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Efficient Waveform Design Method for Target Estimation Under the Detection and Peak-to-Average Power Ratio Constraints in Cognitive Radar

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ABSTRACT This paper addresses the waveform design problem to estimate the target impulse response (TIR) of the temporally correlated extended target in a cognitive radar, subject to a detection constraint and a peak-to-average power ratio constraint. Owing to these types of constraints and the convolution operation of the waveform in the time domain, the formulated optimization problem for minimizing the mean square error of the estimated TIR based on Kalman filtering is a complex non-convex problem. To this end, an auxiliary variable is first introduced to modify the original problem, and the non-convex problem is converted to a convex problem with respect to the matrix variable. Then, a trick is used for replacing the matrix variable with the vector variable by utilizing the properties of the Toeplitz matrix. Moreover, the convex problem is further decomposed into three simple sub-problems which can be solved efficiently. Finally, the optimal waveform can be obtained efficiently through cognitive iteration combined with the nearest neighbor method. The simulation results illustrate that the proposed method is superior to the existing method in terms of the estimation performance and computational complexity when designing the constrained waveform.

INDEX TERMS Waveform design, cognitive radar, extended target, estimation performance, peak-to-average power ratio, Kalman filtering.

I. INTRODUCTION

Cognitive radar (CR) is a new intelligent closed-loop radar system that can perceive the surrounding complicated electromagnetism environment in real time and make reasoning decisions on this basis [1], [2]. In CR, adaptive transmitted waveform design based on the perceived prior knowledge of environment and target is one of the key technologies which can significantly improve the performance of target detection, parameter estimation, recognition, and tracking in complicated environment. Generally, the prior knowledge of the surrounding environment can be obtained by collecting the thermal noise and other interference before designing the waveform [3], [4]. Meanwhile, when the prior knowledge of target is considered to be known, the target can be modeled as a determinant target impulse response (TIR) function [5] or a stochastic process with a known distribution [6], [7].

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As is well known, it may cause the TIR to fluctuate when the relative angle between the target and radar changes [4]. Hence, Dai *et al.* [8] and Zhang and Cui [9] use the wide sense stationary-uncorrelated scattering (WSSUS) model to describe this type of extended target which can be called the temporally correlated target [3], [10]–[12]. In CR, this type of TIR needs to be tracked in real time and the estimated result should be fed back to the transmitter for the optimization of the transmitted waveform. Furthermore, Detection is absolutely an essential prerequisite for any estimation or recognition mission in a radar system [13]–[15].

Yao *et al.* [11] addressed the cognitive radar waveform design problem with such target under the detection performance constraint based on Kalman filtering (KF), then the obtained waveform had better estimation and detection performance. However, the envelope constraint on the transmitted waveform was not considered in this work, which made it difficult to meet the hardware constraints and maximize the power efficiency [16]. Therefore, unimodular or low peak-to-average power ratio (PAR) waveforms should be applied in radar systems [7], [16]. Nevertheless, unimodular waveform may lead to the degradation of waveform performance [17], a more general low-PAR constraint can be used instead of the unimodular constraint to further improve the waveform performance [10], [18], [19].

Owing to the nonhomogeneous inequality constraints of the detection and PAR, and the convolution operation of waveform, it is difficult to tackle the optimization problem in the time domain for minimizing the mean square error (MSE) of the estimated TIR based on KF [12]. Therefore, a frequency domain-based waveform design method was proposed in [3], [10], and [12], in which the semi-definite relaxation (SDR) was used to optimize the cognitive radar waveform. However, it is worth noting that the SDR will result in high complexity and the long sequences could hardly be handled.

In this paper, we propose an efficient cognitive waveform design method which is directly studied in the time domain. Based on KF, the minimization of the mean square error (MSE) of the estimated TIR is taken as the optimization criterion. The original problem can be converted to a convex problem by introducing an auxiliary variable and utilizing the properties of the Toeplitz matrix. With the objective of achieving high efficiency, the convex problem is further decomposed into three simple sub-problems which can be solved efficiently. Combined with the nearest neighbor method, the sub-problems are solved efficiently via cognitive iteration for a given PAR range and detection probability. Compared with the existing methods, the proposed method has lower computational complexity, and the synthesized waveform thus has the better estimation performance.

The remainder of the paper is organized as follows: Section 2 describes the waveform optimization model of the CR. In Sec. 3, the optimal criterion is formulated, an efficient waveform design method is proposed and a detailed computational complexity analysis is also provided. Section 4 presents our simulation results. Finally, the conclusions are summarized in Sec. 5.

Notation: Scalars are represented by italic letters, vectors and matrices are denoted by boldface lowercase and uppercase letters, respectively. The superscripts in $(\cdot)^T$ and $(\cdot)^H$ represent the transpose and Hermitian transpose operations, respectively. $\boldsymbol{A}(m,n)$ denotes the element located in the mth row and nth column of $\boldsymbol{A}$. $F(\cdot)$ denote the Toeplitz matrix mapping function of a vector, $\mathrm{tr}(\cdot)$ denotes the trace of a matrix, $\mathrm{vec}(\cdot)$ denotes vectorization of a matrix. $\Re(\cdot)$, $\Im(\cdot)$, $|\cdot|$, and $\|\cdot\|$ represent the real part, the imaginary part, modulus, and 2-norm of a complex scalar/vector/matrix, respectively. $\mathbb{C}$ is the set of complex numbers. The symbol "$\otimes$" and "$*$" denote the Kronecker product and the convolution operation, respectively. $\mathbb{F}(\cdot)$ is the Fourier transform matrix. Finally, $\mathcal{CN}(0,\boldsymbol{A})$ denotes a circular symmetric complex Gaussian distribution with zero mean and the covariance matrix $\boldsymbol{A}$.

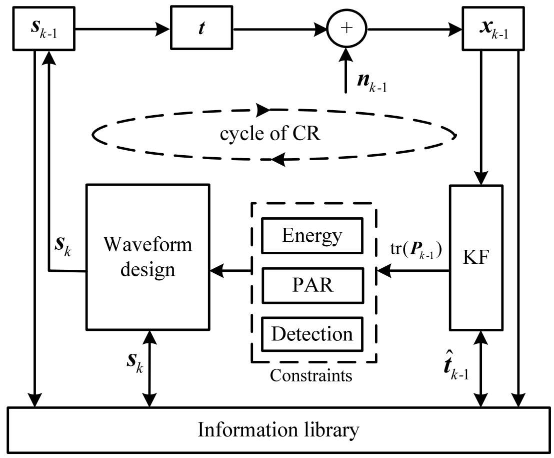

FIGURE 1. Waveform design model of CR.

II. SYSTEM MODEL

In this paper, we consider the waveform design of cognitive radar for temporally correlated target in noise. The scattering characteristic of target is represented by the target impulse response (TIR) [20]. According to the characteristics of the temporally correlated target, the target model [8] can be formulated as

$$\boldsymbol{t}_k = e^{-T_P/\tau}\boldsymbol{t}_{k-1} + z_{k-1}, \tag{1}$$

where $\boldsymbol{t}_k$ is the target vector at kth iteration, z_{k-1} is the fluctuation of TIR, T_P is the pulse repetition interval (PRI), and where τ is the temporal decay constant, which is determined by two factors: the velocity of the target and the angle between the moving direction of the target and the line of sight of radar. Then, $\boldsymbol{t}_k$ and z_{k-1} are mutually independent [20] and $\boldsymbol{t}_k \sim \mathcal{CN}(0,\boldsymbol{R}_t), z_{k-1} \sim \mathcal{CN}(0,\boldsymbol{R}_z)$, where $\boldsymbol{R}_z = \left(1-e^{-2T_P/\tau}\right)\boldsymbol{R}_t$. It is assumed that the influence of sidelobes has been mitigated by sidelobe blanking technology in front of the receiver. Meanwhile, we focus on the analysis of single-input single-output radar in this paper which can be straightforwardly extended to multiple-input multiple-output radar case. Then the waveform design model of CR can be depicted as shown in Fig. 1.

