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Stability and Existence of Anti-Periodic Solution for FCNNs With Time-Varying Delays and Impulsive Impacts

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ABSTRACT This paper deals with the problem of anti-periodic solution for fuzzy cellular neural networks (FCNNs) with time-varying delays and impulsive impacts. Utilizing Krasnoselski's fixed point theorem and contraction principle, developing some appropriate Lyapunov functional, some adequate conditions are set up for the global exponential stability and existence of anti-periodic solutions of FCNNs with time-varying delays and impulsive impacts. In addition, a model is exhibited to represent results setup.

INDEX TERMS Exponential stability, anti-periodic solution, fuzzy cellular neural networks, time-varying delays, impulsive impacts.

I. INTRODUCTION

Celluar neural networks (CNNs) initially presented by Chua and Yang [1], [2] have pulled in much consideration lately. This is generally on the grounds that they have the extensive variety of promising applications in the fields of associated memory, parallel computing, pattern recognition, signal processing and optimization. CNNs are portrayed by essential circuit units called cells. Every unit forms a few information signals and delivers a yield signal which is gotten by different units associated with it including itself. In the execution of a signal or impact going through neural systems, time delays do exist and influence dynamical behavior of a working neural network. As of late there have been a few results on dynamical properties of delayed neural systems including global exponential stability of equilibrium points, periodic solutions and almost periodic solutions [3]–[8].

Other than delay impacts, it has been seen that numerous transformative procedures, including those identified with neural systems, may display incautious impacts. In these developmental procedures, the arrangements of framework are not consistent but rather present hops which can cause shakiness of dynamical frameworks. Thus, numerous neural systems with motivations have been contemplated broadly, and a lot of writing are engaged on the issue of the stability and existence of an equilibrium point [9]–[12]. The stability and existence of periodic solution of neural networks with impulses are researched extensively by many authors [13]–[16].

In 1996, Yang *et al.* [17], [18] first introduced another compose cellular neural networks display called fuzzy cellular neural networks (FCNNs). FCNNs joined fuzzy task (fuzzy AND and fuzzy OR) with cell neural systems. In any case, it is important that Takagi-Sugeno (T-S) fuzzy neural systems are not quite same as FCNNs. T-S fuzzy neural networks depend on an arrangement of fuzzy guidelines to depict nonlinear framework. As of late analysts have discovered that FCNNs are helpful in image processing, and many fascinating results have been introduced on steadiness of FCNNs. For instance, Li *et al.* [19], applying linear matrix inequality (LMI) approach, contemplated existence, uniqueness and global asymptotic stability of fuzzy cell neural systems with leakage delay under impulsive perturbations. Zhang and Xiang [20] obtained the results of asymptotic stability for fuzzy cellular neural networks with time-varying delays. He and Chu [21] studied the stability of fuzzy cellular neural networks having time-varying delay in leakage term deprived of assuming the boundedness of activation function. Other related works readers can refer to [22]–[24].

However, the existence of anti-periodic solutions plays an important role in portraying the conduct of nonlinear differential conditions. For instance, anti-periodic trigonometric polynomials are vital for the investigation of addition issues, against occasional wavelets and simple voltage transmission is frequently against intermittent process, in this way it is profitable to consider anti-periodic solutions. Meanwhile anti-periodic solution, as a special case of periodic solution, has an important research value in dynamic behavior of the neural networks. In recent years, the problem of anti-periodic solution of CNNs, Hopfield neural nets, shunting inhibitory cellular neural networks and recurrent neural networks has been studied by many scholars (see [25]–[32] and references therein). For example, Shao [25] studied the exponential stability and existence of the anti-periodic solutions of intermittent neural systems with time-varying and continuous distributed delays. In [27], Applying inequality and dependent on Lyapunov functional, the creators examined the existence and global exponential stability of anti-periodic solutions for CNNs with delay and impulsive impacts. In any case, to the best of our insight, there are not very many results on the issues of anti-periodic solutions for FCNNs with time-varying delays and impulsive impacts. Ma *et al.* [32] contemplated the global exponential stability and existence of anti-periodic solutions for a class of fuzzy Cohen-Grossberg neural systems with impulsive impacts on time scales. Zhang *et al.* [34] utilizing fundamental solution matrix of coefficients and Lyapunov function, acquired some adequate conditions on the global exponential stability and existence of anti-periodic solution for fuzzy bi-directional memory neural systems with time delays.

