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# IT2-Based Fuzzy Hybrid Decision Making Approach to Soft Computing

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**ABSTRACT** This aims to evaluate the risk appetite of the financial investors in emerging economies using an integrated interval type-2 fuzzy model. For this purpose, eight different criteria are identified with the supporting literature. The interval type-2 fuzzy DEMATEL approach is used to weight these criteria regarding the importance level. In addition, investors are classified into three different groups with respect to the risk appetite which are the aggressive/risk taker, moderate/risk neutral, and conservative/risk averse. Moreover, the interval type-2 fuzzy QUALIFLEX methodology is taken into consideration to rank these investor groups. The novelty of this paper is to propose a hybrid fuzzy decision-making approach to the investors' risk appetite based on the interval type-2 fuzzy sets. The findings show that aggressive investors play the most important role in emerging economies. Therefore, financial products, which offer high returns, should be developed to attract the attention of these aggressive investors. Owing to this aspect, it can be possible for emerging economies to improve their financial systems.

**INDEX TERMS** Interval type-2 fuzzy DEMATEL, interval type-2 QUALIFLEX, financial investment, risk appetite.

## I. INTRODUCTION

Financial system plays a very key role for the development of the economies by combining fund demanders and fund suppliers. Therefore, with an effective working financial system, investors can get a chance to increase their investments. This situation has an increasing effect on the economic growth. Because emerging economies aim to reach the standards of developing countries, they give importance to increase investment amount. Because of this issue, effective financial system is a must especially for emerging economies [1].

Countries generate financial instruments to attract the attention of the financial investors to increase the power of this system. It is accepted that there is a positive correlation between risk and income in financial investments [2]–[4]. Some financial instruments have minimum risk, such as government bonds [5]. Because of this situation, they provide low income by comparing with other products. On the other hand, some other instruments have higher risk by providing higher income opportunity at the same time. Stocks, foreign currencies, precious metals and financial derivatives can be given as the examples of this category [6]. In addition to this aspect,

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the risk profiles of the investors can be different. As general, investors can be classified into three different groups with respect to the risk appetite. Aggressive or risk take investors refer to the investors who love taking risk by making investments. This means that this kind of investors expect to earn higher return in this process [7]. These investors follow the market in a detailed manner and can make speculative investments. Moreover, conservative or risk averse investors do not like taking risk in their investment decisions [8]. In other words, these investors prefer to earn low but predictable return instead of future uncertain high returns. Finally, moderate or risk neutral investors do not have preferences between risk and return [9].

As it can be seen, in order for the financial system to be effective, products that meet the risk profile of investors in the country have to be presented. Otherwise, financial products cannot be preferred by the investors and this situation has a decreasing effect in the performance of this financial system. In this process, the significant point is to identify the risk appetites of the investors. For this purpose, countries should make a detailed analysis so as to understand the risk profile of the investors. Owing to this issue, it can be more possible to attract the attentions of potential investors. The importance of the paper is to construct a novel hybrid



decision making model based on interval type-2 fuzzy sets. Thus, it is aimed to identify the risk appetite of the investors in emerging economies more accurately under the fuzzy environment. Within this framework, 8 different criteria and 3 investment profiles are identified by considering similar studies in the literature. Additionally, interval type-2 fuzzy DEMATEL and fuzzy QUALIFLEX approaches are taken into the consideration. With the help of interval type 2 fuzzy DEMATEL method, these criteria are weighted. In addition to this situation, by using interval type-2 fuzzy QUALIFLEX methodology, investment profiles are ranked according to their importance levels. According to these results, necessary recommendations can be provided in order to improve the financial system of emerging economies.

The proposed model has some outstanding novelties and advantages. Firstly, the proposed model provides a novel hybrid approach to soft computing based on interval type-2 fuzzy sets. Thus, by using hybrid multi-criteria decision making models with interval type-2 fuzzy sets, it is possible to obtain more precious and reliable results than use of one of decision making techniques such as TOPSIS and VIKOR. Additionally, the proposed model is more appropriate for the complex decision making problem and it is accepted that these approaches are very successful in decision making under uncertain environment. In addition to this aspect, interval type-2 fuzzy logic is firstly used in order to identify the risk profiles of the investors. Hence, it has an increasing effect on the originality of this study.

There are five different sections in this study. After this introduction section, the outstanding studies related to risk appetite, interval type-2 fuzzy DEMATEL and QUALIFLEX are detailed in the second section. In addition, the methods used in the analysis process are explained in the third section. Furthermore, the fourth section gives information about the application on emerging economies by using these methodologies. Finally, in the last section, necessary recommendations are provided based on these analysis results.

#### **II. LITERATURE REVIEW**

In the literature, the risk appetite of the investors during financial crisis period also attracted the attention of many researchers. For instance, reference [10] considered risk appetite in emerging economies. With the help of regression methodology, it is determined that risk aversion increases in these markets during the financial crisis period. Reference [11] evaluated the global risk in nine different emerging markets. For this purpose, volatility, credit, and liquidity risks are taken into the consideration with respect to the components of the global risk. They determined that the investors become more risk taker before the financial crisis period. Moreover, reference [12] concluded that financial crisis changed the investors' risk appetite. Reference [13] argues that measurement of risk appetite is a very useful aspect to predict financial crisis. Furthermore, [14] underlined that investors' risk appetite plays a key role in the developments in global financial markets. In this study, regression

methodology is considered for ten years data of seventeen different markets. As a result, they defined that this aspect is also very useful to predict financial crisis. Reference [15] stated that investor sentiment has a strong influence during 2008 global mortgage crisis period.

Also, identifying the determinants of risk appetite is the concern of the researchers. Within this context, [16] evaluated the conditions which affect investors' preferences. They reached the condition that gender has an impact on the risk perception of the investors whereas marital status does not have the same influence. Additionally, [17] tried to find the relationship between global risks and volatility in the financial market. As a result of the analysis made by vector error correction method, it is concluded that volatility index (VIX) is the most outstanding indicator of the investors' risk appetite in the market. Also, [18] stated that demographic variables have an influence on risk tolerance levels of the investors.

On the other hand, [19] created a model regarding bond spreads. For this purpose, many different factors are taken into the consideration, such as risk appetite of the investors. It is identified that there is a strong correlation between liquidity risk and investors' risk appetite. Reference [20] focused on the relationship between monetary policies and financial stability. In this study, the data of 257 different banks in 26 countries is evaluated by using regression methodology. It is identified that monetary policy of US central banks has a strong influence on the risk appetite of investors. Moreover, Furthermore, [21] argued that communication quality of the top managers has an important effect on the risk appetite of the investors. Reference [22] also examined the determinants of investor profiles by using structural equation model. Within this scope, European data for the years between 2003 and 2014 is analyzed. It is defined that political uncertainty plays a key role in this circumstance. In addition, some studies evaluated the impacts of investors' risk appetite. For instance, [23] evaluates financial risk-taking behavior of the investors. They explained that the risk profiles of the investors have a significant influence on the stability of the market. Reference [24] analyzed the role of brokerdealer risk appetite on commodity returns. It is concluded that fluctuations in risk bearing capacity of broker-dealer have an impact on energy returns. Reference [25] also underlined that investors' risk appetite has a strong influence on the financial stability of the countries. In this study, they analyze the details of International Monetary Fund's risk appetite index, but they concluded that the measures of risk appetite do not always give similar results for each country. Additionally, [26] stated that changes in investors' risk appetite is an important signal for the effectiveness of the financial market.

Parallel to these studies, [27] evaluates the relationship between changes in the level of investor fear and financial market returns. It is concluded that investor fear level has a strong influence on the market returns. Furthermore, [28] measures what investors consider before choosing financial instruments. In this context, a survey was conducted with 254 investors. Increasing risk appetite has a positive effect on the



preferences of mutual funds. In addition to them, [29] aimed to identify investor profiles during financial crisis period. For this purpose, investors' behavior before and after 2008 global financial crisis is considered. It is determined that taking too much risks by the investors increases volatility in the market that negatively affects sustainable economic growth.

There are also some studies in the literature which analyzed the different aspects of the risk appetite of the investors. As an example, [30] developed a portfolio selection model by considering the expectations of the investors. Reference [31] developed a new model which measures the risk appetite of the investors. Additionally, [32] underlined the importance of understanding risk profiles of the investors so that financial advisors can serve them more effectively. Also, [33] aimed to understand the risk appetite of international bond investors. It is claimed that classification of the investor risk profile is a very useful way to attract the attention of them. Moreover, [34] underlined the importance of identifying risk profiles of the investors to present financial instruments for them.

Moreover, some studies focused on the risk aversion. For example, [35] measured the relationship between risk aversion and performance of energy markets. A simulation model is taken into the consideration in this study. They reached the conclusion that risk aversion has a negative influence on the reliability of this market. Similarly, Bekaert and Hoerova [36] makes a study to understand the relationship between risk aversion and uncertainty in the market. Within this framework, monthly data of German and US for the periods between 1992 and 2008 is evaluated. They reached the conclusion that credit spreads have a significant effect on risk aversion.

Moreover, the subject of developing products for risk averse investor was also considered in the literature. As an example, [37] proposed a two-stage robust investment model regarding electricity market. In this model, hedging methodology is considered to minimize uncertainty so that financial products can become appropriate for risk averse investors. Parallel to this study, [38] developed a renewable energy investment alternative for risk averse investors by using derivative products. Reference [39] also made a study about the risk appetite of the investors for currency portfolios. They concluded that risk aversion increases rapidly when there is uncertainty in the market.

Internal type-2 fuzzy DEMATEL approach is very popular in the literature. Especially in the last studies, this approach was preferred form many different subjects. Reference [40] focused on the indicators of effective human resource management by using this methodology and identified that education is the most important criterion. Also, [41] developed a new interval type-2 multiple attribute decision making model by combining IT2F-DEMATEL and IT2F-TOPSIS methods. Moreover, [42] considered this methodology to improve customer satisfaction in transportation industry. In addition to these studies, [43] used interval type-2 DEMATEL approach for green supplier selection. Reference [44] also made an

analysis to see causal relationship of knowledge management criteria with the help of this approach. Reference [45] made a study to provide environmental sustainability and interval type-2 DEMATEL method is taken into the consideration in this study.