The information library in Fig. 1 is mainly used to store the prior knowledge of the environment and to update the knowledge of waveform and target. $\hat{\boldsymbol{t}}_{k-1}$ denotes the estimated value of TIR at $(k-1)$th iteration, $\boldsymbol{P}_{k-1}$ is the covariance matrix of the estimation error of estimated TIR based on KF, the symbol "tr" represents the trace of a matrix. Then $\mathrm{tr}(\boldsymbol{P}_{k-1})$ can represent the MSE of the estimated TIR [9], [10], and the smaller $\mathrm{tr}(\boldsymbol{P}_{k-1})$ is, the higher the estimation accuracy is. Thus, we can take minimization of $\mathrm{tr}(\boldsymbol{P}_{k-1})$ as the optimization criterion of waveform design under the transmitted energy, detection performance, and PAR constraints. $\boldsymbol{s} \in \mathbb{C}^{N_s\times 1}$ denotes a transmitted waveform with length N_s, $\boldsymbol{n} \in \mathbb{C}^{N_n\times 1}$ denotes the sum of the noise and the interference, $N_n = N_s + N_t - 1$, $\boldsymbol{n} \sim \mathcal{CN}(0,\boldsymbol{R}_n)$, and N_t is the length of target. Then, the kth echo signal can be expressed as

$$\boldsymbol{x}_k = \boldsymbol{s}_k * \boldsymbol{t}_k + \boldsymbol{n}_k = \boldsymbol{S}_k \boldsymbol{t}_k + \boldsymbol{n}_k = \boldsymbol{T}_k \boldsymbol{s}_k + \boldsymbol{n}_k, \tag{2}$$

where $\boldsymbol{x} \in \mathbb{C}^{N\times 1}$ with the length $N = N_s + N_t - 1$. $\boldsymbol{S}_k\boldsymbol{t}_k = \boldsymbol{T}_k\boldsymbol{s}_k$ can be obtained due to the reciprocity of the convolution operation. The convolution matrices $\boldsymbol{S}$ and $\boldsymbol{T}$ are Toeplitz matrices corresponding to s and $\boldsymbol{t}$, respectively. We use the function "$F(\bullet)$" represents their mapping relationship in this paper, i.e., $\boldsymbol{T} = F(\boldsymbol{t}), \boldsymbol{S} = F(\boldsymbol{s})$. Taking transmitted waveform as an example, the convolution matrix $\boldsymbol{S}$ can be written as

$$\boldsymbol{S} = \begin{bmatrix} \boldsymbol{s}(1) & 0 & \cdots & 0 \\ \vdots & \boldsymbol{s}(1) & \ddots & \vdots \\ s(N_s) & \vdots & \ddots & 0 \\ 0 & s(N_s) & \ddots & s(1) \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & s(N_s) \end{bmatrix} \in \mathbb{C}^{(N_s+N_t-1)\times N_t}. \tag{3}$$

III. WAVEFORM DESIGN METHOD

In this section, an efficient cognitive radar waveform design method for temporally correlated target is proposed. Unlike the existing works [3], [10], [12] for solving the optimization problem based on SDR in frequency domain, which may not only cause the relaxation of the original time domain problem but also has high computational complexity, an efficient optimization method that is directly solved in time domain is devised.

A. PAR CONSTRAINT AND DETECTION CONSTRAINT

Without any loss of generality, it can be assumed that the waveform is energy-limited and the total energy of the transmitted waveform is N_s. Then the PAR can be defined as

$$\mathrm{PAR}(\boldsymbol{s}) \frac{\max_i |\boldsymbol{s}(i)|^2}{\frac{1}{N_s}\sum_{i=1}^{N_s} |\boldsymbol{s}(i)|^2} = \max_i |\boldsymbol{s}(i)|^2 \le \eta, \quad \eta \in [1, N_s], \tag{4}$$

where $\boldsymbol{s}(i)$ is the ith element of $\boldsymbol{s}$, and η is a predefined parameter that denotes the maximum allowed PAR. Note that the PAR constraint is equivalent to a unimodular constraint when $\eta = 1$, whereas it becomes a redundant constraint when $\eta = N_s$.

Besides, the detection probability of the waveform under the constant false alarm rate (CFAR) criterion can be expressed as [21]

$$P_D(\alpha) = Q\left[Q^{-1}(\alpha) - \sqrt{2\hat{\mu}_k}\right], \tag{5}$$

where $\hat{\mu}_k = \sum_{j=0}^{k} s_j^H \hat{\boldsymbol{T}}_j^{\mathrm{H}} \boldsymbol{R}_n^{-1} \hat{\boldsymbol{T}}_j s_j$, $\hat{\boldsymbol{T}}_j = F(\hat{\boldsymbol{t}}_j)$, and $\hat{\boldsymbol{t}}_j$ denotes the estimated value of TIR at jth iteration of KF. α $(0 < \alpha < 1)$ denotes the CFAR, and $Q(\bullet)$ represents right tail probability distribution function of standard Gaussian distribution, $Q(\boldsymbol{x}) = \int_x^\infty p(\boldsymbol{x})\,dx$, $p(\boldsymbol{x})$ is the probability density function of standard Gauss distribution. Assuming that the given detection probability is P_γ, the constraint of detection performance can be formulated as

$$s_k^H \hat{\boldsymbol{T}}_k^{\mathrm{H}} \boldsymbol{R}_n^{-1} \hat{\boldsymbol{T}}_k s_k \ge \varepsilon, \tag{6}$$

where $\varepsilon = \left[Q^{-1}(\alpha) - Q^{-1}(P_\gamma)\right]^2 \big/ 2 - \sum_{j=0}^{k-1} s_j^H \hat{\boldsymbol{T}}_j^H \boldsymbol{R}_n^{-1} \hat{\boldsymbol{T}}_j s_j$.

TABLE 1. The recursion process of Kalman filtering.

