IEEE Access
Multidisciplinary ⋮ Rapid Review ⋮ Open Access Journal

Received January 30, 2019, accepted February 12, 2019, date of publication February 19, 2019, date of current version March 8, 2019.

Digital Object Identifier 10.1109/ACCESS.2019.2900254

Two-Stage Robust Security-Constrained Unit Commitment Model Considering Time Autocorrelation of Wind/Load Prediction Error and Outage Contingency Probability of Units

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This work was supported in part by the National Natural Science Foundation of China under Grant 51777067, and in part by the Research on Green Dispatching Technology of Shanxi Power Grid Project under Grant 52053017000S.

ABSTRACT The conservativeness of unit commitment models-based robust optimization (RO) depends on the modeling of uncertainty sets. This paper proposes a new two-stage robust security-constrained unit commitment (SCUC) model, which aims at minimizing the operating cost in the base scenario while guaranteeing that the robust solution can be adaptively and safely adjusted according to the uncertainties of wind power, load, and N-k fault. This new model has the following characteristics: 1) the temporal correlation of continuous uncertainties (i.e., wind power output and load) are studied and a time-correlation constraint is established to reduce the conservativeness of uncertainty sets in the proposed robust SCUC model; 2) the discrete characteristics of the uncertain set is used to describe the uncertainty of discrete N-k fault; 3) the outage probability of units with different capacity is also considered with a proposed probability criterion; and 4) the constraint approximation is simplified to a linear constraint that can be applied to RO. The proposed model is solved by the Benders decomposition and dual theory. The simulation results on modified IEEE-RTS-96 system show that the proposed method can effectively reduce the conservativeness of uncertain sets and ensure the economic and security of the optimization results.

INDEX TERMS Economic dispatch, robust optimization, uncertainty set, wind/load uncertainty, unit outage contingency.

I. INTRODUCTION

There are more and more uncertainties in today's power system, such as the output of renewable energy generation, the outage probability of generators and transmission lines, which may lead to power shortages, frequency fluctuations and large-scale outages. These uncertainties bring new challenges to power system analysis and operation control. The security-constrained unit commitment (SCUC) model with uncertainties is one of the difficult problems. To cope with this challenge, power systems are in urgent need of new dispatch methods. Robust optimization (RO) methods and random programming methods have been applied to the above problem and achieve good results [1]–[6]. Robust SCUC model uses closed convex sets to describe the uncertainty and solves the optimal problem under the ''worst scenario''. Compared with the random programming methods, Robust SCUC model is not necessary to obtain the distribution of uncertain quantities, and it is not necessary to generate a typical scenario, thus it is easier to be implemented [7], [8]. However, RO methods often pursue the optimal value of objective function in the worst case, whereas the probability of the worst case happening in practice is not large, which leads to the results of RO methods are often too conservative [9]–[11]. In order to overcome this conservatism, accurate modeling of uncertain sets becomes an inevitable requirement.

There are two kinds of uncertainty sets in SCUC problem considering uncertainty, i.e. continuous uncertain sets and discrete uncertain sets. The former mainly refers to the

The associate editor coordinating the review of this manuscript and approving it for publication was Bin Zhou.

uncertainty of wind power output and load; the latter mainly refers to the uncertainty of line interruption and unit failure, etc. For modeling of continuous uncertainty sets, [3] uses the box-type set to describe the uncertainty, which further reduces the conservativeness of the RO method. Due to the linear nature of the box-type set, this uncertain set is widely used in the power system. Reference [11] considers the temporal smoothing effect between different periods of a single wind farm. Reference [12] considers the spatial clustering effect between wind farms and further compresses the uncertainty set of wind power.

For modeling of discrete uncertainty sets, [13], [14] require that the total number of units operating normally is not less than the given limit. As we know, reliability is the basic requirement for the operation of power systems. Occasional unit and transmission line faults may cause a large shortage of power supply, which may even cause large-scale power outages. Therefore, the $N-1$ or $N-2$ security criterion are used to verify the scheduling plan during system operation. In recent years, with the frequent occurrence of power outages caused by multiple independent events in power systems, many scholars have extended the security criterion to more general N-k security criterion that consider k component failures. For discrete N-k security criterion, it is difficult for large systems to list outages one by one. In the robust SCUC model, the worst outage contingency scenario can be found by mathematical optimization method. Reference [6] establishes a static robust SCUC model considering the N-k fault constraint of the units and the transmission lines, ensuring that the power imbalance in the system is not greater than a threshold under any outage contingency. Reference [15] establishes a joint optimization model for units output and reserve capacity that takes into account N-k fault constraint. Reference [16] further expands the unit commitment model considering N-k fault constraint. Due to the low probability of serious outage contingency in the power systems, [16] proposes an improved safety criterion that allows for reduction of conservation by reducing the load when $k > 1$.

The above research has done a very meaningful work. But for modeling of continuous uncertainty sets, most of them assume that the prediction errors among the various scheduling periods are independent of each other. However, this assumption of independence may not correspond to the actual situation in practice and needs to be evaluated through actual data. Taking different scheduling periods as an example, Ref [17]–[19] point out that the wind power output of two adjacent scheduling periods has a correlation. Reference [20] analyzes the wind power data of Irish wind farms and points out that the wind power prediction error sequence has autocorrelation. Reference [21] verifies that the model considering correlation can improve the economics of the robust optimal power flow model. For modeling of discrete uncertainty sets, most of the existing reference does not discuss the outage contingency probability of components, when the outage contingency probability of components is not considered, the resulting uncertainty set is often too conservative. However, the probabilities of failure of different units or transmission lines in the power systems are different, for example, units with larger capacity have higher reliability.

Through the above analysis, the conservativeness of existing robust SCUC models may be attributed to the fact that the time autocorrelation of wind power/load forecasting error (continuous uncertainty sets) and the unit failure probability (discrete uncertainty sets) are not taken into account in the modeling process. If the above two kinds of uncertainty sets are modeled precisely, the conservativeness of robust SCUC model may be improved.

Therefore, this paper proposes a two-stage robust SCUC model considering time autocorrelation of wind/load prediction error and outage contingency probability of units. The main contribution of the paper are twofold: (1) The correlation between wind power prediction errors in adjacent time periods is considered by using a new time autocorrelation constraint of uncertainty prediction error; (2) a new α_{cut} criterion is introduced to describe the uncertainty of occasional N-k perturbations.

The rest of the paper is organized as follows. The uncertain set modeling of continuous variable considering the temporal correlation of prediction errors as well as the discrete variables considering the outage contingency probability are introduced in Section II. The formulation of a new two-stage robust SCUC model is presented in Sections III. Section IV gives the solution methodologies of the proposed model. Section IV presents numerical case study results, and the conclusions are drawn in Section V.

