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# Proportional Fairness-Based User Pairing and Power Allocation Algorithm for Non-Orthogonal Multiple Access System

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**ABSTRACT** In this paper, we proposed a joint user pairing (UP) and power allocation (PA) algorithm in the non-orthogonal multiple access uplink communication systems, aiming at improving the proportional fairness of the users. We first solve the optimization problem in a basic scenario, where the users are distributed in only one base station (BS), which is broadly used in many papers. Subsequently, the algorithm is further extended into a complex scenario that the interfering users are allocated randomly outside the BS by spatial homogeneous Poisson point process (HPPP). The joint UP and PA is an NP-hard problem in both scenarios. To solve the problem efficiently, we decouple the UP and PA part. In the PA part of the basic scenario, according to our analysis, the user pair can be divided into three kinds according to the different relationship between the channel condition and the signal to noise and interference constraint. The different kind of user pair's near optimal PA solution is found in different ways. In the UP part of the basic scenario, a probability-based Tabu search user-pairing algorithm is provided to find the near optimal user pairing solution. While, in the PA part of the complex scenario, the optimization problem is extended into a stochastic programming problem aiming at enhancing proportional fairness in users with the outage rate constraint. We derive the closed form expression of outage rate and average data rate for each possible user pairs according to HPPP. Then, a prediction-based particle swarm optimization algorithm is proposed to solve the stochastic programming problem. The simulation results show that the proposed algorithm provides better proportional fairness comparing to previous algorithms.

**INDEX TERMS** Proportional fairness, NOMA, power allocation, user pairing.

## I. INTRODUCTION

The rapid increasing communication quality requirement in wireless cellular systems is leading to the investigation of 5th generation mobile networks (5G). To provide higher system throughput, non-orthogonal multiple access (NOMA) has been proposed as one of the promising technologies for 5G system [1]–[3]. In NOMA, multiple users in one base station (BS) can use the same resource. In the downlink NOMA, on the transmitter side, the superposition coding is used to transmit multiple user signals [4]. These signals are allocated with different transmission power. On the receiver side, the strongest signal decoded firstly. Then, the successive interference cancellation (SIC) is applied to eliminate the strong signal and decode the rest signals [5]. While in the uplink NOMA, each user transmit information independently. The decoding process is handled at BS by SIC. However, the power allocation scheme in uplink NOMA should be different to OFDMA system. In OFDMA, we want that all users' signal have the similar arrived power. However, in NOMA, we wants the signals come with different received power, so that we can use SIC to decode each of them. As the resources in NOMA system can be used by more than one user, the spectrum efficiency can be higher than traditional OFDMA system, as long as we allocate the resources reasonably [6]. In NOMA, one user's communication quality is effected by the other users who occupy the same resource [7]. In most papers [8]–[13], due to the processing

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complexity in the SIC receiver, the number of multiplexing users is two. Thus, how to pair users and how to control transmission power are two critical factors to improve the communication quality.

The common user pairing (UP) and power allocation (PA) schemes such as iterative water-filling power allocation, fixed power allocation, fractional transmit power allocation, orthogonal pairing algorithm, determinant pairing algorithm, and channel state sorting-pairing algorithm are widely used in the NOMA system [8]. However, the advantage of NOMA can not be made use of adequately by these schemes.

There are a lot of papers proposed to find the optimal solution of UP and PA problem for the NOMA system. In [9], a comprehensive algorithm containing sub-channel Assignment, power allocation, and user scheduling was proposed. But the operation was quite complex. In [10], a new solution of user schedule and PA was provided to enhance the proportional fairness of user data rate. Besides, there are amount of works for uplink transmission optimization problems, too [11]. In [12], an user pairing algorithm for both single antenna and multiple antenna system are proposed. In [13], a power allocation algorithm is proposed to enlarge the system throughput under the constraints of code word error probability, with practical modulation and coding scheme employed to simulate the system. In [14], a low complex general power allocation algorithm is proposed to meet the different data rate request of the users.

Apart from system throughput, fairness is also an important factor in NOMA. The users in a BS should have similar data rate. By adjusting the power allocation coefficient, the fairness between the two users in a pair can be enhanced. Furthermore, by using user pairing algorithm, the fairness among use pairs can be further prompted. In [15], the fairness of NOMA and OMA is compared. In [16], the fairness performance of NOMA in broadcasting network is analyze. In [17], fairness is treated as a constraint of the objective function, which can proved high data rate with certain level of fairness. In [18], a proportional fairness based joint power allocation and user scheduler algorithm is proposed. However, the novel water-filling based power allocation part didn't consider the fairness. In [19], a max-min fairness based resource allocation algorithm is proposed for V2X NOMA network. In [20], an optimal power allocation for $\alpha$-fairness is proposed. In [21], a fairness based bisection search algorithm was proposed for PA, and several low-complex solutions for UP were provided. In this paper, we wants to enhance user data rate and user fairness at the same time, so we use proportional fairness as our optimization objective.

Generally, the problem of joint PA and UP aiming at enhancing the proportional fairness enhancement is regarded as a mixed integer programming problem, which is difficult to obtain the optimal solution directly. Most of above mentioned papers focus on the problem of joint PA and UP by making a lot of compromises to find a solution. However, in this way, the system's performance loss is severe. In this paper, we decouple the PA and UP part. In the power allocation part, the near optimal power allocation solution for each possible pair can be found with a low computational complexity. Specifically, according to the relationship between channel condition and SINR threshold, we classify the user pairs into three different kinds. We provide the power allocation method for each of them. In the UP part, we propose a probability-based Tabu search user pairing algorithm (PTS) with the great exploration ability. This algorithm can jump out the local best point and move forward to the global best position. When compared to the above mentioned work, we can obtain higher proportional fairness performance with a low computational complexity by combining power allocation solution and PTS.

The other deficient of the works mentioned above is that they didn't consider the interference from other users which are located out of the coverage of the BS. However, these interference has a significant influence on the communication quality. Furthermore, due to the mobility of the interfering users, the interference could be time varying, which may lead to high outage probability. By taking into these two factors, we model the interfering users randomly deployed outside the BS coverage area by spatial homogeneous poisson point process (HPPP). In this case, the optimization problem is much more complicated as the interferences are stochastic variables, which turns to be a mixed integer stochastic programming problem. To solve this problem, a prediction-based particle swarm optimization (PBPSO) algorithm is proposed to find the near optimal power allocation solution. By combining PBPSO and PTS, we can get higher proportional fairness performance under the constraints of outage probability with a low computational complexity.

The main contribution of our paper is shown as following.

1) To enhance the proportional fairness, we provide a joint PA and UP algorithm for two different scenarios: one is basic scenario and the other one is complex scenario. In the basic scenario, the users are only distributed in cell coverage area. While, in the complex scenario, the interference users are distributed outside the cell coverage area.
2) Considering the basic scenario, to provide high proportional fairness performance, we decouple the power allocation part and user pairing part. In the PA part, to find the near optimal power allocation solution, we divide the user pairs into three different kinds according to the relationship between user channel gain and SINR constraint. For different kinds of user pairs, we propose different power allocation methods. Furthermore, different decoding order is considered for each pair. In the UP part, to avoid local best solutions, we propose a probability-based Tabu search user-pairing algorithm (PTS), which has great exploration ability to find the global optimal point.
3) With respect to the complex scenario, we propose a prediction-based particle swarm optimization (PBPSO) power allocation algorithm with a low computational complexity to achieve high proportional

fairness performance under the constraints of outage probability. We derive the closed form expression of outage probability and average data rate for user pairs uplink transmission when interfering users are randomly deployed by spatial HPPP. The derived average data rate and outage probability are objective and constraint in PBPSO. By combining PBPSO and PTS, we can obtain high system performance for the complex scenario.

The rest of this paper is organized as follows. In Section II, we present the system model. In Section III, we propose our power allocation and user pairing algorithm for basic scenario and complex scenario. Section IV verifies the simulation results and Section V gives the conclusion.

## II. SYSTEM MODEL

In this paper, we consider an uplink NOMA cellular network. We solve the PA and UP problem in two scenarios. In the basic scenario, the users are only distributed in cell coverage area. While, in the complex scenario, the interference users are distributed outside of the cell. The system environment of these two scenarios are shown in Fig. 1 and in Fig. 2. In both scenario, we assume the number of users in the coverage area is $M$. The user index is denoted as $u_i, i \in \{1, \cdots, M\}$. The frequency resource is consist of $N$ sub-channels. The bandwidth of each sub-channel is $B_0$. We use successive interference cancellation (SIC) at the BS side. Due to the processing complexity in the SIC receiver, the maximum number of users using the same resource is two.

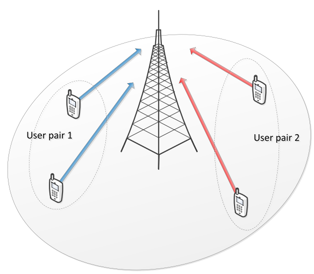


**FIGURE 1.** System environment of basic scenario.

In an user pair, two users' information signal arrive at BS at the same time. The entire signal received by BS contains two users' information signal, the interference from the users outside of the cell, and the noise. After that, the BS decodes these two information signals from the entire signal. The decoding order is important, which is related to the users' data rate. We can use power allocation algorithm to control the decoding order of these two information signals which come from two users. If we want an information signal be decoded firstly at the BS, we will control the information source user to transmit at high power. On the contrary, if we want the information signal be decoded secondly, we will control the information source user to transmit at low power. We denote the signal, which will be decoded firstly, as first signal.

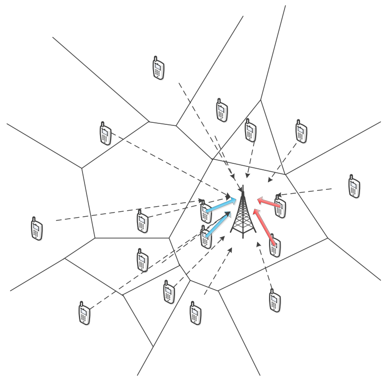

**FIGURE 2.** System environment of complex scenario.

The signal that decoded secondly is denoted as second signal. The first signal and the second signal is named according to the decoding order not the arrive order. The BS receives them at the same time. In the decoding process, the first signal is decoded directly, takeing the second signal as an interference. Then the BS extracts the first decoded signal by SIC and decode the second signal from the rest part of the whole signal afterwards.

