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Application of Electric Field for Augmentation of Ferrofluid Heat Transfer in an Enclosure Including Double Moving Walls

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ABSTRACT Due to employing electric field, nanofluid heat transfer enhances. This phenomenon in a porous geometry has been simulated in this paper. To augment heat transfer, the radiation term is included in the model. Ethylene glycol nanofluid is considered as the homogenous model. Electric field and shape of nanoparticles can affect the properties of ferrofluid. CVFEM is proposed for simulating the treatment of EHD flow. Voltage, permeability, radiation parameters, and nanoparticles' shape are important variables. The results prove that the platelet shape leads to the highest convective flow. As electric force enhances, temperature gradient augments. Greater permeability leads to more convective flow.

INDEX TERMS EHD, nanofluid, radiation, double moving walls, ferro fluid, shape factor, CVFEM, porous cavity.

NOMENCLATURE

k	Thermal conductivity
C_p	Heat Capacity
q	Electric density
Pr	Prandtl number
Rd	Radiation parameter
L_f	Latent Heat of Fusion
T	Fluid temperature
q_r	Radiation parameter
Da	Darcy number

Greek Symbols

ϕ	Concentration of nanofluid
α	Thermal diffusivity
ρ	Density
φ	voltage

Subscripts

nf	Nano enriched PCM
p	solid

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I. INTRODUCTION

Nano science can be mentioned as best way of changing properties of working fluid. Such passive method can augment conductivity and can be utilized in cooling of electronic device.

Currently, sole attracted the attention of different researchers is to analyses the nanofluid, which has been generated by Choi [1] in 25 years ago. Kang *et al.* [2] tried to reach correlations for nanofluid. Sheikholeslami and Chamkha [3] and Sheikholeslami [4] analyzed nanofluid and their uses in porous media and under magnetic field. Khanafer *et al.* [5] examined the Buoyancy-driven heat transfer flow by using nanoparticles. Prandtl number properties with convection in a heat transmission cavity was study by Moallemi and Jang [6].

Magnetohydrodynamics (MHD) study behavior of electrically conducted fluid under the magnetic field. Plasma, salt-H_2O solution and molten metals were the instance of magneto fluids. Alfvén [7] initiated the field of MHD for the first time in 1942. Farady [8] conducted an experiment in 1832 over Waterloo Bridge in London in which he studied the flow of salty water through River Thames under the earth's magnetic field, which produces the potential difference between the two banks of the River Thames. Michael Faraday called this effect Magneto electric conduction. The current was too low to measure with equipment at that time. Newly, Shah *et al.* [9], [10] have deeply scrutinized MHD nanofluids with Hall current in rotating systems.

Ferrofluids is a type of magnetically controllable nanofluids. Ferrofluids are composed of nano-particles of the size 10nm or less. Fe_3O_4, $CoFe_2O_4$, $\gamma - Fe_2O_3$, Co, $Fe - C$ or Fe are the dispersed Ferrofluids. Ferofluid has many applications in field of solar cells (Graphene, Nanowires), medicine (Clinical trial, Drug delivery), food (Cartons, Bottles), electronics (Electronic circuits, Switches, Silicon nanphotonic etc. Additionally, magnetic force can be applied in biomechanics. Researcher employed magnetic source impact on blood flow in vessel. They utilized such forces in order to drug delivery. Motsa *et al.* [11] scrutinized the MHD flow of micro-polar liquids in the occurrence of Hall Effect. Eastman *et al.* [12] examined the Cu − water suspended nanoparticles by studying some preliminary experiements. Hamad [13] analytically examined the magnetohydrodynamic (MHD) nanofluid onver a stretching surface. Sheikholeslami *et al.* [14] observed nanoparticle MHD flow via ANN inside a duct. Ebaid and Sharif [15] deliberated the effect of Lorentz on CNTs treatment. Kandasamy *et al.* [16] deliberated the influence of chemical reaction on the flow of Al2O3, Cu and SWCNTs nanofluid. Khan [17] has examined Buongiorno's model with heat transfer and mass for nanofluid flow. Mahdy and Chamkh [18] have applied Buongiorno's model for nanofluid flows with heat transfer through unsteady contracting cylinder. In [19]–[22] researchers have examined flow of nanofluid through a plate. Sheikholeslami *et al.* [23] have discussed nanofluid movement through a pipe with MHD effect. Sheikholeslami *et al.* [23]–[28] simulated numerically the Nano enhanced PCM charging in existence of extended surfaces. He showed that fin length has effective role.

The furthermost recent investigational and theoretical research on nanofluids can be studied in [29]–[36]. The final aim of current article is study the application of electric field in nanofluid heat transfer. For this aim porous geometry has been simulated in current paper. Ethylene glycol nanofluid is deliberated as homogenous model. Electric field and shape of nanoparticles can affect the properties of ferrofluid. CVFEM is proposed for simulating treatment of the EHD flow. Effects of several important parameters are presented graphically.

II. PROBLEM EXPLANATION

In current research, impact of electric field on treatment of ferrofluid exist in porous geometry (as shown in Fig. 1) has been examined. CVFEM is employed with triangular element (see Fig. 1). The related boundary conditions are demonstrated in Fig. 1. Contour plot of electrical density were demonstrated in Fig. 2. More complex contour has been reported for higher Darcy number.

