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Minimum Initial Marking Estimation in Labeled Petri Nets With Unobservable Transitions

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ABSTRACT In the literature, researchers have been studying the minimum initial marking (MIM) estimation problem in the labeled Petri nets with observable transitions. This paper extends the results to labeled Petri nets with unobservable transitions (with certain special structure) and proposes algorithms for the MIM estimation (MIM-UT). In particular, we assume that the Petri net structure is given and the unobservable transitions in the net are contact-free. Based on the observation of a sequence of labels, our objective is to find the set of MIM(s) that is(are) able to produce this sequence and has(have) the smallest total number of tokens. An algorithm is developed to find the set of MIM(s) with polynomial complexity in the length of the observed label sequence. Two heuristic algorithms are also proposed to reduce the computational complexity. An illustrative example is also provided to demonstrate the proposed algorithms and compare their performance.

INDEX TERMS Labeled Petri nets, initial marking estimation, unobservable transitions, algorithmic complexity.

I. INTRODUCTION

Petri nets (*PNs*) is a useful tool for the modeling and analysis of complex systems [1], [2]. In literature, Petri nets have been used quite widely in a variety of practical systems including microgrid systems, traffic systems, among others [3]–[7]. For the studies of Petri nets, the marking/state estimation is a fundamental problem, which aims at estimating the state of the system model given a PN structure along with some observations. Qin *et al.* [8] proposed an application of Petri net deadlock prevention involving unobservable transitions. Bonhomme [9] studied the problem of marking estimation in an unlabeled P-time Petri net with unobservable transitions. The proposed approach was based on state observer under partial observation. The work in [10] also used the observer techniques to estimate both the markings and the firing vectors. Corona *et al.* [11], [12] presented a technique of estimating Petri net markings based on an observation of transition labels, which does not depend on the observation length under some assumptions. Cabasino *et al.* [13] proposed an approach for marking estimation in a probabilistic setting. Initial marking or a finite set of initial markings were assumed to be known with a priori probabilities, then conditional probabilities of marking estimates given the label observations are obtained.

The aforementioned works contributed significantly in the research area of marking/state estimation in Petri nets. However, they assume that the initial marking of the system is known or partially known. In the MIM problem we study in this paper, we have a different objective, i.e., we aim at estimating the initial marking(s) based on the observation of a sequence of labels that is produced by the system's underlying transition activities. Moreover, the system has some transitions that are unobservable (e.g., their firings can occur without being observed). This is inspired by the initial resource allocation problem in manufacturing systems, where the minimum number of resources need to be planned/scheduled in order to complete a set of pre-assigned processes/tasks [14].

Several researchers have been studying the topic related to the MIM estimation problem. For instance, Yamauchi and Watanabe [15] proposed a heuristic method for MIM estimation in Petri nets. In this study, the structure and firings

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of transitions were assumed to be fully known, but with limited firing times of each transition. The objective is to find the unknown initial marking that has the least total number of tokens. Li and Hadjicostis [16] studied the MIM estimation problem in labeled Petri nets with observable transitions and proposed algorithms to obtain the MIM estimates.

In this paper, we aim at extending the results in [16] to labeled Petri nets with unobservable transitions, that is, minimum initial marking estimation in labeled Petri nets with unobservable transitions (*MIM-UT*). The existence of unobservable transitions makes this problem a challenging task. We show that, under some assumptions on the structure of unobservable subnet, we are able to develop an algorithm that can find the set of MIM-UT estimates with complexity that is polynomial in the length of the label observations. In addition, two heuristic algorithms are proposed to reduce the computational complexity. An example is also provided to illustrate the proposed algorithms and compare their performance.

II. PETRI NET PRELIMINARIES

In this section, we talk about the notation and terminology that are used in this paper briefly. More specific information about Petri nets is provided in [1] and [2].

A Petri net $N_P = (Places, Transitions, Arcs, Weights)$ is a graph with directed arcs connecting from places to transitions (and transitions to places), where $Places = \{p_1, p_2, \ldots, p_n\}$ represents the set of *places* (shown as empty circles) with a finite size n, $Transitions = \{t_1, t_2, \ldots, t_m\}$ represents the set of *transitions* (shown as empty or solid bars) with a finite size m, $Arcs$ is a set of arcs that connecting $Places$ and $Transitions$, and $Weights : Arcs \rightarrow \{1, 2, 3, \ldots\}$ represents the *weighting function* of the arcs.

