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Scheduling Jobs on a Single Machine With Dirt Cleaning Consideration to Minimize Total Completion Time

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ABSTRACT This paper studies a single-machine scheduling problem observed in the wafer manufacturing process, where the machine must receive periodical maintenance so that the dirt generated in the process does not exceed the limit. The objective is to minimize the total completion times. A mixed binary integer programming model is formulated, and, due to its computational intractability for large problems, three effective heuristics are proposed based on our developed properties. The proposed heuristics are evaluated by comparison with exact solutions on small problems and with lower bounds on large problems. The experimental results show that the INDEX-LOE heuristic yields high-quality solutions in comparison with those obtained from the other two heuristics. Furthermore, the impacts of dirt accumulation and cleaning time are discussed in detail.

INDEX TERMS Scheduling, total completion time, machine unavailability, mixed binary integer programming, heuristic algorithms.

I. INTRODUCTION

Many researchers that have investigated single-machine scheduling problems, assumed that the machines are available all the time. However, in practical cases, machine operations have to be interrupted for some engineering needs, such as repairing, changing kits, or cleaning. This paper is motivated by a wafer fabrication process, where dirt, such as particles, organic materials, and metal-salts, on the surface of the wafer will be left in the machine during wafer processing. Once the accumulation of dirt reaches a threshold value, the wafer will be damaged. Thus, the machine has to be stopped to remove the accumulated dirt with a cleaning agent. That is, the cleaning agent cleans the machine so that the accumulated dirt does not exceed the threshold value. In the cleaning period, the machine cannot process any jobs.

This paper studies a single-machine scheduling problem with flexible maintenance; the flexible maintenance in this paper specifies that the starting time of maintenance activity is determined by the amount of dirt accumulated and the

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threshold value. This situation is commonly encountered in a wafer manufacturing company, and these non-available periods of a machine will affect the scheduling and the system's performance. For the problem, we consider the objective of minimizing the total completion times. This objective implies minimizing the work-in-process (WIP) inventory, which is an important internal managerial performance measure for a company. According to the notation [1], the problem is denoted 1 |*cleaning*| $\sum C_j$, where the first field denotes a single machine, the second field denotes the cleaning activities and the third field denotes the total completion time.

In the literature, many studies assumed that maintenance is required in fixed intervals or during some window times. Among them, the first study related to our study was conducted by Yang *et al*. [2]. Other related articles include the following (Schmidt and Sanlaville [3], Qi *et al*. [4], Schmidt [5], Liao and Chen [6], Chen [7], Chen [8], Ji *et al*. [9], Chen [10], Mosheiov and Sarig [11], Low *et al*. [12], Ma *et al*. [13], Yang *et al*. [14], Zammori *et al*. [15], Xu *et al*. [16], Yin *et al*. [17], and Yin *et al*. [18]).

The variable maintenance defines that the duration of maintenance depends on its start time. That is, the sooner the maintenance is started, the shorter the length of the maintenance time is, which is also called deteriorating maintenance. Kubzin and Strusevich [19] added the variable maintenance restriction into the two-machine flow shop and two-machine open shop problems, where the objective is to minimize the makespan. They showed that the open-shop problem is polynomially solvable, while the flow shop problem is binary NP-hard and pseudo-polynomially solvable with dynamic programming. Mosheiov and Sidney [20] also considered the deteriorating maintenance activity. The difference is that the processing time of jobs is affected by the maintenance activity, that is, the processing time of job *j* will decrease if job *j* is processed after the maintenance activity. For the problem, they proposed polynomial time solutions for the problems with different objectives. Yin *et al*. [21] considered position-dependent deteriorating jobs and deteriorating maintenance activities simultaneously on a single-machine scheduling problem, and their objective is to jointly minimize the cost of due-date assignment, and the cost of discarding jobs. The study of Luo *et al*. [22] is inspired by the above two works by Kubzin and Strusevich [19] and Mosheiov and Sidney [20], and proposed two approximation algorithms to minimize the total weighted completion time. Xu *et al*. [23] considered two scheduling problems with a single machine and parallel machine where the maintenance is an increasing linear function and the maintenance activity has to be implemented within a prefixed interval. The objective is to minimize the makespan. They developed two approximation algorithms for the problems. Motivated by the request serving process in a wireless sensor network, Gu *et al*. [24] considered single-machine problems with the machine aging effect, where maintenance is implemented once to recover the service ability. For the problem, they adopted two objectives of minimizing the makespan and the total completion time, respectively, and proposed two dynamic programming algorithms. Luo *et al*. [25] investigated scheduling jobs and variable maintenance activity on a single machine and provided polynomial-time algorithms to solve the problems while minimizing the makespan, total completion time, maximum lateness, and number of tardy jobs. Zhu *et al*. [26] added resources into the maintenance activity in the problem with deteriorating processing time, where the greater the amount of resources allocated to the maintenance activity, the shorter the duration of maintenance. For the problem, they considered different objective functions and proved that the problems are polynomially solvable. Recently, Luo and Liu [27] extended the study of Xu *et al*. [23], where the objective is to minimize the total weighted completion, and they proposed two approximation algorithms for the problem. Ying *et al*. [28] extended the problem studied by Luo *et al*. [25] to consider four different objective functions and proposed an exact algorithm with the computational complexity $O(n^2)$ for each problem. Su and Wang [29] studied a single machine problem with