Inspired with the previous, it is reasonable to proceed the examination of the stability and existence of anti-periodic solutions for FCNNs with time-varying delays and impulsive properties. Here, this paper is concerned with the next model

$$\begin{cases} x_i'(t) = -a_i(t)x_i(t) + \sum_{j=1}^{n} d_{ij}(t)f_j(x_j(t)) \\ \qquad + \bigwedge_{j=1}^{n} \alpha_{ij}(t)g_j(x_j(t-\tau_{ij}(t))) \\ \qquad + \bigvee_{j=1}^{n} \beta_{ij}(t)g_j(x_j(t-\tau_{ij}(t))) \\ \qquad + E_i(t)], \quad t \geq 0,\ t \neq t_k,\ k \in N^+, \\ \Delta(x_i(t_k)) = x_i(t_k^+) - x_i(t_k^-) = I_{ik}(t_k, x_i(t_k)), \\ x_i(t) = \varphi_i(t), \quad t \in [-\tau, 0],\ i = 1, 2, \cdots, n. \end{cases} \tag{1}$$

where n is the amount of elements in the net. $x_i(t)$ is the activations of the i-th neuron at the time t. $a_i(t), d_{ij}(t), \alpha_{ij}(t), \beta_{ij}(t), E_i(t), f_j(t), g_j(t), \tau_{ij}(t)$ are continuous functions on R. $a_i(t) > 0$ represents the amplification function. $d_{ij}(t)$ means the synaptic joining weight of the element j on the element i at time t. $\alpha_{ij}(t)$ and $\beta_{ij}(t)$ are units of fuzzy feedback MIN pattern and fuzzy feedback MAX process, correspondingly. $\bigwedge$ and $\bigvee$ represent the fuzzy AND and fuzzy OR task, individually. $E_i(t)$ represents the i-th element of an outside input source presented in exterior of the system to the ith cell. $\tau_{ij}(t)$ is time-varying delay satisfying $0 \leq \tau_{ij}(t) \leq \tau$, τ is a positive constant. $f_j(\cdot)$ and $g_j(\cdot)$ are the activation functions. $\Delta x_i(t_k) = x_i(t_k^+) - x_i(t_k^-), x_i(t_k^+) = \lim_{h\to 0+} x_i(t_k + h)$, $x_i(t_k^-) = \lim_{h\to 0-} x_i(t_k + h), (i = 1, 2, \cdots, n, k = 1, 2, \cdots)$. $\{t_k\}$ is an array of floating point numbers for which $t_1 < t_2 < \cdots$ and $\lim_{k\to+\infty} t_k = +\infty$.

The fundamental motivation behind this paper is to consider the global exponential stability and existence of anti-periodic solutions of (1). The blueprint of this paper is as per the following. In Sect. 2, we present a few definitions and lemmas. In Sect. 3, we set up new adequate conditions for the existence of the anti-periodic solutions of framework (1). In Sect. 4, by building reasonable Lyapunov functional, we obtain adequate conditions for the global exponential stability of anti-periodic solutions of framework (1). An illustrative model is given to demonstrate the results of framework (1) in Sect. 5. Finally, a discussion is given to conclude this paper in Sect. 6.

