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Weapon-Target Assignment for Multi-to-Multi Interception With Grouping Constraint

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ABSTRACT This paper presents a weapon-target assignment (WTA) method for a multi-to-multi interception with fixed and adaptive grouping constraints. First, to get a better evaluation of the interception performance, the miss-distance under heading error, the time-to-go, and the line-of-sight rate are used to construct an interception probability function which considers the interception efficiency and the required energy. Second, to provide good engagement conditions for cooperative guidance in the case of multiple missiles against multiple targets, and meanwhile, to ensure that each target is allocated with appropriate interception resources, a fixed grouping strategy and an adaptive grouping strategy based on penalty function are proposed for the WTA problem. Then, the artificial bee colony algorithm is employed to solve the WTA problem with grouping constraints. Finally, two scenarios of multi-to-multi interception are simulated to verify the proposed WTA method. Results indicate that the method can realize the optimal allocation schemes which satisfy both the fixed and the adaptive grouping constraints.

INDEX TERMS Weapon-target assignment, grouping constraint, cooperative attack, multi-to-multi interception, artificial bee colony algorithm.

I. INTRODUCTION

With rapid development of detection and defense technology, the traditional attack mode based on single missile is difficult to meet the combat requirements, which leads to the emergence of multi-missiles cooperative operations. The cooperative operation of multi-missiles can be realized through pre-setting the combat requirements or conducting the real-time inter-missile communication. Multi-missiles with cooperative guidance can attack target simultaneously from different directions, which poses a serious threat to the current defense system [1], [2]. Therefore, in the defender's view, an effective multi-to-multi interception is required. On one hand, the number of missiles should be no less than that of the targets. On the other hand, the interceptors should be reasonably allocated to the targets.

In recent years, extensive research on the cooperative guidance problem has been carried out. Jeon *et al.* [3] proposed a homing guidance law with impact-time constraint based on suboptimal control theory, which could realize salvo attack for fixed target. In further research, Kim *et al.* [4] developed an augmented polynomial guidance law that can satisfy both the impact-time and angle constraints. Based on the consistency theory, a cooperative guidance law with fixed and switching communication topology was designed [5]. For the condition with large initial heading error, Kumar and Ghose [6] investigated an impact-time constraint guidance law based on sliding mode control (SMC) theory. In order to attack maneuvering targets, Zhao *et al.* [7], [8] proposed two cooperative guidance laws based on proportion navigation guidance (PNG) with the

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coordination variable time-to-go. For achieve the simultaneous arrival of multiple missiles in the handover position, a cooperative mid-course guidance law with terminal handover constraints was proposed [9]. For maneuvering targets, a three-dimensional cooperative guidance law with fixed impact angle constraints was designed based on finite-time observer [10]. Zhao *et al.* [11] proposed a cooperative guidance law based on extended state observer for intercepting maneuvering target. A three-dimensional cooperative guidance law, which can control the miss-distance and impact-time for moving targets was designed [12]. For the case of multiple inferior missiles against a highly maneuvering target, Su *et al.* [13] developed a cooperative interception strategy. In [14], the cooperative interception can be achieved by optimizing and coordinating the reachable sets of different interceptors. These guidance methods are all for multi-missiles intercepting one target. For the case of multi-to-multi interception, there is a great difference in the engagement scenario among the different missiles and targets. To intercept all the targets, missiles must be grouped in a reasonable way. If the grouping result is not appropriate, the cooperative interception would be difficult. For example, consider that a missile with large initial heading error is allocated to a target. Then, a large error of the coordination variable may exist during the whole course, which may lead to the failure of cooperative interception. Therefore, to improve the combat performance of the multi-to-multi interception, a weapon-target assignment (WTA) method is required. The WTA with multiple constraints is a typical discrete NP-complete problem. The scale of solution space increases exponentially with the number of missiles and targets.

At present, the solving method of WTA problem can be divided into two categories. The first category derives from the traditional solving method, such as the Lagrange relaxation method and the exhaustive method [15], [16]. For cooperative air combat, a WTA method with time series constraints was presented and the traditional integer programming method was used to solve the problem [17]. These methods can effectively solve the small-scale WTA problem. When the number of missiles and targets is large, the solving efficiency would decrease sharply. The second strategy, which has been widely adopted, employs heuristic optimization algorithm. To minimize the expected damage of own-force asset, Lee *et al.* [18] proposed a WTA method based on genetic algorithm and greedy criterion. To solve asset-based dynamic weapon-target assignment problems, an efficient rule-based heuristic was proposed in [19]. For WTA problem of multi-to-multi interception, a WTA method is presented based on the geometric relationship between missile and target, and the discrete particle swarm optimization (PSO) algorithm is used to solve this problem [20]. In [21], the WTA problem for multi-to-multi interception was constructed by the interception revenue model and solved by the particle swarm optimization algorithm. Two heuristic algorithms based on simulated annealing and threshold acceptance were developed [22]. Based on the ant colony algorithm and its improvement, the WTA problem was investigated in [23] and [24]. The heuristic optimization algorithms introduced can adapt to large-scale WTA problems, but the premature stagnation phenomenon always exists, which results in the low computational efficiency. Therefore, in order to improve the robustness and the global convergence for solving large-scale optimization problems, Lee *et al.* [25] systematically proposed an approach based on the artificial bee colony (ABC) algorithm. Then, some scholars improved the ABC algorithm, which significantly enhanced the search speed and the solution accuracy [26], [27]. Results show that the ABC algorithm has simple concept, few control parameters, good convergence, and easy implementation. Most of the existing WTA methods can take the group combat effectiveness as the optimization objective. However, it would be easily to get the optimal solution, but may lead some targets being allocated too many or few interceptors [21]. In the multi-to-multi interception, the threat level of a target varies with the engagement scenario and the battlefield value. Therefore, to ensure that each target is allocated with sufficient interception resources, a reasonable grouping strategy that considers the number of missiles and the engagement scenario condition is required.