multiple unavailability periods, where the machine has to be interrupted to remove dirt. Their objective is to minimize the total absolute deviation of job completion time (TADC).

Some researches attempted to consider uncertain maintenance activities in production scheduling with different optimization objectives in different manufacturing shop floors [30], [31]. Xiong *et al*. [32] consider the machine disruption may occur at a particular time in a single machine, and the maintenance time will last for a period of time with a certain probability. For the problem, they proposed different approximation approaches to minimize the expected integrated cost function including the earliness, tardiness and due date assignment cost. Yin *et al*. [33] extended the study of Xiong *et al*. [32] to consider parallel-machine scheduling problem, and provide polynomial-time approximation schemes to solve the problem of minimizing the expected total completion time.

In this work, we considered the same problem studied by Su and Wang [29]; the difference is that our objective is to minimize the total completion time. Many dominance properties are explored and used in the lower bound calculation and heuristic algorithms. The rest of this paper is organized as follows. Section 2 defines the problem, presents many properties to optimally solve the problem and proposes two lower bounds. A mixed binary integer programming model is developed in Section 3. In Section 4, we present additional properties for the heuristic algorithms and propose three heuristic algorithms. Section 5 gives the computational experiments, and finally, in Section 6, the conclusions are provided along with some future research directions.

II. PROBLEM DEFINITION

This paper considers a nonresumable single-machine scheduling problem; i.e., once a job is started, it cannot be interrupted until its completion. The setup time of the job is sequence independent and included in the processing time. There are *n* jobs to be processed at time zero. Each job has a processing time *pⁱ* and an amount of dirt *tⁱ* left on the machine where $i = 1, 2, \ldots, n$. A cleaning activity with time *w* is carried out before the accumulation of dirt reaches a threshold value *T*, where $t_i \leq T$. The objective is to minimize the total completion time.

A. NOTATION AND PROBLEM SETTING

The following notations will be used throughout the study:

 J_i job number $i(i = 1, 2, \ldots, n);$

 C_j completion time of the job at the j_{th} position in a given sequence $(j = 1, 2, ..., n)$;

 k_i1 , if the cleaning activity is taken immediately following the *jth* position job; 0, otherwise

 x_{ij} 1, if J_i is scheduled at position *j*; 0, otherwise.

In addition, $J_{[j]}$ denotes the job in sequence position *j*, and $p_{[j]}$ and $t_{[j]}$ are defined accordingly.

The objective is to find a schedule that minimizes the total completion time (*TC*), represented as $TC = \sum_{j=1}^{n} C_j$ Denote a schedule π containing a sequence of jobs and several

cleaning activities inserted in the job sequence. In the schedule, those jobs processed between two adjacent cleaning activities form a batch, denoted B_l where $l = 1, ..., L$. Thus, a schedule π can be denoted $\pi = (B_1, w, B_2, w, \ldots, B_L)$. Note that *L* denotes the number of batches and is a decision variable in our problem.

Theorem 1: The problem 1| cleaning $|\sum C_j$ is strongly NP-hard.

Proof: The problem 1|cleaning| $\sum C_j$ is strongly NP-hard because the special case, where each dirt is equal to its corresponding processing time, and the machine must be implemented cleaning activity after a maximum allowed dirt, i.e., 1|cleaning, $t_j = p_j \geq C_j$, is NP-hard. (Qi *et al.* [4]).

Obviously, there are at least $(L'$ -1) cleaning activities in an optimal schedule. It is worth mentioning that the 1|cleaning| $\sum C_j$ problem with agreeable processing time and dirt, i.e., $p_i \leq p_j$ implies that $t_i \leq t_j$ is also NP-hard because the special case of 1|cleaning, $t_j = p_j \sum_i C_j$ is NP-hard.

For the problem 1|cleaning $|\sum C_j|$, the optimal schedule may have a larger number of batches than the schedule that contains the minimum number of batches. Therefore, the optimal schedule may need more cleaning activities to achieve a lower total completion time. The following is a simple example.