Notation I is used to represent the incident matrix of the Petri net and I is defined as $I = I^{out} - I^{in}$, where I^{out} is called the output incident matrix of the Petri net and I^{in} is called the input incident matrix of the Petri net. Each entry I^{out}_{ij}, which is at the i^{th} row, j^{th} column position of matrix I^{out}, denotes the weight of the arc from transition t_j to place p_i; Similarly, each element I^{in}_{ij} in matrix I^{in} denotes the weight of the arc that starts at place p_i and ends up at transition t_j, where $i \in [1, n]$ and $j \in [1, m]$. Note that all the weights on the arcs should be non-negative integer numbers and the value of I^{out}_{ij} (or I^{in}_{ij}) is set to be zero if there is no arc connecting place p_i and transition t_j. Hence, we can also use $N_p = (Places, Transitions, I^{out}, I^{in})$ to represent a Petri net. Then we introduce $^{\rightarrow}p$ ($^{\rightarrow}t$) as the input transitions (places) set of place p (transition t) and $p^{\rightarrow}$ ($t^{\rightarrow}$) as the output transitions (places) set of place p (transition t). Furthermore, notation $^{\rightarrow}p^{\rightarrow} = {}^{\rightarrow}p \bigcup p^{\rightarrow}$ ($^{\rightarrow}t^{\rightarrow} = {}^{\rightarrow}t \bigcup t^{\rightarrow}$) represents the input and output transitions (places) set of place p (transition t).

$M : Places \rightarrow Z_0^+$ is a vector used to denote the marking, which assigns a non-negative integer number of tokens into each place of the Petri net. The tokens are represented as black dots. The total number of tokens in all places is denoted as $|M|$ (i.e., $|M| = \sum_{i=1}^{n} M(p_i)$). The *enabling* condition of a transition t is that when each of its input place p_{in} has no less than $I^{in}(p_{in}, t)$ tokens, where $I^{in}(p_{in}, t)$ represents the arc weight from place p_{in} to transition t. $M[t\rangle$ is used to represent that transition t is enabled at marking M. The enabled transition t can fire. When transition t fires, $I^{in}(p, t)$ tokens should be removed from all of its input places, while all of its output places should be deposited $I^{out}(p, t)$ tokens. These will yield to a new marking, denoted as M' that satisfies $M' = M + I(:, t)$, where $I(:, t)$ denotes the column in the incident matrix I of the Petri net that corresponds to transition t.

A labeled Petri net structure $N_{PL} = (N_P, Labels, \Sigma)$ is a Petri net with each of its transition assigned with a label, which is an element in Σ (or empty label ϵ for an unobservable transition) and $Labels : Transitions \rightarrow \Sigma \cup \{\epsilon\}$ represents the assigning function. Note that the same label could be assigned to two or more different transitions. T_l is used to denote the transition set with label $l \in \Sigma$ while $|T_l|$ represents the number of transitions associated with label l.

In this paper, we consider the case when unobservable transitions exist in the labeled Petri net. Hence, we divide the transition set $Transition$ into two separated sets T_o and T_u, such that $Transition = T_o \cup T_u$ and $T_o \cap T_u = \emptyset$, where T_o is the set of observable transitions (represented with empty bars) and T_u is the set of unobservable transitions (represented with solid bars).

Definition 1: Given a labeled Petri net with unobservable transition subset $T_u \subseteq Transitions$. The *unobservable subnet* of the given labeled Petri net is defined as the net $N_{Pu} = (Places, T_u, I^{in}_u, I^{out}_u)$, where I^{in}_u and I^{out}_u consist of the columns that correspond to unobservable transitions in the input and output incident matrices I^{in} and I^{out}.

In this paper, we assume all unobservable transitions in the net are *contact-free*.

Definition 2 [17]: Contact-free for two transitions t_i and t_j is defined as follows: $^{\rightarrow}t_i^{\rightarrow} \bigcap {}^{\rightarrow}t_j^{\rightarrow} = \emptyset$, $^{\rightarrow}t_i \bigcap t_i^{\rightarrow} = \emptyset$, and $^{\rightarrow}t_j \cap t_j^{\rightarrow} = \emptyset$, i.e., the two transitions don't have common input or output places and don't have self-loops.