Similar to the interval type-2 fuzzy DEMATEL approach, the popularity of QUALIFLEX approach increased especially in the last years. Reference [46] considered this methodology for medical decision-making problem. Reference [47] generated a new hierarchical Pythagorean fuzzy QUALIFLEX method in his study. Reference [48] also used QUALIFLEX method with fuzzy logic. In addition to these studies, [49] and [50] tried to select the best green supplier with the help of this approach. As a result of literature analysis, it is concluded that risk appetite of the investors is a very popular subject in the literature. It was considered in many different researchers in various ways. For example, some studies aimed to understand the risk appetite of the investors during financial crisis period whereas some others focused on the determinants of risk appetite. In addition to them, the impacts of investors' risk appetite and the subject of risk aversion were also taken into the consideration in many different studies. Also, most of these studies were conducted with regression, survey, structural equation model. However, it is obvious that there is a need for a new study which focuses on risk appetite subject with a new methodology. On the other hand, it is also identified that the popularity of the interval type-2 fuzzy DEMATEL and QUALIFLEX approaches increase especially in the last years. Nonetheless, it is understood that these methods were not considered regarding the risk profile of the investors before. Hence, making a new analysis to measure the investors' risk appetite by using interval type-2 fuzzy DEMATEL and QUALIFLEX methods makes a significant contribution to the literature.

# III. METHODOLOGY

# A. IT2 FUZZY SETS

Type-2 fuzzy logic is a new approach of the fuzzy sets. The main purpose is to minimize the minimize the uncertainties in fuzzy system.  $\tilde{A}$  represents a type-2 fuzzy set and  $\mu_{\tilde{A}(x,u)}$  gives information about the membership function [51]. The details are given on equation (1).

$$\tilde{A} = \left\{ \left( (x, u), \mu_{\tilde{A}(x, u)} \right) \mid \forall_x \in X, \forall_u \in J_x \subseteq [0, 1] \right\},\,$$

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$$\tilde{A} = \int_{x \in X} \int_{u \in J_x} \mu_{\tilde{A}}(x, u) / (x, u) \quad J_x \subseteq [0, 1]$$
 (1)

where  $0 \le \mu_{\tilde{A}}(x, u) \le 1$ . In addition to this aspect,  $\int \int$  gives information about the union over all x and u.  $\int$  is replaced with  $\Sigma$  regarding discrete universes. Interval type-2 fuzzy sets  $(\tilde{A})$  can be demonstrated as following.

$$\tilde{A} = \int_{x \in X} \int_{u \in J_x} 1/(x, u) \quad J_x \subseteq [0, 1]$$
 (2)



For this circumstance, all  $\mu_{\tilde{A}}(x,u)$  should be equal to "1". With respect to the interval type-2 fuzzy set  $\tilde{A}_i$ ,  $\tilde{A}_i^U$  gives information about the upper trapezoidal membership function whereas  $\tilde{A}_i^L$  indicates the lower trapezoidal membership function. The details are stated on the equation (3).

$$\tilde{A}_{i} = \left(\tilde{A}_{i}^{U}, \tilde{A}_{i}^{L}\right) = \left(\left(a_{i1}^{U}, a_{i2}^{U}, a_{i3}^{U}, a_{i4}^{U}; H_{1}(\tilde{A}_{i}^{U}), H_{2}\left(\tilde{A}_{i}^{U}\right)\right), \\ \left(a_{i1}^{L}, a_{i2}^{L}, a_{i3}^{L}, a_{i4}^{L}; H_{1}\left(\tilde{A}_{i}^{L}\right), H_{2}\left(\tilde{A}_{i}^{L}\right)\right)\right)$$
(3)

In this equation,  $\tilde{A}_{i}^{U}$  and  $\tilde{A}_{i}^{L}$  indicate type-1 fuzzy sets. On the other side,  $a_{i1}^{U}$ ,  $a_{i2}^{U}$ ,  $a_{i3}^{U}$ ,  $a_{i4}^{U}$ ,  $a_{i1}^{L}$ ,  $a_{i2}^{L}$ ,  $a_{i3}^{L}$ ,  $a_{i4}^{L}$  are the values of interval type-2 fuzzy set. The calculation of the interval type-2 fuzzy sets is shown on the equations (4) and (8).

$$\begin{split} \tilde{A}_{1} \oplus \tilde{A}_{2} &= \left( \tilde{A}_{1}^{U}, \tilde{A}_{1}^{L} \right) \oplus \left( \tilde{A}_{2}^{U}, \tilde{A}_{2}^{L} \right) \\ &= \left( \left( a_{11}^{U} + a_{21}^{U}, a_{12}^{U} + a_{22}^{U}, a_{13}^{U} + a_{23}^{U}, a_{14}^{U} + a_{24}^{U}; \right. \\ & \left. \min \left( H_{1} \left( \tilde{A}_{1}^{U} \right), H_{1} \left( \tilde{A}_{2}^{U} \right) \right), \min \left( H_{2} \left( \tilde{A}_{1}^{U} \right), \right. \\ & \left. H_{2} \left( \tilde{A}_{2}^{U} \right) \right) \right), \left( a_{11}^{L} + a_{21}^{L}, a_{12}^{L} + a_{22}^{L}, a_{13}^{L} + a_{23}^{L}, \right. \\ & \left. a_{14}^{L} + a_{24}^{L}; \min \left( H_{1} \left( \tilde{A}_{1}^{L} \right), H_{1} \left( \tilde{A}_{2}^{L} \right) \right) \right), \\ \left. \min \left( H_{2} \left( \tilde{A}_{1}^{L} \right), H_{2} \left( \tilde{A}_{2}^{L} \right) \right) \right) \right) \\ &= \left( \left( a_{11}^{U} - a_{24}^{U}, a_{12}^{U} - a_{23}^{U}, a_{13}^{U} - a_{22}^{U}, a_{14}^{U} - a_{21}^{U}; \right. \\ \left. \min \left( H_{1} \left( \tilde{A}_{1}^{U} \right), H_{1} \left( \tilde{A}_{2}^{U} \right) \right) \right), \left( min \left( H_{2} \left( \tilde{A}_{1}^{U} \right), \right. \\ \left. H_{2} \left( \tilde{A}_{2}^{U} \right) \right) \right), \left( a_{11}^{L} - a_{24}^{L}, a_{12}^{L} - a_{23}^{L}, a_{13}^{L} - a_{22}^{L}, \right. \\ \left. a_{14}^{L} - a_{21}^{L}; \min \left( H_{1} \left( \tilde{A}_{1}^{L} \right), H_{1} \left( \tilde{A}_{2}^{L} \right) \right) \right), \\ \left. \tilde{A}_{1} \otimes \tilde{A}_{2} = \left( \tilde{A}_{1}^{U}, \tilde{A}_{1}^{L} \right) \otimes \left( \tilde{A}_{2}^{U}, \tilde{A}_{2}^{L} \right) \\ = \left( \left( a_{11}^{U} \times a_{21}^{U}, a_{12}^{U} \times a_{22}^{U}, a_{13}^{U} \times a_{23}^{U}, a_{14}^{U} \times a_{24}^{U}; \right. \\ \left. \min \left( H_{1} \left( \tilde{A}_{1}^{U} \right), H_{1} \left( \tilde{A}_{2}^{U} \right) \right) \right), \left( \tilde{a}_{1}^{L} \times \tilde{a}_{23}^{L}, a_{14}^{L} \times a_{24}^{U}; \right. \\ \left. \min \left( H_{1} \left( \tilde{A}_{1}^{U} \right), H_{1} \left( \tilde{A}_{2}^{U} \right) \right), \min \left( H_{2} \left( \tilde{A}_{1}^{U} \right), \right. \\ \left. H_{2} \left( \tilde{A}_{2}^{U} \right) \right) \right), \left( a_{11}^{L} \times a_{21}^{L}, a_{12}^{L} \times a_{22}^{L}, a_{13}^{L} \times a_{23}^{L}; \right. \\ \left. a_{14}^{L} \times a_{24}^{L}; \min \left( H_{1} \left( \tilde{A}_{1}^{L} \right), H_{2} \left( \tilde{A}_{2}^{U} \right) \right) \right), \left( \tilde{a}_{11}^{L} \times \tilde{a}_{12}^{L}, \left. \tilde{a}_{12}^{L} \times \tilde{a}_{22}^{L}, a_{13}^{L} \times \tilde{a}_{23}^{L}; \right. \\ \left. a_{14}^{L} \times a_{24}^{L}; \min \left( H_{1} \left( \tilde{A}_{1}^{L} \right), H_{2} \left( \tilde{A}_{2}^{L} \right) \right) \right), \left( \tilde{a}_{11}^{L} \times \tilde{a}_{12}^{L}, \left. \tilde{a}_{12}^{L} \times \tilde{a}_{13}^{L}, \left. \tilde{a}_{23}^{L}, \left. \tilde{a}_{23}^{L}, \left. \tilde{a}_{23}^{L}, \left. \tilde{a}_{23}^{L}, \left. \tilde{a}_{23}^{L}, \left. \tilde{a}_{23}^{L}, \left. \tilde$$

#### **B. IT2 FUZZY DEMATEL**

The word of DEMATEL is generated from the first letters of "decision making trial and evaluation laboratory". This method is mainly used to weight different dimensions according to their importance. In addition to this condition, the interdependence among the factors can also be analyzed with the help of this method [52]. This approach is extended by considering interval type-2 fuzzy sets. The details are given below [46].

