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Multiple Model Tracking for Hypersonic Gliding Vehicles With Aerodynamic Modeling and Analysis

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ABSTRACT The multiple model tracking methods for hypersonic gliding vehicles (HGVs), which improves the design of tracking model-set is proposed in this paper. Different from the traditional tracking method with dynamic pressure models, the proposed method refines the model-set by modeling and analyzing the specific aerodynamic characteristics of the HGVs. Based on the aerodynamic model of the vehicle, the novel tracking models are established and the multiple model tracking methods are adopted. The uniform model-sets are designed by determining the representative points from the constrained characteristic parameters to ensure the stability of the tracking system. Moreover, the model-sets are then augmented by the real-time updates of the motion estimations to further improve the tracking accuracy. The simulation studies indicate that the proposed tracking method outperforms the method with dynamic pressure models in both position and velocity estimations, the position estimation accuracy is increased by about 23%–53% and the velocity estimation accuracy is increased by about 42%–56% in the given examples.

INDEX TERMS Hypersonic gliding vehicle, aerodynamic modeling, multiple model tracking, model-set design.

I. INTRODUCTION

The HGVs are a kind of near space aircraft that can make long range hypersonic glide flights with a velocity of more than Mach 5. Benefited from the aerodynamic configuration with a high lift-to-drag ratio, the HGVs may have the ability of gliding for even over ten thousand kilometers without any power [1]. The great mobility and extremely high speed of the HGVs bring serious challenges to the radar tracking systems, thus the research in tracking the HGVs is of great value and has attracted extensive attention.

Successful design for tracking models makes great sense in target tracking and can effectively promote extracting useful information from observations [2]. In general, the motion modes and dynamic behaviors of the maneuvering targets are uncertain to the trackers, thus the models can be built by modeling the kinematic characteristics without considering the cause of maneuvers. The key to effective kinematic modeling lies in the appropriate description of the kinematic variables (generally refer to the acceleration or the angular velocity). In these models, the kinematic variables are usually supposed as random processes of certain properties. Typical examples include the constant-velocity model, the constant-acceleration model, the Singer model, the current model and the constant turn model [2]. Recently, the typical models have been improved to deal with more complex motion modes [3]–[5]. On the basis of this, [6] demonstrated a novel sine tracking model which was specifically designed for the hypersonic targets with periodical skip trajectories. The kinematic models have the advantages of simple structure and less computations. However, because of the complex motion modes of the HGVs, these models may have difficulties in describing the real maneuvers appropriately and this will lead to inaccurate tracking. When more details of the dynamic characteristics are available to the trackers, the models can be designed to be more sophisticated, known as the dynamic models. These models are generally based on the dynamic analysis of the vehicles and can describe the motion modes more accurately [7]–[9]. With sufficient details of the dynamic characteristics, the dynamic models can achieve a better accuracy than the kinematic models. To further improve the tracking performance,

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the multiple model methods have been widely used [10]–[12], where multiple models are utilized simultaneously to estimate one motion mode.

On account of the great maneuverability of the HGVs, the tracking method based on dynamic models is adopted and developed in this paper. The HGVs have some specific dynamic features compared with the traditional aircrafts. For instance, with lifting body technology, the HGVs have super glide ability in the near space. Meanwhile, the aerodynamic characteristics are quite different because of the special atmospheric environment in the near space. Thus the corresponding dynamic models need to be improved.

In this paper, a novel multiple model tracking method for the HGVs with aerodynamic modeling and analysis is proposed. The hypersonic aerodynamics engineering calculation method is utilized to establish a representative aerodynamic model. The characteristic parameters of the motion mode are designed and constrained according to the capability analysis of this model. Furthermore, the representative points of these characteristic parameters are determined based on the number theoretic method. Thus the uniform tracking model-sets are built, and the model-sets are then augmented to match the real motion mode of the HGVs better.

The rest of the paper is organized as follows. Section II presents the basic form and the limitations of the traditional dynamic pressure model. Section III provides the aerodynamic modeling method for the HGVs and designs the characteristic parameters. Section IV gives the novel tracking model-sets and the simulation results are shown in Section V.

II. DYNAMIC PRESSURE MODEL

Traditional tracking models for reentry targets are generally based on the dynamic pressure [7]. The basic form of the dynamic pressure model is deduced in the following.