An example Petri net with unobservable subset can be found in Figure 1.(a) while Figure 1.(b) shows its unobservable subnet. Note that the two unobservable transitions t_5 and t_6 are contact-free.

III. PROBLEM FORMULATION

A. PETRI NETS WITH OBSERVABLE TRANSITIONS

We first consider the case where we are given a labeled Petri net with observable transitions only. Given a Petri net structure with labeling function $Labels$, $N_{PL} = (Places, Transitions, Arcs, Weights, Labels, \Sigma)$ and an label observation sequence $\omega = l_1 l_2 \ldots l_k$ (where $l_j \in \Sigma, j \in \{1, 2, \ldots, k\}$) that has been generated by underlying transition activities, Li and Hadjicostis [16] proposed a method that could find the MIM estimate set with polynomial computational complexity with respect to the length of ω. Here we

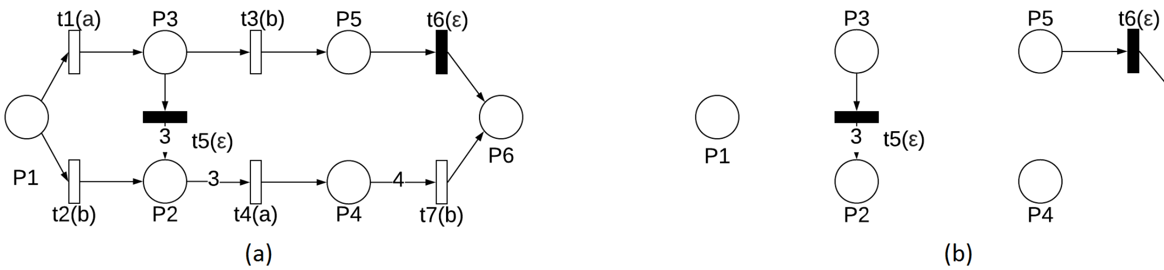

FIGURE 1. A labeled Petri net with unobservable subnet (a), and its unobservable subnet (b).

present some necessary definitions proposed in [16], which are used later to deal with nets with unobservable transitions.

Definition 3 [16]: Given a label observation sequence ω, three quantities are defined with respect to ω: 1) The *initial marking estimates* set $E(\omega)$, 2) The *minimal initial marking estimates* set $E_{minimal}(\omega)$, and 3) The *minimum initial marking estimates* $E_{minimum}(\omega)$, where:

$$E(\omega) = \{M \in (Z_0^+)^n \mid \exists\sigma \in Transitions^* : M[\sigma\rangle \text{ and } Labels(\sigma) = \omega\},$$

$$E_{minimal}(\omega) = \{M \in E(\omega) \mid \nexists M' \in E(\omega) : M' \leq M \text{ and } M' \neq M\},$$

$$E_{minimum}(\omega) = \{M \in E_{minimal}(\omega) \mid |M| \leq |M'| \text{ for all } M' \in E_{minimal}(\omega)\}.$$

In [16], the minimal initial marking is calculated using Equation (1) below.

$$M_0^{j+1} = \max\{M_0^j + I \cdot y_{j-1},\ I^{in}(:, t_{i_j})\} - I \cdot y_{j-1}, \quad (1)$$

where $j = 1, 2, \ldots, k$, y_{j-1} represents the firing vector at the $(j-1)$-th stage of the transition firing sequence $t_{i_1}t_{i_2}\ldots t_{i_{j-1}}$. M_0^1 is an n-dimensional vector with all entries equal to zero (i.e., $\vec{\mathbf{0}}_n$), and y_0 is an m-dimensional vector with all entries equal to zero (i.e., $\vec{\mathbf{0}}_m$). Note that M_0^j (M_0^{j+1}) represents the initial marking estimate before (after) transition t_{i_j} fires.

The *trellis diagram* was then introduced in [16] to calculate the set of minimal initial marking estimates. After all labels are considered, the minimum initial marking set can be found by choosing from these markings, the ones where the sum of the total number of tokens in all places is the minimum.

B. PETRI NETS WITH UNOBSERVABLE SUBNET

Note that in the case of Petri nets with unobservable subnet, the amount of firing sequences that generated from one label observation sequence can grow exponentially, and in the worst case, infinitely. To make our calculations simpler, in this paper we have the following assumptions.

($A1$)The unobservable subnet is contact-free.