In this paper, we assume the channel condition $h = gr^{-\beta}$ contains two parts, rayleigh fading $g$ and path loss $r^{-\beta}$. A single antenna is applied at both BS and user equipment. In this paper, we only consider the joint power allocation and user pairing problem. The resource allocation and user schedule are not included. Thus, In our scenario, we assume that each user pair can only has one sub-channel. The sub-channel's number is always larger than the user pair's number.

From the decoding process we mentioned above, the interference of these two information signals are different. As a result, the data rate of the two users are different, too. We take two users $u_i$ and $u_j$ for example to illustrate that. The user $i$'s data rate could be expressed as function (1), when user $i$ is paired with user $j$. Without loss of generality, we assume the information from $u_i$ is designed to be decoded firstly at the BS. The interference of $u_i$'s information includes the interference from outside the cell which is denoted as $I$, the received information signal from $u_j$, which is expressed as $P_jh_j$, and the noise $\delta$. In this paper, we have assumed each user pair has one sub-channel. The noise $\delta$ is the total noise on sub-channel bandwidth. $P_j$ is the transmission power of $u_j$. $h_j$ is the channel gain between BS and user $j$. The interference of $u_j$'s information only includes $I$ and $\delta$. That is because the information from $u_i$ has already been decoded and extracted by SIC. Thus, increasing $u_i$'s transmission power will not have any influence on decoding $u_j$'s signal. Furthermore, the larger $u_i$ signal's strength is, the higher the data rate of the system will be. So, the $u_i$ can transmit on full power. We let $u_i$ transmit at full power. We set the power allocation coefficient of user $j$ as $0 < a_j \leq 1$. Each user pair has the same bandwidth $B_0$. We denote the data rate of user $i$ and $j$ as

$R_{i,j}$ and $R_{j,i}$, which are shown as

$$R_{i,j} = B_0 \log_2\left(1 + \frac{P_t h_i}{I + \alpha_j P_t h_j + \delta}\right), \tag{1}$$

$$R_{j,i} = B_0 \log_2\left(1 + \frac{\alpha_j P_t h_j}{I + \delta}\right). \tag{2}$$

The data rate (1) and (5) is based on the condition that SIC is working. In the uplink transmission, the BS receives the two users' signal and some interferences. $u_i$'s signal is decoded firstly from the whole signal. Once $u_i$'s signal is decoded successfully, the BS can use SIC to cancel it and continue to decode $u_j$'s signal. The condition of SIC in the uplink transmission is the $u_i$'s signal can be decoded correctly. If we set the decoding SINR threshold as $\Phi$. The SINR of $u_i$ and $u_j$, which are represented as $\Gamma_{i,j}$ and $\Gamma_{j,i}$ must fulfill (3) and (4).

$$\Gamma_{i,j} = \frac{P_t h_i}{I + \alpha_j P_t h_j + \delta} \geq \Phi, \tag{3}$$

$$\Gamma_{j,i} = \frac{\alpha_j P_t h_j}{I + \delta} \geq \Phi \tag{4}$$

It is worth to be mentioned that there is no interfering user outside the BS in the basic scenario. So, in this scenario, $I$ will be 0. On the other hand, in the complex scenario, $I$ depends on the interfering users' density.

## III. PROBLEM FORMULATION AND PROPOSED OPTIMIZATION METHODS

### A. PROBLEM FORMULATION IN BASIC SCENARIO

In this paper, we proposed a joint PA and UP algorithm to improve proportional fairness among the users. Thus, our objective can be optimized by solving function (5).

$$\max_{\zeta_{i,j},\alpha_j} \sum_{i=1}^{M}\sum_{j=1}^{M} \frac{\zeta_{i,j}}{2}(\log_2(R_{i,j}) + \log_2(R_{j,i})) \tag{5a}$$

$$\text{s.t. } 0 < \alpha_j \leq 1, \tag{5b}$$

$$\Gamma_{i,j} \geq \Phi, \tag{5c}$$

$$\Gamma_{j,i} \geq \Phi, \tag{5d}$$

$$\zeta_{i,j} \in \{0, 1\}, \tag{5e}$$

$$\sum_j \zeta_{i,j} = 1, \tag{5f}$$

$$\zeta_{i,j} + \zeta_{j,i} \leq 1, \tag{5g}$$

$$\zeta_{i,i} = 0, \tag{5h}$$

where $\zeta_{i,j} \in \{0, 1\}$ is the user pairing coefficient. $\zeta_{i,j} = 1$, if user $i$ is paired with user $j$ and the signal from user $i$ can be canceled by SIC at the BS. The sum of the logarithmic function is used to improve the proportional fairness of usersąŕ data rate.

To solve it, we proposed a two-step algorithm. In the first step, for each possible user pairing solution, the power allocation coefficient are optimized according to instantaneous channel conditions of the two users. In the second step, the near optimal user pairing solution is calculated based on these power allocation solutions we got in the first step.

### B. POWER ALLOCATION SCHEME IN BASIC SCENARIO

The decoding order between the two users in an user pair is important. The different decoding order with the same two users should be treated as two different combinations. So, there are $M \times (M-1)$ possible user pairs in this system. $M$ is the user number. We will find the near optimal power allocation coefficient $\alpha$ for each pair. We take one user pair containing $u_i$ and $u_j$ for example. Without loss of generality, we suppose the signal from user $i$ decoded firstly. We set $\alpha_j$ as the power allocation coefficient of $u_j$. The proportional fairness optimization for an user pair is given by

$$\max_{\alpha_j} B_0 \log_2(1 + \frac{h_i}{\alpha_j h_j + \widetilde{\delta}}) \times B_0 \log_2(1 + \frac{\alpha_j h_j}{\widetilde{\delta}}) \tag{6a}$$

$$\text{s.t. } 0 < \alpha_j < 1, \tag{6b}$$

$$\frac{h_i}{\alpha_j h_j + \widetilde{\delta}} \geq \Phi, \tag{6c}$$

$$\frac{\alpha_j h_j}{\widetilde{\delta}} \geq \Phi, \tag{6d}$$

where $\widetilde{\delta} = \delta/P_{t,n}$ is the normalized noise power. $\Phi$ is the.

From (6c), we can see that $\alpha_j \leq (h_i - \Phi\widetilde{\delta})/(h_j\Phi)$ must be fulfilled. And from (6d), we need $\alpha_j \geq (\Phi\widetilde{\delta})/h_j$. For each pair, we will check this firstly. If $(\Phi\widetilde{\delta})/h_j > (h_i - \Phi\widetilde{\delta})/(h_j\Phi)$, $u_i$ and $u_j$ can not be paired together.

For each qualified user pair, the range of $\alpha_j$ is $(h_i - \Phi\widetilde{\delta})/(h_j\Phi) \geq \alpha_j \geq (\Phi\widetilde{\delta})/h_j$. To find the optimal $\alpha^{\ddagger}$, we derive the first order derivative of the function (6a) as

$$\frac{dy}{d\alpha_j} = \frac{B_0^2}{\ln(2)} \cdot \frac{h_j}{\alpha_j h_j + \widetilde{\delta}} \cdot \{\log_2(1 + \frac{h_i}{\alpha_j h_j + \widetilde{\delta}}) - \log_2(1 + \frac{\alpha_j h_j}{\widetilde{\delta}}) \cdot \frac{h_i}{\alpha_j h_j + \widetilde{\delta} + h_i}\}. \tag{7}$$

If we set $\widetilde{\alpha} = \alpha_j h_j + \widetilde{\delta}$, according to the qualified range of $\alpha_j$, we get the range of $\widetilde{\alpha}$ is $(1+\Phi)\widetilde{\delta} \leq \widetilde{\alpha} \leq h_i/\Phi$, equation (7) is transformed as

$$\frac{dy}{d\widetilde{\alpha}} = \frac{B_0^2}{\ln(2)} \cdot \frac{1}{\widetilde{\alpha}} \cdot [\log_2(1 + \frac{h_i}{\widetilde{\alpha}}) - \log_2(\frac{\widetilde{\alpha}}{\widetilde{\delta}}) \cdot \frac{h_i}{\widetilde{\alpha} + h_i}]. \tag{8}$$

We want to find the optimal $\widetilde{\alpha}^{\ddagger}$ making $\frac{dy}{d\widetilde{\alpha}} = 0$. To make the problem more clear, we set

$$y^* = \log_2(1 + \frac{h_i}{\widetilde{\alpha}}) - \log_2(\frac{\widetilde{\alpha}}{\widetilde{\delta}}) \cdot \frac{h_i}{\widetilde{\alpha} + h_i}. \tag{9}$$

If we find the $\widetilde{\alpha}^{\ddagger}$ making $y^* = 0$, we find the optimal $\alpha^{\ddagger}$ for $y$. First, we need to make sure whether $\widetilde{\alpha}^{\ddagger}$ exists.

We calculate the value of $y^*$ on the boundary, $\widetilde{\alpha} = (1+\Phi)\widetilde{\delta}$ and $\widetilde{\alpha} = h_i/\Phi$. On the left boundary,

$$y^*((1+\Phi)\widetilde{\delta}) = \log_2(1 + \frac{h_i}{(1+\Phi)\widetilde{\delta}}) - \frac{h_i \log_2(1+\Phi)}{(1+\Phi)\widetilde{\delta} + h_i}. \tag{10}$$

We can prove that $y^*((1+\Phi)\widetilde{\delta}) > 0$ is always true. As $(1+\Phi)\widetilde{\delta} \leq \widetilde{\alpha} \leq h_i/\Phi$. It can be easily proved that $\frac{h_i}{((1+\Phi)\widetilde{\delta})} \geq \Phi$. So, we have $\log_2(1 + \frac{h_i}{((1+\Phi)\widetilde{\delta})}) \geq \log_2(1+\Phi) > \frac{h_i \log_2(1+\Phi)}{((1+\Phi)\widetilde{\delta}) + h_i}$.