Let $\boldsymbol{X}(t) = [x(t), y(t), z(t), \dot{x}(t), \dot{y}(t), \dot{z}(t)]^{\mathrm{T}}$ denote the target state at time t in the East-North-Up (ENU) coordinates with position $[x(t), y(t), z(t)]$ and velocity $\boldsymbol{V}(t) = [\dot{x}(t), \dot{y}(t), \dot{z}(t)]$, let $[a_d(t), a_t(t), a_c(t)]$ denote the aerodynamic acceleration in the Velocity-Turn-Climb (VTC) coordinates, the acceleration in the ENU coordinates can be deduced by

$$\frac{\mathrm{d}\boldsymbol{V}(t)}{\mathrm{d}t} = \boldsymbol{T}_{\mathrm{VTC}}^{\mathrm{ENU}} \begin{bmatrix} a_d(t) \\ a_t(t) \\ a_c(t) \end{bmatrix} + \boldsymbol{g}(t) + \boldsymbol{\omega}(t), \tag{1}$$

where $\boldsymbol{g}(t)$ is the acceleration of gravity, $\boldsymbol{\omega}(t)$ is the process noise, $\boldsymbol{T}_{\mathrm{VTC}}^{\mathrm{ENU}}$ is the transfer matrix from the VTC coordinates to the ENU coordinates. $\boldsymbol{T}_{\mathrm{VTC}}^{\mathrm{ENU}}$ can be calculated according to $\boldsymbol{V}$, that is

$$\boldsymbol{T}_{\mathrm{VTC}}^{\mathrm{ENU}}(\boldsymbol{V}) = \begin{bmatrix} \dot{x}/v & -\dot{y}/v_g & -\dot{x}\dot{z}/(vv_g) \\ \dot{y}/v & \dot{x}/v_g & -\dot{y}\dot{z}/(vv_g) \\ \dot{z}/v & 0 & v_g/v \end{bmatrix}, \tag{2}$$

where $v = \|\boldsymbol{V}\| = \sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2}$, $v_g = \sqrt{\dot{x}^2 + \dot{y}^2}$.

Therefore, the dynamic model can be given by

$$\dot{\boldsymbol{X}}(t) = \begin{bmatrix} \boldsymbol{0}_{3\times 3} & \boldsymbol{I}_{3\times 3} \\ \boldsymbol{0}_{3\times 3} & \boldsymbol{0}_{3\times 3} \end{bmatrix} \boldsymbol{X}(t) + \begin{bmatrix} \boldsymbol{0}_{3\times 3} \\ \boldsymbol{I}_{3\times 3} \end{bmatrix} \left(\boldsymbol{T}_{\mathrm{VTC}}^{\mathrm{ENU}} \begin{bmatrix} a_d(t) \\ a_t(t) \\ a_c(t) \end{bmatrix} + \boldsymbol{g}(t) + \boldsymbol{\omega}(t) \right), \tag{3}$$

where $\boldsymbol{0}_{3\times 3}$ is a three-dimensional null matrix and $\boldsymbol{I}_{3\times 3}$ is a three-dimensional unit matrix.

When the observation interval is small enough, Equation (3) can be discretized according to the piecewise constant acceleration model, which is addressed as

$$\boldsymbol{X}_{k+1} = \begin{bmatrix} \boldsymbol{I}_{3\times 3} & T\boldsymbol{I}_{3\times 3} \\ \boldsymbol{0}_{3\times 3} & \boldsymbol{I}_{3\times 3} \end{bmatrix} \boldsymbol{X}_k + \begin{bmatrix} 0.5T^2\boldsymbol{I}_{3\times 3} \\ T\boldsymbol{I}_{3\times 3} \end{bmatrix} \left(\boldsymbol{T}_{\mathrm{VTC}}^{\mathrm{ENU}} \begin{bmatrix} a_{d,k} \\ a_{t,k} \\ a_{c,k} \end{bmatrix} + \boldsymbol{g}_k + \boldsymbol{\omega}_k \right), \tag{4}$$

In Equation (4), the aerodynamic accelerations $a_{d,k}$, $a_{t,k}$, and $a_{c,k}$ constitute the characteristic parameters of the dynamic model and can determine the motion mode, and different characteristic parameters correspond to different tracking models. In the traditional dynamic models for reentry targets, the aerodynamic accelerations are generally calculated according to the dynamic pressure, the normalized equation is given by

$$a = \xi q = \xi \cdot \frac{1}{2}\rho v^2, \tag{5}$$

where a stands for one of the aerodynamic accelerations in Equation (4), q is the dynamic pressure, ρ is the atmospheric density, ξ is the aerodynamic coefficient of the aircraft and it is equal to the reference area S times a non-dimensional coefficient σ, then divided by the mass m, that is $\xi = \sigma S$. Thus the coefficient ξ determines the mode of the tracking system, and can be used to design the dynamic pressure model. Equation (5) provides a general simplified computing method to calculate the aerodynamic accelerations. However, when the aircraft is in the hypersonic flow, the relationship between the aerodynamic force and the dynamic pressure is much more complicated and the stress condition of the vehicle is quite different. For instance, there are some particular phenomena in the hypersonic flow such as the shock wave and the expansion wave, and the aerodynamic force may even be generated from the static pressure instead of the dynamic pressure [13]. Thus Equation (5) may not perform well in the hypersonic flow which will result in the difficulties and limitations in designing tracking models.