This study focuses on the WTA problem with grouping constraints for multi-to-multi interception. The proposed grouping strategy considers the allocation number or the upper/lower allocation limit to each target according to the battlefield value, combat requirements and other factors, which can enhance the operation effectiveness while ensuring interception requirements. The contributions of this paper are as follows: 1) Compared with the existing WTA methods [15]–[24], the interception probability function proposed in this paper takes into account both the interception efficiency and the required energy. By selecting three factors: miss-distance, time-to-go, and line-of-sight (LOS) rate as the main indicators of the interception probability function, the interception efficiency can be better evaluated. 2) To ensure that each target is allocated with sufficient interception resources and provide a good condition for the cooperative guidance, a fixed and an adaptive grouping constraints are investigated for the WTA problem. 3) The ABC algorithm with fast convergence speed and good global performance is adopted to solve the proposed WTA problem with grouping constraint, and good allocation results can be obtained.

The paper is structured as follows. In Section 2, the interception probability function is designed after analyzing the guidance geometry. In Section 3, the WTA methods with fixed and adaptive grouping are proposed. Section 4 gives the WTA solving method based on the ABC algorithm. In Section 5, numerical simulation is carried out, and the results under two grouping strategies are analyzed and compared. Finally, Section 6 summarizes the whole work.

II. MODELING

To achieve an optimal intercept efficiency for the multi-to-multi interception, interceptors need to be reasonably allocated at the beginning of terminal guidance phase.

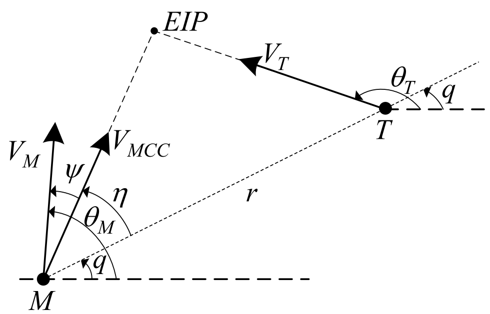

FIGURE 1. Guidance geometry.

Miss-distance is an important factor to measure whether a missile can intercept a target effectively. In terminal guidance phase, the interception effectiveness of the missile can be measured by evaluating the miss-distance at current time according to the engagement scenario. The time-to-go, another importance factor, is the time required for the missile to intercept the target. Smaller time-to-go means that less flight energy is required, and the interception is more favorable. In addition, most of the homing-guidance laws achieve interception by converging the LOS rate. Smaller the LOS rate means that less normal acceleration and flight energy are needed, which is expected by the interception. Therefore, the miss-distance, the time-to-go, and the LOS rate are selected as the indicators to construct the interception probability function.

A. GUIDANCE SYSTEM

The guidance geometry model for interception is shown in Fig. 1, where q is the LOS angle, and r is the relative distance between the missile and the target. V_M and V_T are velocities of the missile and the target, respectively. *EIP* is the expected intercept point under the collision course. V_{MCC} is the velocity vector which satisfies the collision triangle condition. θ_M and θ_T are heading angles of the missile and the target, respectively. η is the lead angle between V_{MCC} and the LOS:

$$\eta = \arcsin\left[\frac{V_T}{V_M}\sin(\theta_T - q)\right] \tag{1}$$

The heading error ψ is defined as the error between the missile's velocity and V_{MCC}, which is given by:

$$\psi = \theta_M - q - \eta \tag{2}$$

A heading error represents that the missile's heading deviates from the collision triangle. The magnitude of heading error affects the miss-distance.

The Adjoint system is widely used in the design and analysis of guidance system due to the advantages of accuracy and calculation speed. The adjoint homing loop of a single-lag guidance system is shown in Fig. 2. The target acceleration and the heading error are two inputs of the homing loop. At the time of interception $t = t_f$, the output of the homing loop is the miss-distance ΔS.

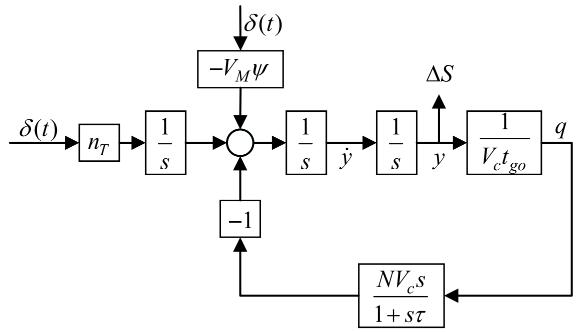

FIGURE 2. Adjoint system of homing loop.

In the homing loop, τ is time constant of the first-order lag element, and t_{go} is approximately given by

$$t_{go} \approx -\frac{r}{V_c} \tag{3}$$

where V_c denotes the closing speed of the missile and the target, which is given by

$$V_c = V_T\cos(\theta_T - q) - V_M\cos(\theta_M - q) \tag{4}$$

The guidance system employs the proportional navigation guidance (PNG) law, which is given by [29]

$$n^c = NV_M\dot{q} \tag{5}$$

where N is the navigation constant, and $\dot{q}$ is the LOS rate.

Assume that the target is not maneuverable and $N = 4$ is utilized by the PNG. Then, the miss-distance can be calculated by [29]

$$\Delta S = \left|\psi V_M t_{go} e^{-\frac{t_{go}}{\tau}}\left(1 - \frac{t_{go}}{\tau} + \frac{t_{go}^2}{6\tau^2}\right)\right| \tag{6}$$

When intercepting a moving target, the traditional time-to-go calculation method for stationary target is difficult to achieve an accurate estimation [30]. In order to improve the calculation accuracy and evaluate the required flight energy effectively, the time-to-go estimation method [31] is modified for moving targets and expressed as

$$T_{go} = \frac{r}{V_M}\left[1 + \frac{\sin^2(\theta_M - q)}{2(2N-1)} + \frac{3\sin^4(\theta_M - q)}{8(4N-3)} + \frac{5\sin^6(\theta_M - q)}{16(6N-5)} + \frac{35\sin^8(\theta_M - q)}{128(8N-7)}\right]\left(\frac{V_M}{V_M + V_T}\right) \tag{7}$$

B. INTERCEPTION PROBABILITY

The interception probability considers three indicators and each indicator has complex impacts. For instance, consider the LOS rate. Under a certain initial LOS rate, a large r is beneficial to the missile for correcting its heading error and enhancing the interception probability but may result in a large time-to-go. When r is small, the missile may be unable to enter the collision triangle and then miss the target. Therefore, these interception probability indicators are mutually reinforcing and balancing each other. In order to achieve the maximization of combat effectiveness, each indicator needs to be compromised, which also reflects the cooperative operation between the missiles.