II. MATHEMATICAL PRELIMINARIES

Let us present the following:

$$a_i^- = \min_{t\in[0,\omega]} |a_i(t)|, \quad a^+ = \max_{1\leq i\leq n} \max_{t\in[0,\omega]} |a_i(t)|,$$
$$\overline{d}_{ij} = \max_{t\in[0,\omega]} |d_{ij}(t)|, \quad \overline{d} = \max_{1\leq i,j\leq n} \max_{t\in[0,\omega]} |d_{ij}(t)|,$$
$$\overline{\alpha}_{ij} = \max_{t\in[0,\omega]} |\alpha_{ij}(t)|, \quad \overline{\alpha} = \max_{1\leq i,j\leq n} \max_{t\in[0,\omega]} |\alpha_{ij}(t)|,$$
$$\overline{\beta}_{ij} = \max_{t\in[0,\omega]} |\beta_{ij}(t)|, \quad \overline{\beta} = \max_{1\leq i,j\leq n} \max_{t\in[0,\omega]} |\beta_{ij}(t)|,$$
$$\overline{E} = \max_{1\leq i\leq n} \max_{t\in[0,\omega]} |E_i(t)|, \quad \zeta_i = e^{\int_0^\omega a_i(\theta)d\theta}.$$

Here, the next assumptions are made

$(A1)$ For $i, j = 1, 2, \cdots, n, k = 1, 2, \cdots$, there is $\omega > 0$ for which $v \in R$

$$a_i(t+\omega) = a_i(t), \quad \tau_{ij}(t+\omega) = \tau_{ij}(t),$$
$$\alpha_{ij}(t+\omega)g_j(-v) = -\alpha_{ij}(t)g_j(v),$$
$$\beta_{ij}(t+\omega)g_j(-v) = -\beta_{ij}(t)g_j(v),$$
$$d_{ij}(t+\omega)f_j(-v) = -d_{ij}(t)f_j(v),$$
$$E_i(t+\omega) = -E_i(t), \quad I_{ik}(t+\omega, v) = -I_{ik}(t, -v).$$

$(A2)$ $f_j(\cdot), g_j(\cdot) \in C(R \times R, R)$, and there are positive real numbers $M_f, M_g, \mu_j, \nu_j (j = 1, 2, \cdots, n)$ such that, for $u, v \in R$,

$$f_j(0) = 0, \quad |f_j(t,u)| \leq M_f, \ |f_j(u) - f_j(v)| \leq \mu_j|u - v|,$$
$$g_j(0) = 0, \quad |g_j(t,u)| \leq M_g, \ |g_j(u) - g_j(v)| \leq \nu_j|u - v|.$$

$(A3)$ For $i, j = 1, 2, \cdots, n, k = 1, 2, \cdots$, there is a positive integer q such that

$$I_{i(k+q)} = I_{ik}, \quad t_{k+q} = t_k + \omega.$$

$(A4)$ For $i, j = 1, 2, \cdots, n, k = 1, 2, \cdots$, there is $c_{ik} > 0$,

$$|I_{ik}(t,u) - I_{ik}(t,v)| \leq c_{ik}|u - v|, \quad \forall t \in [0,\omega], u, v \in R.$$

Remark 2.1: In assumption (A2), the activating functions $f_j, g_j, j = 1, 2, \cdots, n$, are typically expected to be bounded, Lipchtiz continuous and do not have to be differential.

Let $x(t) = (x_1(t), x_2(t), \cdots, x_n(t))^T \in R^n$, where T means the transposition. The intial assumptions based on (1) are determined by:

$$x(t) = \varphi(t), \quad t \in [-\tau, 0],$$

where $\varphi(t) = (\varphi_1(t), \varphi_2(t), \cdots, \varphi_n(t))^T \in R^n, \varphi_i(i = 1, 2, \cdots, n)$ are continuous with norm

$$\|\varphi\| = \sup_{t\in[-\tau,0]} (\sum_{i=1}^{n} |\varphi_i(t)|^2)^{\frac{1}{2}}.$$

Definition 2.1: A solution $x(t)$ of (1) is called an ω anti-periodic solution, if

$$x(t+\omega) = -x(t), \quad t \neq t_k.$$
$$x(t_k+\omega)^+ = -x(t_k^+), \quad k = 1, 2, \cdots,$$

and the minimum positive number ω is ω anti-periodic.

