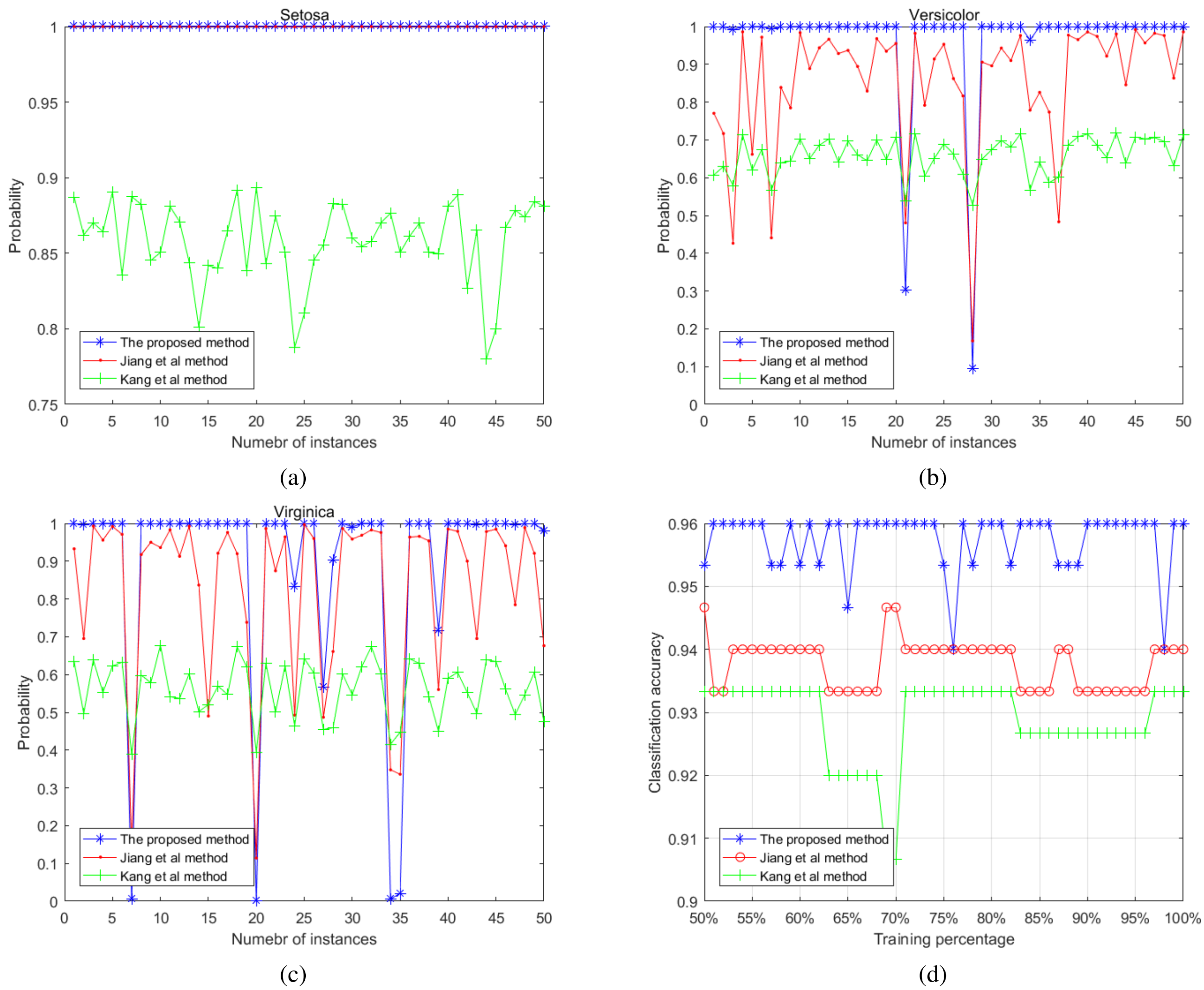

FIGURE 4. Probability of three classes (a-c) and recognition accuracy (d) in Iris data set without noise.