The miss-distance is selected as an indicator to evaluate the interception efficiency. The time-to-go for moving target and the LOS rate are taken as indicators to evaluate the required energy for interception. For the terminal guidance phase, the smaller these three indicators are, the greater the interception probability is. The negative exponential function is used to define the interception probability function under three indicators:

$$\begin{cases} P_{\Delta S(i,j)} = P_0^{\Delta S} e^{-\frac{1}{2}\left(\frac{\Delta S(i,j)}{\delta_{\Delta S}}\right)^2} \\ P_{T_{go}(i,j)} = P_0^{T_{go}} e^{-\frac{1}{2}\left(\frac{T_{go}(i,j)}{\delta_{T_{go}}}\right)^2} \\ P_{\dot{q}(i,j)} = P_0^{\dot{q}} e^{-\frac{1}{2}\left(\frac{\dot{q}(i,j)}{\delta_{\dot{q}}}\right)^2} \end{cases} \tag{8}$$

where i and j represent the label of the missile and the target, respectively. The miss-distance $\Delta S(i,j)$, the time-to-go $T_{go}(i,j)$, and LOS rate $\dot{q}(i,j)$ denote the three indicators in the case of the ith missile intercepting the jth target. $P_{\Delta S(i,j)}$, $P_{T_{go}(i,j)}$, and$P_{\dot{q}(i,j)}$ are the interception probabilities corresponding to the three indicators, with $P_0^{\Delta S}$, $P_0^{T_{go}}$, and $P_0^{\dot{q}}$ being the default values. It can be seen that $P_{\Delta S(i,j)}$, $P_{T_{go}(i,j)}$, and $P_{\dot{q}(i,j)}$ would increase when the of corresponding indicators decrease. $\delta_{\Delta S}$, $\delta_{T_{go}}$, and $\delta_{\dot{q}}$ are the average interception probabilities that all m missiles intercepting n targets, which is given by

$$\begin{cases} \delta_{\Delta S} = \dfrac{\sum_{j=1}^{n}\sum_{i=1}^{m}\Delta S(i,j)}{mn} \\ \delta_{T_{go}} = \dfrac{\sum_{j=1}^{n}\sum_{i=1}^{m}T_{go}(i,j)}{mn} \\ \delta_{\dot{q}} = \dfrac{\sum_{j=1}^{n}\sum_{i=1}^{m}\dot{q}(i,j)}{mn} \end{cases} \tag{9}$$

where m is the number of missiles, and n is the number of targets. The multi-to-multi interception generally requires $m > n$. In Eq. (8), one can get $P_{\Delta S(i,j)} = 0.61\ P_0^{\Delta S}$ for the case $\Delta S(i,j) = \delta_{\Delta S}$ (similarly results for $P_{T_{go}(i,j)}$ and $P_{\dot{q}(i,j)}$).

With the three indicators, the interception probability is defined by:

$$P_{ij} = \beta_{\Delta S} P_{\Delta S(i,j)} + \beta_{T_{go}} P_{T_{go}(i,j)} + \beta_{\dot{q}} P_{\dot{q}(i,j)} \tag{10}$$

where $\beta_{\Delta S}$, $\beta_{T_{go}}$, and $\beta_{\dot{q}}$ are the weights of the three interception probabilities, which satisfy

$$\beta_{\Delta S} + \beta_{T_{go}} + \beta_{\dot{q}} = 1 \tag{11}$$

In general, the miss-distance is an important indicator to evaluate whether the missile hits the target, and also an important indicator to evaluate the feasibility of interception. Therefore, $\beta_{\Delta S}$, should be greater than $\beta_{T_{go}}$ and $\beta_{\dot{q}}$.

III. WTA METHOD WITH GROUPING CONSTRAINT

A. WTA PROBLEM FOR MULTI-TO-MULTI INTERCEPTION

Let P_{ij} be the interception probability of the ith missile against jth target, and X_{ij} be the allocation result of the ith missile to the jth target. In details, $X_{ij} = 1$ represents that the ith missile is allocated to the jth target, and $X_{ij} = 0$ represents that the ith missile is not allocated to the jth target. The WTA problem is to maximize the total interception probability by determining the allocation scheme for each target, which is given by

$$\min \sum_{j=1}^{n} \left\{ V_j \prod_{i=1}^{m} \left(1 - P_{ij}\right)^{X_{ij}} \right\} \tag{12}$$

where V_j is the battlefield value of the jth target.

According to the operational requirements, each missile must be allocated to one target. Meanwhile, each target should be allocated at least one missile, and at most B_j missiles. Though a target with higher threat level is allowed to be intercepted by multiple missiles, the allocation number should be no greater than the upper limit. When the allocation number to one target exceeds a certain range, the total interception probability may become worse in severe cases. Therefore, the allocation number to each target should be limited. Then, the WTA problem is rewritten by

$$\begin{aligned} &\min \sum_{j=1}^{n} \left\{ V_j \prod_{i=1}^{m} \left(1 - P_{ij}\right)^{Xij} \right\} \\ &\text{s.t} \begin{cases} 1 \le \sum_{i=1}^{n} X_{ij} \le B_j, & j = 1, 2, \ldots, n \\ \sum_{j=1}^{n} X_{ij} = 1, & i = 1, 2, \ldots, m \end{cases} \end{aligned} \tag{13}$$

B. GROUPING STRATEGY

In the interception combat, different targets have different threats levels due to the attribute and the engagement scenario condition. To meet the interception requirements and improve the operational efficiency, it is necessary to synthesize the engagement scenarios and the scale of missiles to achieve grouping attacks on different targets. Most of the existing WTA methods aim at achieving optimal target allocation without grouping constraints. It may lead to the allocation results that some targets are allocated too many missiles and some targets are not allocated to any missiles. In that case, the multi-to-multi interception would fail. The WTA problem is a discrete NP complete problem with multi-parameters and multi-constraints. If the grouping requirements are added, the number of constraints would increase and the problem difficulty would further improved. For a constrained optimization problem, infeasible solutions may be contained in the initial point set and iterative process. Thus, to solve the WTA problem with grouping constraints, the penalty function method is utilized to transform the constrained optimization problem into an unconstrained one. In this subsection, two WTA problems with fixed and adaptive grouping constraints are considered.