($A2$) For each label observation, one and only one unobservable transition can fire before the observable transition that corresponds to the label.

Based on assumption ($A2$), after label l_j is observed, the potential firing sequences we need to consider are in the form of t or t_ut, where $t \in T_o$ and $Labels(t) = l_j$, and $t_u \in T_u$. Equation (1) can be applied to these sequences after each label observation to obtain the corresponding firing vectors y as well as the associated MIM estimates.

Example 1: Consider the labeled Petri net with unobservable subnet shown in Figure 1.(a). Our objective is to obtain the MIM estimates after label sequence $\omega = \{aa\}$ is observed.

TABLE 1. List of minimal initial marking estimates calculated in example 1.

Labels	Sequences	Minimal marking estimates	Firing vectors
aa	t_1t_1	$[2\,0\,0\,0\,0\,0]^T$	$[2\,0\,0\,0\,0\,0\,0]^T$
	t_1t_4	$[1\,0\,3\,0\,0\,0]^T$	$[1\,0\,0\,1\,0\,0\,0]^T$
	t_4t_4	$[0\,0\,6\,0\,0\,0]^T$	$[0\,0\,0\,2\,0\,0\,0]^T$
	$t_6t_1t_1$	$[2\,0\,0\,0\,1\,0]^T$	$[2\,0\,0\,0\,0\,1\,0]^T$
	$t_1t_5t_1$	$[2\,0\,0\,0\,0\,0]^T$	$[2\,0\,0\,0\,1\,0\,0]^T$
	$t_1t_6t_1$	$[2\,0\,0\,0\,1\,0]^T$	$[2\,0\,0\,0\,0\,1\,0]^T$
	$t_6t_1t_4$	$[1\,3\,0\,0\,1\,0]^T$	$[1\,0\,0\,1\,0\,1\,0]^T$
	$t_1t_5t_4$	$[1\,0\,0\,0\,0\,0]^T$	$[1\,0\,0\,1\,1\,0\,0]^T$
	$t_5t_4t_4$	$[0\,3\,1\,0\,0\,0]^T$	$[0\,0\,0\,2\,1\,0\,0]^T$
	$t_6t_4t_4$	$[0\,6\,0\,0\,1\,0]^T$	$[0\,0\,0\,2\,0\,1\,0]^T$
	$t_5t_1t_5t_4$	$[1\,0\,1\,0\,0\,0]^T$	$[1\,0\,0\,1\,2\,0\,0]^T$
	$t_6t_1t_5t_4$	$[1\,0\,0\,0\,1\,0]^T$	$[1\,0\,0\,1\,1\,1\,0]^T$
	$t_6t_1t_6t_4$	$[1\,3\,0\,0\,2\,0]^T$	$[1\,0\,0\,1\,0\,2\,0]^T$
	$t_5t_1t_5t_1$	$[2\,0\,1\,0\,0\,0]^T$	$[2\,0\,0\,0\,2\,0\,0]^T$
	$t_6t_1t_5t_1$	$[2\,0\,0\,0\,1\,0]^T$	$[2\,0\,0\,0\,1\,1\,0]^T$
	$t_6t_1t_6t_1$	$[2\,0\,0\,0\,2\,0]^T$	$[2\,0\,0\,0\,0\,2\,0]^T$
	$t_5t_4t_5t_4$	$[0\,0\,2\,0\,0\,0]^T$	$[0\,0\,0\,2\,2\,0\,0]^T$
	$t_5t_4t_6t_4$	$[0\,3\,1\,0\,1\,0]^T$	$[0\,0\,0\,2\,1\,1\,0]^T$
	$t_6t_4t_6t_4$	$[0\,6\,0\,0\,2\,0]^T$	$[0\,0\,0\,2\,0\,2\,0]^T$

If we only consider observable transitions, it is not difficult to see that the minimum initial marking estimate after observing label sequence aa is $[2\ 0\ 0\ 0\ 0\ 0]^T$ (by transition firing sequence t_1t_1 with firing vector $y = [2\ 0\ 0\ 0\ 0\ 0\ 0]^T$). When unobservable transitions exist, however, there can be a significantly larger number of transition firing sequences to consider. For instance, for each label observation a, we need to consider all possible firing sequences $\{t_1, t_4, t_5t_1, t_6t_1, t_5t_4, t_6t_4\}$. By applying Equation (1) to these firing sequences, we can find the minimal initial marking estimate set, and from which, we can obtain the minimum initial marking estimate(s). The set of minimal initial marking estimates and their associated firing vectors are shown in Table 1. Note that due to space considerations, Table 1 do not include all transition firing sequences we considered since those marking estimates that are not minimal were removed.