1: Initial state
$\hat{\boldsymbol{t}}_{0\vert 0} = \left(\boldsymbol{S}_0^{\mathrm{H}}\boldsymbol{R}_n^{-1}\boldsymbol{S}_0\right)^{-1}\boldsymbol{S}_0^{\mathrm{H}}\boldsymbol{R}_n^{-1}\boldsymbol{x}_0,$ (7)
$\boldsymbol{P}_{0\vert 0} = \left(\boldsymbol{S}_0^{\mathrm{H}}\boldsymbol{R}_n^{-1}\boldsymbol{S}_0\right)^{-1},$ (8)
2: Time update
$\hat{\boldsymbol{t}}_{k\vert k-1} = e^{-T/\tau}\hat{\boldsymbol{t}}_{k-1\vert k-1},$ (9)
$\boldsymbol{P}_{k\vert k-1} = e^{-2T/\tau}\boldsymbol{P}_{k-1\vert k-1} + \boldsymbol{R}_u,$ (10)
3: Measurement update
$\boldsymbol{K}_k = \boldsymbol{P}_{k\vert k-1}\boldsymbol{S}_k^{\mathrm{H}}\left(\boldsymbol{R}_n + \boldsymbol{S}_k\boldsymbol{P}_{k\vert k-1}\boldsymbol{S}_k^{\mathrm{H}}\right)^{-1},$ (11)
$\hat{\boldsymbol{t}}_{k\vert k} = \hat{\boldsymbol{t}}_{k\vert k-1} + \boldsymbol{K}_k\left(\boldsymbol{x}_k - \boldsymbol{S}_k\hat{\boldsymbol{t}}_{k\vert k-1}\right),$ (12)
$\boldsymbol{P}_{k\vert k} = \boldsymbol{P}_{k\vert k-1} - \boldsymbol{K}_k\boldsymbol{S}_k\boldsymbol{P}_{k\vert k-1},$ (13)

B. PROBLEM FORMULATION

Since the prior information of the target is unknown, the target should be estimated before waveform design. According to [9], the KF can be used to estimate and predict the TIR, and the recursion process is shown in Table 1.

In Table 1, $\boldsymbol{P}_{k|k}$ is the MSE matrix which can be obtained by using the $k+1$ measurement data (including $k = 0$) and $\hat{\boldsymbol{t}}_{k|k}$ denotes the estimation of t. $\hat{\boldsymbol{t}}_{0|0}$ and $\boldsymbol{P}_{0|0}$ can be obtained by using the minimum variance unbiased (MVU) estimating method [22]. Then, the problem of waveform design can be turned to the minimization of $\mathrm{tr}\left(\boldsymbol{P}_{k|k}\right)$, which can be expressed as

$$\min_{\boldsymbol{S}_k} \mathrm{tr}(\boldsymbol{P}_{k|k}) = \min_{\boldsymbol{S}_k} \mathrm{tr}\left(\boldsymbol{P}_{k|k-1} - \boldsymbol{P}_{k|k-1}\boldsymbol{S}_k^{\mathrm{H}} \times \left(\boldsymbol{R}_n + \boldsymbol{S}_k\boldsymbol{P}_{k|k-1}\boldsymbol{S}_k^{\mathrm{H}}\right)^{-1}\boldsymbol{S}_k\boldsymbol{P}_{k|k-1}\right). \tag{14}$$

As we can see, the expression in (14) is complicated which can be simplified by utilizing the matrix inversion lemma [23], then we can get

$$\boldsymbol{P}_{k|k-1} - \boldsymbol{P}_{k|k-1}\boldsymbol{S}_k^{\mathrm{H}}\left(\boldsymbol{R}_n + \boldsymbol{S}_k\boldsymbol{P}_{k|k-1}\boldsymbol{S}_k^{\mathrm{H}}\right)^{-1}\boldsymbol{S}_k\boldsymbol{P}_{k|k-1} = \left(\boldsymbol{P}_{k|k-1}^{-1} + \boldsymbol{S}_k^{\mathrm{H}}\boldsymbol{R}_n^{-1}\boldsymbol{S}_k\right)^{-1}. \tag{15}$$

Therefore, the optimization criterion can be turned to

$$\min_{\boldsymbol{S}_k} \mathrm{tr}\left(\boldsymbol{P}_{k|k-1}^{-1} + \boldsymbol{S}_k^{\mathrm{H}}\boldsymbol{R}_n^{-1}\boldsymbol{S}_k\right)^{-1}. \tag{16}$$

Thus, under the energy, detection, and PAR constraints, the optimization problem can be written as

$$\begin{cases} \min_{s_k} \operatorname{tr}\left(\boldsymbol{P}_{k|k-1}^{-1} + \boldsymbol{S}_k^{\mathrm{H}} \boldsymbol{R}_n^{-1} \boldsymbol{S}_k\right)^{-1} \\ \text{s.t. } \boldsymbol{s}_k^H \boldsymbol{s}_k \leq E_s \\ \quad |\boldsymbol{s}_k(i)|^2 \leq \eta, \quad i = 1, 2, \cdots, N_s \\ \quad \boldsymbol{s}_k^H \hat{\boldsymbol{T}}_k^{\mathrm{H}} \boldsymbol{R}_n^{-1} \hat{\boldsymbol{T}}_k \boldsymbol{s}_k \geq \varepsilon \\ \quad \boldsymbol{S}_k = F(\boldsymbol{s}_k). \end{cases} \tag{17}$$

We can note that the objective function of the problem in (17) is non-convex, the first two quadratic inequality constraints are nonhomogeneous and the third constraint is non-convex. Hence, this optimization problem is a complex non-convex problem which is difficult to tackle [24]. One possible way to tackle this problem is that to convert it to frequency domain and to apply the convex optimization method based on SDR [3], [10], [12] which has an approximate computational complexity of $\mathrm{O}\left(N_s^{4.5}\right)$ at each iteration [25]. As a result, SDR will bring a high computation cost especially when N_s is large. Therefore, an efficient optimization method is needed.

C. WAVEFORM DESIGN

With the objective of reducing the computation cost and making the synthesized waveform closer to the optimal solution of the original problem, we tackle the problem of (17) with a more efficient method. Firstly, we should convert the detection constraint to a convex set. According to the rank-one approximation method [25], let $\boldsymbol{W} = \hat{\boldsymbol{T}}_k^{\mathrm{H}} \boldsymbol{R}_n^{-1} \hat{\boldsymbol{T}}_k$, we can get that $\boldsymbol{W} \approx \theta_{\max} \boldsymbol{w}_{\max} \boldsymbol{w}_{\max}^H$, where $\boldsymbol{w}_{\max}$ is the eigenvector corresponding to the maximum eigenvalue $\theta_{\max}$ of $\boldsymbol{W}$. Then we can have

$$\Re(\boldsymbol{s}_k^{\mathrm{H}} \boldsymbol{w}_{\max}) \geq \sqrt{\varepsilon / \theta_{\max}}. \tag{18}$$

It can be seen that the detection constraint in (18) is a convex set. Then, the optimization problem in (17) can be recast as

$$\begin{cases} \min_{s_k} \operatorname{tr}\left(\boldsymbol{P}_{k|k-1}^{-1} + \boldsymbol{S}_k^{\mathrm{H}} \boldsymbol{R}_n^{-1} \boldsymbol{S}_k\right)^{-1} \\ \text{s.t. } \boldsymbol{s}_k^H \boldsymbol{s}_k \leq E_s \\ \quad |\boldsymbol{s}_k(i)|^2 \leq \eta, \quad i = 1, 2, \cdots, N_s \\ \quad \Re(\boldsymbol{s}_k^{\mathrm{H}} \boldsymbol{w}_{\max}) \geq \sqrt{\varepsilon / \theta_{\max}} \\ \quad \boldsymbol{S}_k = F(\boldsymbol{s}_k). \end{cases} \tag{19}$$