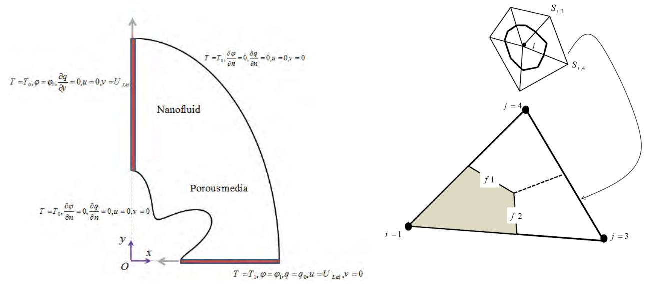


FIGURE 1. **Problem and boundary conditions.**

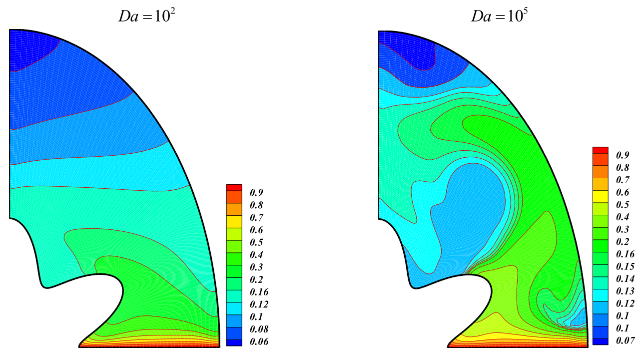


FIGURE 2. **q contours for $\Delta\varphi = 10kV$, $\phi = 0.05$, $Rd = 0.8$, $Re = 3000$.**

III. LEADING EQUATIONS AND CVFEM

A. FORMULATION

The first 4 equations get us the electric field [3]:

$$\vec{E} = -\nabla\varphi \tag{1}$$

$$q = \nabla \cdot \varepsilon \vec{E} \tag{2}$$

$$\vec{J} = q\vec{V} + \sigma\vec{E} - D\nabla q \tag{3}$$

$$\nabla \cdot \bar{J} = -\frac{\partial q}{\partial t} \tag{4}$$

Governing of this problem is [3]:

$$\begin{cases} \nabla.\vec{V} = 0, \\ -\dfrac{\nabla p}{\rho_{nf}} + \dfrac{\mu_{nf}}{\rho_{nf}}\nabla^2\vec{V} - \dfrac{\vec{V}\mu_{nf}}{K\rho_{nf}} + \dfrac{q\vec{E}}{\rho_{nf}} \\ = \left(\left(\vec{V}.\nabla\right)\vec{V} + \dfrac{\partial\vec{V}}{\partial t}\right), \\ \left(\left(\vec{V}.\nabla\right)T + \dfrac{\partial T}{\partial t}\right) = \dfrac{k_{nf}}{\left(\rho C_p\right)_{nf}}\nabla^2 T + \dfrac{\vec{J}.\vec{E}}{\left(\rho C_p\right)_{nf}} \\ -\left(\rho C_p\right)_{nf}^{-1}\dfrac{\partial q_r}{\partial y}, \\ \left[\, T^4 \cong 4T_c^3 T - 3T_c^4,\ q_r = -\dfrac{4\sigma_e}{3\beta_R}\dfrac{\partial T^4}{\partial y} \,\right] \\ \dfrac{\partial q}{\partial t} + \nabla.\vec{J} = 0,\ -\nabla\varphi = \vec{E},\ q - \nabla.\varepsilon\vec{E} = 0, \end{cases} \tag{5}$$

μ_{nf}, $\left(\rho C_p\right)_{nf}$ & ρ_{nf} are [4]:

$$\begin{aligned} \mu &= A_1 + A_2\left(\Delta\varphi\right) + A_3\left(\Delta\varphi\right)^2 + A_4\left(\Delta\varphi\right)^3, \\ \left(\rho C_p\right)_{nf} &= \left(1-\phi\right)\left(\rho C_p\right)_f + \phi\left(\rho C_p\right)_s, \\ \rho_{nf} &= \rho_f(1-\phi) + \rho_s\phi \end{aligned} \tag{6}$$

Here k_{nf} is estimated including shape factor impact:

$$\frac{k_{nf}}{k_f} = \frac{kk\phi + mk_f - mkk\phi + k_p + k_f}{mk_f + k_f + k_p + kk\phi}, \quad (kk = k_f - k_p) \tag{7}$$

Finally equations (5) are reduced to the form as:

$$\begin{cases} \nabla \cdot \vec{V} = 0, \\ \left(\left(\vec{V} \cdot \nabla\right)\theta + \frac{\partial \theta}{\partial t}\right) = \frac{1}{\Pr Re} \frac{k_{nf}/k_f}{(\rho C_p)_{nf} / (\rho C_p)_f} \nabla^2 \theta + \\ Ec\left(\vec{J} \cdot \vec{E}\right) S_E \frac{(\rho C_p)_f}{(\rho C_p)_{nf}} + \frac{4}{3}\left(\frac{k_{nf}}{k_f}\right)^{-1} Rd \frac{\partial^2 \theta}{\partial Y^2} \\ \frac{S_E}{\rho_{nf}/\rho_f} q\vec{E} + \frac{1}{Re}\frac{\rho_{nf}/\rho_f}{\mu_{nf}/\mu_f}\nabla^2 \vec{V} - \nabla p \\ -\frac{1}{Re\,Da}\frac{\mu_{nf}}{\mu_f}\left(\frac{\rho_{nf}}{\rho_f}\right)^{-1}\vec{V} = \left(\left(\vec{V} \cdot \nabla\right)\vec{V} + \frac{\partial \vec{V}}{\partial t}\right) \\ \nabla \cdot \vec{J} + \frac{\partial q}{\partial t} = 0, q = \nabla \cdot \varepsilon \vec{E}, \vec{E} + \nabla \varphi = 0, \end{cases} \tag{8}$$

$$\overline{\varphi} = \frac{\varphi - \varphi_0}{\nabla \varphi}, \quad (\overline{u}, \overline{v}) = \frac{(u, v)}{U_{Lid}}, \quad \overline{q} = \frac{q}{q_0},$$
$$\overline{E} = \frac{E}{E_0}, \quad \overline{t} = U_{Lid}\frac{t}{L}, \overline{p} = \frac{P}{\rho U_{Lid}^2}, \quad \theta = \frac{T - T_0}{\nabla T},$$
$$\nabla T = T_1 - T_0, \quad (\overline{y}, \overline{x}) = \frac{(y, x)}{L}, \quad \nabla \varphi = \varphi_1 - \varphi_0, \tag{9}$$

By inserting following equation, we can discard pressure terms.

$$v = -\frac{\partial \psi}{\partial x}, \quad \omega = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y},$$
$$\frac{\partial \psi}{\partial y} = u, \quad \Omega = \frac{\omega}{LU_{Lid}}, \quad \Psi = \frac{\psi L}{U_{Lid}} \tag{10}$$