To solve the problem above, a novel method to design the tracking models for the HGVs is proposed in the following sections. A typical aerodynamic model is established to help make the tracking models more suitable and representative for the HGVs, and the coefficient ξ is no longer used to design the tracking models.

III. AERODYNAMIC MODELLING FOR HGVS

The previous sections indicate that the traditional kinematic or dynamic models may have difficulties in tracking the HGVs for the lack of aerodynamic analysis of the vehicles. In this section, the aerodynamic characteristics of the HGVs are analyzed and utilized to modify the tracking models in order to make the models more suitable for tracking the HGVs.

The purpose of aerodynamic modeling is to find some characteristic parameters which can be used to design the tracking models. The aerodynamic model should ensure the accuracy as well as the efficiency because the HGVs are with super velocities and the tracking models must perform well in the real-time tasks. In this section, the simplified aerodynamic configuration of a certain HGV is given to make the model more efficient and two hypersonic mechanical theories are used to calculate the pressure distributions of the vehicle. Consequently the characteristic parameters can be determined according to the process of the calculations.

The Hypersonic Technology Vehicle 2 (HTV-2) is a typical example of the HGVs and is taken as the research object of this work. The simplified aerodynamic configuration of the HTV-2 is shown in Figure 1.

The air is assumed to be the perfect gas. To calculate the pressure distributions on the aircraft in the hypersonic flow, the oblique shock theory and the Prandtl-Meyer expansion flow theory are utilized [14]. The vehicle in the hypersonic flow is shown in Figure 2.

Here M_∞ is the Mach number of the freestream, α is the attack angle, δ is the flow turn angle, θ_s is the shock angle, τ_u and τ_l are the structural bending angles.

On the windward side, the flow is compressed and the oblique shock occurs. Let the pressure behind the oblique shock and the pressure of the freestream be denoted as P_s and P_∞, respectively, the relationship between P_s and θ_s can be addressed as

$$P_s = P_\infty \left[1 + \frac{2\gamma}{\gamma+1}(M_\infty^2 \sin^2\theta_s - 1)\right], \tag{6}$$

where $\gamma = 1.4$ is the specific heat of the perfect gas.

The pressure of the freestream can be calculated according to the standard atmosphere model, and the shock angle can be calculated by the following equation:

$$\sin^6\theta_s + b\sin^4\theta_s + c\sin^2\theta_s + d = 0, \tag{7}$$

where

$$\begin{cases} b = -\dfrac{M_\infty^2+2}{M_\infty^2} - \gamma\sin^2\delta \\ c = \dfrac{2M_\infty^2+1}{M_\infty^4} + \left(\dfrac{(\gamma+1)^2}{4} + \dfrac{\gamma-1}{M_\infty^2}\right)\sin^2\delta \\ d = -\dfrac{\cos^2\delta}{M_\infty^4}. \end{cases} \tag{8}$$

By setting $s = \sin^2\theta$, Equation (7) can be considered as a cubic equation for s. Generally, there are three roots for this cubic equation, the smallest root is physically unrealizable, while the largest root corresponds to the strong shock and it is difficult for the back pressures to support it. Thus these two roots are discarded, and the remaining root is utilized to calculate θ_s. Once θ_s is determined, P_s can be calculated by Equation (6).

On the leeward side, the flow is expanded and the pressure after the expansion can be calculated according to the Prandtl-Meyer expansion flow theory. Set M_1 be the Mach number of the flow after the expansion, the Prandtl-Meyer equation is given by

$$v(M_1) = v(M_\infty) + \delta, \tag{9}$$

where

$$v(M) = \sqrt{\frac{\gamma+1}{\gamma-1}}\arctan\sqrt{\frac{\gamma-1}{\gamma+1}(M^2-1)} - \arctan\sqrt{M^2-1}. \tag{10}$$

The physical meaning of $v(M)$ is the turn angle through which the flow is expanded from Mach 1 to Mach M. Thus there will be a maximum turn angle as $M \to \infty$, $v_{\max}$ is calculated to be 130.4° according to Equation (10). This implies that the solution of Equation (9) may not be convergent when M_∞ is big enough. To solve this problem, when the right side of Equation (9) is greater than $v_{\max}$, set $v(M)$ be slightly smaller than $v_{\max}$ which corresponds to the high Mach number and the low air pressure, and the leeward side is close to vacuum. Let P_o denote the pressure after the expansion, once M_1 is determined, P_o can be calculated by

$$P_o = P_\infty \left\{\frac{1+[(\gamma-1)/2]M_\infty^2}{1+[(\gamma-1)/2]M_1^2}\right\}^{\gamma/(\gamma-1)}. \tag{11}$$