1) FIXED GROUPING

For the fixed grouping, the allocation number to each target is set to a fixed value according to the threat level. In this case, the WTA problem needs to satisfy two equality constraints:

$$\begin{cases} \sum_{i=1}^{m} X_{ij} = A_j \\ \sum_{j=1}^{m} X_{ij} = 1 \end{cases} \tag{14}$$

where A_j is the number of missiles allocated to the jth target, which satisfies $\sum_{j=1}^{n} A_j = m$. The second constraint $\sum_{j=1}^{n} X_{ij} = 1$ represent that each missile intercepts only one target during the whole interception phase.

After processing these two equality constraints by penalty function method, the new objective function can be obtained:

$$\min \left(\sum_{i=1}^{m} \left\{ V_j \prod_{i=1}^{m} (1 - P_{ij})^{X_{ij}} + Q_j G_j \right\} + \sum_{i=1}^{m} S_i H_i \right) \tag{15}$$

where Q_j and S_i are the penalty factors for the jth target and the ith missile:

$$\begin{cases} G_j = \left| \sum_{i=1}^{m} X_{ij} - A_j \right|, & j = 1, 2, \ldots, n \\ H_j = \left| \sum_{j=1}^{n} X_{ij} - 1 \right|, & i = 1, 2, \ldots, m \end{cases} \tag{16}$$

In Eq. (15), when the penalty function terms $Q_j G_j$ and $S_i H_i$ are both equal to 0, the objective function with fixed grouping constraint is the same as that of the original WTA problem in Eq. (12). These two terms represent the constraint violation degree for the infeasible solution. The stronger the constraint violation degree is, the higher the objective function is.

2) ADAPTIVE GROUPING

In the multi-to-multi interception scenario, each target should be allocated to an appropriate number of interceptors. For the fixed grouping, the allocation number to each target is only determined by threat level or battlefield value of the target, it may cause the poor objective function value and the low combat effectiveness. Therefore, in order to intercept each target and avoid some targets being allocated to too many or too few interceptors, an adaptive grouping strategy is developed.

The WTA problem should satisfy two inequality constraints and one equality constraint:

$$\begin{cases} \sum_{i=1}^{m} X_{ij} \le B_j, & j = 1, 2, \cdots, n \\ \sum_{i=1}^{m} X_{ij} \ge C_j, & j = 1, 2, \cdots, n \\ \sum_{j=1}^{n} X_{ij} = 1, & i = 1, 2, \cdots, m \end{cases} \tag{17}$$

where B_j and C_j are the maximum and minimum allocation number for the jth target, respectively.

With the penalty function method for these three constraints, the objective function for the adaptive grouping is expressed as

$$\min \left(\sum_{j=1}^{n} \left\{ V_j \prod_{i=1}^{m} (1 - P_{ij})^{X_{ij}} + U_j E_j + L_j F_j \right\} + \sum_{i=1}^{m} S_i H_i \right) \tag{18}$$

where $U_j E_j$, $L_j F_j$, and $S_i H_i$ are the penalty function terms, with L_j and U_j as the penalty function factors of lower and upper bounds for the jth target, respectively. Variables E_j, F_j, and H_i are defined by

$$\begin{cases} E_j = \max \left[0, \sum_{i=1}^{m} X_{ij} - B_j \right], & j = 1, 2, \cdots, n \\ F_j = \max \left[0, C_j - \sum_{i=1}^{m} X_{ij} \right], & j = 1, 2, \cdots, n \\ H_i = \left| \sum_{j=1}^{n} X_{ij} - 1 \right|, & i = 1, 2, \cdots, m \end{cases} \tag{19}$$

Here, the maximum logic is used to construct the penalty function for inequality constraints.

It can be seen that when the solution of the problem does not satisfy the inequality constraints, the value of the new objective function is greater than that of the original problem. According to the minimum penalty value principle, the penalty function factors should be configured by small values. If the penalty function factors are too large, some useful information would be lost, and the solution obtained may not optimal. However, if the penalty function factors are too small, the penalty effect would be poor, and the solution obtained may not satisfy the grouping constraints. Therefore, the setting of penalty function factors should consider information such as the number of missiles and targets, so as to ensure the quality of the allocation results.

C. ABC ALGORITHM FOR WTA SOLVING

In ABC algorithm, the artificial bee colony consists of employed bees, onlooker bees, and scout bees[20]. The solution of the optimization problem and the corresponding function value are abstracted as the location of honey source and the quality of honey. The process of the bee colony searching for the best honey source can be described as follows: Employed bees find and memorize the current honey source, then search for new sources near the old ones and select the better sources according to the honey's quality. After that, all employed bees return to the dance area after completing neighborhood search and share the information, which is proportional to the quality of the honey source. Onlooker bees get the honey source information from employed bees colony, and then select honey source according to the fitness value with certain probability. Similar to that of the employed bees, onlooker

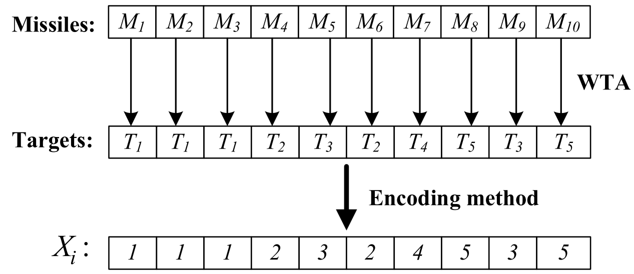

FIGURE 3. **Illustration of encoding method.**

bees also perform a neighborhood search and retain better solutions. However, if the quality of the honey source has not change after several searches, the corresponding employed bees or onlooker bees would become the scout bees. And then the new honey source obtained by random search is used to replace the old one.

The WTA for interception is an integer programming problem. Therefore, individuals in the ABC algorithm need to be coded to establish a mapping with actual problem. When solving WTA problem with ABC algorithm, it is necessary to round the solution X_i in each iteration process, so as to ensure that the allocation results obtained in each step are integer sequences. For m missiles and n targets, each solution X_i is an integer vector of m dimension: $X_i = \left(x_i^1, x_i^2, \ldots, x_i^m\right)$. Here, $x_i^k (k = 1, 2, \ldots, m)$ is an integer between 1 and n, which means that the kth missile is allocated to the target labeled by x_i^k. Assuming $m = 10$ and $n = 5$, an illustration of the integer encoding method is shown in Fig. 3, where $X_i = [1, 1, 1, 2, 3, 2, 4, 5, 3, 5]$ is the encoding result. The allocation results for the interception can be obtained after each honey source updated.