II. UNCERTAIN SET MODELING FOR SCUC MODEL

A. MODELING OF CONTINUOUS UNCERTAIN SETS

In the unit commitment (UC) problem considering uncertainty, wind power and load are both uncertainties, and the same modeling method is generally used for uncertain sets of wind power output and load. In this section, taking the wind power output as an example, the temporal correlation of prediction error will be taken into account when modeling uncertain sets of wind power output.

1) THE TEMPORAL CORRELATION OF PREDICTION ERROR

The prediction error of wind power mainly stems from the inability to accurately predict meteorological conditions. Considering the continuity of meteorological changes, the prediction error of wind power output prediction in adjacent time segments may have some correlation, the temporal correlation of the prediction error will be verified below before modeling the uncertainty set of wind power output.

The relative prediction error of the t-th period can be expressed as

$$e_t = \frac{w_t^e - w_t}{w_t^w} \tag{1}$$

where, w_t^e and w_t are the predicted output and actual output of wind power, respectively; w_t^u and w_t^l are the upper and lower bounds of the prediction error at period t, $w_t^w = (w_t^u + w_t^l)/2$, $t = 1, \cdots, T$, $T = 24$.

The relative error sequence is

$$\begin{aligned} \boldsymbol{s}_1 = [&e_{1,1}, \cdots e_{n,1}, \cdots e_{N,1}, \cdots, e_{1,T-1}, \cdots, \\ &e_{n,T-1}, \cdots, e_{N,T-1}] \end{aligned} \tag{2}$$

$$\begin{aligned} \boldsymbol{s}_2 = [&e_{1,2}, \cdots e_{n,2}, \cdots e_{N,2}, \cdots, e_{1,T}, \cdots, \\ &e_{n,T}, \cdots, e_{N,T}] \end{aligned} \tag{3}$$

where, n indicates the number of the predicting data sample, $n = 1, \cdots, N$, N represents the total number of samples; A data sample represents one day with 24 periods; e_t in the Eq. (1) is the value of $e_{n,t}$ in the Eq. (2) and (3) in the period t of the specific sample n. $\boldsymbol{s}_2$ is a sequence in which $\boldsymbol{s}_1$ is shifted backward by a period of time. The time interval here is consistent with the calculation step size of the unit commitment.

The sample of prediction error sequences is the wind power access data from October 2017 to September 2018, provided by Belgian transmission system operator Elia [22]. Figure 1 shows the scatter plots drawn by the sequences $\boldsymbol{s}_1$ and $\boldsymbol{s}_2$. In figure 1, the abscissa and the ordinate correspond to the $e_{n,t}$ and $e_{n,(t+1)}$ of the same position in the $\boldsymbol{s}_1$ and the $\boldsymbol{s}_2$, respectively.

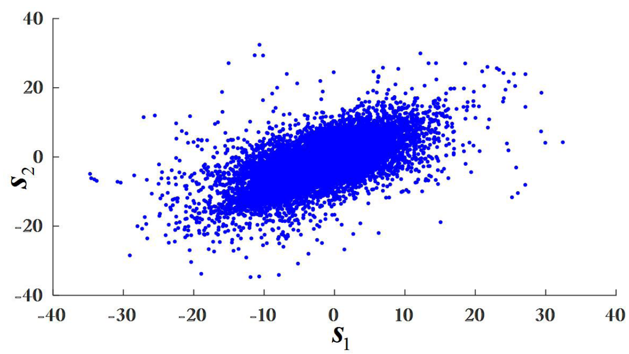

FIGURE 1. Prediction error sequence scatter plot.

It can be seen that the scatter distribution in figure 1 is very concentrated and appears elliptical, thereby demonstrating that $\boldsymbol{s}_1$ and $\boldsymbol{s}_2$ are correlated. It is found that the Pearson correlation coefficient (PCC) of $\boldsymbol{s}_1$ and $\boldsymbol{s}_2$ is 0.67, which furtherly verifies that the wind power output prediction errors of adjacent time sections have a significant correlation.

To further illustrate the effect of the time interval on the prediction error correlation, figure 2 shows the correlation coefficient matrix for the prediction error for each of the 24 periods (the time interval is 1 hour). In figure 2, the horizontal and vertical coordinates respectively correspond to random vectors of prediction errors in all scheduling periods. Each square represents the prediction error correlation coefficient of the corresponding two periods. The analysis shows that the correlation coefficients of the prediction error of each scheduling period are mostly between 0.2∼1.0, and the closer the scheduling period is, the stronger the correlation is.

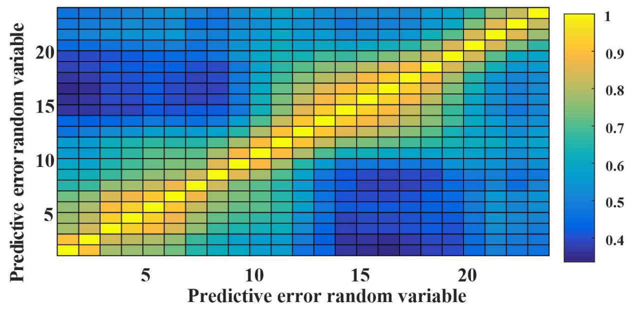

FIGURE 2. Wind power prediction error correlation matrix of different time periods of single wind farm.

Through the analysis in this section, it can be concluded that that there is a correlation between wind power prediction errors of adjacent scheduling periods.

2) UNCERTAIN SET MODELING

In the UC problem, a box-type uncertainty set is generally adopted. Taking the wind farm as an example, the predicted output at time point t is given by

$$w_t^e - w_t^l \le w_t \le w_t^e + w_t^u \tag{4}$$

$$\sum_t \frac{\left|w_t - w_t^e\right|}{w_t^w} \le \Gamma_T \tag{5}$$

where, Γ_T is the budget of the uncertainty.

The value of Γ_T plays a crucial role in the results of robust SCUC. If the value of Γ_T is too large, the result is extremely conservative; If the value of Γ_T is too small, the effect of uncertainty on scheduling cannot be fully reflected. According to the limitebery-levy central limit theorem proposed in [7], the appropriate value of Γ_T is given by

$$\Gamma_T = T\mu + \phi^{-1}(\alpha)\sqrt{T\sigma} \tag{6}$$

where, α is the confidence level, $\phi^{-1}(\cdot)$ is the cumulative probability distribution function of the normal distribution, T is the total scheduling period, μ and σ are the expected and variance of $\left|w_t - w_t^e\right|/w_t^w$, respectively. If the corresponding statistics are missing, it can be assumed that $\left|w_t - w_t^e\right|/w_t^w$ satisfies the normal distribution and then finds μ AND σ. As we know, the actual prediction error does not necessarily satisfy the normal distribution, at this time the actual distribution of the prediction error $\left|w_t - w_t^e\right|/w_t^w$ can also be fitted by historical data.

