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Route Selection of the Arctic Northwest Passage Based on Hesitant Fuzzy Decision Field Theory

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ABSTRACT The hesitant fuzzy set (HFS) is widely applied in actual multi-attribute group decision-making (MAGDM) problems. It can depict experts' hesitant evaluation information with the membership degree consisting of several possible values. Most existing methods based on HFSs only focus on the final integrated information by different kinds of aggregation operators but fail to provide detailed comparisons between alternatives. They are essentially result-oriented static decision-making methods, based on which, the decision-making results may be inconsistent with reality. However, there is no process-oriented research on hesitant fuzzy information. The decision field theory (DFT) is a dynamic decision-making method and can better simulate the uncertain decision-making process. Thus, this paper integrates the HFS into the DFT and proposes a new decision-making method named as hesitant fuzzy decision field theory (HFDFT) to fill this vacancy. First, we define the hesitant fuzzy momentary preference function and other parameters in HFDFT. After that, for the MAGDM problems with incompletely known attribute weight information, the programming model is used to determine the weights of attributes. Then, the group decision-making method based on HFDFT is presented. Moreover, we apply the proposed HFDFT method to a case about route selection of the Arctic Northwest Passage. Two traditional methods based on the score function and the correlation coefficient, respectively, are further implemented for comparisons to illustrate the validity of the proposed HFDFT method.

INDEX TERMS Hesitant fuzzy set, decision field theory, multi-attribute group decision making, route selection.

I. INTRODUCTION

With the widespread uncertainty and ever-increasing complexity in actual decision-making problems, there will be more difficulties in depicting the experts' preferences and cognition accurately. Actually, when evaluating alternatives, it is not easy for experts in a group to reach a certain consensus or provide a common measure of the membership degree with sound reliability. In order to manage such cases and avoid the loss of information, Torra and Narukawa [1], Torra [2] proposed the concept of hesitant fuzzy set (HFS) with the membership degree consisting of several possible values, which can depict the hesitant preferences and uncertain knowledge more comprehensively. After that, Xia and Xu [3] defined the mathematical expression of HFS and introduced the concept of hesitant fuzzy element (HFE). As the basic component of HFS, the HFE is a concise means to convey and depict the evaluation values of each attribute clearly.

Since the excellent properties of HFSs in quantitative decision-making problems, many scholars have studied HFS theory and obtained a series of research achievements, including basic operation laws [3], [4], aggregation operators [3], information measures [5]–[8] and consistency measures [9]–[11]. For example, Yu *et al.* [12] defined a hesitant fuzzy Choquet integral operator and applied it to solve MAGDM problems with unknown weight, Wei [13] proposed a priority integration operator for hesitant fuzzy information, and Yu *et al.* [14] put forward a generalized hesitant fuzzy Bonferroni method to solve the MAGDM problems. Besides,

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Chen *et al.* [15] proposed a hesitant fuzzy Elimination Et Choix Tradulsant la REaltite (ELECTRE) I method and applied it to deal with the MAGDM problems under hesitant fuzzy environment. The method is developed based on the concept of hesitant fuzzy concordance and hesitant fuzzy discordance, which are provided based on score function and deviation degree. These theoretical studies are widely applied in the fields of decision making [16]–[19], such as risk evaluation [20], classification [21], data analysis [22], [23], pattern recognition [24], [25] and medical diagnosis [26]–[28].

As for fuzzy MAGDM, Chen and Hwang [29] pointed out that the key of solving it is to determine the weights of attributes, and select the appropriate information integration operators to calculate the fuzzy utility values of alternatives. After that, the optimal alternative is obtained by comparing and sorting the fuzzy utility values. Most existing studies on hesitant fuzzy MAGDM methods are carried out based on Chen and Hwang's summary. The development of various hesitant fuzzy integration operators lays the mathematical foundation of the integration of hesitant fuzzy data. Besides, the introduction of various HFS sequencing theories makes it possible to sort and compare hesitant fuzzy information.

Most existing methods based on HFSs only focus on the final integrated information by different kinds of aggregation operators. They ignore the influence of time of consideration on the decision-making results and fail to provide the detailed comparisons between alternatives. They are essentially result-oriented static decision-making methods. However, many studies indicate that there is a functional relationship between preference intensity and time [30], [31]. Besides, experts need to consider decision-making time, decision scenarios and variation of different factors, which will affect the results over time. Thus, compared with traditional static decision-making methods, dynamic decision-making methods are more applicable and logical.

The research on human dynamic decision making based on psychological theory [32] and computer simulations of complicated decision-making tasks [33] has promoted the development of dynamic decision-making theory, which is important to solve the MAGDM problems. Scholars have carried out some studies on the dynamic decision-making processes and put forward the corresponding models and theories. For instance, Gonzalez *et al.* [34] proposed the instance-based learning theory (IBLT), which improves the accumulation of cases based on the results of the action and makes decisions based on the accumulated experience. They also provided the decision-making process of dynamic decision tasks. Saaty [35] elaborated his idea for the development of dynamic decision making. Busemeyer and Pleskac [36] deeply studied the connection and application range of various theories and methods in dynamic decision making, including Expected Multi Utility Theory, Game Theory, Bayesian Inference, Decision Tree, Markov Logic Network and so on.

The research findings on dynamic decision making can be mainly divided into two types: normative decision making and behavioral decision making [37]. Normative decision-making theory demonstrates the possibility of the optimal decision by theoretical analysis. The studies on behavioral decision making focus on exploring the behavioral characteristics and laws of experts in the dynamic process through empirical methods. Combining normative decision-making theory and behavioral decision-making theory, Busemeyer and Townsend [38] proposed the concept of decision field theory (DFT), which applied the diffusion to the study of human decision-making behaviors. The DFT method is a process-oriented dynamic decision-making method, and it can simulate the motivational process and cognitive process of uncertain decision making [39], [40]. Besides, the DFT method can accurately predict the selection probability and present the relationship between preference intensity and time. It has been applied to stock trading, military command control and traffic control [41]–[43]. However, when the preference information or uncertain knowledge is expressed by the form of HFS, we cannot use the traditional DFT to address it efficiently. Considering the prominent advantages of the HFS and the DFT, in this paper, we try to integrate the HFS into the DFT and propose a new process-oriented dynamic decision-making method named as hesitant fuzzy decision field theory (HFDFT). We define the hesitant fuzzy momentary preference function and other parameters in HFDFT. After that, the group decision-making method based on HFDFT is presented. We also provide a specific implementation process for HFDFT method to manage actual evaluation problems. The advantages of the proposed method are summarized as follows:

a) The HFDFT method can not only delicately depict the experts' preferences, but also describe the background information of alternatives and illustrate the original decision-making process.

b) The HFDFT method is more effective and reasonable to solve the complicated practical problems than the existing hesitant fuzzy decision-making methods that only depend on the final score function values and the correlation coefficient values by integration operators.

c) The HFDFT method can make use of the original data and experts' evaluation information effectively, and it depicts the process of comparison between different alternatives clearly.

d) The HFDFT method can simulate the process of experts' consideration and comparison by the contrast matrix and feedback matrix.

The reminder of this paper is organized as follows: Section 2 reviews the basic knowledge of the HFS. In Section 3, based on the concept of classical DFT, we propose a new process-oriented dynamic decision-making method named as HFDFT. For the MAGDM problems with incompletely known attribute weight information, the programming model is used to determine the weights of attributes. Then, the group decision-making method based on HFDFT is presented. A specific implementation process of the general MAGDM process is also illustrated. In Section 4, the proposed method is applied to solve the route selection problem of the Arctic Northwest Passage. Furthermore, two existing methods based on score function and correlation coefficient respectively are implemented to solve the application case. The comparisons between the results by using our method and the results by using the existing methods illustrate the validity of our method. Finally, we end this paper with some conclusions in Section 5.