80% of instances are selected from each class at random and used to construct the target model of triangular fuzzy number without noise. To express clearly, 50 instances of each class are used as test data. The results of probability are shown in Figure 4 (a-c). As shown in Figure 4 (a-c), the proposed method keeps high probabilities for most of instances. In Figure 4 (a), for the class *Setosa*, both the proposed method and Jiang *et al*.'s method recognize the instances of *Setosa* with high probabilities. Compared with other methods, Kang *et al*.'s method also identifies the correct category but its probabilities are much lower. In Figure 4 (b), for the class *Versicolour*, the proposed method recognizes most of instances with much higher probabilities than Jiang *et al*.'s method. Whereas, Kang *et al*.'s method still keeps far lower probabilities. In Figure 4 (c), for the class *Virginica*, the proposed method has a number of higher probabilities and a better performance. The average probabilities are given in Table 8. As can be seen from Table 8, for the class *Setosa*,

TABLE 8. The average probabilities for three methods in Iris data set without noise.

Method	*Setosa*	*Versicolour*	*Virginica*
The proposed method	1.0000	0.9668	0.9002
Jiang *et al.*'s method [55]	1.0000	0.8539	0.8325
Kang *et al.*'s method [50]	0.8581	0.6578	0.5615

Versicolour and *Virginica*, the proposed method keeps high average probabilities with the values 1.0000, 0.9668 and 0.9002, respectively. Although Jiang *et al.*'s method has high average probabilities, its average probabilities of class *Versicolour* and *Virginica* are much lower than the proposed method. For the class *Setosa*, *Versicolour* and *Virginica*, the average probabilities of Jiang *et al.*'s method are 1.0000, 0.8539 and 0.8325, respectively. Kang *et al.*'s method also performs well. However, compared with other methods, its average probabilities are only 0.8581, 0.6578 and 0.5615, respectively.

To compare the recognition accuracy, the percentage of training data is set from 50% to 100%. The result of recognition accuracy obtained by different methods is shown in Figure 4 (d). The average recognition accuracy generated by different methods is shown in Table 9. According to this table, the average recognition accuracy of the proposed method is 95.75%. Furthermore, the average recognition accuracy of Jiang *et al.*'s method is 93.78%. The average recognition accuracy of Kang *et al.*'s method is 92.89%. Obviously, the proposed method performs better under the environment without noise.

TABLE 9. The average recognition accuracy for three methods in Iris data set without noise.

Method	Average
The proposed method	95.75%
Jiang *et al.*'s method [55]	93.78%
Kang *et al.*'s method [50]	92.89%

2) RESULT COMPARISON IN IRIS DATA SET UNDER THE ENVIRONMENT WITH NOISE

The noise or other factors may cause conflict and influence the result of combination. In this part, white Gaussian noise is used to simulate the influence of real noise. The white Gaussian noise is added into the Iris data set randomly. The results of probability are shown in Figure 5 (a-c). In Figure 5 (a), due to the influence of noise, the probabilities of Kang *et al.*'s method decrease. But the proposed method and Jiang *et al.*'s method still keep high probabilities for the class *Setosa*. In Figure 5 (b), for the class *Versicolour*, some probabilities of all the methods become far lower because of the noise. The proposed method still identifies the correct category with a much higher probability than other methods in most cases. In Figure 5 (c), for the class *Virginica*, there are some approximate probabilities for the related works. However, the proposed method still keeps a much higher probability than other methods in general. The average probabilities obtained by different methods are given in Table 10. As shown in Table 10, the proposed method still keeps high average probabilities even in a noisy environment. For the class *Setosa*, *Versicolour* and *Virginica*, the average probabilities of the proposed method are 1.0000, 0.9358 and 0.9303, respectively. Additionally, for the class *Setosa*, *Versicolour* and *Virginica*, the average probabilities of Jiang *et al.*'s method are 1.0000, 0.8512 and 0.8391, respectively. Meanwhile, for the class *Setosa*, *Versicolour* and *Virginica*, Kang *et al.*'s method has far lower probabilities with the values 0.8402, 0.6631 and 0.5552, respectively.

TABLE 10. The average probabilities for three methods in Iris data set with noise.