Based on the above time correlation analysis of prediction error and the corresponding uncertainty set model, time correlation constraints are added to the uncertainty set

$$\left\{\boldsymbol{e} \middle| \boldsymbol{e} = \frac{|\boldsymbol{w} - \boldsymbol{w}^e|}{\boldsymbol{w}^w}, C(\boldsymbol{s}_1, \boldsymbol{s}_2) \ge \gamma\right\} \tag{7}$$

where, $C(\cdot)$ is the calculation function of PCC, γ is the lower limit of time correlation of the uncertain scenario and the predicted scenario. The smaller the value of γ, the more conservative the uncertainty set is.

Reference [3] pointed out that in the process of solving to model, the worst scenario corresponds to the boundary of the uncertain set. In this case, the uncertainty set of Eq. (4) and (5) can be expressed as

$$W^u = \left\{ \begin{array}{l} w_t = w_t^e + z_t^+ w_t^u - z_t^- w_t^l \\ \sum_t \left(z_t^+ + z_t^-\right) \le \Gamma_T z_t^+ + z_t^- \le 1, \forall t \\ z_t^+, z_t^- \in \{0, 1\} \end{array} \right\} \tag{8}$$

When the form of the uncertainty set is changed from the continuous type (4)-(5) to the discrete type (8), the prediction error sequence can be expressed as $\boldsymbol{e} = |\boldsymbol{z}^+ - \boldsymbol{z}^-|$. The certification process is as follows.

Substitute Eq. (8) into Eq. (1), then we have

$$\boldsymbol{e} = \frac{|\boldsymbol{w} - \boldsymbol{w}^e|}{\boldsymbol{w}^w} = \frac{\left|\boldsymbol{w}^e + \boldsymbol{z}^+\boldsymbol{w}^u - \boldsymbol{z}^-\boldsymbol{w}^l - \boldsymbol{w}^e\right|}{\boldsymbol{w}^w} = \frac{\left|\boldsymbol{z}^+\boldsymbol{w}^u - \boldsymbol{z}^-\boldsymbol{w}^l\right|}{\boldsymbol{w}^w} \tag{9}$$

In Eq. (9), $\boldsymbol{w}^w = (\boldsymbol{w}^u + \boldsymbol{w}^l)/2$, $\boldsymbol{w}^u = \boldsymbol{w}^l$, SO $\boldsymbol{e} = |\boldsymbol{w} - \boldsymbol{w}^e|/\boldsymbol{w}^w$ can be converted to $\boldsymbol{e} = |\boldsymbol{z}^+ - \boldsymbol{z}^-|$.

Therefore, the time correlation constraint of $\boldsymbol{s}_1$ and $\boldsymbol{s}_2$ in Eq. (7) can be changed to the constraints on $\boldsymbol{z}^+$ and $\boldsymbol{z}^-$. in Eq (8), $\boldsymbol{z}^+$ and $\boldsymbol{z}^-$ are symmetrical. Considering that the prediction errors of adjacent periods are correlated, the time correlation constraint after simplification is discussed by taking $\boldsymbol{z}^+$ as an example. Two sequences in $\boldsymbol{z}^+$ separated by one time interval are expressed as

$$\boldsymbol{s}_1^+ = \left[z_1^+, z_2^+, \cdots, z_{T-1}^+\right] \tag{10}$$

$$\boldsymbol{s}_2^+ = \left[z_2^+, z_3^+, \cdots, z_T^+\right] \tag{11}$$

According to Eq. (10) and (11), the change flag variable $\boldsymbol{\upsilon}^+$ is defined as

$$\left\{\boldsymbol{\upsilon}^+ | \upsilon_t^+ = \left\{ \begin{array}{ll} 0 & z_t^+ = z_{t+1}^+ \\ 1 & z_t^+ \ne z_{t+1}^+ \end{array} \right\} \right. \tag{12}$$

The sum of all the dimensions of vector $\boldsymbol{\upsilon}^+$ is recorded as Λ^+, Λ^+ is called the "variation budget" on the time series. Λ^+ represents the number of $\boldsymbol{z}^+$ fluctuations in the time series. The larger Λ^+ is, the smaller the prediction error correlation coefficient $C\left(\boldsymbol{s}_1^+, \boldsymbol{s}_2^+\right)$. at the same time, the value of $C\left(\boldsymbol{s}_1^+, \boldsymbol{s}_2^+\right)$ is related to Γ_T and Λ^+, and it is independent of the specific values of $\boldsymbol{z}^+$ and $\boldsymbol{\upsilon}^+$.

In summary, the uncertainty set considering the temporal correlation of wind power prediction error can be expressed as

$$W^u = \left\{ \begin{array}{l} w_t = w_t^e + z_t^+ w_t^u - z_t^- w_t^l \\ \sum_t \left(z_t^+ + z_t^-\right) \le \Gamma_T, z_t^+ + z_t^- \le 1, \forall t \\ z_t^+ - z_{t+1}^+ \le \upsilon_t^{d+}, -z_t^+ + z_{t+1}^+ \le \upsilon_t^{u+} \\ z_t^- - z_{t+1}^- \le \upsilon_t^{d-}, -z_t^- + z_{t+1}^- \le \upsilon_t^{u-} \\ \sum_t \left(\upsilon_t^{d+} + \upsilon_t^{u+} + \upsilon_t^{d-} + \upsilon_t^{u-}\right) \le \Lambda \\ z_t^+, z_t^-, \upsilon_t^{d+}, \upsilon_t^{u+}, \upsilon_t^{d-}, \upsilon_t^{u-} \in \{0, 1\} \end{array} \right\} \tag{13}$$

where, Λ is the total uncertainty budget, and the superscripts U and D of Λ indicate that z_t to z_{t+1} are from 0 to 1 and from 1 to 0, respectively.

B. MODELING OF DISCRETE UNCERTAIN SETS

The conservativeness of the robust SCUC model is not only related to the uncertainty set of the wind power output and load, but also related to the discrete uncertainty set of the occasional failure of the component. Those discrete uncertainty sets are usually expressed by the N-k security criterion, which could be expressed as [13], [14]

$$Z = \left\{ \sum_i A_{it} \ge Nunits - k_G, A_{it} \in \{0, 1\}, \forall t \right\} \tag{14}$$

where, A_{it} indicates whether unit i is faulty during *t-th* period, 0 indicates outage, 1 indicates normal; *Nunits* denotes the total number of units; k_G denotes the maximum number of unit faults.

The conservativeness of the uncertain set can be changed by adjusting the parameter k_G in Eq. (14). However, in the worst scenario, the probability of outage of all units is the same, which may result in multiple units with large capacity being in outage status at the same time, even if the operational reliability of these units is high. At this point, the uncertain set of N-k outage contingency constraints is often too conservative. In the actual power systems, the probability of outage contingencies of units of different capacities or types is different. Generally, the unit with a larger capacity has higher operational reliability. Therefore, it is necessary to consider probability information in the uncertain sets. The formula for calculating the outage contingency probability of the unit is given by [24]

$$p_{it} = \frac{\lambda_i}{\lambda_i + \mu_i} = \frac{MTTR_i}{MTTF_i + MTTR_i} \tag{15}$$

where, p_{it} denotes the failure probability of unit i; λ_i denotes the failure rate of unit i; μ_i denotes the repair rate of unit i; $MTTF_i$ denotes the average time for the unit to operate without failure of unit i; $MTTR_i$ denotes the average of the time period from the failure of the unit to the end of maintenance of unit i.

