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Adaptive Neural Command Filtered Tracking Control for Flexible Robotic Manipulator With Input Dead-Zone

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ABSTRACT In this paper, an adaptive neural network (NN) command filtered tracking control method is developed for a flexible robotic manipulator with dead-zone input. To deal with the input dead-zone nonlinearity, it is viewed as a combination of a linear part and bounded disturbance-like term. The Neural networks (NNs) are used to estimate the uncertain nonlinearities appeared in the control system. By using the command filter technique, the problem of 'explosion of complexity' is overcome. The proposed controller guarantees that all the closed-loop signals are bounded and the system output can track the given reference signal. The simulation results are provided to demonstrate the effectiveness of the proposed controller.

INDEX TERMS Adaptive neural network control, robotic manipulator, dead-zone, command-filter technique, backstepping.

I. INTRODUCTION

Since the control problem of single-link robotic manipulator widely exist in industry and engineering fields, the investigation on single-link robotic manipulator has caused quite a lot of attention during the past two decades. So far, various interesting control approaches have been developed, such as sliding control [1]–[4], backstepping technique [5], [6] and intelligent control method [7]–[16]. Backstepping design method has been considered as one of the most popular and effective control methods for non-linear systems with strict feedback form, i.e. those that do not meet the matching conditions. In [5], a backstepping design approach is presented for single-link flexible robotic manipulator. Noted that the backstepping method in [5] is only suitable for solving the control system being known accurately. However, it is well known that uncertain nonlinearities exist widely in practical engineering, this means that the precise system model of flexible joint manipulator is unavailable. Under this circumstance, many important results have been achieved by combining fuzzy/neural control together with backstepping and adaptive control approach, see, for example [7]–[16]. Nevertheless, adaptive neural network or fuzzy backstepping control [17] based on approximation cannot solve the problem of 'explosion of complexity' caused by the repeated differential of virtual input [7]. To overcome this shortcoming, the command filter backstepping method is first proposed in [18], and is extended to the adaptive backstepping control of strict-feedback systems in [19]. As indicated in [19], the problem of 'explosion of complexity' can be eliminated by using the output of command filter to approximate the derivative of the virtual control at each step of the backstepping method. By introducing the compensation signal, the error caused by the command-filter can be reduced. More recently, the command filter control algorithm is used to deal with the control design problem of flexible robotic manipulator with input saturation [12].

Dead-zone is one of the nonsmooth nonlinear characteristics in many industrial processes. The existence of dead-zone can severely affect the control performance and even result in instability of the system. Therefore, dead-zone should be considered in the design and analysis of control system, and there exist series of researches for the control systems with dead-zone [20]. Recker *et al.* [21] putted forward an adaptive

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control technique for nonlinear systems with input dead-zone. In recent years, more and more researchers have considered the control problem of nonlinear systems with dead zone input [22]–[32]. For instance, in [22], an adaptive dead zone inverse algorithm is proposed to control a class of nonlinear systems with unknown dead zone. In [23], a compensation scheme is presented for general nonlinear actuator dead zone of unknown width. The compensator uses two neural networks, one to estimate the unknown dead zone and another to provide adaptive compensation in the feedforward path. In [25], the adaptive control of sandwich nonlinear system with unknown dead zone between linear dynamic blocks is studied. In [27], an adaptive output feedback control problem for a class of uncertain nonlinear systems with unknown asymmetrical dead zone is studied. As far as we know, there are few results on adaptive neural command-filter control for flexible manipulator with dead-zone input, which motivates us to carry out this research.

In this paper, the problem of output tracking control is considered for flexible robotic manipulator with input dead-zone. By employing adaptive neural control combined with command filter technique, an output tracking control scheme is presented for a single-link flexible manipulator with input dead-zone. The proposed adaptive neural network control approach can ensure that all the variables in the closed-loop systems are bounded, and the trajectory tracking error can be made as small as possible for all bounded initial conditions. The simulation results are given to verify the effectiveness of the proposed controller. The main advantage of the developed scheme is that the command filtered adaptive neural network backstepping control can overcome the problem of the classical backstepping for the nonlinear systems with input dead-zone. It also can alleviate the online calculation burden.

The rest of this article is organized as follows. Section 2 gives the problem statement and preliminaries. Adaptive neural network control design and the stability analysis are presented in Section 3. Section 4 gives simulation results to demonstrate the effectiveness of the proposed scheme. The conclusion is included in Section 5.

II. PROBLEM STATEMENT AND PRELIMINARIES

A. SYSTEM DESCRIPTIONS

Consider a single-link robotic manipulator coupled to a brushed direct current motor with a nonrigid joint, the dynamic equation is expresses as follows

$$\begin{cases} J_1\ddot{q}_1 + F_1\dot{q}_1 + K(q_1 - \dfrac{q_2}{N}) + mgd\cos q_1 = 0, \\ J_2\ddot{q}_2 + F_2\dot{q}_2 - \dfrac{K}{N}(q_1 - \dfrac{q_2}{N}) = K_t i, \\ L\dot{i} + Ri + K_b\dot{q}_2 = u(v), \end{cases} \tag{1}$$

where J_1 and J_2 denote the inertias, q_1 is the angular positions of the link, q_2 displays the motor shaft, R and L express the armature resistance and inductance respectively. i shows the armature current, K denotes the spring constant, K_t denotes the torque constant, $u(v)$ is the armature voltage, g displays the acceleration of gravity, d expresses the position of the link's center of gravity, F_1 and F_2 are the viscous friction constants, K_b denotes the back-emf constant, M denotes the link mass, and N denotes the gear ratio.

By introducing the state variables, $x_1 = q_1, x_2 = \dot{q}_1, x_3 = q_2, x_4 = \dot{q}_2, x_5 = i$, and defining $K_tK = J_1J_2NL$, the dynamic equation (1) changes into

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = \delta_2(x_1, x_2, x_3) + x_3, \\ \dot{x}_3 = x_4, \\ \dot{x}_4 = \delta_4(x_1, x_2, x_3, x_4, x_5) + x_5, \\ \dot{x}_5 = \delta_5(x_1, x_2, x_3, x_4, x_5) - \frac{1}{L}u, \\ y = x_1, \end{cases} \tag{2}$$

where $\delta_2(x_1, x_2, x_3) = -\frac{mgd}{J_1}\cos x_1 - \frac{F_1}{J_1}x_2 - \frac{K}{J_1}(x_1 - \frac{x_3}{N}) - x_3$, $\delta_4(x_1, x_2, x_3, x_4, x_5) = \frac{K}{J_2N}(x_1 - \frac{x_3}{N}) - \frac{F_2}{J_2}x_4 + \frac{K_t}{J_2}x_5 - x_5$, $\delta_5(x_1, x_2, x_3, x_4, x_5) = -\frac{R}{L}x_5 - \frac{K_b}{L}x_4$.

Note that system (2) is in non-strict-feedback form.