Method	*Setosa*	*Versicolour*	*Virginica*
The proposed method	1.0000	0.9358	0.9303
Jiang *et al.*'s method [55]	1.0000	0.8512	0.8391
Kang *et al.*'s method [50]	0.8402	0.6631	0.5552

Similarly, the percentage of training data is set from 50% to 100% to compare the difference between three methods. The result of recognition accuracy is shown in Figure 5 (d). From these results, some conclusions are obtained. The proposed method performs better and has much higher recognition accuracy than Jiang *et al.*'s method and Kang *et al.*'s method even in a noisy situation. The average recognition accuracy is shown in Table 11. Due to the noise, the average of above methods drops. The average of the proposed method decreases to 94.41%, which is the highest value among methods. The average of Jiang *et al.*'s method decreases to 93.28% and the average of Kang *et al.*'s method decreases to 92.69%. In a word, the proposed method has a better performance than other related works even in a noisy situation.

TABLE 11. The average recognition accuracy for three methods in Iris data set with noise.

Method	Average
The proposed method	94.41%
Jiang *et al.*'s method [55]	93.28%
Kang *et al.*'s method [50]	92.69%

B. THE WINE QUALITY DATA SET

In the applications, a large data set Wine Quality is used to analyze the performance of the related works. The experimental settings are as same as the Iris data set experiment. This data set includes two classes: *red wine* and *white wine*. There are 1599 instances of *red wine* and 4898 instances of *white wine*. Furthermore, this data set has 11 physicochemical attributes which are shown in Table 12. Based on the physicochemical tests, the data set is aimed at modeling wine quality.

1) RESULT COMPARISON IN WINE QUALITY DATA SET UNDER THE ENVIRONMENT WITHOUT NOISE

Firstly, 80% of the instances are selected from each class and used to establish the target model. The results of probability

