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The Strategy Evolution in Double Auction Based on the Experience-Weighted Attraction Learning Model

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ABSTRACT The double auction is a widely applicable trading mechanism used to converge to competitive equilibrium in different markets from which multiple equilibriums and incomplete information may arise. Therefore, different learning models have been applied to facilitate bidding strategies for buyers and sellers in the market. However, due to the existence of problems in double auction markets such as individual bounded rationality and information incompleteness, it is still necessary to explore a more general learning model to depict the learning mechanism in double auction markets and predict the evolution processes of bidding strategies for both sides. Therefore, this paper aims at introducing the use of the experience-weighted attraction (EWA) model for double auction because it combines reinforcement learning with belief learning that then converts EWA in a suitable and interesting learning model for describing and improving individuals' learning behavior. It can become an effective learning model for bidding strategies in the double auction. In addition to the use of the EWA for strategy evolution in the double auction, the impact of its different bidding strategy performance parameters will be also analyzed and compared with other learning models.

INDEX TERMS Bidding strategies, double auction, EWA learning, parameters selection.

I. INTRODUCTION

Double auction is an important trading mechanism that can solve the problem of asymmetric information between buyers and sellers in the market, thus effectively solving problems such as colluding and malicious bidding in unilateral auctions. However, many experiments and practices show that double auction markets can quickly converge to equilibrium despite there being only are few participants and insufficient information in the market, which being contrary to traditional economic theory, has attracted many scholars' attention.

In 1962 Smith [1] successfully introduced the experimental method in double auction in order to reach competitive equilibrium as predicted by neoclassical economics, even with incomplete information. It has attracted the interest of many researchers in experimental economics [2]–[4] as has its application to real-world markets such as electricity markets, cognitive radio networks, and spectrum auction [5]–[7].

Due to the fact that double auction market information is often not fully available, and also often incomplete and scarce, different proposals have applied game theory with the aim of reaching competitive equilibrium [8], [9]. Also, the difficulty of reaching a quick consensus in the game with incomplete information and multiple equilibrium, makes that the participants in the double auction market follow an iterative learning process to be able to form their bidding strategy and reach the final equilibrium [10]. So multiple learning processes have been proposed to depict strategy dynamic and equilibrium formation for both sides (seller/buyer) with asymmetric information in double auction markets based on different models such as reinforcement learning [11]–[13], genetic learning [14]–[16] and other learning algorithms [17]–[19].

It is common that individuals adjust their bidding strategies by using several learning models such as reinforcement, and

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belief learning. Therefore, a more general learning model that analyzes the strategy evolution in a double auction market seem to be worth studying. A promising choice to fulfil this necessity is the Experience-Weighted Attraction learning model (EWA) proposed by Camerer and Ho Teck [20] that combines elements of reinforcement learning and weighted fictitious pay (belief learning). The EWA model not only generalizes the reinforcement and belief learning models, including them as special cases, but also combines both learning models which provides a more accurate process to describe individual learning behaviors than either the reinforcement or the belief model alone. Due to these features, the EWA learning model has been researched and applied to predict behaviors in games [21]–[23] and other applications such as stimulating the operations of wholesale electricity markets [24], analyzing the navigation of mobile robots [25] and meeting cognitive radio application's practical requirements [26]. In recent years, the EWA learning also has been applied in auction research regarding its functional attributes [27]–[30].

EWA has provided successful results in the improvement of (accuracy) prediction of individuals' behavior, because it includes the main characteristics of belief and reinforcement learning and chooses strategies with weights obtained in the learning process. Therefore, in this paper we will extend the EWA approach to double auction markets in order to describe the evolution strategies and learning behaviors of bounded rational individuals in such markets. Also, we will adjust the values of the three key parameters of the EWA model: (i) the weight on foregone payoffs, (ii) the decay rate of the previous attraction and (iii) the growth rate of attraction; to depict the different combinations of market information structure and individual rational characteristics, which can better show the flexibility of the EWA learning model and further reveal the impacts of this learning model on the transaction evolution mechanism for both sides in double auction markets. Eventually, according to the experimental results, we will design a mechanism that can make the double auction market reach a final equilibrium quickly, thus saving the time of participants and the cost of organizers in the market.

The remaining parts of the paper are set up as follows. In section 2, we review the basics of the double auction model and of the EWA learning model that will be applied to our proposal. In section 3 we introduce the double auction model with EWA learning and simulate the double auctions based on EWA. Thereafter, we compare the results of the double auction based EWA learning with another two pure learning models in section 4. Eventually, section 5 concludes the paper pointing out the conclusions obtained.