The key of the proposed method is to covert the non-convex problem in (19) to a convex problem which can be solved efficiently. To this end, we introduce an auxiliary variable $\Delta \boldsymbol{S}_k$, which is the incremental matrix of $\boldsymbol{S}_k$, (viz, $\boldsymbol{S}_k = \boldsymbol{S}_{k-1} + \Delta \boldsymbol{S}_k$) to modify the optimization problem. $\Delta \boldsymbol{S}_k$ is also a convolution matrix which has same structure as shown in (3) and $|\Delta \boldsymbol{S}_k(i)| \leq \delta, i = 1, 2, \cdots, NN_t$, where δ is a small real value. S_{k-1} is a known convolution matrix of waveform at $(k-1)$th iteration. Then, the objective function can be reformulated as

$$\begin{aligned} &\left(\boldsymbol{P}_{k|k-1}^{-1} + \boldsymbol{S}_k^{\mathrm{H}} \boldsymbol{R}_n^{-1} \boldsymbol{S}_k\right)^{-1} \\ &\quad = \left(\boldsymbol{P}_{k|k-1}^{-1} + (\boldsymbol{S}_{k-1} + \Delta \boldsymbol{S}_k)^H \boldsymbol{R}_n^{-1} (\boldsymbol{S}_{k-1} + \Delta \boldsymbol{S}_k)\right)^{-1} \\ &\quad = \left(\boldsymbol{P}_{k|k-1}^{-1} + \boldsymbol{S}_{k-1}^{\mathrm{H}} \boldsymbol{R}_n^{-1} \boldsymbol{S}_{k-1} + \boldsymbol{S}_{k-1}^{\mathrm{H}} \boldsymbol{R}_n^{-1} \Delta \boldsymbol{S}_k \right. \\ &\qquad \left. + \Delta \boldsymbol{S}_k^{\mathrm{H}} \boldsymbol{R}_n^{-1} \boldsymbol{S}_{k-1} + \Delta \boldsymbol{S}_k^{\mathrm{H}} \boldsymbol{R}_n^{-1} \Delta \boldsymbol{S}_k\right)^{-1} \\ &\quad \approx \left(\boldsymbol{P}_{k|k-1}^{-1} + \boldsymbol{S}_{k-1}^{\mathrm{H}} \boldsymbol{R}_n^{-1} \boldsymbol{S}_{k-1} + \boldsymbol{S}_{k-1}^{\mathrm{H}} \boldsymbol{R}_n^{-1} \Delta \boldsymbol{S}_k \right. \\ &\qquad \left. + \Delta \boldsymbol{S}_k^{\mathrm{H}} \boldsymbol{R}_n^{-1} \boldsymbol{S}_{k-1}\right)^{-1}. \end{aligned} \tag{20}$$

Define $\boldsymbol{Y}_k = \boldsymbol{P}_{k|k-1}^{-1} + \boldsymbol{S}_{k-1}^{\mathrm{H}} \boldsymbol{R}_n^{-1} \boldsymbol{S}_{k-1}$, $\Delta \boldsymbol{Y}_k = \boldsymbol{S}_{k-1}^{\mathrm{H}} \boldsymbol{R}_n^{-1} \Delta \boldsymbol{S}_k + \Delta \boldsymbol{S}_k^{\mathrm{H}} \boldsymbol{R}_n^{-1} \boldsymbol{S}_{k-1}$, then we have

$$\begin{aligned} &(\boldsymbol{Y}_k + \Delta \boldsymbol{Y}_k)^{-1} \\ &\quad = \left[\boldsymbol{Y}_k \left(\boldsymbol{I} + \boldsymbol{Y}_k^{-1} \Delta \boldsymbol{Y}_k\right)\right]^{-1} = \left(\boldsymbol{I} + \boldsymbol{Y}_k^{-1} \Delta \boldsymbol{Y}_k\right)^{-1} \boldsymbol{Y}_k^{-1}. \end{aligned} \tag{21}$$

According to [26], $(\boldsymbol{I} + \boldsymbol{A})^{-1}$ is equal to $\boldsymbol{I} - \boldsymbol{A}$ if $\boldsymbol{A}$ is small, viz.

$$(\boldsymbol{I} + \boldsymbol{A})^{-1} \cong \boldsymbol{I} - \boldsymbol{A}. \tag{22}$$

Then we have

$$\begin{aligned} &\left(\boldsymbol{I} + \boldsymbol{Y}_k^{-1} \Delta \boldsymbol{Y}_k\right)^{-1} \boldsymbol{Y}_k^{-1} \\ &\cong \left(\boldsymbol{I} - \boldsymbol{Y}_k^{-1} \Delta \boldsymbol{Y}_k\right) \boldsymbol{Y}_k^{-1} \\ &= \boldsymbol{Y}_k^{-1} - \boldsymbol{Y}_k^{-1} \boldsymbol{S}_{k-1}^{\mathrm{H}} \boldsymbol{R}_n^{-1} \Delta \boldsymbol{S}_k \boldsymbol{Y}_k^{-1} - \boldsymbol{Y}_k^{-1} \Delta \boldsymbol{S}_k^{\mathrm{H}} \boldsymbol{R}_n^{-1} \boldsymbol{S}_{k-1} \boldsymbol{Y}_k^{-1}. \end{aligned} \tag{23}$$

where $\boldsymbol{Y}_k$ is a known matrix. Let $\boldsymbol{Z}_k = \boldsymbol{S}_{k-1}^{\mathrm{H}} \boldsymbol{R}_n^{-1}$, ignoring the constant term and the optimization criterion in (16) can be reformulated as

$$\min_{\Delta \boldsymbol{S}_k} \operatorname{tr}\left(-\boldsymbol{Y}_k^{-1} \boldsymbol{Z}_k \Delta \boldsymbol{S}_k \boldsymbol{Y}_k^{-1} - \boldsymbol{Y}_k^{-1} \Delta \boldsymbol{S}_k^{\mathrm{H}} \boldsymbol{Z}_k \boldsymbol{Y}_k^{-1}\right). \tag{24}$$

It can be seen that the objective function in (24) is a convex function with respect to matrix variable $\Delta \boldsymbol{S}_k$ [24]. Then, we show that we can recast the objective function of (24) as a function with respect to the vector variable $\Delta \boldsymbol{s}_k (\Delta \boldsymbol{S}_k = \mathrm{F}(\Delta \boldsymbol{s}_k))$. By using the identities that $\operatorname{tr}(\boldsymbol{ABCD}) = \operatorname{vec}^T\left(\boldsymbol{A}^T\right)\left(\boldsymbol{D}^T \otimes \boldsymbol{B}\right) \operatorname{vec}(\boldsymbol{C}) = \operatorname{vec}^T\left(\boldsymbol{B}^T\right)\left(\boldsymbol{A}^T \otimes \boldsymbol{C}\right) \operatorname{vec}(\boldsymbol{D})$ [27], we can rewrite the objective of (24) as

$$-\boldsymbol{u} \operatorname{vec}(\Delta \boldsymbol{S}_k) - \operatorname{vec}^T\left(\left(\Delta \boldsymbol{S}_k^H\right)^T\right) \boldsymbol{v}, \tag{25}$$

where $\boldsymbol{u} \in \mathbb{C}^{1 \times NN_t}$, and $\boldsymbol{u} = \operatorname{vec}^T\left(\left(\boldsymbol{Y}_k^{-1}\right)^T\right) \left(\left(\boldsymbol{Y}_k^{-1}\right)^T \otimes \boldsymbol{Z}_k\right)$, $\boldsymbol{v} \in \mathbb{C}^{NN_t \times 1}$, and $\boldsymbol{v} = \left(\left(\boldsymbol{Y}_k^{-1}\right)^T \otimes \boldsymbol{Z}_k\right) \operatorname{vec}\left(\boldsymbol{Y}_k^{-1}\right)$.