In the first step, expert evaluations are obtained. These linguistic evaluations are converted to internal fuzzy sets. This situation is demonstrated on Table 4. In the second step, initial direct relation matrix is created by considering the evaluation results collectively. Additionally, the average scores of pairwise comparisons are calculated. In this process, interval type-2 fuzzy numbers are taken into the consideration. With the help of these issues, the initial direct-relation fuzzy matrix  $\tilde{Z}$  is developed. In this context, the degree of the influence is presented by

$$\tilde{Z}_{ij} = \left(a_{ij}, b_{ij}, c_{ij}, d_{ij}; H_1\left(z_{ij}^U\right), H_2\left(z_{ij}^U\right)\right),$$

$$\left(e_{ij}, f_{ij}, g_{ij}, h_{ij}; H_1\left(z_{ij}^L\right), H_2\left(z_{ij}^L\right)\right).$$

The details of this matrix are given on equation (9).

$$\tilde{Z} = \begin{bmatrix}
0 & \tilde{z}_{12} & \cdots & \tilde{z}_{1n} \\
\tilde{z}_{21} & 0 & \cdots & \tilde{z}_{2n} \\
\vdots & \vdots & \ddots & \cdots \\
\tilde{z}_{n1} & \tilde{z}_{n2} & \cdots & 0
\end{bmatrix}$$
(9)

In order to construct the initial direct-relation matrix, the average fuzzy scores are considered which are detailed on equation (10).

$$\tilde{Z} = \frac{\tilde{Z}^1 + \tilde{Z}^2 + \tilde{Z}^3 + \cdots \tilde{Z}^n}{n} \tag{10}$$

In the third step, the pairwise matrix is normalized. In this process, equations (11), (12) and (13) are used.

$$\tilde{X} = \begin{bmatrix} \tilde{x}_{11} & \tilde{x}_{12} & \cdots & \tilde{x}_{1n} \\ \tilde{x}_{21} & \tilde{x}_{22} & \cdots & \cdots & \tilde{x}_{2n} \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \tilde{x}_{n1} & \tilde{x}_{n2} & \cdots & \cdots & \tilde{x}_{nn} \end{bmatrix}$$
(11)

where

$$\tilde{x}_{ij} = \frac{\tilde{z}_{ij}}{r} = \left(\frac{Z_{a'_{ij}}}{r}, \frac{Z_{b'_{ij}}}{r}, \frac{Z_{c'_{ij}}}{r}, \frac{Z_{d'_{ij}}}{r}; H_1\left(z^U_{ij}\right), H_2\left(z^U_{ij}\right)\right),$$

$$\left(\frac{Z_{e'_{ij}}}{r}, \frac{Z_{f'_{ij}}}{r}, \frac{Z_{g'_{ij}}}{r}, \frac{Z_{h'_{ij}}}{r}; H_1\left(z^L_{ij}\right), H_2\left(z^L_{ij}\right)\right)$$

$$r = \max\left(\max_{1 \le i \le n} \sum_{j=1}^{n} Z_{d'_{ij}}, \max_{1 \le i \le n} \sum_{j=1}^{n} Z_{d'_{ij}}\right)$$
(12)



The fourth step is related to the development of the total influence fuzzy matrix (T). The equations (14)-(18) represent this process.

$$X_{a'} = \begin{bmatrix} 0 & a'_{12} & \cdots & \cdots & a'_{1n} \\ a'_{21} & 0 & \cdots & \cdots & a'_{2n} \\ \vdots & \vdots & \ddots & \cdots & \ddots \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a'_{n1} & a'_{n2} & \cdots & \cdots & 0 \end{bmatrix}, \dots,$$

$$X_{h'} = \begin{bmatrix} 0 & h'_{12} & \cdots & \cdots & h'_{1n} \\ h'_{21} & 0 & \cdots & \cdots & h'_{2n} \\ \vdots & \vdots & \ddots & \cdots & \cdots \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ h'_{n1} & h'_{n2} & \cdots & \cdots & 0 \end{bmatrix}$$

$$\tilde{T} = \lim_{k \to \infty} \tilde{X} + \tilde{X}^2 + \cdots + \tilde{X}^k$$

$$\begin{bmatrix} \tilde{t}_{12} & \tilde{t}_{12} & \cdots & \tilde{t}_{1n} \\ \tilde{t}_{2n} & \tilde{t}_{2n} & \cdots & \tilde{t}_{2n} \end{bmatrix}$$
(15)

$$\tilde{T} = \begin{bmatrix}
\tilde{t}_{11} & \tilde{t}_{12} & \cdots & \tilde{t}_{1n} \\
\tilde{t}_{21} & \tilde{t}_{22} & \cdots & \tilde{t}_{2n} \\
\vdots & \vdots & \ddots & \dots
\end{bmatrix}$$
(16)

$$\tilde{T} = \begin{bmatrix}
\tilde{t}_{11} & \tilde{t}_{12} & \cdots & \cdots & \tilde{t}_{1n} \\
\tilde{t}_{21} & \tilde{t}_{22} & \cdots & \cdots & \tilde{t}_{2n} \\
\vdots & \vdots & \ddots & \cdots & \vdots \\
\tilde{t}_{n1} & \tilde{t}_{n2} & \cdots & \cdots & \tilde{t}_{nn}
\end{bmatrix}$$
(16)

where

$$\tilde{t}_{ij} = \left(a_{ij}^{"}, b_{ij}^{"}, c_{ij}^{"}, d_{ij}^{"}; H_1\left(\tilde{t}_{ij}^{U}\right), H_2\left(\tilde{t}_{ij}^{U}\right)\right),$$

$$\left(e_{ij}^{"}, f_{ij}^{"}, g_{ij}^{"}, h_{ij}^{"}; H_1\left(\tilde{t}_{ij}^{L}\right), H_2\left(\tilde{t}_{ij}^{L}\right)\right) \tag{17}$$

$$\begin{bmatrix} a''_{ij} \end{bmatrix} = X_{a'} \times (I - X_{a'})^{-1}, \dots, \begin{bmatrix} h''_{ij} \end{bmatrix} = X_{h'} \times (I - X_{h'})^{-1}$$
(18)

In the last step, the influence degrees are calculated. For this purpose, the sums of all vector rows and columns of the total relation matrix are taken into the consideration. This situation is detailed on equations (19) and (20).

$$\tilde{D}_i = \left[ \sum_{i=1}^n \tilde{t}_{ij} \right]_{n \times 1} \tag{19}$$

$$\tilde{R}_i = \left[\sum_{i=1}^n \tilde{t}_{ij}\right]_{1 \times n}^{\prime} \tag{20}$$

In these equations,  $\tilde{D}_i$  shows the sum of all vector rows. On the other hand,  $\tilde{R}_i$  refers to the sum of all vector columns. Thus,  $(\tilde{D}_i + \tilde{R}_i)$  gives information about the total degree of the influence among criteria. Owing to this issue, when this value is higher, it means that the criterion becomes closer to the central point of the object.

### C. IT2 FUZZY QUALIFLEX

The QUALIFLEX is generated to develop a flexible multiple criteria decision-making method by Paelinck in 1976 and it is a generalization form of Jacquet-Lagreze's permutation [53]. The method aims to the flexibility with the correct treatment of cardinal and ordinal information and the preferences with the concordance results [54], [55]. There are several extensions of the method as seen in the literature. But, the use of interval type 2 is extremely limited for the QUALIFLEX. The extended method is summarized as 46.

Step 1: Construct the decision matrix: IT2 fuzzy decision matrix is developed by using the averaged values of k decision makers' criteria evaluations for each alternative as follows.

$$D = \begin{bmatrix} X1 & X2 & X3 & \cdots & Xn \\ A_1 & A_{12} & A_{13} & & \cdots & A_{1n} \\ A_2 & A_{21} & A_{22} & A_{23} & & \cdots & A_{2n} \\ A_{31} & A_{32} & A_{33} & & \cdots & A_{3n} \\ \vdots & \vdots & \vdots & \ddots & & \vdots \\ A_m & A_{m2} & A_{m3} & & \cdots & A_{mn} \end{bmatrix}$$
(21)

where

$$A_{ij} = \frac{1}{k} \left[ \sum_{e=1}^{k} A_{ij}^{e} \right]$$
 (22)

Step 2: Compute the signed distance: Signed distance  $d(A_{ij}, 0)$  is calculated for each  $A_{ij}$  in the decision matrix by the formula (23)

$$d\left(A_{ij},\tilde{0}\right)$$

$$= \frac{1}{8} \left(a_{1ij}^{L} + a_{2ij}^{L} + a_{3ij}^{L} + a_{4ij}^{L} + 4a_{1ij}^{U} + 2a_{2ij}^{U} + 2a_{3ij}^{U} + 4a_{4ij}^{U} + 3(a_{2ij}^{U} + a_{3ij}^{U} - a_{1ij}^{U} - a_{4ij}^{U}) \frac{h_{ij}^{L}}{h_{ii}^{U}}\right)$$
(23)

where

$$A_{ij} = \begin{bmatrix} A_{ij}^{L}, A_{ij}^{U} \end{bmatrix} = \begin{bmatrix} \left( a_{1ij}^{L}, a_{2ij}^{L}, a_{3ij}^{L}, a_{4ij}^{L}; h_{ij}^{L} \right), \\ \left( a_{1ij}^{U}, a_{2ij}^{U}, a_{3ij}^{U}, a_{4ij}^{U}; h_{ij}^{U} \right) \end{bmatrix}$$
(24)

Step 3: Calculate the concordance/discordance index: The index  $I_i^l$  is employed for each pair of alternatives  $(A_\rho, A_\beta)$ with m alternatives, m! permutations of the ranking of the alternatives exist:

$$I_{j}^{l} = \sum_{A_{\rho}, A_{\beta} \in A} I_{j}^{l} \left( A_{\rho}, A_{\beta} \right)$$

$$= \sum_{A_{\rho}, A_{\beta} \in A} \left( d \left( A_{\rho j}, \tilde{0}_{1} \right) - d \left( A_{\beta j}, \tilde{0}_{1} \right) \right) \quad (25)$$

$$\begin{split} A_{\rho j} &= \left[ A_{\rho j}^{L}, A_{\rho j}^{U} \right] = \left[ \left( a_{1\rho j}^{L}, a_{2\rho j}^{L}, a_{3\rho j}^{L}, a_{4\rho j}^{L}; h_{\rho j}^{L} \right), \\ \left( a_{1\rho j}^{U}, a_{2\rho j}^{U}, a_{3\rho j}^{U}, a_{4\rho j}^{U}; h_{\rho j}^{U} \right) \right] \end{split}$$

$$\begin{split} A_{\beta j} &= \left[ A_{\beta j}^{L}, A_{\beta j}^{U} \right] = \left[ \left( a_{1\beta j}^{L}, a_{2\beta j}^{L}, a_{3\beta j}^{L}, a_{4\beta j}^{L}; h_{\beta j}^{L} \right), \\ & \left( a_{1\beta j}^{U}, a_{2\beta j}^{U}, a_{3\beta j}^{U}, a_{4\beta j}^{U}; h_{\beta j}^{U} \right) \right] \end{split} \tag{26}$$

$$P_l = (\dots, A_\rho, \dots, A_\beta \dots)$$
 for  $l = 1, 2, \dots, m!$  (27)

Step 4: Compute the comprehensive concordance/ discordance index. Weights of the criteria based on IT2 fuzzy



numbers are considered in the concordance/discordance index in the equation (28)

$$I^{l} = \sum_{A_{\rho}, A_{\beta} \in A} \sum_{j=1}^{n} I_{j}^{l} \left( A_{\rho}, A_{\beta} \right) . W_{j}$$

$$= \sum_{A_{\rho}, A_{\beta} \in A} \sum_{j=1}^{n} \left( d \left( A_{\rho j}, \tilde{0}_{1} \right) - d \left( A_{\beta j}, \tilde{0}_{1} \right) \right) . W_{j}$$

$$(28)$$

The comprehensive index results are listed for all permutations and the maximum value defines the optimal ranking order of the alternatives.

