

Received October 30, 2018, accepted December 20, 2018, date of publication February 20, 2019, date of current version March 1, 2019.

Digital Object Identifier 10.1109/ACCESS.2019.2893542

Performance Evaluation of Advanced Relaying Protocols in Large Wireless Networks

ANDRÉS ALTIERI^{1,2}, (Member, IEEE), AND PABLO PIANTANIDA^{3,4}, (Senior Member, IEEE)

¹Department of Electronics, Universidad de Buenos Aires, Buenos Aires C1063ACV, Argentina

²CSC-CONICET, Buenos Aires C1425FQD, Argentina

³CentraleSupélec—French National Center for Scientific Research—Université Paris-Sud, 91190 Gif-sur-Yvette, France

⁴Montreal Institute for Learning Algorithms, Université de Montréal, Montreal, QC H2S 3H1, Canada

Corresponding author: Andrés Altieri (aaltieri@fi.uba.ar)

This work was partially supported by a Level II Bilateral Cooperation Project funded by the CNRS, MinCyT, and CONICET. The work of P. Piantanida was supported by the PICS MoSiME through CNRS.

ABSTRACT This paper studies the performance of some state-of-the-art cooperative full-duplex relaying protocols in the context of a large wireless network modeled using stochastic geometry tools. We investigate the outage behavior for different cooperative schemes, namely decode-and-forward, noisy-network coding, and mixed noisy-network coding, considering fading, path loss, and interference from other sources and relays. Due to the high complexity of the network topology and the protocols considered, a closed-form analysis is not possible, so our study is performed through extensive but careful numerical simulations, sweeping a large number of relevant parameters. Several scenarios of particular interest are investigated. In this way, insightful conclusions are drawn regarding the network regimes in which relay-assisted cooperation is most beneficial and the potential gains that could be achieved through it.

INDEX TERMS Cooperative communications, decode-and-forward, compress-and-forward, noisy network coding, full duplex, outage probability, Poisson point process, stochastic geometry.

I. INTRODUCTION

For many years, cooperative wireless communications have been an active area of research, showing promising gains in terms of throughput and reliability. One of the most interesting scenarios is that in which cooperation takes place through the use of wireless relays, which aid a transmitter-receiver pair either in a full-duplex or half-duplex fashion [1], [2]. In their seminal work, Cover and Gamal [3] introduced the main cooperative relaying schemes, and since then, these schemes have been improved upon and new ones have been derived (e.g. see [2], [4], [5] and references therein). In many cases, the evaluation of the performance of these protocols via information-theoretic tools is restricted to a small number of nodes affected by uncorrelated Gaussian noise. However, in a more realistic scenario, there will be a large number of source-destination pairs transmitting and the effects of the interference will be larger than the effects of the noise. Furthermore, the interference at the receivers will be correlated because the interference comes from the same sources. Finally, in a large wireless network users interact and may cause adverse interference conditions to each other. In this context, stochastic geometry [6], [7] has emerged as a useful

tool to model and study different aspects of large wireless networks. In a stochastic geometry setting, the random distribution of nodes in a network is modeled through a spatial point process and different, spatially-correlated magnitudes such as the interference at several points of the network can be considered. Thus, averaging over all possible realizations of the node's positions, the average behavior of the network can be studied. Many interesting problems involving multiple nodes and spatially correlated interference have been studied in this framework and elegant closed-form results have been found [8]–[10]. Nevertheless, as more complex network architectures, protocols and interactions between the nodes are considered, closed form or approximate solutions become increasingly hard to obtain, and bounds or approximations become inaccurate. Another common situation is that as more complex communication protocols are considered, the complexity and the number of error events involved in the decoding procedure grow very fast, making it hard or impossible to evaluate the performance of a protocol in closed-form. This is specially true in the context of advanced relaying protocols which may use many relays and sophisticated decoding strategies. For this reason, in some of these

complex scenarios it may be interesting to perform numerical simulations, with the aim of gaining insight and drawing qualitative conclusions regarding certain problems.

In this paper, we propose to study the performance of advanced relaying protocols in the context of a large wireless network modeled using stochastic geometry tools. These protocols are the state-of-the-art in cooperative relaying strategies drawn from information theory and can be interpreted as a benchmark in terms of performance. The complexity of these protocols, specially when multiple relays are involved, makes it almost impossible to evaluate their performance with a closed-form expressed in terms of simple functions. It is nevertheless interesting to explore the performance that could be achieved through these protocols. The network is composed of source-destination pairs which attempt to communicate with the help of full-duplex relays. The transmissions in the network are affected by path-loss and slow fading, and each source and its relays cause interference to other relays and destinations in the network. This implies that, as more relays are added, the overall interference increases. Also, the relays that help each source are drawn from a spatial model, meaning that as more relays help a source-destination pair, the further away they will be and their contribution to improving the quality of the links will (on average) diminish. These two simultaneous effects introduce a balance between cooperation and interference which would also be present in a real network. The main metric to evaluate the performance of the protocols is the *outage probability* (OP) that is, the probability that, due to instantaneous conditions, the channel cannot support the rate attempted by a transmitting user. The protocols considered are decode-and-forward [3], noisy network coding [4], which is an extension of the well-known compress-and-forward scheme [3], and mixed noisy-network coding [11], which combines both protocols, allowing some of the relays to perform decode-and-forward and others noisy network coding. The transmissions in the network are decoded treating other transmissions, which are not helpful, as noise, and each destination can choose which subset of its helping relays to use to decode the message from its source. We study the performance of these protocols under different network setups, considering relevant parameters such as relay density, position, and transmission power, and conclusions are drawn about the possible gains that could be achieved, and which scenario is the most favorable.

A. RELATED WORKS

There have been some works investigating cooperative communication with relays in wireless networks under the effects of interference. In general half-duplex DF has drawn the most attention [12]–[15]. For example, in [12], the authors consider a selection half-duplex DF protocol in which there is one transmitter which is concurrently transmitting and may cause interference. The node distribution is not random but fixed and the source cannot directly reach the destination. Papers which consider stochastic geometry models are for example [13]–[15]. In [13] the authors perform an outage

probability analysis of a half-duplex DF protocol in which only one of several relays retransmits a message to a destination. If multiple relays are available they are assumed to be superimposed at the same fixed position, that is there is no spatial model for the relay distribution, like a point process model. In [14] the authors considered an outage and diversity-order analysis of a half-duplex selection decode-and-forward protocol. The relay is in a fixed position on the line between the source and the destination. In [16] we obtained the OP of full-duplex and half-duplex decode-and-forward and compress-and-forward schemes for a single relay affected by interference. In all these scenarios it is assumed that only a single relay is active at once, either in full-duplex or half-duplex fashion, and interference is caused by nodes distributed as a Poisson point process (PPP). The interfering nodes are not allowed to use relays, so the analysis performed does not take into account that in real-world networks as more users request relays there will be an overall increase of interference up to the point where cooperation may become useless or even detrimental. Besides this, the relays' positions are assumed to be fixed and in some cases superimposed which does not consider that as more relays are added to communicate, they will likely be further away from the source and destination, reducing the possible improvement of cooperation. In [17] some of the above mentioned assumptions were addressed where we considered a decentralized wireless network in which each source-destination pair has a relay that can be active or not according to some probability. Cooperation can only take place using decode-and-forward, and the authors derive the optimal relay activation probability and the gains that can be achieved in terms of the OP according to the position of the relays.

The main focus of these works was at finding closed form results which are afterwards validated through simulation. Although whenever it is possible closed-form expressions are very valuable, this approach limits the possibility of studying complex scenarios in which closed form expressions or tight bounds cannot be found. The setup proposed in this paper can be considered as an extension of the setup of [17] which was described in the previous paragraph. However, in the present work each source-destination pair can take advantage of several different relays and the cooperative protocols considered are the state of the art and thus these are expected to perform better. Furthermore, several additional networks parameters are introduced and studied.

B. MAIN CONTRIBUTIONS

The main contribution of this work is studying the performance of some state-of-the-art full-duplex cooperative relaying protocols in a large interference-limited wireless network. These protocols can be interpreted as a benchmark for performance in terms of error probabilities and their performance has not been studied, to the best of our knowledge, in the context of a wireless network with multiple relays in which correlated interference is present between the nodes. In this study we aim at studying how well these protocols actually

perform when compared to a direct transmission and in which network operating regimes they bring the greatest advantages in terms of outage, and how large the improvements are. The conclusions drawn cannot be completely general because only simulations, over a wide operation condition, are considered, but we believe that it is at least interesting to study the performance of these protocols in well-established modeling scenarios, as compared to not knowing how well these protocols might actually perform. This setup is very different to the usual AWGN case in which communications are hampered by uncorrelated Gaussian noise at each receiver. In our setup, additional relays create more interference in the network, and also, since the relays come from a spatial model, activating more relays implies that the relays will on average be further away and the benefits of their activation will be smaller. In this framework, we consider three representative protocols, namely, opportunistic decode-and-forward (ODF) [1], noisy network coding (NNC) [4] and mixed noisy-network coding (MNNC) [11]. For each of these protocols we consider two versions: a standard version in which all the relays of each source-destination pair can transmit independently of their channel qualities towards the source or destination, and *interference aware* versions, in which the relays can transmit only if their channel towards its destination or its source are above a certain threshold. These interference aware protocols aim at reducing the interference in the network by turning off relays which most likely would not help their corresponding source-destination pairs. Since closed forms or close approximations are almost impossible to obtain, we have performed extensive simulations on the network and provide conclusions regarding the dependence of the OP of each protocol with respect to the relay density, the relative transmission powers between relay and sources, the number of active relays for each source-destination pair and the position of the relays. Also, we compare the performance of each of the protocols in their standard and interference aware forms.