Thus, we can prove $y^*(\widetilde{\alpha} = (1+\Phi)\widetilde{\delta}) > 0$. Then, we calculate the right boundary $y^*(h_i/\Phi)$.

$$y^*(h_i/\Phi) = \log_2(1+\Phi) - \log_2(\frac{h_i}{\Phi\widetilde{\delta}}) \cdot \frac{\Phi}{\Phi+1}. \quad (11)$$

$y^*(h_i/\Phi)$ is not always positive or negative. It is related to the value of each parameter in (11). So, we can have two cases. In **case 1**, $y^*(h_i/\Phi) \leq 0$. And in **case 2**, $y^*(h_i/\Phi) > 0$. In **case 1**, as the sign of these two boundary value is different, there must be a zero point $\widetilde{\alpha}^\ddagger$.

However, in **case 2**, we need to check the first order derivative of $y^*$

$$\begin{aligned}\frac{dy^*}{d\widetilde{\alpha}} &= -\frac{2h_i}{\widetilde{\alpha}^2\ln 2} \cdot \frac{\widetilde{\alpha}}{\widetilde{\alpha}+h_i} + \frac{h_i}{(\widetilde{\alpha}+h_i)^2} \cdot \log_2(\frac{\widetilde{\alpha}}{\widetilde{\delta}}) \\ &= \frac{h_i}{\ln 2(\widetilde{\alpha}+h_i)^2} \cdot [\ln(\frac{\widetilde{\alpha}}{\widetilde{\delta}}) - 2(1+\frac{h_i}{\widetilde{\alpha}})]. \end{aligned} \quad (12)$$

We use $y^\dagger$ to find the positive and negative domain of $\frac{dy^*}{d\widetilde{\alpha}}$.

$$y^\dagger = \ln(\frac{\widetilde{\alpha}}{\widetilde{\delta}}) - 2(1+\frac{h_i}{\widetilde{\alpha}}). \quad (13)$$

From (13), $y^\dagger$ is a monotone increasing function. As $\widetilde{\alpha} \geq (1+\Phi)\widetilde{\delta}$, $y^\dagger((1+\Phi)\widetilde{\delta})$ is the minimum value of $y^\dagger$. As $\Gamma_i = \frac{h_i}{\alpha_j h_j + \widetilde{\delta}} \geq \Phi$, the maximum value of $y^\dagger((1+\Phi)\widetilde{\delta})$ is (14), which is negative for all $\Phi$.

$$\max_{h_i,\widetilde{\delta}} y^\dagger((1+\Phi)\widetilde{\delta}) = \ln(1+\Phi) - 2(1+\Phi). \quad (14)$$

The maximum value of $y^\dagger$ is $y^\dagger(h_i/\Phi)$, which is shown as

$$y^\dagger(h_i/\Phi) = \log_2(\frac{h_i}{\Phi\widetilde{\delta}}) - 2(1+\Phi), \quad (15)$$

which could be positive or negative according to the relationship among $h_i$, $\Phi$, and $\widetilde{\delta}$.

Thus, $y^\dagger$ is either always negative or negative in front and then positive. So, $y^*$ is either monotone decreasing or decreasing first, then increasing.

In **case 1**, no matter which $y^*$ is, there will be only one zero point. In **case 2**, we assume **case 2.1** as $y^*$ is monotone decreasing. There will be no zero point on $y^*$. The best solution is $\widetilde{\alpha} = h_i/\Phi$. We assume **case 2.2** as $y^*$ is negative in front and then positive.

According to (11), in **case 2** we have

$$\log_2(1+\Phi) \cdot \frac{\Phi+1}{\Phi} > \log_2(\frac{h_i}{\Phi\widetilde{\delta}}). \quad (16)$$

According to (15), in **case 2.2**, we can get

$$\log_2(\frac{h_i}{\Phi\widetilde{\delta}}) > 2(1+\Phi). \quad (17)$$

However, according to $\log_2(1+\Phi) < 2\Phi$, we have

$$\log_2(1+\Phi) \cdot \frac{\Phi+1}{\Phi} < 2(1+\Phi). \quad (18)$$

(16), (17), and(18) are contradictory. So, **case 2.2** is always false. We can say that, in **case 2**, the best solution is $\widetilde{\alpha} = h_i/\Phi$.

After we analyze the existence of zero point. We need to find it for **case 1**. We give the second derivative of $y^*$ as

$$\begin{aligned}\frac{d^2y^*}{d\widetilde{\alpha}^2} &= -\frac{2h_i}{\ln 2(\widetilde{\alpha}+h_i)^3} \cdot [\ln(\frac{\widetilde{\alpha}}{\widetilde{\delta}}) - 2(1+\frac{h_i}{\widetilde{\alpha}})] \\ &\quad + \frac{h_i}{\ln 2(\widetilde{\alpha}+h_i)^2}[\frac{1}{\widetilde{\alpha}} + (\frac{h_i}{\widetilde{\alpha}^2})] \\ &= \frac{h_i}{\ln 2(\widetilde{\alpha}+h_i)^3} \cdot [(\frac{h_i}{\widetilde{\alpha}})^2 + 6(\frac{h_i}{\widetilde{\alpha}}) + 5 - 2\ln(\frac{\widetilde{\alpha}}{\widetilde{\delta}})]. \end{aligned} \quad (19)$$

$\frac{d^2y^*}{d\widetilde{\alpha}^2}$ is a monotonically decreasing function of $\widetilde{\alpha}$. The maximum value is $\frac{d^2y^*}{d\widetilde{\alpha}^2}((1+\Phi)\widetilde{\delta})$. From (20), we can see $\frac{d^2y^*}{d\widetilde{\alpha}^2}((1+\Phi)\widetilde{\delta})$ is always positive.

$$\begin{aligned}&\min_{h_i,\widetilde{\delta}} \frac{d^2y^*}{d\widetilde{\alpha}^2}((1+\Phi)\widetilde{\delta}) \\ &= \frac{h_i}{\ln 2(\widetilde{\alpha}+h_i)^3} \cdot [(\Phi)^2 + 6(\Phi) + 5 - 2\ln(1+\Phi)] \\ &= \frac{h_i}{\ln 2(\widetilde{\alpha}+h_i)^3} \cdot [(\Phi+1)^2 + 4(\Phi) + 4 - \ln((1+\Phi)^2)]. \end{aligned} \quad (20)$$

In **case 1**, there is a zero point $\widetilde{\alpha}^\ddagger$ making $y^*(\widetilde{\alpha}^\ddagger) = 0$.

$$y^*(\widetilde{\alpha}^\ddagger) = \log_2(1+\frac{h_i}{\widetilde{\alpha}^\ddagger}) - \log_2(\frac{\widetilde{\alpha}^\ddagger}{\widetilde{\delta}}) \cdot \frac{h_i}{\widetilde{\alpha}^\ddagger + h_i}. \quad (21)$$

We can prove that $y^*$ in the domain $[(1+\Phi)\widetilde{\delta}, \widetilde{\alpha}^\ddagger]$ is convex function. The second derivative on $\widetilde{\alpha}^\ddagger$

$$\begin{aligned}\frac{d^2y^*}{d\widetilde{\alpha}^2}(\widetilde{\alpha}^\ddagger) &= \frac{h_i}{\ln 2(\widetilde{\alpha}^\ddagger + h_i)^3} \times [(\frac{h_i}{\widetilde{\alpha}^\ddagger})^2 + 6(\frac{h_i}{\widetilde{\alpha}^\ddagger}) \\ &\quad + 5 - 2\ln(1+\frac{h_i}{\widetilde{\alpha}^\ddagger})\frac{\widetilde{\alpha}^\ddagger + h_i}{h_i}]. \end{aligned} \quad (22)$$

As the derivative of $\ln(1+\frac{h_i}{\widetilde{\alpha}^\ddagger})\frac{\widetilde{\alpha}^\ddagger + h_i}{h_i}$ is $\frac{1}{h_i}\ln(1+\frac{h_i}{\widetilde{\alpha}^\ddagger}) - \frac{1}{\widetilde{\alpha}^\ddagger}$, which is negative. $\frac{d^2y^*}{d\widetilde{\alpha}^2}(\widetilde{\alpha}^\ddagger)$ is increasing function of $\frac{h_i}{\widetilde{\alpha}^\ddagger}$. As $\lim_{x\to 0}\ln(1+x)(1+1/x) = 1$, the minimum value is $\lim_{\frac{h_i}{\widetilde{\alpha}^\ddagger}=\Phi\to 0}\frac{d^2y^*}{d\widetilde{\alpha}^2}(\widetilde{\alpha}^\ddagger) = 6$. As we have already prove the $\frac{d^2y^*}{d\widetilde{\alpha}^2}$ is decreasing function, we can prove $\frac{d^2y^*}{d\widetilde{\alpha}^2}$ in the domain $[(1+\Phi)\widetilde{\delta}, \widetilde{\alpha}^\ddagger]$ is always positive.

So that we can use Newton method to solve it. To begin with, we introduce a special point of $\widetilde{\alpha}$. We name it as $\widetilde{\alpha}^*$. The property of $\widetilde{\alpha}^*$ is

$$\log_2(1+\frac{h_i}{\widetilde{\alpha}^*}) - \log_2(\frac{\widetilde{\alpha}^*}{\widetilde{\delta}}) = 0. \quad (23)$$

And from (8), $\frac{dy}{d\widetilde{\alpha}}(\widetilde{\alpha}^*)$ is positive. If we can prove $\widetilde{\alpha}^*$ always exist under the constraints (6c,d) for any $h_j$, $\Phi$, and $\widetilde{\delta}$, we can use $\widetilde{\alpha}^*$ as a start point to implement Newton method. From (23), we can get $\widetilde{\alpha}^*$,

$$\widetilde{\alpha}^* = \frac{\widetilde{\delta} + \sqrt{\widetilde{\delta}^2 + 4h_i\widetilde{\delta}}}{2}. \quad (24)$$

Proving $\widetilde{\alpha}^*$ always existing under the constraints (6c,d) includes two steps. First, we prove $\widetilde{\alpha}^*/\widetilde{\delta} \geq (1+\Phi)$. Then, we prove $h_i/\widetilde{\alpha}^* \geq \Phi$.

$$\begin{aligned}\frac{\widetilde{\alpha}^*}{\widetilde{\delta}} &= \frac{\widetilde{\delta}+\sqrt{\widetilde{\delta}^2+4h_i\widetilde{\delta}}}{2\widetilde{\delta}} \\ &= \frac{1+\sqrt{1+4h_i/\widetilde{\delta}}}{2} \\ &\overset{(a)}{\geq} \frac{1+\sqrt{1+4\Phi(\Phi+1)}}{2} > \Phi, \end{aligned} \tag{25}$$

where (a) comes from $h_i \geq \Phi(\Phi+1)\widetilde{\delta}$. That is because $h_i/\widetilde{\alpha} \geq \Phi$ and $\widetilde{\alpha}/\widetilde{\delta} \geq (1+\Phi)$.