II. PRELIMINARIES

This section revises the basic concepts and performance of the basic double auction model and the EWA learning model, because these concepts are necessary to be able to develop and understand our proposal.

A. THE BASIC DOUBLE AUCTION MODEL

Double auction is an important trading mechanism which can converge to competitive equilibrium steadily and quickly [1], it addresses the interactive procedure during the biding processes and how the mechanism works out different outcomes with various conditions. In a double auction market buyers and sellers submit their bidding in any order, for a single good, the bidding strategies of buyers are underbidding its value, and the bidding strategies of sellers is overbidding its cost. Once, there is an agent, either a buyer or seller, who accepts the bidding from another side; a deal is made between both of them, and a new bidding round for another product begins. There are multiple trading sessions, and the transaction value is always between the initial bidding values of the buyers and the sellers [2].

Therefore, in the basic double auction model, for a single and indivisible good, the buyer's valuation is noted as V_b and the seller's valuation as V_s, both assessments follow a uniform distribution in [0,1]. And the buyer's and seller's biddings are noted as P_b and P_s respectively. If $P_b > P_s$ then both sides make a deal with the price of $P = (P_b + P_s)/2$, and the buyer's and seller's utilities are $U_b = V_b - (P_b + P_s)/2$ and $U_s = (P_b + P_s)/2 - V_s$ respectively. If $P_b < P_s$, neither the buyer nor the seller make a deal and both utilities are 0.

Under the trading mechanism of double auction, if the information is complete, there will be a Nash game demand when V_b and V_s are common knowledge [8]. If $V_b > V_s$ then there is continuous pure-strategy and Pareto efficient equilibrium in this game, i.e., the buyer's and seller's biddings are the same, $P_b = P_s = P \in [V_s, V_b]$, and both sides can obtain a positive surplus at this point. But if $P_b < P$ or $P_s > P$, then neither the buyer nor the seller make a deal.

However, if the information is incomplete, the valuations of V_b and V_s are private information for both sides, so that neither buyer nor seller can achieve a rational expectation estimation of the other's bidding. However, it can be assumed that V_b and V_s follow a uniform distribution in [0,1], and thus there will be a static Bayesian game. In this game, the buyer's bidding is the function of their valuation V_b, i.e., $P_b(V_b)$; and analogously for the seller's bidding the function of their valuation V_s, is $P_s(V_s)$. There is a Bayesian Nash equilibrium in the game [8], hence the optimal response of the buyer to the seller's strategy for any $V_b \in [0, 1]$, $P_b(V_b)$ is obtained by Eq. (1):

$$\pi_b = \max_{P_b}[V_b - (P_b + E[P_s(V_s)\,|P_b \geq P_s(V_s)])/2]P\{P_b \geq P_s(V_s)\} \quad (1)$$

And the optimal response of the seller to the buyer's strategy for any $V_s \in [0, 1]$, $P_s(V_s)$ is solved by Eq. (2):

$$\pi_s = \max_{P_b}[(P_s + E[P_b(V_b)|P_b(V_b) \geq P_s])/2 - V_s]P\{P_b(V_b) \geq P_s\} \quad (2)$$

B. EXPERIENCE-WEIGHTED ATTRACTION (EWA) LEARNING MODEL

Therefore, our aim is to study the application of the experience-weighted attraction leaning model (EWA) [20] to evaluate its impact on the strategy evolution in the double auction. Thus, it is convenient to revise different concepts about this learning model that combine reinforcement learning with belief learning models.

In a n-person normal-form game, players are indexed by $i = 1, \ldots, n$, and player i's strategy space is $S_i = \{s_i^1, s_i^2, \ldots, s_i^j, \ldots, s_i^{m_i-1}, s_i^{m_i}\}$, which indicates that i has m_i discrete choices. Therefore, the strategy space of the game is $S = S_1 \times \ldots \times S_n$, and $s = (s_1, \ldots, s_n) \in S$ is a strategy combination for each player, which consists of n strategies. $s_i \in S_i$ denotes player i's strategy, and $s_{-i} = (s_1, \ldots, s_{i-1}, s_{i+1}, \ldots, s_n)$ is a strategy combination of all players except i. Denote the actual strategy chosen by player i and all other players in period t by $s_i(t)$ and $s_{-i}(t)$, and player i's payoff in period t by $\pi_i = (s_i(t), s_{-i}(t))$. EWA learning assumes that each strategy has a numerical attraction, which is updated by experience and determines the probability of choosing that strategy. There are two core variables in the EWA learning model, which are updated after each round:

- The first variable is experience weight $N(t)$ which is interpreted as the number of *observation-equivalents* of past experience.
- The second variable is attraction $A_i^j(t)$ which represents the player $i's$ attraction of strategy s_i^j after period t has taken place.