Based on the ABC algorithm, steps for solving the WTA problem with grouping constraints are as follows:

Step 1: Select the grouping strategy (fixed or adaptive) according to the combat requirements, and determine the allocation number or the upper/lower allocation limit to each target according to the battlefield value, combat environment and other factors;

Step 2: Set the population of artificial bee colony as $2S$ and the repeat restriction times of algorithm as *limit*. Randomly generate the bee colony of $2S$ and determine the employed the onlooker bee colonies according to the fitness value. Then, set the optimization record variable *trial* $(i) = 0$;

Step 3: Conduct the local search for each employed bees and calculate the fitness value;

Step 4: If the fitness value of the new solution v_i is better than the old solution x_i, replace x_i with v_i and set *trial* $(i) = 0$; Otherwise, replace *trial* (i) with *trial* $(i) + 1$;

Step 5: Calculate the selection probability p_i for each solution, select new solution for onlooker bees according to p_i, and calculates the corresponding fitness value;

Step 6: If the fitness value of the new solution is better than the old one, replace x_i with v_i and set *trial* $(i) = 0$; Otherwise, replace *trial* (i) with *trial* $(i) + 1$;

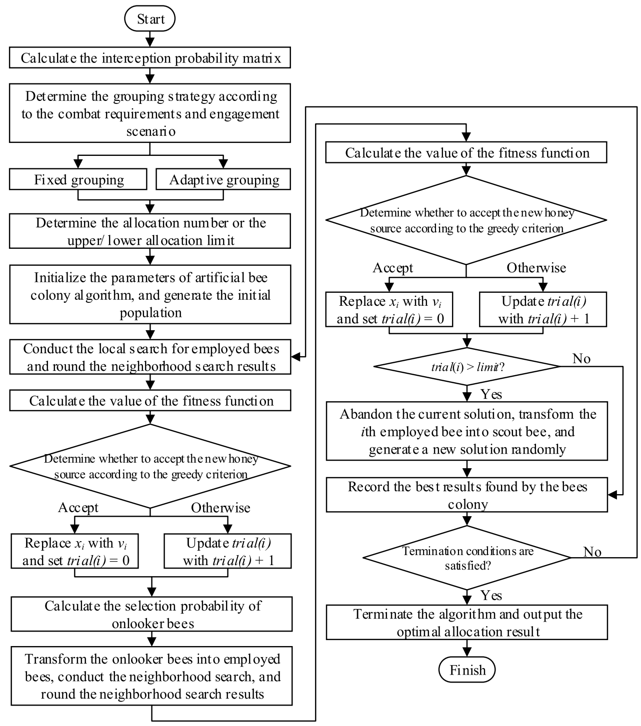

FIGURE 4. **The process for solving WTA problem with grouping constraints under ABC algorithm.**

Step 7: If *trial* (i) is greater than *limit*, go to **Step 8**; Otherwise, go to **Step 9**;

Step 8: Abandon the current solution, change the ith employed bee to a scout bee, and randomly generate a new solution in the solution space;

Step 9: Record the best results found by the bees colony. If the termination conditions are satisfied, end the algorithm and output the result; otherwise, return to **Step 3**.

As shown in Fig. 4, the process for solving the WTA problem with grouping constraints using ABC algorithm is presented.

IV. SIMULATIONS

A. SIMULATION CONDITIONS

The numerical simulations are conducted on a PC with Intel(R) Core (TM) i5-7200U CPU @ 2.50GHz. All the codes have been written under MATALB 2016b. In order to verify the effectiveness of the WTA method with grouping constraint, two cases are considered for the multi-to-multi interception. Suppose that the missile group intercepts the target group in a head-on scheme. The parameters of the 10 missiles (numbered by M1, M2, ..., M10) and the 5 targets (numbered by T1, T2, ..., T5) are shown in Table 1 and Table 2, respectively. In **Case 1**, the 10 missiles are used to intercept 3 targets: T1, T3, and T5. In **Case 2**, all the 5 targets need to be intercepted. The computation complexity for **Case 2** is larger than that of **Case 1**, but since **Case 1** and **Case 2** have different number of targets and different grouping constraints under fixed and adaptive grouping, so the simulation of **Case 1** is necessary. Values of parameters for WTA method are given

TABLE 1. Parameters of the 10 missiles.

Missile	X (km)	Y (km)	Heading angle (deg)	Velocity (m/s)
M1	15	65	-10	750
M2	14	59	5	780
M3	16	54	-25	725
M4	10	50	10	820
M5	4	45	-15	690
M6	6	38	25	765
M7	10	35	2	770
M8	12	25	-25	795
M9	8	18	10	745
M10	11	5	20	755

TABLE 2. Parameters of the 5 targets.

Target	X (km)	Y (km)	Heading angle (deg)	Velocity (m/s)	Value
T1	50	52	170	580	0.85
T2	48	45	175	560	0.69
T3	50	40	195	535	0.72
T4	46	34	170	550	0.95
T5	52	25	182	565	0.80

in Table 3. In Table 4, parameters of the ABC algorithm are given. The dimension of the solution is determined according to the number of missiles. The value of the initial population of bee colonies and the termination condition are selected according to [28]. These values are modified appropriately according to the actual situation of this paper. The repeated restriction times of algorithm are determined through multiple experiments.

B. RESULTS FOR CASE1

According to parameters of the missiles and the targets given in **Case 1**, the interception probability matrix P can be calculated using the method proposed in Section 2. The result is shown in Table 5.

1) FIXED GROUPING

The allocation number to the jth target can be set according to the target's battlefield value and the engagement scenario. In this study, we assume that the allocation numbers to targets T1, T3, and T5 are 4, 4, and 2, respectively, i.e. $A = [4, 4, 2]$.

Figure 5 shows the position and heading angle of the missiles and the targets. The allocation result, which is illustrated by connecting lines, satisfies the fixed grouping constraint. The optimal value of the objective function for the WTA problem are shown in Fig. 6. According to Eq. (15), the smaller the objective function is, the higher the group interception probability is. In Fig. 6, the horizontal axis (cycle number) represents the number of iterations used by the algorithm, and the vertical axis represents the optimal value in the current cycle. The optimal value curve of the objective function converges rapidly. Result indicates that the optimal solution, which satisfies the fixed grouping constraint, has a good computational performance. In order to evaluate the

TABLE 3. Parameter setting for the WTA method.