At present, the modeling of the N-k uncertainty set considering the probability of equipment failure has not yet been carried out. Based on the above analysis, this paper proposes a α_{cut} criterion to reduce the conservativeness of RO method by adding the probability information of units outage contingencies to the uncertainty set. The worst scenario generated by this α_{cut} criterion includes only the case where the probability is greater than the given α_{cut} threshold, and the α_{cut} criterion constrained can be expressed as

$$\prod_{i=1}^{Nunits} p_{it}^{(1-A_{it})} \prod_{i=1}^{Nunits} (1 - p_{it})^{A_{it}} \ge \alpha_{cut} \tag{16}$$

Eq. (16) is a nonlinear equation and cannot be directly applied to the robust SCUC problem. A linear formula can be obtained by taking the logarithm of both sides of the equation (15).

$$\sum_{i=1}^{Nunits} (1-A_{it}) \cdot \log(p_{it}) + \sum_{i=1}^{Nunits} A_{it} \cdot \log(1-p_{it}) \geq \log(\alpha_{cut}) \tag{17}$$

Eq. (16) can be transformed into the following form

$$\sum_{i=1}^{Nunits} A_{it} \cdot (\log(1-p_{it}) \log(p_{it})) \geq \log(\alpha_{cut}) - \sum_{i=1}^{Nunits} \log(p_{it}) \tag{18}$$

From Eq. (18), it can be concluded that if the value of α_{cut} is small, the resulting uncertain set is conservative; the greater the value of α_{cut}, the smaller the uncertainty.

III. ROBUST SCUC MODEL

A. DETERMINISTIC SCUC MODEL

The deterministic SCUC model has been modeled as a MILP problem, and its model can be expressed as Eq. (19)-(30). The objective (19) is to minimize the total operation costs for the base case, including energy production cost, startup/shutdown cost over the entire scheduling horizon. Prevailing constraints include system power balance (20), generation capacity limits of thermal units (21)-(23) and wind farms (24), minimum ON/OFF time limits (25)-(26), startup/shutdown costs (27)-(28), ramping up/down limits (29)-(30), and transmission network security constraint (31).

$$\underset{P_{it}^b, I_{it}^b, P_{wt}^b, P_{it}^u(\bullet)}{Min} \sum_t \sum_i [\sum_k c_{ik} \cdot P_{ikt}^b + SU_{it}^b + SD_{it}^b] \tag{19}$$

$$\sum_i P_{it}^b + \sum_w P_{wt}^b = \sum_d P_{dt}^b \tag{20}$$

$$P_i^{\min} \cdot I_{it}^b \leq P_{it}^b \leq P_i^{\max} \cdot I_{it}^b \tag{21}$$

$$0 \leq P_{ikt}^b \leq I_{it}^b \cdot P_{ik}^{\max} \tag{22}$$

$$P_{it}^b = \sum\nolimits_k P_{ikt}^b \tag{23}$$

$$\leq P_{wt}^b \leq w_{wt}^e \tag{24}$$

$$\left[X_{on,i(t-1)}^b - T_{on,i}\right] \cdot \left[I_{i(t-1)}^b - I_{it}^b\right] \geq 0 \tag{25}$$

$$\left[X_{off,i(t-1)}^b - T_{off,i}\right] \cdot \left[I_{it}^b - I_{i(t-1)}^b\right] \geq 0 \tag{26}$$

$$SU_{it}^b \geq su_i \cdot (I_{it}^b - I_{i(t-1)}^b), \mathrm{SU}_{it}^b \geq 0 \tag{27}$$

$$SD_{it}^b \geq sd_i \cdot (I_{i(t-1)}^b - I_{it}^b), \mathrm{SD}_{it}^b \geq 0 \tag{28}$$

$$P_{it}^b - P_{i(t-1)}^b \leq UR_i \cdot I_{i(t-1)}^b + P_i^{\min} \cdot (I_{it}^b - I_{i(t-1)}^b) + P_i^{\max} \cdot (1 - I_{it}^b) \tag{29}$$

$$P_{i(t-1)}^b - P_{it}^b \leq DR_i \cdot I_{it}^b + P_i^{\min} \cdot (I_{i(t-1)}^b - I_{it}^b) + P_i^{\max} \cdot (1 - I_{i(t-1)}^b) \tag{30}$$

$$\left|\sum_m SF_{l,m}\left(\sum_{i\in U(m)} P_{it}^b + \sum_{w\in W(m)} P_{wt}^b - \sum_{d\in D(m)} P_{dt}^b\right)\right| \leq PL_l^{\max} \tag{31}$$

where, c_{ik} denotes the incremental cost of segment k of unit i; P_{ikt}^b denotes the base case dispatch of unit i at time t in segment k; P_{it}^b, I_{it}^b are the decision variables, which are the output of the generator set and the start and stop state respectively; SU_{it}^b and SD_{it}^b are the on-off costs; P_{wt}^b denotes the dispatch of wind farm w at time t; P_{dt}^b is the load forecast of load d at time t; $P_i^{\max}$ and $P_i^{\min}$ denote the upper and lower limits of the output of unit i; w_{wt}^e denotes the predicted value of wind power; $X_{on,it}^b$, $X_{off,it}^b$ denote the on/off time counters of unit i at time t; $T_{on,i}$, $T_{off,i}$ denote the minimum start-up and downtime limit; su_i, sd_i is the start-stop cost of unit i; UR_i, DR_i are the unit climbing power limit; $PL_l^{\max}$ denotes the maximum power flow constraint of the line; $SF_{l,m}$ denotes the node power transfer factor; $D(m)$ denotes the set of load demands located at bus m; $U(m)$ denotes the set of thermal units located at bus m; $W(m)$ denotes the set of wind farms located at bus m.

B. ROBUST CONSTRAINTS UNDER UNCERTAINTY

The proposed robust SCUC model includes robust security constraints (32)-(41) for handling continuous uncertainty and discrete uncertainty as defined by the uncertainty set (13), (14) and (18). In the proposed robust SCUC model, the uncertainty set $\mathbf{U}$ include wind power forecast w_{wt}^u, load forecast P_{dt}^u, as well as unit outage contingency statuses A_{it}. For the sake of discussion, the compact form $\mathbf{S}$ is used to represent the set of uncertain load/wind and outage contingency parameters. That is, $\mathbf{S} = \left\{P_{dt}^u, w_{wt}^u, A_{it}\right\} \in \mathbf{U}$.

The proposed robust SCUC model is further explained as follows.