Numerical Example 1: Let $n = 4$, $p_1 = 1$, $p_2 = 3$, $p_3 = 1$, $p_4 = 3$, $t_1 = 1$, $t_2 = 2$, $t_3 = 1$, $t_4 = 2$, and $T = 3$. The schedule π_1 with the minimum number of batches is $\pi_1 = (J_1, J_2, w, J_3, J_4)$ with $L = 2$ and $TC = 18 + 2w$. Consider the schedule $\pi_2 = (J_1, J_3, w, J_2, w, J_4)$ with $L = 3$ and $TC = 16 + 3w$. Then we can see that the second schedule that encounters two cleaning activities is better than the first schedule with one cleaning activity when $w < 2$.

Therefore, in the optimal schedule, the cleaning activity may be carried out even if the machine can process more jobs within the batch.

In the following Theorem, we show that the total completion time in a schedule π containing both jobs and cleaning activities can be calculated in a more efficient way.

Lemma 1: The TC value of a given schedule π containing jobs and cleaning activities can be calculated as

TC = $\sum_{j=1}^{n} (n-j+1)p_{[j]} + w \sum_{j=1}^{n-1} (n-j)k_j$, where the value of k_j equals 1 if the cleaning activity is conducted immediately following the *jth* position job; otherwise, it equals 0.

Proof: Let the number of jobs in the k_{th} batch be β_k .

$$
TC = \sum_{j=1}^{n} C_j
$$

= $\sum_{j=1}^{n} (n-j+1)p_{[j]} + \sum_{k=1}^{B_{L-1}} (n - \sum_{i=1}^{k} \beta_i)w$
= $\sum_{j=1}^{n} (n-j+1)p_{[j]} + w \sum_{j=1}^{n-1} (n-j)k_j,$ (1)

where the former factor $(n-j + 1)$ is the positional weight independent of p_j and the latter factor $(n - j) \times k_j$ is the positional weight of the cleaning activity immediately following job sequence *j*.

Numerical Example 2: The data in example 1 is considered. The job sequence $\pi = (J_1, J_3, w, J_2, w, J_4)$ can be scheduled as follows.

$$
TC = 4 \times p_1 + 3 \times p_3 + 2 \times p_2
$$

+ 1 \times p_4 + (4 - 2)w + (4 - 3)w
= 16 + 3w

According to Lemma 1, the positional weight of the cleaning activity is equal to that of the job immediately following it and thus the problem can be solved polynomially when each cleaning activity is carried out after a fixed number of jobs.

B. CALCULATION OF LOWER BOUND

Two lower bounds are proposed as benchmarks to evaluate the heuristics. Based on the fact that the *SPT* rule gives an optimal solution to the problem $1 \parallel \sum C_j$ and that the positional index of the cleaning activity's contribution to the total completion time is in descending order due to Lemma 1, we now combine these properties with the assumption that the dirt accumulation is resumable, i.e., the dirt accumulation interrupted by a cleaning activity can resume, in order to obtain the lower bound of the total completion time *LB*(*TC*).

Denote *e* as the elapsed time between the end time of the last cleaning activity and the completion of the incumbent job. Additionally, let *Z* be the lower bound of the cleaning activity's contribution to the total completion time. The procedure to obtain *LB*(*TC*) is as follows.

Step 1. Sequence the processing time p_i in ascending order and the dirt t_i in descending order of all *n* jobs, where $i = 1$, 2,..,*n*. Set $i = 1, e = 0, Z = 0$.

Step 2. If $i > n$, then set $LB(TC) = \sum_{j=1}^{n} (n-j+1) p_j + Z$ and *Stop*; else, go to *Step 3*.

Step 3. If $(e + t_i) > T$, then $Z = Z + w (n - i)$ and $e = e + t_i - T$; else, $e = e + t_i$. Set $i = i + 1$ and return to *Step 2*.

The lower bound $LB(TC)$ obtained with complexity $O(2n \cdot$ log *n*) is very efficient but may not be so tight. It is proposed as a benchmark when evaluation of our heuristics in a very short time is essential. To evaluate the effectiveness of our heuristics precisely, a tighter lower bound obtained using IBM ILOG CPLEX Optimization Studio Ver. 12.6.1 is proposed. First, the global time limit is changed from the default value of infinity to the specified value (we set 600 seconds in our case). Then, the script code based on ILOG Script is developed to obtain the upper and the lower bounds of the objective function value, as well as the CPU time. If the CPU time does not exceed the time limit, then the upper bound obtained is the optimal solution. Otherwise, the lower bound obtained is used for evaluating the effectiveness of the proposed heuristics.