II. PRELIMINARIES

The HFS can deal with the situations in which the experts hesitate to provide their evaluations and preferences. In this section, we review some basic knowledge about HFS, including basic concepts, operational laws, aggregation operators and distance measures.

The mathematical expression of the HFS, the score and the standard deviation degree of the HFE are presented as follows:

Definition 1 [3]: Let $H = \{x, h(x) | x \in X\}$ be a HFS, in which $h(x)$ consists of a set of some different values in [0, 1]. Xu and Xia named $h(x)$ as a hesitant fuzzy element (HFE), and it denotes the membership degree consisting of several possible values.

Definition 2 [3]: Let $h(x) = \{\gamma_i | i = 1, 2, \ldots, \#h\}$ be a HFE, then the score and the standard deviation degree of $h(x)$ are defined respectively as:

$$s(h(x)) = \frac{1}{\#h} \sum_{i=1}^{\#h} \gamma_i \tag{1}$$

$$\sigma(h(x)) = \sqrt{\frac{1}{\#h} \sum_{i=1}^{\#h} (\gamma_i - s(h))^2} \tag{2}$$

where $\#h$ indicates the number of the elements in $h(x)$.

Based on the score and standard deviation functions, the comparison between two HFEs can be conducted. Taking two HFEs $h_1(x)$ and $h_2(x)$ as example, we have:

(1) If $s(h_1(x)) > s(h_2(x))$, then $h_1(x) > h_2(x)$;
(2) If $s(h_1(x)) < s(h_2(x))$, then $h_1(x) < h_2(x)$;
(3) If $s(h_1(x)) = s(h_2(x))$ and $\sigma(h_1(x)) < \sigma(h_2(x))$, then $h_1(x) > h_2(x)$;
(4) If $s(h_1(x)) = s(h_2(x))$ and $\sigma(h_1(x)) > \sigma(h_2(x))$, then $h_1(x) < h_2(x)$;
(5) If $s(h_1(x)) = s(h_2(x))$ and $\sigma(h_1(x)) = \sigma(h_2(x))$, then we define that $h_1(x)$ is equivalent to $h_2(x)$, denoted as $h_1(x) \sim h_2(x)$.

What's more, some basic operations and aggregation operators for hesitant fuzzy information are presented as follows:

Definition 3 [2], [3]: Let h, h_1 and h_2 be three HFEs, and h^c be the complementary set of h, $\lambda > 0$, then

(1) $h^c = \cup_{\gamma \in h} \{1 - \gamma\}$;
(2) $h_1 \cup h_2 = \cup_{\gamma_1 \in h_1, \gamma_2 \in h_2} \max\{\gamma_1, \gamma_2\}$;
(3) $h_1 \cap h_2 = \cup_{\gamma_1 \in h_1, \gamma_2 \in h_2} \min\{\gamma_1, \gamma_2\}$;
(4) $h^\lambda = \cup_{\gamma \in h} \{\gamma^\lambda\}$;
(5) $\lambda h = \cup_{\gamma \in h} \{1 - (1-\gamma)^\lambda\}$;
(6) $h_1 \oplus h_2 = \cup_{\gamma_1 \in h_1, \gamma_2 \in h_2} \{\gamma_1 + \gamma_2 - \gamma_1\gamma_2\}$;
(7) $h_1 \otimes h_2 = \cup_{\gamma_1 \in h_1, \gamma_2 \in h_2} \{\gamma_1\gamma_2\}$.

Definition 4 [3]: Let h_i $(i = 1, 2, \ldots, n)$ be a set of HFEs, then the hesitant fuzzy weighted averaging (HFWA) operator and the hesitant fuzzy weighted geometric (HFWG) operator are defined respectively as:

$$\begin{aligned} &HFWA(h_1, h_2, \ldots, h_n) \\ &= \overset{n}{\underset{i=1}{\oplus}} (\omega_i h_i) = \cup_{\gamma_1 \in h_1, \gamma_2 \in h_2, \ldots, \gamma_n \in h_n} \left\{1 - \prod_{i=1}^{n} (1-\gamma_i)^{\omega_i}\right\} \end{aligned} \tag{3}$$

$$\begin{aligned} &HFWG(h_1, h_2, \ldots, h_n) \\ &= \overset{n}{\underset{i=1}{\otimes}} (\omega_i h_i) = \cup_{\gamma_1 \in h_1, \gamma_2 \in h_2, \ldots, \gamma_n \in h_n} \left\{\prod_{i=1}^{n} \gamma_i^{\omega_i}\right\} \end{aligned} \tag{4}$$

where $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$ is the corresponding weight vector of HFEs with $\omega_i \in [0, 1]$ and $\sum_{i=1}^{n} \omega_i = 1$.

Xu and Xia [5], [7] developed different kinds of distance measures for HFSs, which satisfy the condition that HFEs in the same position have the same length and the values in them are arranged in ascending order. If two HFEs in the same position are not in the same length, according to the pessimistic principle, the shorter one will be expanded by increasing the minimum value until they have the same length.

Definition 5 [5]: Let $X = \{x_1, x_2, \cdots, x_n\}$ be a discrete universe, $H = \{x, h(x) | x \in X\}$ and $M = \{x, m(x) | x \in X\}$ be two HFSs, then the distance between H and M satisfies the following properties:

(1) $0 \le d(H, M) \le 1$;
(2) $d(H, M) = 0$ if and only if $H = M$;
(3) $d(H, M) = d(M, H)$.

The generalized hesitant weighted distance is depicted as:

$$d_1(H, M) = \left[\sum_{i=1}^{n} \omega_i \left(\frac{1}{\#h_{x_i}} \sum_{j=1}^{\#h_{x_i}} \left|h^{\rho(j)}(x_i) - m^{\rho(j)}(x_i)\right|^\lambda\right)\right]^{1/\lambda} \tag{5}$$

where $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$ is the corresponding weight vector of HFEs with $\omega_i \in [0, 1]$, $i = 1, 2, \ldots, n$, $\sum_{i=1}^{n} \omega_i = 1$, $h^{\rho(j)}(x_i)$ and $m^{\rho(j)}(x_i)$ are the jth largest values in $h(x_i)$ and $m(x_i)$, respectively.

Remark 1: If $\lambda = 1$, the generalized hesitant weighted distance is reduced to the hesitant fuzzy weighted Hamming distance $d_2(H, M) = \left[\sum_{i=1}^{n} \omega_i \left(\frac{1}{\#h_{x_i}} \sum_{j=1}^{\#h_{x_i}} \left|h^{\rho(j)}(x_i) - m^{\rho(j)}(x_i)\right|\right)\right]$. If $\lambda = 2$, the generalized hesitant weighted distance is reduced to the hesitant weighted Euclidean distance $d_3(H, M) = \left[\sum_{i=1}^{n} \omega_i \left(\frac{1}{\#h_{x_i}} \sum_{j=1}^{\#h_{x_i}} \left|h^{\rho(j)}(x_i) - m^{\rho(j)}(x_i)\right|^2\right)\right]^{1/2}$. If $\omega = \left(\frac{1}{n}, \frac{1}{n}, \cdots, \frac{1}{n}\right)^T$, then $d_2(H, M)$ is reduced to the normalized hesitant fuzzy Hamming distance $d_4(H, M) = \frac{1}{n} \sum_{i=1}^{n} \left(\frac{1}{\#h_{x_i}} \sum_{j=1}^{\#h_{x_i}} \left|h^{\rho(j)}(x_i) - m^{\rho(j)}(x_i)\right|\right)$ and $d_3(H, M)$ is reduced to the normalized hesitant fuzzy Euclidean distance $d_5(H, M) = \frac{1}{n} \sum_{i=1}^{n} \left(\frac{1}{\#h_{x_i}} \sum_{j=1}^{\#h_{x_i}} \left|h^{\rho(j)}(x_i) - m^{\rho(j)}(x_i)\right|^2\right)^{1/2}$.