Considering that $\Delta \boldsymbol{S}_k$ is a convolution matrix with Toeplitz structure (shown in (3)) which consists of $\Delta \boldsymbol{s}_k$, we can recast the objective function of (25) as

$$-\boldsymbol{u}'\Delta \boldsymbol{s}_k - \Delta \boldsymbol{s}_k^H \boldsymbol{v}', \tag{26}$$

where $\boldsymbol{u}' \in \mathbb{C}^{1\times N_s}$, $\boldsymbol{u}' = \sum_{i=1}^{N_t} \boldsymbol{u}_i'$, $\boldsymbol{u}_i' = \big[u((i-1)*N+i), u((i-1)*N+i+1), \cdots, u((i-1)*N+i+N_s-1)\big]$, and $\boldsymbol{v}' \in \mathbb{C}^{N_s\times 1}$, $\boldsymbol{v}' = \sum_{i=1}^{N_t} \boldsymbol{v}_i'$, $\boldsymbol{v}_i' = [v((i-1)*N+i), v((i-1)*N+i+1), \cdots, v((i-1)*N + i+N_s-1)]^T$. Then, the optimization criterion can be reformulated as

$$\min_{\Delta \boldsymbol{s}_k} \boldsymbol{f}\Delta \boldsymbol{s}_k, \tag{27}$$

where

$$\boldsymbol{f} = -\boldsymbol{u}' - \left(\boldsymbol{v}'\right)^H. \tag{28}$$

Therefore, the optimization problem in (19) can be rewritten as

$$\begin{cases} \min\limits_{\Delta \boldsymbol{s}_k} \boldsymbol{f}\Delta \boldsymbol{s}_k \\ \text{s.t. } \boldsymbol{s}_k^H \boldsymbol{s}_k \le E_s \\ \qquad |\boldsymbol{s}_k(i)|^2 \le \eta, \quad i = 1, 2, \cdots, N_s \\ \qquad \Re(\boldsymbol{s}_k^{\mathrm{H}} \boldsymbol{w}_{\max}) \ge \sqrt{\varepsilon/\theta_{\max}} \\ \qquad |\Delta \boldsymbol{s}_k(i)| \le \delta, \quad \boldsymbol{s}_k = \boldsymbol{s}_{k-1} + \Delta \boldsymbol{s}_k, \end{cases} \tag{29}$$

where $\boldsymbol{s}_{k-1}$ is a known vector at $(k-1)$th iteration. So far, the problem of (29) becomes a convex problem. One possible way to tackle this problem is applying the interior point method with CVX toolbox [28]. Denote $\boldsymbol{s}_k^*$ is the solution of (29), it is worth noting that $\boldsymbol{s}_k^*$ may not be the true optimal solution at kth iteration, because the available region of the $\Delta \boldsymbol{s}_k$ is small. That is to say, the true optimal solution at kth iteration can be obtained by solving the problem of (29) repeatedly until the termination criterion is met to ensure that all the available region of $\boldsymbol{s}_k$ can be searched. However, the interior point method has a computational complexity of $\mathrm{O}\left(N_s^{3.5} \log \xi^{-1}\right)$ (where ξ is the accuracy of algorithm) [25], and it would be performed repeatedly until the termination criterion is met in the process from the $(k-1)$th iteration to the kth iteration. Although its computational complexity is lower than the method in [3], [10], and [12], it still brings a heavy computational burden. To this end, a fast optimization method is developed.

D. WAVEFORM DESIGN FAST HIERARCHICAL OPTIMIZATION METHOD

In this subsection, we shall present a fast hierarchical optimization method which includes inner iteration and outer iteration to tackle the problem in (29).

1) INNER ITERATION

Since the objective function in (29) is a linear function with respect to $\Delta \boldsymbol{s}_k$, and $|\Delta \boldsymbol{s}_k(i)| \le \delta$ is a linear constraint with respect to $\Delta \boldsymbol{s}_k$. So first we can construct a standard linear programming problem, viz., sub-problem 1, which can be formulated as

$$\begin{cases} \min\limits_{\Delta \boldsymbol{s}_{k,q}} \boldsymbol{f}\Delta \boldsymbol{s}_{k,q} \\ \text{s.t. } \left|\Delta \boldsymbol{s}_{k,q}(i)\right| \le \delta, \quad i = 1, 2, \cdots, N_s, \end{cases} \tag{30}$$

where the subscript k and q denote the serial number of outer iteration and inner iteration, respectively. As is well known, the sub-problem 1 is a standard linear programming problem, so it can be solved efficiently by using the simplex method [29] with the function "linprog" in MATLAB. Moreover, since the "linprog" is only used to deal with real-valued problems, so (30) is reformulated as

$$\boldsymbol{SP}_1 \begin{cases} \min\limits_{\tilde{\Delta} \boldsymbol{s}_{k,q}} \tilde{\boldsymbol{f}}\tilde{\Delta} \boldsymbol{s}_{k,q} \\ \text{s.t. } \left|\tilde{\Delta} \boldsymbol{s}_{k,q}(i)\right| \le \delta, \quad i = 1, 2, \cdots, N_s, \end{cases} \tag{31}$$

where $\tilde{\boldsymbol{f}} = \left[\Re\left(\boldsymbol{f}^T\right) \Im\left(\boldsymbol{f}^T\right)\right]^T$, $\tilde{\Delta} \boldsymbol{s}_{k,q} = \left[\Re\left(\Delta \boldsymbol{s}_{k,q}^T\right) \Im\left(\Delta \boldsymbol{s}_{k,q}^T\right)\right]^T$. Denote the solution of $\boldsymbol{SP}_1$ is $\tilde{\Delta} \boldsymbol{s}_{k,q}^*$, and its complex-valued can be denoted as $\Delta \boldsymbol{s}_{k,q}^*$. Therefore, the candidate waveform at kth inner iteration can be written as

$$\bar{\boldsymbol{s}}_{k,q}^* = \boldsymbol{s}_{k,q-1} + \Delta \boldsymbol{s}_{k,q}^*, \tag{32}$$

where $\boldsymbol{s}_{k,q-1}$ is the known vector at $(k, q-1)th$ iteration.

Next, considering the transmitted energy and PAR constraint, the sub-problem 2 can be expressed as

$$\boldsymbol{SP}_2 \begin{cases} \min\limits_{\boldsymbol{s}_{k,q}} \left\|\boldsymbol{s}_{k,q} - \bar{\boldsymbol{s}}_{k,q}^*\right\|_2^2 \\ \text{s.t. } \boldsymbol{s}_{k,q}^H \boldsymbol{s}_{k,q} \le E_s \\ \qquad \left|\boldsymbol{s}_{k,q}(i)\right|^2 \le \eta, \quad i = 1, 2, \cdots, N_s. \end{cases} \tag{33}$$

It can be seen that $\boldsymbol{SP}_2$ is a convex problem which can be solved by using the interior point method. However, we can find that the form of objective function and constraints of $\boldsymbol{SP}_2$ are the same as the nearest neighbor method with complexity of $\mathrm{O}\left(N_s^2\right)$ [30]. Thus, $\boldsymbol{SP}_2$ can be tackled efficiently through the nearest neighbor method. Then, we can get the optimal solution $\boldsymbol{s}_{k,q}^*$ at kth inner iteration until the termination criterion is met by solving $\boldsymbol{SP}_1$ and $\boldsymbol{SP}_2$ iteratively.