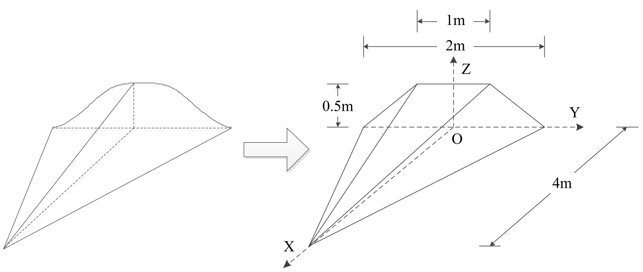

FIGURE 1. Simplified configuration of the HTV-2.

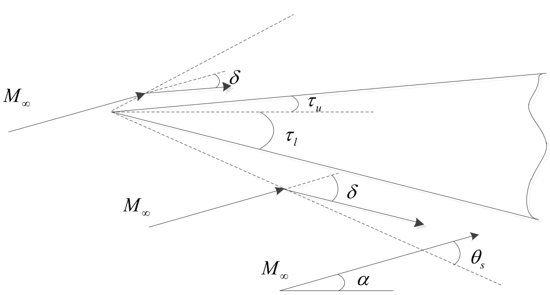

FIGURE 2. Vehicle in hypersonic flow.

According to the configuration of the vehicle in Figure 1 and the pressures calculated by Equation (6) and Equation (11), the aerodynamic force on the vehicle can be calculated. Then the accelerations in the VTC coordinates can be further determined by the attitude and the mass of the vehicle. The sideslip angle of the HGVs is generally supposed to be zero, thus the attitude is determined by the attack angle and the bank angle.

Therefore, the accelerations of the vehicle can be denoted as the functions of the target state $\boldsymbol{X}_k$, the attack angle α, the bank angle β, and the mass m, that is

$$\begin{cases} a_{d,k} = f_1(\boldsymbol{X}_k, \alpha, \beta, m) \\ a_{t,k} = f_2(\boldsymbol{X}_k, \alpha, \beta, m) \\ a_{c,k} = f_3(\boldsymbol{X}_k, \alpha, \beta, m). \end{cases} \tag{12}$$

The specific expressions of f are complicated, hence only the non-analytical equations are given. Equation (12) shows that the mode space is determined by the parameters α, β, and m, which are chosen to be the characteristic parameters to design the model-set in the following section. Therefore, with the given aerodynamic modeling method, the design of tracking models translates into the appropriate description of the parameters α, β, and m, hence the tracking models can be modified by the aerodynamic characteristics of the vehicles.

IV. TRACKING MODEL-SET DESIGN FOR HGVS

If the dynamics information of the vehicle is totally unknown to the trackers, the kinematic model will be a better choice for tracking. In this paper, it is supposed that the trackers can obtain the basic configuration information of the HGVs from the public reports or the existing observations, and the vehicles can be successfully classified by the early warning system when it is non-cooperative. Therefore, the simplified configuration and the aerodynamic model in Section III are available and the tracking model-set can be established as follows.

A. ANALYSIS OF CHARACTERISTIC PARAMETERS

Equation (12) shows that the mode space of the vehicle is determined by three characteristic parameters. The model-set design method should be aimed at approximating the mode space effectively. The distributions of the characteristic parameters are unknown, thus they are considered to be uniformly distributed and the representative points can be selected using the number theoretic method. Before selecting the representative points, the ranges of the characteristic parameters are given first, and the ranges are appropriately extended considering the inaccuracy of the aerodynamic model built in Section III.

The ranges of attack angle and bank angle for the HGVs are not explicit, [8] set these ranges as $\alpha \in [6°, 12°]$ and $\beta \in [-20°, 20°]$, respectively. To obtain more suitable ranges for the aerodynamic model in Section III, the capability of this model is analyzed in the following.

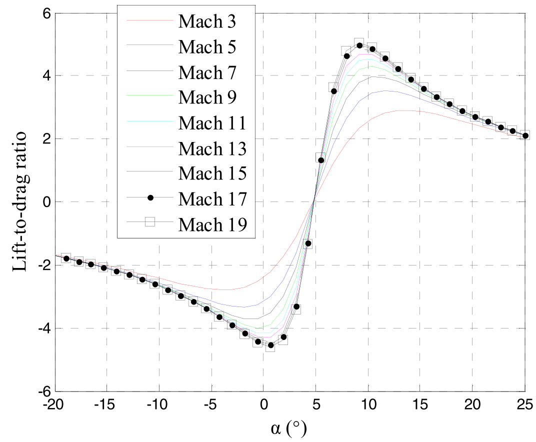

FIGURE 3. Relationship between attack angle and lift-to-drag ratio.