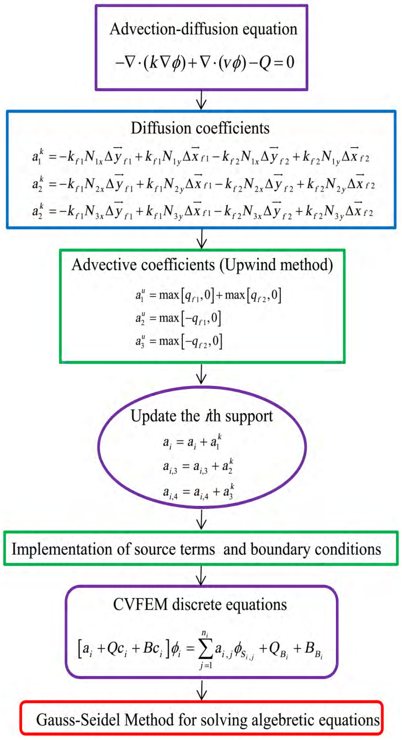
Nu_{loc} and Nu_{ave} are:

$$Nu_{loc} = \left(\frac{k_{nf}}{k_f}\right)\left(1 + \frac{4}{3}Rd\left(\frac{k_{nf}}{k_f}\right)^{-1}\right)\frac{\partial \Theta}{\partial X} \tag{11}$$

$$Nu_{ave} = \frac{1}{L}\int_0^L Nu_{loc}\, dY \tag{12}$$

B. CVFEM PROCEDURE

Newly, one of the greatest researcher developed new method (CVFEM) for hydrothermal simulation. His name is Sheikholeslami [3] and he published a reference book. In mentioned method, both FVM and FEM have been involved for reaching more accurate method. The producer with detail is given as



IV. CODE VALIDATION AND MESH ANALYSIS

To achieve confidence about correctness of FORTRAN code, the data of previous published articles [3], [4] were reproduced. As displayed in Fig. 3, we can be sure about accuracy. Moreover, the reliable data should not dependent on mesh size. Thus, all states should have mesh analysis step. For instance, table 4 illustrated outputs of different meshes for certain case.

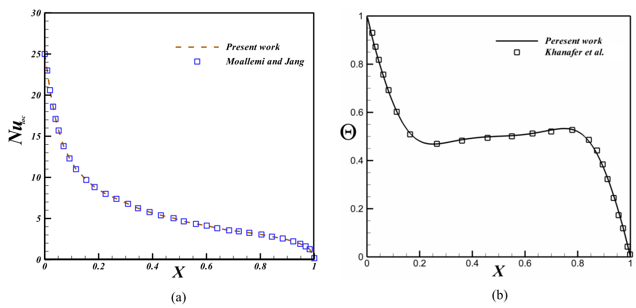


FIGURE 3. Verification for (a) lid driven cavity [5] and (b) nanofluid convection [6]. (a) Pr=1, Re=500, and Ri=0.4. (b) $Gr = 10^4$, $\phi = 0.1$.

V. DISCUSSIONS

In this section we have discussed the influence of main factors affecting on ferro-nanofluid. Influence of electric filed on ferrofluid ($Fe_3O_4 - C_2H_6O_2$ nanofluid) hydrothermal styles was simulated. The permeable cavity is full of Fe_3O_4 – $C_2H_6O_2$. Roles of supplied voltage ($\Delta\varphi = 0$ to $10kV$), Darcy number ($Da = 10^2$ to 10^5), shape factor ($m = 3$ to 5.7), Radiation parameter ($Rd = 0$ to 0.8), volume fraction of

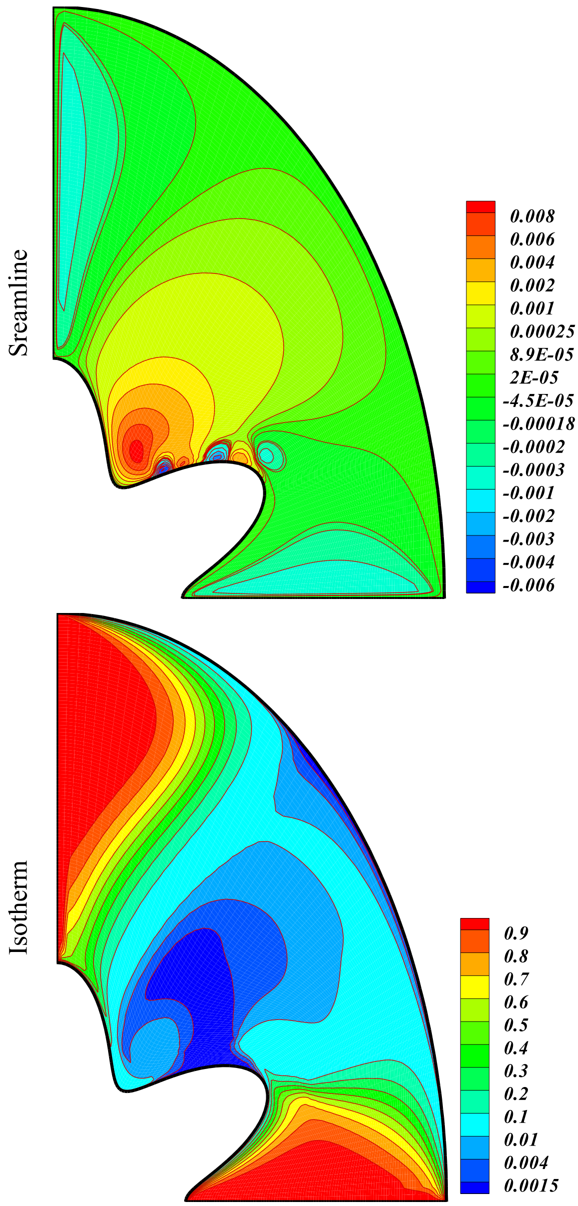


FIGURE 4. **Contours plots for $Da = 10^2$, $Re = 3000$, $\Delta\varphi = 0kV$, $\phi = 0.05$, $Rd = 0.8$.**