III. MIXED BINARY INTEGER PROGRAMMING (MBIP) MODEL

In this section, an *MBIP* model for optimally solving the problem $1 |cleaning| \sum C_j$ is formulated

as follows:

Minimize
$$
\sum_{j=1}^{n} (n-j+1)p_{[j]} + w \sum_{j=1}^{n-1} (n-j)k_j
$$
 (2)

Subject to :
$$
\sum_{j=1}^{n} x_{ij} = 1 \quad \forall i = 1, 2, 3, ..., n
$$
 (3)

$$
\sum_{i=1}^{n} x_{ij} = 1 \quad \forall j = 1, 2, 3, ..., n \tag{4}
$$

$$
p_{[j]} = \sum_{i=1}^{n} (p_i \dots x_{ij}) \quad \forall j = 1, 2, 3, \dots, n
$$
 (5)

$$
C_1 = \sum_{i=1}^{n} (p_i \dots x_{i1})
$$
 (6)

$$
C_{j-1}+\sum_{i=1}^n (p_i \ldots x_{ij})+w\leq C_j
$$

+
$$
M(1-k_{j-1})
$$
 $\forall j = 2, 3, 4, ..., n$ (7)
 $C_{j-1} + \sum_{i=1}^{n} (p_i ... x_{ij}) \le C_j$

$$
y_{j-1} + \sum_{i=1}^{n} (x_i + \cdots + x_j) = 0
$$

+ $M \dots k_{j-1} \quad \forall j = 2, 3, 4, \dots, n$ (8)

$$
t_{[1]} = \sum_{i=1}^{n} (t_i \dots x_{i1})
$$
(9)

$$
t_{[i-1]} + \sum_{i=1}^{n} (t_i \dots x_{ii}) \le t_{[i]} + M \dots k_{i-1}
$$

$$
t_{[j-1]} + \sum_{i=1}^{n} (t_i \dots x_{ij}) \le t_{[j]} + M \dots k_{j-1}
$$

\n
$$
\forall j = 2, 3, 4, \dots, n \quad (10)
$$

\n
$$
\sum_{i=1}^{n} (t_i \dots x_{ij}) \le t_{[j]} + M \dots (1 - k_{j-1})
$$

$$
\sum_{i=1}^{n} \frac{y_i \dots y_j}{y_j} = \frac{y_{j+1} \dots (1 - y_{j-1})}{y_j}
$$

$$
\forall j = 2, 3, 4, \dots, n \quad (11)
$$

$$
T \ge t_{[j]} - M \dots (1 - k_j)
$$

$$
\forall j = 1, 2, 3, \dots, n - 1 \quad (12)
$$

$$
T \ge t_{[j]} - M \dots k_j \quad \forall j = 1, 2, 3, \dots, n - 1
$$
\n(13)

$$
t_{[j]} \ge 0
$$
, $C_j \ge 0 \quad \forall j = 1, 2, 3, ..., n$ (14)
 x_{ij} is binary, $\forall i = 1, 2, 3, ..., n;$

$$
j = 1, 2, 3, ..., n
$$
 (15)
 k_j is binary, $\forall j = 1, 2, 3, ..., n - 1$ (16)

The model mentioned above is modified from the one proposed by Su and Wang [29], in which their objective function minimizes the total absolute deviation of the job completion time, whereas in our model, the objective function [\(2\)](#page-3-0) describes the minimum *TC* according to Lemma 1. Constraints [\(3\)](#page-3-0) and [\(4\)](#page-3-0) dictate that each job must be placed at one position, and that each position can only perform one job. Constraint [\(5\)](#page-3-0) defines the processing time of $J_{[j]}$. Constraint [\(6\)](#page-3-0) specifies the completion time of the job at the first position. If a cleaning activity is performed immediately after the $(j-1)_{th}$ job, then the cleaning time *w* is added to the completion time of the j_{th} job. Constraints [\(7\)](#page-3-0) and [\(8\)](#page-3-0) together define the completion times of the jobs processed after the first one by combining the binary variable *kj*−¹ with an extremely large positive number *M*. Constraints (9-11) define the dirt accumulation between the completion of the last cleaning activity and the completion of the *jth* job. Constraints (12-13) ensure that the accumulation of dirt in each batch cannot exceed the threshold value. Finally, constraints (14-16) define the nonnegativity of $t_{[j]}$ and C_j , and the binary restrictions for *xij* and *k^j* .

IV. HEURISTIC ALGORITHMS

Since our considered problem is strongly NP-hard, it is difficult to apply the mixed BIP to optimally solve largescale problems due to the considerable computational burden. Therefore, three heuristics are proposed. We first introduce some properties of the optimal schedule for the heuristics.

A. SOLUTION PROPERTIES

Recall that the shortest processing time (*SPT*) rule solves the problem $1 | \sum C_j$. For the $1 | \text{cleaning} | \sum C_j$ problem, we derive the following property to solve it.

Property 1: There exists an optimal schedule in which jobs in each batch are sequenced in *SPT* order.