To obtain the range of the attack angle, the relationship between the attack angle and the lift-to-drag ratio with different Mach numbers is analyzed in Figure 3. The height is set to be 30 kilometers, the hypersonic aerodynamics engineering calculation method and the configuration in Figure 1 are used to calculate the lift-to-drag ratio. It is reasonable that the HGVs should keep high level of lift-to-drag ratio in most conditions to acquire plenty of lift force and maintain a long range glide in the near space,

In order to obtain the range of the bank angle, the reachable footprint of the vehicle with different bank angles is shown in Figure 4. The HGVs are supposed to adopt the bank-to-turn control technology. It can be seen that the value of bank angle can obviously affect the glide distance of the vehicle, thus the bank angle should be constrained at most of the time to ensure a long range glide.

Considering the above simulation results and the ranges given in [8], the ranges of attack angle and bank angle are set as $\alpha \in [7°, 20°]$ and $\beta \in [-30°, 30°]$, respectively. In fact, the real change laws of α and β for the HTV-2 are confidential and the exact ranges are unavailable. The value of α and β may be out of the given ranges at some time in a real case.

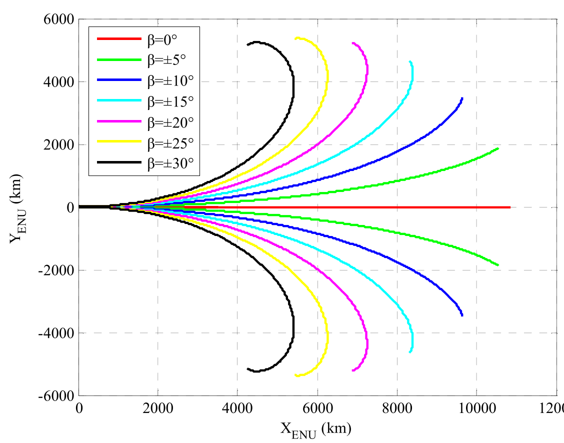

FIGURE 4. Reachable footprint of the vehicle.

However, for most of the time, α and β should be within the given ranges so that the HTV-2 can make long range glide flights. Therefore, the given approximate ranges are sufficient for designing tracking model-set as they can match the real motion mode of the vehicle in most cases. In addition, the interactive multiple model (IMM) algorithm which will be used in Section V is a robust tracking method and it can adjust the probability of each model in the model-set to match the real motion mode as far as possible [12].

According to the existing reports, the mass of the HTV-2 is roughly 450 *kg*. In general, it is difficult to get the exact mass of a non-cooperative HGV and the range of m should be appropriately extended to contain the real value. In this paper, the range of the mass is set to be $m \in [150kg, 750kg]$ to ensure that the real mass of the HTV-2 is within this range.

B. MODEL-SET DESIGN

The uniform model-set can be established according to the number theoretic method, and to make the model-set match the real motion mode better, the model-set is then augmented by the mode estimation at each time point.

The characteristic parameters are supposed to be uniformly distributed without prior distribution information of the mode space. The mode space should be well represented by the designed model-set, which means that the uniformity of the model-set should be high and the distance between the mode space and the model-set should be small, so that the physical behaviors of the vehicle can be described precisely. The model-set with the minimum discrepancy is called the uniform model-set, and the number theoretic method is an effective approach to generate a uniform model-set [15]. The number theoretic method is a mathematical method to uniformly extract discrete points from the numerical space. And it is used to determine the representative points from the given characteristic parameters in the following, thus the uniform model-sets are established.

Table 1 and Table 2 show two examples of uniform model-sets with 6 models and 12 models according to the number theoretic method, respectively.

Based on the designed model-sets, the IMM algorithm and the unscented Kalman filter (UKF) are adopted in the tracking algorithm, hence the trackers can get the state estimation at each time point. In the IMM algorithm, the probability of each model is updated in real time, and the mode estimation

TABLE 1. Uniform model-set with 6 models.

Model	Para. 1	Para. 2	Para. 3
1	1	3	2
2	2	6	4
3	3	2	6
4	4	5	1
5	5	1	3
6	6	4	5

TABLE 2. Uniform model-set with 12 models.