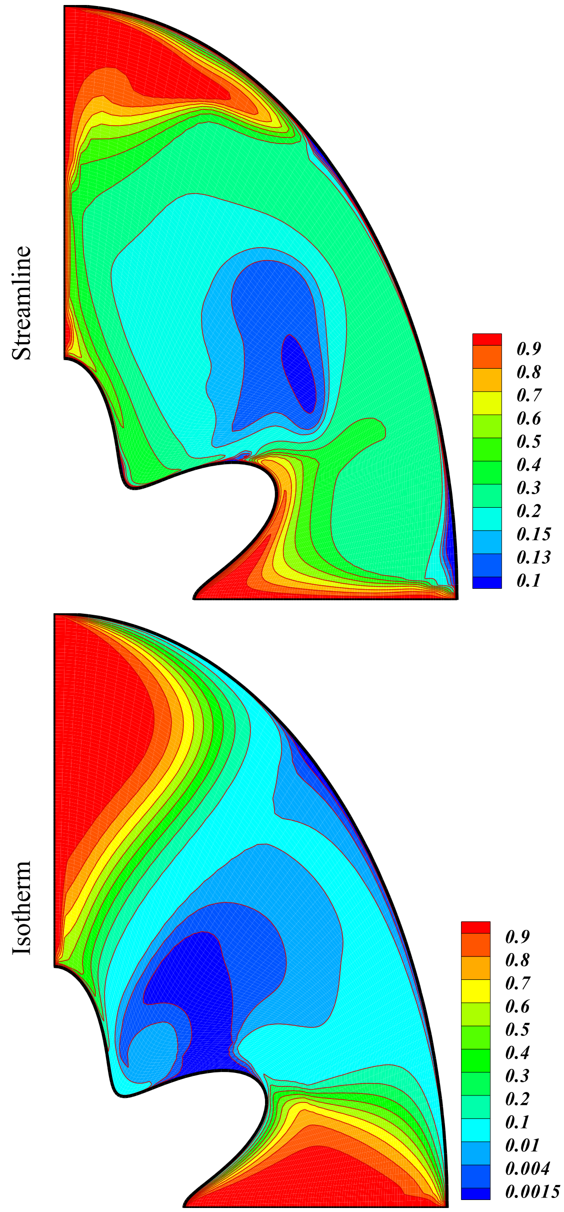


FIGURE 5. **Contours plots for $Da = 10^5$, $Re = 3000$, $\Delta\varphi = 0kV$, $\phi = 0.05$, $Rd = 0.8$.**

Fe_3O_4 ($\phi = 0\%$ to 5%) are depicted graphically. The geometry of the nanofluid flow related with boundary conditions is shown in Fig. 1. Contour plot of electrical density were demonstrated in Fig. 2 when $\Delta\varphi = 10kV$, $\phi = 0.05$, $Rd = 0.8$, $Re = 3000$.More complex contour has been reported for higher Darcy number. Fig. 3. Show the validation for (a) forced convection [3] $Ri = 0.4$, $\Pr = 1$, $Re = 500$ and (b) nanofluid flow [4] at $Gr = 10^4$, $\phi = 0.1$. We proved that the current data were in excellent agreement with previous one. Contours for streamlines and isotherm at $\phi = 0.05$, $\Delta\varphi = 0kV$ and, $Rd = 0.8$, $Da = 100$, $Re = 3000$ is shown in Figs. (4- 7) varying Da and $\Delta\varphi$ for other outcomes. We can observe that there is a once clock-wise vortex in the streamlines. An improvement in Darcy number causes to produce the second vortex which rotates in a counterclockwise direction and the first vortex moves to the upper side. When electric applies, then it causes the central vortex to become stronger and moves to the upper side. According to the isotherms, it is found that they are more disturbed when Coulomb force is not zero. Further, an augment in Darcy number tends to enhance the significantly. It can also be observed that the strong presence of coulomb forces tends to reduce the secondary diminishes whereas the central eddy increases.

Fig. 8 shows that the behavior of Nu_{ave} versus*m, Da,* and, *Rd at* $\log(Da) = 3.5$, $Rd = 0.4$, $\phi = 0.05$, $Rd = 0.4$,

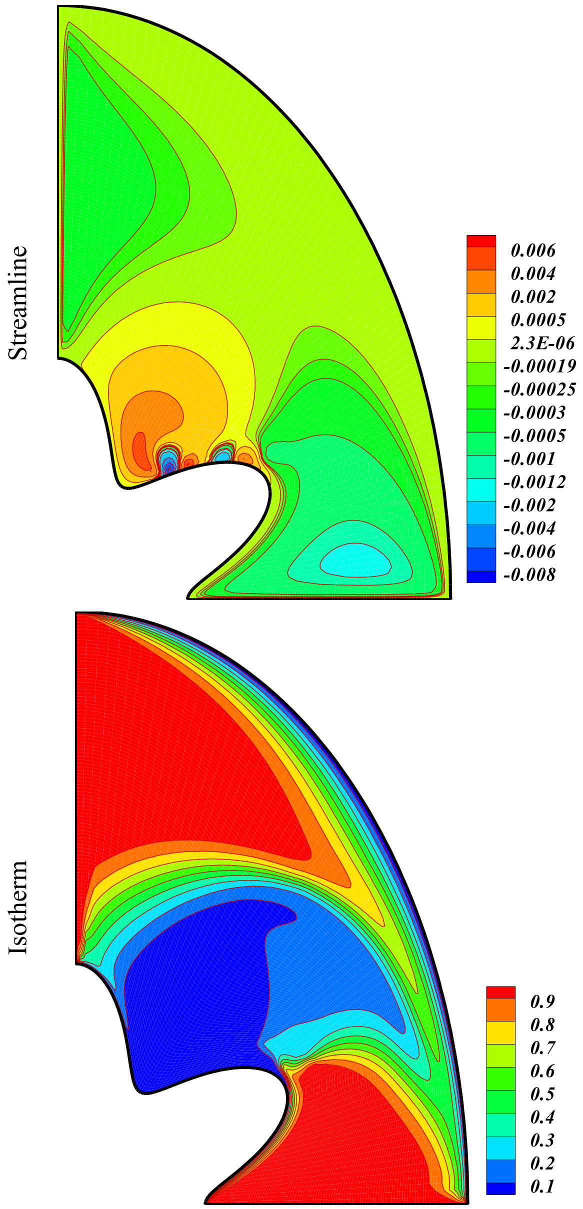


FIGURE 6. Contours plots for $Da = 10^2$, $Re = 3000$, $\Delta\varphi = 10kV$, $\phi = 0.05$, $Rd = 0.8$.