According to [30], the dead-zone with input $v(t)$ and output $u(t)$ is described by

$$u = D(v) = \begin{cases} m_r(v - b_r), & v \geq b_r \\ 0, & b_l < v < b_r \\ m_l(v - b_l), & v \leq b_l \end{cases} \tag{3}$$

where $u \in R$ denotes the control signal of the system, and it is also the output of an uncertain dead-zone. $D(\cdot)$ denotes a piecewise function with three zones.

For ease of control design and analysis, the following assumptions and lemmas are introduced.

Assumption 1 [30]: The dead-zone output u is not available for measurement.

Assumption 2 [30]: The dead-zone slopes in positive and negative region are same, *i.e.* $m_r = m_l = m$.

Assumption 3 [30]: The dead-zone parameters b_r, b_l, and m are unknown, but their signs are known: $b_r > 0, b_l < 0, m > 0$.

Assumption 4 [30]: The dead-zone parameters b_r, b_l, and m are bounded, *i.e.* there exist known constants b_{rmin}, b_{rmax}, b_{lmin}, b_{lmax}, m_{min}, m_{max} such that $b_r \in [b_{rmin}, b_{rmax}]$, $b_l \in [b_{lmin}, b_{lmax}]$, and $m \in [m_{min}, m_{max}]$.

Remark 1: From a practical point of view, we can redefine model (3) as

$$u = D(v) = mv + d(v) \tag{4}$$

where m is called the general slope of the dead-zone, $d(v(t))$ can be calculated from (3) and (4) as

$$d(v) = \begin{cases} -mb_r, & v \geq b_r \\ -mv, & b_l < v < b_r \\ -mb_l, & v \leq b_l \end{cases} \tag{5}$$

From Assumptions 2 and 4, one can conclude that $d(v)$ is bounded.

The purpose of this paper is to construct an adaptive neural controller as follows:

(1) The system output y can track the given reference signal y_r as close as possible.

(2) All variables within the closed-loop system are bounded.

Before proceeding to the next step, let's introduce the following lemma.

Lemma 1 [18]: The command filter is defined as

$$\begin{cases} \dot{\omega}_i = \omega_n \omega_{i,2} \\ \dot{\omega}_{i,2} = -2\varsigma\omega_n\omega_{i,2} - \omega_n(\omega_i - \alpha_{i-1}) \end{cases} \tag{6}$$

where $\omega_n > 0$ and $\varsigma \in (0.1]$ are positive design parameters that are the same for all command filters. α_{i-1} and ω_i are the input and output of the command filter. The initial value of ω_i is equal to $\alpha_{i-1}(0)$, $\omega_{i,2}(0) = 0$.

B. NEURAL NETWORK

In this research, the radial basis function (RBF) neural network will be used to approximate the continuous function $f(x) : R^n \to R$. The RBF neural networks is described by:

$$f_{nn}(x) = \theta^T \varphi(x) \tag{7}$$

where $x \in \Omega_x \subset R^q$ denotes input vector, $\theta = [\theta_1, \cdots, \theta_l]^T \in R^l$ with $l > 1$ denotes weight vector, and $\varphi(x) = [\varphi_1(x), \cdots, \varphi_l(x)]^T$ means the basis function vector with $\varphi_i(x)$ being the Gaussian function in the form

$$\varphi_i(x) = \exp[-\frac{(x-\mu_i)^T(x-\mu_i)}{\eta^2}] \tag{8}$$

where $\mu_i = [\mu_{i1}, \cdots, \mu_{iq}]^T$ for $i = 1, \cdots, l$ is the center of the receptive field and η is the width of Gaussian function.

Lemma 2 [33]: For given accuracy $\varepsilon > 0$, with sufficiently large node number l the RBF NN (7) can approximate any continuous function $f(x)$ over compact set $\Omega_x \subset R^q$ such that

$$f(x) = \theta^{*T}\varphi(x) + \varepsilon(x), \forall x \in \Omega_x \in R^q \tag{9}$$

where $\varepsilon(x)$ denotes the approximation error and satisfies $|\varepsilon(x)| \le \varepsilon^*$, θ^* denotes the ideal constant weight vector and defined as

$$\theta^* = \arg\min_{\theta \in \bar{R}^l}\{\sup_{x\in\Omega_x} |f(x) - \theta^T\varphi(x)|\} \tag{10}$$

Lemma 3 [34]: Let $S(\bar{x}_q) = [S_1(\bar{x}_q), \cdots, S_l(\bar{x}_q)]^T$ be the basis function vector of a RBF NN and $\bar{x}_q = [x_1, \cdots, x_q]^T$ be the input vector. Then, for any positive integer $k \le q$, let $\bar{x}_k = [x_1, \cdots, x_k]^T$, the following inequality holds:

$$\|S(\bar{x}_q)\|^2 \le \|S(\bar{x}_k)\|^2 \tag{11}$$

III. ADAPTIVE NEURAL NETWORK CONTROL DESIGN AND STABILITY ANALYSIS

In this section, an adaptive neural network state-feedback controller, the compensation signal and the parameter adaptive law are obtained via command filter.

The 5-step adaptive neural network backstepping state feedback control devise is obtained by the following coordinate changes

$$\begin{cases} \lambda_i = x_i - \omega_i \\ \nu_i = \lambda_i - r_i \end{cases} \tag{12}$$

where $\omega_1 = y_r$, for $i = 1, \cdots, 5$, λ_i is the tracking error for command filter, ω_i indicates the output of command filter, r_i denotes the compensating signal of command filter, y_r displays the desired trajectory, ν_i expresses the compensating tracking error signal.

Step 1: From the coordinate transformation (12), the time derivative of ν_1 is

$$\begin{aligned} \dot{\nu}_1 &= \dot{\lambda}_1 - \dot{r}_1 \\ &= \dot{x}_1 - \dot{y}_r - \dot{r}_1 \\ &= x_2 - \dot{y}_r - \dot{r}_1 \\ &= \lambda_2 + \omega_2 - \dot{y}_r - \dot{r}_1 \\ &= \nu_2 + r_2 + \omega_2 - \dot{y}_r - \dot{r}_1 \end{aligned} \tag{13}$$

Choose a Lyapunov function candidate as

$$V_1 = \frac{1}{2}\nu_1^2 \tag{14}$$

The time derivative of V_1 is

$$\dot{V}_1 = \nu_1(\nu_2 + r_2 + \alpha_1 - \alpha_1 + \omega_2 - \dot{y}_r - \dot{r}_1) \tag{15}$$

Next, consider a virtual control signal α_1 and a compensating signal $\dot{r}_1$ as

$$\alpha_1 = -c_1\lambda_1 + \dot{y}_r \tag{16}$$

$$\dot{r}_1 = -c_1 r_1 + r_2 + (\omega_2 - \alpha_1) \tag{17}$$

where $c_1 \ge 0$ is a design parameter. By substituting (16)-(17) into (15), we obtain

$$\begin{aligned} \dot{V}_1 &= \nu_1(\nu_2 + r_2 - c_1\lambda_1 + \dot{y}_r - \alpha_1 + w_2 \\ &\quad - \dot{y}_r + c_1 r_1 - r_2 - \omega_2 + \alpha_1) \\ &= -c_1\nu_1^2 + \nu_1\nu_2 \end{aligned} \tag{18}$$