Model	Para. 1	Para. 2	Para. 3
1	1	4	3
2	2	8	6
3	3	12	9
4	4	3	12
5	5	7	2
6	6	11	5
7	7	2	8
8	8	6	11
9	9	10	1
10	10	1	4
11	11	5	7
12	12	9	10

at time $k-1$ can be given by

$$m_{k-1}^{N+1} = \sum_{n=1}^{n=N} p_{k-1}^{n} \cdot m^{n}, \tag{13}$$

where m_{k-1}^{N+1} is the mode estimation, N is the cardinality of the uniform model-set, p_{k-1}^{n} is the probability of each model, n is the serial number of the models, m^n is one of the source models generated according to the number theoretic method.

Compared with the designed model-sets, the statistical distance between the mode estimation and the real mode is shorter. Thus the model-set at time k can be augmented by

$$\boldsymbol{M}_k = \boldsymbol{M}' \cup m_{k-1}^{N+1}, \tag{14}$$

where $\boldsymbol{M}_k$ is the augmented model-set, $\boldsymbol{M}'$ is the source models which ensure the stability of the tracking system. The mode estimation m_{k-1}^{N+1} improves the accuracy of the model-sets.

V. SIMULATIONS AND DISCUSSIONS

In this section, the performance of the proposed multiple model tracking method for the HGVs is compared with the traditional method via two simulation examples shown in Figure 5 and Figure 6. The attack angle of the vehicle is set to be 10° so that the vehicle can get nearly maximal lift-to-drag ratio. And the change laws of the bank angle are given as follows.

Trajectory 1: the bank angle is changing as a sine wave, the mean value is 5°, the amplitude is 20° and the period is 100s.

Trajectory 2: the bank angle is changing as a square wave, the mean value is −5°, the amplitude is 20° and the period is 100s.

It is supposed that the vehicle can successfully change its attitude in a very short time. The change laws of the bank angle above ensure that the trajectories contain various maneuvering modes. The air is assumed to be the perfect gas and the position of the vehicle can be seen from Figure 5 and Figure 6.

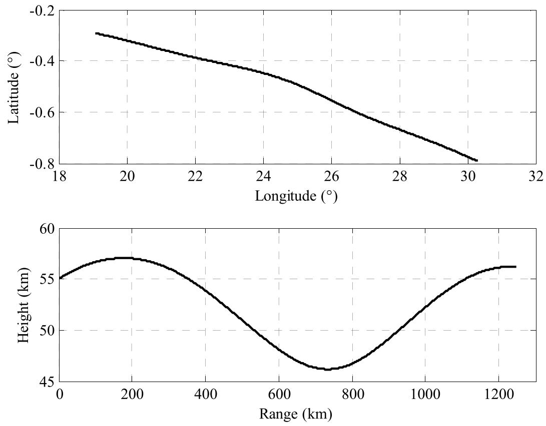

FIGURE 5. Trajectory 1 for the vehicle.

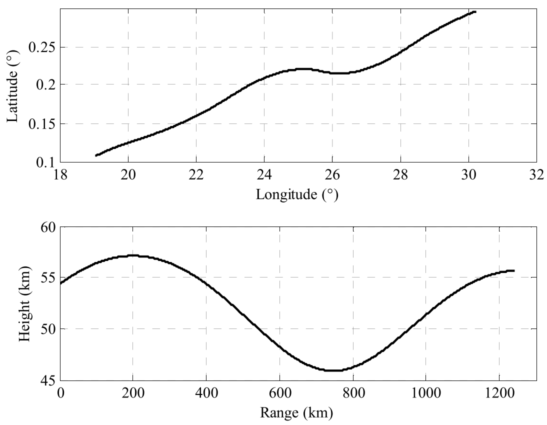

FIGURE 6. Trajectory 2 for the vehicle.

The performance of the proposed tracking method is compared with the traditional dynamic pressure based tracking method presented in Section II. To ensure the accuracy of the dynamic pressure model, the range of the coefficient ξ in Equation (5) is calculated based on the trajectories given in Figure 5 and Figure 6 and the hypersonic aerodynamics engineering calculation method given in Section III. Let $[\xi_d, \xi_t, \xi_c]$ denote the coefficient ξ in the Velocity-Turn-Climb (VTC) coordinates The results are as follows: $\xi_d \in [-7\times10^{-5}, -8\times10^{-6}]$, $\xi_t \in [-6\times10^{-5}, 6\times10^{-5}]$, $\xi_c \in [5\times10^{-5}, 3\times10^{-4}]$.