Another secondary contribution is the release of the software that has been developed to evaluate the performance of these protocols, which can be further used to explore other scenarios involving multiple relays with different setups or even different protocols. Testing these complex network models and protocols is a very difficult task that requires large-scale computer-based simulations whose intrinsic difficulty should not be underestimated. This is because the outage events become increasingly complex for more advanced protocols, and also the number of events grows exponentially with the number of relays. Finally, the interference time signals are correlated by the spatial distribution of the nodes in a very complex fashion.

The paper is organized as follows. In Section II, we present the mathematical model of the network, and in Section III we present the protocols considered and their outage events. In Section IV, we show how the outage events are evaluated for the network, and in Section V we present the numerical results. Finally, in Section VI we summarize some conclusions and comments.

NOTATION

T denotes transpose and $*$ complex conjugation for scalars and transpose-conjugate for matrices or vectors. $I(\cdot; \cdot)$ and $I(\cdot; \cdot | \cdot)$ denote mutual information and conditional mutual information respectively [18]. $h(\cdot, \cdot)$ denotes differential entropy [18].

II. NETWORK MODEL

We consider a planar network model in which source nodes attempt to communicate a message to their destination with the cooperation of other nearby nodes which act as relays. Relays are assumed to work in full-duplex mode on the same time slots and frequency bands as the sources. The main modeling assumptions are the following:

- The spatial distribution of the sources is modeled as a homogeneous PPP Φ_s of intensity λ_s . Each source has a destination which is located at a distance D from the source in a random uniform direction from the source.
- Each source-destination pair has a set of n_r potential relays. In order to define the position of the relays we propose the following model: we assume that the nodes which may act as relays are distributed in space as a homogeneous PPP of intensity λ_r . Then, for each source-destination pair, a point which lies on the line between them is chosen. For example, for a source at x and a destination at d_x , a point c_x is chosen as:

$$c_x = x + \varepsilon (d_x - x), \quad (1)$$

where $0 \leq \varepsilon \leq 1$ (the same for all sources). Finally, the n_r potential relays of the pair are found as the points of Φ_r which are closest to c_x . Notice that ε is a network setup parameter which allows us to control whether the relays will be chosen closer to the source or the destination on average.

This model involves the intrinsic difficulty that, since all the relays come from Φ_r , it is possible that different source-destination pairs –which are close to each other– will choose the same potential relay. However, this event is very unlikely provided that the density of potential relays is much larger than the density of sources ($\lambda_r \gg \lambda_s$), which is likely to occur. However, this complicates the implementation of the network because this possibility of sharing relays has to be considered. For this reason, we introduce the following simplifying assumption: the marginal distribution of the position of the potential relays of each cluster are the same as if selected from Φ_r by the above procedure, but are independent among clusters. With this assumption, we can determine the density of the positions of the potential relays for each cluster and then generate the potential relays for each source-destination pair independently, without needing to explicitly draw the PPP Φ_r , and exhaustively search for the potential relays for each source-destination pairs. We refer to each group formed by a source-destination pair and their corresponding n_r potential relays a *cluster*. The following lemma gives

this distribution and allows the simulation of n_r closest neighbors of each source-destination pair:

Lemma 2.1: Let X_1, X_2, \dots be the positions of the nearest nodes of a homogeneous PPP in \mathbb{R}^2 to a fixed point x , relative to this point, in order of increasing distance. Then $\{\|X_1\|^2, \|X_2\|^2, \dots\}$ forms a homogenous PPP of intensity: $\lambda_r \pi$ in $(0, \infty)$, and the phases of these points are independent uniform random variables in $[0, 2\pi)$, independent of the process of distances.

Proof: See for example [19]. □

- The sources and the relays use Gaussian signaling, that is, the codebooks used are generated as draws of independent complex zero-mean Gaussian random variables of variance P_s for sources and P_r for relays. The communication channels are narrow-band with flat-fading, and transmissions are attenuated both by path loss and independent Rayleigh fading, that is, the channel between to points x and y is:

$$g_{x,y} = h_{x,y} \sqrt{l(x,y)}, \quad (2)$$

where $h_{x,y}$ is a complex circularly symmetric Gaussian (CCSG) signal fading coefficient with zero-mean and unit variance; $l(x,y) \triangleq \|x - y\|^{-\alpha}$ denotes the power path loss function ($\alpha > 2$), and the channels are independent between points. This means that the power fading coefficients $|h_{x,y}|^2$ are independent unit-mean exponential random variables.

We assume that a destination is located at the origin, with its source located at $p_s = (-D, 0)$, and they are called the typical destination and source, respectively. The n_r potential relays of the typical source-destination pair are centered around the point c_s according to (1) and are denoted, in order of distance to this point, as $\{r_1, \dots, r_{n_r}\}$. We call the ‘‘typical cluster’’ the group of users formed by the destination at the origin, its source and its potential relays. The channel gains from the source to its potential relays and the destination are denoted as $g_{s,i}$ ($i = 1, \dots, n_r$) and $g_{s,d}$, respectively. Channel gains from the potential relays to the other relays and the destination are denoted as $g_{i,j}$ and $g_{i,d}$, ($i, j = 1, \dots, n_r, i \neq j$), respectively. Channel gains from an interfering source at x to the typical relays and the destination are denoted as $\tilde{g}_{x,i}$, $i = 1, \dots, n_r$, and $\tilde{g}_{x,d}$, respectively. Finally, the channel gains from the i -th relay of a source at x to the j -th relay and the destination of the typical cluster are denoted as $\tilde{g}_{x,i,j}$, and $\tilde{g}_{x,i,d}$, $i, j = 1, \dots, n_r$, respectively. In Fig. 1 we can see a representation of the network and the relevant channels.

In the interference aware protocols, a destination may have some of its n_r potential relays turned off, if their channels toward their destination or from their source are not considered to be strong enough. For a source located at x we denote by $\mathcal{A}_x \subseteq \{1, \dots, n_r\}$ the set of the indexes of the potential relays that have not been turned off due to bad channels while for the typical cluster, we denote this set by \mathcal{A}_s . Some of these relays which have not been turned off will be transmitting,

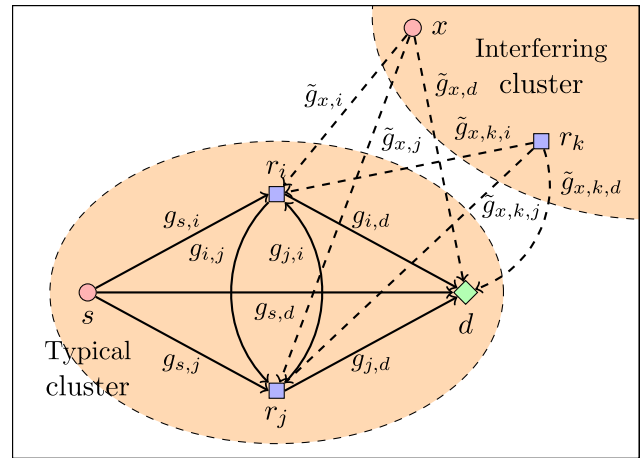


FIGURE 1. Representation of the network and relevant channel involved. d denotes the typical destination at origin, and s is the typical source at $p_s = (-D, 0)$.

depending on the protocol employed. For example, in ODF, only the relays which have not been turned off and can decode the message from the source will transmit. We denote by $\mathcal{B}_x \subseteq \mathcal{A}_x$ the relays of the source at a point x which are transmitting, and by \mathcal{B}_s the relays of the typical cluster which are transmitting.

With this model we can now determine the signals received in each node of the typical cluster, and the power and correlation of their corresponding interference signals. We denote by $Y_{d,k}$ and $Y_{i,k}$ the signals received at the destination and the i -th relay of the typical cluster at the time instant k , respectively, which we may write as:

$$Y_{d,k} = g_{s,d}X_{s,k} + \sum_{m=1}^{n_r} \mathbb{1}_{\{m \in \mathcal{B}_s\}} g_{m,d}X_{m,k} + Z_{d,k}, \quad (3)$$

$$Y_{i,k} = g_{s,i}X_{s,k} + \sum_{m=1, m \neq i}^{n_r} \mathbb{1}_{\{m \in \mathcal{B}_s\}} g_{m,i}X_{m,k} + Z_{i,k}, \quad (4)$$

where $X_{s,k}$ and $X_{m,k}$ denote the symbols transmitted by the typical source and its m -th relay at time k . $Z_{d,k}$ and $Z_{i,k}$ are the interference time signals, which we may write as:

$$Z_{d,k} = \sum_{x \in \Phi_s} \tilde{g}_{x,d} \tilde{X}_{x,k} + \sum_{m=1}^{n_r} \mathbb{1}_{\{m \in \mathcal{B}_x\}} \tilde{g}_{x,m,d} \tilde{X}_{x,m,k}, \quad (5)$$

$$Z_{i,k} = \sum_{x \in \Phi_s} \tilde{g}_{x,i} \tilde{X}_{x,k} + \sum_{m=1}^{n_r} \mathbb{1}_{\{m \in \mathcal{B}_x\}} \tilde{g}_{x,m,i} \tilde{X}_{x,m,k}, \quad (6)$$

where $\{\tilde{X}_{x,k}\}$ and $\{\tilde{X}_{x,m,k}\}$ are the symbols transmitted by the interfering sources and their relays at time k . If we condition on the point process and the fading coefficients, under the independent Gaussian signalling hypothesis, the received signals in Eqs. (3) and (4) and the interferences, given by (5) and (6), are Gaussian, and their distribution is the same for every time instant k . So we only need to find the conditional variance and correlation between the interference time signals