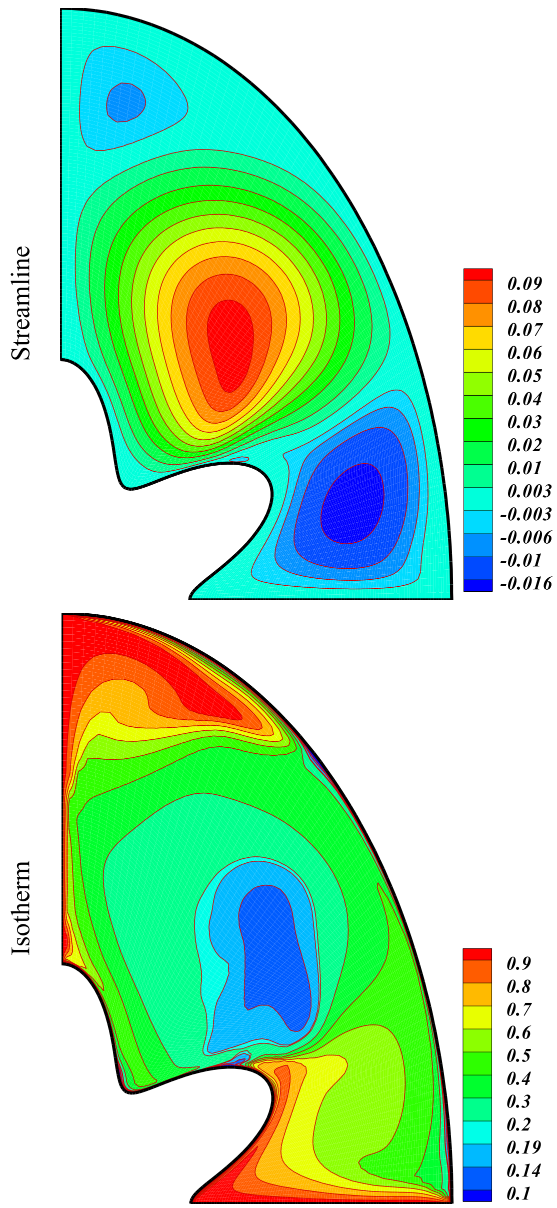


FIGURE 7. Contours plots for $Da = 10^5$, $Re = 3000$, $\Delta\varphi = 10kV$, $\phi = 0.05$, $Rd = 0.8$.

TABLE 1. Characteristics of nanoparticles and ethylene glycol.

	$C_p(j/kgk)$	$k(W/m.k)$	$\rho(kg/m^3)$
Ethylene glycol	2400	0.26	1110
Fe_3O_4	670	6	5200

TABLE 2. A_i for Eq. (6).

Coefficient values	$\phi = 0$	$\phi = 0.05$
A_4	-1.1876E-008	-4.1344E-008
A_2	-2.698E-003	-3.4119E-003
A_3	2.9082E-006	5.5228E-006
A_1	1.0603E+001	9.5331

$\Delta\varphi = 5, \phi = 0.05$, by varying $\Delta\varphi, \phi$ and m. Platelet shape nanoparticles have a augmented rate of heat transmission as compared with other shaped nanoparticles. Coulomb force is beneficial to augment the convective mode. On the other hand, thermal radiation is helpful to improve the convective flow. The behavior of permeability is similar as compared with the Rd. Therefore, Nu_{ave} behaves as a rising function for the Darcy number and Radiation parameter.