2) OUTER ITERATION

Since the detection constraint is not considered in inner iteration, the solution $\boldsymbol{s}_{k,q}^*$ may not meet the requirements of detection performance. Thus, let $\bar{\boldsymbol{s}}_k^* = \boldsymbol{s}_{k,q}^*$ be the candidate solution, the sub-problem 3 can be formulated as

$$\boldsymbol{SP}_3 \begin{cases} \min\limits_{\boldsymbol{s}_k} \left\|\boldsymbol{s}_k - \bar{\boldsymbol{s}}_k^*\right\|_2^2 \\ \text{s.t. } \tilde{\boldsymbol{s}}_k^H \tilde{\boldsymbol{s}}_k \le E_s \\ \qquad |\boldsymbol{s}_k(i)|^2 \le \eta, \quad i = 1, 2, \cdots, N_s \\ \qquad \tilde{\boldsymbol{s}}_k^{\mathrm{H}} \tilde{\boldsymbol{w}}_{\max} \ge \sqrt{\varepsilon/\theta_{\max}}, \end{cases} \tag{34}$$

where $\tilde{\boldsymbol{s}}_k = \left[\Re\left(\boldsymbol{s}_k^T\right) \Im\left(\boldsymbol{s}_k^T\right)\right]^T$, $\tilde{\boldsymbol{w}}_{\max} = \left[\Re\left(\boldsymbol{w}_{\max}^T\right) \Im\left(\boldsymbol{w}_{\max}^T\right)\right]^T$. $\boldsymbol{SP}_3$ is a convex problem which can be solved by using

TABLE 2. **The proposed waveform design method for target estimation.**

Step 0: $k=0$, generate a random waveform $\boldsymbol{s}_k$ with $\boldsymbol{s}_k^{\mathrm{H}}\boldsymbol{s}_k = E_s$, let $\boldsymbol{s}_{k,0}=\boldsymbol{s}_k$ and $\boldsymbol{S}_k = \mathrm{F}(\boldsymbol{s}_k)$. Also, initialize σ_1, σ_2, κ_1 and κ_2.

Step 1: Compute $\boldsymbol{P}_{k+1|k}$ and $\boldsymbol{f}$ via (10) and (28).

Step 2: $q=1$.

Step 2-1: Solve $\boldsymbol{SP}_1$ to obtain $\tilde{\Delta}\boldsymbol{s}_{k,q}^*$, further obtain the complex-valued $\Delta\boldsymbol{s}_{k,q}^*$. Compute $\overline{\boldsymbol{s}}_{k,q}^*$ via (32).

Step 2-2: Solve $\boldsymbol{SP}_2$ to obtain $\boldsymbol{s}_{k,q}^*$. Let $\boldsymbol{s}_{k,q}=\boldsymbol{s}_{k,q}^*$. $q=q+1$.

Step 2-3: Repeat steps 2-1 and 2-2 until $\tau_1 \le \sigma_1$ or $q \ge \kappa_1$.

Step 3: Denote the solution obtained in step 2 by $\overline{\boldsymbol{s}}_k^*$ and solve $\boldsymbol{SP}_3$ to obtain $\boldsymbol{s}_k^*$. Let $\boldsymbol{s}_{k+1}=\boldsymbol{s}_{k+1,0}=\boldsymbol{s}_k^*$. $k=k+1$.

Step 4: Repeat step 1 to step 3 until $\tau_2 \le \sigma_2$ or $k \ge \kappa_2$.

interior point method (only one time) to get the optimal solution $\boldsymbol{s}_k^*$ with complexity of $\mathrm{O}\left(N_s^{3.5}\right)$ [25].

We now discuss the termination criterion. By substituting the optimal waveforms $\boldsymbol{s}_{k,q}^*$ and $\boldsymbol{s}_{k,q-1}^*$ of the $(k,q)th$ and $(k,q-1)th$ inner iteration into (13), we can get $\boldsymbol{P}_{k,q}$ and $\boldsymbol{P}_{k,q-1}$, respectively. Then, the termination criterion of inner iteration can be denoted by $\tau_1 = \left|\mathrm{tr}\left(\boldsymbol{P}_{k,q}\right) - \mathrm{tr}\left(\boldsymbol{P}_{k,q-1}\right)\right|^2 \le \sigma_1$. By substituting the optimal waveforms $\boldsymbol{s}_k^*$ and $\boldsymbol{s}_{k-1}^*$ of the kth and outer iteration into (13), we can get $\boldsymbol{P}_k$ and $\boldsymbol{P}_{k-1}$, respectively. Then, termination criterion of outer iteration can be denoted by $\tau_2 = |\mathrm{tr}(\boldsymbol{P}_k) - \mathrm{tr}(\boldsymbol{P}_{k-1})|^2 \le \sigma_2$. Here σ_1 and σ_2 are termination tolerances of inner and outer iteration, respectively. In addition, the maximum iterative number of inner and outer iteration are denoted by κ_1 and κ_2, respectively.

According to the above steps, the proposed method is summarized in Table 2.

3) COMPUTATIONAL COMPLEXITY ANALYSIS

The original optimization problem can be converted into three sub-problems that can be solved efficiently. The first two sub-problems are solved in inner iteration, in which the simplex method and nearest neighbor method are used with complexity of $\mathrm{O}(N_s)$ and $\mathrm{O}\left(N_s^2\right)$, respectively. Since the convergence rate of inner iteration is difficult to analyze theoretically, the average iteration times denoted by L is used to express the convergence rate. Through a large number of Monte Carlo experiments, it is found that L is always smaller than 30 (which is demonstrated in Fig. 2). The third sub-problem is solved in outer iteration, in which the interior point method is applied to tackle the MSE problem [25] and eigenvalue decomposition is also used, with complexity of $\mathrm{O}\left(N_s^{3.5}\right)$ and $\mathrm{O}\left(N_s^3\right)$, respectively. Therefore, the total computation complexity of the proposed method is $\mathrm{O}\left(LN_s + LN_s^2 + N_s^{3.5} + N_s^3\right)$ in each iteration. For comparison, we employed the convex optimization method (COM) in [3], [10], and [12]. Since this kind of method is based on Kalman filtering, for convenience, it is denoted by

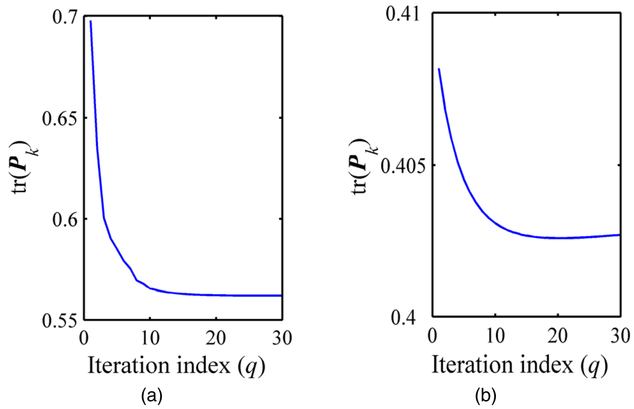

FIGURE 2. **The convergence of inner iteration. (a) k = 1. (b) k = 2.**

"COM-KF" which means the "Convex Optimization Method based on Kalman filtering" in this paper. It needs to tackle the problems of SDR and MSE with interior point method, and to compute eigenvalue decomposition twice. Their computational complexity are $\mathrm{O}\left(N_s^{4.5}\right)$, $\mathrm{O}\left(N_s^{3.5}\right)$ and $\mathrm{O}\left(2N_s^3\right)$, respectively. In summary, the computation complexity of the COM-KF is $\mathrm{O}\left(N_s^{4.5} + N_s^{3.5} + 2N_s^3\right)$ in each iteration.