1) System balance constraint

$$\sum_i P_{it}^u(\mathbf{S}) + \sum_w P_{wt}^u(\mathbf{S}) = \sum_d P_{dt}^u \tag{32}$$

where, P_{it}^u and P_{wt}^u denote the adaptive dispatch adjustment of unit i and wind farm w at time t in response to uncertain intervals; P_{dt}^u denotes the uncertain load demand of load d at time t.

(2) Generation limits of thermal units and wind farms under uncertainties.

$$P_i^{\min} \cdot IA_{it} \leq P_{it}^u(\mathbf{S}) \leq P_i^{\max} \cdot IA_{it} \tag{33}$$

$$IA_{it} \leq I_{it}^b \tag{34}$$

$$I_{it}^b + A_{it} - 1 \leq IA_{it} \tag{35}$$

$$0 \leq P_{wt}^u(\mathbf{S}) \leq w_{wt}^u \tag{36}$$

where, IA_{it} is auxiliary binary variable, which indicates the value of the unit commitment I_{it}^b obtained in the deterministic SCUC model under contingency outage.

(3) Dispatch adjustments of thermal units in response to uncertainty sets are restricted by their corrective capabilities and generation dispatches in the base case.

$$P_{it}^u(\mathbf{S}) - P_{it}^b \leq R_i^{up} \cdot IA_{it} - \left(I_{it}^b - IA_{it}\right) \cdot P_i^{\min} \tag{37}$$

$$P_{it}^b - P_{it}^u(\mathbf{S}) \leq R_i^{down} \cdot IA_{it} + \left(I_{it}^b - IA_{it}\right) \cdot P_i^{\max} \tag{38}$$

where, R_i^{up}, R_i^{down} are the up/down corrective action limits of unit i; corrective capabilities R_i^{up}, R_i^{down} refer to the 10-min spinning reserve capacities of generations units.

(4) Hourly ramping up and down limits of thermal units under uncertainty sets.

$$P_{it}^{u}(\mathbf{S}) - P_{i(t-1)}^{u}(\mathbf{S}) \leq UR_i \cdot IA_{i(t-1)} + P_i^{\min} \cdot (IA_{it} - IA_{i(t-1)}) + P_i^{\max} \cdot (1 - IA_{it}) \quad (39)$$

$$P_{i(t-1)}^{u}(\mathbf{S}) - P_{it}^{u}(\mathbf{S}) \leq \mathrm{D}R_i \cdot IA_{it} + P_i^{\min} \cdot (IA_{i(t-1)} - IA_{it}) + P_i^{\max} \cdot (1 - IA_{i(t-1)}) \quad (40)$$

(5) Transmission network constraint

$$\left| \sum_m SF_{l,m} (\sum_{i \in U(m)} P_{it}^{u}(\mathbf{S}) + \sum_{w \in W(m)} P_{wt}^{u}(\mathbf{S}) - \sum_{d \in D(m)} P_{dt}^{u}) \right| \leq PL_l^{\max} \quad (41)$$

For the sake of discussion, the compact matrix formulation (42) is used to represent the above robust SCUC model. In (42), matrix inequality (a) represents constraints in base scenario; matrix inequality (b) represents constraints in uncertain scenarios.

$$\underset{\boldsymbol{I}^b, \boldsymbol{P}^b}{Min}\ \boldsymbol{N}^T \cdot \boldsymbol{I}^b + \boldsymbol{c}^T \cdot \boldsymbol{P}^b$$
$$s.t. \begin{cases} \boldsymbol{X} \cdot \boldsymbol{I}^b + \boldsymbol{Y} \cdot \boldsymbol{P}^b \leq \boldsymbol{g}^b & \text{(a)} \\ \boldsymbol{Q} \cdot \boldsymbol{I}^b + \boldsymbol{W} \cdot \boldsymbol{P}^b + \boldsymbol{R} \cdot \boldsymbol{P}^u(S) \leq \boldsymbol{g}^u(S) & (b) \\ \boldsymbol{P}^b \geq 0, \boldsymbol{P}^u \geq 0, \boldsymbol{I}^b \in \{0, 1\} & (c) \end{cases} \quad (42)$$

where, $\boldsymbol{I}^b$ represents commitment related decisions I_{it}^b, SU_{it}^b and SD_{it}^b; $\boldsymbol{P}^b$ represents dispatch related decisions P_{it}^b and P_{wt}^b; $\boldsymbol{P}^u(S)$ represents dispatch related decisions P_{it}^u and P_{wt}^u in response to uncertainties.

IV. SOLUTION METHODOLOGY

The proposed robust SCUC model (42) can be decompose into a master UC problem in first stage and security checking subproblem under various uncertainties in two stage by the Benders decomposition (BD).

A. MASTER UNIT COMMITMENT PROBLEM

The objective of master *UC* problem is Eq. (19), the constraints include Eq. (20)-(31) and all Benders cuts obtained from the security checking subproblem in the two stage.

$$\underset{\boldsymbol{I}^b, \boldsymbol{P}^b}{Min}\ \boldsymbol{N}^T \cdot \boldsymbol{I}^b + \boldsymbol{c}^T \cdot \boldsymbol{P}^b$$
$$s.t. \begin{cases} \boldsymbol{X} \cdot \boldsymbol{I}^b + \boldsymbol{Y} \cdot \boldsymbol{P}^b \leq \boldsymbol{g}^b \\ \textit{All Benders cuts obtained so far} \\ \boldsymbol{P}^b \geq 0, \boldsymbol{I}^b \in \{0, 1\} \end{cases} \quad (43)$$

B. SECURITY EVALUATION FOR UNCERTAINTY SETS

The security subproblem is to verify whether the $\boldsymbol{I}^b$ and $\boldsymbol{P}^b$ obtained by the master UC problem in the base scenario is feasible in the uncertainty interval. The security check constraint in the uncertain scenario includes (32)-(41). The security evaluation subproblem is solved via two steps in sequence.

1) IDENTIFY THE WORST SCENARIO WITH THE LARGEST SECURITY VIOLATION

The problem of determining the worst scenario can be expressed as a *Max-Min* problem (39). In order to make the constraints in the uncertain scenario feasible, it is necessary to introduce a slack variable, where the slack variable can be understood as a security violation in the worst scenario, so the security violation needs to be minimized.

$$\underset{\mathbf{S}}{Max}\ \underset{\boldsymbol{P}^u, \boldsymbol{v}}{Min}\ \mathbf{1}^T \cdot \boldsymbol{v}$$
$$s.t. \begin{cases} \boldsymbol{R} \cdot \boldsymbol{P}^u + \boldsymbol{v} \leq \boldsymbol{g}^u(\mathbf{S}) - \boldsymbol{Q} \cdot \hat{\boldsymbol{I}}^b - \boldsymbol{W} \cdot \hat{\boldsymbol{P}}^b \\ \boldsymbol{v} \geq \mathbf{0}, \boldsymbol{P}^u \geq \mathbf{0} \end{cases} \quad (44)$$

where, $\boldsymbol{v}$ are vector of slack variables; $\mathbf{1}^T \cdot \boldsymbol{v}$ represents the largest security violation.