Step 2: The time derivative of ν_2 is

$$\begin{aligned} \dot{\nu}_2 &= \dot{\lambda}_2 - \dot{r}_2 \\ &= \dot{x}_2 - \dot{w}_2 - \dot{r}_2 \\ &= \delta_2(\bar{x}_3) + x_3 - \dot{\omega}_2 - \dot{r}_2 \end{aligned} \tag{19}$$

Choose the following Lyapunov function

$$V_2 = V_1 + \frac{1}{2}\nu_2^2 + \frac{1}{2\eta_2}\tilde{\theta}_2^2 \tag{20}$$

where $\eta_2 > 0$ is a parameter to be designed, $\tilde{\theta}_2 = \theta_2 - \hat{\theta}_2$ displays the approximation error. The time derivative of V_2 is

$$\dot{V}_2 = -c_1\nu_1^2 + \nu_1\nu_2 + \nu_2\Big(\delta_2(\bar{x}_3) + x_3 - \dot{\omega}_2 - \dot{r}_2\Big) - \frac{1}{\eta_2}\tilde{\theta}_2\dot{\hat{\theta}}_2 \quad (21)$$

Since $\delta_2(\bar{x}_3)$ is unknown, and cannot be applied to design virtual control signal α_2 directly. Neural network $\theta_2^{*T}\varphi_2(\bar{x}_3)$ is thus used to estimate $\delta_2(\bar{x}_3)$, such that, for any given positive constant $\varepsilon_2^* > 0$.

$$\delta_2(\bar{x}_3) \le \theta_2^{*T}\varphi_2(\bar{x}_3) + \varepsilon_2(\bar{x}_3) \quad (22)$$

where $\varepsilon_2(\bar{x}_3)$ satisfies $|\varepsilon_2(\bar{x}_3)| \le \varepsilon_2^*$. Then, with the consideration of (22), we can rewrite (21) as

$$\begin{aligned}\dot{V}_2 &\le -c_1\nu_1^2 + \nu_1\nu_2 + \nu_2\Big(\theta_2^{*T}\varphi_2(\bar{x}_3) + \varepsilon_2 + x_3 - \dot{\omega}_2 - \dot{r}_2\Big) - \frac{1}{\eta_2}\tilde{\theta}_2\dot{\hat{\theta}}_2 \\ &\le -c_1\nu_1^2 + \nu_1\nu_2 + \nu_2\Big(\theta_2^{*T}\varphi_2(\bar{x}_3) + \varepsilon_2 + \lambda_3 + \omega_3 - \dot{\omega}_2 - \dot{r}_2\Big) - \frac{1}{\eta_2}\tilde{\theta}_2\dot{\hat{\theta}}_2 \\ &\le -c_1\nu_1^2 + \nu_1\nu_2 + \nu_2\Big(\theta_2^{*T}\varphi_2(\bar{x}_3) + \varepsilon_2 + \nu_3 + r_3 + \omega_3 - \dot{\omega}_2 - \dot{r}_2\Big) - \frac{1}{\eta_2}\tilde{\theta}_2\dot{\hat{\theta}}_2 \end{aligned} \quad (23)$$

By applying Young's inequality and according to Lemma 3, we conclude

$$\begin{aligned}\nu_2\theta_2^{*T}\varphi_2(\bar{x}_3) &\le \frac{1}{2a_2^2}\nu_2^2\theta_2\varphi_2^T(\bar{x}_3)\varphi_2(\bar{x}_3) + \frac{1}{2}a_2^2 \\ &\le \frac{1}{2a_2^2}\nu_2^2\theta_2\varphi_2^T(\bar{x}_2)\varphi_2(\bar{x}_2) + \frac{1}{2}a_2^2 \end{aligned} \quad (24)$$

$$\nu_2\varepsilon_2 \le \frac{1}{2}\nu_2^2 + \frac{1}{2}\varepsilon_2^{*2} \quad (25)$$

where $\|\theta_2^*\|^2 = \theta_2$. By substituting (24)-(25) into (23), we have

$$\dot{V}_2 \le -c_1\nu_1^2 + \nu_2\Big(\frac{1}{2a_2^2}\nu_2\theta_2\varphi_2^T(\bar{x}_2)\varphi_2(\bar{x}_2) + \alpha_2 - \alpha_2 + \nu_3 + r_3 + \omega_3 - \dot{\omega}_2 - \dot{r}_2 + \nu_1 + \frac{1}{2}\nu_2\Big) + \frac{1}{2}a_2^2 + \frac{1}{2}\varepsilon_2^{*2} - \frac{1}{\eta_2}\tilde{\theta}_2\dot{\hat{\theta}}_2 \quad (26)$$

Choose a virtual control signal α_2, the command filter $\dot{\omega}_2$ and the compensating signal $\dot{r}_2$ as

$$\alpha_2 = -c_2\lambda_2 - \frac{1}{2a_2^2}\nu_2\hat{\theta}_2\varphi_2^T(\bar{x}_2)\varphi_2(\bar{x}_2) - \frac{1}{2}\nu_2 - \nu_1 + \dot{\omega}_2 \quad (27)$$

$$\dot{\omega}_2 = \omega_n\omega_{2,2} \quad (28)$$

$$\dot{\omega}_{2,2} = -2\varsigma\omega_n\omega_{2,2} - \omega_n(\omega_2 - \alpha_1) \quad (29)$$

$$\dot{r}_2 = -c_2r_2 + r_3 + (\omega_3 - \alpha_2) \quad (30)$$

By substituting (27)-(30) into (26), we can get

$$\begin{aligned}\dot{V}_2 &\le -c_1\nu_1^2 + \nu_2\Big(\frac{1}{2a_2^2}\nu_2\theta_2\varphi_2^T(\bar{x}_2)\varphi_2(\bar{x}_2) - c_2\lambda_2 - \frac{1}{2a_2^2}\nu_2\hat{\theta}_2\varphi_2^T(\bar{x}_2)\varphi_2(\bar{x}_2) - \frac{1}{2}\nu_2 \\ &\quad - \nu_1 + \dot{\omega}_2 - \alpha_2 + \nu_3 + r_3 + \omega_3 - \dot{\omega}_2 + c_2r_2 - r_3 - \omega_3 + \alpha_2 + \nu_1 + \frac{1}{2}\nu_2\Big) \\ &\quad + \frac{1}{2}a_2^2 + \frac{1}{2}\varepsilon_2^{*2} - \frac{1}{\eta_2}\tilde{\theta}_2\dot{\hat{\theta}}_2 \\ &\le -c_1\nu_1^2 - c_2\nu_2^2 + \nu_2\nu_3 + \frac{1}{2}a_2^2 + \frac{1}{2}\varepsilon_2^{*2} + \frac{1}{2a_2^2}\nu_2\tilde{\theta}_2\varphi_2^T(\bar{x}_2)\varphi_2(\bar{x}_2) - \frac{1}{\eta_2}\tilde{\theta}_2\dot{\hat{\theta}}_2 \\ &\le -c_1\nu_1^2 - c_2\nu_2^2 + \nu_2\nu_3 + \frac{1}{2}a_2^2 + \frac{1}{2}\varepsilon_2^{*2} + \frac{1}{\eta_2}\tilde{\theta}_2\Big(\frac{\eta_2}{2a_2^2}\nu_2^2\varphi_2^T(\bar{x}_2)\varphi_2(\bar{x}_2) - \dot{\hat{\theta}}_2\Big) \end{aligned} \quad (31)$$