$\{Z_{d,k}, Z_{1,k}, \dots, Z_{n_r,k}\}$ to fully characterize their joint conditional distribution. Thus, the random interference powers at the nodes of the typical cluster are (for any time k):

$$I_d \triangleq \mathbb{E} \left[|Z_{d,k}|^2 \right] \tag{7}$$

$$= \sum_{x \in \Phi_s} |\tilde{g}_{x,d}|^2 \mathbb{E} \left[|\tilde{X}_{x,k}|^2 \right]$$

$$+ \sum_{m=1}^{n_r} \mathbb{1}_{\{m \in \mathcal{B}_x\}} |\tilde{g}_{x,m,d}|^2 \mathbb{E} \left[|\tilde{X}_{x,m,k}|^2 \right] \tag{8}$$

$$= \sum_{x \in \Phi_s} \left[P_s |\tilde{g}_{x,d}|^2 + P_r \sum_{m=1}^{n_r} \mathbb{1}_{\{m \in \mathcal{B}_x\}} |\tilde{g}_{x,m,d}|^2 \right], \tag{9}$$

$$I_{r_i} \triangleq \mathbb{E} \left[|Z_{i,k}|^2 \right] \tag{10}$$

$$= \sum_{x \in \Phi_s} \left[P_s |\tilde{g}_{x,i}|^2 + P_r \sum_{m=1}^{n_r} \mathbb{1}_{\{m \in \mathcal{B}_x\}} |\tilde{g}_{x,m,i}|^2 \right] \tag{11}$$

with $i = 1, \dots, n_r$. The correlation between the interference time signals is (for any time instant):

$$\beta_{r_i,r_j} \triangleq \mathbb{E} \left[Z_{i,k} Z_{j,k}^* \right] \tag{12}$$

$$= \sum_{x \in \Phi_s} \tilde{g}_{x,i} \tilde{g}_{x,j}^* \mathbb{E} \left[|X_{x,k}|^2 \right]$$

$$+ \sum_{m=1}^{n_r} \mathbb{1}_{\{m \in \mathcal{B}_x\}} \tilde{g}_{x,m,i} \tilde{g}_{x,m,j}^* \mathbb{E} \left[|X_{x,m,k}|^2 \right] \tag{13}$$

$$= \sum_{x \in \Phi_s} \left[P_s \tilde{g}_{x,i} \tilde{g}_{x,j}^* + P_r \sum_{m=1}^{n_r} \mathbb{1}_{\{m \in \mathcal{B}_x\}} \tilde{g}_{x,m,i} \tilde{g}_{x,m,j}^* \right]$$

$$i, j = 1, \dots, n_r, i \neq j, \tag{14}$$

$$\beta_{r_i,d} \triangleq \sum_{x \in \Phi_s} \tilde{g}_{x,i} \tilde{g}_{x,d}^* \mathbb{E} \left[|X_{x,k}|^2 \right]$$

$$+ \sum_{m=1}^{n_r} \mathbb{1}_{\{m \in \mathcal{B}_x\}} \tilde{g}_{x,m,i} \tilde{g}_{x,m,d}^* \mathbb{E} \left[|X_{x,m,k}|^2 \right] \tag{15}$$

$$= \sum_{x \in \Phi_s} \left[P_s \tilde{g}_{x,i} \tilde{g}_{x,d}^* + P_r \sum_{m=1}^{n_r} \mathbb{1}_{\{m \in \mathcal{B}_x\}} \tilde{g}_{x,m,i} \tilde{g}_{x,m,d}^* \right]$$

$$i = 1, \dots, n_r. \tag{16}$$

III. INFORMATION-THEORETIC RATES OF SELECTIVE COOPERATIVE RELAYING

In this section we describe the protocols under study and their corresponding achievable rates from an information-theoretic perspective. We define the following sets:

- $\mathcal{D} \subseteq \mathcal{A}_s$: relays from the typical cluster that can decode the transmission from the typical source treating the transmissions of all the other typical relays and from the other clusters as noise.
- $\mathcal{A}_s \setminus \mathcal{D}$ is the set of typical relays that cannot decode the transmission from the source.

We assume that the source is unaware of the channel coefficients towards the destination and the relays, meaning that it cannot adapt its transmission rate to guarantee a successful

transmission, and has to choose a rate R and attempt to communicate at that rate. For a chosen protocol, the transmission will fail and an outage will be declared whenever R is larger than the rate that is achievable for the particular realization of the network. We now define the protocols and their respective outage events:

- *Opportunistic Decode-and-Forward [1]*: in this protocol, the relays which can decode the transmission from the source, cooperate as a set of distributed antennas, forwarding the message to the destination. A relay in \mathcal{A}_s , that is, which has not been turned off due to a bad channel, will forward the message to the destination when it can decode the point-to-point transmission from the source while treating all other transmissions in the network as noise. That is, the i -th relay of the typical cluster will belong to \mathcal{D} whenever the attempted rate R satisfies:

$$R < I(X_s; Y_i), \tag{17}$$

where Y_i is the signal received by the relay. Each time a relay is turned on or off it affects the possibility of decoding of all the other relays in the network, so determining the best set of relays in the whole network that can decode the message of their source is too computationally demanding because we have to test all possible combinations of active and inactive relays. For this reason, to determine which relays of the typical cluster are able to decode we assume that all the relays in the network, including the ones in the typical cluster, are transmitting at the same time. Therefore in Y_i in (17) we assume that all the relays (in \mathcal{A}_s and $\mathcal{A}_x, \forall x \in \Phi_s$) and sources are transmitting, and their transmissions are treated as noise for decoding. After the set \mathcal{D} of relays which can decode the message is obtained, they will forward the message to the destination. An outage will be declared whenever the attempted rate R does not satisfy:

$$R < I(X_s, X_{\mathcal{D}}; Y_d), \tag{18}$$

where $X_{\mathcal{D}}$ is a set of random variables, one for each relay in \mathcal{D} , such that $\{X_s, X_{\mathcal{D}}\}$ has the same joint distribution (Gaussian) as the transmitted symbols of the source and the relays in \mathcal{D} . For the same reasons invoked for the outage event in (17), to evaluate the outage event (18) we consider that all the relays outside the typical cluster are transmitting ($\mathcal{A}_x = \mathcal{B}_x, \forall x \in \Phi_s$). This effectively gives an upper bound to the best OP attainable through ODF. However, since in MNNC the relays also perform ODF, its performance will be an approximate lower bound for ODF. Therefore we can estimate the actual performance of ODF using both simulations.

- *Noisy Network Coding [4], [20]*: in this protocol, it is not required that a relay decode the transmission from the source in order to cooperate. When the i -th relay receives the transmission from the source Y_i , it will generate a compressed representation \hat{Y}_i of this signal.

In our case this representation is obtained adding noise to the received signal, that is:

$$\hat{Y}_i = Y_i + Z_{c,i}, \quad (19)$$

where $Z_{c,i}$ is a complex circularly symmetric white Gaussian noise of zero-mean and variance n_c , independent of everything else. Since conditioned on the point process and the fading coefficients the random variables $\{Y_1, \dots, Y_{n_r}\}$ are jointly Gaussian, then $\{\hat{Y}_1, \dots, \hat{Y}_{n_r}\}$ are also Gaussian. Each relay will forward information about this compressed representation to the destination, which will use this information together with the transmission from the source to decode the message. The destination is not forced to use all the signals sent by the relays, it is able to choose any subset of these signals while treating the rest as noise. This increases the probability of successfully decoding the message. Since all the relays which have not been turned off due to bad channels will transmit, for this protocol we have $\mathcal{B}_s = \mathcal{A}_s$. An outage event is declared when the attempted rate R does not satisfy:

$$R < \max_{\mathcal{T} \in 2^{\mathcal{A}_s}} \min_{\mathcal{S} \in 2^{\mathcal{T}}} \{I(X_S, X_{\mathcal{S}}; \hat{Y}_{\mathcal{S}^c}, Y_d | X_{\mathcal{S}^c}) - I(\hat{Y}_{\mathcal{S}}; Y_{\mathcal{S}} | X_S, X_{\mathcal{T}}, \hat{Y}_{\mathcal{S}^c}, Y_d)\}, \quad (20)$$

where $2^{\mathcal{A}_s}$ denotes the power set of \mathcal{A}_s . The outer maximum over all the subsets of relays is equivalent to testing all possible combinations of relays for decoding. From (20) we see that each set \mathcal{S} is a subset of $\{1, \dots, n_r\}$; then, for each of the indexes in \mathcal{S} the random sets $\hat{Y}_{\mathcal{S}}$ and $\hat{Y}_{\mathcal{S}^c}$ are made of the variables \hat{Y}_i whose indexes belong to \mathcal{S} and \mathcal{S}^c , respectively. The sets $X_{\mathcal{S}}$ and $X_{\mathcal{T}}$ are constructed as $X_{\mathcal{D}}$ in the ODF protocol.

- *Mixed Noisy Network Coding [11]*: this protocol is a combination of the two above. The relays in \mathcal{D} , which can decode the transmission from the source, cooperate as a set of distributed antennas, while the other ones in $\mathcal{A}_s \setminus \mathcal{D}$ use NNC. In this case, determining \mathcal{D} by the procedure indicated in ODF does not provide an upper bound to the OP, because all the relays in the network are always transmitting. The destination will use all the signals from the relays which are using ODF and chooses which subset of user using NNC (the ones in $\mathcal{A}_s \setminus \mathcal{D}$) to use for decoding the message, treating the rest as noise. An outage event will take place whenever the attempted rate R does not satisfy:

$$R < \max_{\mathcal{T} \in 2^{\mathcal{A}_s \setminus \mathcal{D}}} \min_{\mathcal{S} \in 2^{\mathcal{T}}} \{I(X_S, X_{\mathcal{D}}, X_{\mathcal{S}}; \hat{Y}_{\mathcal{S}^c}, Y_d | X_{\mathcal{S}^c}) - I(\hat{Y}_{\mathcal{S}}; Y_{\mathcal{S}} | X_S, X_{\mathcal{D}}, X_{\mathcal{T}}, \hat{Y}_{\mathcal{S}^c}, Y_d)\}. \quad (21)$$

All the definitions of the sets are the same as in the previous protocols.