The initial value of experience weight is $N(0)$, it will be updated according to $N(t) = \varphi(1-k)N(t-1) + 1$, and its restricted condition is $N(t) \leq 1/[1-\varphi(1-k)]$, so $N(t)$ is increasing. The initial value of attraction is $A_i^j(0)$, and $A_i^j(t)$ is updated by the sum of a depreciated, experience-weighted previous attraction $A_i^j(t-1)$ plus the weighted payoff from period t:

$$A_i^j(t) = \frac{\varphi N(t-1) A_i^j(t-1) + [\delta + (1-\delta) I(s_i^j, s_i(t))] \pi_i(s_i^j, s_{-i}(t))}{N(t)} \quad (3)$$

where $I(x, y)$ is an indicator function which is equal to 1 if $x = y$ and 0 if $x \neq y$. Intuitively, the formula (3) represents that the attraction is equal to the sum of decayed (φ), experience-weighted ($N(t-1)$) lagged attraction and reinforcement for the received payoff (if $s_i^j = s_i(t)$) or δ times reinforcement for the foregone payoff (if $s_i^j \neq s_i(t)$), and then divided by the updated experience weight. Note that there are δ, φ and k that are three key parameters in the EWA learning model, and these three parameters all have intuitive interpretations:

- The weight on foregone payoffs, δ. This is the imagination of foregone payoffs, and could be considered to be the responsiveness to opportunity costs or regrets. Besides, this parameter also can be interpreted as an endogenous aspiration level against which payoffs are compared.
- The decay rate to the previous attraction, φ. This shows when the learning environment changes constantly; the previous attraction will decay because of the oblivion or intentional abjuration for old experiences. In other words, it can be seen as the degree to which participants realize others are adapting, so that old observations on what others did become useless.
- The growth rate of attraction, k.This reflects how quickly the participants lock in a strategy and shift from exploring an environment to exploiting what they have learned. When $k = 0$, the attraction is the weighted average of lagged attraction and past payoff; when $k = 1$, the attraction is accumulation of past payoffs. With the increase of k, the strategy which is chosen more frequently and yields positive payoffs will build up a large lead against unchosen strategies, causing bried exploration times and allowing participants to lock in certain strategies faster.

The EWA learning model combines reinforcement learning with belief learning, which is a blend of unconscious learning and conscious learning. Therefore, the EWA learning model can be regarded as a general learning model which can better describe the strategy evolution for both sides in double auction markets, and it can be expressed as the following learning types using proper parameter selection. When $k = 0, \delta = 1$, the EWA learning will degrade into weighted fictitious play which includes Cournot best-response dynamics ($\varphi = 0$) and fictitious play ($\varphi = 1$); and when $\delta = 0, k = 0$ *or* 1, it will degrade into the form of an averaged or cumulative reinforcement.

III. DOUBLE AUCTION BASED ON EWA LEARNING MODEL

In the introduction we pointed out the necessity of improving not only the double auction models in their evolution forming strategies, but also evolution mechanisms and the efficiency of achieving equilibrium. Therefore, in this section our proposal introduces the application of the EWA learning model in double auction markets to better describe the learning behavior of individuals with bounded rationality and explore its evolution mechanism for forming biddings that rely on three key functional parameters. Afterwards, the simulation process of double auction based on the EWA learning model is described, which is focused on the simulation targets, sequential steps, initialization and design of parameters set

A. APPLYING EWA LEARNING TO DOUBLE AUCTION MARKETS

In this subsection, the original EWA learning model is modified by introducing the bidding function and utility function of the double auction model into the EWA learning model's attraction function, and so the modified EWA learning model becomes more suitable for describing the learning process and strategy evolution of both sides in a double auction market. In addition, in order to evaluate the performance of the double auction based on EWA learning model, we further obtain the double auction based on reinforcement learning and belief learning by changing the values of key parameters. The double auction based on EWA learning has experience weight $N(t)$ and attraction $A_b^i(t)$ or $A_s^j(t)$ two core variables. The initial value of the experience weight is $N(0)$. The initial attraction values are $A_b^i(0)$ and $A_s^j(0)$, they will be updated according to the equations (4) and (6). The attractions must determine the probabilities of choosing strategies, so they can be taken in the logit form (the formula (5) and (7)) to decide the probability value.