Then, we prove $h_i/\widetilde{\alpha}^* \geq \Phi$.

$$\begin{aligned}\frac{h_i}{\widetilde{\alpha}^*} &= \frac{2h_i}{\widetilde{\delta}+\sqrt{\widetilde{\delta}^2+4h_i\widetilde{\delta}}} \\ &= \frac{2h_i/\widetilde{\delta}}{1+\sqrt{1+4h_i/\widetilde{\delta}}} \\ &= \frac{2h_i(\sqrt{1+4h_i/\widetilde{\delta}}-1)/\widetilde{\delta}}{(\sqrt{1+4h_i/\widetilde{\delta}}-1)(1+\sqrt{1+4h_i/\widetilde{\delta}})} \\ &= \frac{1}{2}(\sqrt{1+4h_i/\widetilde{\delta}}-1) \geq \Phi. \end{aligned} \tag{26}$$

Thus, $\widetilde{\alpha}^*$ always exist under the constraints (6c,d). So, $\widetilde{\alpha}^*$ is closer to $\widetilde{\alpha}^\ddagger$ than $\widetilde{\alpha} = (1+\Phi)\widetilde{\delta}$ to start Newton method to find the near optimal $\widetilde{\alpha}^\ddagger$.

In summary, there are three kinds of user pairs.

1. $(\Phi\widetilde{\delta})/h_j > (h_i - \Phi\widetilde{\delta})/(h_j\Phi)$, $u_i$ and $u_j$ can not be paired together.
2. $(\Phi\widetilde{\delta})/h_j \leq (h_i - \Phi\widetilde{\delta})/(h_j\Phi)$, and $y^*(h_i/\Phi) \leq 0$, we find the near optimal solution by Newton method starting from $\widetilde{\alpha}^*$.
3. $(\Phi\widetilde{\delta})/h_j \leq (h_i - \Phi\widetilde{\delta})/(h_j\Phi)$, and $y^*(h_i/\Phi) > 0$, the optimal solution is $\widetilde{\alpha}^\ddagger = h_i/\Phi$. The whole procedure is shown as Algorithm 1.

**Algorithm 1** Power Allocation Algorithm in the Basic Scenario

```
For a user pair compose of u_i and u_j
if (Φ·δ̃)/h_j ≤ ((h_i − Φ·δ̃))/h_j then
    u_i and u_j can not be paired together
else
    if y*(h_i/Φ) ≤ 0 then
        Finding α̃‡ by Newton Method, starting at α̃*
    else
        if y*(h_i/Φ) > 0 then
            α̃‡ = h_i/Φ
        end if
    end if
end if
α_j‡ = (α̃‡ − δ̃)/h_j
```

### C. PROBABILITY BASED TABU SEARCH ALGORITHM FOR USER PAIRING

Based on the power allocation coefficient $\alpha^\ddagger$ for all the possible user pairs, we will solve the user pairing problem.

$$\max_{\zeta_{i,j}} \sum_{i=1}^{M}\sum_{j=1}^{M} \frac{\zeta_{i,j}}{2}(\log_2(R_{i,j}) + \log_2(R_{j,i})) \tag{27a}$$
$$\text{s.t. } \zeta_{i,j} \in \{0,1\}, \tag{27b}$$
$$\sum_j \zeta_{i,j} = 1, \tag{27c}$$
$$\zeta_{i,j} + \zeta_{j,i} \leq 1, \tag{27d}$$
$$\zeta_{i,i} = 0. \tag{27e}$$

The user pairing problem is a 0-1 integer programming problem. $\zeta \in 0, 1$ is the user pairing parameter. We set the user pairing matrix as $Z_t$. $Z_t(i,j) = \zeta_{i,j}(t)$. $t$ is the iteration number. The elements in $Z_t$ may change iteration by iteration. In each row of $Z_t$, only one element can be set as 1 and the rest elements are set as 0.

The algorithms like exhaust search, and dynamic programming method can give the optimal solution. However, these algorithms require a lot of computations. So many heuristic algorithms are proposed to find a suboptimal solution with low computation complexity. Tabu search is an useful algorithm [22], which can provide fine solution with low computational complexity. However, it can easily stuck in local optimal solution. Enlightened by quantum-inspired evolutionary algorithm [23], we bring randomness to improve the exploration ability.

The general process of conventional Tabu search algorithm (TS) including the following steps [22]. In step 1, We create a possible solution as candidate. We calculate and record its fitness value. In step 2, we discover several neighbours of the candidate and calculate their fitness value. In step 3, we compare the neighbours and the candidate. If the best neighbour's fitness value is better then the candidate, the candidate is replaced by the best neighbour. Otherwise, the candidate doesn't change. In step 4, the element changes between the new candidate and the original candidate are recorded in a Tabu list. These recorded elements can not be changed for several following iterations to prevent endless loop. The iterative process which is composed of step 2-4 keeps going until the candidate doesn't change for few iterations. Then the algorithm stops. The last candidate is the best solution we can obtain.

Inspired by quantum search method [23]. we bring randomness in the initialization and the discovering neighbour steps to improve the exploration ability. In our algorithm, each element of $Z_t$ could have a chance to be set as 1. We set the probability as $pr_{i,j}(t)$. $pr_{i,j}$ can change iteration by iteration. Obviously, we have $pr_{i,i}(t) = 0$, and $\sum_{j\neq i} pr_{i,j} = 1$. We set the probability matrix as $\Lambda_t$, $\Lambda_t(i,j) = pr_{i,j}(t)$. To begin with, the elements in $\Lambda_t$ are set as the same value. $\Lambda_t(i,j) = 1/M, \forall i, \forall j$. In another word, each element of $Z_1$ has the same probability to be 1.

Unlike the traditional Tabu search, we remove the step 1 from the conventional TS. We provide $N_m$ possible neighbours directly. The neighbours are generated by the following procedure. For each user $i$, $\forall i \in [1, M]$, we set a random parameter $\chi(i) = rand(0, 1)$, which follows Uniform distribution from 0 to 1. If $\sum_1^{j-1} \Lambda_t(i,j) < \chi(i) \le \sum_1^{j} \Lambda_t(i,j)$, $i$ is paired with $j$. As one user can only be paired one time, we build a set $Q$ to contain the users which are already paired. The users in $Q$ can not participate in the next procedure any more. Besides, if the $j$-th user is chosen by $i$-th user, we will set all the elements in the $j$-th column of $\Lambda$ as 0, to prevent the $j$-th user from being chosen again by other users. The initialization procedure is shown in Algorithm 2.

**Algorithm 2** Initial Procedure of PTS in $t$-th Iteration

$n_m = 1$
**for** $n_m \le N_m$ **do**
  INPUT $\Lambda_t$
  let $\Lambda_t'$=$\Lambda_t$
  Establish the chosen set $Q$ as an empty set.
  **for** $i \in [1, M]$&$i \notin Q$ **do**
    Normalization the $i$-th row of $\Lambda_t'(i,j)$
    Set $\chi(i) = rand(0, 1)$
    **if** $\sum_1^{j-1} \Lambda_t'(i,j) < \chi(i) \le \sum_1^{j} \Lambda_t'(i,j)$ **then**
      put user pair $(i, j)$ in $neighbour_{n_m}$.
      $i$ and $j$ are included by $Q$
      Set the $j$-th column of $\Lambda_t'(i,j)$=0, $\forall i$.
    **end if**
  **end for**
  $n_m = n_m + 1$
  OUTPUT $neighbour_{n_m}$
**end for**

For each of these $N_m$ neighbours, we evaluate the fitness function based on (27a). We choose the best one $\Theta_b(t)$ and the worst one $\Theta_w(t)$. After that we get into the Tabu step. Dislike the traditional Tabu search, the changes of $\zeta_{i,j}$ are stored in Tabu list. In our algorithm, the changes of probability, $pr_{i,j}$, of variable are stored in Tabu list.

When we got the the best neighbour $\Theta_b(t)$ and the worst neighbour $\Theta_w(t)$. Firstly, we need to compare the best neighbour $\Theta_b(t)$ and the candidate we got in the last iteration. In the first iteration, we set the candidate $C = \Theta_b(t = 1)$. In each following iteration, we compare the fitness value of $\Theta_b(t)$ and $C(t-1)$. If $\Theta_b(t) > C(t-1)$, $\Theta_b(t)$ replace the original $C(t-1)$. $C(t) = \Theta_b(t)$. Otherwise, the candidate doesn't change,$C(t) = C(t-1)$. We want the neighbours in next iteration to be generated closer to $C(t)$ and further from $\Theta_w(t)$. So we double the probability of the pairs in $C(t)$. And we cut the probability of the pairs in $\Theta_w(t)$ by half. The elements in $C(t)$ and $\Theta_w(t)$ are stored in the Tabu list. For example, if user pair $(u_i, u_j)$ is a element of $C(t)$, $\Lambda_{t+1}(i,j) = \Lambda_t(i,j) \times 2$. $(u_i, u_j)$ is stored in Tabu list. There is a special case, if user pair $(u_i, u_j)$ is the element of both $C(t)$ and $\Theta_w(t)$. $(u_i, u_j)$ will not be stored in the Tabu list. These elements in Tabu list can't change reversely in next iteration. For example, if a user pair $(u_i, u_j)$ in $C(t)$ is an element of $\Theta_w(t+1)$, the probability of this pair doesn't reduce by $\Delta pr$ after iteration $t+1$-th iteration, $\Lambda_{t+2}(i,j) = \Lambda_{t+1}(i,j)$. However, if this pair $(u_i, u_j)$ in $C(t)$ is still an element of $C(t+1)$, the probability increase by $\Delta pr$ again. $\Lambda_{t+2}(i,j) = \Lambda_{t+1}(i,j) \times 2$.

We set terminating iteration counting index ($TCI$) as a termination parameter. $TCI$ is set as 0 initially. Once the candidate doesn't change between two iteration, $TCI = TCI + 1$. However, Once the $C(t)$ changes, no matter how large $TCI$ is, $TCI$ will be set as 0 and start over again. The termination condition of our algorithm is $TCI = TCI_{max}$ or $t = t_{max}$.