# IV. AN APPLICATION ON THE EMERGING ECONOMIES A. PROPOSED MODEL

Integrated model is summarized in the following steps as seen in figure 1.

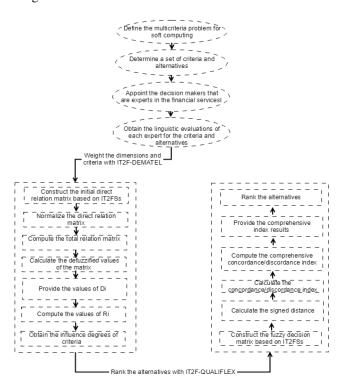


FIGURE 1. The flowchart of the model.

Step 1: Define the decision-making problem: Investors perception and their risk appetite are analyzed for the emerging economies based on the integrated interval type-2 fuzzy approach. For this issue, 8 criteria are defined with the supported literature in Table 1. Additionally, conservative, moderate, and aggressive investors are considered as a set of alternatives.

Table 1 explains 8 different criteria. Transactional confidence (C1) refers to the security of financial transactions against the threats, such as hacking and competitive cost (C2) defines having low costs in case of purchasing financial products. Additionally, variety of financial instruments (C3) explains the diversity of financial instruments and presenting different financial products. Similarly, functionality (C4) means ease of use of financial products with

TABLE 1. Determinants of investors' risk appetite.

Criteria	Supported Literature
Transactional	[56], [57]
Confidence (C1)	[50], [57]
Competitive cost (C2)	[58], [59]
Variety of financial	[60] [61]
instruments (C3)	[60], [61]
Functionality (C4)	[62], [63]
Transaction Speed (C5)	[64], [65]
Legal Easiness (C6)	[66], [67]
Macroeconomic	[60] [60]
Performance (C7)	[68],[69]
Political Stability (C8)	[70], [71]

**TABLE 2.** Linguistic scales and interval type-2 trapezoidal fuzzy numbers for the criteria and alternatives.

Criteria	Alternatives	IT2TrFNs		
Absolutely	Absolutely	((0.0,0.0,0.0,0.0;1.0),		
Low (AL)	Poor (AP)	(0.0,0.0,0.0,0.0;1.0))		
Very Low	Very Poor	((0.0075, 0.0075, 0.015, 0.0525;0.8),		
(VL)	(VP)	(0.0,0.0,0.02,0.07;1.0))		
Low (L)	Poor (P)	((0.0875, 0.12, 0.16, 0.1825;0.8),		
		(0.04,0.10,0.18,0.23;1.0))		
Medium Low	Medium	((0.2325, 0.255, 0.325, 0.3575;0.8),		
(ML)	Poor (MP)	(0.17,0.22,0.36,0.42;1.0))		
Medium (M)	Fair (F)	((0.4025, 0.4525, 0.5375,		
		0.5675;0.8),		
		(0.32,0.41,0.58,0.65;1.0))		
Medium	Medium	((0.65, 0.6725, 0.7575, 0.79;0.8),		
High (MH)	Good (MG)	(0.58,0.63,0.80,0.86;1.0))		
High (H)	Good (G)	((0.7825, 0.815, 0.885, 0.9075;0.8),		
		(0.72,0.78,0.92,0.97;1.0))		
Very High	Very Good	((0.9475, 0.985, 0.9925, 0.9925;0.8),		
(VH)	(VG)	(0.93,0.98,1.0,1.0;1.0))		
Absolutely	Absolutely	((1.0, 1.0, 1.0, 1.0; 1.0), (1.0, 1.0,		
High (AH)	Good (AG)	1.0, 1.0; 1.0))		

the help of availability of different alternative distribution channels. On the other side, transaction speed (C5) gives information about the speed of a financial transaction to be actualized. In this process, the speed of internet plays a key role. Moreover, legal easiness (C6) explains whether there are legal obstacles to trading or the laws in the country make it difficult to trade. In other words, this criterion gives information about the legal convenience because investors prefer to enter this market in such an environment. Furthermore, macroeconomic performance (C7) identifies whether economic indicators of the country are in a good condition to attract the attention of the investors. Finally, political stability (C8) refers to the positive performance of the government to manage the country so that there is a minimum risk of government collapse.



**TABLE 3.** Dependency degrees among the criteria of investors' risk appetite.

	C1			C2			СЗ			C4		
	D M1	D M2	D M3									
C 1	-	-	-	M H	Н	VH	ML	M	M H	ML	M	ML
C 2	Н	VH	Н	-	-	-	Н	VH	M H	M H	M	M
C 3	ML	M	ML	VL	L	M	-	-	-	VL	VL	L
C 4	ML	M	ML	VL	L	L	ML	L	ML	-	-	-
C 5	ML	L	L	M H	Н	VH	M	ML	M	M H	M	M H
C 6	L	L	ML	VL	L	M	M	M H	M	M	M	M
C 7	ML	M H	ML	M H	M	ML	ML	M	M	VL	M	M
C 8	ML	M	М	M	М	M H	M	M	M H	L	M	M H
	C5			C6			C7			C8		
	D M1	D M2	D M3									
C 1	VL	L	ML	M	M	ML	VL	VL	L	VL	L	L
C 2	M H	M H	ML	M H	Н	Н	ML	M	M	ML	M H	M
C 3	M H	M	ML	ML	М	M	L	L	ML	L	ML	ML
C 4	M	M	ML	ML	М	M	ML	VL	M	ML	M	M
C 5	-	-	-	M	ML	ML	M	M	M H	M H	M H	M
C 6	М	ML	M H	-	-	-	VL	L	M	M	ML	ML
C 7	VL	L	L	ML	М	M		-	-	M	ML	M
С												

**TABLE 4.** Linguistic evaluations of the alternatives.

C. 21. 2. (A):	(coi	A1 nservat	ive)	A2 (moderate)			A3 (aggressive)		
Criteria/Alter natives	D M 1	D M 2	D M 3	D M 1	D M 2	D M 3	D M 1	D M 2	D M 3
Transactional Confidence (C1)	V G	V G	A G	G	G	M G	M G	F	G
Competitive cost (C2)	M G	F	M G	G	G	G	V G	V G	A G
Variety of financial instruments (C3)	F	F	M G	M G	V G	G	V G	A G	V G
Functionality (C4)	M G	F	M G	M G	M G	G	V G	V G	V G
Transaction Speed (C5)	F	F	M G	F	M G	M G	V G	V G	A G
Legal Easiness (C6)	G	F	M G	G	G	M G	G	V G	V G
Macroeconom ic Performance (C7)	V G	V G	V G	V G	G	G	G	M G	M G
Political Stability (C8)	V G	V G	V G	G	G	V G	G	M G	M G

Step 2: Provide the linguistic evaluations: 3 experts are appointed to obtain the linguistic preferences for the criteria and alternatives. The experts have ten year experiences in