For buyers:

$$A_b^i(t) = \frac{\varphi N(t-1)A_b^i(t-1)+[\delta+(1-\delta)I(P_b^i, P_b(t))]\pi_b(P_b(t), P_s(t))}{N(t)} \tag{4}$$

$$p_b^i(t+1) = e^{\lambda A_b^i(t)} / \sum\nolimits_{i=1}^{m_i} e^{\lambda A_b^i(t)} \tag{5}$$

For sellers:

$$A_s^j(t) = \frac{\varphi N(t-1)A_s^j(t-1)+[\delta+(1-\delta)I(P_s^j, P_s(t))]\pi_s(P_s(t), P_b(t))}{N(t)} \tag{6}$$

$$p_s^j(t+1) = e^{\lambda A_s^j(t)} / \sum\nolimits_{j=1}^{m_j} e^{\lambda A_s^j(t)} \tag{7}$$

Here, we can further correlate the three key parameters δ, φ and k with market information structure and individual rational characteristics in the context of a double auction market:

- The weight on foregone payoffs δ rises with the increase of information on unchosen strategies or others' strategies and their payoffs provided by the double auction market. The bounded rational individuals are always more sensitive to losses than to gains [31], so they respond more strongly to opportunity costs or regrets. If the individuals in markets know about strategies' foregone payoffs and find that the actual payoffs of chosen strategies are lower than the foregone payoffs of unchosen strategies, then they will move to higher forgone payoffs and away from lower actual payoffs. Besides, if δ is applied to others' actual payoffs instead of its own foregone payoffs, EWA learning can be used to describe imitation behaviors. This means that individuals will adjust their own strategies by imitating the good strategies of others.
- The decay rate of the previous attraction φ declines with the increase of the complete historical information provided by the double auction market. There is a cognitive phenomenon which the bounded rational individuals tend to deliberately forget, which is to discount old experience when the market environment is changing constantly. One way to raise the decay rate is by providing complete historical information, which can promote buyers and sellers to make better bidding strategies in double auction markets.
- There is an inverse relationship between the growth rate of attraction k and the individual rationality in the double auction market. When k rises with the decrease of individual rationality, the exploration for market environment becomes simpler. It means that individuals with lower rationality will quickly lock in a strategy which is chosen more frequently and yields positive payoffs without comparing it to other strategies, which may cause them to miss better strategies. On the contrary, the individuals with higher rationality will flexibly adjust their strategies rather than quickly lock in a frequently chosen strategy. In this case, the value of k is relatively lower.

Under the restriction of different parameter values, EWA learning will degrade into reinforcement learning or belief learning. Take the buyers for example, and the attractions for sellers can be modified in the same way.

When $\delta = 0, k = 1$, then $N(t) = 1$, and the value of attraction will be updated according to the formula (8). This is a form of cumulative choice reinforcement.

$$A_b^i(t) = \varphi A_b^i(t-1) + I(P_b^i, P_b(t))\pi_b(P_b(t), P_s(t)) \tag{8}$$

When $\delta = 0, k = 0$, then the value of attraction will be updated according to the formula (9). This is a form of averaged choice reinforcement.

$$A_b^i(t) = \varphi N(t-1) A_b^i(t-1) + (1-\varphi)[\delta + (1-\delta)] \times I(P_b^i, P_b(t))\pi_b(P_b(t), P_s(t)) \tag{9}$$

When $\delta = 1, k = 0$, its renewal equation is the formula (10). And we can conclude that this renewal equation is extremely similar to weighted fictitious play using algebraic analyzing.

$$A_b^i(t) = \frac{\varphi N(t-1) A_b^i(t-1) + \pi_b(P_b(t), P_s(t))}{\varphi N(t-1) + 1} \tag{10}$$

B. THE SIMULATION OF DOUBLE AUCTION BASED ON THE EWA LEARNING MODEL

Here the procedure for carrying out the experimental simulation of double auction based on the EWA learning model is addressed and described. First, however, it is necessary to design the experiment procedure in which the comparison of our proposal with previous proposals will be based. This procedure consists of three phases: (i) *Targets* that fix the objectives of the simulation, (ii) *Simulation steps* that show the processes of bidding and learning for both sides in a double auction market, (iii) *Parameter assignment* that ensures the execution of simulation. Below we describe these phases in further detail.