In this paper, dual theory is adopted to solve (44) with integer variables in the inner minimization problem. The inner minimization problem is an LP problem, and dual theory can be used to transfer (44) into a single level problem (45).

$$\max \boldsymbol{\lambda}^T \left(\boldsymbol{g}^u\left(\mathbf{S}\right) - \boldsymbol{Q} \cdot \hat{\boldsymbol{I}}^b - \boldsymbol{W} \cdot \hat{\boldsymbol{P}}^b \right)$$
$$s.t. \begin{cases} \boldsymbol{\lambda}^T \boldsymbol{R} \leq \mathbf{0}^T \\ -\mathbf{1}^T \leq \boldsymbol{\lambda}^T \leq \mathbf{1} \\ \boldsymbol{\lambda}^T \leq \mathbf{0}^T \end{cases} \quad (45)$$

where, $\boldsymbol{\lambda}$ are optimal dual solutions of constraints in (44).

Due to the existence of quadratic form $\boldsymbol{\lambda}^T \cdot \boldsymbol{g}^u(\mathbf{S})$ in the objective function, model (45) is a typical bilinear programming problem and generally it is not a convex optimization problem. The auxiliary variable method in [7] can be applied to solve (45), thereby deriving the worst scenario $\mathbf{S}^{worst} = \hat{\mathbf{S}}$ corresponding to the largest security violation.

2) GENERATE BENDERS CUTS CORRESPONDING TO THE WORST SCENARIO WITH THE LARGEST SECURITY VIOLATION

If the largest security violation $\mathbf{1}^T \cdot \boldsymbol{v}$ in the worst scenario $\hat{\mathbf{S}}^{worst}$ is higher than the predefined threshold, the threshold for security evaluation is set to $10^{-3} MWh$. We can directly generating valid Benders cut (46) to the master UC problem, and the Benders cut is feed backed to the master UC problem to find robust UC solutions that can alleviate security violations.

$$\mathbf{1}^T \cdot \boldsymbol{v} + \boldsymbol{\lambda}^T \cdot \left[\boldsymbol{Q} \cdot \left(\boldsymbol{I}^b - \hat{\boldsymbol{I}}^b \right) + \boldsymbol{W} \cdot \left(\boldsymbol{P}^b - \hat{\boldsymbol{P}}^b \right) \right] \leq \mathbf{0} \quad (46)$$

In summary, the proposed robust SCUC model is solved by the following steps.

Step 1) The master UC model is solved. The optimal solutions $\boldsymbol{I}^b$ and $\boldsymbol{P}^b$ are passed on the security evaluation subproblem for uncertainties.

Step 2) This step checks the security checking subproblem for the uncertainty set, which includes two steps. The first substep transfer (44) into a single level problem (45) by dual theory and identify the worst scenario $\hat{\mathbf{S}}^{worst}$ that would lead to the largest security violation.

Step 3) If the largest security violation in the worst scenario $\hat{\mathbf{S}}^{worst}$ is higher than the predefined threshold, and generates Benders cut (46) to the master UC problem. The iterative procedure stops when the $\boldsymbol{I}^b$ and $\boldsymbol{P}^b$ of first stage satisfies all security violation checks.

V. CASE STUDIES

To illustrate the economical and reliability of the proposed method, numerical cases are conducted on one area of the modified IEEE-RTS-96 system [23], which includes 24 buses and 32 units. The IEEE RTS-96 system provides outage probability information for units with different capacities. This paper slightly adjusts the parameters in the case to make the units with larger capacity have high power supply reliability. The proposed two-stage robust SCUC model is solved using YALMIP and Gurobi-7.5.2 on MATLAB 2015b.

A. ROBUST SCUC MODEL CONSIDERING CONTINUOUS UNCERTAIN PREDICTION ERRORS

The maximum load in the improved IEEE-RTS-96 system is approximately 2500 MW. A wind farm is connected to the bus 10, the maximum wind power permeability is about 33%, and the average wind power permeability is 21%. Wind power and load access data are from Elia.

TABLE 1. Comparison of scheduling results of three models.

model	solution time/s	cost/$	Security check
SCUC	7.06	564406	not pass
RUC	103.7488	582134	pass
TRUC	81.50	579030	pass

To verify the validity of the proposed model, the following three models are compared in table 1. Parameters are estimated based on historical data. When $\Gamma^T = 12$, $\Lambda = 3$, the UC decisions under the three models are calculated and the security check is performed. The security check is whether the dispatch plan can satisfy the minimum safety violation in equation (42) below the given safety threshold under the actual output of wind power and load. The three UC models are explained as follow

a. Traditional SCUC model. The deterministic optimization algorithm is adopted, and the wind power and load prediction errors are not considered.

b. Two-stage robust unit commitment (RUC). Consider wind power and load forecasting errors, but do not consider time correlation.

c. Two-stage robust unit commitment that takes into account the temporal correlation of the prediction error (TRUC).

As can be seen from table 1, the deterministic SCUC model has the lowest operating cost and the fastest calculation speed. When considering the uncertainty, deterministic UC decision can not guarantee the security of the power system. Whereas the RUC and TRUC models can ensure the security operation of the power system. Compared with the RUC model, the TRUC model reduces the conservativeness of uncertainty set, thereby reducing the operating cost and improving the calculation efficiency.

To further explain the mechanism of the TRUC model, the "wind power worst scenario" calculated by RUC and TRUC is labeled in figure 3.

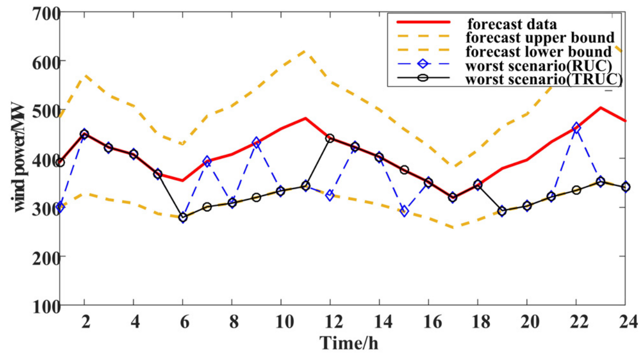

FIGURE 3. Wind power worst scenario solution results of RUC and TRUC.

The worst scenario of RUC fluctuates many times between the predicted value and the predicted lower bound, and multiple wind climb events occur. According to the time correlation analysis of the previous wind output, the probability of such scenario is low. In the TRUC model, due to the variation budget Λ, the fluctuation of wind power prediction error is limited, so the scene with low probability of occurrence is eliminated, the number of climb of thermal units is reduced, as a result the economics of robust optimization is improved.

The optimization results of the TRUC model are mainly affected by the uncertainty budget Γ^T and the change budget Λ. Figure 4 shows the results of TRUC model under different Γ^T and Λ.