**TABLE 5.** Initial direct relation matrix for the criteria.

	C1	C2	С3	C4
C 1	((0.0,0.0,0.0,0.0;1.0 ), (0.0,0.0,0.0,0.0;1.0)	((0.79,0.82,0.88,0. 90;0.80), (0.74,0.80,0.91,0.9 4;1.00))	((0.43,0.46,0.54,0.5 7;0.80), (0.36,0.42,0.58,0.6 4;1.00))	((0.29,0.3 2,0.40,0. 43;0.80), (0.22,0.2 8,0.43,0. 50;1.00))
C 2	((0.84,0.87,0.92,0.9 4;0.80), (0.79,0.85,0.95,0.9 8;1.00))	((0.0,0.0,0.0,0.0;1. 0), (0.0,0.0,0.0,0.0;1.0 ))	((0.79,0.82,0.88,0.9 0;0.80), (0.74,0.80,0.91,0.9 4;1.00))	((0.49,0.5 3,0.61,0. 64;0.80), (0.41,0.4 8,0.65,0. 72;1.00))
C 3	((0.29,0.32,0.40,0.4 3;0.80), (0.22,0.28,0.43,0.5 0;1.00))	((0.17,0.19,0.24,0. 27;0.80), (0.12,0.17,0.26,0.3 2;1.00))	((0.0,0.0,0.0,0.0;1.0 ), (0.0,0.0,0.0,0.0;1.0)	((0.03,0.0 5,0.06,0. 10;0.80), (0.01,0.0 3,0.07,0. 12;1.00))
C 4	((0.29,0.32,0.40,0.4 3;0.80), (0.22,0.28,0.43,0.5 0;1.00))	((0.06,0.08,0.11,0. 14;0.80), (0.03,0.07,0.13,0.1 8;1.00))	((0.18,0.21,0.27,0.3 0; 0.80), (0.13,0.18,0.30,0.3 6;1.00))	((0.0,0.0, 0.0,0.0;1. 0), (0.0,0.0,0 .0,0.0;1.0
C 5	((0.14,0.17,0.22,0.2 4; 0.80), (0.08,0.14,0.24,0.2 9;1.00))	((0.79,0.82,0.88,0. 90;0.80), (0.74,0.80,0.91,0.9 4;1.00))	((0.35,0.39,0.47,0.5 0;0.80), (0.27,0.35,0.51,0.5 7;1.00))	((0.57,0.6 0,0.68,0. 72;0.80), (0.49,0.5 6,0.73,0. 79;1.00))
C 6	((0.14,0.17,0.22,0.2 4; 0.80), (0.08,0.14,0.24,0.2 9;1.00))	((0.17,0.19,0.24,0. 27;0.80), (0.12,0.17,0.26,0.3 2;1.00))	((0.49,0.53,0.61,0.6 4;0.80), (0.41,0.48,0.65,0.7 2;1.00))	((0.40,0.4 5,0.54,0. 57;0.80), (0.32,0.4 1,0.58,0. 65;1.00))
C 7	((0.37,0.39,0.47,0.5 0;0.80), (0.31,0.36,0.51,0.5 7;1.00))	((0.43,0.46,0.54,0. 57;0.80), (0.36,0.42,0.58,0.6 4;1.00))	((0.35,0.39,0.47,0.5 0;0.80), (0.27,0.35,0.51,0.5 7;1.00))	((0.27,0.3 0,0.36,0. 40;0.80), (0.21,0.2 7,0.39,0. 46;1.00))
C 8	((0.35,0.39,0.47,0.5 0;0.80), (0.27,0.35,0.51,0.5 7;1.00))	((0.49,0.53,0.61,0. 64;0.80), (0.41,0.48,0.65,0.7 2;1.00))	((0.49,0.53,0.61,0.6 4;0.80), (0.41,0.48,0.65,0.7 2;1.00))	((0.38,0.4 2,0.49,0. 51;0.80), (0.31,0.3 8,0.52,0. 58;1.00))
	C5	C6	C7	C8
C 1	((0.11,0.13,0.17,0.2 0;0.80), (0.07,0.11,0.19,0.2 4;1.00))	((0.35,0.39,0.47,0. 50;0.80), (0.27,0.35,0.51,0.5 7;1.00))	((0.03,0.05,0.06,0.1 0;0.80), (0.01,0.03,0.07,0.1 2;1.00))	((0.06,0.0 8,0.11,0. 14;0.80), (0.03,0.0 7,0.13,0. 18;1.00))
C 2	((0.51,0.53,0.61,0.6 5;0.80), (0.44,0.49,0.65,0.7 1;1.00))	((0.74,0.77,0.84,0. 87;0.80), (0.67,0.73,0.88,0.9 3;1.00))	((0.35,0.39,0.47,0.5 0;0.80), (0.27,0.35,0.51,0.5 7;1.00))	((0.43,0.4 6,0.54,0. 57;0.80), (0.36,0.4 2,0.58,0. 64;1.00))
C 3	((0.43,0.46,0.54,0.5 7;0.80), (0.36,0.42,0.58,0.6 4;1.00))	((0.35,0.39,0.47,0. 50;0.80), (0.27,0.35,0.51,0.5 7;1.00))	((0.14,0.17,0.22,0.2 4; 0.80), (0.08,0.14,0.24,0.2 9;1.00))	((0.18,0.2 1,0.27,0. 30; 0.80), (0.13,0.1 8,0.30,0. 36;1.00))
C 4	((0.35,0.39,0.47,0.5 0;0.80), (0.27,0.35,0.51,0.5 7;1.00))	((0.35,0.39,0.47,0. 50;0.80), (0.27,0.35,0.51,0.5 7;1.00))	((0.21,0.24,0.29,0.3 3;0.80), (0.16,0.21,0.32,0.3 8;1.00))	((0.35,0.3 9,0.47,0. 50;0.80), (0.27,0.3 5,0.51,0. 57;1.00))
C 5	((0.0,0.0,0.0,0.0;1.0), (0.0,0.0,0.0;1.0)	((0.29,0.32,0.40,0. 43;0.80), (0.22,0.28,0.43,0.5 0;1.00))	((0.49,0.53,0.61,0.6 4;0.80), (0.41,0.48,0.65,0.7 2;1.00))	((0.57,0.6 0,0.68,0. 72;0.80), (0.49,0.5 6,0.73,0. 79;1.00))



TABLE 5. (Continued.) Initial drect relation matrix for the criteria.

C 6	((0.43,0.46,0.54,0.5 7;0.80), (0.36,0.42,0.58,0.6 4;1.00))	((0.0,0.0,0.0,0.0;1. 0), (0.0,0.0,0.0,0.0;1.0 ))	((0.17,0.19,0.24,0.2 7;0.80), (0.12,0.17,0.26,0.3 2;1.00))	((0.29,0.3 2,0.40,0. 43;0.80), (0.22,0.2 8,0.43,0. 50;1.00))
C 7	((0.06,0.08,0.11,0.1 4;0.80), (0.03,0.07,0.13,0.1 8;1.00))	((0.35,0.39,0.47,0. 50;0.80), (0.27,0.35,0.51,0.5 7;1.00))	((0.0,0.0,0.0,0.0;1.0 ), (0.0,0.0,0.0,0.0;1.0)	((0.35,0.3 9,0.47,0. 50;0.80), (0.27,0.3 5,0.51,0. 57;1.00))
C 8	((0.09,0.12,0.16,0.1 8;0.80), (0.04,0.10,0.18,0.2 3;1.00))	((0.35,0.39,0.47,0. 50;0.80), (0.27,0.35,0.51,0.5 7;1.00))	((0.35,0.39,0.47,0.5 0;0.80), (0.27,0.35,0.51,0.5 7;1.00))	((0.0,0.0, 0.0,0.0;1. 0), (0.0,0.0,0 .0,0.0;1.0

the field of finance and are selected within ten candidates for providing more accurate results. Table 2 represents the linguistic scales and their interval type-2 trapezoidal fuzzy numbers for measuring the weights of the criteria and ranking the alternatives.

Table 3 and 4 show that the linguistic opinions of each decision maker for the criteria and alternatives respectively.

Step 3: Weight the criteria of investors' risk appetite: IT2 fuzzy DEMATEL are used for calculating the relative importance of each criterion by using the formulas (9)-(20). Table 5-8 illustrate the analysis results for weighting the determinants of the investors' risk appetite in the emerging economies.

Step 4: Rank the alternatives: QUALIFLEX method based on interval type 2 fuzzy sets are considered to analyze the alternatives that define the risk choices of investors traded in the emerging economies with the equations (21)-(28). The results are provided in table 9-13 accordingly.

#### B. ANALYSIS

In the first stage of the analysis, the criteria of investors' risk appetite are weighted by using DEMATEL based on interval type 2 fuzzy sets. Initially, the direct relation matrix has been constructed by converting the linguistic scales to the interval type-2 trapezoidal fuzzy numbers with the equations (9) and (10). The matrix can be seen in Table 5.

The initial direct relation matrix has been normalized by the equations (11)-(13). The results are shown in Table 6.

Table 7 represents the total relation matrix with the equations (14)-(18).

The sums of all vector rows  $\tilde{D}_i$  and columns  $\tilde{R}_i$  of the total relation matrix are used for measuring the relative importance of the criteria with the equations (19) and (20).  $\left(\tilde{D}_i + \tilde{R}_i\right)$  defines the impact degrees and weights of the criteria. Table 8 illustrates the interval type 2 fuzzy numbers for weighting the criteria.

The second phase of the analysis continues with the QULIFLEX method for ranking the investors' behaviors on the risky investments in the emerging economies. For this purpose, initially, linguistic evaluations of each alternative define the risk profile for investors have been converted to the IT2 fuzzy numbers and averaged values of decision makers

TABLE 6. Normalized initial direct relation matrix.