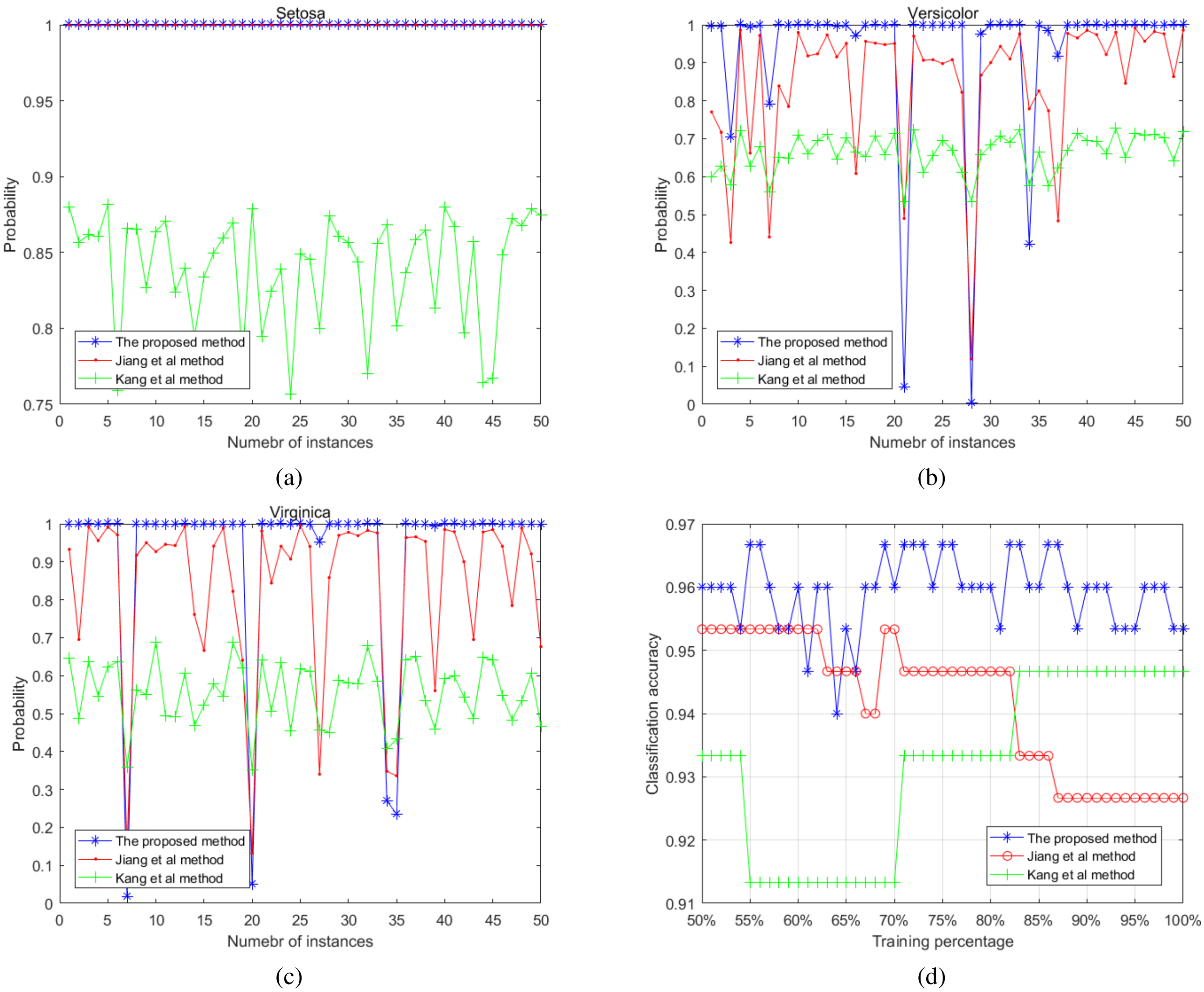

FIGURE 5. Probability of three classes (a-c) and recognition accuracy (d) in Iris data set with noise.

TABLE 12. The attributes of the Wine Quality data set.

No.	Attribute
1	fixed acidity
2	volatile acidity
3	citric acid
4	residual sugar
5	chlorides
6	free sulfur dioxide
7	total sulfur dioxide
8	density
9	pH
10	sulphates
11	alcohol

are shown in Figure 6 (a-b). Here 1000 instances are selected randomly to compare the performances of different methods. Obviously, the instances are recognized as correct category by the proposed method with much higher probabilities than Jiang *et al*.'s method. In Figure 6 (a), for the class *red wine*, the proposed method can identify the correct category with high probabilities. As shown in the figure, the probabilities of Jiang *et al*.'s method are mostly lower than the proposed method. Kang *et al*.'s method is stable but its probabilities are much lower. In Figure 6 (b), for the class *white wine*, the proposed method also has a better performance than other methods. What's more, Jiang *et al*.'s method becomes more unstable and some probabilities reach below 0.5 or even lower, that is, more instances fail to be recognized. Furthermore, Kang *et al*.'s method still keeps low probabilities. The average probabilities are shown in Table 13. As shown in this table, the average probabilities of the proposed method are much higher than other methods. For the class *red wine*, the average probability of the proposed method is 0.9551; for the class *white wine*, the average probability of the proposed method is 0.9556. Moreover, for the class *red wine* and *white wine*, the average probabilities of Jiang *et al*.'s method are 0.8910 and 0.7619, respectively. Meanwhile, Kang *et al*.'s