Property 2: Consider a sequence where *J*[*j*] is to be shifted to the position immediately preceding $J_{[i]}$, where $i < j$, without violating the dirt constraint and suppose that there are *k* cleaning activities between *J*_[*i*] and *J*_[*j*]. If $\Delta = (j - i) p_{[j]} \sum_{u=i}^{j-1} p_{[u]} k w < 0$, then *TC* is decreased by this shift.

Proof: When $J_{[j]}$ is to be moved immediately before $J_{[i]}$, the change in the cost is

$$
\Delta_1 = (n - i + 1) p_{[j]} - (n - j + 1) p_{[j]}
$$

= $(j - i) p_{[j]}$

The jobs from the i_{th} position to the $(j - 1)_{th}$ position are moved backward one position as $\Delta_2 = \sum_{u=1}^{j-1} [(n-u) (n - u + 1)$] $p_{[u]} = -\sum_{u=i}^{j-1} p_{[u]}$

Each cleaning activity in between is moved backward one position as

$$
\Delta_3 = -kw
$$

$$
\Delta = \Delta_1 + \Delta_2 + \Delta_3
$$

This completes the proof.

Property 3: Given σ the list of the remaining unscheduled jobs in *SPT* order are indexed as $J_{\sigma_1}, J_{\sigma_2}, \ldots, J_{\sigma_r}$. If J_{σ_1} cannot be scheduled at the *ith* job sequence position in the current batch B_l due to $\sum_{k \in B_l} t_k + t_{\sigma_1} > T$, then job J_{σ_i} , $i = 2, 3, \ldots, r$ where $\sum_{k \in B_l} t_k + t_{\sigma_1} \leq T$ and $(i - 1) p_{\sigma_i} - \sum_{k=1}^{i-1} p_{\sigma_k} - w \leq 0$ should be scheduled in *B^l* .

Proof: If J_{σ_i} is available to be scheduled in the current batch B_l at the *i*th sequence position, then job J_{σ_i} contributes $\Delta_1 = (i - 1)p_{\sigma_i}$ to *TC*.

The jobs from J_{σ_1} to $J_{\sigma_{i-1}}$ in σ are moved backward one position and the total completion time is changed as

$$
\Delta_2 = -\sum_{k=1}^{i-1} p_{\sigma_k}
$$

The cleaning activity is moved backward one position and the total completion time is changed as

$$
\Delta_3 \ge -w
$$

$$
\Delta = \Delta_1 + \Delta_2 + \Delta_3
$$

This completes the proof.

Numerical Example 3: Suppose that there are four unscheduled jobs in *SPT* order. Their processing times are $p_1 = 6, p_2 = 7, p_3 = 7,$ and $p_4 = 8$, and their dirt amounts

are $t_1 = 2$, $t_2 = 2$, $t_3 = 2$, and $t_4 = 1$. Suppose the current batch is B_l and $T - \sum_{k \in B_l} t_k = 1$. To schedule with the *SPT* rule, the unscheduled jobs will be processed as schedule I in Figure 1, but it could be better in schedule II. With $w = 5$, the total completion time is $67 + 4w$ in schedule I, which equals 87. Meanwhile, in schedule II, the total completion time is $71 + 3w$, which equals 86.

The *LOE* (last-only-empty) approach, a well-known and commonly encountered approach in practice, is introduced.

LOE: batching in which the accumulated dirt of each batch is kept full whenever possible except for the last one.

Property 4: If the sequence of all jobs is fixed, then the *LOE* rule gives an optimal solution.

Proof: According to Lemma 1, the *TC* value of a given schedule π containing jobs and cleaning activities can be calculated as

 $TC = \sum_{j=1}^{n} (n-j+1)p_{[j]} + w \sum_{j=1}^{n-1} (n-j)k_j$, where the value of *k^j* equals 1 if a cleaning activity occurs immediately following the *jth* position job; otherwise, it equals 0. Since the sequence of all jobs is fixed, the term $\sum_{j=1}^{n} (n-j+1)p_{[j]}$ is constant. Another term $\sum_{j=1}^{n-1} (n-j)k_j$ can be recalculated as $\sum_{j=1}^{n-1} (n-j)k_j = (n-1)k_1 + (n-2)k_2 + (n-3)k_3 + \ldots$ $k_{n-1} = n \sum_{i=1}^{n-1} k_i - \sum_{i=1}^{n-1} i k_i$

To minimize the value of $n \sum_{i=1}^{n-1} k_i$, the number of batches is as small as possible; and in order to maximize $\sum_{i=1}^{n-1} i k_i$, the cleaning activity is taken as late as possible. Therefore, the *LOE* rule gives an optimal solution.