IV. SIMULATION RESULTS

Several numerical simulations were performed to demonstrate the performance of the proposed method. Let the length of the transmitted waveform be $N_s = 10$ and the total transmitted energy be $E_s = N_s$. The initial transmitted waveform $\boldsymbol{s}_0$ was generated by a random phase-coded signal and the waveform convolution matrix $\boldsymbol{S}_0 = \mathrm{F}(\boldsymbol{s}_0)$. Supposing the noise to be white Gaussian, we let noise covariance matrix be $\boldsymbol{R_n} = \sigma_n^2\boldsymbol{I}_N$, where σ_n^2 denotes the variance of noise, and the signal-to-noise ratio (SNR) of the echo signal is 7 dB. Meanwhile, set $N_t = 10$, the target covariance matrix $\boldsymbol{R_t} = \sigma_t^2\boldsymbol{R}_{(t)}$, where $\sigma_t^2 = 1$ and $\boldsymbol{R}_{(t)} = \boldsymbol{U_t}\boldsymbol{\Lambda}_t\boldsymbol{U}_t^H$ was the normalized covariance matrix. According to [31], $\boldsymbol{\Lambda_t} \in \mathbb{C}^{N_t \times N_t}$ is a diagonal matrix and $\boldsymbol{U_t}$ is the unitary matrix with its $(n,m)\,th$ entry given by

$$\frac{1}{\sqrt{N_t}}\exp\left[\frac{-j2\pi\,(n-1)\,(m-1)}{N_t}\right], \quad \forall n,m \in [1,N_t]. \quad (35)$$

In addition, we performed 300 Monte Carlo trials for each combination of parameters. We set PRI $T_P = 1ms$ and temporal decay constant $\tau = 0.1$s. The termination tolerances were $\sigma_1 = \sigma_2 = 10^{-3}$, and the maximum iterative numbers were $\kappa_1 = 30$ and $\kappa_2 = 20$. Furthermore, the false alarm probability was $\alpha = 0.05$ and the detection probability was $P_\gamma = 0.95$. The Matlab 2013b version was used to perform the simulations with a standard PC (CPU Core i5-3230M 2.6GHz and 4GB RAM).

A. EFFECTIVENESS VERIFICATION

In this subsection, we demonstrate the effectiveness of the proposed method. We set PAR to $\eta = 2$, and the available

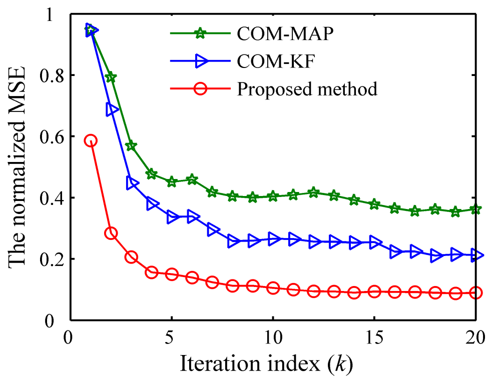

FIGURE 3. Normalized MSE of different methods versus outer iteration number k.

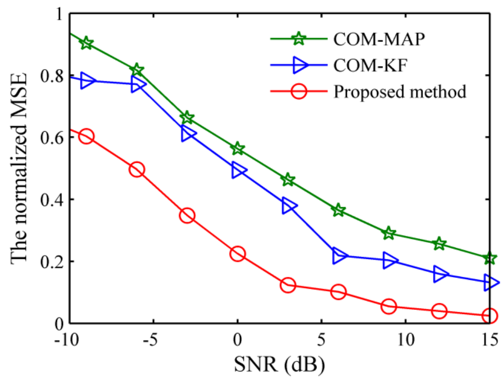

FIGURE 4. Normalized MSE of different methods versus SNR.

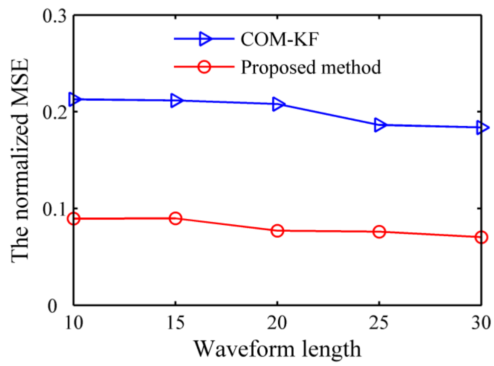

FIGURE 5. Normalized MSE of different methods versus waveform length.

region of $\Delta \boldsymbol{s}_k$ to be $\delta = 0.1$. respectively. Figs. 2(a) and (b) show the convergence performance of the proposed method in inner iteration when the sequence numbers of outer iteration are $k = 1$ and $k = 2$, respectively. The MSE matrix $\boldsymbol{P}_k$ can be obtained via (13). It can be seen that both the curves are monotonically convergent within 30 iterations, which indicate that the estimation accuracy of the target is gradually improved as the number of iterations increases. Thus, the effectiveness of the internal iteration is verified.

Next, the proposed method is compared with existing methods. For convenience, another "Convex Optimization Method based on Maximum A Posteriori criterion" in [3], [10], and [12] is denoted by "COM-MAP" in this paper. As we can see, both the COM-KF and COM-MAP are studied based on frequency domain. In order to make a comparison under the same standard, the target estimation value $\hat{t}_k$ obtained by the proposed method is converted into the frequency domain through Fourier transform. Let $\hat{\boldsymbol{g}}_k = \mathbb{F}\left(\hat{t}_k\right), \boldsymbol{g}_k = \mathbb{F}(\boldsymbol{t}_k)$, the symbol "$\mathbb{F}$" denotes the Fourier transform matrix. In order to average the simulation results, 50 random targets are generated to perform the experiments. Then, estimation performance can be evaluated with the normalized MSE criterion

$$n\text{MSE} = \left\|\hat{g}_k - g_k\right\|_2^2 \Big/ \|g_k\|_2^2. \tag{36}$$

Fig. 3 shows the normalized MSE of TIR estimation based on the proposed method, COM-KF and COM-MAP versus the iteration number. One can see that the normalized MSE based on the proposed method is smaller than that of COM-KF and COM-MAP. This may be because that these two methods relax the original time domain optimization problem to the frequency domain, which leads to the deviation of the obtained waveform from the original problem. As the proposed method directly studies in the time domain, the proposed CR system adapts its transmitted waveform better to the fluctuating TIR and the synthesized waveform is closer to the optimal solution of the original problem. In addition, the estimation performance of COM-KF is better than that of COM-MAP, because the temporal correlation of TIR can be fully utilized in KF. Fig. 4 shows the normalized MSE versus the SNR based on the three methods, we can see that the estimation performance is improved as SNR increases and the proposed method is superior to COM-KF and COM-MAP.