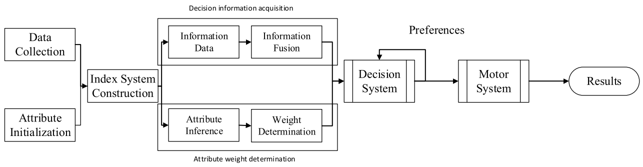


FIGURE 1. **The detailed process of DFT.**

III. THE INTEGRATED HESITANT FUZZY DECISION FIELD THEORY

The DFT is a dynamic decision-making method and can better simulate the uncertain decision-making process. Sometimes, when evaluating alternatives, experts can't reach a certain consensus or provide a common measure of the membership degree with sound reliability. In such cases, the HFS is an effective tool to depict hesitant preferences and uncertain knowledge of the experts. Therefore, to avoid the loss of information, we integrate the HFS into the DFT and propose the hesitant fuzzy decision field theory (HFDFT). Then, the group decision-making method based on HFDFT is presented and a specific implementation process for the HFDFT method is also illustrated.

A. THE CLASSICAL DFT METHOD

The DFT method, which can capture experts' cognitive decision-making behaviors, is a dynamic decision-making method. It can simulate the uncertain decision-making process and the cognitive process better. What is more, it can also accurately predict the relationship between the probability of selection and preference with time. Besides, it is widely applied in the fields of military command control and traffic control [43].

The main idea of DFT method is to calculate the momentary valences of alternatives based on the weights and the given attributes, which are inputted into the decision system. Then, the momentary preferences are obtained by integrating the decision system and valences along with the time. The final result is produced by the motor system after accumulating the preferences. The detailed process is illustrated in Figure 1.

Suppose that $P(t)$ is the preference information at the time t, then we can obtain the preference information at the next moment $t+s$ by the following equation [38]:

$$P(t+s) = SP(t) + V(t+s) \tag{6}$$

in which s is a little time step.

The feedback matrix S, satisfying $0 \leq S_{ij} \leq 1$, indicates the competitive effects between alternatives in the process of decision making. It is symmetric and reflects the interactive relationship between alternatives. In order to ensure the stability and convergence of computations, the eigenvalues of S should be less than 1 in general.

The valence vector represents the psychological expectations of experts for each alternative. The momentary valence $V_i(t)$ reflects the advantages or disadvantages of the ith alternative to the others in a certain attribute at the time t. We can obtain the valence vector by the following formula:

$$V(t) = CMW(t) \tag{7}$$

where the contrast matrix $C = \begin{cases} 1, i=j \\ -\frac{1}{n-1}, i \neq j \end{cases}$, the decision matrix M consists of complete preference information, W is the corresponding weight vector.

B. THE HESITANT FUZZY DECISION FIELD THEORY

Uncertainty and imprecision widely exist in practical decision-making problems. To depict the randomness and hesitant information more comprehensively, the HFS is proposed with the membership degree consisting of several possible values.

Similar to traditional DFT method, we elaborate the basic process of HFDFT method. To begin with, we identify the main attributes and establish an index system. Some experts are invited to evaluate the alternatives from different aspects by using HFSs. Then, in the situations that attribute weights are uncertain, we need to calculate the weight vector based on the hesitant fuzzy information. Finally, the feedback matrix and the corresponding parameters are redefined and calculated under hesitant fuzzy environment, presented as follows:

Definition 6: Let C be the contrast matrix, M^* be the hesitant fuzzy decision matrix that contains the preference information of the experts, $W(t)$ be the weight vector of attributes, then the hesitant fuzzy valence vector is defined as:

$$V^*(t) = C \otimes M^* \otimes W(t) \tag{8}$$

where $C = \begin{cases} 1, i=j \\ -\frac{1}{n-1}, i \neq j \end{cases}$ and $M^* \otimes W(t)$ is the weighted evaluation information of the alternatives, calculated by the basic operation laws of HFSs.

Definition 7: The feedback matrix S^* under the hesitant fuzzy environment depicts the memorizing effect of competitive relationship between different alternatives, consisting of self-connection and interconnection. The diagonal elements indicate the degree of self-influences for a specific alternative, and the off-diagonal elements represent the competitive influences between alternatives. Let I be the identity matrix, D^* be the distance degree matrix, φ and δ be the related parameters, then the feedback matrix based on Gaussian function under hesitant fuzzy environment is defined as:

$$S^* = I - \varphi \cdot e^{-\delta \cdot D^{*2}} \tag{9}$$

where φ indicates the competitive influence between alternatives and belongs to [0,1], δ depicts the discriminable capability and belongs to [0.01,1000] [42]. The more similar the alternatives are, the larger the value of δ should be. What's more, the distance degree matrix D^* is calculated by using the hesitant fuzzy weighted Euclidean distance as introduced in *Definition 5*.

Definition 8: Let $P^*(t)$ be the hesitant fuzzy preference information at the time t, S^* be the feedback matrix under hesitant fuzzy environment, and V^* be the hesitant fuzzy valence vector, then the hesitant fuzzy preference information at the next moment $t+s$ is defined as follows:

$$P^*(t+s) = S^* P^*(t) + V^*(t+s) \tag{10}$$

in which s is a little time step. It is obvious that the hesitant fuzzy preference $P^*(t)$ can be calculated by the dynamic process with time. The positive preference value indicates a tendency for a certain alternative, and the largest preference value corresponds to the best alternative.

C. GROUP DECISION MAKING BASED ON THE HESITANT FUZZY DECISION FIELD THEORY

When dealing with practical decision-making problems, we usually need to invite a group of experts to evaluate the alternatives from different aspects. To improve the application of the HFDFT method in solving the MAGDM problems, the group decision-making method based on the HFDFT is developed. As for the process-oriented decision theory in the group decision-making process, the key is to integrate preference information of different experts effectively. After integrating the preference information of different experts, the group decision-making method based on the HFDFT can reflect the original decision-making process and the final decision results is also obtained. The general group decision-making process based on the HFDFT is presented in detail as follows:

For a MAGDM problem, we assume that $P = \{p_1, p_2, \ldots, p_k\}$ is a group of experts, and $\zeta = (\zeta_1, \zeta_2, \ldots, \zeta_k)^T \left(\zeta \geq 0, \sum_{i=1}^{k} \zeta_i = 1\right)$is the corresponding weight vector of the experts. Then, $S = \{Si \,|i = 1, 2, \cdots, m\}$ is a finite set of alternatives, and $c = \{c_1, c_2, \ldots, c_n\}$ is a set of attributes. The corresponding weight vector of attributes is $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T \left(\omega \geq 0, \sum_{i=1}^{n} \omega_n = 1\right)$. By collecting the hesitant fuzzy information provided by the experts, we can construct the hesitant fuzzy decision matrix $H_l = \left(h_{ij}^{(l)}\right)_{m \times n} (l = 1, 2, \ldots, k)$, where $h_{ij}^{(l)}$ is the evaluation value of the alternative Si on the attribute c_j provided by the lth expert. $h_{ij}^{(l)}$ is the HFE consisting of several possible values.

With the individual hesitant fuzzy decision matrix of each expert, the HFWA operator in Section 2 is adopted to integrate the preference information provided by all the experts, denoted as $\dot{h}_{ij} = HFWA\left(h_{ij}^{(1)}, h_{ij}^{(2)}, \ldots, h_{ij}^{(k)}\right)$ $(i = 1, 2, \ldots, m; j = 1, 2, \ldots, n)$. The collective hesitant fuzzy decision matrix is $\dot{H} = \left(\dot{h}_{ij}\right)_{m \times n}$.