B. HEURISTC ALGORITHMS

We develop three heuristics, *SPT-LOE, SPCT-LOE,* and *INDEX-LOE*, to solve the 1 |*cleaning* | $\sum C_j$ problem. Initially, in each heuristic, a greedy algorithm is applied to sequence both the jobs and cleaning activities. An improvement algorithm is then applied. If, at any time, the processing time of a job is greater than that of the job in the following batches and the interchange these two jobs does not violate the dirt capacity constraint, then the interchange is performed. The detailed steps of the three heuristics are outlined as follows:

1) HEURISTIC SPT-LOE

The algorithm commonly used in the real world applies the *SPT* rule to sort the job and follows the *SPT* order to form the

batch using the *LOE* rule. If a job is added to batch *B^l* , resulting in the total amount of dirt accumulation exceeding *T* , then the job is added to the next batch B_{l+1} . An improvement algorithm as previously described is then applied. The steps of the heuristic algorithm are as follows.

Step 1. Sequence all *n* jobs in *SPT* order and break any ties in favor of less dirt.

*Step 2***.** Use the *LOE* rule to insert the cleaning activities.

Step 3. Apply the improvement algorithm to interchange the jobs in each batch with the job having the larger processing time being placed in the following batches provided that the interchange does not violate the dirt constraint. The procedure is continued until the last batch has been considered.

Step 4. Apply Property 1 to sequence the jobs in each batch in *SPT* order.

2) HEURISTIC SPCT-LOE

Since the processing time of a job and the dirt left on a machine do not agreeable, the processing time of a job is adapted by adding the estimated cleaning time proportional to the amount of dirt. The heuristic *SPCT-LOE* is outlined as follows.

*Step 1***:** Calculate the modified processing time for each job $\lim_{t \to \infty} p_j = p_j + (t_j/T) \, w,$ wherej = 1, ..., *n*

Step 2: Find a job sequence by ordering all jobs in nondecreasing order of the modified processing time, p'_i *j* , and break any ties in favor of the smaller processing time.

*Step 3***:** Assign unscheduled jobs one by one to the machine until a job cannot be scheduled to the current batch due to the dirt constraint. In this case, assign a job that has not scheduled yet according to Property 3 to the last position of the current batch. If no such job is found, arrange a cleaning activity and build a new batch as the current batch. The step is continued until all jobs have been scheduled.

*Step 4***.** Use Property 1 to sequence the jobs in each batch based on the *SPT* rule.

3) HEURISTIC INDEX-LOE

An index for each job *j* in each iteration is developed by the following expression:

$$
I_j = (n - iteration + 1) \times p_j + (n - iteration) \times w
$$

$$
\times \left(\frac{e + t_j}{T} + b\right),
$$

where the value of *e* denotes the amount of accumulated dirt in the current batch before job J_i and b equals 1 if $e + t_j > T$; otherwise, it equals 0. The job with the smallest *I^j* is selected to be scheduled to the machine. If the selected job cannot be scheduled to the current batch due to the dirt constraint, then a cleaning activity is inserted. The steps of the *INDEX-LOE* heuristic are outlined as follows.

*Step 1***:** Set $e = 0$, Iteration = 1. Let *S* denote the set of all scheduled jobs and *U* the set of all unscheduled jobs. Initially, $S = \emptyset$ and $U = \{J_1, J_2, ..., J_n\}$.

Step 2: If $U = \emptyset$, go to *Step* 5. Otherwise, calculate the index of all jobs in *U* and find the smallest one, denoted *I^j* . If $e + t_i \leq T$, go to *Step* 3; otherwise, go to *Step* 4.

*Step***3:** Schedule *J^j* to the last position of the current batch. Let $S \leftarrow S \cup J_j$, $U \leftarrow U \setminus J_j$, $e = e + t_j$, Iteration $=$ Iteration $+ 1$ and go to *Step* 2.

Step 4: Schedule a cleaning activity after the current batch and assign job *J_j* to a new batch. Let $S \leftarrow S \cup J_j$, $U \leftarrow$ $U\setminus J_i$, $e = t_i$, Iteration = Iteration + 1 and go to *Step* 2.

Steps **5** and **6** are the same as *Steps* 3 and 4 in the *SPT-LOE* heuristic.

Numerical Example 4: The data in example 1 is considered. Let $w = 1$.

Iteration 1: $e = 0$, $S = \emptyset$, and $U = \{J_1, J_2, J_3, J_4\}.$ $I_1 = 4 \times 1 + 3 \times 1 \times (1/3) = 5, I_2 = 4 \times 3 + 3 \times 1 \times (2/3) =$ $14, I_3 = 4 \times 1 + 3 \times 1 \times (1/3) = 5, I_4 = 4 \times 3 + 3 \times 1 \times (2/3) =$ 14. The smallest index is that with J_1 and J_3 . Job J_1 is selected arbitrarily and is placed in position 1.