In the present, we design an adaptive law $\dot{\hat{\theta}}_2$ as

$$\dot{\hat{\theta}}_2 = \frac{\eta_2}{2a_2^2}\nu_2^2\varphi_2^T(\bar{x}_2)\varphi_2(\bar{x}_2) - \sigma_2\hat{\theta}_2 \quad (32)$$

By substituting (32) into (31), we have

$$\dot{V}_2 \le -c_1\nu_1^2 - c_2\nu_2^2 + \frac{1}{2}a_2^2 + \frac{1}{2}\varepsilon_2^{*2} + \nu_2\nu_3 + \frac{\sigma_2}{\eta_2}\tilde{\theta}_2\hat{\theta}_2 \quad (33)$$

Step 3: The time derivative of ν_3 is

$$\begin{aligned}\dot{\nu}_3 &= \dot{\lambda}_3 - \dot{r}_3 \\ &= \dot{x}_3 - \dot{\omega}_3 - \dot{r}_3 \\ &= x_4 - \dot{\omega}_3 - \dot{r}_3 \\ &= \lambda_4 + \omega_4 - \dot{\omega}_3 - \dot{r}_3 \\ &= \nu_4 + r_4 + \omega_4 - \dot{\omega}_3 - \dot{r}_3 \end{aligned} \quad (34)$$

Choose a Lyapunov function candidate as

$$V_3 = V_2 + \frac{1}{2}\nu_3^2 \quad (35)$$

The time derivative of V_3 is

$$\begin{aligned}\dot{V}_3 &= \dot{V}_2 + \nu_3\dot{\nu}_3 \\ &= \dot{V}_2 + \nu_3(\nu_4 + r_4 + \omega_4 - \dot{\omega}_3 - \dot{r}_3) \\ &\le -c_1\nu_1^2 - c_2\nu_2^2 + \frac{1}{2}a_2^2 + \frac{1}{2}\varepsilon_2^{*2} + \frac{\sigma_2}{\eta_2}\tilde{\theta}_2\hat{\theta}_2 + \nu_3(\nu_4 + \nu_2 + r_4 + \omega_4 + \alpha_3 - \alpha_3 - \dot{\omega}_3 - \dot{r}_3) \end{aligned} \quad (36)$$

Choose a virtual control signal α_3, the command filter $\dot{\omega}_3$ and the compensating signal $\dot{r}_3$ as

$$\alpha_3 = -c_3\lambda_3 - \nu_2 + \dot{\omega}_3 \quad (37)$$

$$\dot{\omega}_3 = \omega_n\omega_{3,2} \quad (38)$$

$$\dot{\omega}_{3,2} = -2\varsigma\omega_n\omega_{3,2} - \omega_n(\omega_3 - \alpha_2) \quad (39)$$

$$\dot{r}_3 = -c_3r_3 + r_4 + (\omega_4 - \alpha_3) \quad (40)$$

Substituting (37)-(40) into (36) results in

$$\begin{aligned}\dot{V}_3 \le& -c_1\nu_1^2 - c_2\nu_2^2 + \frac{1}{2}a_2^2 + \frac{1}{2}\varepsilon_2^{*2} + \frac{\sigma_2}{\eta_2}\tilde{\theta}_2\hat{\theta}_2 \\ &+ \nu_3(\nu_4 + \nu_2 + r_4 + \omega_4 - c_3\lambda_3 - \nu_2 + \dot{\omega}_3 \\ &- \alpha_3 - \dot{\omega}_3 + c_3 r_3 - r_4 - \omega_4 + \alpha_3) \\ \le& -c_1\nu_1^2 - c_2\nu_2^2 - c_3\nu_3^2 + \frac{1}{2}a_2^2 + \frac{1}{2}\varepsilon_2^{*2} \\ &+ \nu_3\nu_4 + \frac{\sigma_2}{\eta_2}\tilde{\theta}_2\hat{\theta}_2 \qquad (41)\end{aligned}$$

Step 4: The derivative of ν_4 can be expressed as

$$\dot{\nu}_4 = \delta_4(\bar{x}_5) + x_5 - \dot{\omega}_4 - \dot{r}_4 \qquad (42)$$

Choose a Lyapunov function candidate as

$$V_4 = V_3 + \frac{1}{2}\nu_4^2 + \frac{1}{2\eta_4}\tilde{\theta}_4^2 \qquad (43)$$

From (42)-(43), we can get the time derivative of V_4 as follows

$$\begin{aligned}\dot{V}_4 \le& -c_1\nu_1^2 - c_2\nu_2^2 - c_3\nu_3^2 + \frac{\sigma_2}{\eta_2}\tilde{\theta}_2\hat{\theta}_2 \\ &+ \frac{1}{2}a_2^2 + \frac{1}{2}\varepsilon_2^{*2} + \nu_4\Big(\delta_4(\bar{x}_5) + x_5 \\ &- \dot{\omega}_4 - \dot{r}_4\Big) - \frac{1}{\eta_4}\tilde{\theta}_4\dot{\hat{\theta}}_4 \qquad (44)\end{aligned}$$

Since $\delta_4(\bar{x}_5)$ is unknown, and cannot be applied to design virtual control signal α_4 directly. Neural network $\theta_4^{*T}\varphi_4(\bar{x}_5)$ is used to estimate $\delta_4(\bar{x}_5)$, such that, for any given positive constant $\varepsilon_4^* > 0$.

$$\delta_4(\bar{x}_5) \le \theta_4^{*T}\varphi_4(\bar{x}_5) + \varepsilon_4(\bar{x}_5) \qquad (45)$$

where $|\varepsilon_4(\bar{x}_5)| \le \varepsilon_4^*$. By taking (45) into account, one has

$$\begin{aligned}\dot{V}_4 \le& -c_1\nu_1^2 - c_2\nu_2^2 - c_3\nu_3^2 + \frac{\sigma_2}{\eta_2}\tilde{\theta}_2\hat{\theta}_2 + \frac{1}{2}a_2^2 \\ &+ \frac{1}{2}\varepsilon_2^{*2} + \nu_4\Big(\theta_4^{*T}\varphi_4(\bar{x}_5) + \alpha_4 - \alpha_4 + x_5 \\ &+ \varepsilon_4 - \dot{\omega}_4 - \dot{r}_4 + \nu_3\Big) - \frac{1}{\eta_4}\tilde{\theta}_4\dot{\hat{\theta}}_4 \qquad (46)\end{aligned}$$