It is not difficult to see that, the MIM estimate after observing label sequence aa is $[1\ 0\ 0\ 0\ 0\ 0]^T$ via transition firing sequence $t_1t_5t_4$. This MIM estimate has a smaller total number of tokens compared to the MIM estimate obtained by considering only observable transitions.

IV. DESCRIPTIONS OF PROPOSED ALGORITHMS

A. MAIN ALGORITHM

In this section, a recursive algorithm has been proposed as the method of generating the minimum initial marking set based on an length k label observation sequence $\omega = l_1\ l_2 \ldots l_k$ in labeled Petri nets with unobservable subnet.

In Algorithm 1, during each label observation, we consider observable transitions that are related to the observed label, as well as the sequences of exactly one unobservable transition followed by the observable transition (based on assumption A2). The firing vectors and their corresponding minimal initial marking estimate sets are calculated recursively. *Flag* is the indicator of the current transition under consideration, i.e., $Flag == TRUE$ if it is an observable transition and $Flag == FALSE$ if it is an unobservable transition.

According to [16], the number of firing vectors at the j-th stage is given by $n_j = j^b$ where b is a structural parameter of the given Petri net. For unobservable transitions, we can fire one and only one unobservable transition when each label is observed. Hence the total number of different unobservable firing vectors is m_u, where m_u is the number of unobservable transitions. Combining the observable and unobservable firing vectors, at the j-th stage, there are $n_j = (m_u j)^b$ different firing vectors. At the $(j-1)$-th stage, each firing vector can generate at most $m_u \times m_o$ different firing vectors, where m_o is the number of observable transitions. So at j-th stage, the number of new generated firing vectors is given by $m_u \times m_o \times n_{j-1} = O(m_o m_u (m_u j)^b)$. The maximum number of the comparisons decide the uniqueness for each of these firing vectors is represented by n_j and each comparison has a complexity of $O(m_u + m_o) = O(m)$. If it is not unique, the maximum number of comparisons that we need to compare each current marking estimate with the existing minimal initial marking estimates is represented by q_j, while the complexity for each comparison is $O(n)$.

Parameter q_j is used to represent the number of minimal initial marking estimates associated with each firing vector at stage j. We are trying to find markings that are not comparable with the same firing vector. For observable cases, we know that the q_j is bounded by $O(j^n)$. For unobservable cases, since each unobservable transition can fire at most once, the firing sequence under consideration is with length $2j$. Thus, q_j in our case is bounded by $O((2j)^n) = O(j^n)$.

From analysis above, it is not difficult to see the total computational complexity of Algorithm 1 is $\sum_{j=1}^{k}[O(n_{j-1} \times m_o \times m_u \times (m \times n_j + q_{j-1} \times q_j \times n))] = \sum_{j=1}^{k}[O(m_o m_u (m_u j)^b \times [m m_u (m_u j)^b + j^n \times n \times j^n])]$, which can be simplified as $O(m_o m m_u^{2b+2} k^{2b+1} + m_o n m_u^{b+1} k^{2n+b+1}) = O(k^{2b+1} + k^{2n+b+1})$. Clearly, Algorithm 1 has complexity that is polynomial in k, which is the length of the observed label sequence. However, Algorithm 1 has exponential complexity on some structural parameters of the Petri net (such as n and b).

Algorithm 1 MIM Estimation in Labeled Petri Nets With Contact-Free Unobservable Transitions

Require: A labeled Petri net with contact-free unobservable transitions and an label observation sequence $\omega = l_1\ l_2 \ldots l_k$ of length k.
Ensure: Minimum initial marking estimate(s).