The objective that we introduce probability into our algorithm is to promote the exploration ability. Unlike the traditional Tabu search algorithm, which changes only few elements from the best candidate to form the neighbours in a new iteration, the neighbors in our algorithm could be very different from best candidate in the last iteration. However, even through the algorithm extending the searching area, the of fitness value of $\Theta_b(t)$ progressively increase iteration by iteration effectively. That comes from the changing of the probability. When the probability of a pair increase, this user pair will have more chance to show up in the neighbours in this iteration. When the probability of one pair decrease, the pair is harder to show up in the following iterations. The whole procedure of PTS is shown as Algorithm 3.

**Algorithm 3** Entire Procedure of PTS

$TCI = 0$,$t = 1$
**while** $TCI \le TCI_{max}$ and $t \le t_{max}$ **do**
  $t = t + 1$
  Create $N_m$ neighbours by procedure in Algorithm 2.
  Find $\Theta_b(t)$ and $\Theta_w(t)$ according to (27a).
  **if** $t = 1$ **then**
    Record $C(1) = \Theta_b(1)$
  **else**
    **if** $C(t-1) > \Theta_b(t)$ **then**
      $C(t) = C(t-1)$ which is not changed.
      $TCI = TCI + 1$
    **else**
      $C(t) = \Theta_b(t)$
      $TCI = 0$
      Update probability matrix $\Lambda$
      Update Tabu list
    **end if**
  **end if**
**end while**

### D. PROBLEM FORMULATION IN COMPLEX SCENARIO

Different from the basic scenario we analyze above, in the complex scenario, the BS can received the interference from users in the adjacent BSs. The interference can greatly affect the users' communication quality. As the interfering user may move time to time, the interfering user's number and distribution could be time varying. Thus, from the perspective of one BS, the interferences from the users in other BSs are variable, which are related to user density. The interfering user outside the BS can be seen as a random variable $I$, which is related to user distribution density. We model the interfering users are distributed as spatial HPPP with intensity $\lambda$.

As the interference is an estimated value. Sometimes, the SINR may not be large enough for normal communication. In another word, outage may happen. Thus, our target is enlarging the proportional fairness with the outage probability constraints. To maintain normal communication service, we constraint the outage probability must smaller than threshold $\eta$. The objective function is proposed as,

$$\max_{\alpha_j,\zeta_{i,j}} \sum_{i=1}^{M}\sum_{j=1}^{M}\frac{\zeta_{i,j}}{2}\left(\log_2\left(\bar{R}_{i,j}\right)+\log_2\left(\bar{R}_{j,i}\right)\right) \quad (28a)$$

$$\text{s.t. } \zeta_{i,j}\in\{0,1\}, \quad (28b)$$

$$0<\alpha_j\le 1, \quad (28c)$$

$$\sum_j \zeta_{i,j}=1, \quad (28d)$$

$$\zeta_{i,j}+\zeta_{j,i}\le 1, \quad (28e)$$

$$\zeta_{i,i}=0, \quad (28f)$$

$$Pr_{outage}(1,i)<\eta, \quad (28g)$$

$$Pr_{outage}(1,j)<\eta. \quad (28h)$$

To get the power allocation and pairing solution, we need to analyse the outage probability and average data rate of each possible user pair in the BS. We randomly choose two user $u_i$ and $u_j$. The channel condition of the two users is know by the BS. The transmit power of $u_i$ is assume as $p_i$, and $p_j$ is the transmit power of $u_j$. We define the maximum power is $P_t$. As the channel coefficient of $u_i$ and $u_j$ are already known, there could be two different situations, which are $p_i > p_j$ and $p_i < p_j$. No matter in which situation, SIC can works. However, the outage probability and average data rate will be different in these two situations. For each two users in BS, we will analyse the outage probability and average data rate in both situations. We first find the near optimal power allocation solution for these two situation. Then, we compare these two power allocation solutions and find the better one.

### *E. OUTAGE RATE IN COMPLEX SCENARIO*

We assume $u_i$ is more closer to the BS than $u_j$, which means $r_i < r_j$. While, in the basic scenario, we allocated the power based on the instant channel condition. However, in the complex scenario, we wants to analyse the outage probability and average data rate in a long term. In this case, $u_i$ has the probability that his channel condition is worse than $u_j$, due to the rayleigh fading. In this section, we divide the power allocation problem in two different situations. We define **situation 1** as $p_i$ is larger than $p_j$, SIC is operated on $u_i$'s signal. $\beta$ is path-loss parameter. We define the interference as $I$.

Then the SINR of $u_i$ and $u_j$ in **situation 1** can be expressed as

$$\Gamma_{1,i}=\frac{p_i g_i r_i^{-\beta}}{I+p_j r_j^{-\beta}+\delta}, \quad (29)$$

$$\Gamma_{1,j}=\frac{p_j g_j r_j^{-\beta}}{I+\delta}. \quad (30)$$

We define **situation 2** as $p_i$ is smaller than $p_j$, SIC is operated on $u_j$'s signal. Then the SINR $\Gamma$ of the signal from $u_i$ and $u_j$ in **situation 2** can be expressed as

$$\Gamma_{2,i}=\frac{p_i g_i r_i^{-\beta}}{I+\delta}, \quad (31)$$

$$\Gamma_{2,j}=\frac{p_j g_j r_j^{-\beta}}{I+p_i g_i r_i^{-\beta}+\delta}. \quad (32)$$

In $\Gamma_{1,j}$ and $\Gamma_{2,i}$, there is no intra-interference. That is because intra-interference is canceled by SIC. However, the operation of SIC is conditional. The BS can receive the signal from $u_i$, $u_j$ and the interfering users outside the BS. Taking *situation*1 for example, the condition SIC applied successfully, is $\Gamma_{1,j} > \Phi$. We solve the problem in these two situation separately, then choose the better situation as the final solution.

In *situation*1, $u_i$'s outage condition is $\Gamma_{1,i} < \Phi$. The outage probability is defined as

$$Pr_{outage}(1,i)=Pr\{\Gamma_{1,i}<\Phi\}, \quad (33)$$

where $\Phi$ is the SINR requirement of $u_i$.

The outage condition of $u_j$ is $\Gamma_{1,j} < \Phi$, whose probability is defined as

$$Pr_{outage}(1,j)=Pr_{outage}(1,i) + \left(1-Pr_{outage}(1,i)\right)Pr\{\Gamma_{1,i}<\Phi\}, \quad (34)$$

where $\Phi$ is the SINR requirement of $u_j$.

In *situation*2, the outage condition of $u_i$ is $\Gamma_{2,i} < \Phi$, whose probability is defined as

$$Pr_{outage}(2,i)=Pr_{outage}(2,j) +(1-Pr_{outage}(2,j))Pr\{\Gamma_{2,i}<\Phi\}. \quad (35)$$

$u_j$'s outage condition is $\Gamma_{2,j} < \Phi$, which is defined as

$$Pr_{outage}(2,j)=Pr\{\Gamma_{2,j}<\Phi\}. \quad (36)$$

The complete expression of $Pr\{\Gamma_{1,i} < \Phi\}$, $Pr\{\Gamma_{1,j} < \Phi\}$, $Pr\{\Gamma_{2,i} < \Phi\}$, and $Pr\{\Gamma_{2,j} < \Phi\}$ are similar. We take $Pr\{\Gamma_{2,j} < \Phi\}$ for example to illustrate the procedure.

$$\begin{aligned} &Pr\{\Gamma_{2,j}<\Phi\}\\ &=1-\Pr\{\frac{p_j g_j r_j^{-\beta}}{I+p_i g_i r_i^{-\beta}+\delta}\ge\Phi\}\\ &=1-\Pr\{g_j\ge\Phi(I+\delta+p_i r_i^{-\beta})/(p_j r_j^{-\beta})\}\\ &\overset{(a)}{=}1-\int_I\int_{g_j=\Phi(I+\delta+p_i r_i^{-\beta})/(p_j r_j^{-\beta})}^{\infty} f(I)e^{-g_j}dg_j dI\\ &=1-\int_I \exp[-\Phi(I+\delta+p_i r_i^{-\beta})/(p_j r_j^{-\beta})]f(I)dI. \end{aligned} \quad (37)$$

where (a) comes from $g$ follows the exponential distribution with coefficient as 1. That is because the Rayleigh fading follows zero mean cyclic symmetric complex gaussian distribution, $\mathbb{CN}(0,1)$. And $I$ is a stochastic variable as the the interfering users are randomly distributed. Thus, we have

$$\begin{aligned}
&\int_I \exp[-\Phi(I+\delta+p_i r_i^{-\beta})/(p_j r_j^{-\beta})] f(I)dI \\
&\overset{(a)}{=} \mathbb{E}_I\left[e^{-s_{2,j}I+k_{2,j}}\right] \\
&= e^{k_{2,j}}\mathbb{E}_{\Xi,g_z}\left[\exp\{-s_{2,j}\sum_{z\in\Xi/b_o} g_z r_z^{-\beta}P_t\}\right] \\
&= e^{k_{2,j}}\mathbb{E}_{\Xi}\left[\prod_{z\in\Xi/b_o}\mathbb{E}_{g_z}\left(\exp(-s_{2,j}g_z r_z^{-\beta}P_t)\right)\right] \\
&\overset{(b)}{=} e^{k_{2,j}}\exp\{-2\pi\lambda\int_d^{\infty}\left[1-\mathbb{E}_{g_z}\left(\exp(-s_{2,j}g_z\rho^{-\beta}P_t)\right)\right]\rho d\rho\} \\
&= e^{k_{2,j}}\exp[-2\pi\lambda\int_d^{\infty}\left(1-\frac{1}{1+s_{2,j}\rho^{-\beta}P_t}\right)\rho d\rho]. \qquad (38)
\end{aligned}$$

where $f(I)$ is the probability density function of $I$. $d$ is the radius of the coverage of the BS. In the above (a), $s_{2,j}$ is set as $\Phi/(p_j r_j^{-\beta})$. $k_{2,j} = -\Phi(\delta+p_i r_i^{-\beta})/(p_j r_j^{-\beta})$. (b) follows from the probability generating functional (PGFL) of HPPP [24], which states that $= \mathbb{E}[\prod_{x\in\Xi}] = \exp(-\lambda\int_{\mathbb{R}^2}(1-f(x))dx)$. In this way, we get the outage probability as

$$\begin{aligned}
&Pr\{\Gamma_{2,j}<\Phi\} \\
&\quad = 1-e^{k_{2,j}}\exp\left(-2\pi\lambda\int_{rd}^{\infty}\left(1-\frac{1}{1+s_{2,j}\rho^{-\beta}P_t}\right)\rho d\rho\right). \qquad (39)
\end{aligned}$$

If we set $\beta = 4$, we can transfer (39) into (40)

$$\begin{aligned}
&Pr\{\Gamma_{2,j}<\Phi\} \\
&= 1-e^{k_{2,j}}\exp\left(-2\pi\lambda\int_{rd}(\frac{1}{1+(\frac{\rho^2}{\sqrt{s_{2,j}P_t}})^2})\rho d\rho\right) \\
&\overset{(a)}{=} 1-\exp\left(k_{2,j}-2\pi\lambda\sqrt{s_{2,j}P_t}(\frac{\pi}{2}-arctan(\frac{rd^2}{\sqrt{s_{2,j}P_t}}))\right). \qquad (40)
\end{aligned}$$

where (a) comes from $\int\frac{1}{1+x^2} = arctan(x)$.