For each of these basic protocols we consider two versions according to how the relays are activated:

- *Standard versions of ODF, NNC and MNNC*: all the potential relays in a cluster may transmit, meaning that $\mathcal{A}_x = \{1, \dots, n_r\} = \mathcal{A}_s$ for all $x \in \Phi_s$.
- *Interference aware versions*: a threshold activation scheme for activating the relays in all clusters is employed, and this threshold can be used in the source-relay or relay-destination channels. These thresholds attempt to mitigate the interference by turning off relays which may not improve the performance of their cluster. If the source-relay threshold is active, then each relay will be active if the channel from its source exceeds a predefined threshold. On the other hand, if the relay-destination threshold is active, then each relay will be active provided that its channel towards its destination exceeds the threshold.

IV. PROCEDURE FOR THE NUMERICAL EVALUATION OF OUTAGE EVENTS

In what follows we describe the outage events corresponding to each of the protocols evaluated for the network model detailed in the previous sections. As mentioned before, the software implementation of the network is available together with the paper. The procedure to perform the Monte Carlo simulation of the outage probabilities of each protocol consists in drawing multiple realizations of the network parameters, i.e., node positions and fading coefficients, then finding the mutual informations which appear in the outage events and checking if the selected rate R is above or below the achievable rate given for each realization. In order to do this, we perform the following procedure:

- 1) We draw a realization of the network and the typical cluster according to the model in Section II.
- 2) Given the network realization and the chosen protocol, the mutual informations in the outage events for Section III can be computed and the outage condition can be determined. To do this, the mutual informations are computed in terms of differential entropies using the following standard identities [18]:

$$I(X; Y) = h(X) + h(Y) - h(X, Y), \quad (22)$$

$$I(X; Y|Z) = h(X, Z) + h(Y, Z) - h(X, Y, Z) - h(Z). \quad (23)$$

where X, Y, Z are continuous random variables and $h(\cdot)$ denotes differential entropy [18]. Since the nodes use Gaussian signaling, conditioned on the realization of the network, all the random variables involved in the mutual informations are CCSG random variables. It is well known that for a CCSG random vector \mathbf{x} with covariance matrix \mathbf{Q}_x , the entropy is [21]:

$$h(\mathbf{x}) = \log \det(\pi e \mathbf{Q}_x). \quad (24)$$

Therefore, to determine if an outage event occurs we must compute the mutual informations in the outage events, which are written in terms of joint entropies

and require all the covariance matrices of the random variables appearing in the joint entropies.

The large computational burden of this procedure can be reduced by noticing that it is not necessary to calculate a covariance matrix for each differential entropy. If we compute the covariance matrix of all the random variables involved in the typical cluster and the chosen protocol we can find the other ones which are required by deleting some of its rows and columns. This is the covariance matrix with largest dimensions for a given setup, and it is necessary to find it to calculate some of the mutual informations involved.

In what follows we focus on the typical cluster and describe how to find the largest covariance matrix of all the random variables involved for a chosen protocol. First, from the n_r potential relays of the source, we have a subset \mathcal{A}_s of the relays which can transmit (all of them if the protocol is not interference aware). Among these, only a subset $\mathcal{D} \subseteq \mathcal{A}_s$ will be able to decode the message of the source and act as secondary antennas. To determine those relays, we must evaluate for which relays condition (17) is met. This condition using (22) and (24), can be written for the i -th relay as:

$$R < \log_2 \left(1 + \frac{|g_{s,i}|^2 P_S}{I_{r_i} + \sum_{m=1, m \neq i}^{n_r} \mathbb{1}_{\{m \in \mathcal{A}_s\}} |g_{m,i}|^2 P_r} \right).$$

The relays which do not fulfill this condition (cannot decode) may perform NNC (if the NNC or MNNC protocols are employed) or remain silent. Let us define the set of relays which perform NNC as:

$$\mathcal{C}_s \triangleq \{t_1, \dots, t_{N_{\text{nnnc}}}\} \subseteq \mathcal{A}_s \setminus \mathcal{D}. \quad (25)$$

Notice that we cannot say that $\mathcal{C}_s = \mathcal{A}_s \setminus \mathcal{D}$ because for example in ODF, $\mathcal{C}_s = \emptyset$ while $\mathcal{A}_s \setminus \mathcal{D}$ will have the relays that cannot decode the message from the source. We define the set of indexes of the relays of the typical cluster which will be transmitting using either ODF or NNC as:

$$\mathcal{D} \cup \mathcal{C}_s = \{u_1, \dots, u_{N_a}\} \subseteq \{1, \dots, n_r\}. \quad (26)$$

In order to find the correlation matrix of the random variables appearing in the outage events, we define a random vector that has the signals which are transmitted and received by the nodes of the typical cluster, and, for NNC, the noise signals which are added at each relay according to (19):

$$\mathbf{u} \triangleq [X_s, X_{u_1}, \dots, X_{u_{N_a}}, Z_{u_1}, \dots, Z_{u_{N_a}}, Z_d, Z_{c,t_1}, \dots, Z_{c,t_{N_{\text{nnnc}}}}]^T. \quad (27)$$

We do not include the time instant, since, as mentioned before, the distribution of the vector does not depend on it. In the case of ODF, since $\mathcal{C}_s = \emptyset$ there is no need for compression noise random variables. Since all the transmitters in the network use independent Gaussian signaling the correlation matrix \mathbf{Q}_u of the vector \mathbf{u} is block diagonal (see (28) at the bottom of the next page), where:

- \mathbf{I}_n denotes an identity matrix of $n \times n$;
- $\mathbf{0}_{n,m}$ denotes a block of zeros of $n \times m$;

- \mathbf{Q}_Z is a square matrix of side $N_a + 1$ containing the correlation between the interference random variables:

$$\mathbf{Q}_Z \triangleq \begin{bmatrix} I_{u_1} & \beta_{u_1, u_2} & \dots & \beta_{u_1, u_{N_a}} & \beta_{u_1, d} \\ \beta_{u_2, u_1} & I_{u_2} & \dots & \beta_{u_2, u_{N_a}} & \beta_{u_2, d} \\ \vdots & & \ddots & & \vdots \\ \beta_{u_{N_a}, u_1} & \beta_{u_{N_a}, u_2} & \dots & I_{u_{N_a}} & \beta_{u_{N_a}, d} \\ \beta_{d, u_1} & \beta_{d, u_2} & \dots & \beta_{d, u_{N_a}} & I_d \end{bmatrix}. \quad (29)$$

Then we define a random vector containing all involved random variables that appear in the outage events:

$$\mathbf{v} \triangleq [X_s, X_{u_1}, \dots, X_{u_{N_a}}, Y_{u_1}, \dots, \quad (30)$$

$$Y_{u_{N_a}}, Y_d, \hat{Y}_{t_1}, \dots, \hat{Y}_{t_{N_{\text{nnnc}}}}]^T. \quad (31)$$

Now we find the matrix which allows us to write \mathbf{v} in terms of \mathbf{u} . We define a fading coefficient matrix between the source and the relays towards the destination and the relays:

$$\mathbf{H} \triangleq \begin{bmatrix} h_{s, u_1} & 0 & h_{u_2, u_1} & h_{u_3, u_1} & \dots & h_{u_{N_a}, u_1} \\ h_{s, u_2} & h_{u_1, u_2} & 0 & h_{u_3, u_2} & \dots & h_{u_{N_a}, u_2} \\ \vdots & & & \ddots & & \\ \vdots & & & & \ddots & \\ h_{s, u_{N_a}} & h_{u_1, u_{N_a}} & \dots & & & 0 \end{bmatrix}. \quad (32)$$

With this we may write:

$$\mathbf{v} = \tilde{\mathbf{H}} \mathbf{u} \quad (33)$$

with $\tilde{\mathbf{H}}$ a matrix:

$$\tilde{\mathbf{H}} \triangleq \begin{bmatrix} \mathbf{I}_{N_a+1} & \mathbf{0}_{N_a+1, N_a+N_{\text{nnnc}}+1} \\ \mathbf{H} & \begin{bmatrix} \mathbf{I}_{N_a} & \mathbf{0}_{N_a, N_{\text{nnnc}}+1} \\ \mathbf{0}_{1, N_a} & 1 & \mathbf{0}_{1, N_{\text{nnnc}}} \end{bmatrix} \\ \mathbf{H}_{(t_1, \dots, t_{N_{\text{nnnc}}})} & \begin{bmatrix} \mathbf{I}_{N_a(t_1, \dots, t_{N_{\text{nnnc}}})} & \mathbf{0}_{N_{\text{nnnc}}, 1} & \mathbf{I}_{N_{\text{nnnc}}} \end{bmatrix} \end{bmatrix} \quad (34)$$

where:

- $\mathbf{H}_{(t_1, \dots, t_{N_{\text{nnnc}}})}$ is a matrix containing the rows of \mathbf{H} indicated in the vector $[t_1, \dots, t_{N_{\text{nnnc}}}]$, that is:

$$[\mathbf{H}_{(t_1, \dots, t_{N_{\text{nnnc}}})}]_{i,j} = \mathbf{H}_{t_i, j}. \quad (35)$$

- $\mathbf{I}_{N_a(t_1, \dots, t_{N_{\text{nnnc}}})}$ is matrix of size $N_{\text{nnnc}} \times N_a$ obtained by taking the identity matrix of size N_a and keeping the rows indicated in the vector $[t_1, \dots, t_{N_{\text{nnnc}}}]$, that is:

$$[\mathbf{I}_{N_a(t_1, \dots, t_{N_{\text{nnnc}}})}]_{i,j} = \begin{cases} 1 & \text{if } j = t_i, \\ 0 & \text{otherwise.} \end{cases} \quad (36)$$

Then the covariance matrix of the vector \mathbf{v} can be found as:

$$\mathbf{Q}_v = \mathbb{E} \left[(\tilde{\mathbf{H}} \mathbf{u})(\tilde{\mathbf{H}} \mathbf{u})^* \right] = \tilde{\mathbf{H}} \mathbf{Q}_u \tilde{\mathbf{H}}^*. \quad (37)$$

As mentioned above, the matrix \mathbf{Q}_v is calculated only once for each realization of the network and is used to find all the entropies required to evaluate the outage events. To evaluate the joint entropy between any of the variables in \mathbf{v} one must take the matrix \mathbf{Q}_v and delete the rows and columns corresponding to the elements of \mathbf{v} whose entropy one does not need to calculate. Then, applying (24), the joint entropy, and

hence the mutual information between any of the variables in \mathbf{v} can be found.