Set the longitude, latitude and height of the radar in the geodetic coordinates be 26.27°, 3.60° and 100 m, respectively. The observation at time k can be modeled by

$$\begin{aligned}\boldsymbol{Z}_k &= \boldsymbol{H}(\boldsymbol{X}_k) + \boldsymbol{\varepsilon}_k \\ &= \begin{bmatrix} \sqrt{x_k^2 + y_k^2 + z_k^2} \\ \arctan\left(z_k \Big/ \sqrt{x_k^2 + y_k^2} \right) \\ \arctan\left(y_k \big/ x_k \right) \end{bmatrix} + \begin{bmatrix} \varepsilon_{r,k} \\ \varepsilon_{\theta,k} \\ \varepsilon_{\varphi,k} \end{bmatrix}, \end{aligned} \tag{15}$$

where $\boldsymbol{Z}_k$ is the observation, $[\varepsilon_{r,k}, \varepsilon_{\theta,k}, \varepsilon_{\varphi,k}] \sim N(\cdot; 0, \boldsymbol{R}_k)$ is the measurement noise, $\boldsymbol{R}_k = \mathrm{diag}\left(\left[(30\mathrm{m})^2, (0.1°)^2, (0.1°)^2\right]\right)$.

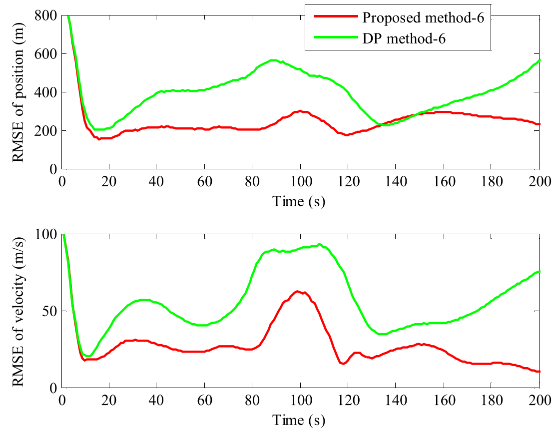

FIGURE 7. RMSE for the proposed and DP based tracking methods with 6 source models for Trajectory 1.

200 Monte Carlo runs are performed for the proposed tracking method and the dynamic pressure based tracking method (labeled as DP method in the figures). The root mean square errors (RMSE) of the position and the velocity estimations are used to evaluate the performance of each method.

Figure 7 and Figure 8 show the simulation results for Trajectory 1, the results indicate that the proposed tracking method outperforms the dynamic pressure based tracking method in both position and velocity estimations. The tracking errors for these two methods with 6 source models are shown in Figure 7 and the tracking errors for these two methods with 12 source models are shown in Figure 8. The source models have been given in Table 1 and Table 2. It can be seen that the proposed tracking method with 6 source models and that with 12 source models have roughly the same accuracy, whereas the dynamic pressure based tracking method with 12 source models performs better than that with 6 source models. More models may not always improve the tracking accuracy because of the possible model competition, whereas more models generally lead to more calculations. The RMSE curves grow and reach a wave crest between about 80s to 120s, this is due to the fact that

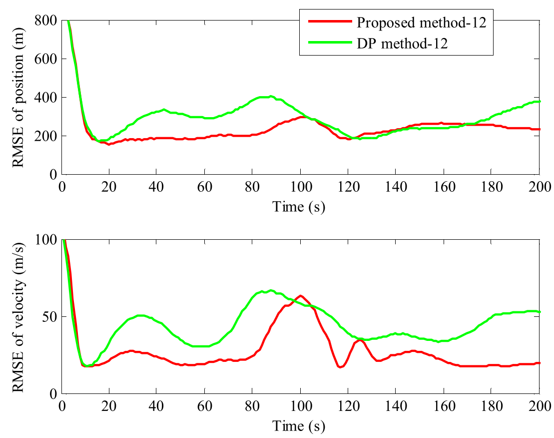

FIGURE 8. RMSE for the proposed and DP based tracking methods with 12 source models for Trajectory 1.

TABLE 3. RMSE for each tracking method.

Trajectory \ Method		Proposed method with 6 source models	Proposed method with 12 source models	DP based method with 6 source models	DP based method with 12 source models
Trajectory 1	RMSE of position (m)	234.83	222.92	407.31	289.43
	RMSE of velocity (m/s)	26.04	26.38	58.78	45.20
Trajectory 2	RMSE of position (m)	204.70	196.88	434.81	281.95
	RMSE of velocity (m/s)	27.15	27.36	58.75	48.10

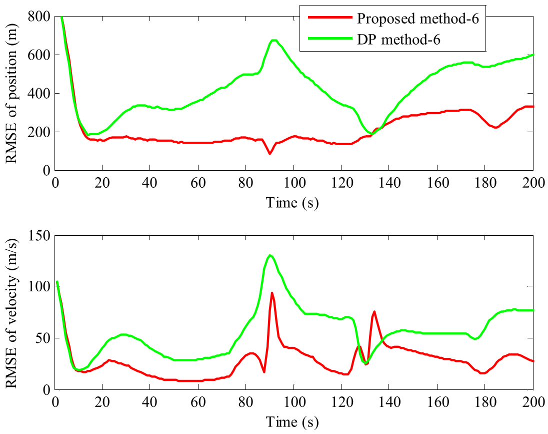

FIGURE 9. RMSE for the proposed and DP based tracking methods with 6 source models for Trajectory 2.