Iteration 2: $e = 1$, $S = \{J_1, U = \{J_2, J_3, J_4\}.$

 $I_2 = 3 \times 3 + 2 \times 1 \times (3 \cdot 3) = 11, I_3 = 3 \times 1 + 2 \times 1 \times (2 \cdot 3) =$ 4.3, $I_4 = 3 \times 3 + 2 \times 1 \times (3/3) = 11$. Job J_3 is placed in position 2.

Iteration 3: $e = 2$, $S = \{J_1, J_3, U = \{J_2, J_4\}.$

 $I_2 = 2 \times 3 + 1 \times 1 \times \{[(2+2)/3]+1\} = 8.3,$ $I_4 = 2 \times 3 + 1 \times 1 \times \{[(2+2)/3] + 1\} = 8.3$. Job J_2 is selected arbitrarily and is placed in position 3, and the remaining job, *J*4, is placed in position 4. This results in the sequence $\pi = (J_1, J_3, w, J_2, w, J_4)$, which is the optimal schedule.

V. COMPUTATIONAL EXPERIMENTS

The computational experiments aim to examine the performances of the proposed algorithms, including the Mixed Binary Integer Programming (MBIP) model and the heuristic algorithm. The proposed MBIP model is implemented by IBM ILOG CPLEX Optimization Studio Ver. 12.6.1 software and the heuristics as well as the lower bound are coded in C++. All experiments are run on a PC with an i3-530 CPU. Two sets of experiments are carried out. The first set aims to evaluate the efficiency of the MBIP model and the effectiveness of the heuristic algorithm for small problems. Another set evaluates the performances of the heuristics using the lower bound as a benchmark for large problems.

The parameter settings *n*, p_i , t_i , *T*, and *w* for small and large problems are shown in Table 1, in which the parameters α , β and γ control the amount of dirt left by each job, the length of the cleaning time, and the dirt threshold, respectively. The experimental procedure consists of a design of dirt with two settings of α (α = 0.2 *and* 0.4) and two settings of γ (γ = 2 and 4). The two settings of $\beta(\beta = 1.5 \text{ and } 2.5)$, which controls the cleaning times, refer to the maintenance times of Liao and Chen [6].

For each combination of n , p_i , t_i , T , and w , ten instances are generated, yielding 1440 instances for the problem. The formula, $dev(\%) = (heuristic - optimum) / optimum$, is used to determine the deviation of our heuristic

TABLE 1. Data for computational experiments.

solution over the optimal solution for small problem instances. On the other hand, the formula $dev(\%) =$ (*heurisitc* − *LowerBound*) *LowerBound* is employed to determine the deviation of our heuristics over the Lower Bound.

A. COMPARISON OF OUR HEURISTIC WITH BIP FOR SMALL PROBLEMS

For small problems with $n = 8, 10, 15, 20, 30, 40$ and 50, the computational results are shown in Tables 2 and 3 for the processing time distributions *U*(1,10) and *U*(1,100), respectively. Both tables show the average CPU time in seconds for the MBIP model and the heuristic algorithm, as well as the average deviation of the heuristic solution over the optimal solution. Table 4 further summarized the average deviation of each heuristic over the optimal solution.

The results in Tables 2 and 3 show that the MBIP model solves all the instances within four seconds when the number of jobs *n* is not larger than 20. However, the required computational time increases drastically when *n* increases to 50. The instance of widely dispersed job processing times *U*(1,100) needs much more execution time than that of *U*(1,10). For example, when $n = 50$, the average execution time for the job processing time distribution *U*(1,10) is 371.553 seconds, while it is 4852.296 seconds for $U(1,100)$. The execution times of all three heuristics are almost zero when $n < 50$. With regard to the quality of the heuristics, the summary results are given in Table 4. The average deviations of the three heuristics, *SPT-LOE*, *SPCT-LOE* and *INDEX-LOE*, from the optimal solution are 2.308%, 0.317%, and 0.281%, respectively, for the processing time distributions $U(1,10)$. For the processing time distributions $U(1,100)$, the deviations of *SPT-LOE*, *SPCT-LOE* and *INDEX-LOE* from the optimal solution are 5.252%, 0.741%, and 0.669% on average, respectively. The influences of the two processing time distributions associated with the three heuristics on the solution quality are shown in Figure 2. The heuristic solutions perform slightly worse as the dispersion of the job processing times widens. From Figure 2 or Table 4, it is clear that *INDEX-LOE* outperforms the other two heuristics, especially *SPT-LOE*, which is commonly used in practice. The impacts of the

TABLE 2. The results of $p_i = U$ [1,10] for small problems.

FIGURE 2. The influences of processing time distribution, α , β and γ on the solution quality for small problems.

parameters α , β and γ on the solution quality are analyzed and depicted in Figure 2.

The parameter α controls the amount of dirt left by a job on the machine. The larger the value of α is, the more

dirt left by a job. Figure 2 shows that the deviation of the heuristic from the optimal solution increases as the value of α increases. The parameter β represents the influence of the cleaning time on solution quality. The larger the value of β is,

TABLE 3. The results of $p_i = U$ [1,100] for small problems.