Following this procedure the outage events for the selected protocol can be evaluated and it can be determined if an outage has taken place for the realization of the network.

V. NUMERICAL RESULTS AND DISCUSSIONS

A large number of parameters $((\lambda_r, n_r, P_r, n_c)$ among others) where swept for each of the protocols considered, and, for each setup, a large number ($>10^5$) of realizations was drawn. Given this and the complexity of the outage events involved, a standard desktop computer could not be employed to perform the simulations. For this reason, the Tupac supercomputer cluster hosted at CSC-CONICET (<http://tupac.conicet.gov.ar>) was employed. With this machine we are able to perform the large-scale simulations required for these complex networks and protocols. Given the large amount of data available, attention was paid in order to present the main observations with the minimal number of figures and plots per figure.

In what follows we describe the simulation setup. The density of sources is $\lambda_s = 10^{-4}$ nodes/unit area. The destination is located at the origin and its source is located at $p_s = (-10, 0)$. The relays are chosen as the nearest neighbors of a point c_{p_s} which lies on the line between the source and the destination, according to (1), where $\epsilon = 0$ implies the relays are centered around the source, and $\epsilon = 1$ means they are centered around the destination. The density of potential relays λ_r is chosen as a multiple of the density of sources. Each source can use at most n_r relays (the same for all clusters). The sources transmit with unit power $P_s = 1$ and the relays with a fixed power $0 \leq P_r \leq P_s$. For the case of protocols involving NNC, the compression noise variance n_c is optimized for each network setup (the optimized value of n_c was always in the range $[10^{-8}, 10^{-2}]$). The attempted rate in all cases is $R = 1$ bit/use, and the path loss exponent is $\alpha = 4$. Finally, each Monte Carlo simulation was obtained by averaging at least 10^5 realizations of the network. We compare the OP of the protocols with a point-to-point transmission without involving relays in the network, which is [22] $P_{\text{out,DT}}(R) = 1 - e^{-\lambda_s C T^{2/\alpha} \|p_s\|^2}$, where $T = 2^R - 1$, $C = \frac{2\pi}{\alpha} \Gamma\left(\frac{2}{\alpha}\right) \Gamma\left(1 - \frac{2}{\alpha}\right)$ and $\Gamma(z) = \int_0^\infty t^{z-1} e^{-t} dt$ is the standard Gamma function.

In what follows we consider three main questions regarding the three protocols: the behavior of the OP as a function of the relative density between sources and relays, the behavior of the OP as a function of the relay transmission power, and the dependence of the OP with the point around which the relays are chosen. After observing this behavior we analyze

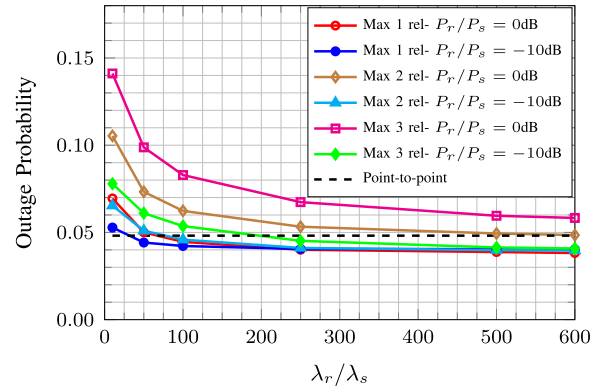


FIGURE 2. Outage probability for ODF as a function of the relative density between relays and sources, for different relay-source relative transmission powers. $\lambda_s = 10^{-4}$, $R = 1$ b/use, $\alpha = 4$. Relays are chosen centered around the source ($\epsilon = 0$).

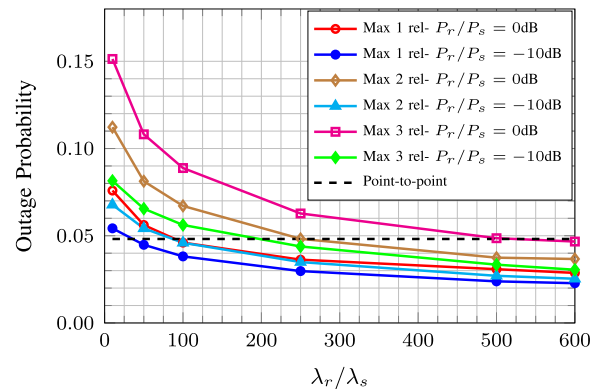


FIGURE 3. Outage probability for NNC as a function of the relative density between relays and sources, for different relay-source relative transmission powers. $\lambda_s = 10^{-4}$, $R = 1$ b/use, $\alpha = 4$. Relays are chosen centered around the destination ($\epsilon = 1$). OP is optimized w.r.t. the noise compression variance.

the OP that can be obtained by optimizing the relay transmission power and using interference aware relays, which turn themselves off if the channel amplitude towards their destination or source do not exceed a predefined threshold.

A. DEPENDENCE OF THE OP WITH THE RELAY DENSITY

In Figs. 2, 3 and 4 we plot the OP as a function of the relative relay density λ_r/λ_s , for different number of relays and fixed relative transmission powers P_r/P_s for ODF, NNC and MNNC, respectively. The relays are chosen centered around the source ($\epsilon = 0$) for ODF, around the destination ($\epsilon = 1$) for NNC, and in the middle ($\epsilon = 0.5$) for MNNC. The values of ϵ were chosen according to what is supposed to be the best option for each protocol considering their characteristics and results for other setups presented in other papers. ODF is expected to work better when the relays are,

$$\mathbf{Q}_{\mathbf{u}} \triangleq \mathbb{E}[\mathbf{u}\mathbf{u}^*] = \begin{bmatrix} P_s & \mathbf{0}_{1,2N_a+N_{\text{nnc}}+1} \\ \mathbf{0}_{2N_a+N_{\text{nnc}}+1,1} & \begin{bmatrix} P_r \mathbf{I}_{N_a} & \mathbf{0}_{N_a,N_a+N_{\text{nnc}}+1} \\ \mathbf{0}_{N_a+N_{\text{nnc}}+1,N_a} & \mathbf{QZ} \end{bmatrix} \end{bmatrix}, \quad (28)$$

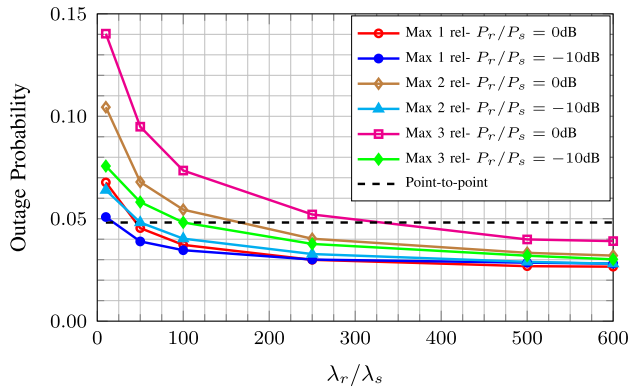


FIGURE 4. Outage probability for MNNC as a function of the relative transmission power between relays and sources, for different relative densities. $\lambda_s = 10^{-4}$, $R = 1\text{b/use}$, $\alpha = 4$. Relays are chosen centered around the midpoint between a source and its destination ($\epsilon = 0.5$). The OP is optimized w.r.t. the noise compression variance.

on average, closer to the source because this increases the chances for relays to decode the transmission, while NNC will perform better if the relays are closer to the destination which receives a compressed version of the observation of the relays. Finally, MNNC is a combination of both protocols and thus, it is expected to outperform the other when the relays are on average midway between the source and the destination.

For all the protocols it is interesting to observe that cooperation is more beneficial when the density of potential relays is much larger than the density of sources (100 times or more according to the scenario), that is, the OP is decreasing with λ_r/λ_s in all cases. Also, the performance is either improved or does not decrease if the relays use a smaller transmission power than the source ($P_r/P_s = -10\text{dB}$ in the plot). It is also worth to mention that using more relays does not improve the OP with respect to using a single one. This is because the additional relays in the other clusters increase the interference at the typical clusters, and because the second and third relays are further away than the first one, so that the benefits of cooperation are reduced by path loss. Furthermore, the destination chooses the best set of relays for decoding and treats the rest as noise. Although cycling through all the combinations of relays improves the chances of decoding, the interference generated by treating the rest of the relays and noise, added to the interference from other clusters, does not result in any benefits in the OP. In the case of ODF, this conclusion may be affected by the assumption that we made in order to keep the problem tractable that relays in the other clusters always remain on.

Although the OP is decreasing in λ_r/λ_s , the gains, however, are not the same in this regime. For the case of ODF the gains are not substantial in this regime, that is, when the relays are chosen to be located around the source. However, for NNC and MNNC a reduction of the OP close to 50% is feasible when the relays are chosen closer to the destination.