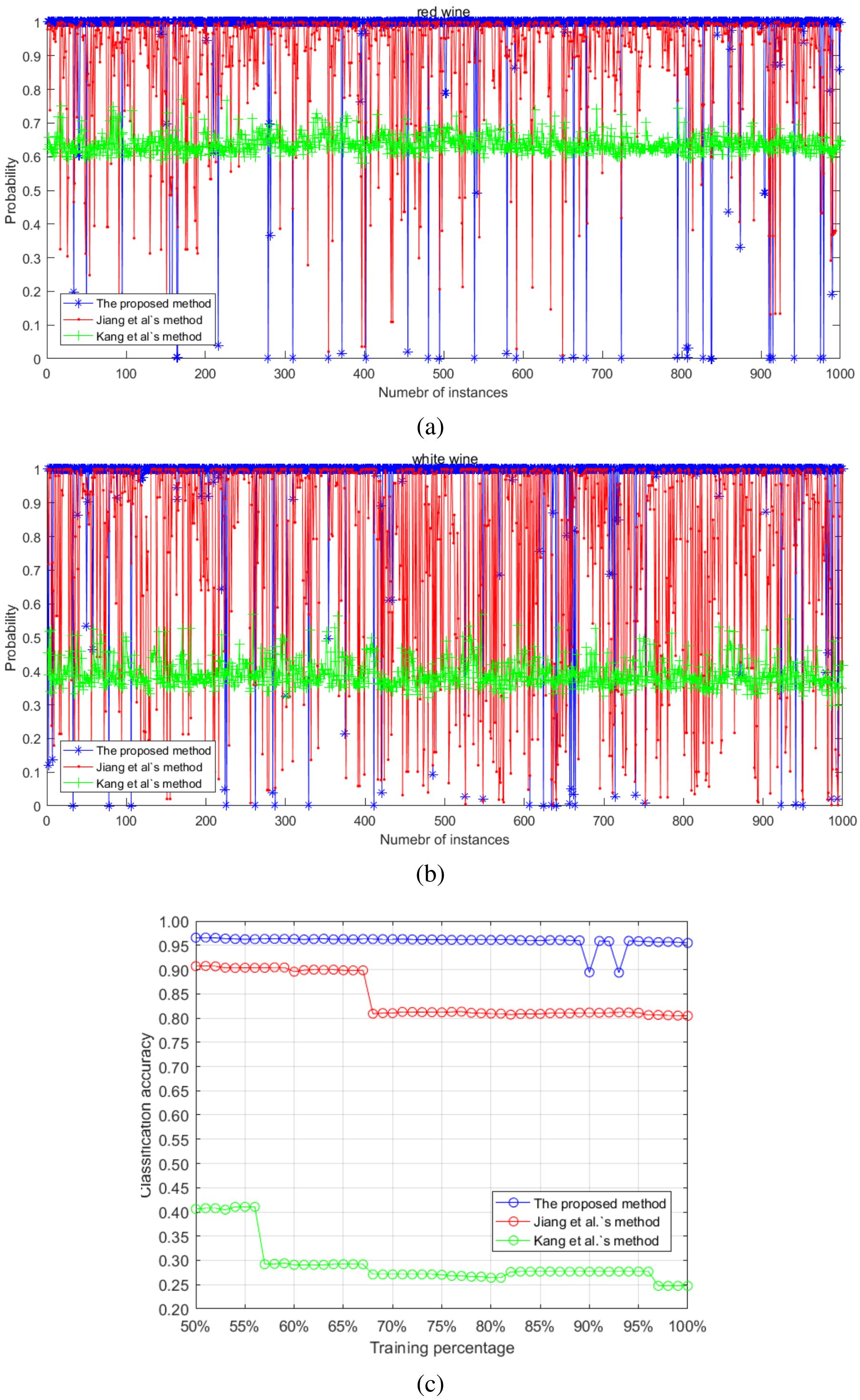

FIGURE 6. Probability of two classes (a-b) and recognition accuracy (c) in Wine Quality data set without noise.

method has much lower average probabilities with the values 0.6389 and 0.3891, respectively.

After that, the percentage of training data is set from 50% to 100% to compare the recognition accuracy. The result of recognition accuracy is shown in Figure 6 (c). Apparently, in Figure 6 (c), the proposed method keeps a much higher recognition accuracy than Jiang *et al*.'s method. The average recognition accuracy is given in Table 14. Basically, the gap of recognition accuracy is obvious. As can be seen from this table, the average recognition accuracy of the proposed