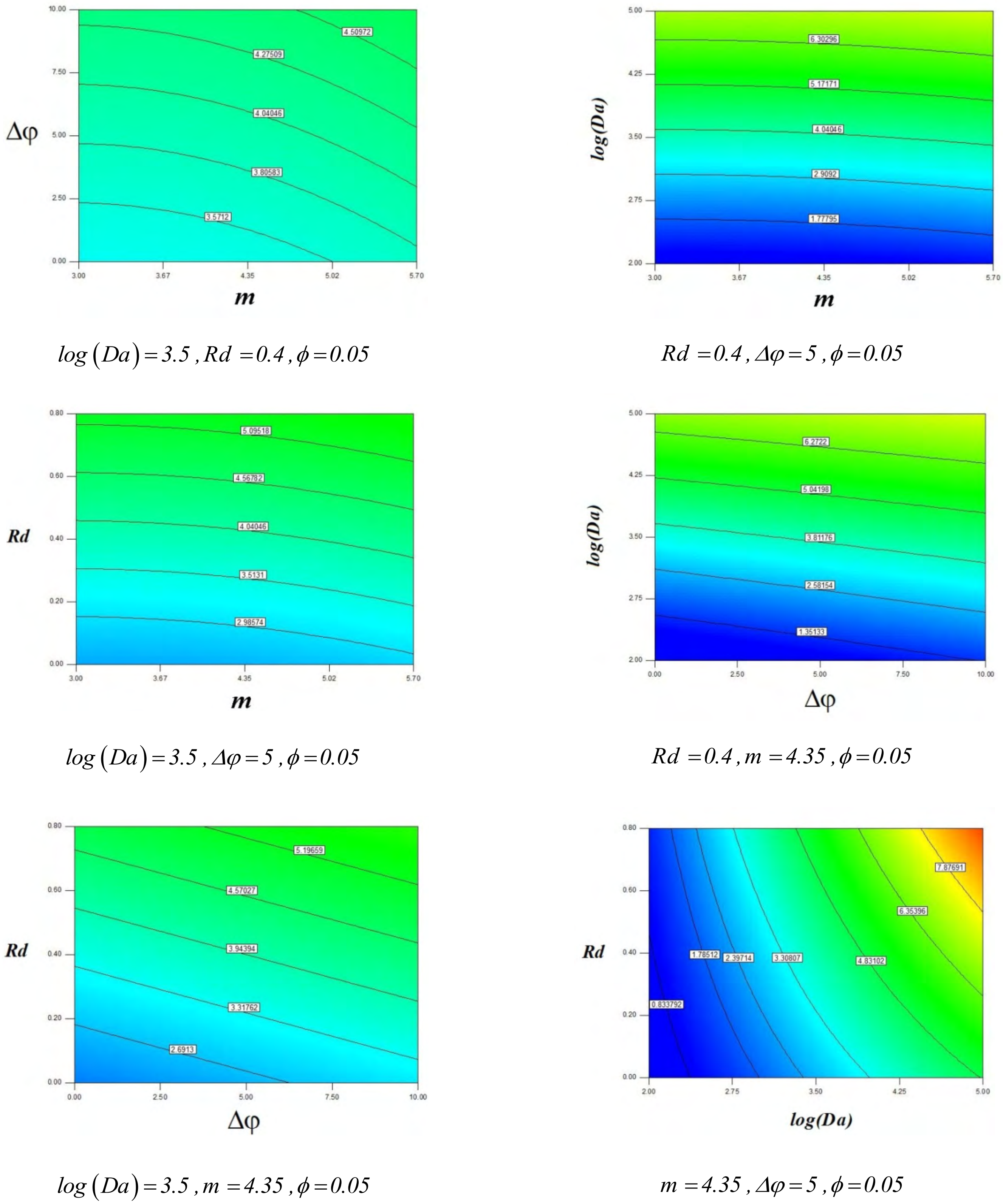


FIGURE 8. Various Da, $\Delta\varphi$, Rd, m and obtained Nu_{ave}.

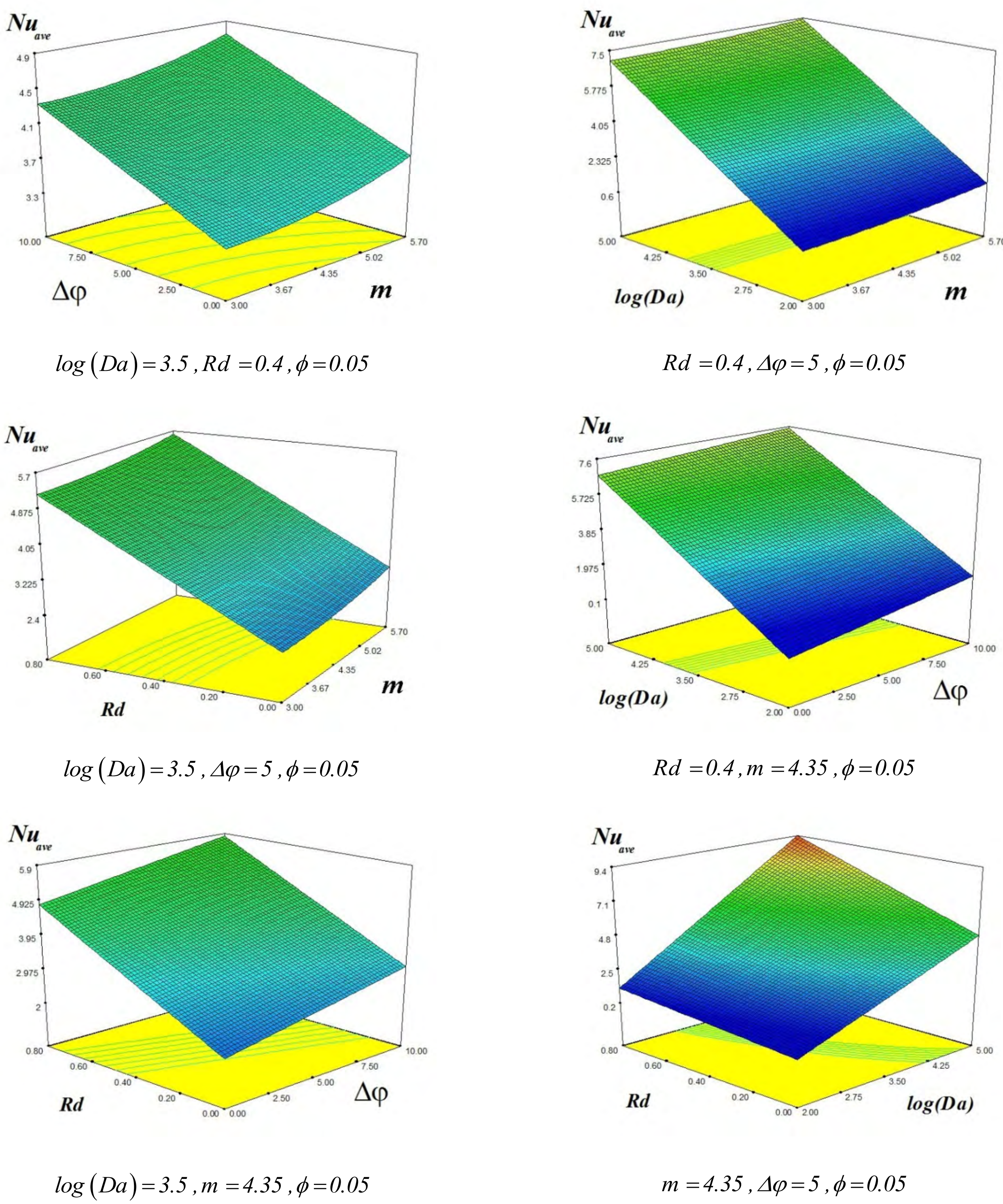


FIGURE 8. *(Continued.)* Various Da, $\Delta\varphi$, Rd, m and obtained Nu_{ave}.

Changes of Nu_{ave} respect to variables are displayed in Fig.8 and Eq.(13) can presented these relationships.

$$\begin{aligned} Nu_{ave} = {} & 3.94 + 0.2m + 0.5\Delta\varphi \\ & + 3.19 \log Da + 1.38Rd \\ & - 0.13\Delta\varphi (\log Da) \\ & + 0.89 (\log Da) Rd + 0.097m^2 \end{aligned} \tag{13}$$

A. TABLE DISCUSSION

Tables 1, 2 and 3 illustrate characteristics, various shape factors, related coefficient and, respectively [4].Variation of Nu_{ave} with change of mesh size at $Da = 10^5$, $\Delta\varphi = 10$, $\phi = 0.05$ and $Rd = 0.8$, $Re = 3000$ is shown in table.4 For instance, table 4 illustrated results of various meshes for certain case.