TABLE 4. The summary of each scenario in average.

the slightly smaller the deviation of the heuristic over the optimum. Finally, γ controls the value of the dirt threshold. Figure 2 shows that the smaller the value of γ is, the slightly better the quality of the heuristic.

B. COMPARISON OF THE HEURISTICS WITH LOWER BOUND FOR LARGE PROBLEMS

We compare the three heuristics with the *Lower Bound* obtained by the IBM ILOG CPLEX Optimization Studio

Ver. 12.6.1 for large instances with $n = 100$ and 200. The computational results are shown in Tables 5 and 6 for the processing time distributions *U*(1,10) and *U*(1,100), respectively. The deviations of the three heuristics over the lower bound are also summarized in Table 4.

The results in Tables 5 and 6 indicate that the average CPU times for IBM ILOG CPLEX Optimization Studio Ver. 12.6.1 to obtain the lower bound are 472.122 seconds and 573.373 seconds when $n = 100$ and 200, respectively.

TABLE 5. The results of $p_i = U$ [1, 10] for large problems.

TABLE 6. The results of $p_i = U[1, 100]$ for large problems.

The instance of the widely dispersed job processing time distribution $U(1,100)$ needs more execution time and incurs greater solution deviation. The execution times of all three heuristics are less than 0.039 seconds and 0.083 seconds for the processing time distributions $U(1,10)$ and $U(1,100)$, respectively, when $n = 200$. With regard to the qualities of the heuristics, the average deviations of three heuristics, *SPT-LOE*, *SPCT-LOE* and *INDEX-LOE*, over the lower bound are 1.671%, 0.183%, and 0.296%, respectively, for the processing time distribution $U(1,10)$. For the processing time distribution *U*(1,100), the deviations of *SPT-LOE*, *SPCT-LOE* and *INDEX-LOE*, over the optimal solution are 6.385%, 1.330%, and 1.088% on average, respectively. Since the lower bound value instead of the optimal solution is used to evaluate the performances of the heuristics, the actual values of the deviation of the heuristics from the optimal solution will be lower than those shown in Tables 4, 5 and 6.

The influences of the two processing time distributions associated with the three heuristics on the solution quality are shown in Figure 3. From Figure 3 and Table 4, it is clear that *INDEX-LOE* outperforms the other two heuristics, especially *SPT-LOE*, which is commonly used in practice. The impacts of the parameters α , β and γ on the solution quality are analyzed and depicted in Figure 3.

The influences of the parameters α , β , and γ on the solution quality are shown in Figure 3. From Figure 3, we observe that the deviation of the heuristic from the lower bound increases as the value of α increases, and the larger the value of β is, the smaller the deviation of the heuristic over the lower bound. Finally, the smaller the value of γ is, the slightly better

FIGURE 3. The influences of processing time distribution, α , β , and γ on the solution quality for large problems.

the quality of the heuristic. Although the three parameters α , β , and γ have impacts on the solution quality of the heuristics, the influence is rather small. Therefore, we can assure that the heuristics are robust and pragmatic.

VI. CONCLUSION

This study focuses on the problem of minimizing the total completion time on a single machine with nonresumable jobs and machine unavailability. This problem is motivated by a wafer fabrication process, where the machine has to be stopped to remove the accumulated dirt with a cleaning agent avoiding damage to the quality of a wafer. The objective is to minimize the total completion times.

The considered problem is *NP-hard*. Many properties for optimally solving the problem are developed. Based on these properties, an MBIP model is implemented to find optimal solutions with the commercial solver of IBM ILOG CPLEX Optimization Studio Ver. 12.6.1, and a lower bound and three simple and effective heuristics are also proposed. From the computational results with problem sizes $n \leq 50$, the best heuristic *INDEX-LOE* is 0.281% and 0.669% on average over the optimal solution for the processing time distributions $U(1,10)$ and $U(1,100)$, respectively. When $n = 100$ and 200, the *INDEX-LOE* is 0.30% and 1.088% on average over the lower bound for the processing time distributions with *U*(1,10) and *U*(1,100), respectively. The complexities of the heuristic are very low, but the algorithms can quickly obtain a near-optimal or optimal schedule to satisfy the quick response requirement in a real world environment.

The main contribution of this paper is that another type of machine unavailability that stems from a common practice in IC manufacturing industry, which has seldom been discussed in the scheduling literature, is considered. Another direction for future research is to extend the problem to other machine environments, such as identical or unrelated parallel machines. Otherwise, considering a due date related objective function is important because meeting due dates are also a concern in practical situations. Finally, developing a

meta-heuristic to solve the multiple machine problem will be worthwhile.

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