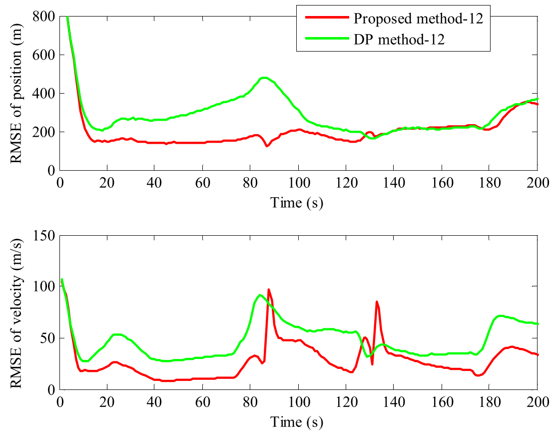

FIGURE 10. RMSE for the proposed and DP based tracking methods with 12 source models for Trajectory 2.

during this period, the height of the vehicle is relatively low and the atmospheric pressure and density get larger, thus the aerodynamic force has greater impact on the vehicle which leads to the inaccuracy in aerodynamic modelling.

Figure 7 and Figure 8 also show that the RMSE of the dynamic pressure based tracking method become larger at the last period of the simulations, this is because the vehicle is further away from the radar during this period and the observation errors increase. Therefore, the dynamic pressure based method is more sensitive to the observation errors compared with the proposed tracking method.

Figure 9 and Figure 10 show the simulation results for Trajectory 2. The results indicate that the proposed tracking method still performs better than the dynamic pressure based tracking method in both position and velocity estimations. For Trajectory 1, the bank angle of the vehicle keeps changing all the time but it changes gently, while for Trajectory 2, the bank angle only changes at several time points but the changes are violent and will greatly affect the tracking accuracy. The velocity estimation accuracy is more sensitive to the sudden changes of the bank angle because the vehicle attitude strongly influences the force on the vehicle. It can be seen that the RMSE curves of the velocity grow larger about every 50 seconds which correspond to the change period of the bank angle for Trajectory 2.

Table 3 gives the value of the time-averaged RMSE for each tracking method.

The results indicate that the proposed tracking method achieves a better accuracy and increases position estimation accuracy by about 23%–53% and velocity estimation accuracy by about 42%–56% in the given examples.

VI. CONCLUSIONS AND FUTURE WORK

The key to establish the traditional motion models for the reentry targets lies in the appropriate modelling of the dynamic pressure, which is not suitable for the HGVs because the dynamic characteristics of the vehicles and the atmospheric environment are different. In this paper, a multiple model tracking method for the HGVs with aerodynamic modelling and analysis is proposed. Compared with the traditional dynamic pressure models, the proposed method establishes more suitable and accurate motion models for the HGVs by analyzing the aerodynamic characteristics of the vehicles in the near space. The main work of the paper can be concluded as follows.

1) The aerodynamic modeling method for the HGVs is proposed. The aerodynamic force of the vehicle is calculated according to its simplified configuration and the hypersonic aerodynamics engineering calculation method. The characteristic parameters are then decided.

2) Based on the aerodynamic model, the novel tracking model-sets are established. The characteristic parameters are constrained by analyzing the capability of the aerodynamic model and the model-sets are designed according to the

number theoretic method. The model-sets are also augmented to further match the real motion mode of the vehicle.

3) Simulations are conducted to compare the performance of the proposed tracking method and the dynamic pressure based tracking method. The results show that the proposed method performs better in both position and velocity estimations.

To make the tracking models more suitable for the HGVs, the aerodynamic models of the vehicles should be precise enough. However, a complex model may lead to huge quantity of calculation and poor efficiency. The efficiency of the tracking method for the HGVs is very important because of the super velocities of the vehicles. Therefore, a topic of the future work is to establish more precise and efficient aerodynamic models for the HGVs.

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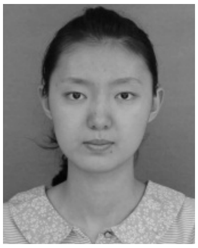

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