In fact, (40) can be seen as a general outage probability function for $Pr\{\Gamma_{1,i}<\Phi\}$, $Pr\{\Gamma_{1,j}<\Phi\}$, $Pr\{\Gamma_{2,i}<\Phi\}$, and $Pr\{\Gamma_{2,j}<\Phi\}$. We just need to replace $s_{2,j}$ and $k_{2,j}$ with the following coefficients.

$$s_{1,i}=\Phi/(p_i r_i^{-\beta}),\quad k_{1,i}=-\Phi(\delta+p_j r_j^{-\beta})/(p_i r_i^{-\beta}), \qquad (41a)$$
$$s_{1,j}=\Phi/(p_j r_j^{-\beta}),\quad k_{1,j}=-\Phi\delta/(p_j r_j^{-\beta}), \qquad (41b)$$
$$s_{2,i}=\Phi/(p_i r_i^{-\beta}),\quad k_{2,i}=-\Phi\delta/(p_i r_i^{-\beta}), \qquad (41c)$$
$$s_{2,j}=\Phi/(p_j r_j^{-\beta}),\quad k_{2,j}=-\Phi(\delta+p_i r_i^{-\beta})/(p_j r_j^{-\beta}). \qquad (41d)$$

### F. AVERAGE DATA RATE IN COMPLEX SCENARIO

Next, we will derive the average data rate of these two users. In **situation 1**, the average data rate of $u_i$ and $u_j$ are $\bar{R}_{1,i} = \mathbb{E}[\log_2(1+\Gamma_{1,i})]$ and $\bar{R}_{1,j} = \mathbb{E}[\log_2(1+\Gamma_{1,j})]$. The transmit power of $u_i$ is $P_t$. The transmit power of $u_j$ is $\alpha_j P_t$.

$$\begin{aligned}
\bar{R}_{1,i} &= B_0\mathbb{E}\left[\log_2(1+\Gamma_{1,i})\right] \\
&\overset{(a)}{=} B_0\int_{R=\log_2(1+\Phi)}^{\log_2(1+\frac{P_i r_i^{-\beta}}{\delta})} Pr\{\log_2(1+\Gamma_{1,i})>R\}dR \\
&= B_0\int_{R=\log_2(1+\Phi)}^{\log_2(1+\frac{P_i r_i^{-\beta}}{\delta})} 1-Pr\{\log_2(\Gamma_{1,i})<2^R-1\}dR \\
&= B_0\int_{\phi=\Phi}^{\frac{P_i r_i^{-\beta}}{\delta}}\frac{1-Pr\{\Gamma_{1,i}<\phi\}}{\ln 2\times(1+\phi)}d\phi, \qquad (42)
\end{aligned}$$

where (a) is according to $\mathbb{E}[X] = \int_{x>0}1-F_X(x)dx$. $F_X(x) = P(X<x)$ is the cumulative distribution function of $X$, which is a strictly monotonic continuous function. We prove $\mathbb{E}[X] = \int_{x>0}1-F_X(x)dx$ is always satisfied. As we all know, $\mathbb{E}[X] = \int_{F_X(x)}xdF_X(x)$. According to

$$\begin{aligned}
\mathbb{E}[X] &= \int xdF_X(x) \\
&= xF_X(x)|_{x_{min}}^{x_{max}} - \int F_X(x)dx \\
&= \int 1-F_X(x)dx. \qquad (43)
\end{aligned}$$

Based on (38), we can get the average data rate of $u_i$. And the data rate of $u_j$ is

$$\bar{R}_{1,j} = B_0\int_{\phi=\Phi}^{\frac{\alpha_j P_t r_j^{-\beta}}{\delta}}\frac{1-Pr\{\Gamma_{1,j}<\phi\}}{\ln 2\times(1+\phi)}d\phi. \qquad (44)$$

In **situation 2**, the average data rate of $u_i$ and $u_j$ are $\bar{R}_{2,i} = \mathbb{E}[\log_2(1+\Gamma_{2,i})]$ and $\bar{R}_{2,j} = \mathbb{E}[\log_2(1+\Gamma_{2,j})]$.

$$\bar{R}_{2,i} = B_0\int_{\phi=\Phi}^{\frac{\alpha_i P_t r_i^{-\beta}}{\delta}}\frac{1-Pr\{\Gamma_{2,i}<\phi\}}{\ln 2\times(1+\phi)}d\phi, \qquad (45)$$

$$\bar{R}_{2,j} = B_0\int_{\phi=\Phi}^{\frac{P_j r_j^{-\beta}}{\delta}}\frac{1-Pr\{\Gamma_{2,j}<\phi\}}{\ln 2\times(1+\phi)}d\phi. \qquad (46)$$

where $\Gamma_{2,i}$ and $\Gamma_{2,j}$ are the SINR of $u_i$ and $u_j$ in **situation 2**. They have already been given under function (40). (45) and (46) can be solved by calculation software, when we give any reasonable $p_i$ and $p_j$. Also, we give a approximated expression (47), which is according to Chebyshev-Gauss quadrature.

$$\begin{aligned}
\bar{R}_{2,j} &= B_0\int_{\phi=\Phi}^{\frac{P_j r_j^{-\beta}}{\delta}}\frac{1-Pr\{\Gamma_{2,j}<\phi\}}{\ln 2\times(1+\phi)}d\phi \\
&= \int_{-1}^{1}\frac{2B_0(1-Pr\{\Gamma_{2,j}<\phi^*\})}{(\ln 2\times(1+\phi^*))(\frac{P_j r_j^{-\beta}}{\delta}-\Phi)}d\phi^* \\
&= \int_{-1}^{1}\frac{2B_0(1-Pr\{\Gamma_{2,j}<\phi^*\})\sqrt{1-\phi^*}}{\sqrt{1-\phi^*}(\ln 2\times(1+\phi^*))(\frac{P_j r_j^{-\beta}}{\delta}-\Phi)}d\phi^*
\end{aligned}$$

$$\approx \sum_{n_c}^{N_c} \omega_{n_c} \frac{2B_0(1 - Pr\{\Gamma_{2,j} < \phi_{n_c}^*\})\sqrt{1-\phi_{n_c}^*}}{(\ln 2 \times (1+\phi^*))(\frac{P_j r_j^{-\beta}}{\delta} - \Phi)}, \quad (47)$$

where $\omega_{n_c} = \pi/n_c$, $\phi_n^* = \cos(\frac{2n_c-1}{2N_c}\pi)$. $N_c$ is the number of Chebyshev nodes. The larger $N_\phi$, the more accurate it will be.

As we did in basic scenario, we find the near optimal power allocation solution for each possible pair. In power allocation part, we find the near optimal power allocation in **situation 1** and **situation 2** separately. Then we compare and choose the solution with better objective value. So, in **situation 1**, the objective function (28) can be transformed into

$$\max_{\alpha_j} \ \log_2(\bar{R}_{1,i}) + \log_2(\bar{R}_{1,j}) \quad (48a)$$
$$\text{s.t. } P_i = P_t, \quad (48b)$$
$$0 < \alpha_j \leq 1, \quad (48c)$$
$$Pr_{outage}(1,i) < \eta, \quad (48d)$$
$$Pr_{outage}(1,j) < \eta. \quad (48e)$$

In **situation 2**, $p_i < p_j$, the objective function (28) can be transformed into

$$\max_{\alpha_i} \ \log_2(\bar{R}_{2,i}) + \log_2(\bar{R}_{2,j}) \quad (49a)$$
$$\text{s.t. } P_j = P_t, \quad (49b)$$
$$0 < \alpha_i \leq 1, \quad (49c)$$
$$Pr_{outage}(2,i) < \eta, \quad (49d)$$
$$Pr_{outage}(2,j) < \eta. \quad (49e)$$

### G. POWER ALLOCATION SCHEME IN COMPLEX SCENARIO

First, we find the domain of $\alpha_i$ and $\alpha_j$. The procedure of finding the domain of $\alpha_i$ and $\alpha_j$ are the same. We take $\alpha_i$ for example. It can be easily proved that (40) is a monotone decreasing function of $\alpha_i$. Also, we can prove that $Pr\{\Gamma_{2,i} < \Phi\}$ is a monotone increasing function of $\alpha_i$. As these functions' derivative can be easily calculated, we can solve $Pr_{outage}(2,i) = \eta$, and $Pr_{outage}(2,j) = \eta$ with secant method. The domain of $\alpha_i$ is found after that. These analysis also works with **situation 1**.

After that, we will find the near optimal $\alpha_i$ for objective function (45a) based on the domain of $\alpha_i$ we just found. Our objective function (45a) with $\bar{R}$ is a very complex function of $\alpha_i$. As there is only one dimension in our problem, we wants to find the optimal points more quickly and accurately. There are some conventional schemes like secant method, method of bisection, or Newton method. However, they need to ensure that there is only one maximum point in the domain. So, we solve the problem by a prediction based Particle Swarm Optimization algorithm (PBPSO).