TABLE 3. Different values of m.

Platelet	Spherical	Cylinder	Spherical	Brick	shape
5.7	3	4.8	3	3.7	m

TABLE 4. Various grids for $Rd = 0.8$, $Re = 3000$, $Da = 10^5$, $\Delta\varphi = 10$, $\phi = 0.05$.

51×151	61×181	71×211	81×241	91×271	101×301
10.0667	10.0778	10.0843	10.0922	10.0936	10.0968

VI. CONCLUSIONS

In this article application of electric field is presented. This phenomenon in a porous geometry has been simulated in current paper. Voltage, permeability, radiation parameters and nanoparticles' shape are important variables. Influence of electric filed on ferrofluid ($Fe_3O_4 - C_2H_6O_2$ nanofluid) hydrothermal styles was simulated. The porous cavity is full of $Fe_3O_4 - C_2H_6O_2$. Roles of supplied voltage ($\Delta\varphi = 0$ to $10kV$), Darcy number ($Da = 100$ to 10^5), shape factor ($m = 3$ to 5.7), Radiation parameter ($Rd = 0$ to 0.8), volume fraction of Fe_3O_4 ($\phi = 0\%$ to 5%) are depicted graphically. To augment heat transfer, radiation term is included in the model. Ethylene glycol nanofluid is considered as homogenous model. The main concluded point are given below

- Due to employing electric field, nanofluid heat transfer enhances.
- It is prove that Platelet shape leads to highest convective flow.
- As electric force enhances, temperature gradient augments.
- Greater permeability leads to more convective flow.
- Table 4 illustrated results of various meshes for certain case.
- Electric field and shape of nanoparticles affect the properties of ferrofluid.
- CVFEM is proposed for simulating treatment of EHD flow and excellent result is obtained

COMPETING INTERESTS STATEMENT

The author declares that they have no competing interests.

REFERENCES

[1] S. U. S. Choi and J. A. Estman, "Enhancing thermal conductivity of fluids with nanoparticles," in *Proc. ASME Int. Mech. Eng. Congr. Expo.*, San Francisco, CA, USA, 1995, pp. 99–105.

[2] H. U. Kang, S. H. Kim, and J. M. Oh, "Estimation of thermal conductivity of nanofluid using experimental effective particle volume," *Exp. Heat Transf.*, vol. 19, pp. 181–191, Sep. 2006.

[3] M. Sheikholeslami and J. A. Chamkha, "Flow and convective heat transfer of a ferro-nanofluid in a double-sided lid-driven cavity with a wavy wall in the presence of a variable magnetic field," *Numer. Heat Transf. A, Appl.*, vol. 69, no. 10, pp. 1186–1200, 2016. doi: 10.1080/10407782.2015.1125709.

[4] M. Sheikholeslami, *Application of Control Volume Based Finite Element Method (CVFEM) for Nanofluid Flow and Heat Transfer*. Amsterdam, The Netherlands: Elsevier, 2019.

[5] K. Khanafer, K. Vafai, and M. Lightstone, "Buoyancy-driven heat transfer enhancement in a two-dimensional enclosure utilizing nanofluids," *Int. J. Heat Mass Transf.*, vol. 46, no. 19, pp. 3639–3653, 2003.

[6] M. K. Moallemi and K. S. Jang, "Prandtl number effects on laminar mixed convection heat transfer in a lid-driven cavity," *Int. J. Heat Mass Transf.*, vol. 35, no. 8, pp. 1881–1892, 1992.

[7] H. Alfvén, "Existence of electromagnetic-hydrodynamic waves," *Nature*, vol. 150, pp. 405–406, Oct. 1942.

[8] M. Farady, *Experimental Researches in Electricity*, vol. 1, 2nd ed. London, U.K.: Richard and John Edward Taylor, 1849, p. 55.

[9] Z. Shah, S. Islam, H. Ayaz, and S. Khan, "Radiative heat and mass transfer analysis of micropolar nanofluid flow of Casson fluid between two rotating parallel plates with effects of Hall current," *ASME J. Heat Transf.*, vol. 141, no. 2, 2018, Art. no. 022401. doi: 10.1115/1.4040415.

[10] Z. Shah, S. Islam, T. Gul, E. Bonyah, and M. A. Khan, "The electrical MHD and hall current impact on micropolar nanofluid flow between rotating parallel plates," *Results Phys.*, vol. 9, pp. 1201–1214, Jun. 2018. doi: 10.1016/j.rinp.2018.01.064.

[11] S. S. Motsa and S. Shatery, "The effects of chemical reaction, Hall and ion-slip currents on MHD flow with temperature dependent viscosity and thermal diffusivity," *J. Appl. Math.*, vol. 2012, Nov. 2012, Art. no. 689015.

[12] J. A. Eastman, S. U. S. Cho, S. Li, G. Soyez, L. J. Thompson, and R. J. Dimelfi, "Novel thermal properties of nanostructured materials," *J. Metastable Nanocryst. Mater.*, vol. 2, pp. 629–637, Jul. 1999.

[13] M. A. A. Hamad, "Analytical solution of natural convection flow of a nanofluid over a linearly stretching sheet in the presence of magnetic field," *Int. Commun. Heat Mass Transf.*, vol. 38, no. 4, pp. 487–492, 2011.

[14] M. Sheikholeslami, M. B. Gerdroodbary, R. Moradi, A. Shafee, and Z. Li, "Application of neural network for estimation of heat transfer treatment of Al_2O_3-H_2O nanofluid through a channel," *Comput. Methods Appl. Mech. Eng.*, vol. 344, pp. 1–12, Feb. 2019.

[15] A. Ebaid and M. A. A. Sharif, "Application of Laplace transform for the exact effect of a magnetic field on heat transfer of carbon nanotubes-suspended nanofluids," *Z. Naturforsch. A*, vol. 70, pp. 471–475, Jun. 2015.