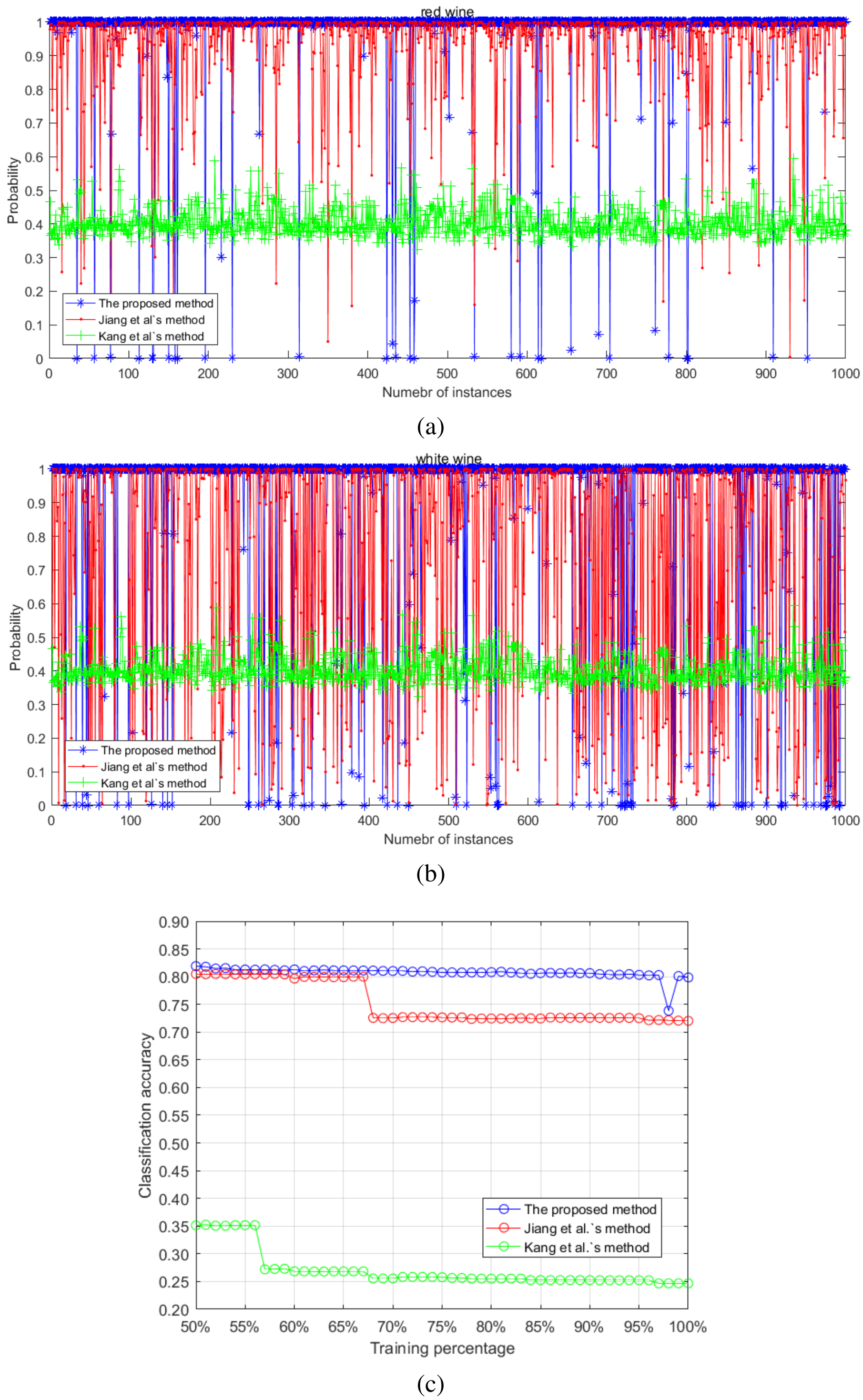

FIGURE 7. Probability of two classes (a-b) and recognition accuracy (c) in Wine Quality data set with noise.

method is 95.87%. Additionally, the average recognition accuracy of Jiang *et al.*'s method is 84.23% and the average recognition accuracy of Kang *et al.*'s method is 29.36%. To sum up, the proposed method is capable of handling the large data set which performs better without noise.

2) RESULT COMPARISON IN WINE QUALITY DATA SET UNDER THE ENVIRONMENT WITH NOISE

In this section, white Gaussian noise is added into the Wine Quality data set to test the anti-noise ability of the related works. 80% of the instances from each class are selected to

TABLE 13. The average probabilities for three methods in Wine Quality data set without noise.

Method	*red wine*	*white wine*
The proposed method	0.9551	0.9556
Jiang *et al.*'s method [55]	0.8910	0.7619
Kang *et al.*'s method [50]	0.6389	0.3891

TABLE 14. The average recognition accuracy for three methods in Wine Quality data set without noise.

Method	Average
The proposed method	95.87%
Jiang *et al.*'s method [55]	84.23%
Kang *et al.*'s method [50]	29.36%

TABLE 15. The average probabilities for three methods in Wine Quality data set with noise.