1) SIMULATION TARGETS

Firstly, as mentioned above, the EWA learning model is a more flexible and general learning model, which combines the main features of reinforcement and belief learning and includes them as special cases. In order to illustrate whether the EWA learning model can better describe the bounded rational individuals' behaviors in double auction markets rather than other traditional learning models, we simulate the different evolution processes of bidding strategies for both sides in double auctions based on reinforcement, belief and EWA learning.

Meanwhile, note that the weight on foregone payoffs, the decay parameter and the growth rate of attraction, or the three key parameters are obviously different among the individuals with different rational structures in the double auction markets with diverse information structures, which will have an impact on the attractions determining the probabilities of choosing strategies. To further discuss the impacts of these three key parameters on the strategy evolution for both sides with EWA learning, and to explain how the flexibility of the EWA learning model depends on these three parameters; we also simulate the double auctions based on EWA learning with different values for these three key parameters.

2) SIMULATION STEPS

Supposing there are buyers and sellers (two groups) and the size of each group is N. For a single product, the valuation of a buyer is V_b, the valuation of a seller is V_s, and $V_b \geq V_s$. Setting the initial bidding of buyer i at P_{i0}^b and the initial bidding of seller j at P_{j0}^s, in which $i \in [1, N], j \in [1, N]$ then the simulation steps perform as follows:

Step 1 (Initialization): Buyers and sellers deal with each other in sequential rounds. In each round, randomly make both sides pairs for simple double auction, and record the profit of each individual;

Step 2 (Compute the Total Profits): With a period of m rounds, it holds that the total profits of the buyer group is π_{it}^b (t is the period number) and the total profits of the seller group is π_{jt}^s in each period;

Step 3 (Adjust Bidding Strategies): When the period t is finished, all individuals will adjust their biddings based on EWA learning model:

a) According to EWA learning, the attractions of strategies in the period t chosen by buyer i and seller j are calculated with the formula (4) and (6);

b) Taking the attractions in the Logit model (formulas (5) and (7)) to decide the probability value, and obtaining the bidding and learning result $P_{it}^{b'}$ of buyer i in the period *t+1*, similarly, we can also see that the learning result of seller j is $P_{jt}^{s'}$ following this process;

Step 4 (Obtain the Ultimate Biddings): We can then obtain the stable bidding strategies for both sides after M periods.

Step 5 (Repeat Steps): Next, changing the input of δ, φ or k according to the value series of parameters and by repeating the steps above. Particularly, when $\delta = 0, k = 1$ or $\delta = 1, k = 0$, all individuals will adjust their biddings based on cumulative choice reinforcement or weighted fictitious play in step 3, the attractions of buyers are updated according to formulas (8) or (10), and the attractions of sellers can be modified in the same way.

3) PARAMETER ASSIGNMENT

To perform the simulation, the parameters are set as follows: both group sizes are $N = 50$, for a certain product, buyers' valuation at its value is $V_b = 100$ and sellers' valuation at its cost is $V_s = 0$, the initial bidding of each individual is randomly generated in the interval [0,100] and the learning weight is randomly generated in the interval [0,1], and $\lambda = 0.1$, $\varphi = 0.5$ with a period of 5 rounds, conducting simulation experiments 10 times.

To explore the different strategy evolution in double auctions under EWA, reinforcement and belief learning, we conduct the different bidding processes to simulate the various learning behaviors by setting specific parameters in Experiment 1. Firstly, we set $\delta = 0.5$, $k = 0.9$ and obtain a general EWA learning model, then change the value of δ and k to $\delta = 0, k = 1$ or $\delta = 1, k = 0$ and get cumulative choice reinforcement or weighted fictitious play. Based on the above parameters, the simulation results are shown in Figure 1.

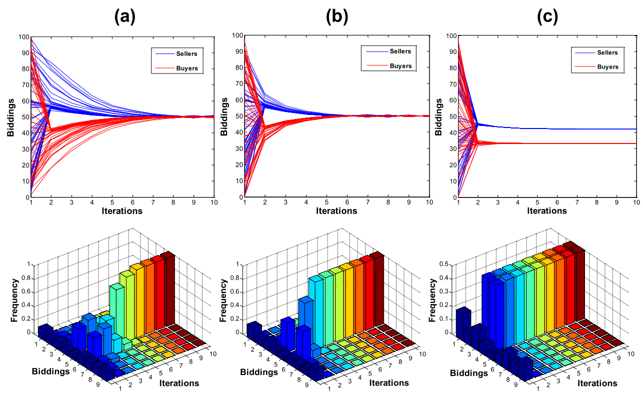

FIGURE 1. The strategy evolution for both sides in the double auction based on (a) cumulative choice reinforcement, (b) EWA learning and (c) weighted fictitious play.