[16] R. Kandasamy, R. Mohamad, and M. Ismoen, "Impact of chemical reaction on Cu, Al_2O_3 and SWCNTs–nanofluid flow under slip conditions," *Int. J. Eng. Sci. Technol.*, vol. 19, no. 2, pp. 700–709, 2016.

[17] W. A. Khan, A. Aziz, and N. Uddin, "Buongiorno model for nanofluid Blasius flow with surface heat and mass fluxes," *J. Thermophys. Heat Transf.*, vol. 27, no. 1, pp. 134–141, 2013.

[18] A. Mahdy and A. Chamkha, "Heat transfer and fluid flow of a non-Newtonian nanofluid over an unsteady contracting cylinder employing Buongiorno's model," *Int. J. Numer. Method Heat Fluid Flow*, vol. 25, pp. 703–723, May 2015.

[19] M. Sheikholeslami, "New computational approach for exergy and entropy analysis of nanofluid under the impact of Lorentz force through a porous media," *Comput. Methods Appl. Mech. Eng.*, vol. 344, pp. 319–333, Feb. 2019.

[20] T. Hayat, M. B. Ashraf, S. A. Shehzad, and E. I. Abouelmaged, "Three-dimensional flow of Eyring Powell nanofluid over an exponentially stretching sheet," *Int. J. Numer. Method Heat Fluid Flow*, vol. 25, pp. 333–357, Apr. 2015.

[21] M. Sheikholeslami, "Numerical approach for MHD Al_2O_3-water nanofluid transportation inside a permeable medium using innovative computer method," *Comput. Methods Appl. Mech. Eng.*, vol. 344, pp. 306–318, Feb. 2019.

[22] A. B. Rosmila, R. Kandasamy, and I. Muhaimin, "Lie symmetry group transformation for MHD natural convection flow of nanofluid over linearly porous stretching sheet in presence of thermal stratification," *Appl. Math. Mech. Eng. Ed.*, vol. 33, no. 5, pp. 593–604, 2012.

[23] M. Sheikholeslami, R.-U. Haq, A. Shafee, and Z. Li, "Heat transfer behavior of Nanoparticle enhanced PCM solidification through an enclosure with V shaped fins," *Int. J. Heat Mass Transf.*, vol. 130, pp. 1322–1342, Mar. 2019.

[24] M. Sheikholeslami, S. A. Shehzad, Z. Li, and A. Shafee, "Numerical modeling for alumina nanofluid magnetohydrodynamic convective heat transfer in a permeable medium using Darcy law," *Int. J. Heat Mass Transf.*, vol. 127, pp. 614–622, Dec. 2018.
[25] M. Sheikholeslami and O. Mahian, "Enhancement of PCM solidification using inorganic nanoparticles and an external magnetic field with application in energy storage systems," *J. Cleaner Prod.*, vol. 215, pp. 963–977, Apr. 2019.
[26] M. Sheikholeslami and M. M. Bhatti, "Active method for nanofluid heat transfer enhancement by means of EHD," *Int. J. Heat Mass Transf.*, vol. 109, pp. 115–122, Jun. 2017.
[27] M. Sheikholeslami, "Numerical investigation of nanofluid free convection under the influence of electric field in a porous enclosure," *J. Mol. Liquids*, vol. 249, pp. 1212–1221, Jan. 2018.
[28] M. Sheikholeslami and H. B. Rokni, "Numerical simulation for impact of Coulomb force on nanofluid heat transfer in a porous enclosure in presence of thermal radiation," *Int. J. Heat Mass Transf.*, vol. 118, pp. 823–831, Mar. 2018.
[29] S. Madruga and G. S. Mischlich, "Melting dynamics of a phase change material (PCM) with dispersed metallic nanoparticles using transport coefficients from empirical and mean field models," *Appl. Therm. Eng.*, vol. 124, pp. 1123–1133, Sep. 2017.
[30] M. Arıcı, E. Tütüncü, M. Kan, and H. Karabay, "Melting of nanoparticle-enhanced paraffin wax in a rectangular enclosure with partially active walls," *Int. J. Heat Mass Transf.*, vol. 104, pp. 7–17, Jan. 2017.
[31] M. Sheikholeslami and M. M. Bhatti, "Forced convection of nanofluid in presence of constant magnetic field considering shape effects of nanoparticles," *Int. J. Heat Mass Transf.*, vol. 111, pp. 1039–1049, Aug. 2017.
[32] Z. Shah, A. Dawar, S. Islam, I. Khan, and D. L. C. Ching, "Darcy-Forchheimer flow of radiative carbon nanotubes with microstructure and inertial characteristics in the rotating frame," *Case Stud. Therm. Eng.*, vol. 12, pp. 823–832, Sep. 2018.
[33] Z. Shah, A. Dawar, S. Islam, I. Khan, D. L. C. Ching, and A. Z. Khan, "Cattaneo-Christov model for electrical magnetite micropoler Casson ferrofluid over a stretching/shrinking sheet using effective thermal conductivity model," *Case Stud. Therm. Eng.*, vol. 13, Mar. 2019, Art. no. 100352. doi: 10.1016/j.csite.2018.11.003.
[34] A. Khan, Z. Shah, S. Islam, S. Khan, W. Khan, and Z. A. Khan, "Darcy–Forchheimer flow of micropolar nanofluid between two plates in the rotating frame with non-uniform heat generation/absorption," *Adv. Mech. Eng.*, vol. 10, no. 10, pp. 1–16, 2018. doi: 10.1177/1687814018808850.
[35] M. Jawad, Z. Shah, S. Islam, E. Bonyah, and Z. A. Khan, "Darcy-Forchheimer flow of MHD nanofluid thin film flow with Joule dissipation and Navier's partial slip," *J. Phys. Commun.*, vol. 2, Nov. 2018, Art. no. 115014. doi: 10.1088/2399-6528/aaeddf.
[36] N. Khan, S. Zuhra, Z. Shah, E. Bonyah, W. Khan, and S. Islam, "Slip flow of Eyring-Powell nanoliquid film containing graphene nanoparticles," *AIP Adv.*, vol. 8, Nov. 2018, Art. no. 115302. doi: 10.1063/1.5055690.

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