By using Young's inequality and according to Lemma 3, we obtain

$$\begin{aligned}\nu_4\theta_4^{*T}(\bar{x}_5) \le& \frac{1}{2a_4^2}\nu_4^2\theta_4\varphi_4^T(\bar{x}_5)\varphi_4(\bar{x}_5) + \frac{1}{2}a_4^2 \\ \le& \frac{1}{2a_4^2}\nu_4^2\theta_4\varphi_4^T(\bar{x}_4)\varphi_4(\bar{x}_4) + \frac{1}{2}a_4^2 \qquad (47)\end{aligned}$$

$$\nu_4\varepsilon_4 \le \frac{1}{2}\nu_4^2 + \frac{1}{2}\varepsilon_4^{*2} \qquad (48)$$

By substituting (47)-(48) into (46), we can get

$$\begin{aligned}\dot{V}_4 \le& -c_1\nu_1^2 - c_2\nu_2^2 - c_3\nu_3^2 + \frac{\sigma_2}{\eta_2}\tilde{\theta}_2\hat{\theta}_2 + \frac{1}{2}\varepsilon_2^{*2} \\ &+ \frac{1}{2}a_2^2 + \nu_4\Big(\frac{1}{2a_4^2}\nu_4\theta_4\varphi_4^T(\bar{x}_4)\varphi_4(\bar{x}_4) \\ &+ \frac{1}{2}\nu_4 + \nu_3 + \nu_5 + r_5 + \omega_5 + \alpha_4 - \alpha_4 \\ &- \dot{\omega}_4 - \dot{r}_4\Big) - \frac{1}{\eta_4}\tilde{\theta}_4\dot{\hat{\theta}}_4 \qquad (49)\end{aligned}$$

Choose a virtual control signal α_4, the command filter $\dot{\omega}_4$ and the compensating signal $\dot{r}_4$ as

$$\begin{aligned}\alpha_4 =& -c_4\lambda_4 - \frac{1}{2a_4^2}\nu_4\hat{\theta}_4\varphi_4^T(\bar{x}_4)\varphi_4(\bar{x}_4) \\ &- \frac{1}{2}\nu_4 - \nu_3 + \dot{\omega}_4 \qquad (50)\end{aligned}$$

$$\dot{\omega}_4 = \omega_n\omega_{4,2} \qquad (51)$$

$$\dot{\omega}_{4,2} = -2\varsigma\omega_n\omega_{4,2} - \omega_n(\omega_4 - \alpha_3) \qquad (52)$$

$$\dot{r}_4 = -c_4 r_4 + r_5 + (\omega_5 - \alpha_4) \qquad (53)$$

Substituting (50)-(53) into (49) results in

$$\begin{aligned}\dot{V}_4 \le& -c_1\nu_1^2 - c_2\nu_2^2 - c_3\nu_3^2 + \frac{\sigma_2}{\eta_2}\tilde{\theta}_2\hat{\theta}_2 + \frac{1}{2}\varepsilon_2^{*2} \\ &+ \frac{1}{2}a_2^2 + \nu_4\Big(\frac{1}{2a_4^2}\nu_4\theta_4\varphi_4^T(\bar{x}_4)\varphi_4(\bar{x}_4) \\ &+ \frac{1}{2}\nu_4 + \nu_3 + \nu_5 - \frac{1}{2a_4^2}\nu_4\hat{\theta}_4\varphi_4^T(\bar{x}_4)\varphi_4(\bar{x}_4) \\ &+ r_5 + \omega_5 - c_4\lambda_4 - \nu_3 - \dot{\omega}_4 - \frac{1}{2}\nu_4 - \alpha_4 \\ &- \dot{\omega}_4 + c_4 r_4 - r_5 - \omega_5 + \alpha_4\Big) + \frac{1}{2}a_4^2 \\ &+ \frac{1}{2}\varepsilon_4^{*2} - \frac{1}{\eta_4}\tilde{\theta}_4\dot{\hat{\theta}}_4 \\ \le& -c_1\nu_1^2 - c_2\nu_2^2 - c_3\nu_3^2 - c_4\nu_4^2 + \frac{\sigma_2}{\eta_2}\tilde{\theta}_2\hat{\theta}_2 \\ &+ \frac{1}{2}a_2^2 + \frac{1}{2}\varepsilon_2^{*2} + \nu_4\nu_5 + \frac{1}{2}a_4^2 + \frac{1}{2}\varepsilon_4^{*2} \\ &+ \frac{1}{\eta_4}\tilde{\theta}_4\Big(\frac{\eta_4}{2a_4^2}\nu_4^2\varphi_4^T(\bar{x}_4)\varphi_4(\bar{x}_4) - \dot{\hat{\theta}}_4\Big) \qquad (54)\end{aligned}$$

In the present, we design an adaptive law $\dot{\hat{\theta}}_4$ as

$$\dot{\hat{\theta}}_4 = \frac{\eta_4}{2a_4^2}\nu_4^2\varphi_4^T(\bar{x}_4)\varphi_4(\bar{x}_4) - \sigma_4\hat{\theta}_4 \qquad (55)$$

Substituting (55) into (54) results in

$$\begin{aligned}\dot{V}_4 \le& -c_1\nu_1^2 - c_2\nu_2^2 - c_3\nu_3^2 - c_4\nu_4^2 + \frac{1}{2}a_2^2 + \frac{1}{2}\varepsilon_2^{*2} \\ &+ \frac{\sigma_2}{\eta_2}\tilde{\theta}_2\hat{\theta}_2 + \frac{\sigma_4}{\eta_4}\tilde{\theta}_4\hat{\theta}_4 + \frac{1}{2}a_4^2 + \frac{1}{2}\varepsilon_4^{*2} + \nu_4\nu_5 \qquad (56)\end{aligned}$$

Step 5: According to (2) and (12), we can get

$$\dot{\nu}_5 = \delta_5(\bar{x}_5) - \frac{1}{L}u - \dot{\omega}_5 - \dot{r}_5 \qquad (57)$$

Choose the following Lyapunov function

$$V_5 = V_4 + \frac{1}{2}\nu_5^2 + \frac{1}{2\eta_5}\tilde{\theta}_5^2 \qquad (58)$$

From (57)-(58), we obtain

$$\dot{V}_5 \leq -c_1\nu_1^2 - c_2\nu_2^2 - c_3\nu_3^2 - c_4\nu_4^2 + \frac{\sigma_2}{\eta_2}\tilde{\theta}_2\hat{\theta}_2 + \frac{\sigma_4}{\eta_4}\tilde{\theta}_4\hat{\theta}_4 + \frac{1}{2}a_2^2 + \frac{1}{2}\varepsilon_2^{*2} + \frac{1}{2}a_4^2 + \frac{1}{2}\varepsilon_4^{*2} + \nu_5\Big(\delta_5(\bar{x}_5) - \frac{1}{L}u - \dot{\omega}_5 - \dot{r}_5\Big) - \frac{1}{\eta_5}\tilde{\theta}_5\dot{\hat{\theta}}_5 \quad (59)$$