	C1	C2	CZ	C4
	Cı	C2	C3	((0.06,0.0
C 1	((0.0,0.0,0.0,0.0;1.0), (0.0,0.0,0.0,0.0;1.0))	((0.16,0.16,0.17,0.18; 0.80), (0.15,0.16,0.18,0.19; 1.00))	((0.08,0.09,0.11,0.11; 0.80), (0.07,0.08,0.11,0.13; 1.00))	6,0.08,0.0 8;0.80), (0.04,0.06, 0.09,0.10; 1.00))
C 2	((0.17,0.17,0.18,0.19; 0.80), (0.16,0.17,0.19,0.19; 1.00))	((0.0,0.0,0.0,0.0;1.0), (0.0,0.0,0.0,0.0;1.0))	((0.16,0.16,0.17,0.18; 0.80), (0.15,0.16,0.18,0.19; 1.00))	((0.10,0.1 0,0.12,0.1 3;0.80), (0.08,0.10, 0.13,0.14; 1.00))
C 3	((0.06,0.06,0.08,0.08; 0.80), (0.04,0.06,0.09,0.10;	((0.03,0.04,0.05,0.05; 0.80), (0.02,0.03,0.05,0.06;	((0.0,0.0,0.0,0.0;1.0), (0.0,0.0,0.0,0.0;1.0))	((0.01,0.0 1,0.01,0.0 2;0.80), (0.00,0.01, 0.01,0.02; 1.00))
C 4	((0.06,0.06,0.08,0.08; 0.80), (0.04,0.06,0.09,0.10; 1.00))	((0.01,0.02,0.02,0.03; 0.80), (0.01,0.01,0.03,0.03; 1.00))	((0.04,0.04,0.05,0.06; 0.80), (0.03,0.04,0.06,0.07; 1.00))	((0.0,0.0,0 .0,0.0;1.0), (0.0,0.0,0 0,0.0;1.0))
C 5	((0.03,0.03,0.04,0.05; 0.80), (0.02,0.03,0.05,0.06; 1.00))	((0.16,0.16,0.17,0.18; 0.80), (0.15,0.16,0.18,0.19; 1.00))	((0.07,0.08,0.09,0.10; 0.80), (0.05,0.07,0.10,0.11; 1.00))	((0.11,0.1 2,0.14,0.1 4;0.80), (0.10,0.11, 0.14,0.16; 1.00))
C 6	((0.03,0.03,0.04,0.05; 0.80), (0.02,0.03,0.05,0.06; 1.00))	((0.03,0.04,0.05,0.05; 0.80), (0.02,0.03,0.05,0.06; 1.00))	((0.10,0.10,0.12,0.13; 0.80), (0.08,0.10,0.13,0.14; 1.00))	((0.08,0.0 9,0.11,0.1 1;0.80), (0.06,0.08, 0.11,0.13; 1.00))
C 7	((0.07,0.08,0.09,0.10; 0.80), (0.06,0.07,0.10,0.11; 1.00))	((0.08,0.09,0.11,0.11; 0.80), (0.07,0.08,0.11,0.13; 1.00))	((0.07,0.08,0.09,0.10; 0.80), (0.05,0.07,0.10,0.11; 1.00))	((0.05,0.0 6,0.07,0.0 8;0.80), (0.04,0.05, 0.08,0.09; 1.00))
C 8	((0.07,0.08,0.09,0.10; 0.80), (0.06,0.07,0.10,0.11; 1.00))	((0.10,0.10,0.12,0.13; 0.80), (0.08,0.10,0.13,0.14; 1.00))	((0.10,0.10,0.12,0.13; 0.80), (0.08,0.10,0.13,0.14; 1.00))	((0.08,0.0 8,0.10,0.1 0;0.80), (0.06,0.08, 0.10,0.11; 1.00))
	C5	C6	C7	C8
C 1	((0.02,0.03,0.03,0.04; 0.80), (0.01,0.02,0.04,0.05; 1.00))	((0.07,0.08,0.09,0.10; 0.80), (0.05,0.07,0.10,0.11; 1.00))	((0.01,0.01,0.01,0.02; 0.80), (0.00,0.01,0.01,0.02; 1.00))	((0.01,0.0 2,0.02,0.0 3;0.80), (0.01,0.02, 0.04,0.05; 1.00))
C 2	((0.10,0.11,0.12,0.13; 0.80), (0.09,0.10,0.13,0.14; 1.00))	((0.15,0.15,0.17,0.17; 0.80), (0.13,0.14,0.17,0.18; 1.00))	((0.07,0.08,0.09,0.10; 0.80), (0.05,0.07,0.10,0.11; 1.00))	((0.08,0.0 9,0.11,0.1 1;0.80), (0.11,0.12, 0.17,0.19; 1.00))
C 3	((0.08,0.09,0.11,0.11; 0.80), (0.07,0.08,0.11,0.13; 1.00))	((0.07,0.08,0.09,0.10; 0.80), (0.05,0.07,0.10,0.11; 1.00))	((0.03,0.03,0.04,0.05; 0.80), (0.02,0.03,0.05,0.06; 1.00))	((0.04,0.0 4,0.05,0.0 6;0.80), (0.04,0.05, 0.09,0.11; 1.00))
C 4	((0.07,0.08,0.09,0.10; 0.80), (0.05,0.07,0.10,0.11; 1.00))	((0.07,0.08,0.09,0.10; 0.80), (0.05,0.07,0.10,0.11; 1.00))	((0.04,0.05,0.06,0.06; 0.80), (0.03,0.04,0.06,0.08; 1.00))	((0.07,0.0 8,0.09,0.1 0;0.80), (0.08,0.10, 0.15,0.17; 1.00))
C 5	((0.0,0.0,0.0,0.0;1.0), (0.0,0.0,0.0,0.0;1.0))	((0.06,0.06,0.08,0.08; 0.80), (0.04,0.06,0.09,0.10; 1.00))	((0.10,0.10,0.12,0.13; 0.80), (0.08,0.10,0.13,0.14; 1.00))	((0.11,0.1 2,0.14,0.1 4;0.80), (0.15,0.17, 0.22,0.23; 1.00))
C 6	((0.08,0.09,0.11,0.11; 0.80), (0.07,0.08,0.11,0.13; 1.00))	((0.0,0.0,0.0,0.0;1.0), (0.0,0.0,0.0;0.0;1.0))	((0.03,0.04,0.05,0.05; 0.80), (0.02,0.03,0.05,0.06; 1.00))	((0.06,0.0 6,0.08,0.0 8;0.80), (0.07,0.08, 0.13,0.15; 1.00))
C 7	((0.01,0.02,0.02,0.03; 0.80), (0.01,0.01,0.03,0.03; 1.00))	((0.07,0.08,0.09,0.10; 0.80), (0.05,0.07,0.10,0.11; 1.00))	((0.0,0.0,0.0,0.0;1.0), (0.0,0.0,0.0,0.0;1.0))	((0.07,0.0 8,0.09,0.1 0;0.80), (0.08,0.10, 0.15,0.17; 1.00))
C 8	((0.02,0.02,0.03,0.04; 0.80), (0.01,0.02,0.04,0.05; 1.00))	((0.07,0.08,0.09,0.10; 0.80), (0.05,0.07,0.10,0.11; 1.00))	((0.07,0.08,0.09,0.10; 0.80), (0.05,0.07,0.10,0.11; 1.00))	((0.0,0.0,0 .0,0.0;1.0), (0.0,0.0,0. 0,0.0;1.0))

have been used for the fuzzy decision matrix based on the equations (21)-(22). Table 9 presents the averaged values of the decision matrix.



**TABLE 7.** Total relation matrix.

7.	Total relation n	natrix.		
	C1	C2	C3	C4
C 1	((0.06,0.08,0.12,0.15; 0.80), (0.04,0.06,0.18,0.31; 1.00))	((0.20,0.22,0.27,0.31; 0.80), (0.17,0.20,0.34,0.48; 1.00))	((0.15,0.17,0.24,0.28; 0.80), (0.12,0.15,0.32,0.48; 1.00))	((0.11,0.1 3,0.19,0.2 2;0.80), (0.07,0.11, 0.25,0.39; 1.00))
C 2	((0.24,0.27,0.35,0.39; 0.80), (0.20,0.25,0.45,0.65; 1.00))	((0.11,0.14,0.21,0.25; 0.80), (0.08,0.12,0.31,0.52; 1.00))	((0.26,0.29,0.39,0.44; 0.80), (0.22,0.27,0.51,0.75; 1.00))	((0.18,0.2 1,0.30,0.3 5;0.80), (0.14,0.19 0.40,0.62; 1.00))
C 3	((0.09,0.11,0.16,0.20; 0.80), (0.06,0.10,0.23,0.36; 1.00))	((0.08,0.10,0.15,0.19; 0.80), (0.06,0.09,0.22,0.36; 1.00))	((0.05,0.07,0.12,0.15; 0.80), (0.03,0.06,0.19,0.33; 1.00))	((0.05,0.0 7,0.11,0.1 5;0.80), (0.03,0.05 0.17,0.31; 1.00))
C 4	((0.09,0.11,0.16,0.20; 0.80), (0.06,0.10,0.23,0.36; 1.00))	((0.07,0.09,0.14,0.17; 0.80), (0.04,0.07,0.21,0.36; 1.00))	((0.09,0.11,0.17,0.21; 0.80), (0.06,0.10,0.26,0.43; 1.00))	((0.04,0.0 6,0.10,0.1 3;0.80), (0.03,0.05 0.17,0.31; 1.00))
C 5	((0.11,0.14,0.21,0.26; 0.80), (0.08,0.12,0.31,0.50; 1.00))	((0.23,0.25,0.33,0.37; 0.80), (0.20,0.24,0.43,0.63; 1.00))	((0.17,0.20,0.29,0.34; 0.80), (0.13,0.18,0.41,0.64; 1.00))	((0.18,0.2 1,0.29,0.3 3;0.80), (0.15,0.19, 0.39,0.59; 1.00))
C 6	((0.07,0.10,0.15,0.19; 0.80), (0.04,0.08,0.22,0.38; 1.00))	((0.09,0.11,0.17,0.20; 0.80), (0.06,0.10,0.24,0.40; 1.00))	((0.15,0.17,0.25,0.29; 0.80), (0.11,0.16,0.33,0.51; 1.00))	((0.12,0.1 5,0.21,0.2 5;0.80), (0.09,0.13, 0.28,0.44; 1.00))
C 7	((0.12,0.14,0.21,0.24; 0.80), (0.09,0.13,0.28,0.44; 1.00))	((0.14,0.16,0.22,0.26; 0.80), (0.10,0.14,0.31,0.47; 1.00))	((0.13,0.16,0.23,0.28; 0.80), (0.10,0.14,0.33,0.51; 1.00))	((0.10,0.1 2,0.18,0.2 2;0.80), (0.07,0.11, 0.26,0.42; 1.00))
C 8	((0.13,0.15,0.22,0.26; 0.80), (0.09,0.13,0.29,0.45; 1.00))	((0.15,0.18,0.25,0.29; 0.80), (0.11,0.16,0.32,0.49; 1.00))	((0.16,0.19,0.27,0.32; 0.80), (0.12,0.17,0.36,0.54; 1.00))	((0.13,0.1 5,0.22,0.2 6;0.80), (0.09,0.13 0.29,0.45; 1.00))
	C5	C6	C7	C8
C 1	((0.08,0.09,0.14,0.18; 0.80), (0.05,0.08,0.19,0.32; 1.00))	((0.13,0.16,0.22,0.26; 0.80), (0.09,0.13,0.29,0.45; 1.00))	((0.05,0.06,0.10,0.13; 0.80), (0.03,0.05,0.15,0.27; 1.00))	((0.06,0.0 8,0.13,0.1 6;0.80), (0.05,0.09, 0.27,0.46; 1.00))
C 2	((0.18,0.20,0.28,0.32; 0.80), (0.14,0.18,0.37,0.56; 1.00))	((0.24,0.27,0.36,0.42; 0.80), (0.19,0.25,0.48,0.71; 1.00))	((0.13,0.15,0.23,0.27; 0.80), (0.09,0.14,0.31,0.50; 1.00))	((0.16,0.1 9,0.27,0.3 2;0.80), (0.17,0.24, 0.54,0.83; 1.00))
C 3	((0.11,0.13,0.18,0.21; 0.80), (0.09,0.11,0.23,0.35; 1.00))	((0.11,0.13,0.19,0.23; 0.80), (0.08,0.11,0.26,0.41; 1.00))	((0.06,0.07,0.11,0.14; 0.80), (0.03,0.06,0.16,0.28; 1.00))	((0.07,0.0 9,0.14,0.1 7;0.80), (0.07,0.11, 0.29,0.47; 1.00))
C 4	((0.10,0.12,0.17,0.20; 0.80), (0.07,0.10,0.23,0.36; 1.00))	((0.11,0.14,0.20,0.24; 0.80), (0.08,0.12,0.28,0.44; 1.00))	((0.07,0.09,0.13,0.16; 0.80), (0.05,0.08,0.19,0.32; 1.00))	((0.10,0.1 2,0.18,0.2 1;0.80), (0.11,0.16, 0.36,0.56; 1.00))
C 5	((0.07,0.09,0.15,0.18; 0.80), (0.04,0.07,0.22,0.39; 1.00))	((0.15,0.18,0.26,0.31; 0.80), (0.11,0.16,0.38,0.60; 1.00))	((0.15,0.17,0.24,0.28; 0.80), (0.12,0.16,0.32,0.49; 1.00))	((0.18,0.2 0,0.28,0.3 2;0.80), (0.21,0.27, 0.54,0.81; 1.00))
C 6	((0.12,0.14,0.20,0.23; 0.80), (0.09,0.13,0.26,0.40; 1.00))	((0.05,0.07,0.13,0.16; 0.80), (0.03,0.06,0.20,0.37; 1.00))	((0.07,0.09,0.13,0.17; 0.80), (0.05,0.07,0.19,0.33; 1.00))	((0.10,0.1 2,0.18,0.2 2;0.80), (0.10,0.15 0.36,0.58; 1.00))
C 7	((0.06,0.08,0.13,0.17; 0.80), (0.03,0.06,0.19,0.33; 1.00))	((0.13,0.15,0.22,0.26; 0.80), (0.09,0.13,0.31,0.49; 1.00))	((0.04,0.05,0.09,0.12; 0.80), (0.02,0.04,0.15,0.27; 1.00))	((0.11,0.1 3,0.19,0.2 3;0.80), (0.11,0.17, 0.38,0.61; 1.00))
C 8	((0.07,0.09,0.15,0.19; 0.80), (0.04,0.08,0.21,0.35; 1.00))	((0.13,0.16,0.24,0.28; 0.80), (0.09,0.14,0.32,0.50; 1.00))	((0.10,0.13,0.18,0.22; 0.80), (0.07,0.11,0.24,0.38; 1.00))	((0.05,0.0 7,0.12,0.1 6;0.80), (0.04,0.08 0.26,0.47;