$M_0^1 = \vec{0}_n$, $y_0 = \vec{0}_m$, $Flag == TRUE$.
$nodes = (M_0^1, y_0, Flag)$
for $i = 1$ to k **do**
  Search nodes to find all nodes with $t_i \in T_{l_i}$.
  for each *node* in $nodes(:, i)$ with $t_i \in T_{l_i}$ **do**
    for each transition $t \in T_{l_i} \cup T_u$ **do**
      Update M_0^{i+1} using Equation (1) from information M_0^i and y_{j-1} stored in *node*.
      Update firing vector y_i using y_{i-1} and the current transition t.
      if y_i has already appeared **then**
        Compare M_0^{i+1} with its minimal initial marking estimates $M_{minimal}$
        if M_{i+1}^0 is not comparable with $M_{minimal}$ **then**
          Store *node* in $nodes(:, i)$
        else if $M_0^{i+1} \leq M_{minimal}$ **then**
          Store *node* in $nodes(:, i)$ and delete the original *node* in $M_{minimal}$
        end if
      end if
      if $t \in T_u$ **then**
        Set $Flag == FALSE$
      else
        Set $Flag == TRUE$
      end if
      if $Flag == FALSE$ **then**
        for each $t \in T_{l_i}$ **do**
          Goto 7
        end for
      end if
    end for
  end for
end for
Output the minimum initial marking estimate set

B. HEURISTIC METHODS

In some cases, we can further reduce the complexity of solving the MIM estimation problem when high accuracy is not required. We propose two heuristic methods in this section that can improve the calculation speed, while only a subset or an approximation of MIM estimates could be found.

The first heuristic algorithm we propose is that, after our calculation of minimal initial marking estimate set in each iteration, only the marking(s) that has (have) the smallest token sum will be kept for further calculations. The second heuristic algorithm is to only consider observable transitions, namely, all unobservable transitions are ignored during our calculations.

We will show the performance of these three algorithms in the following illustrative example.

V. ILLUSTRATIVE EXAMPLE

In this section, we illustrate our algorithm with a manufacturing system example modeled by Petri nets. Consider the labeled Petri net model shown in Figure 2, which has 10 different places $Places = \{p_1, p_2, \ldots, p_{10}\}$ and 12 different transitions $Transitions = \{t_1, t_2, \ldots, t_{12}\}$. The labeling function is shown as: $Labels(t_3) = Labels(t_5) = a$, $Labels(t_6) = b$, $Labels(t_7) = Labels(t_8) = c$, $Labels(t_9) = d$, $Labels(t_1) = e$, $Labels(t_2) = f$, $Labels(t_{11}) = g$, $Labels(t_{12}) = h$ and $Labels(t_4) = Labels(t_{10}) = \epsilon$. Note that two unobservable transitions t_4 and t_{10} are contact-free.

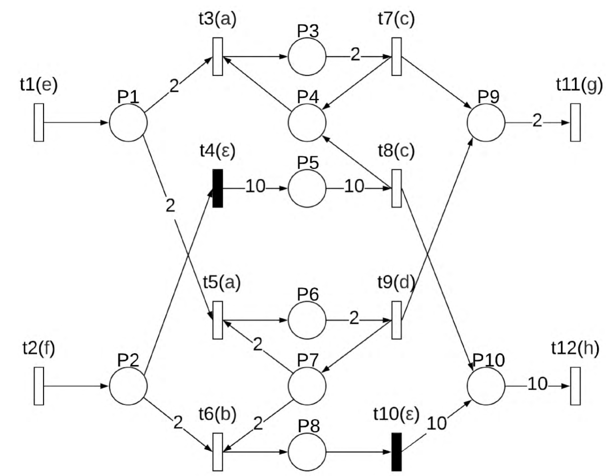

FIGURE 2. Petri net model of the illustrative example.

We use the sequence of labels

$$\omega = efabcdcbgh$$

of length 10 as our observation. We run Algorithm 1 and two heuristic algorithms on this example. Performance are compared with respect to: 1) The number of minimal initial marking estimates calculated from each label observation, and 2) The minimum initial marking estimate(s) obtained after each observation iteration.

Table 2 summarizes the amount of minimal initial marking estimates calculated by three algorithms for each label observation, where Main stands for Algorithm 1, HM1 stands for Heuristic Method 1, and HM1 stands for Heuristic Method 2. It is clear that the amount of minimal initial marking calculated by Algorithm 1 is much larger than HM1 and HM2. Figure 3 shows the amount of minimal initial marking estimates calculated to the length of label observations for Algorithm 1, HM1, and HM2, respectively. To see that the algorithmic complexity is polynomial in the length of the label observations, we also added the function $O(k^4)$ in Figure 4 for comparisons. Clearly, we can see that the number of minimal initial marking considered by three algorithms are bounded by $O(k^4)$, which is a polynomial function in the length of label observations k.