Similarly, neural network $\theta_5^{*T}\varphi_5(\bar{x}_5)$ is used to estimate $\delta_5(\bar{x}_5)$, such that

$$\delta_5(\bar{x}_5) \leq \theta_5^{*T}\varphi_5(\bar{x}_5) + \varepsilon_5(\bar{x}_5) \quad (60)$$

where $|\varepsilon_5(\bar{x}_5)| \leq \varepsilon_5^*$, with ε_5^* being a positive constant. Therefore, we choose $-\frac{1}{L} = p$ and according to (4), we get

$$\dot{V}_5 \leq -c_1\nu_1^2 - c_2\nu_2^2 - c_3\nu_3^2 - c_4\nu_4^2 + \frac{\sigma_2}{\eta_2}\tilde{\theta}_2\hat{\theta}_2 + \frac{\sigma_4}{\eta_4}\tilde{\theta}_4\hat{\theta}_4 + \frac{1}{2}a_2^2 + \frac{1}{2}\varepsilon_2^{*2} + \frac{1}{2}a_4^2 + \frac{1}{2}\varepsilon_4^{*2} + \nu_5\Big(\theta^{*T}\varphi_5(\bar{x}_5) + \nu_4 + pm\nu + pd(\nu) + \varepsilon_5 - \dot{\omega}_5 - \dot{r}_5\Big) - \frac{1}{\eta_5}\tilde{\theta}_5\dot{\hat{\theta}}_5 \quad (61)$$

By using Young's inequality, we can get

$$\nu_5\theta_5^{*T}\varphi_5(\bar{x}_5) \leq \frac{1}{2a_5^2}\nu_5^2\hat{\theta}_5\varphi_5^T(\bar{x}_5)\varphi_5(\bar{x}_5) + \frac{1}{2}a_5^2 \quad (62)$$

$$\nu_5\varepsilon_5 \leq \frac{1}{2}\nu_5^2 + \frac{1}{2}\varepsilon_5^{*2} \quad (63)$$

$$\nu_5 pd(\nu) \leq \frac{1}{2}\nu_5^2 + \frac{1}{2}D_1^2 \quad (64)$$

where let $|pd(\nu)| < D_1$. By substituting (62)-(64) into (61), we can conclude

$$\dot{V}_5 \leq -c_1\nu_1^2 - c_2\nu_2^2 - c_3\nu_3^2 - c_4\nu_4^2 + \frac{\sigma_2}{\eta_2}\tilde{\theta}_2\hat{\theta}_2 + \frac{\sigma_4}{\eta_4}\tilde{\theta}_4\hat{\theta}_4 + \frac{1}{2}a_2^2 + \frac{1}{2}\varepsilon_2^{*2} + \frac{1}{2}a_4^2 + \frac{1}{2}\varepsilon_4^{*2} + \nu_5\Big(\frac{1}{2a_5^2}\nu_5\theta_5\varphi_5^T(\bar{x}_5)\varphi_5(\bar{x}_5) + \nu_4 + pm\nu + \frac{1}{2}\nu_5 - \dot{\omega}_5 + \frac{1}{2}\nu_5 - \dot{r}_5\Big) + \frac{1}{2}a_5^2 + \frac{1}{2}\varepsilon_5^{*2} + \frac{1}{2}D_1^2 - \frac{1}{\eta_5}\tilde{\theta}_5\dot{\hat{\theta}}_5 \quad (65)$$

Choose a controller ν, the command filter $\dot{\omega}_5$ and the compensating signal $\dot{r}_5$ as

$$\nu = \frac{1}{pm}\Big(-c_5\lambda_5 - \frac{1}{2a_5^2}\nu_5\hat{\theta}_5\varphi_5^T(\bar{x}_5)\varphi_5(\bar{x}_5) - \nu_5 - \nu_4 + \dot{\omega}_5\Big) \quad (66)$$

$$\dot{\omega}_5 = \omega_n\omega_{5,2} \quad (67)$$

$$\dot{\omega}_{5,2} = -2\varsigma\omega_n\omega_{5,2} - \omega_n(\omega_5 - \alpha_4) \quad (68)$$

$$\dot{r}_5 = -c_5 r_5 \quad (69)$$

Substituting (66)-(69) into (65), we have

$$\begin{aligned}\dot{V}_5 \leq& -c_1\nu_1^2 - c_2\nu_2^2 - c_3\nu_3^2 - c_4\nu_4^2 + \frac{1}{2}a_2^2 + \frac{1}{2}\varepsilon_2^{*2} + \frac{\sigma_2}{\eta_2}\tilde{\theta}_2\hat{\theta}_2 + \frac{\sigma_4}{\eta_4}\tilde{\theta}_4\hat{\theta}_4 + \frac{1}{2}a_4^2 \\ &+ \frac{1}{2}\varepsilon_4^{*2} + \nu_5\Big(\frac{1}{2a_5^2}\nu_5\theta_5\varphi_5^T(\bar{x}_5)\varphi_5(\bar{x}_5) + \nu_4 - c_5\lambda_5 - \frac{1}{2a_5^2}\nu_5\hat{\theta}_5\varphi_5^T(\bar{x}_5)\varphi_5(\bar{x}_5) \\ &- \nu_5 - \nu_4 + \dot{\omega}_5 + \nu_5 - \dot{\omega}_5 + c_5 r_5\Big) + \frac{1}{2}a_5^2 + \frac{1}{2}\varepsilon_5^{*2} + \frac{1}{2}D_1^2 - \frac{1}{\eta_5}\tilde{\theta}_5\dot{\hat{\theta}}_5 \\ \leq& -c_1\nu_1^2 - c_2\nu_2^2 - c_3\nu_3^2 - c_4\nu_4^2 - c_5\nu_5^2 + \frac{1}{2}a_2^2 + \frac{1}{2}\varepsilon_2^{*2} + \frac{\sigma_2}{\eta_2}\tilde{\theta}_2\hat{\theta}_2 + \frac{\sigma_4}{\eta_4}\tilde{\theta}_4\hat{\theta}_4 \\ &+ \frac{1}{2}a_4^2 + \frac{1}{2}\varepsilon_4^{*2} + \frac{1}{2}a_5^2 + \frac{1}{2}\varepsilon_5^{*2} + \frac{1}{2}D_1^2 + \frac{1}{\eta_5}\tilde{\theta}_5\Big(\frac{\eta_5}{2a_5^2}\nu_5^2\varphi_5^T(\bar{x}_5)\varphi_5(\bar{x}_5) - \dot{\hat{\theta}}_5\Big)\end{aligned} \quad (70)$$

In the present, we design an adaptive law $\dot{\hat{\theta}}_5$ in the following

$$\dot{\hat{\theta}}_5 = \frac{\eta_5}{2a_5^2}\nu_5^2\varphi_5^T(\bar{x}_5)\varphi_5(\bar{x}_5) - \sigma_5\hat{\theta}_5 \quad (71)$$