Parameters	Meaning	Values
N	Navigation constant	4
τ	Time constant of flight control system	1
$P_0^{\Delta S}$	Default interception probability for miss-distance	0.9
$P_0^{T_{go}}$	Default interception probability for time-to-go	0.9
$P_0^{\dot{q}}$	Default interception probability for LOS rate	0.9
$\beta_{\Delta S}$	Weight of intercepting probability of Miss-distance	0.5
$\beta_{T_{go}}$	Weight of intercepting probability of time-to-go	0.25
$\beta_{\dot{q}}$	Weight of intercepting probability of LOS rate	0.25
Qj	Penalty factor for jth target	0.1
Si	Penalty factor for ith missile	0.04
Uj	Upper bound penalty factor for jth target	0.1
Lj	Lower bound penalty factor for jth target	0.1

TABLE 4. Parameter setting for the ABC algorithm.

Parameters	Meaning	Values
D	Dimension of the solutions	M
S	Initial population of the employed bees and onlooker bees	100
limit	Repeated restriction times of algorithm	100
TC	Termination condition of ABC algorithm	1000*D

TABLE 5. Interception probability for Case 1.

	T1	T2	T3
M1	0.7515	0.7629	0.6742
M2	0.7629	0.3118	0.2169
M3	0.6742	0.7043	0.4969
M4	0.8342	0.4611	0.2842
M5	0.5643	0.7888	0.6628
M6	0.7732	0.6151	0.3855
M7	0.4635	0.8265	0.6631
M8	0.1669	0.3935	0.6917
M9	0.3082	0.6612	0.8100
M10	0.2575	0.5149	0.6930

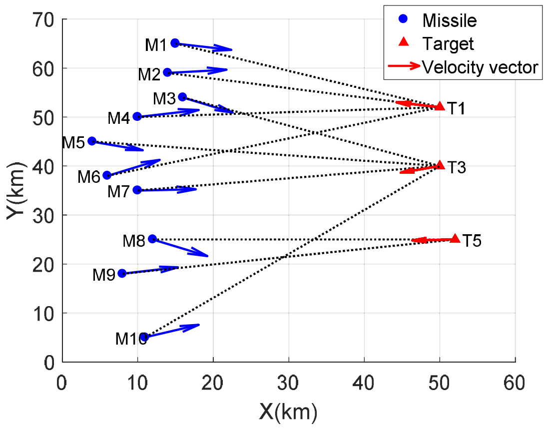

FIGURE 5. Allocation results for Case 1 with fixed grouping strategy.

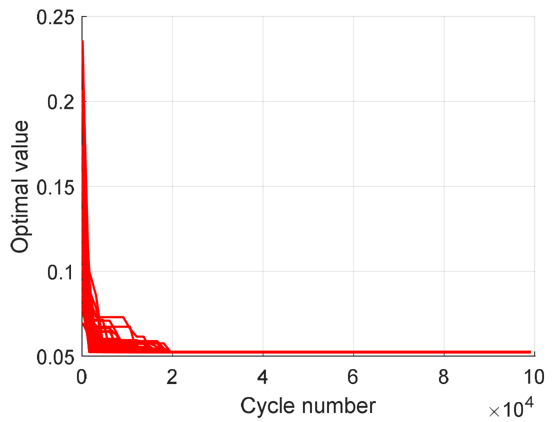

FIGURE 7. Objective function for Case 1 with fixed grouping strategy (50 runs).

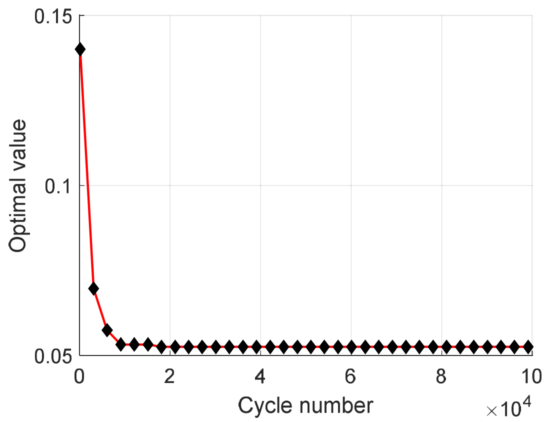

FIGURE 6. Objective function for Case 1 with fixed grouping strategy.

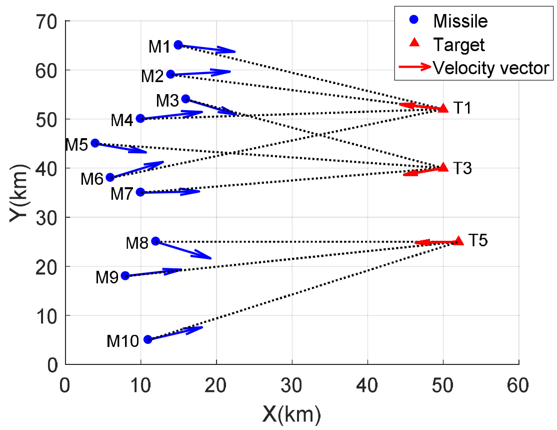

FIGURE 8. Allocation results for Case 1 with adaptive grouping strategy.

robustness of the WTA method, 50 runs are performed for **Case 1**. Figure 7 shows that the objective functions all converge to the same value, and the allocation results are the same as those in Fig. 5. Thus, the WTA method is indicated to be robust for the case of fixed grouping.

2) ADAPTIVE GROUPING

For the adaptive grouping condition, the allocation number is an interval and the solution space is much larger than that of the fixed grouping condition. The global optimal solution under the adaptive grouping constraint can be obtained by adjusting the allocation number for each target adaptively. Assume that the maximum and minimum allocated number to each target are 4 and 1, respectively, i.e. $B = [4, 4, 4]$ and $C = [1, 1, 1]$.

The allocation results for the WTA problem with adaptive grouping constraint are given in Fig. 8. The results show that the allocation number to each target simultaneously meets the upper and lower limits. By comparing the allocation results in Figs. 5 and 8, one can observe that M10 is allocated to T2 in the case of fixed grouping, while it is allocated to T3 in the case of adaptive grouping. In addition, the penalty factor of each penalty function term in the adaptive grouping condition is consistent with that in the fixed grouping condition. In fact, if the solutions for the two cases are same and both satisfy the grouping constraints, then the corresponding objective function values are equal. Figure 9 shows the optimal value of the objective function during the iteration. The optimal value of the objective function is 0.0525 under fixed grouping and 0.0241 under adaptive grouping. It can be seen that the allocation results under the adaptive grouping is superior to that under the fixed grouping. Similarly, the objective functions for 50 runs are given in Fig. 10. The allocation results and the optimal value are same as those in Figs. 8 and 9. The comparison of Figs. 7 and 10 shows that the objective function for the adaptive grouping converges faster. Therefore, compared with the case of the fixed grouping, the WTA with the adaptive grouping constraint can get better allocation results under the premise of meeting the combat requirements.