Thereafter a further analysis of the impacts of different δ, φ and k on the strategy evolution in double auction based on EWA learning is necessary in Experiment 2. For such a goal, the initial values of the parameters are set as $\delta = 0.5, \varphi = 0.5, k = 0.9$ according to the EWA parameters estimated by Camerer *et al.* [32], randomly fixing two of them, and changing another one in the interval [0.1, 0.9]. We carry out the simulations in the range of 0.1 to 0.9 repeatedly and reach the most typical results when their values are equal to 0.1, 0.5 and 0.9 respectively, as displayed in Figure 2. Meanwhile, when the values of δ, φ and k are equal to 0.1, 0.5 and 0.9 respectively, there are another 18 parameter combinations besides the 9 parameter combinations shown in Figure 2. In order to comprehensively explore whether the strategy evolution will be affected by other parameter combinations, we also simulate and compare the results based on all 27 parameter combinations. The most representative results of the above simulations are also shown in Figure 2, and we have found that other combinations also lead to similar results. Therefore, we mainly focus on the results shown in Figure 2 and analyze the strategy evolution in a double auction based on EWA learning with different values for the three key parameters.

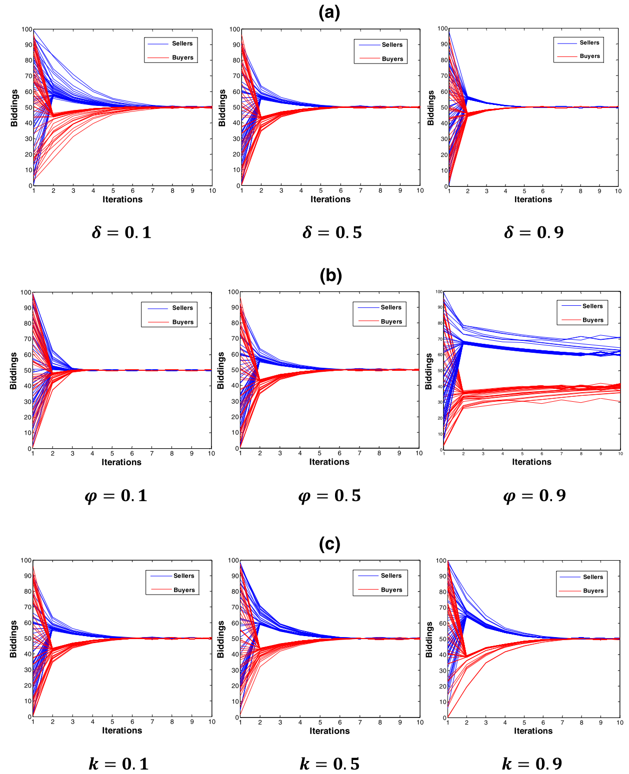

FIGURE 2. The strategy evolution for both sides in double auction based on EWA learning with the series of values of δ, φ and k. (a) When $\varphi = 0.5$, $k = 0.9$. (b) When $\delta = 0.5$, $k = 0.9$. (c) When $\delta = 0.5$, $\varphi = 0.5$.

IV. RESULTS AND ANALYSIS

In this section, the simulation results of double auctions based on the three learning models reinforcement, belief and EWA and the double actions based on EWA learning with the series values of the corresponding parameters, δ, φ and k, will be analyzed and compared.

A. THE COMPARISON AMONG REINFORCEMENT, BELIEF AND EWA LEARNING

In order to explore whether EWA learning, which combines the elements of reinforcement learning and belief learning, can better describe the bounded rational individuals' behavior in double auction markets rather than other learning models, and to specify the different impacts of EWA learning on the strategy evolution in double auction, we stimulate the evolution process of players' bidding strategies in the double auction based on EWA learning and compare it with the double auction based on cumulative choice reinforcement and weighted fictitious play. The results of the evolution processes and corresponding three-dimensional diagrams are shown in Figure 1. The comparison graphs illustrate that the strategy evolution based on EWA learning is an intermediate state of the strategy evolutions based on reinforcement learning and belief learning, which shows that the EWA learning model is more flexible when describing the formation of ultimate bidding strategies for both sides in a double auction market as compared to the other two learning models.