As can be seen from figure 4, as the budget uncertainty Γ^T increases, the uncertainty set becomes more and more conservative, the optimization result increases, and the probability of passing the security check is also higher.

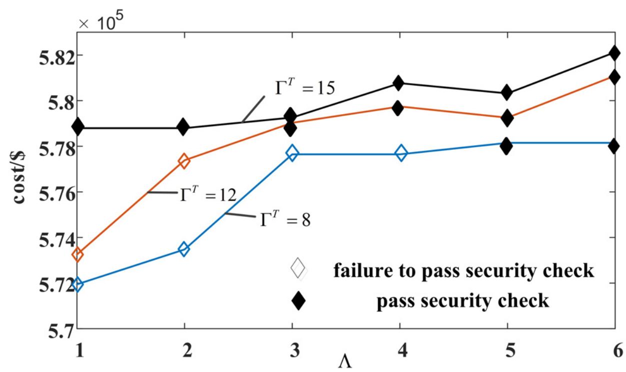

FIGURE 4. Robust optimization costs under different Γ^T and Λ.

Further comparisons of the effects of changes in Λ under different conditions Γ^T on the optimization results show that when Γ^T is small, the results changes greatly with the increase of Λ; when Γ^T is large, the variation of the robust optimization results with the change of Λ is small. Through analysis, it can be concluded that under a small Λ, the uncertainty prediction error is too concentrated, and the actual prediction error cannot be fully reflected, resulting in an ideal optimization result. When Γ^T is large, the uncertain budget can fully reflect the error of the predicted value over the entire scheduling period, so the RO cost is less affected by Λ. Therefore, in RUC problem, when Γ^T is reasonably selected, Λ can be used as an auxiliary parameter, and the conservatism of RUC model can be changed by fine-tuning Λ.

From the above analysis, we can see that the change of Λ will adjust the robust optimization results. In order to test the influence of the Λ on the robustness of the TRUC model, the robust security check is performed on the UC solution under different Λ. The adjusted Elia's historical data from January 2018 to June 2018 (including forecast data, real data and forecast upper and lower limits) is used as the input data. In the test, Γ^T is set to 12, Λ is changed for robust testing. The robust test results of the above three UC models are shown in Table 2. In Table 2, total cost represents the sum of costs in all historical scenarios, and α_{robust} denotes the proportion of passing the robust test. As can be seen from Table 2, the traditional SCUC model has the poorest robust performance. Compared with the RUC model, TRUC model improves economics while ensuring robustness. When $\Lambda = 6$, the α_{robust} of TRUC is 93.44%, and the α_{robust} of the RUC is 93.99%, the robustness of TRUC and RUC is approximately the same, but the total operating cost of TRUC model is reduced by approximately 46,000$ compared with that of RUC model.

TABLE 2. Robust test results based on historical data.

model		total cost(10^6 $)	α_{robust}
SCUC		129.607968	50.81%
RUC($\Gamma^T = 12$)		130.631054	93.99%
TRUC($\Gamma^T = 12$)	Λ=1	130.436212	81.87%
	Λ=3	130.570276	87.43%
	$\Lambda = 6$	130.585309	93.44%

B. CASE STUDIES OF N-k OUTAGE CONTINGENCIES

In order to demonstrate how the α_{cut} criterion exclude unit outage contingencies with very low probability from the uncertainty set, this section analyzes the test results by changing the parameters k_G and α_{cut} on the IEEE RTS-96 system. The data of *MTTF* and *MTTR* are shown in Table 3.

When the α_{cut} criterion is not considered, the RUC decision is only affected by the N-k outage contingency constraint. Table 4 gives the RO cost under different k_G. As can be seen from Table 4, as k_G increases, the cost of RUC model in the worst scenario also increases. When k_G is equal to 4, the RUC model has no optimal solution. This is because the spinning reserve capacities of the units that can operate normally in the system cannot meet the load demand.

TABLE 3. MTTF and MTTR for different capacity generators.

Unit capacity	MTTF(hours)	MTTR(hours)
12MW	2940	60
20MW	450	50
50MW	495	20
76MW	1960	40
100MW	1200	50
155MW	960	40
197MW	950	50
350MW	3000	100
400NW	8000	150

TABLE 4. Robust optimization cost under different k_G.

$N-k_G$	N-0	N-1	N-2	N-3	N-4
Cost/$	564406	566288	579367	587391	/

In order to verify that the N-k criterion can effectively ensure that the system operates security under different outage contingencies scenario, it is necessary to perform security check on the RUC decision. Randomly generate 500 sets of outage contingencies scenarios under different outage numbers, and perform safety tests under different N-k outage constraints to obtain the average value of the expected energy not supplied (EENS) under different fault numbers, as shown in Table 5.

TABLE 5. EENS test under different unit failure numbers (MWH).

$N-k_G$	Units failure numbers			
	1	2	3	4
N-0	145.6	1331.9	3176.5	5679.8
N-1	0	179.8	775.6	1885.2
N-2	0	0	24.3	173.1
N-3	0	0	0	34.8

It can be seen from Table 5 that with the increase of k_G, the robustness of the UC decisions are improved, and the average value of the EENS under different outage decreases. When the number of unit failures is equal to k_G, the N-k security criterion can be guarantee that the UC decision feasible in worst scenario.

Under the N-k security criterion, this paper proposes a α_{cut} criterion that takes into account the outage probability of the units. This subsection focuses on investigating how the α_{cut} criterion affects the performance of UC decisions. Figure 5 shows the cost of RUC model for N-k security criterion at different α_{cut}.

It can be seen from figure 5 that the cost of robust CCUC model under different N-k security criterion will decrease with the increase of α_{cut}. The larger k_G is, the more the optimization cost is affected by α_{cut}. Through analysis, it can be concluded when $\alpha_{cut} = 10^{-7}$ and 10^{-6}, the uncertainty

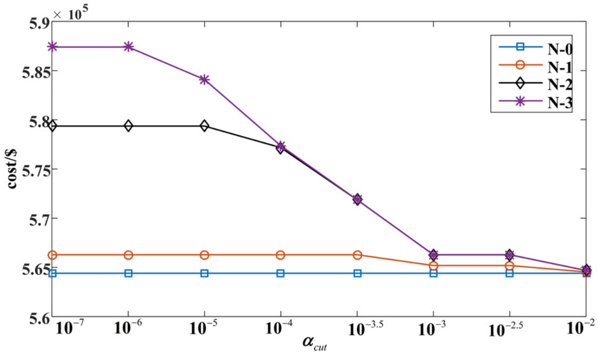

FIGURE 5. ***N-k*** **outage contingencies constrained robust optimization cost under different α_{cut}.**

set is quite conservative that contains extremely outage contingencies that are very unlikely to occur, resulting in the cost of RUC model is high. As α_{cut} increases, due to the limitation of the α_{cut} criterion, some extreme scenarios will be excluded from the uncertainty set, and the conservativeness of the uncertain set is reduced. When $\alpha_{cut} \geq 10^{-4}$, $k_G = 2$ and 3, the cost of RUC model are the same. This is because the α_{cut} criterion limits the worst scenario in the uncertainty set, so the worst scenario are the same when $k_G = 2$ and 3.