Perhaps, the most interesting conclusion from this setup is that cooperation appears to be most useful in networks in which relays from a dense network of low-power nodes (compared to the sources), such as sensor or cellphone networks.

B. DEPENDENCE OF THE OP WITH THE RELAY TRANSMISSION POWER

In Figs. 5, 6, and 7 we plot the OP for ODF, NNC and MNNC as a function of the relative transmission power between relays and sources, for different relative relay-source densities. As in the previous section, the relays are chosen centered around the source ($\epsilon = 0$) for ODF, around the destination ($\epsilon = 1$) for NNC, and in the middle ($\epsilon = 0.5$) for MNNC. For the case of NNC (Fig. 6) we see that the OP is increasing in the relay transmission power for moderate or large relay densities. On the other hand, for ODF or MNNC the OP is also increasing in general, except when a single relay is used and the density of relays is large. In that case the OP is decreasing in the relay transmission power, but the gains are marginal to warrant the increase in relay transmission power.

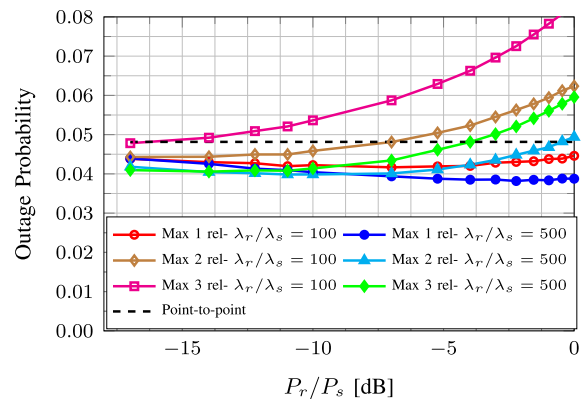


FIGURE 5. Outage probability for ODF as a function of the relative transmission power between relays and sources, for different relative densities. $P_r/P_s = -10\text{dB}$, $\lambda_s = 10^{-4}$, $R = 1\text{b/use}$, $\alpha = 4$. Relays are chosen centered around the sources ($\epsilon = 0$).

As we have seen before, without any optimization in the relay transmission power or position we see that large gains in terms of OP can be achieved through NNC or MNNC while in the case of ODF this setup is not the most convenient.

C. DEPENDENCE OF THE OP WITH THE RELAY POSITION

In Figs. 8, 9, and 10 we plot the OP as a function ϵ , which indicates the center point around which the relays are chosen on the line between the source and the destination, for different relative relay-source densities. In all cases we set the relative transmission power $P_r/P_s = -10\text{dB}$, which has shown to be reasonable in previous plots.

For the case of ODF (Fig. 8) we see that the biggest gains can be obtained when the center point is chosen near the midpoint between the source and destination, but closer to the destination. This is because at this position, on average, the quality of the source-relay and relay-destination channels are balanced, together with the different source and relay transmission powers, and the OP is minimized. In that case, if a single low-power relay is used, the OP is reduced by more than 40% when $\lambda_r/\lambda_s \geq 500$, compared to a transmission without cooperation. In addition, a similar reduction can be achieved even if the relative density is smaller ($\lambda_r/\lambda_s \geq 250$).

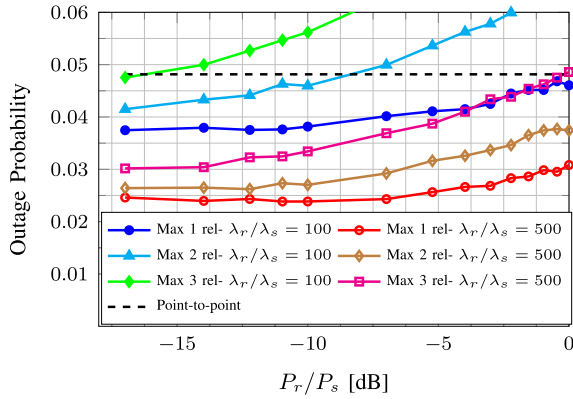


FIGURE 6. Outage probability for NNC as a function of the relative transmission power between relays and sources, for different relative densities. $P_r/P_s = -10\text{dB}$, $\lambda_s = 10^{-4}$, $R = 1\text{b/use}$, $\alpha = 4$, $d = 10$. Relays are chosen centered around the destination ($\epsilon = 1$). OP is optimized w.r.t. the noise compression variance.

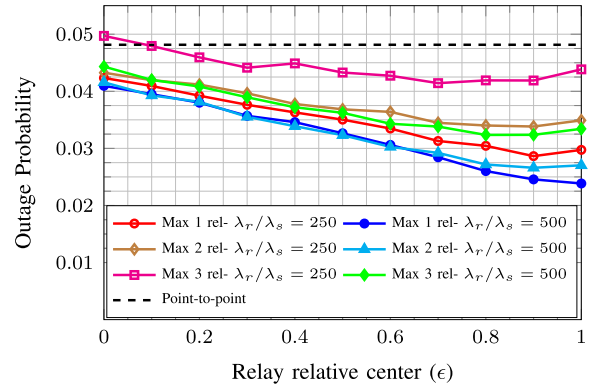


FIGURE 9. Outage probability for NNC as a function of the relay relative center, for different relative densities. $P_r/P_s = -10\text{dB}$, $\lambda_s = 10^{-4}$, $R = 1\text{b/use}$, $\alpha = 4$, $d = 10$. OP is optimized w.r.t. the noise compression variance.

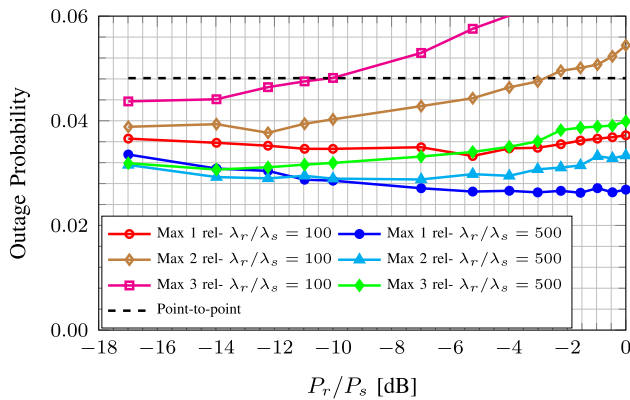


FIGURE 7. Outage probability for MNNC as a function of the relative transmission power between relays and sources, for different relative densities. $P_r/P_s = -10\text{dB}$, $\lambda_s = 10^{-4}$, $R = 1\text{b/use}$, $\alpha = 4$. Relays are chosen centered around the midpoint between a source and its destination ($\epsilon = 0.5$). The OP is optimized w.r.t. the noise compression variance.

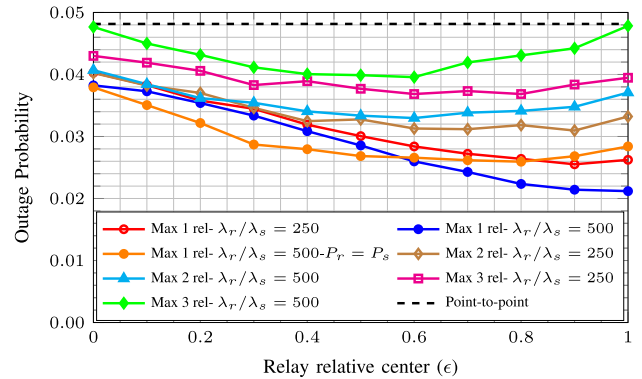


FIGURE 10. Outage probability for MNNC as a function of the relay relative center, for different relative densities. $P_r/P_s = -10\text{dB}$, $\lambda_s = 10^{-4}$, $R = 1\text{b/use}$, $\alpha = 4$. OP is optimized w.r.t. the noise compression variance.

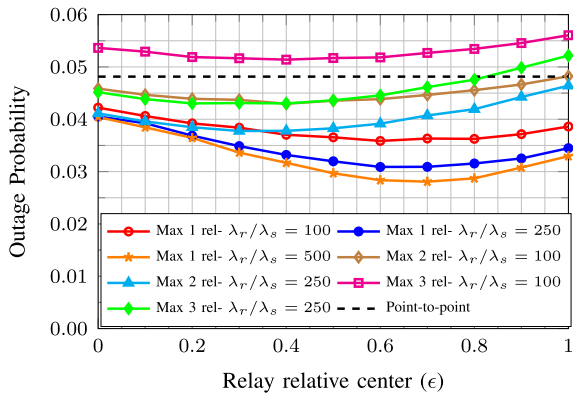


FIGURE 8. Outage probability for ODF as a function of the relay relative center, for different relative densities. $P_r/P_s = -10\text{dB}$, $\lambda_s = 10^{-4}$, $R = 1\text{b/use}$, $\alpha = 4$.

For NNC (Fig. 9) the OP appears to be decreasing provided that $0 \leq \epsilon \leq 1$, implying that the best would be to choose the relays centered around the destination. In this case the performance is very similar if one or two low-power relays are used and the dispersion in performance is smaller when compared to ODF. The potential gains of NNC are larger than

that of ODF, but similar. For example, when $\lambda_r/\lambda_s = 500$ a reduction of more than 50% in the OP can be achieved by using a single relay.