After ten simulations, Figure 1 shows that with the increase of iterations, the ultimate bidding in double auctions based on cumulative choice reinforcement will be stabilized in the interval [45], [55], the ultimate bidding in double auctions based on EWA learning also will be stabilized in the interval [45], [55], but the ultimate biddings of buyers and sellers in double auctions based on weighted fictitious play will be respectively stabilized in the interval [35], [45] and [25], [35].

According to the above experimental results, we can find that the ultimate biddings for both sides in the double auction market will reach a stable state when the continuous learning process is applied, except for the double auction based on belief learning, in which the buyers' biddings are slightly higher than the sellers' biddings. In the two other double auctions based on reinforcement learning and EWA learning, both sides' biddings are roughly same. Although the evolutionary process and result are different in each simulation, each sides ultimate bid is very close to the midpoint of their valuations, that is 50, and the surplus is distributed equally by both sides.

The previous simulation results show that through the continuous learning process in groups, there is a fair result of consensus between buyers and sellers in the double auction market. Although both sides cannot achieve the optimal response because they cannot predict the bidding strategy of each other at the beginning, and there is no convention like the sense of fairness for both sides to coordinate their behaviors. Significantly this study demonstrates that the bounded rational individuals can reach consensus on a fairer bidding result in a double auction market based on reinforcement learning and EWA learning instead of belief learning.

Meanwhile, according to the simulation results, Figure 1 also shows that the ultimate bidding strategies in double auction based on reinforcement learning begin to stabilize as of the seventh iteration, while the ultimate bidding strategies in double auction based on EWA learning begin to stabilize as of the fifth iteration. These results demonstrate that the update speeds of the individuals with reinforcement learning are slower than those of the individuals with EWA learning, because the latter are more insensitive to foregone payoffs than the former. Hence the efficiency of EWA learning is more advantageous than that of reinforcement learning, although the ultimate bidding strategies based on those two learning types are similar.

To conclude, the strategy evolution in double auction based on EWA learning is an intermediate state of that based on reinforcement learning and belief learning. The double auction based on EWA learning can achieve a fairer result than that based on belief learning, and it also can achieve ultimate equilibrium more quickly than that based on reinforcement learning. Therefore, EWA learning is a superior and flexible learning mechanism for both sides in the double auction market,reaching consensus using a fairer bidding strategy with higher efficiency than reinforcement learning and belief learning.

B. THE IMPACTS OF δ, φ AND k ON STRATEGY EVOLUTION WITH EWA LEARNING

Moreover, the flexibility of the EWA learning model used to describe the strategy evolution in double auction markets depends on three key parameters, namely the weight on foregone payoffs, the decay rate to the previous attraction and the growth rate of attraction. To further analyze the impacts of the different parameter combinations, which correspond to different combinations of market information structure and individual rational characteristics, on the formation of bidding strategies for both sides with EWA learning in double auction market, we conduct a simulation of strategy evolution for both sides with the different values of δ, φ and k respectively, and the most typical results of evolution processes are shown in Figure 2. This illustrates that the strategy evolution in double auction based on EWA learning model will change flexibly as the values of three key parameters change.

Firstly, simulations are carried out with the set of three key parameters as following $\varphi = 0.5, k = 0.9$ and three levels of the weight on foregone payoffs $\delta = 0.1, 0.5, 0.9$. The above results show that the ultimate bidding strategies begin to stabilize as of the seventh, fifth and fourth iteration when $\delta = 0.1, 0.5, 0.9$ respectively. With the increase of the weight on foregone payoffs δ, buyers and sellers separately form their own similar bidding strategies more quickly. This indicates that when the weight on foregone payoffs δ rises with the increase of information on unchosen strategies or others' strategies and their payoffs, both sides in a double auction market will accelerate the formation of their ultimate unified bidding.

Furthermore, the next experiment is conducted with different parameter values, such as $\delta = 0.5$, $k = 0.9$ and the decay rate to the previous attraction $\varphi = 0.1, 0.5, 0.9$ respectively. The results show that the ultimate bidding strategies begin to stabilize as of the third and fifth iteration when the decay rate is set as $\varphi = 0.1, 0.5$ respectively, while the ultimate bidding for both sides cannot reach a stable value which is close to the midpoint of their valuations. When the above results indicate that the decay rate to the previous attraction φ rises with the decrease of complete historical information provided by a double auction market, the update speeds for both sides to form the ultimate unified bidding will slow down.