Method	*red wine*	*white wine*
The proposed method	0.9637	0.8979
Jiang *et al.*'s method [55]	0.9346	0.7491
Kang *et al.*'s method [50]	0.4052	0.4053

establish target model. The results of probability are shown in Figure 7 (a-b). In Figure 7 (a), for the class *red wine*, the proposed method still identifies most of instance with correct category. In Figure 7 (b), for the class *white wine*, the proposed method still performs better than Jiang *et al.*'s method and Kang *et al.*'s method. The average probabilities are given in Table 15. As can be seen from Table 15, the proposed method still keeps high average probabilities even in a noisy environment. For the class *red wine* and *white wine*, the average probabilities of the proposed method are 0.9637 and 0.8979, respectively. Jiang *et al.*'s method also has a good performance. Its average probability of the class *red wine* is 0.9346. Additionally, its average probability of the class *white wine* is 0.7491. Whereas Kang *et al.*'s method still keeps far lower average probabilities compared with other methods for the class *red wine* and *white wine*, where its average probabilities are 0.4052 and 0.4053, respectively.

Then, the same experimental settings are taken in this experiment. The result of recognition accuracy is shown in Figure 7 (c). In Figure 7 (c), due to the impact of noise, the recognition accuracy slips to a lower level. The average recognition accuracy is given in Table 16. The average recognition accuracy of the proposed method decreases to 80.74%. Furthermore, the average recognition accuracy of Jiang *et al.*'s method decreases to 75.21%. The average recognition accuracy of Kang *et al.*'s method becomes lower, it decreases to 27.06%. Obviously, the proposed method recognizes instances more accurately than Jiang *et al.*'s method and Kang *et al.*'s method. In general, the proposed method has a better performance even under a noisy environment. Additionally, it has the better anti-noise capability compared with other methods.

C. DISCUSSION

The experiments illustrate that the proposed method has a better performance in generating the BBAs and keeping the high recognition accuracy rate for both of the cases in large data sets and small data sets. Furthermore, the proposed method has the effective capacity of anti-noise no matter the attributes are high dimensional or low dimensional. Although Jiang *et al.*'s method also performs well, it is not as obvious as the proposed method. Meanwhile, Kang *et al.*'s method has a good performance with small data sets. However, it has low capability to handle the recognition problem in the large data set with high dimensional attributes.

TABLE 16. The average recognition accuracy for three methods in Wine Quality data set with noise.

Method	Average
The proposed method	80.74%
Jiang *et al.*'s method [55]	75.21%
Kang *et al.*'s method [50]	27.06%

As can be seen from the above results, the main differentiating factor between the proposed method and Jiang *et al.*'s method is the construction of TFN model. The proposed method constructs the TFN model based on the k-means++ method. In practical environment, especially in noisy environment, data is often influenced. Whereas, the clustering centers obtained by k-means++ method can be regarded as the mass centers of data. Thus the influence of inaccurate data is balanced. Jiang *et al.*'s method constructs the TFN model based on the extremum and average values. Although Jiang *et al.*'s method has high recognition accuracy, its performance is much lower than the proposed method. On the other hand, because Kang *et al.*'s method uses interval number model to construct the target model and does not consider the conflict evidence, it does not perform well. In summary, these are the reasons why the proposed method has a better performance comparing with other related works no matter in a noisy environment or not.

VI. CONCLUSIONS

In this paper, an improved method based on triangular fuzzy number was proposed. Compared to the existing methods, it used k-means++ clustering method to construct a more reliable triangular fuzzy number model. Some features of triangular fuzzy number could be used to determine the difference and similarity between target model and sample. After normalizing the similarity matrix, the initial BBAs were obtained. Because of the conflict, the result of combination might be inaccurate. Thus the discount coefficient method was needed to resolve the conflict when conflict exceeded threshold. Finally, the final BBA was obtained by using Dempster's combination rule. The proposed method took advantages of D-S theory, k-means++ clustering method and fuzzy set, so it had the ability of anti-noise. In particular, the proposed method made contributions to the improvement of the construction of TFN. It performed better than the existing method even in a noisy situation. Additionally, the proposed method had the ability to obtain the reliable BBA even for the large data set with the high dimensional attributes.

Through the experimental analysis, we found some interesting problems to be solved. When some attributes are too similar, that is, their triangular fuzzy numbers have large area of intersection, the existing methods are hard to recognize them. Hence, the process of pre-selection will be considered in the future work. Furthermore, some attributes may have the value under zero in certain circumstances. Therefore, we will consider this issue as the other future work.

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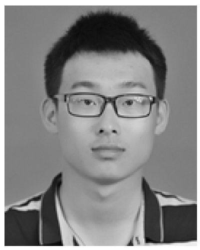

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