In the case of MNNC (Fig. 10), in addition to the plots with one relay and $P_r/P_s = -10\text{dB}$, we also plot a curve with $P_s = P_r$, because in Fig. 7 we saw that this may be better for this protocol when at most one relay is used. The behavior of the OP is similar to that of NNC in the sense that it is more convenient to choose the relays closer to the destination. When using a single relay, the potential gains of MNNC are better but similar to the gains obtained with NNC; for $\lambda_r/\lambda_s \geq 500$ a reduction greater than 55% in the OP can be achieved when the relay uses a low power. For the case of single relay, when $P_r = P_s$ a larger reduction of the OP is obtained near the source, with a loss of performance near the destination as compared to using a low-power relay. This may be because MNNC is a combination of ODF and NNC; near the destination NNC will be dominant, which does not benefit from setting $P_r = P_s$, while near the midpoint ODF will be dominant and will benefit from $P_r = P_s$. This results in that the OP remains almost constant when $0.3 \leq \epsilon \leq 1$ with a reduction of 45% in the OP compared to a point-to-point transmission.

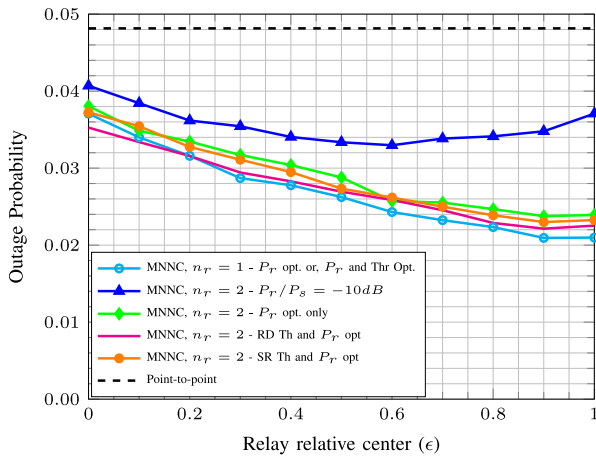


FIGURE 11. Outage probability for MNNC with optimized relay power and thresholds. $\lambda_r/\lambda_s = 500$, $\lambda_s = 10^{-4}$, $R = 1\text{b/use}$, $\alpha = 4$.

D. EFFECT OF POWER OPTIMIZATION AND INTERFERENCE-AWARE RELAYS

In this section we study the performance of the protocols and the optimal number of relays when the transmission power is optimized and when interference-aware relays are employed. When interference aware relays are used, we consider that only one of the thresholds (source-relay or source-destination) is used to activate the relays. In the previous section we saw that when fixed power is used, it is best to use a single relay. Since there are several parameters to consider for each protocol (number of relays, relay power, thresholds) and for space reasons, we focus first on MNNC which has the best performance. Afterwards we compare the performance of ODF and NNC to MNNC to see how close they are to MNNC when their respective parameters are optimized.

In Fig. 11 we plot the OP of MNNC under different parameter optimizations when at most one or two relays are used. It can be seen that in the case of at most one relay ($n_r = 1$), it is the same to optimize the relay power P_r or to optimize at the same time the relay power and either of the thresholds, which implies that the thresholds are not necessary for this protocol in terms of OP, and that the relay could remain on all the time. In the case of at most two relays ($n_r = 2$), optimizing only the relay power brings the OP very close to that of using a single relay with optimized power. If, in addition, the thresholds are optimized as well, the performance becomes the same as using one relay. Nevertheless, the advantage of using two or more relays is that the transmission power of each relay can be reduced with respect to employing only one relay, as it is shown in Fig. 12.

Since the best performance with MNNC is obtained by simply optimizing the relay power, in the following plots of ODF and NNC we use MNNC with $n_r = 1$ and optimized P_r as a benchmark comparison. In Fig. 13 we plot the OP of ODF for different number of relays when the power and the thresholds are optimized. For the case of one relay, we see that by optimizing the transmission power only or both the transmission and the thresholds, a performance similar to

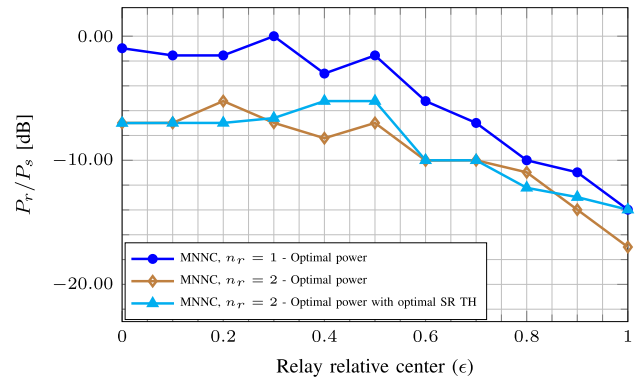


FIGURE 12. Optimal power for MNNC corresponding to the OP of Fig. 11. Using more than one relay allows the reduction of the relay power.

$\lambda_r/\lambda_s = 500$, $\lambda_s = 10^{-4}$, $R = 1\text{b/use}$, $\alpha = 4$.

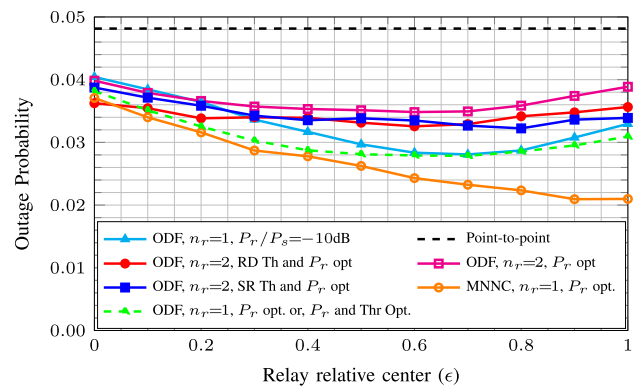


FIGURE 13. Outage probability for ODF with optimized relay power and thresholds, compared to the best performance obtained with MNNC.

$\lambda_r/\lambda_s = 500$, $\lambda_s = 10^{-4}$, $R = 1\text{b/use}$, $\alpha = 4$.

that of MNNC can be achieved when the relay is chosen close to the source ($\epsilon < 0.4$), while the performance of MNNC is much better than that of ODF near the destination. On the other hand, and in contrast with MNNC, when at most two relays are used, the performance does not improve substantially by optimizing the relay transmission power or the thresholds. This is probably because the second relay is on average further and has a smaller probability of decoding the message of the source, and also because we assume that the relays outside the typical cluster are always on in the case of ODF even if they cannot decode.

Finally, in Fig. 14 we consider the optimization of the power and thresholds for NNC. We plot this together with the plots of MNNC and ODF for a single relay with optimized power. We optimized the power and the thresholds when using at most one or two relays, and found that the best option was to use an optimized RD threshold and power, though the gains were marginal compared to using a small fixed power. In the case of using up to two relays the performance improved notoriously compared to using a constant power, and the best option was to optimize both the transmission power and using an RD threshold. Similar to the case of MNNC, the performance obtained using two relays with optimized parameters was similar to those corresponding to

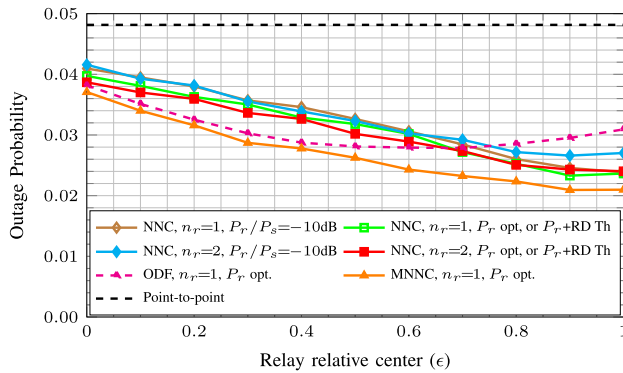


FIGURE 14. Outage probability for NNC with optimized relay power and thresholds, compared to the best performance obtained with MNNC and ODF. $\lambda_r/\lambda_s = 500$, $\lambda_s = 10^{-4}$, $R = 1$ b/use, $\alpha = 4$. OP is optimized w.r.t. the noise compression variance.

a single relay, but each relay can use a smaller transmission power.

As a conclusion from this section we see that MNNC performs better than the other two protocols, which is reasonable since it can be interpreted as a combination of both, while ODF comes close when the relay is chosen near the source, and NNC comes closer provided that the relay is near the destination. In the three protocols we observed that using a single relay and optimizing the relay power is enough to attain the best performance in terms of the OP for each case. Furthermore, using more relays and optimizing the relay transmission power in the case of MNNC, or NNC, does not improve the OP compared to employing only one relay, but reduces the power consumption of each relay. In the case of ODF, using more relays does not reduce the OP even when the transmission power or thresholds are activated. This conclusion could be affected by the simplifying assumption that the relays in the other clusters are always active for ODF, even if they cannot decode.

VI. SUMMARY AND CONCLUDING REMARKS

In this paper we studied the performance of three advanced full-duplex relaying protocols, namely ODF, NNC and MNNC in the context of a large wireless network limited by interference. These protocols can be considered a benchmark for performance in the context of wireless cooperation with relays. The main difference in setup with other works is that they considered either half-duplex decode-and-forward or full-duplex protocols with a single relay active in the whole network and a limited set of parameters, while in this work we considered full-duplex protocols, multiple relays and a larger number of setup parameters. The following main observations can be made:

- In general, cooperation was most useful when the density of relays was much larger than the density of sources, i.e., $\lambda_r \gg \lambda_s$ and a single relay is used. In regard to this it is interesting to observe the following: as the density of potential relays increases it is more likely that several potential relays can be found near the optimal point which maximizes the performance of

each protocol. In this context, if the other sources in the network are not allowed to use relays, it is clear that OP will decrease when we increase the relay density and the number of active relays (this was observed in other works involving half-duplex DF in which there are no relays in the interference). If all the sources in the network can use relays, then for a fixed number of relays we still observe that increasing the density reduces the OP, because the relays become concentrated around the optimal point. However, no matter how large the potential relay density λ_r is, increasing the number of relays does not lead to a smaller OP even if we optimize their transmission power. In fact, the best OP in all the three protocols was obtained by using a single relay with an optimal power selection. This conclusion is similar to the one presented in [23] albeit under different assumptions (half-duplex DF under background noise). Also, a large reduction of the OP (around 50%) was obtained by simply using a single relay with a fixed low transmission power (compared to the source). This means that the added interference generated by using more relays is more detrimental than the potential gains of using them. This is worse when the density λ_r is not very large because each new relay is further away from the optimal point and thus helps the destination much less.