The attribute weights are usually directly provided by the experts. However, in the process of emergency decision making, sometimes the weight vector of attributes is either uncertain or incompletely known. Hence, it is significant to study the MAGDM problems with completely unknown or incompletely known attribute weight information.

According to the relationship and constraints of different attributes, we can obtain the weight information of attributes by a linear programming model [44] based on the score function, which is presented as follows:

Model 1:

$$\begin{cases} \max\left(s_i(\omega)\right) = \sum_{j=1}^{n} \omega \cdot s_{ij} \\ s.t. \quad \omega = \Lambda \\ \sum_{j=1}^{n} \omega_j = 1 \end{cases} \tag{11}$$

where $s_{ij} = s\left(\dot{h}_{ij}\right) = \frac{1}{\# h_{ij}} \sum_{\sigma=1}^{\# h_{ij}} \gamma_\sigma (i = 1, 2, \ldots, m; j = 1, 2, \ldots, n)$, γ_σ is the basic value of $\dot{h}_{ij}$. Λ represents all possible weight sets that can be determined by the known weight information.

In general, there are several kinds of relationships among the weights of attributes as follows [45]:

$$\begin{cases} \{\omega_i \geq \omega_j\} & \\ \{\omega_i - \omega_j \geq \delta_i\} & \delta_i > 0 \\ \{\omega_i \geq \delta_i \omega_j\} & 0 \leq \delta_i \leq 1 \\ \{\delta_i \leq \omega_i \leq \delta_i + \varepsilon_i\} & 0 \leq \delta_i < \delta_i + \varepsilon_i \leq 1 \end{cases} \tag{12}$$

By solving Model 1, the optimal weight solution corresponding to the alternative Si is obtained: $\omega^{(i)} = \left(\omega_1^{(i)}, \omega_2^{(i)}, \ldots, \omega_n^{(i)}\right)^T$. After obtaining the group hesitant fuzzy decision matrix and the corresponding weight vector of the attributes, we acquire the feedback matrix under hesitant fuzzy environment and the hesitant fuzzy valence vector. Subsequently, the hesitant fuzzy preference information of each alternative is calculated. Based on the preference values of alternatives, we can choose the optimal alternative. The specific implementation process is illustrated in Figure 2.

The HFDFT method combines the advantages of the HFSs in describing fuzzy information and the advantages of the DFT in dealing with cognitive decision-making problems, which will surely provide more reliable decision-making results. On the one hand, the introduction of HFSs can make up for the deficiency of the DFT in dealing with group dynamic decision-making problems to some extent. On the other hand, the HFDFT has transformed the traditional hesitant fuzzy MADM from single information integration to a dynamic comparative reasoning process. And the hesitant fuzzy information is fully utilized in the comparison process at every moment. In addition, the HFDFT method also takes into account the influence of the background information of each alternative on the experts' preferences.

IV. APPLICATION TO ROUTE SELECTION OF THE ARCTIC NORTHWEST PASSAGE

In order to solve the MAGDM problem concerning route selection of the Arctic Northwest Passage, we first establish an index system of route risk evaluation. Then, the proposed HFDFT method is used to evaluate the route risk of the Arctic Northwest Passage. A comparative analysis is further conducted to illustrate the advantages of our method.

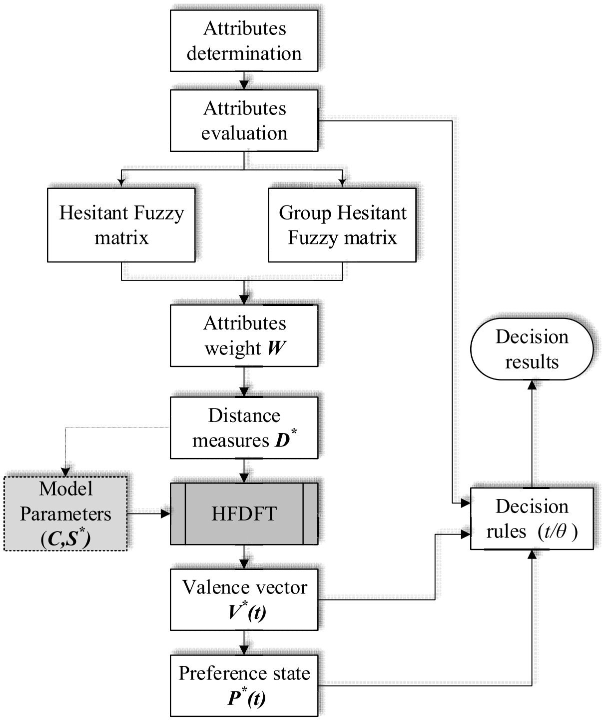


FIGURE 2. The general implementation procedure for the HFDFT.

TABLE 1. Routes of the arctic northwest passage.

Route number	Passing area (from east to west)
S1	Lancaster Sound---Barrow Strait---Viscant Melville Sound---Prince of Wales Strait---Amundsen Gulf
S2	Lancaster Sound---Barrow Strait---Viscant Melville Sound---M'Clure Strait
S3	Lancaster Sound---Barrow Strait---Peel Sound---Larsen Strait---Victoria Strait---Queen Maud Bay---Dease Strait---Coronation G.---Dolphin Channel---Amundsen Gulf
S4	Lancaster Sound---Prince Regent Inlet---The Strait of Peter---Victoria Strait---Queen Maud Bay---Dease Strait---Coronation G.---Dolphin Channel---Amundsen Gulf
S5	Foxe Channel---Foxe Basin---Gulf of Boothia---The Strait of Peter
S6	Lancaster Sound---Barrow Strait---Viscant Melville Sound---McClintock Chan---Victoria Strait---Dease Strait---Coronation G.---Dolphin Channel---Amundsen Gulf

TABLE 2. Index system for evaluating route risk of the northwest passage.

Object	Attribute
Route risk evaluation	Sea ice c_1
	Visibility c_2
	Strong wind c_3
	Islands and reefs c_4
	Water depth and width c_5

A. CASE STUDY

The Arctic is the most sensitive area to global climate change. In recent years, with the accelerated melting of the sea ice in Arctic [46], the impact of ice barriers on the opening of the Arctic waterways has gradually weakened. The Northwest Passage is a channel connecting the Pacific and the Atlantic oceans through the Arctic Ocean along the northern coast of North America and Canadian Arctic Islands. It is the shortest route that connects Asia and eastern North America. What's more, the Northwest Passage can shorten the transport route prominently, which will bring huge economic benefits. Therefore, the opening of the Northwest Passage is of great importance to the shipping industry. At present, some scholars have studied the navigation environment, navigation management, strategic planning and laws of the Northwest Passage. However, there are few studies on emergency response to marine emergencies in the Arctic region. For the long-term development and utilization of the Arctic region, it is necessary to evaluate risk of the routes of the Arctic Northwest Passage.

The Northwest Passage has been divided into six major routes by the Arctic Council in The Arctic marine shipping assessment 2009 report [47], and the specific route planning is presented in Table 1.

The identification and determination of indicators is the primary task, which directly affects the rationality of the evaluation process. There are many factors that influence the navigation of ships. In this paper, we mainly focus on the marine environment and geographical environment, which are complicated and changeable. From the perspective of the marine environment, the Northwest Passage is located in the cold area of high latitude which is covered with ice and snow most of the year, and most straits are blocked by sea ice all the year round. Meanwhile, the Arctic with low temperature harms ships and crews. Besides, strong winds, thick fog and other bad weather in the Arctic threaten the navigation of ships. Geographically, the Northwest Passage, along with its intricate islands, straits, bays, and icebergs, is considered to be one of the most dangerous waterways in the world. Complex terrain and other uncertainties also increase the safety risks to navigation.