Substituting (71) into (70), we can figure that out

$$\begin{aligned}\dot{V}_5 \leq& -c_1\nu_1^2 - c_2\nu_2^2 - c_3\nu_3^2 - c_4\nu_4^2 - c_5\nu_5^2 + \frac{1}{2}a_2^2 + \frac{1}{2}\varepsilon_2^{*2} + \frac{\sigma_2}{\eta_2}\tilde{\theta}_2\hat{\theta}_2 + \frac{\sigma_4}{\eta_4}\tilde{\theta}_4\hat{\theta}_4 \\ &+ \frac{\sigma_5}{\eta_5}\tilde{\theta}_5\hat{\theta}_5 + \frac{1}{2}a_4^2 + \frac{1}{2}\varepsilon_4^{*2} + \frac{1}{2}a_5^2 + \frac{1}{2}\varepsilon_5^{*2} + \frac{1}{2}D_1^2 \\ \leq& -\sum_{i=1}^{5}c_i\nu_i^2 + \sum_{j=2,4,5}\frac{\sigma_j}{\eta_j}\tilde{\theta}_j\hat{\theta}_j + \sum_{j=2,4,5}\frac{1}{2}a_j^2 + \sum_{j=2,4,5}\frac{1}{2}\varepsilon_j^{*2} + \frac{1}{2}D_1^2\end{aligned} \quad (72)$$

Through the above analysis and design, we can get the following main result.

Theorem 1: Consider the single-link flexible robotic manipulator (1) with Assumptions 1-4. For bounded initial conditions, the proposed adaptive neural network control scheme can ensure that all the signals in the closed-loop system is bounded. Moreover, the tracking error can be made arbitrarily small by choosing appropriate design parameters.

Proof: According to Young's inequality, one has

$$\tilde{\theta}_j\hat{\theta}_j = \tilde{\theta}_j(\theta_j - \tilde{\theta}_j) \leq -\frac{1}{2}\tilde{\theta}_j^2 + \frac{1}{2}\theta_j^2 \quad (73)$$

Substituting (73) into (72) produces

$$\dot{V}_5 \le -(2c_i \sum_{i=1}^{5} \frac{1}{2}\nu_i^2 + 2\sigma_j \sum_{j=2,4,5} \frac{1}{\eta_j}\tilde{\theta}_j^2) + \sum_{j=2,4,5} (\frac{\sigma_j}{2\eta_j}\theta_j^2 + \frac{1}{2}a_j^2 + \frac{1}{2}\varepsilon_j^{*2}) + \frac{1}{2}D_1^2 \tag{74}$$

Then, (72) can be rewritten as

$$\dot{V}_5 \le -CV_5 + D \tag{75}$$

where $C = min\{2c_i, 2\sigma_j, i = 1 \cdots 5, j = 2, 4, 5\}$,

$$D = \sum_{j=2,4,5} (\frac{\sigma_j}{2\eta_j}\theta_j^2 + \frac{1}{2}a_j^2 + \frac{1}{2}\varepsilon_j^{*2}) + \frac{1}{2}D_1^2.$$

By integrating (75) over $[0, t]$, one has

$$0 \le V_5(t) \le \Big(V_5(0) - \frac{D}{C}\Big)e^{-Ct} + \frac{D}{C} \tag{76}$$

which means that

$$\lim_{t\to\infty} V_5(t) \le \frac{D}{C} \tag{77}$$

According to (76), we can get

$$|\nu_1| \le \sqrt{2\Big(V_5(0)e^{-Ct} + \frac{D}{C}\Big)} \tag{78}$$

which implies that

$$\lim_{t\to\infty} |\nu_1| \le \sqrt{2D/C} \tag{79}$$

This means ν_1 is bounded. In a similar way, ν_i, $i = 2, 3, 4, 5$ is also bounded.

To guarantee the boundedness of output tracking error $\lambda_1 = \nu_1 + r_1$, the convergence of r_1 should be considered. Similarly, in order to obtain the boundedness of λ_i, it is necessary to study the property of r_i. To this end, consider the system consisting of the error compensating signals defined in equations (17), (30), (40), (53), (69).

$$\dot{r}_i = -c_i r_i + r_{i+1} + (\omega_{i+1} - \alpha_i), \quad i = 1, \cdots, 4 \tag{80}$$

$$\dot{r}_5 = -c_5 r_5 \tag{81}$$

The following lemma shows that the compensating signals are bounded.

Lemma 4 [19] [35]: The system defined in (80)-(81), whose states are bounded by

$$\lim_{t\to\infty} V_r(t) \le \frac{b_0\xi^2\beta^2}{a_0} \tag{82}$$

where b_0 and a_0 are positive constants. For bounded input α_i, which satisfies $\|g_i(\omega_{i+1} - \alpha_i)\| \le \xi\beta$, $i = 1, 2, 3, 4$. g_i represents the control coefficient of the command filter error, that is, $g_1 = 1$, $g_2 = 1$, $g_3 = 1$, $g_4 = 1$. ξ is the upper bound of command filter error. β is the upper bound of g_i.

Proof: For (80)-(81), we can construct the following Lyapunov function $V_r = \sum_{i=1}^{5} \frac{1}{2} r_i^2$. Taking the time derivative of V_r with the help of (80)-(81) and Young's inequality yields

$$\begin{aligned}\dot{V}_r &= \sum_{i=1}^{5} r_i \dot{r}_i \\ &= r_1(-c_1 r_1 + r_2 + \omega_2 - \alpha_1) \\ &\quad + r_2(-c_2 r_2 + r_3 + \omega_3 - \alpha_2) \\ &\quad + r_3(-c_3 r_3 + r_4 + \omega_4 - \alpha_3) \\ &\quad + r_4(-c_4 r_4 + r_5 + \omega_5 - \alpha_4) \\ &\quad + r_5(-c_5 r_5) \\ &\le -\sum_{i=1}^{5} c_i r_i^2 + \xi\beta(r_1 + r_2 + r_3 + r_4) \\ &\le -\sum_{i=1}^{4} (c_i - \frac{1}{2}) r_i^2 - c_5 r_5^2 + 2\xi^2\beta^2 \\ &\le -a_0 V_r + b_0 \xi^2 \beta^2 \end{aligned} \tag{83}$$

where $a_0 = \min\{2(c_i - \frac{1}{2}), 2c_5\}$, $i = 1, 2, 3, 4$, and $b_0 = 2$. By choosing appropriate a_0 and solving (83), the following result can be obtained

$$\lim_{t\to\infty} V_r(t) \le \frac{b_0\xi^2\beta^2}{a_0} \tag{84}$$

From (12) and (84), it can be concluded that the signal λ_i is bounded because of the boundedness of ν_i and r_i in (77) and (84), respectively. To sum up, it can be shown that all the signals in the closed-loop system are bounded.

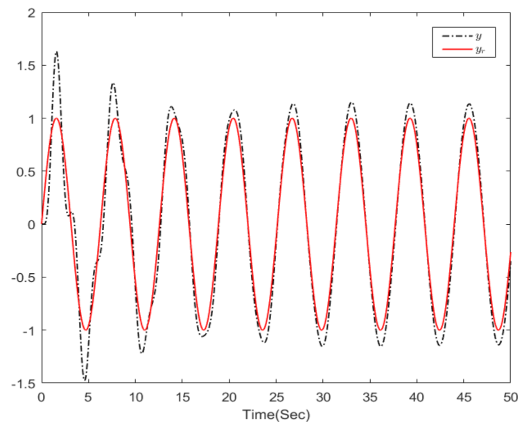

FIGURE 1. The system output y and reference signal y_r.