This observation supports the use of relays specially in some scenarios. An example are networks in which there are high power base stations which establish links to a large network of low power nodes which may cooperate with each other, such as cellular networks or sensor networks. Another possible scenario is a network of similar nodes in which some nodes may help others by sometimes acting as relays, but when they do so they save energy by transmitting at a fraction of their nominal power.

- In the case of NNC and MNNC it was possible to match the performance of a single relay when using two relays if the optimal power was chosen for two relays. The advantage was that the optimal power of each relay was smaller than the optimal power used by a single relay (see Fig. 12). However, for example, in MNNC using a single relay with a fixed power smaller than the optimal ($P_r/P_s = -10$ dB, Fig. 10) gave almost the same performance as using two relays with a smaller and fixed transmission power. Therefore using two relays does not seem to be useful. In the case of ODF it was not possible to use two relays and match the performance of using one relay; however, this may be affected by the simplifications used to simulate the OP of the protocol.
- MNNC was shown to outperform NNC and ODF in all cases. NNC without optimizations was shown to perform better than ODF without optimizations near the destination, while ODF was better near the midpoint between the source and destination. Similar conclusions regarding the optimal relay position,

with different assumptions, were found for example in [13] for half-duplex DF and in [16] for full-duplex DF (please see the Related Works section). The best performance of each protocol was obtained by using a single relay and optimizing its transmission power. Using a threshold based activation scheme was not necessary. When the relay transmission power was optimized or the threshold-based activation schemes were considered, it was observed that NNC can only beat ODF in a close proximity of the destination, while in the other positions, ODF was preferred. Of the three protocols, ODF is, so far, more relevant from a practical viewpoint, while the others remain interesting from a theoretical perspective. Bearing this in mind, it is interesting to observe that ODF can perform almost as well as MNNC when the relay is between the source and the midpoint between the source and the destination ($0 \leq \epsilon \leq 0.5$, see Fig. 14).

Although a closed-form analysis of the protocols considered in this work may not be possible in this context, we believe that the simulation framework introduced allowed us to gain some insight into their performance. In addition, the framework is flexible enough to allow for more complex simulation scenarios to be considered. This may lead to exploring setups with different channel models or node distributions in which different conclusions may be drawn, for example regarding the optimal number of relays for each protocol.

ACKNOWLEDGMENT

The authors would like to thank Dr. B. Blaszczyszyn for valuable and inspiring discussions at the early stage of this work. All numerical simulations were ran in the TUPAC supercomputer at CSC-CONICET (<http://tupac.conicet.gov.ar>).

REFERENCES

- [1] J. N. Laneman, D. N. C. Tse, and G. W. Wornell, "Cooperative diversity in wireless networks: Efficient protocols and outage behavior," *IEEE Trans. Inf. Theory*, vol. 50, no. 12, pp. 3062–3080, Dec. 2004.
- [2] G. Kramer, M. Gastpar, and P. Gupta, "Cooperative strategies and capacity theorems for relay networks," *IEEE Trans. Inf. Theory*, vol. 51, no. 9, pp. 3037–3063, Sep. 2005.
- [3] T. M. Cover and A. A. El Gamal, "Capacity theorems for the relay channel," *IEEE Trans. Inf. Theory*, vol. IT-25, no. 5, pp. 572–584, Sep. 1979.
- [4] S. H. Lim, Y.-H. Kim, A. El Gamal, and S.-Y. Chung, "Noisy network coding," *IEEE Trans. Inf. Theory*, vol. 57, no. 5, pp. 3132–3152, May 2011.
- [5] B. Nazer and M. Gastpar, "Compute-and-forward: Harnessing interference through structured codes," *IEEE Trans. Inf. Theory*, vol. 57, no. 10, pp. 6463–6486, Oct. 2011.
- [6] D. Stoyan, W. S. Kendall, and J. Mecke, *Stochastic Geometry and its Applications*, 2nd ed. Hoboken, NJ, USA: Wiley, 2009.
- [7] P. M. Haenggi, *Stochastic Geometry for Wireless Networks*. Cambridge, U.K.: Cambridge Univ. Press, 2012.
- [8] M. Haenggi, J. G. Andrews, F. Baccelli, O. Dousse, and M. Franceschetti, "Stochastic geometry and random graphs for the analysis and design of wireless networks," *IEEE J. Sel. Areas Commun.*, vol. 27, no. 7, pp. 1029–1046, Sep. 2009.
- [9] H. S. Dhillon, R. K. Ganti, F. Baccelli, and J. G. Andrews, "Modeling and analysis of K-tier downlink heterogeneous cellular networks," *IEEE J. Sel. Areas Commun.*, vol. 30, no. 3, pp. 550–560, Apr. 2012.
- [10] H. Elsayy, E. Hossain, and M. Haenggi, "Stochastic geometry for modeling, analysis, and design of multi-tier and cognitive cellular wireless networks: A survey," *IEEE Commun. Surveys Tuts.*, vol. 15, no. 3, pp. 996–1019, Jun. 2013.
- [11] A. Behboodi and P. Piantanida, "Mixed noisy network coding and cooperative unicasting in wireless networks," *IEEE Trans. Inf. Theory*, vol. 61, no. 1, pp. 189–222, Jan. 2015.
- [12] A. Bletsas, A. G. Dimitriou, and J. N. Sahalos, "Interference-limited opportunistic relaying with reactive sensing," *IEEE Trans. Wireless Commun.*, vol. 9, no. 1, pp. 14–20, Jan. 2010.
- [13] A. Crismani, S. Toumpis, U. Schilcher, G. Brandner, and C. Bettstetter, "Cooperative relaying under spatially and temporally correlated interference," *IEEE Trans. Veh. Technol.*, vol. 64, no. 10, pp. 4655–4669, Oct. 2015.
- [14] R. Tanbourgi, H. Jäkel, and F. K. Jondral, "Cooperative relaying in a poisson field of interferers: A diversity order analysis," in *Proc. IEEE Int. Symp. Inf. Theory*, Jul. 2013, pp. 3100–3104.
- [15] Y. Zhou and W. Zhuang, "Performance analysis of cooperative communication in decentralized wireless networks with unsaturated traffic," *IEEE Trans. Wireless Commun.*, vol. 15, no. 5, pp. 3518–3530, May 2016.
- [16] A. Altieri, L. R. Vega, P. Piantanida, and C. G. Galarza, "On the outage probability of the full-duplex interference-limited relay channel," *IEEE J. Sel. Areas Commun.*, vol. 32, no. 9, pp. 1765–1777, Sep. 2014.
- [17] A. Altieri, L. Rey Vega, P. Piantanida, and C. G. Galarza, "Analysis of a cooperative strategy for a large decentralized wireless network," *IEEE/ACM Trans. Netw.*, vol. 22, no. 4, pp. 1039–1051, Aug. 2014.
- [18] T. M. Cover and J. A. Thomas, *Elements of Information Theory*, 2nd ed. New York, NY, USA: Wiley, 2006.
- [19] S. I. Resnick, *Adventures in Stochastic Processes*. Basel, Switzerland: Birkhäuser Basel, 2002.
- [20] J. Hou and G. Kramer, "Short message noisy network coding with a decode-forward option," *IEEE Trans. Inf. Theory*, vol. 62, no. 1, pp. 89–107, Jan. 2016.
- [21] I. E. Telatar, "Capacity of multi-antenna Gaussian channels," *Eur. Trans. Telecommun.*, vol. 10, no. 6, pp. 585–595, 1999.
- [22] B. F. Baccelli, B. Blaszczyszyn, and P. Muhlethaler, "An Aloha protocol for multihop mobile wireless networks," *IEEE Trans. Inf. Theory*, vol. 52, no. 2, pp. 421–436, Feb. 2006.
- [23] A. Bletsas, H. Shin, and M. Z. Win, "Outage-optimal cooperative communications with regenerative relays," in *Proc. 40th Annu. Conf. Inf. Sci. Syst.*, Mar. 2006, pp. 632–637.



ANDRÉS ALTIERI (S'12–M'15) received the B.Sc. and M.Sc. degrees (Hons.) in electrical engineering from the University of Buenos Aires, Argentina, in 2009, and the joint Ph.D. degree from the University of Buenos Aires and the École Supérieure d'Électricité, France, in 2014. Since 2016, he has been an Assistant Professor with the Universidad de Buenos Aires and an Assistant Researcher with the Centro de Simulación Computacional CONICET. His research interests

include cooperative wireless networks, signal processing, and microwave system design, especially with applications to ultrawideband systems.



PABLO PIANTANIDA (S'04–M'08–SM'16) received the B.Sc. in electrical engineering and the M.Sc. degree (Hons.) from the University of Buenos Aires, Argentina, in 2003, and the Ph.D. degree from Université Paris-Sud, Orsay, France, in 2007. Since 2007, he has been with the Laboratoire des Signaux et Systèmes, Centrale-Supélec, together with CNRS (UMR 8506) and Université Paris-Sud as an Associate Professor of network information theory. He is currently with

the Montreal Institute for Learning Algorithms, Université de Montréal. His research interests include information theory and its interactions with other fields, including multi-terminal information theory, Shannon theory, machine learning and representation learning, statistical inference, cooperative communications, and communication mechanisms for security and privacy. He is a General Co-Chair of the 2019 IEEE International Symposium on Information Theory.

...