PSO is a heuristic algorithm [25]. The optimal solution is found in an iterative way. Firstly, $B$ particles are generated. The position of each particle represents a possible solution. The fitness value of each particle is calculated by the objective function. In each iteration, the position of particle may change to a different place. The best position that a particle has ever been and the best position that the whole swarm have ever found are recorded by each particle. As the particle try to find a better position iteration by iteration. The particle will update the best position in their record if it find a new place better than the personal best position in the last iterations. In this way, they can always find a better position. However, the searching direction that the particles is not totally random. In a new iteration, a particle's moving direction and velocity are decided based on three factors, its position in the last iteration, its best position, and the swarm's best position. If we set the position of particle $b$ in the $t$-th iteration as $\vec{\psi}_b(t)$. The velocity of the movement of the particle in the $t$-th iteration is set as $\vec{v}_b(t)$. Then $\vec{\psi}_b(t)$ comes from

$$\vec{\psi}_b(t) = \vec{\psi}_b(t-1) + \vec{v}_b(t), \quad (50)$$
$$\vec{v}_b(t) = \vec{v}_b(t-1) + \omega_1 \times rand_1 \times (\vec{L}_b - \vec{\psi}_b(t-1)) + \omega_2 \times rand_2 \times (\vec{L}_g - \vec{\psi}_b(t-1)), \quad (51)$$

where $\vec{L}_g$ is the global best position and $\vec{L}_b$ is the particle $b$'s best position. The iteration stops on either of the two conditions, the iteration reaches the maximum number, or each particles gets to the local optimal position. however, the convergency speed of PSO is slow. So, to get the global optimal result more efficiently, we propose a prediction based PSO (PBPSO).

We assume the domain of $\alpha_i$ is $[D_{min}, D_{max}]$, which can be calculated by (40). We generate $B$ particles distributed evenly in $[D_{min}, D_{max}]$. The edge point, $D_{min}$and $D_{max}$, must be included in these particles.

To promote the search efficiency, we invite reference point for each particle. We assume the $b$-th particle's position in the $t$-th iteration is $\vec{\psi}_b(t)$. The position of its reference point $b^*$ is $\vec{\psi}_{b^*}(t) = \vec{\psi}_b(t) + \Delta\psi$. We calculate not only the fitness value of a particle itself, but also the fitness value of the particle's reference point. The fitness value of $b$ and $b^*$ in the $i$-th iteration are $\Upsilon_b(t)$ and $\Upsilon_{b^*}(t)$. For the best position and its reference point, the fitness values are $\Upsilon_{Lb}(t)$ and $\Upsilon_{Lb^*}(t)$. And the the fitness value of the global best position is $\Upsilon_{Lg^*}(t)$

The velocity should take care both exploitation and exploration. We use $|\frac{\Upsilon_{Lb}(t)-\Upsilon_{Lb^*}(t)}{\Delta\psi}|$ as an adjuster. When $|\frac{\Upsilon_{Lb}(t)-\Upsilon_{Lb^*}(t)}{\Delta\psi}|$ is large, the particle knows clearly the direction to a better position. We increase the exploitation ability to help it to find the local best position faster. But when $|\frac{\Upsilon_{Lb}(t)-\Upsilon_{Lb^*}(t)}{\Delta\psi}|$ is small, the particle almost reach the local best position. We need to increase the exploration ability for it to jump out the local best position. The velocity in this paper contains two parts, exploitation part, which is denoted as $\vec{v}_{task1,b}(t)$, and exploration part, which is denoted as $\vec{v}_{task2,b}(t)$. We use a random number $rand$ following uniform distribution between 0 and 1 to decide $\vec{v}_b(t)$.

$$\vec{v}_b(t) = \begin{cases} \vec{v}_{task1,b}(t) \\ \quad rand < 1 - \exp(-|\frac{\Upsilon_{Lb}(t) - \Upsilon_{Lb^*}(t)}{10\Delta\psi}|) \\ \vec{v}_{task2,b}(t) \\ \quad rand \geq 1 - \exp(-|\frac{\Upsilon_{Lb}(t) - \Upsilon_{Lb^*}(t)}{10\Delta\psi}|) \end{cases} \quad (52)$$

To promote the exploitation ability, we use reference point $b^*$ to conduct the moving direction of the particle. We set an orientation factor $O_b(t)$ for $b$, which is shown as function (53).

$$O_b(t) = \begin{cases} 1 & \Upsilon_{b^*}(t) > \Upsilon_b(t) \\ 0 & \Upsilon_{b^*}(t) \leq \Upsilon_b(t) \end{cases} \quad (53)$$

If $O_b(t) = 1$, the nearest local best position is on the right, otherwise, on the left. We use its two neighbours' best position to calculate the velocity. At the beginning, in the first iteration, we check $dr_b(1)$. The particle $b$ goes towards to the nearest neighbours best position. When $O_b(1) = 1$, $b$ should goes forward to $L_{b+1}$. Otherwise, it should go towards $L_{b-1}$. However, in the following iterations, if $\vec{L}_b(t)$ doesn't change, the pace in the last iteration is so large that the particle get across the local best point. There must be a local best point between $\vec{L}_b(t)$ and $\vec{\psi}_b(t)$. The particle should goes back to somewhere between $\vec{L}_b(t)$ and $\vec{\psi}_b(t)$ in next iteration. So, we set a coefficient $\tau_b$ to trigger that. If $\vec{L}_b(t) = \vec{L}_b(t-1)$, $\tau_b = 1$. Otherwise, $\tau_b = 0$. Once, the particle find a better $\vec{L}_b$, it will consult $O_b$ again to decide the searching location in next iteration. The exploitation part $v_{task1}$ is shown as,

$$\begin{aligned} &\vec{v}_{task1,b}(t) \\ &= O_b(t) \times [\tau_b \times \frac{(\vec{L}_{b+1} - \vec{\psi}_b(t))(3-\sqrt{5})}{2}] \\ &\quad + O_b(t) \times [(1-\tau_b) \times \frac{(\vec{L}_b - \vec{\psi}_b(t))(3-\sqrt{5})}{2}] \\ &\quad + (1-O_b(t)) \times [\tau_b \times \frac{(\vec{L}_{b-1} - \vec{\psi}_b(t))(3-\sqrt{5})}{2}] \\ &\quad + (1-O_b(t)) \times [(1-\tau_b) \times \frac{(\vec{L}_b - \vec{\psi}_b(t))(3-\sqrt{5})}{2}]. \end{aligned} \quad (54)$$

In exploration part $\vec{v}_{task2,b}(t)$, we use the edge point $D_{max}$ and $D_{min}$. When the particles trapped in their local best position, much area in the domain is not explored. There may be a better position. We invite random factor $rand_1$ and $rand_2$ to enhance the exploration ability.

$$\begin{aligned} \vec{v}_{task2,b}(t) &= O_b(t) \times rand_1 \times (D_{max} - \vec{\psi}_b(t)) \\ &\quad + (1-O_b(t)) \times rand_2 \times (D_{min} - \vec{\psi}_b(t)). \end{aligned} \quad (55)$$

where $\frac{3-\sqrt{5}}{2}$ is the golden section ratio, which will accelerate the convergence rate, according to the golden section search algorithm. Here, we set a stop condition for particles. If $\Upsilon_{Lb}(t) - \Upsilon_{Lb}(t-1) \leq \Upsilon^\sharp$, we call the particle reach the best position. $\Upsilon^\sharp$ is the threshold, which is set as 0.01 in this paper. In this case, the terminating iteration counting index ($TCI_b$) starts to count. We set the largest number of $TCI$ is $TCI_{max}$. When all the $TCI_b$, $\forall b$ reach $TCI_{max}$, the algorithm finished. The entire power allocation scheme is shown as Algorithm 4.

**Algorithm 4** Power Allocation in Scenario 2

1: **Step 1. Initialization.**
2: Calculate the interval of domain $[D_{min}, D_{max}]$ of $\alpha$ from (40)by secant method.
3: Generate $B$ particles evenly distributed in $[D_{min}, D_{max}]$.
4: **Step 2. Prediction based PSO algorithm**
5: **while** $TCI_b \leq TCI_{max}, \forall b$ **do**
6: Calculate the fitness value $\Upsilon_b(t)$ for $\vec{\psi}_b(t), \forall b$ and $\Upsilon_{b*}(t)$ for the reference points $\vec{\psi}_{b^*}(t), \forall b$ by (42)-(46).
7: Update the particles position by (54)
8: **end while**

### H. COMPUTATIONAL COMPLEXITY

In the basic scenario, we first calculate the power allocation solution for $M \times (M-1)$ possible user pair. As we find a special point $\widetilde{\alpha}^*$, the Newton method only needs few iterations starting from $\widetilde{\alpha}^*$. So, the power allocation part needs $O(M \times (M-1))$ computational complex. In the user pairing part, in initial procedure, $O(N_m M^2)$ calculations are needed. The entire procedure of PTS needs maximum $t_{max}$ times of initial procedure. And the rest part only need few computations. So, the maximum computational complexity of PTS is $O(N_m M^2 t_{max})$. While, the exhaustive search needs $O(\frac{M!}{2^{M/2}(M/2)!})$. The GA needs $O(N_m^2 t_{max})$ and TS needs $O(N_m^2 t_{max})$. In the complex scenario, for each pair, our algorithm PBPSO operate the procedure on $B$ particles. In each iteration, only few computations are needed. Thus, the maximum computational complexity is $O(BM^2 t_{max})$. The PSO needs $O(BM^2 t_{max})$. GA needs $O(B^2 M^2 t_{max})$.

## IV. IMPLEMENTATION AND PERFORMANCE ANALYSIS

We consider a NOMA simulation scenario consisting of one BS and several users. The users are randomly distributed in the coverage area. The radius of BS $d$ is set as 200m. User's maximum transmit power is 23dBm [14]. The channel is modeled as path-loss $r^{-4}$ [24] and rayleigh fading $\mathbb{CN}(0, 1)$ [14]. The noise power spectral density is $-173$dBm/Hz [14]. We assume each user pair is using 1MHz bandwidth. The SINR threshold $\Phi$ is set as 1. The outage probability limit is set as 0.1 [24]. We use Monte Carlo method to execute 10000 times simulation to evaluate the performance.