According to the analysis above, we can determine the main factors influencing the route selection of the Northwest Passage in the Arctic. The constructed index system is presented in Table 2.

From the perspective of decision-making analysis, the route selection of the Northwest Passage involves many uncertainties. It is difficult to provide accurate quantitative preference information. In many situations, it depends on the consultation information provided by the experts. Therefore, we make full use of the advantages of the HFS in uncertain information description and use the HFDFT method to evaluate the routes of the Arctic Northwest Passage.

We note that the attribute indicators are divided into two types: positive attribute and negative attribute. The larger the value of the positive attribute is, the safer the route should be. In contrast, the higher the value of the negative attribute is, the more dangerous the route should be. These two types of attributes usually have different measurement. We need

TABLE 3. The Hesitant fuzzy decision matrix H_1 of the expert P_1.

	c_1	c_2	c_3	c_4	c_5
S1	{0.3}	{0.1,0.2}	{0.2}	{0.3}	{0.4,0.6}
S2	{0.2,0.3}	{0.1}	{0.4}	{0.2,0.4}	{0.8}
S3	{0.5}	{0.2}	{0.4,0.5}	{0.2}	{0.5,0.8}
S4	{0.8}	{0.6}	{0.7}	{0.5}	{0.6}
S5	{0.5,0.6}	{0.8}	{0.5,0.6 }	{0.4}	{0.2}
S6	{0.7}	{0.5}	{0.7}	{0.2}	{0.6}

TABLE 4. The Hesitant fuzzy decision matrix H_2 of the expert P_2.

	c_1	c_2	c_3	c_4	c_5
S1	{0.2}	{0.1}	{0.6}	{0.5}	{0.2}
S2	{0.1}	{0.4,0.5}	{0.5}	{0.4}	{0.6}
S3	{0.2}	{0.5}	{0.2}	{0.4,0.6}	{0.3}
S4	{0.6}	{0.8}	{0.4}	{0.6}	{0.4}
S5	{0.7}	{0.4,0.5}	{0.5}	{0.2,0.4}	{0.5}
S6	{0.4}	{0.3}	{0.5,0.6}	{0.3}	{0.2}

to transform them into the dimensionless indicators or the indicators with the same dimension to ensure the consistency and compatibility. To prevent the adverse consequences of different types of attributes, we define the transformation function to normalize the hesitant fuzzy information.

Definition 9 [2], [3]: Given a hesitant fuzzy set $H = \{x, h(x) | x \in X\}$, then the normalized HFS is defined as:

$$H(x)^N = f(H(x)) = \begin{cases} H(x) & \text{for the positive attribute} \\ neg(H(x)) & \text{for the negative attribute} \end{cases} \quad (13)$$

where $neg(H(x)) = \text{neg}\{x, h(x) | x \in X\} = \{x, 1 - h(x) | x \in X\}$ is the negation operation of $H(x)$.

For convenience, we assume that all the HFSs discussed in the following sections have been normalized. The hesitant fuzzy decision matrix consisting of normalized HFSs is denoted as normalized hesitant fuzzy decision matrix.

We denote the set of the attribute indicators as $c = \{c_j | j = 1, 2, \cdots, n\}$ and the set of the routes as $S = \{Si | i = 1, 2, \cdots, m\}$. According to the data of sea ice and wind speed [48], the data of islands and reefs [49], the data of water depth and width [49] and the data of visibility [50], four experts are invited to evaluate six major routes of the Arctic Northwest Passage concerning all the indicators. Their assessment values are expressed as HFSs. Then we construct the original hesitant fuzzy decision matrix of each expert $H_l = \left(h_{ij}^{(l)}\right)_{m\times n}$ $(l = 1, 2, 3, 4)$(See Tables 3-6). Besides, the attributes in Tables 3-6 are normalized according to *Definition 10:* The weight vector of the four experts is $\zeta = (0.2, 0.3, 0.3, 0.2)^T$. We integrate the decision matrices of each expert by the HFWA operator and construct the collective hesitant fuzzy decision matrix $\dot{H}$(See Table 7).

For actual group decision-making problems, one of the most important considerations is the weight vector of the attributes. After discussions and consultations, four experts reach a consensus for weight information of the attributes,

TABLE 5. The Hesitant fuzzy decision matrix H_3 of the expert P_3.

	c_1	c_2	c_3	c_4	c_5
S1	{0.4,0.5}	{0.2}	{0.3}	{0.3}	{0.5}
S2	{0.3}	{0.5}	{0.5}	{0.6}	{0.7}
S3	{0.5}	{0.4,0.6}	{0.6}	{0.5}	{0.4}
S4	{0.7}	{0.6}	{0.5}	{0.6}	{0.5,0.6}
S5	{0.8}	{0.3}	{0.7}	{0.4}	{0.3}
S6	{0.5}	{0.4}	{0.6}	{0.4}	{0.6}

TABLE 6. The Hesitant fuzzy decision matrix H_4 of the expert P_4.

	c_1	c_2	c_3	c_4	c_5
S1	{0.6}	{0.5}	{0.3,0.4}	{0.1}	{0.2}
S2	{0.4}	{0.6}	{0.3}	{0.2}	{0.6}
S3	{0.6}	{0.3}	{0.5}	{0.4}	{0.2}
S4	{0.5,0.6}	{0.6}	{0.4}	{0.6}	{0.5}
S5	{0.3}	{0.4}	{0.5}	{0.4}	{0.6}
S6	{0.4,0.5}	{0.5,0.6}	{0.4}	{0.4,0.6}	{0.5,0.6}

TABLE 7. The collective Hesitant fuzzy decision matrix $\dot{H}$.

	c_1	c_2	c_3	c_4	c_5
S1	{0.378, 0.411}	{0.228, 0.246}	{0.392, 0.411}	{0.335}	{0.344, 0.395}
S2	{0.248, 0.268}	{0.432, 0.462}	{0.445}	{0.404, 0.437}	{0.681}
S3	{0.449}	{0.379, 0.451}	{0.442, 0.462}	{0.398, 0.467}	{0.358, 0.466}
S4	{0.665, 0.681}	{0.675}	{0.505}	{0.582}	{0.495, 0.528}
S5	{0.651, 0.667}	{0.496, 0.522}	{0.571, 0.589}	{0.346, 0.4}	{0.419}
S6	{0.505, 0.523}	{0.416, 0.441}	{0.562, 0.59}	{0.334, 0.386}	{0.485, 0.508}

which is depicted by a set of linear inequality as follows:

$$\Delta = \begin{cases} \omega_1 \le 0.25 \\ 0.1 \le \omega_2 \le 0.2 \\ 0.2 \le \omega_3 \le 0.3 \\ \omega_5 \le 0.2 \\ \omega_3 - \omega_2 \ge \omega_4 - \omega_5 \\ \omega_4 \ge \omega_1 \\ \omega_3 - \omega_1 \le 0.1 \\ 0.1 \le \omega_4 \le 0.4 \\ \sum_{i=1}^{n} \omega_i = 1\, (i = 1, 2, 3, 4, 5) \end{cases}$$

Therefore, we can calculate the optimal weight vector of the attributes based on Model 1, and the calculation result is$\omega = (0.2113, 0.1801, 0.2315, 0.2618, 0.1153)^T$. Based on the collective hesitant fuzzy decision matrix $\dot{H}$, we can obtain that the contrast matrix as:

$$C = \begin{bmatrix} 1 & -0.2 & -0.2 & -0.2 & -0.2 & -0.2 \\ -0.2 & 1 & -0.2 & -0.2 & -0.2 & -0.2 \\ -0.2 & -0.2 & 1 & -0.2 & -0.2 & -0.2 \\ -0.2 & -0.2 & -0.2 & 1 & -0.2 & -0.2 \\ -0.2 & -0.2 & -0.2 & -0.2 & 1 & -0.2 \\ -0.2 & -0.2 & -0.2 & -0.2 & -0.2 & 1 \end{bmatrix}$$

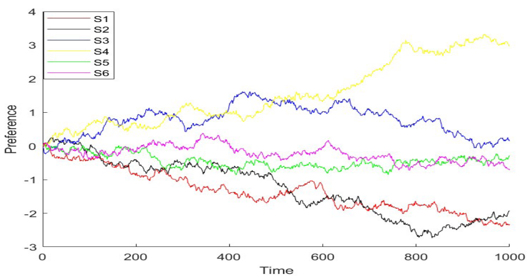


FIGURE 3. The hesitant fuzzy decision field theory predictions for the six the six alternative routes.