C. RESULTS FOR CASE2

In **Case 2**, more targets are considered. With the number of missiles unchanged, the number of allocation schemes would increase exponentially with the targets number. The interception probability matrix P is given in Table 6.

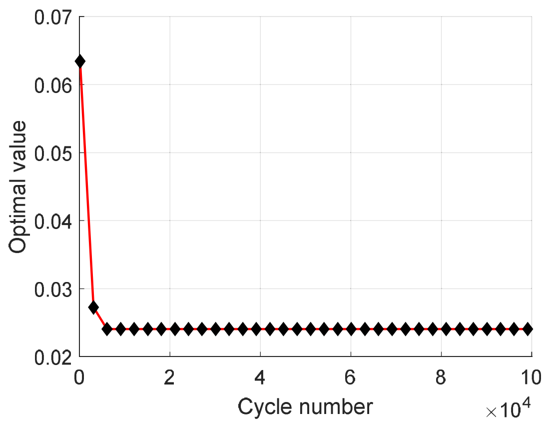

FIGURE 9. Objective function for Case 1 with adaptive grouping strategy.

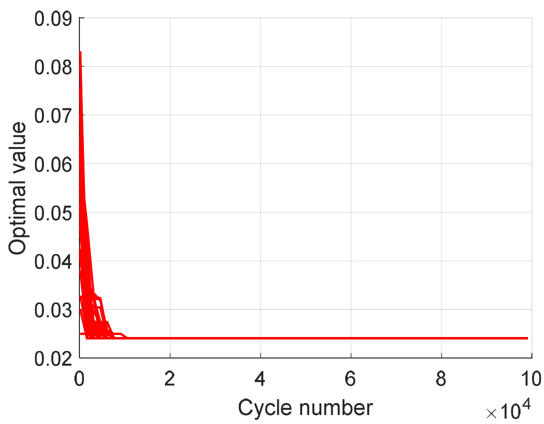

FIGURE 10. Objective function for Case 1 with adaptive grouping strategy (50 runs).

TABLE 6. Interception probability for case 2.

	T1	T2	T3	T4	T5
M1	0.7467	0.5097	0.3330	0.3262	0.2468
M2	0.7583	0.5007	0.3023	0.2922	0.2086
M3	0.6691	0.8415	0.6988	0.6726	0.4873
M4	0.8300	0.7263	0.4522	0.4450	0.2749
M5	0.5552	0.7339	0.7831	0.8030	0.6544
M6	0.7674	0.8101	0.6072	0.6498	0.3753
M7	0.4543	0.6539	0.8220	0.8348	0.6560
M8	0.1605	0.2068	0.3840	0.3243	0.6855
M9	0.2981	0.3986	0.6533	0.5588	0.8050
M10	0.2481	0.3042	0.5046	0.3827	0.6860

1) FIXED GROUPING

Assume that the allocation numbers to T1, T2, T3, T4, and T5 are $A = [2, 1, 2, 3, 2]$. The simulation results of the WTA problem with the fixed grouping constraint are given in Figs. 11 and 12. The allocation results in Fig. 11 show that the allocation number to each target is exactly the same as the expected value. Compared with the fixed grouping in **Case 1**, the allocation result has been widely changed.

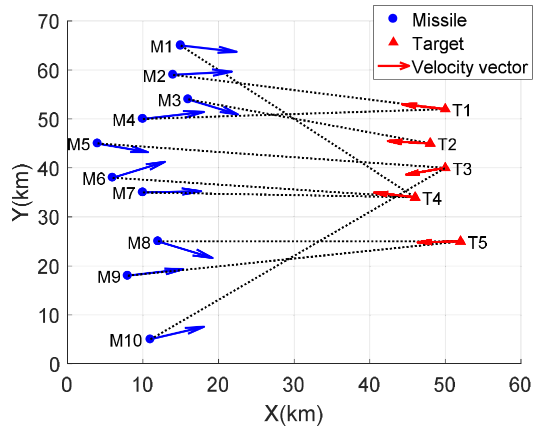

FIGURE 11. Allocation results for Case 2 with fixed grouping strategy.

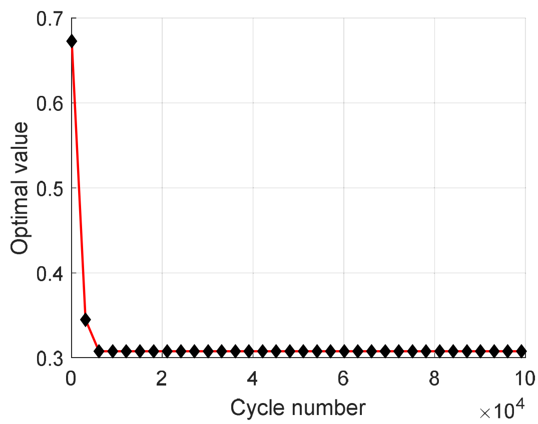

FIGURE 12. Objective function for Case 2 with fixed grouping strategy.

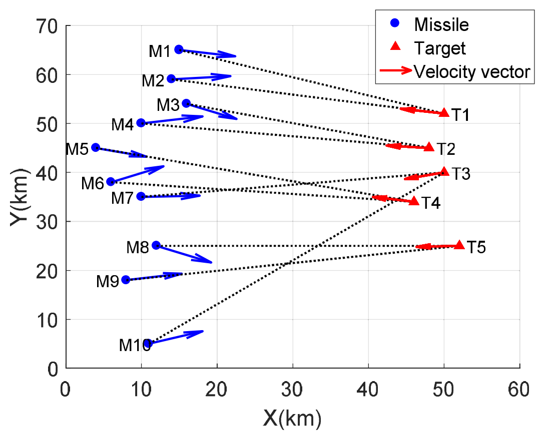

FIGURE 13. Allocation results for Case 2 with adaptive grouping strategy.