In the basic scenario, we compare our algorithm PTS with three algorithms, exhaustive search, Genetic algorithm (GA) [26] and Tabu search (TS) [27]. In the complex scenario, we show the convergency ability of PBPSO in front, compared with conventional PSO [28]. Then, we evaluate the how the interfering user's intensity $\lambda$ effect the performance.

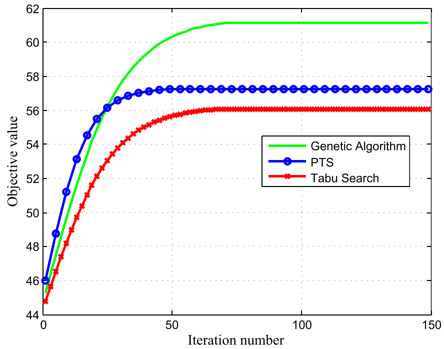


**FIGURE 3.** **Convergency analysis of PTS.**

Fig. 3 shows the convergence ability comparing with two other intelligence methods, GA and conventional TS. 20 users is deployed in the coverage area. We set the population as 200 for The GA and our PTS algorithm. The crossover and mutant rate of GA is set as 0.3 and 0.01. From Fig. 1, we can see our PTS has similar growth rate with GA, as iteration number grows. However, our PTS has more powerful exploration ability, which can help the solutions jump out of the local optimal point. Comparing with the conventional TS, our method has better growth rate and better result.

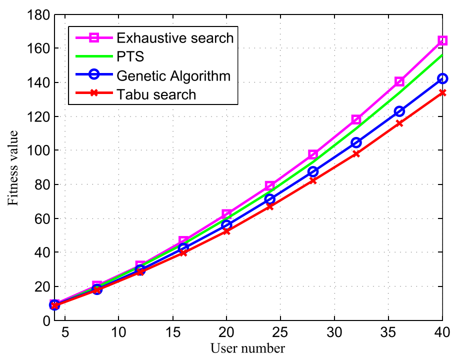


**FIGURE 4.** **Fitness value vs. user number in basic scenario.**

In Fig. 4, the fitness value vs user number is shown. We compare PTS with exhaustive search method, GA and conventional TS. According to the exhaustive search method, the growth rate of the fitness value increase when more users access to the system. The growth is due to the user pairing solution. When the user number is large, an user has more options when he choose his partner. And user pairing algorithm can help the particles to find the optimal one from these options. However, when the user number is low, there won't so many choices for the user pairing algorithm to choose from. Thus, the growth rate of the fitness value increase when more users access to the system. Besides, if we pair the users randomly, the fitness value will linearly increase along with the number of users. Comparing to the exhaustive search, the other three algorithms has worse performance. That is because they can not always get the global optimal solution. GA and TS have worse performance than PTS. That is because they can easily trapped in local optimal solution. When more user associated with the BS, the dimension of the problem goes up and more local optimal point could exist. So, when the user number get lager, the performance gap becomes wider. Our PTS has better exploration ability, which can make the solution has more chances to get the global optimal point.

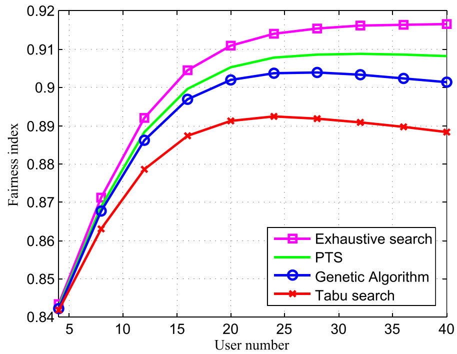


**FIGURE 5.** **Fairness vs. user number in basic scenario.**

In Fig. 5, we show the fairness performance. We use Jain's fairness index as the measurement index [29]. We can see all of these algorithms can have high fairness value, that is because the power allocation part promoting the fairness between two users in a pair. In the ideal condition, the power allocation solution can provide very high fairness among the two users. However, if an users' channel condition is bad, he could not get the similar data rate as his pairing partner even if he transmits at full power. The user pairing solution can find him an other partner to alleviate the problem. The gap between the exhaustive search method and other method is related to the probability of reaching the global optimal solution. Our PTS has more chance to jump out of a local optimal point. So the fairness is better than the other two method.

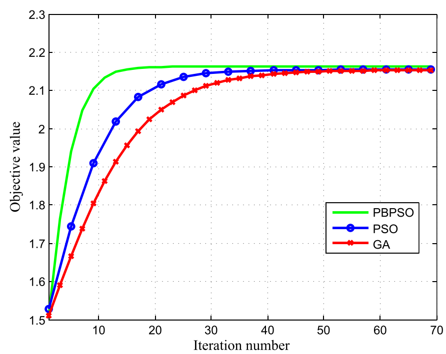


**FIGURE 6.** **Convergency analysis of PBPSO.**

In Fig. 6, We shows the convergency ability of our power allocation PBPSO in the complex scenario where randomly distributed interfering users are deployed. We generate 10 particles in the algorithm. As we use an orientation factor to help the particle to reach the local best point faster, PBPSO can find the local best solution faster than the other two algorithm. Also, the exploration ability is enhance to discover the other possible solution when a particle reach a local best point. Thus, if we set the same iteration number limit, our PBPSO will have better performance than PSO and GA.

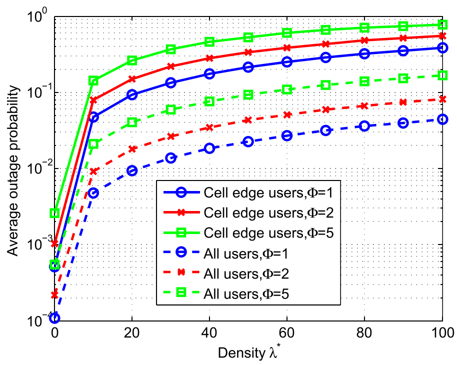


**FIGURE 7.** Average outage probability vs. $\lambda^*$ in complex scenario.

In Fig. 7, we shows the average outage probability vs. interfering users' distribution density. We set $\lambda^* = \lambda\pi(2000)^2$ to represent the density of the interfering users. The meaning of $\lambda^\dagger$ is the average number of interfering users deployed in the area of $\pi(2000)^2$ square meter. We can see from Fig. 7 that the interference from outside the cell coverage area influence communication quality significantly. The solid line represents the average outage probability of the cell edge users. While, the dash line represents the average outage probability of all users. As we can see, the performance degradation is more severe for the cell edge user. As the interfering users are deployed randomly outside the cell coverage area, they have a chance to be allocated close to the cell edge. The interference's strength received by BS can be as strong as the cell edge user's information signal. As a result the outage happens. Also, we can see from Fig. 7, the SINR threshold $\Phi$ has great impact on the average outage probability,too. If we choose a large $\Phi$, many users will not fulfill the outage rate limit.

Fig. 8 shows the fitness value vs. interfering users' distribution density. We set $\lambda^* = \lambda\pi(2000)^2$ to represent decide the density of the interfering users. The meaning of $\lambda^\dagger$ is the average number of interfering users deployed in the area of $\pi(2000)^2$ square meter. $\lambda^*$ is chosen based on Fig. 7, which can't be set too large. Otherwise, the outage probability will be definitely larger than 0.1. The data rate of the users will decrease when $\lambda^*$ gets larger. When $\lambda^*$ is small, the increase of $\lambda^*$ can effect SINR of the communicating users significantly. However, when $\lambda^*$ is large, the SINR of the communicating user is already low. A little increase of $\lambda^*$ will not make big difference. Also from Fig. 8 we can see that combining PBPSO and PTS can provide better performance.

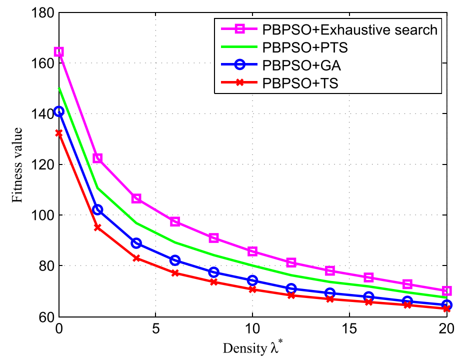


**FIGURE 8.** Fitness value vs. $\lambda^*$ in complex scenario.

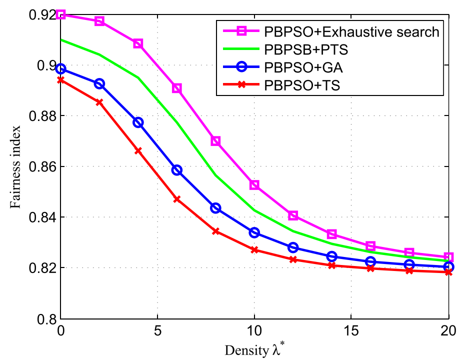


**FIGURE 9.** Fitness value vs. $\lambda^*$ in complex scenario.

In Fig. 9, the fairness vs $\lambda^*$ is shown. At the beginning, the fairness doesn't change much. That is because most users can communication normally. After that, we can see that the fairness of user decrease when $\lambda^*$ increase. That is because when interference is more stronger, the users on the cell edge will have less power allocation option to maintain the outage probability probability threshold. When the interference user density is huge, the fairness changes slowly. That is because, in this case, more users transmit at full power. Comparing to other algorithms, our algorithm combining PBPSO and PTS has better fairness performance.

## V. CONCLUSION

In this paper, we have proposed a proportional-fairness based user pairing and power allocation algorithm for NOMA uplink transmission. We proposed algorithm for two scenarios. In the basic scenario, classify the possible users into three kind according to the relationship between channel condition and SINR constraints. For each kind, the near optimal power allocation solution is obtained in different ways. Furthermore, the different decoding orders are considered in the power allocation process. Then combine the power allocation solution and the proposed user pairing algorithm PTS, which have great exploration ability, we can obtain great proportional fairness performance with SINR constraints. In the complex scenario, stochastic interference from outside the cell coverage area is considered. The difficulty brought from the stochastic interference is handled by the proposed PBPSO power allocation algorithm. Specifically, we first derived the close-form expression of outage probability and average data rate as the function of channel statement, outage probability threshold, and interfering user density. Based on that, PBPSO algorithm can find the optimal power allocation solution for proportional fairness with the outage probability limits. Besides, the different decoding orders are considered and compared. Numerical results demonstrated that combining PTS and PBPSO can achieve good proportional fairness performance for the complex scenario.

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