The parameters of the feedback matrix under hesitant fuzzy environment are set to $\varphi = 0.15$ and $\delta = 20$, then the feedback matrix S^* is obtained as

$$S^* = \begin{bmatrix} 0.75 & -0.03 & -0.03 & -0.01 & -0.02 & -0.02 \\ -0.03 & 0.75 & -0.02 & -0.01 & -0.01 & -0.01 \\ -0.03 & -0.02 & 0.75 & -0.02 & -0.015 & -0.033 \\ -0.01 & -0.01 & -0.02 & 0.75 & -0.04 & -0.02 \\ -0.02 & -0.01 & -0.015 & -0.04 & 0.75 & -0.038 \\ -0.02 & -0.01 & -0.033 & -0.02 & -0.038 & 0.75 \end{bmatrix}$$

In this case, the threshold can be set as 2.8. After 1000 times' simulations, we obtain the preferences of six alternative routes for the Northwest Passage in the Arctic, and the prediction results are illustrated in Figure 3.

It is obvious that the route S4 is the best alternative, which is consistent with The Arctic marine shipping assessment 2009 report published by the Arctic Council. The preference probabilities of different alternative routes vary with time, and the preference of route S4 is more obvious over time. We can find that the proposed HFDFT method can not only depict the hesitant information and experts' preferences more clearly, but also describe the background information of alternatives and illustrate the original decision-making process. It is a process-oriented dynamic decision-making method, which is closer to the actual decision-making circumstance and can provide the decision-making results accurately.

B. THE COMPARISONS WITH THE EXISTING METHODS FOR HFS

To illustrate the advantages and effectiveness of the proposed HFDFT method, we compare it with two existing methods under hesitant fuzzy environment. The one is to compare the routes by the scores of collective hesitant fuzzy information, the other one is to compare the routes by the correlation coefficients between each route and the ideal alternative. For comparing effectively and understanding conveniently, in this section we adopt the collective hesitant fuzzy decision matrix H_c in Table 7 and the corresponding weight vector $\omega = (0.2113, 0.1801, 0.2315, 0.2618, 0.1153)^T$ as the evaluation information.

First, we obtain the overall preferences of the collective hesitant fuzzy information for the six routes through the HFWA operator mentioned in Definition 4 (See Table 8). After aggregating the evaluation information expressed as

TABLE 8. The Preferences of the collective Hesitant Fuzzy information for the routes by the HFWA operator.

Routes	Overall preference values
S1	{0.3413,0.3441,0.3462,0.3475,0.3489,0.3489,0.3502,0.3516,0.352 2,0.3536,0.3549,0.355,0.3564,0.3577,0.3596,0.3624}
S2	{0.432,0.4352,0.4375,0.4404,0.4407,0.4436,0.4458,0.449}
S3	{0.4118,0.4168,0.4242,0.4248,0.429,0.4296,0.4303,0.4351,0.4368 ,0.4416,0.4423,0.4428,0.447,0.4475,0.4545,0.4591}
S4	{0.5948,0.598,0.599,0.6021}
S5	{0.5111,0.5157,0.5159,0.5159,0.5205,0.5205,0.5207,0.522,0.5252 ,0.5265,0.5267,0.5267,0.5312,0.5312,0.5314,0.5358}
S6	{0.4618,0.4646,0.466,0.466,0.4688,0.4688,0.4699,0.4702,0.4727, 0.473,0.4731,0.4741,0.4741,0.4759,0.4768,0.4769, 0.4772,0.4772,0.4782,0.48,0.48,0.481,0.4811,0.4813,0.4838,0.48 41,0.4852,0.4852,0.4879,0.4879,0.4892,0.4919}

TABLE 9. The scores of the collective Hesitant fuzzy information for the routes.

Routes	S1	S2	S3	S4	S5	S6
Scores	0.3519	0.4405	0.4358	0.5985	0.5236	0.477

HFEs in Table 8, the scores of each route are calculated according to Definition 2 and presented in Table 9. It can be seen that the route S4 is the best alternative.

To compare all the routes by the correlation coefficients, we first review some basic concepts about the novel correlation coefficient between HFSs [51]. The novel correlation coefficient doesn't add any value so that it can reserve the original information to the greatest extent. Moreover, the values of the novel correlation matrix vary from negative values to positive ones, and the novel correlation coefficient can depict the relationship between different alternatives better.

Definition 10 [51]: Let $H = \{x_i, h(x_i) | x_i \in X, i = 1, 2, \ldots, n\}$ and $M = \{x_i, m(x_i) | x_i \in X, i = 1, 2, \ldots, n\}$ be two HFSs on the reference set X, and $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$ be the corresponding weight vector of $x_i \in X$ $(i = 1, 2, \ldots, n)$ with $\omega_i \in [0, 1]$, $i = 1, 2, \ldots, n$ and $\sum_{i=1}^{n} \omega_i = 1$. The weighted correlation coefficient between the HFSs H and M is defined as:

$$\rho_{\omega HFS}(H, M) = \sum_{k=1}^{n} \left(\omega_i \overline{h}(x_i) - \overline{H}_\omega\right) \cdot \left(\omega_i \overline{m}(x_i) - \overline{M}_\omega\right) \tag{14}$$

where $\overline{h}(x_i) = \frac{1}{l_{Hi}} \sum_{j=1}^{l_{Hi}} \gamma_{Hij}$, $\overline{m}(x_i) = \frac{1}{l_{Mi}} \sum_{j=1}^{l_{Mi}} \gamma_{Mij}$, $\overline{H}_\omega = \sum_{i=1}^{n} \omega_i \overline{h}(x_i)$ and $\overline{M}_\omega = \sum_{i=1}^{n} \omega_i \overline{m}(x_i)$.

The ideal route is denoted as RT^* with the evaluation information $\{(1)(1), (1), (1), (1)\}$. Then, we can calculate the weighted correlation coefficients between the ideal route RT^* and each route. The calculation results are shown in Table 10. We can see that the route S4 is the best one.

By comparing the proposed HFDFT method with the two existing methods, we note that the best route derived by our method is the same as that derived by the traditional methods. The proposed method can not only depict the uncertain information more delicately, but also display the dynamic decision-making process. However, the traditional

TABLE 10. The weighted correlation coefficient between the ideal route and the six alternative routes.

$\rho(S1)$	$\rho(S2)$	$\rho(S3)$	$\rho(S4)$	$\rho(S5)$	$\rho(S6)$
1.1226	1.3677	1.3978	1.9018	1.6416	1.5109

methods only depend on the final score function values and the correlation coefficient values. Besides, the proposed method can make use of the original data and the experts' evaluation information effectively, and it depicts the process of comparison between different routes clearly. While the two traditional methods are result-oriented static decision-making methods and only compare different alternatives by the final integrated information. The results derived by the two traditional methods are too absolute and inconsistent with reality. Therefore, our method can simulate the process of the experts' consideration and comparison by the contrast matrix and the feedback matrix. The results derived by our method, containing much more probabilistic information and varying with time, are more logical and reasonable.