In **Case 1**, M1 and M6 are allocated to T1, and M3 and M7 are allocated to T3. However, in **Case 2**, M3 is allocated to T2, and M1, M6 and M7 are allocated to T4. In addition, due to the increase of the number of targets, the optimal value of the objective function in **Case 2** is larger than that in **Case 1**. As can be seen from Fig. 12, the value of the objective function quickly converges to the optimal value.

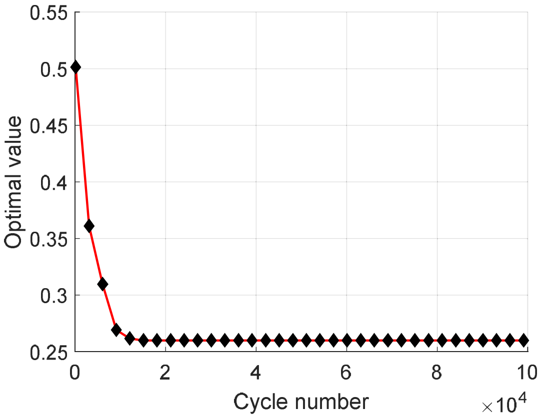

FIGURE 14. Objective function for Case 2 with adaptive grouping strategy.

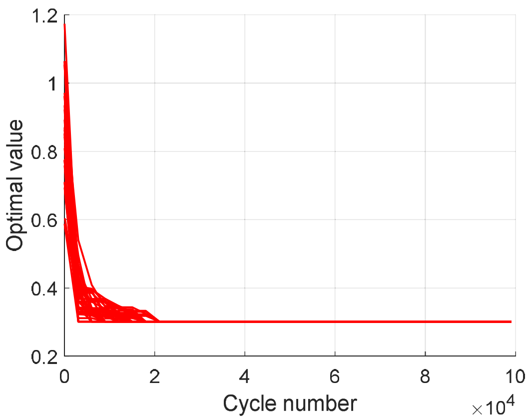

FIGURE 15. Objective function for ABC algorithm with fixed grouping strategy (50 runs).

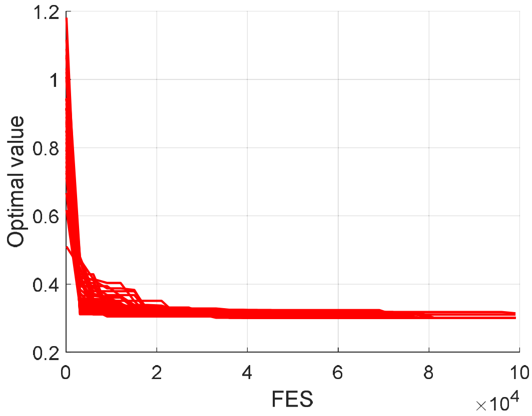

FIGURE 16. Objective function for PSO algorithm with fixed grouping strategy (50 runs).

2) ADAPTIVE GROUPING

Similarly, assume that the maximum and minimum allocation numbers to each target are 1 and 3, respectively, i.e., $B = [3, 3, 3, 3, 3]$ and $C = [1, 1, 1, 1, 1]$. The simulation results are given in Figs. 13 and 14. It is seen that the allocation results meet the upper and lower limits. Meanwhile, the optimal solution can be quickly obtained under the adaptive grouping constraint. Comparing the results in Figs. 12 and 14, one can find that the optimal value of the objective function for the adaptive grouping strategy is better than that for the fixed one.

In general, on the premise that the grouping constraints are set reasonably, the feasible solution range for the adaptive grouping strategy is always larger than that of the fixed one. Therefore, when it is difficult to determine the fixed number of missiles allocated to each target, the adaptive grouping strategy is a better choice for calculating the optimal solution of the WTA problem.

3) STATISTIC AND COMPARISON

To evaluate the convergence property and computational efficiency of the proposed WTA method, the comparative simulation of the ABC algorithm and the PSO algorithm [20] is conducted. With the parameters in **Case 2**, 50 simulations are carried out for the two algorithms under fixed and adaptive grouping constraints. Two indexes are concerned. One is the number of cases that convergence to the optimal solution, and the other is the convergence time. In the case of fixed grouping strategy, for example, the objective functions for 50 runs under ABC algorithm and PSO algorithm are given in Fig. 15 and Fig. 16. Results for the two algorithms are shown in Table 7. Besides, using either the fixed or adaptive grouping strategy, the convergence probability (the ratio of convergence cases to total cases) of the PSO algorithm is below 70%. In contrast, the convergence probability of the ABC algorithm is 100% for both grouping strategies. In addition, the average convergence time is calculated for the convergence cases. The comparison shows that the ABC algorithm has a higher convergence speed to achieve the optimal solution. Therefore, results indicate that the proposed WTA method using the ABC algorithm can stably and rapidly obtain the allocation results which satisfy the grouping constraints.

TABLE 7. Comparison of ABC and PSO algorithm for Case 2.

Index	Adaptive Grouping		Fixed Grouping	
	ABC	PSO	ABC	PSO
Number of convergence cases	50	33	50	31
Average convergence time (s)	0.29	1.30	0.31	1.43

V. CONCLUSION

To enhance the effectiveness for multi-to-multi interception combat, a weapon-target assignment (WTA) method with fixed and adaptive grouping constraints has been developed. The miss-distance under heading error, the time-to-go for moving target, and the LOS rate are used to construct the interception probability function, which is suitable for the multi-to-multi interception. Simulation results show that the proposed WTA method with the fixed and adaptive grouping constraints can both achieve the corresponding optimal allocation results. The optimal value converges rapidly, and the optimal WTA results satisfying the corresponding grouping constraints can be quickly obtained. In different combat cases, the allocation results can meet the grouping constraints and operational requirements simultaneously, which indicates that the proposed WTA method has strong adaptability. Additionally, the comparison indicates that the optimal allocation result is relatively easy to be obtained by the adaptive grouping strategy. In future work, the attack zone and the seeker look angle constraints can be considered in the interception probability function.

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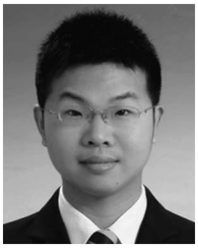

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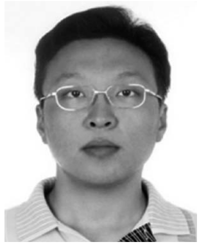

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