The unit commitment under different α_{cut} are compared as follows. Figure 6 shows the unit commitment and the outage contingencies condition of the units in the worst scenario at different α_{cut} when k_G is equal to 3. In figure 6, the abscissa denotes the scheduling period and the ordinate denotes the unit number. It can be seen from the figure 6 that when α_{cut} is equal to 10^{-7}, since the value of α_{cut} is too small, the outage contingency probability fails to affect the uncertain sets; the maximum number of outages in this case is three; the worst scenario obtained at this case is the most serious outage scenario of the system. In this worst scenario, the EENS is 1150 MW, so the UC decision is quite conservative, it is necessary to open a large number of units to cope with the uncertainty of the fault. As α_{cut} increase to be 10^{-5}, the uncertainty set is affected by the α_{cut} criterion. The outage units in the worst scenario still is three, but the conservativeness is limited, so there is no need to turn on expensive 12MW and 20MW units during the operation horizon. When α_{cut} increases to 10^{-4}, the N-k constraint is no longer a key factor affecting the uncertain set. At this time, there are two units in the worst scenario, so in the base case, it is not necessary to turn on all the units to cope with the uncertainty. When α_{cut} becomes $10^{-2.5}$, only one unit outage in the worst scenario. However, at this time the UC decision can not very well protect the system against uncertainty because some outage contingencies with high impact are excluded from the uncertainty set by tight α_{cut} constraints.

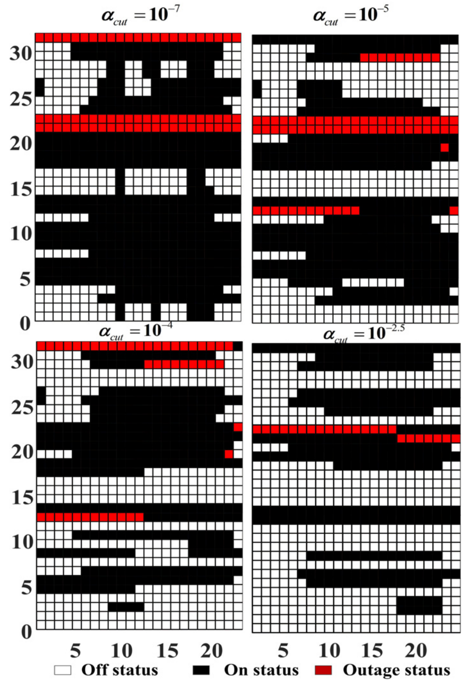

FIGURE 6. **Unit commitment decision under different α_{cut}.**

TABLE 6. **Robust optimization cost with all uncertainties ($).**

k_G / α_{cut}		$N-k_G$								
		$N-3$			$N-2$			$N-1$		
Γ^T	Λ	10^{-7}	10^{-5}	$10^{-2.5}$	10^{-7}	10^{-5}	$10^{-2.5}$	10^{-7}	10^{-5}	$10^{-2.5}$
15	6	/	598495.6	585489.2	594446.2	594446.2	585489.2	585489.2	585489.2	585489.2
	3	/	598068.1	584355.5	592492.6	592492.6	584355.5	584355.5	584355.5	584355.5
	1	/	596834.4	583776.2	592447.7	592447.7	583776.2	583776.2	583776.2	583776.2
12	6	/	597629.7	582944.1	591616.5	591616.5	582944.1	582944.1	582944.1	582944.1
	3	/	596103.6	583626.2	590900.4	590900.4	583626.2	583626.2	583626.2	583626.2
	1	/	592814.2	579234.2	589313.1	589313.1	579234.2	579234.2	579234.2	579234.2
8	6	/	592115.5	582281.3	589935.3	589935.3	582281.3	582281.3	582281.3	582281.3
	3	/	590211.8	581239.8	587368.7	587368.7	581239.8	581239.8	581239.8	581239.8
	1	/	585705.9	576318.9	582539.8	582539.8	576318.9	576318.9	576318.9	576318.9

C. CASE STUDIES CONSIDERING CONTINUOUS UNCERTAINTY AND N-k OUTAGE CONTINGENCIES CONSTRAINT

Based on the analysis of the section A considering the uncertainties prediction error time correlation and the section B considering the unit outage probability of N-k security criterion, this section tests the effects of three uncertainties of wind power, load and outage contingency of unit on the optimization cost of the robust SCUC model. The results are shown in Table 6.

It can be seen from table 6 that when $k_G = 3$ and $\alpha_{cut} = 10^{-7}$, regardless of the values of Γ^T and Λ, there is no feasible robust solution in the worst scenario. According to the analysis, there are three outage units in this case, and considering the uncertainty of wind power and load, even if all the units in the power system are turned on at the same time and they cannot meet the maximum security violation. When α_{cut} is equal to 10^{-5}, it is limited by α_{cut} criterion, and there is a feasible solution under the N-3 security criterion. As with the analysis of section A and section B, the optimization cost decreases with the decrease of uncertainty budget Γ^T. When the value of Γ^T is unchanged, the optimization cost decreases as the value of Λ decreases. Comparing the impact of different α_{cut} on the N-k security criterion, when k_G is large, the optimization cost is greatly affected by α_{cut}. When the value of k_G is small, the optimization cost is less affected by α_{cut}. According to the comprehensive comparison, compared with the uncertainty of wind power and load, the impact of unit outage contingencies on system operation is more serious.

VI. CONCLUSIONS

In order to reduce the conservativeness of the robust unit commitment model, this paper proposes a two-stage robust SCUC model considering time autocorrelation of wind/load prediction error and outage contingency probability of units. The main conclusions of this paper are as follows.:

1) For the conservative of uncertainty set of wind/load, the prediction error time correlation constraint proposed in this paper reduces the conservativeness of uncertainty set under the premise of ensuring robustness.

2) Because of the uncertainty set of N-k contingency constraint is too conservative, this paper proposes a α_{cut} criterion considering the outage contingency probability of unit. Under the premise of ensuring the reliable operation of the power system, the proposed α_{cut} criterion effectively eliminates the fault scenarios with low probability of occurrence and improves the economics of robust CCUC model.

3) Considering wind/load uncertainty and N-k outage contingency uncertainty at the same time, it is concluded that compared with the uncertainty of wind/load, the impact of the outage of units on the operation of the power systems is more serious.

It should be pointed out that although the α_{cut} criterion considering the outage contingency probability of unit proposed in this paper effectively reduces the conservativeness of the uncertainty set, however, the method of selecting the value of α_{cut} has not been discussed in detail, we will leave this topic to the next paper.

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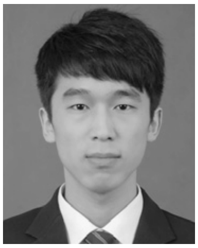

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