The HFDFT method conducts a comprehensive comparative analysis of different alternatives by the input decision-making matrix and various parameters. After that, it obtains and accumulates the dynamic preference values with decision-making time of different alternatives, which avoids the shortcomings of the existing methods that just depend on the information integration by integration operators at a certain time. By analyzing the principles of the existing methods, we can find that they usually ignore the longitudinal dynamic comparisons between alternatives concerning different attributes, and it will result in the local optimal decision-making results easily. The process-oriented dynamic DFT method can make up for this deficiency. Therefore, the HFDFT method, which combines the HFSs and the DFT, has more advantages in dealing with the MAGDM problems.

What's more, we need to consider the choice of two main decision rules for the proposed HFDFT method. If the decision rule is the decision time, then we should stop to analyze and calculate when the accumulation time of the decision process reaches the specified time threshold. At this time, the alternative with the largest preference value is chosen as the optimal one. If the decision rule is the preference threshold, then we should stop to calculate and provide the corresponding best alternative when the preference value reaches the specified preference threshold. At present, there is no general standard and in-depth theoretical research on the choice of threshold in practical applications. Actually, the threshold usually depends on the empirical information of the experts, which may result in some contingency and randomness. To improve the practicality of the HFDFT method, we set a larger time threshold for less time-sensitive MCGDM problems. Although this may spend some computation time, it can ensure that the HFDFT method provides more accurate and reliable decision-making results.

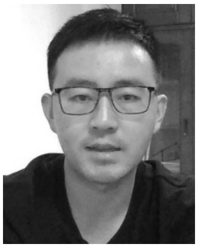
V. CONCLUSIONS

The traditional DFT method is a process-oriented dynamic decision-making method, which can simulate the motivational process and the cognitive process of uncertain decision making. Besides, the DFT method can accurately predict the selection probability and the relationship between preference intensity and time, which has been widely applied in decision-making problems. However, it is never easy for the experts in a group to reach a certain consensus of the membership degree with sound reliability when evaluating the alternatives. In order to avoid the loss of information, we introduce the HFS theory, with the membership degree consisting of several possible values, which can depict the uncertain knowledge and hesitant fuzzy preferences more comprehensively and accurately. It is important to study the cognitive decision mechanism and develop the cognitive decision making model for the hesitant fuzzy decision theory. Therefore, we integrate the HFS into the DFT and propose a new decision-making method named as HFDFT. We also define the hesitant fuzzy momentary preference function and other parameters in the HFDFT. Then, the group decision-making method based on the HFDFT is presented. A specific implementation process for the HFDFT method to manage with actual evaluation problems is illustrated. The HFDFT can not only depict the experts' preferences more delicately, but also describe the background information of alternatives at the same time. The proposed HFDFT model shifts the conception of hesitant fuzzy information making from single information aggregation to dynamic comparison and reason process. It illustrates the original decision-making process better and is more reasonable than traditional hesitant fuzzy decision-making methods. Besides, the proposed HFDFT method can make full use of the evaluation information and depict the process of comparison between different alternatives clearly. Since our method can simulate the process of experts' consideration and comparison by the contrast matrix and feedback matrix, it is obvious that our results are more logical, which contain more probabilistic information and vary with time. Finally, we apply the proposed method to the route selection problem of Arctic Northwest Passage, which demonstrates the accuracy and rationality of our method in dealing with practical MAGDM problems. Besides, two traditional methods based on the score function and the correlation coefficient of the hesitant fuzzy information are introduced for comparisons to further illustrate the advantages of our method.

In the future, we will mainly focus on the exploration of the DFT in other fuzzy sets to improve the evaluation results of the uncertain information. The extension of the DFT to other forms of fuzzy sets is very significant to the MAGDM problems by deliberating the decision-making process dynamically. Besides, we will try to combine the proposed HFDFT method with other models to solve actual problems.