B. COMPARISON OF COMPUTATION COMPLEXITY

Since the computation complexity of COM-KF and COM-MAP in [3], [10], and [12] are same, we just use the COM-KF as the comparison in this subsection. Let the total available transmitted energy $E_s = N_s$, SNR = 7dB, and $\eta = 2$. Fig. 5 shows the normalized MSE obtained using the proposed method and the COM-KF versus the waveform length N_s. It can be seen that the estimation performance obtained using the proposed method is superior to that of CMO-KF as N_s increases. Moreover, we can note that curves of the two methods have almost no change since the SNR is a fixed value. Fig. 6 depicts the corresponding run times of the two methods, revealing that the proposed method is several orders of magnitude faster than the COM-KF, because the proposed method can convert the original problem into several smaller and easily solved convex sub-problems that can be solved efficiently.

C. PAR AND DETECTION CONSTRAINTS

In this subsection, we discuss the influence of the PAR and detection constraints on the synthesized waveform.

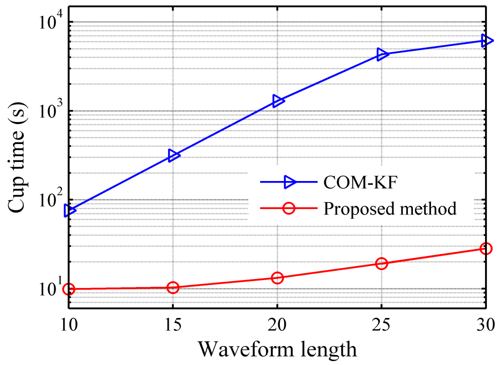

FIGURE 6. Run time of different methods versus waveform length.

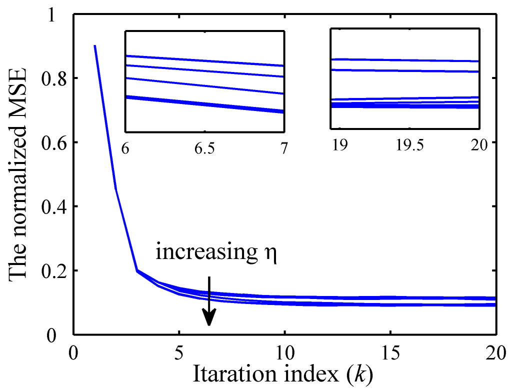

FIGURE 7. Comparison of the waveforms under different PAR constraints, $\eta = [1, 1.2, 1.5, 2, 3, N_s]$.

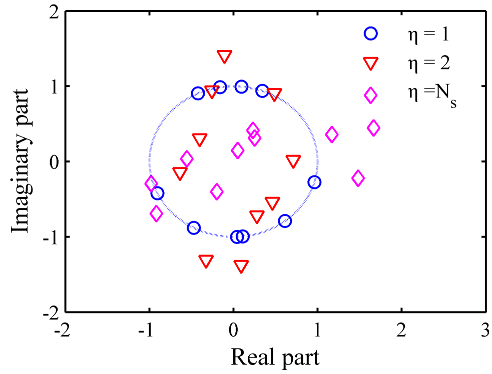

FIGURE 8. Real and imaginary parts of the waveforms with $\eta = [1, 2, N_s]$.

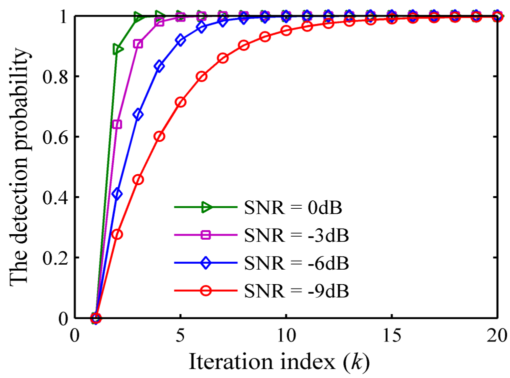

FIGURE 9. Probability of target detection with different SNR.

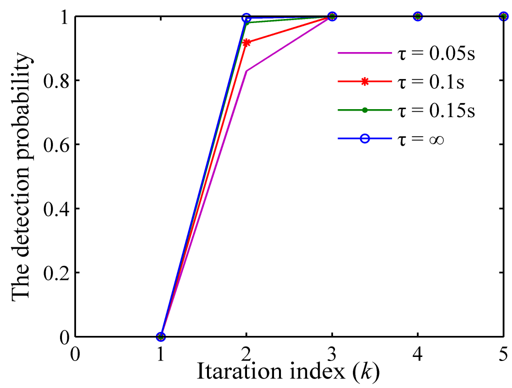

FIGURE 10. Probability of target detection with different τ.

Fig. 7 shows the normalized MSE with the synthesized waveforms under different PAR values. We can see that the curves can converge to their respective stationary values which become smaller as η increases, this is because the feasible set region in $\boldsymbol{SP}_2$ and $\boldsymbol{SP}_3$ becomes larger as η increases. However, since the energy of the transmitted waveform is limited, the waveform performance has its upper bound. Then, we can also see that the curves can be monotonically convergent to the stationary value which are very close and the curves almost overlap when $\eta \geq 2$. Fig. 8 shows the real and imaginary parts of the waveforms under different PAR constraints. When $\eta = N_s$, the distribution radii of the corresponding points are large, which is not favorable for practical applications. In contrast, the results obtained with $\eta = 1$ are unimodular and lie on the unit circle. Meanwhile, the distribution radii of the waveform with $\eta = 2$ are close to those of the waveform with $\eta = 1$, and the performance is very close to that of the waveform with $\eta = N_s$, as shown in Fig. 7. This result indicates that the low-PAR waveform (for example $\eta = 2$) not only meet the hardware constraints but also have better estimation performance than a unimodular waveform. Hence, the low-PAR waveform is more suitable for practical applications.

Fig. 9 shows the detection performance of the synthesized waveform under different SNR. It can be seen that the larger the SNR, the fewer iterations are needed to optimize the waveform to reach the given detection probability. Meanwhile, the detection performance of the synthesized waveforms gradually increase and reach the given detection probability as iterations increases. Thus, the synthesized waveform also can meet the requirement of detection performance when estimating the target.

Let SNR = 0dB, $\eta = 2$. Fig. 10 shows detection performance of the synthesized waveform under different temporal decay τ. It is worth noting that $\tau = \infty$ means the value of

TIR is not changed during the radar pulse interval. Note that although the curves of detection performance under different τ can converge to the same value, the larger τ has better convergence quality. This is because the faster the velocity and view angle change rate of target, the smaller τ is, so the detection performance of the synthesized waveform would be better for the slow-moving target whose value of τ is large.

V. CONCLUSION

In this paper, we proposed an efficient cognitive radar waveform design method for the TIR estimation under the PAR and detection constraints. In order to tackle the original non-convex problem in time domain, which is commonly translated to the frequency domain and solved by the convex optimization method based on SDR, a fast hierarchical optimization method that is directly solved in time domain is proposed. The performance of the proposed method in terms of the estimation performance, computational complexity, and detection performance are evaluated in simulation experiments. The simulation results illustrate that the optimal waveform can be obtained efficiently within the given low-PAR range and detection probability. Compared with the existing method, the proposed method has effectively reduced computational complexity and the synthesized waveform can provide a better estimation performance. Moreover, the proposed method can be used in the waveform design of cognitive radar systems since the high computational efficiency will enable real-time waveform changes.

In this paper, we mainly consider the energy and PAR constraints of the transmitted signal. However, it is worth noting that time-width and bandwidth are also limited by hardware which may affect the detection performance, so the possible future research content include the extension to the cases with time-width and bandwidth constraints. Furthermore, the low autocorrelation sidelobes and spectral constraints in the presence of signal-dependent clutter are also should be considered in the next research content.

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