IV. SIMULATION RESULTS

In this section, to illustrate the effectiveness of the presented control scheme, the simulation is carried out for the control system (1), where $J_1 = J_2 = 30Kgm^2$, $K_t = 8Nm/A$, $K_b = 1Nm/A$, $g = 9.8N/Kg$, $M = 0.5Kg$, $F_1 = F_2 = 5Nms/rad$, $R = 10\Omega$, $K = 5$, $L = 5H$, $N = 1$, $d = 0.5m$, $m = 1$. The Dead-zone model $u = D(v)$ is defined as

$$u = D(v) = \begin{cases} m(v - 2.5), & v \ge 2.5 \\ 0, & -2 < v < 2.5 \\ m(v + 2), & v \le -2 \end{cases} \tag{85}$$

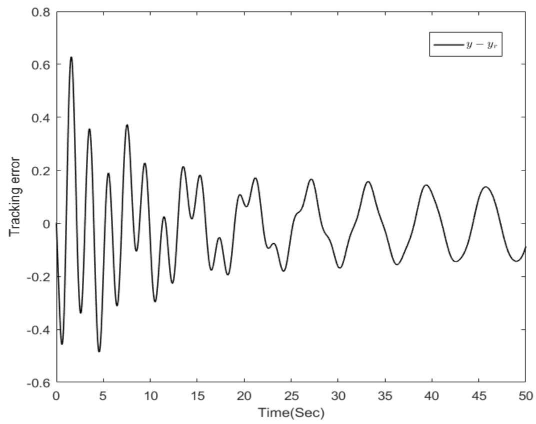

FIGURE 2. The tracking error $y - y_r$.

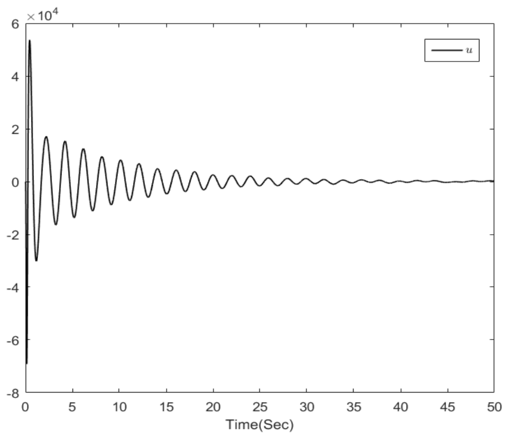

FIGURE 3. The actual control input u.

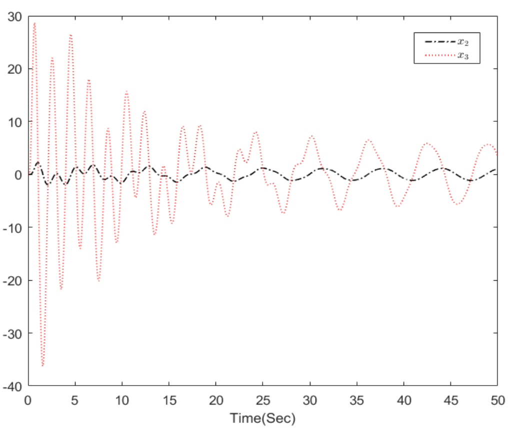

FIGURE 4. The state variables x_2, x_3.

The design parameters in the presented control scheme are chosen as $\omega_n = [10 \quad 20 \quad 30 \quad 30]^T$, $\varsigma = 0.85$, $a_2 = a_4 = a_5 = 10$, $c_1 = 6.8$, $c_2 = c_5 = 5$, $c_3 = 15$, $c_4 = 10$, $\eta_2 = \eta_4 = \eta_5 = 0.1$, $\sigma_2 = \sigma_4 = \sigma_5 = 1$.

The desired trajectory is selected as $y_r = \sin t$, the initial conditions of the states are selected as $x_1(0) = 0.003$, $x_2(0) = 0.001$, the initial conditions of the adaptive law are selected as $\hat{\theta}_2(0) = 0.001$, $\hat{\theta}_4(0) = 0.001$, $\hat{\theta}_5(0) = 0.001$, and other initial conditions are zero.

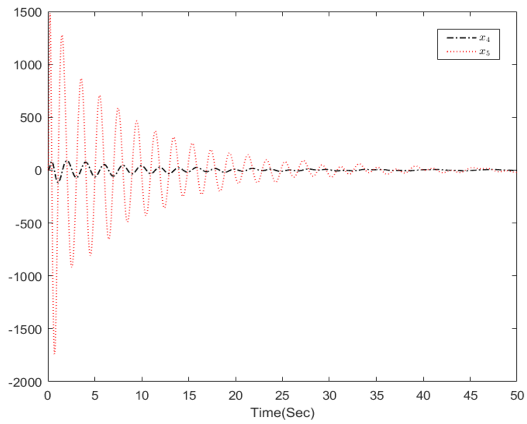

FIGURE 5. The state variables x_4, x_5.

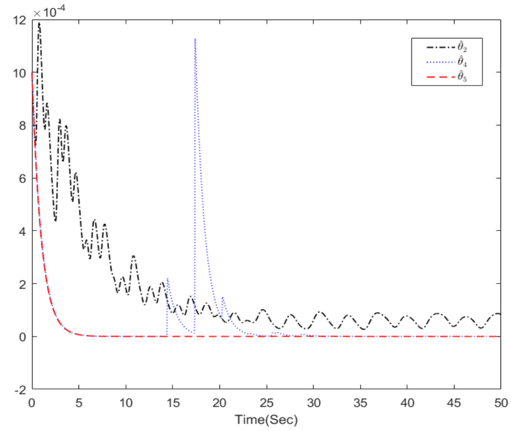

FIGURE 6. The adaptive laws $\hat{\theta}_2$, $\hat{\theta}_4$, $\hat{\theta}_5$.

The simulations results are indicated by Figs. 1-6. Fig. 1 indicates the tracking trajectories of output and reference signal. Fig. 2 displays the tracking error. Fig. 3 shows the actual control input. Fig. 4-5 show the state variables $x_2, x_3, x_4.x_5$. Fig. 6 shows the system adaptive laws. Apparently, simulation results show that all the signals in the closed-loop system are bounded.

V. CONCLUSION

In this paper, an adaptive neural network command filter control method has been presented for a single-link robotic manipulator with input dead-zone. The presented control scheme ensures that all the signals in the closed-loop system are bounded, and the tracking error eventually enters into a small area around the origin. The problem of 'explosion of complexity' existing in the conventional backstepping control method is avoided by introducing the command filter technique. Both the theory analysis and simulation results have illustrated the feasibility and effectiveness of the proposed scheme.

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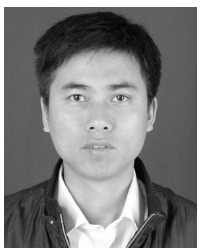

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