In Table 3, we show the set of minimum initial marking estimate(s) after this label sequence is observed. As expected,

TABLE 2. Number of minimal initial markings calculated by three algorithms.

Label Length	Main	HM1	HM2
1	5	5	1
2	11	5	1
3	39	8	2
4	62	5	2
5	184	8	4
6	310	5	4
7	468	8	7
8	694	5	7
9	1,140	5	7
10	1,318	5	7

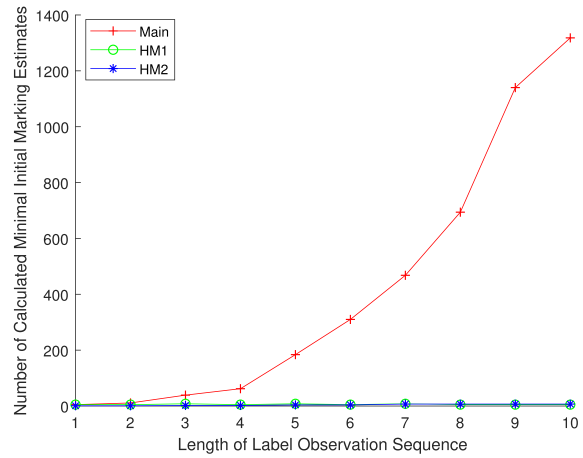

FIGURE 3. Number of minimal initial marking estimates calculated by three algorithms.

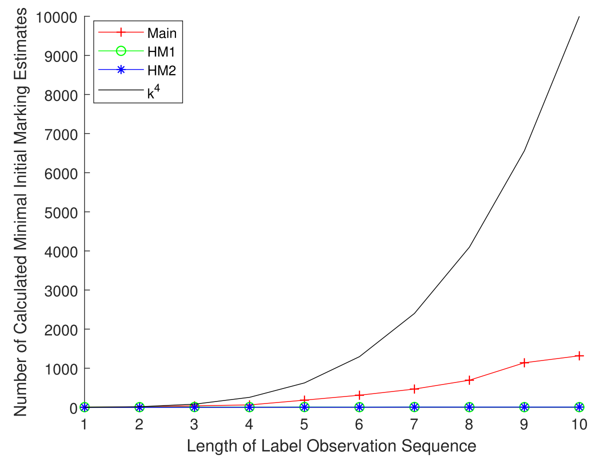

FIGURE 4. Number of minimal initial Marking estimates calculated by three algorithms compared with a polynomial function of k^4.

TABLE 3. Minimum initial marking estimates obtained by three algorithms.

Main	HM1	HM2
$[1\,4\,1\,2\,0\,2\,3\,0\,0\,0]^T$	$[1\,4\,1\,2\,0\,2\,3\,0\,0\,0]^T$	$[1\,3\,4\,0\,0\,1\,5\,0\,0\,10]^T$
$[1\,4\,2\,0\,0\,1\,5\,0\,0\,0]^T$		
$[1\,5\,0\,0\,0\,1\,5\,0\,1\,0]^T$		

we see that although Algorithm 1 considers more minimal initial marking estimates during its calculation, it is able to find a complete set of three minimum initial marking estimate(s) that have a token sum of 13. HM1 and HM2 might consider a less number of minimal initial markings (so the calculation time is faster), while only a subset or an approximation of the final minimum initial marking estimates could be found. For instance, HM1 is able to find a subset of true MIM estimates (one marking that has a token sum of 13) and HM2 can only find an approximation of true MIM estimates (one marking that has a token sum of 24). Thus, Algorithm 1 is able to obtain a complete set of MIM estimates with a higher computational cost. Two heuristic methods are able to find a partial or an approximation of the set of MIM estimates with a lower computational cost.

VI. CONCLUSIONS AND FUTURE WORK

In this paper, we studied the problem of minimum initial marking estimation in labeled Petri nets with contact-free unobservable transitions. An algorithm was developed to find the set of minimum initial markings with the complexity that is polynomial in the length of the observed label sequence. In addition, we proposed two heuristic algorithms that are able to find a partial/approximated set of solutions, but with a lower computational cost.

One of the future research direction is to further reduce the complexity of the proposed algorithms by considering special structures of the unobservable sub-net. It is also interesting to generalize the results into a stochastic or timed setting.

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