The signed distance  $d(A_{ij}, \tilde{0}_1)$  for each  $A_{ij}$  in the decision matrix has been assigned to calculate the concordance/discordance index with the equations (23)-(27). The signed distance results are seen in Table 10.

**TABLE 8.** Impact degrees of the criteria.

Criteria	$\left(\widetilde{D}_i + \widetilde{R}_i\right)$
C1	((1.76,2.09,3.00,3.58;0.80), (1.31,1.86,4.18,6.64;1.00))
C2	((2.55,2.97,4.11,4.81;0.80), (2.06,2.76,5.76,8.83;1.00))
СЗ	((1.79,2.15,3.13,3.74;0.80), (1.32,1.92,4.44,7.06;1.00))
C4	((1.59,2.93,2.87,3.45;0.80), (1.16,1.74,4.15,6.68;1.00))
C5	((2.02,2.39,3.45,4.08;0.80), (1.57,2.21,4.93,7.72;1.00))
C6	((1.83,2.21,3.24,3.87;0.80), (1.35,1.98,4.63,7.36;1.00))
C7	((1.49,1.81,2.70,3.26;0.80), (1.08,1.63,3.93,6.37;1.00))
C8	((1.77,2.14,3.16,3.77;0.80), (1.53,2.26,5.30,8.43;1.00))

**TABLE 9.** Averaged IT2 fuzzy decision matrix.

	A1	A2	A3
С	((0.97,0.99,1.00,1.00;0.80)	((0.74,0.77,0.84,0.87;0.80	((0.61,0.65,0.73,0.76;0.80)
1	(0.95,0.99,1.00, 1.00;1.00))	(0.67,0.73,0.88,0.93;1.00)	(0.54,0.61,0.77,0.83;1.00))
С	((0.57,0.60,0.68,0.72;0.80)	((0.78,0.82,0.89,0.91;0.80	((0.97,0.99,1.00,1.00;0.80)
2	(0.49,0.56,0.73,0.79;1.00))	(0.72,0.78,0.92,0.97;1.00)	(0.95,0.99,1.00,1.00;1.00))
С	((0.49,0.53,0.61,0.64;0.80)	((0.79,0.82,0.88,0.90;0.80	((0.97,0.99,1.00,1.00;0.80)
3	(0.41,0.48,0.65,0.72;1.00))	(0.74,0.80,0.91,0.94;1.00)	(0.95,0.99,1.00,1.00;1.00))
С	((0.57,0.60,0.68,0.72;0.80)	((0.69,0.72,0.80,0.83;0.80	((0.95,0.99,0.99,0.99;
4	(0.49,0.56,0.73,0.79;1.00))	(0.63,0.68,0.84,0.90;1.00)	0.80), (0.93,0.98,1.00,1.00;1.00))
С	((0.49,0.53,0.61,0.64;	((0.57,0.60,0.68,0.72;0.80	((0.97,0.99,1.00,1.00;0.80)
5	0.80), (0.41,0.48,0.65,0.72;1.00))	(0.49,0.56,0.73,0.79;1.00)	(0.95,0.99,1.00,1.00;1.00))
С	((0.61,0.65,0.73,0.76;	((0.74,0.77,0.84,0.87;0.80	((0.89,0.93,0.96,0.96;0.80)
6	0.80), (0.54,0.61,0.77,0.83;1.00))	(0.67,0.73,0.88,0.93;1.00)	(0.86,0.91,0.97,0.99;1.00))
С	((0.95,0.99,0.99,0.99;0.80)	((0.84,0.87,0.92,0.94;0.80	((0.69,0.72,0.80,0.83;0.80)
7	(0.93,0.98,1.00,1.00;1.00))	(0.79,0.85,0.95,0.98;1.00)	(0.63,0.68,0.84,0.90;1.00))
С	((0.95,0.99,0.99,0.99;0.80)	((0.84,0.87,0.92,0.94;0.80	((0.69,0.72,0.80,0.83;0.80)
8	(0.93,0.98,1.00,1.00;1.00))	(0.79,0.85,0.95,0.98;1.00)	(0.63,0.68,0.84,0.90;1.00))

**TABLE 10.** Signed distance  $d(A_{ij}, \tilde{0}_1)$ .

	A1	A2	A3
C1	1.976	1.609	1.371
C2	1.283	1.697	1.976
C3	1.133	1.698	1.976
C4	1.283	1.521	1.965
C5	1.133	1.283	1.976
C6	1.371	1.609	1.875
C7	1.965	1.786	1.521
C8	1.965	1.786	1.521

6 permutations of the ranking for the alternatives have been provided as  $P_1 = (A_1, A_2, A_3)$ ,  $P_2 = (A_1, A_3, A_2)$ ,  $P_3 = (A_2, A_1, A_3)$ ,  $P_4 = (A_2, A_3, A_1)$ ,  $P_5 = (A_3, A_1, A_2)$ ,



**TABLE 11.** Concordance/discordance index  $I_i^I(A_\rho, A_\beta)$ .

$P_1$	$I_j^1(A_1, A$	$l_j^1(A_1, A$	$I_j^1(A_2, A$	$P_2$	$I_j^2(A_1, A$	$l_j^2(A_1, A$	$I_j^2(A_3,A$	$P_3$	$I_j^3(A_2,A$	$I_j^3(A_2,A$	$I_j^3(A_1, A$
C 1	0.368	0.605	0.238	C 1	0.605	0.368	0.238	C 1	0.368	0.238	0.605
C 2	0.413	0.693	0.280	C 2	0.693	0.413	0.280	C 2	0.413	0.280	0.693
C 3	0.565	0.843	0.278	C 3	0.843	0.565	0.278	C 3	0.565	0.278	0.843
C 4	0.238	0.681	- 0.444	C 4	0.681	0.238	0.444	C 4	0.238	- 0.444	0.681
C 5	0.150	0.843	0.693	C 5	0.843	0.150	0.693	C 5	0.150	0.693	0.843
C 6	0.238	0.504	0.266	C 6	0.504	0.238	0.266	C 6	0.238	0.266	0.504
C 7	0.179	0.444	0.265	C 7	0.444	0.179	0.265	C 7	0.179	0.265	0.444
C 8	0.179	0.444	0.265	C 8	0.444	0.179	0.265	C 8	- 0.179	0.265	0.444
$P_4$	$I_j^4(A_2,A$	$I_j^4(A_2,A$	$I_j^4(A_3,A$	$P_5$	$I_j^5(A_3,A$	$I_j^5(A_3,A$	$l_j^5(A_1, A$	$P_6$	$I_j^6(A_3,A$	$I_j^6(A_3,A$	$I_j^6(A_2,A$
C 1	0.238	0.368	0.605	C 1	0.605	0.238	0.368	C 1	0.238	0.605	0.368
C 2	0.280	0.413	0.693	C 2	0.693	0.280	0.413	C 2	0.280	0.693	0.413
C 3	0.278	0.565	0.843	C 3	0.843	0.278	0.565	C 3	0.278	0.843	0.565
C 4	0.444	0.238	0.681	C 4	0.681	0.444	0.238	C 4	0.444	0.681	0.238
C 5	0.693	0.150	0.843	C 5	0.843	0.693	0.150	C 5	0.693	0.843	0.150
C 6	0.266	0.238	0.504	C 6	0.504	0.266	0.238	C 6	0.266	0.504	0.238
C 7	0.265	0.179	- 0.444	C 7	- 0.444	0.265	0.179	C 7	0.265	- 0.444	0.179
C 8	0.265	0.179	- 0.444	C 8	0.444	0.265	0.179	C 8	0.265	- 0.444	0.179