Finally, analyzing the experimental results with the three parameters set as $\delta = 0.5, \varphi = 0.5$ and the growth rate of attraction $k = 0.1, 0.5, 0.9$ we can see that the ultimate bidding strategies begin to stabilize as of the fifth, sixth and seventh iteration when the growth rate of attraction is set as $k = 0.1, 0.5, 0.9$ respectively. Therefore, when the growth rate of attraction k rises with the decrease of individual rationality, both sides in a double auction market will quickly lock in a frequently chosen strategy rather than flexibly adjusting their strategies, which will increase the bidding gap between buyers and sellers and slow down the formation of the ultimate unified bidding.

V. CONCLUSIONS

This paper extends the EWA learning model to double auction, which could better predict the evolution paths of bounded rational individuals' strategies in double auction markets than the other two traditional learning models. This study enriches the academic findings in double auction and benefits those practitioners with managerial implication.

A. THEORETICAL FINDINGS

Various learning models have different effects on the strategy evolution in double auction. A comparison study of three learning models is carried out to highlight the superiority and flexibility of the EWA learning model for depicting the strategy dynamic and equilibrium formation for both sides with asymmetric information in double auction markets. Subsequently, we further discuss the specific impacts of three key parameters in the EWA learning model, which correspond to different combinations of market information structure and individual rational characteristics, on the formation of the ultimate unified bidding for both sides. We can draw the following conclusions.

(1) the EWA learning model can more accurately and flexibly describe the strategy evolution for both sides in double auction than reinforcement and belief learning models. Although the bounded rational individuals in double auction markets with incomplete information cannot accurately predict each other's bidding strategies at the beginning, they can still quickly achieve the ultimate unified bidding close to the midpoint of their valuations through the EWA learning process.

(2) Both sides with EWA learning in a double auction market will accelerate the formation of their ultimate unified bidding in the following cases. a) when the weight on foregone payoffs δ rises with the increase of information on unchosen strategies or others' strategies and their payoffs. b) when the decay rate to the previous attraction φ decreases with the increase of complete historical information provided by the double auction market. c) when the growth rate of attraction k decreases with the increase of individual rationality.

B. MANAGERIAL IMPLICATION

According to previous findings, the managerial implication can be proposed for those practitioners in the double auction market which need to design a mechanism that can make the market reach final equilibrium quickly, thus saving the time of auction participants and the cost of auction organizers.

(1) If participants could know more about opportunity costs by observing unchosen strategies or the strategy selections of others, then they will have greater responsiveness to opportunity costs during the EWA learning processes. The above situation will also then result in the weight on foregone payoffs

increasing and both sides forming the ultimate biddings more rapidly. Therefore, more information on unchosen strategies or others' strategies and their payoffs should be provided to the participants in double auction markets.

(2) The oblivion or intentional abjuration of previous experiences will decrease when participants observe more complete historical information regarding strategy selection and corresponding payoffs, so that the decay rate to the previous attraction will still decrease although the learning environment has been changing constantly. Therefore, the integrity of complete historical information in the double auction market should be maintained to induce the market to achieve ultimate equilibrium with higher efficiency.

(3) Managers in the double auction market should provide participants with advice on strategy selection, such as preventing the bounded rational individuals from blindly following a frequently chosen strategy and encouraging them to flexibly adjust strategies based on actual market conditions. The growth rate of attraction will decline in the above case, which means the bidding gap between buyers and sellers will decrease and the formation of ultimate unified bidding for both sides will accelerate.

The EWA learning mechanism considers multiple factors, which can help the bounded rational individuals in the double auction market to better choose their bidding strategies through continuous learning processes. This study provides several promising initial ideas for designing an effective double auction mechanism and predicting participants' behavior. However, EWA learning as a learning rule will change with some parameter variation, and there may be some deviations between the empirical parameters of simulation and the actual data of the real market. Therefore, in further research, the parameters can be correctly set based on actual data and thus used to design a practical mechanism of double auction based on EWA learning, which could provide some points of reference for participants in the real double auction market to help make strategy selection more efficient.

ETHICAL STATEMENTS

We certify that this manuscript is original and has not been published and will not be submitted elsewhere for publication while being considered by *soft computing*. And the study is not split up into several parts to increase the quantity of submissions and submitted to various journals. The submission has been received explicitly from all co-authors. And authors whose names appear on the submission have contributed sufficiently to the scientific work and therefore share collective responsibility and accountability for the results.

Compliance with Ethical Standards

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The authors declare that they have no conflict of interest.

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This article does not contain any studies with human participants or animals performed by any of the authors.

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