REFERENCES

[1] V. Torra and Y. Narukawa, "On hesitant fuzzy sets and decision," in *Proc. Fuzz-IEEE*, Aug. 2009, pp. 1378–1382.
[2] V. Torra, "Hesitant fuzzy sets," *Int. J. Intell. Syst.*, vol. 25, no. 6, pp. 529–539, 2010.
[3] M. M. Xia and Z. S. Xu, "Hesitant fuzzy information aggregation in decision making," *Int. J. Approx. Reason*, vol. 52, no. 3, pp. 395–407, 2011.
[4] H. C. Liao and Z. S. Xu, "Subtraction and division operations over hesitant fuzzy sets," *J. Intell. Fuzzy Syst.*, vol. 27, no. 1, pp. 65–72, 2014.
[5] Z. S. Xu and M. M. Xia, "Distance and similarity measures for hesitant fuzzy sets," *Inf. Sci.*, vol. 181, no. 11, pp. 2128–2138, 2011.
[6] F. Y. Meng and X. H. Chen, "Correlation coefficients of hesitant fuzzy sets and their application based on fuzzy measures," *Cogn. Comput.*, vol. 7, no. 4, pp. 445–463, 2015.
[7] Z. S. Xu and M. M. Xia, "On distance and correlation measures of hesitant fuzzy information," *Int. J. Intell. Syst.*, vol. 26, no. 5, pp. 410–425, 2011.
[8] X. L. Zhang and Z. S. Xu, "The TODIM analysis approach based on novel measured functions under hesitant fuzzy environment," *Knowl. Based Syst.*, vol. 61, pp. 48–58, May 2014.
[9] B. Zhu, "Studies on consistency measure of hesitant fuzzy preference relations," *Pro. Comput. Sci.*, vol. 17, pp. 457–464, Jan. 2013.
[10] X. Feng, L. Zhang, and C. Wei, "The consistency measures and priority weights of hesitant fuzzy linguistic preference relations," *Appl. Soft Comput.*, vol. 65, pp. 79–90, Apr. 2018. doi: 10.1016/j.asoc.2017.12.050.
[11] B. Zhu, Z. Xu, and J. Xu, "Deriving a ranking from hesitant fuzzy preference relations under group decision making," *IEEE Trans. Cybern.*, vol. 44, no. 8, pp. 1328–1337, Aug. 2014.
[12] D. Yu, Y. Wu, and W. Zhou, "Multi-criteria decision making based on Choquet integral under hesitant fuzzy environment," *J. Comput. Inf. Syst.*, vol. 7, no. 12, pp. 4506–4513, 2011.
[13] G. Wei, "Hesitant fuzzy prioritized operators and their application to multiple attribute decision making," *Knowl. Based Syst.*, vol. 31, pp. 176–182, Jul. 2012.
[14] D. Yu, Y. Wu, and W. Zhou, "Generalized hesitant fuzzy Bonferroni mean and its application in multi-criteria group decision making," *J. Comput. Inf. Syst.*, vol. 9, no. 2, pp. 267–274, 2012.
[15] N. Chen, Z. S. Xu, and M. M. Xia, "The ELECTRE I multi-criteria decision-making method based on hesitant fuzzy sets," *Int. J. Inf. Technol. Decis.*, vol. 14, no. 3, pp. 621–657, 2015.
[16] M. W. Lin, Z. S. Xu, and Y. L. Zhai, "Multi-attribute group decision-making under probabilistic uncertain linguistic environment," *J. Oper. Res. Soc.*, vol. 69, no. 2, pp. 157–170, 2018.
[17] R. M. Rodriguez, L. Martinez, and F. Herrera, "Hesitant fuzzy linguistic term sets for decision making," *IEEE Trans. Fuzzy Syst.*, vol. 20, no. 2, pp. 1651–1670, Feb. 2012.
[18] J. Q. Wang, J. T. Wu, and J. Wang, "Interval-valued hesitant fuzzy linguistic sets and their applications in multi-criteria decision-making problems," *Inf. Sci.*, vol. 288, pp. 55–82, Dec. 2012.
[19] R. M. Rodriguez *et al.*, "A position and perspective analysis of hesitant fuzzy sets on information fusion in decision making. Towards high quality progress," *Inf. Fusion*, vol. 29, pp. 89–97, May 2016.
[20] L. Z. Yang, R. Zhang, T. P. Hou, Z. N. Hao, and J. Liu, "Hesitant cloud model and its application in the risk assessment of 'The Twenty-First Century Maritime Silk Road,'" *Math. Problems Eng.*, vol. 6, pp. 1–11, 2016.
[21] C. Q. Li, H. Zhao, and Z. Xu, "Kernel C-means clustering algorithms for hesitant fuzzy information in decision making," *Int. J. Fuzzy Syst.*, vol. 20, no. 1, pp. 141–154, 2017.
[22] H. P. Kriegel, P. Kroger, E. Schubert, and A. Zimek, "A general framework for increasing the robustness of PCA-based correlation clustering algorithms," in *Scientific and Statistical Database Management* (Lecture Notes in Computer Science), vol. 5069. 2008, pp. 418–435.
[23] N. Chen, Z. S. Xu, and M. M. Xia, "Correlation coefficients of hesitant fuzzy sets and their applications to clustering analysis," *Appl. Math. Model.*, vol. 37, no. 4, pp. 2197–2211, 2013.
[24] Y. Xu, A. Xu, and H. Wang, "Hesitant fuzzy linguistic linear programming technique for multidimensional analysis of preference for multi-attribute group decision making," *Int. J. Mach. Learn. Cybern.*, vol. 7, no. 5, pp. 845–855, 2016.
[25] F. Zhang, J. Li, and J. Chen, "Hesitant distance set on hesitant fuzzy sets and its application in urban road traffic state identification," *Eng. Appl. Artif. Intell.*, vol. 61, pp. 57–64, May 2017.
[26] Y. L. Zhai, Z. S. Xu, and H. C. Liao, "Measures of probabilistic interval-valued intuitionistic hesitant fuzzy sets and the application in reducing excessive medical examinations," *IEEE Trans. Fuzzy Syst.*, vol. 26, no. 3, pp. 1651–1670, Jun. 2018.
[27] C. Zhang, D. Li, and Y. Yan, "A dual hesitant fuzzy multigranulation rough set over two-universe model for medical diagnoses," *Comput. Math. Methods Med.*, vol. 6, pp. 1–11, 2016.
[28] H. Choi, K. Han, and K. Choi, "A fuzzy medical diagnosis based on quantiles of diagnostic measures," *J. Intell. Fuzzy Syst.*, vol. 31, no. 1, pp. 3197–3202, Jan. 2016.
[29] S. J. Chen and C. L. Hwang, "Multiple attribute decision making—An overview," in *Fuzzy Multiple Attribute Decision Making: Methods and Applications*. Berlin, Germany: Springer, 1992.
[30] J. W. Payne, J. R. Bettman, and E. J. Johnson, "Adaptive strategy selection in decision," *J. Exp. Psychol. Learn.*, vol. 14, no. 3, pp. 534–552, 1988.
[31] G. D. Jamieson and W. M. Petrusic, "Preference and the time to choose," *Organ. Behav. Hum. Perform.*, vol. 19, pp. 56–67, 1977.
[32] M. Toda, "The design of a fungus-eater: A model of human behavior in an unsophisticated environment," *Syst. Res. Behav. Sci.*, vol. 7, no. 2, pp. 164–183, 2010.
[33] B. Scheibehenne, J. Rieskamp, and E.-J. Wagenmakers, "Testing adaptive toolbox models: A Bayesian hierarchical approach," *Psychol. Rev.*, vol. 120, no. 1, pp. 39–64, 2013.
[34] C. Gonalez, J. F. Lerch, and C. Lebiere, "Instance-based learning in dynamic decision making," *Cognit. Sci.*, vol. 27, no. 4, pp. 591–635, 2010.
[35] T. L. Saaty, "Making and validating complex decisions with the AHP/ANP," *J. Syst. Sci. Syst. Eng.*, vol. 14, no. 1, pp. 1–36, 2005.
[36] J. R. Busemeyer and T. J. Pleskac, "Theoretical tools for understanding and aiding dynamic decision making," *J. Math. Psychol.*, vol. 53, no. 3, pp. 126–138, 2009.
[37] Z. N. Hao and Z. S. Xu, "Novel intuitionistic fuzzy decision making models in the framework of decision field theory," *Inf. Fusion*, vol. 33, pp. 57–70, Jan. 2017.
[38] J. R. Busemeyer and J. T. Townsend, "Decision field theory: A dynamic-cognitive approach to decision making in an uncertain environment," *Psychol. Rev.*, vol. 100, pp. 432–459, Jan. 1993.
[39] J. R. Busemeyer and A. Diederich, "Survey of decision field theory," *Math. Soc. Sci.*, vol. 43, no. 3, pp. 345–370, 2002.
[40] R. M. Roe, J. R. Busemeyer, and J. T. Townsend, "Multialternative decision field theory: A dynamic connectionst model of decision making," *Psychol. Rev.*, vol. 108, no. 2, pp. 370–392, 2001.
[41] J. M. Hotaling, J. R. Busemeyer, and J. Li, "Theoretical developments in decision field theory: Comment on tsetsos, usher, and chater (2010)," *Psychol. Rev.*, vol. 117, no. 4, pp. 1294–1298, 2010.
[42] N. A. Berkowitsh, B. Scheibehenne, and J. Rieskamp, "Rigorously testing multi-alternative decision field theory against random utility models," *J. Exp. Psychol. Gen.*, vol. 143, no. 3, pp. 1331–1348, 2013.
[43] T. O. Hancock, S. Hess, and C. F. Choudhury, "Decision field theory: Improvements to current methodology and comparisons with standard choice modelling techniques," *Transport. Res. B, Method*, vol. 107, pp. 18–40, Jan. 2018.
[44] Z. S. Xu, "Multi-person multi-attribute decision making models under intuitionistic fuzzy environment," *Fuzzy Optim. Decis. Making*, vol. 6, no. 3, pp. 221–236, 2007.
[45] Z. S. Xu and X. Q. Cai, *Intuitionistic Fuzzy Information Aggregation*. Berlin, Germany: Springer, 2012.
[46] T. Sou and G. Flato, "Sea ice in the Canadian arctic archipelago: Modeling the past (1950–2004) and the future (2041–60)," *J. Climate*, vol. 22, no. 8, pp. 2181–2198, 2009.
[47] L. Brigham and R. Mccalla, *Arctic Marine Shipping Assessment 2009 Report*. London, U.K.: Cambridge Univ. Press, 2009.
[48] The European Center for Medium-Range Weather Forecasts (ECMEF). (Jan. 1979). *ERA-Interim*. Accessed: 2017. [Online]. Available: http://apps.ecmwf.int/datasets/
[49] The National Geophysical Data Center (NGDC). *ETOPO1*. Accessed: 2017. [Online]. Available: http://maps.ngdc.noaa.gov/viewers/wcs-client/
[50] The National Oceanic and Atmospheric Administration (NOAA). *ISCCP Cloud Data*. Accessed: 2017. [Online]. Available: https://www.ncdc.noaa.gov/data-access/satellite-data/satellite-data-access-datasets
[51] H. Liao, Z. Xu, and X.-J. Zeng, "Novel correlation coefficients between hesitant fuzzy sets and their application in decision making," *Knowl. Based Syst.*, vol. 82, pp. 115–127, Jul. 2015.



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