**TABLE 12.** Weighted concordance/discordance index for  $P_1$ .

$P_1$	$I_j^1(A_1, A_2). W_j$	$I_j^1(A_1, A_3). W_j$	$I_j^1(A_2, A_3). W_j$
С	((0.65,0.77,1.10,1.32;0.8 0),	((1.06,1.26,1.82,2.17;0.8 0),	
1	(0.48,0.68,1.54,2,44;1.00	(0.79,1.13,2.53,4.02;1.00	((0.42,0.50,0.71,0.85;0.80), (0.31,0.44,0.99,1.58;1.00))
C 2	((-1.99,-1.70,-1.23,- 1.06;0.80), (-3.65,-2.38,-1.14,-	(-3.33,-2.85,-2.06,- 1.77;0.80), (-6.12,-3.99,-	((-1.34,-1.15,-0.83,- 0.71;0.80), (-2.47,-1.61,-
	0.85;1.00)) ((-2.11,-1.77,-1.21,-	1.92,-1.43;1.00)) ((-3.15,-2.64,-1.81,-	0.77, -0.58;1.00)) ((-1.04,-0.87,-0.60,-
C 3	1.01;0.80), (-3.99,-2.51,- 1.09,-0.75;1.00))	1.51;0.80), (-5.95,-3.74,- 1.62,-1.12))	0.50;0.80), (-1.96,-1.24,- 0.53,-0.37;1.00))
C 4	((-0.82,-0.68,-0.46,- 0.38;0.80), (-1.59,-0.99,- 0.41,-0.28;1.00))	((-2.35,-1.95,-1.32,- 1.08;0.80), (-4.55,-2.83,- 1.18,-0.79;1.00))	((-1.53,-1.27,-0.86,- 0.71;0.80), (-2.96,-1.84,- 0.77, -0.52;1.00))
C 5	((-0.61,-0.52,-0.36,- 0.30;0.80), (-1.16,-0.74,- 0.33,-0.24;1.00)	((-3.44,-2.91,-2.02,- 1.71;0.80), (-6.50,-4.16,- 1.86,-1.33;1.00))	((-2.83,-2.39,-1.40,- 1.66;0.80), (-5.35,-3.42,- 1.09,-1.53;1.00))
C 6	((-0.92,-0.77,-0.53,- 0.44;0.80), (-1.75,-1.10,- 0.47,-0.32;1.00))	((-1.95,-1.64,-1.11,- 0.92;0.80), (-3.71,-2.34,- 1.00,-0.68;1.00))	((-1.03,-0.86,-0.59,- 0.49;0.80), (-1.96,-1.23,- 0.53,-0.36;1.00))
C 7	((0.27,0.32,0.48,0.58;0.8 0), (0.19,0.29,0.70,1.14;1.00	((0.66,0.80,1.20, 1.45;0.80), (0.48,0.72,1.74,2.83;1.00	((0.39,0.48,0.71 ,0.87;0.80), (0.29,0.43,1.04,1.69;1.00))
С	((0.32,0.38,0.56,0.67;0.8 0),	((0.79,0.95,1.40,1.67;0.8 0),	, , , , , , , , , , , , , , , , , , , ,
8	(0.27,0.40,0.95,1.51;1.00	(0.68,1.00,2.35,3.74;1.00	((0.47,0.57,0.84,1.00;0.80), (0.41,0.60,1.40,2.23;1.00))

 $P_6 = (A_3, A_2, A_1)$ . And, the index results for each pair of alternatives in the permutation with respect to each criterion have been defined in Table 11.

IT2 fuzzy numbers defining the weights of criteria have been multiplied with the index results and the weighted index results have been provided. Table 12 gives an example for  $P_1$ .

TABLE 13. Comprehensive concordance/discordance index.

	71(4 4 )	71(4 4 )	71(4 4)	r1	1(11
$P_1$	$I_j^1(A_1, A_2)$	$I_j^1(A_1, A_3)$	$I_j^1(A_2, A_3)$	$I^1$	$d(I^1)$
	((-5.22,-3.96,-1.64,- 0.61; 0.80), (- 11.18,-6.34,- 0.26,2.65;1.00))	((-11.72,-8.96,- 3.90,-1.71;0.80), (- 24.89,-14.20,- 0.95,5.24;1.00))	((-6.50,-5.00,-2.01,- 1.35;0.80), (- 13.70,-7.86,- 0.26,2.15;1.00))	((- 23.44,- 17.93,- 7.55,- 3.68;0.8 0), (- 49.78,- 28.40,- 1.46;10. 05;1.00))	30 .9 4
$P_2$	$I_j^2(A_1,A_3)$	$I_j^2(A_1, A_2)$	$I_j^2(A_3,A_2)$	$I^2$	$d(I^2)$
	((-11.72,-8.96,- 3.90,-1.71;0.80), (- 24.89,-14.20,- 0.95,5.24;1.00))	((-5.22,-3.96,-1.64,- 0.61; 0.80), (- 11.18,-6.34,- 0.26,2.65;1.00))	((1.10,2.27,5.00,6.5 0;0.80), (- 2.59,0.70,7.86,13.7 0;1.00))	((- 15.85,- 10.66,- 0.54,4.1 7;0.80), (-38.67,- 19.84,6. 66,21.60 ;1.00))	13 .5 2
$P_3$	$I_j^3(A_2,A_1)$	$I_j^3(A_2,A_3)$	$I_j^3(A_1,A_3)$	$I^3$	$d(I^3)$
	((0.61,1.64,3.96,5.2 2:0.80), (- 2.65,0.26,6.34,11.1 8;1.00))	((-6.50,-5.00,-2.01,- 1.35;0.80), (- 13.70,-7.86,- 0.26,2.15;1.00))	((-11.72,-8.96,- 3.90,-1.71;0.80), (- 24.89,-14.20,- 0.95,5.24;1.00))	((- 17.60,- 12.33,- 1.95,2.1 6;0.80), (-41.25,- 21.81,5. 13,18.58 ;1.00))	17 .4 2
$P_4$	$I_j^4(A_2, A_3)$	$I_j^4(A_2,A_1)$	$I_j^4(A_3, A_1)$	$I^4$	$d(I^4)$
	((-6.50,-5.00,-2.01,- 1.35;0.80), (- 13.70,-7.86,- 0.26,2.15;1.00))	((0.61,1.64,3.96,5.2 2:0.80), (- 2.65,0.26,6.34,11.1 8; 1.00))	((1.71,3.90,8.96,11.72;0.80), (-5.24,0.95,14.20,24.89;1.00))	((- 4.17,0.5 4,10.92, 15.59;0. 80), (- 21.60,- 6.66,20. 28,38.23 ;1.00))	13 .6 7
P <sub>5</sub>	$I_j^5(A_3, A_1)$	$I_j^5(A_3, A_2)$	$I_j^5(A_1, A_2)$	$I^5$	$d(I^5)$
	((1.71,3.90,8.96,11. 72;0.80), (-	((1.10,2.27,5.00,6.5 0;0.80), (-	((-5.22,-3.96,-1.64,- 0.61;0.80), (-9.44,-	((- 2.42,2.2 1,12.33,	17 .7 3

Table 13 provides the comprehensive concordance/ discordance index results once the weighted index scores are summed according to the equation (28).

Maximum value in the comprehensive concordance/ discordance index indicates the optimal ranking order in a set of permutation for the alternatives. For that,  $P_6$  has the best index value and optimal ranking order is determined as A3 (Aggressive/Risk taker), A2 (Moderate/Risk neutral), and A1 (Conservative/Risk averse) respectively. This situation gives information that aggressive investor plays more important role for emerging economies with respect to these 8 criteria.

Aggressive investors refer to the investors who love to take risk in their investments. Because of this situation, it can be said that while generating financial products, this condition should be taken into the consideration. In other words, companies should develop financial products which offers high gains and for the investors. Therefore, it can be possible to attract the attention of the aggressive investors. This situation can provide a chance to improve the financial markets of emerging economies. Similar to this conclusion, [72] and [73] underlined the importance of generating financial products for risk taker investors.



In addition to this situation, moderate/risk neutral investors (A2) have the second highest importance. On the other side, conservative/risk averse investors (A1) is the last rank. These results provide a chance to emerging economies to develop their financial systems. While considering these aspects, financial products should be adopted so that the efficiency of the financial systems can be increased. Because emerging economies seek the opportunity to improve their economies, these results can serve this purpose.

#### V. DISCUSSION AND CONCLUSIONS

Emerging economies aim to improve their financial systems to get the opportunity to improve their economies. The main reason behind this condition is that with an effective financial system, it can be possible to increase investment amount so that sustainable economic growth can be achieved. However, financial instruments should be generated according to the expectations of the investors. Because of this issue, identifying the risk profile of the investors in these countries play a very key role.

The main purpose of this study is to identify the risk profiles of the investors in emerging economies. For this purpose, eight different criteria and 3 investment profiles are defined. Within this context, interval type-2 fuzzy DEMATEL and fuzzy QUALIFLEX approaches are considered. In addition to this situation, interval type-2 fuzzy DEMATEL methodology is used to weight the criteria. On the other side, fuzzy QUALIFLEX approach is taken into the consideration to rank these risk profiles.

It is concluded that aggressive/risk taker investors have the highest importance for emerging economies by considering these eight criteria. On the other hand, moderate/risk neutral investor has the second highest significance whereas conservative/risk averse investors are on the last rank. While considering these results, it is recommended that financial products, which offers high returns, should be generated for the risk taker investors because they love taking risks in spite of the high risks. With the help of this situation, it can be possible to improve the financial systems so that economic growth can be provided for emerging economies.

It is thought that this study makes a contribution to the literature by evaluating a significant topic for emerging economies. In addition to this issue, using interval type-2 fuzzy QUALIFLEX approach firstly increases the originality of this study. Nevertheless, a new study can also be made by focusing on developed economies. It is believed that this study is also very beneficial for the literature.

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