Define $PC(R^n) = \{x(t) = (x_1(t), x_2(t), \cdots, x_n(t))^T : R \to R^n, x|_{(t_k, t_{k+1}]} \in C((t_k, t_{k+1}], R^n), x(t_k^+), x(t_k)$ exist, and $x(t_k^-) = x(t_k), k = 1, 2, \cdots\}$. Set $X = \{x : x \in PC(R^n), x(t+\omega) = -x(t), t \in R\}$. It is easy to see X is a Banach space with norm $\|x\| = \sup_{t\in[-\tau,0]}(\sum_{i=1}^n |x_i(t)|^2)^{\frac{1}{2}}$.

Next, It is similar to [27], we have the following lemma.

Lemma 2.1: Let $x(t) = (x_1(t), x_2(t), \cdots, x_n(t))^T$ be an ω anti-periodic solution of system (1), then

$$\begin{aligned} x_i(t) = \int_t^{t+\omega} H_i(t,s) \Bigg[&\sum_{j=1}^n d_{ij}(s) f_j(x_j(s)) \\ &+ \bigwedge_{j=1}^n \alpha_{ij}(s) g_j(x_j(s-\tau_{ij}(s))) + E_i(s) \\ &+ \bigvee_{j=1}^n \beta_{ij}(s) g_j(x_j(s-\tau_{ij}(s))) \Bigg] ds \\ &+ \sum_{t_k\in[t,t+\omega]} H_i(t,t_k) I_{ik}(t_k, x_i(t_k)), \end{aligned} \tag{2}$$

where, for $i = 1, 2, \cdots, n$,

$$H_i(t,s) = -\frac{e^{\int_t^s a_i(\theta)d\theta}}{e^{\int_0^\omega a_i(\theta)d\theta} + 1}, \quad s \in [t, t+\omega]. \tag{3}$$

Lemma 2.2 [35]: Let Ω be a nonempty and closed convex subset of a Banach space X, suppose that Φ, Ψ are the operators, for which
(i) $\Phi x + \Psi y \in \Omega$, for $x, y \in \Omega$;
(ii) Φ is continuous and compact;
(iii) Ψ is a contraction function.
Then there is $z \in \Omega$ for which $z = \Phi z + \Psi z$.

Lemma 2.3 [27]: Let $p, q, \tau, c_k, k = 1, 2, \cdots$, be constants, $q \geq 0, \tau > 0, c_k > 0$, and assume that $x(t)$ is piece continuous nonnegative function sustaining

$$\begin{cases} D^+x(t) \leq px(t) + q\bar{x}(t), & t \geq t_0, \ t \neq t_k, \\ x(t_k^+) \leq c_k(x(t_k)), & k = 1, 2, \cdots, \\ x(t) = \varphi(t), & t \in [t_0-\tau, t_0]. \end{cases} \tag{4}$$

If there exists γ such that for $k = 1, 2, \cdots$,

$$\ln c_k \leq \gamma(t_k - t_{k-1}). \tag{5}$$

and

$$p + cq + \gamma < 0. \tag{6}$$

Then

$$x(t) \leq c \sup_{t\in[t_0-\tau,t_0]} |\varphi(t)| e^{-\lambda(t-t_0)}, \tag{7}$$

where $\bar{x}(t) = \sup_{s\in[t-\tau,t]} x(s)$,

$$c = \sup_{1\leq k<+\infty} \left\{ e^{\gamma(t_k-t_{k-1})}, \frac{1}{e^{\gamma(t_k-t_{k-1})}} \right\},$$

λ is a sole nonnegative solution of $\lambda + p + cqe^{\lambda\tau} + \gamma = 0$.

Lemma 2.4 [17]: Let u and v be two states of system (1), then

$$\left| \bigwedge_{j=1}^n \alpha_{ij}(t) g_j(u) - \bigwedge_{j=1}^n \alpha_{ij}(t) g_j(v) \right| \leq \sum_{j=1}^n |\alpha_{ij}(t)| |g_j(u) - g_j(v)|,$$

and

$$\left| \bigvee_{j=1}^n \beta_{ij}(t) g_j(u) - \bigvee_{j=1}^n \beta_{ij}(t) g_j(v) \right| \leq \sum_{j=1}^n |\beta_{ij}(t)| |g_j(u) - g_j(v)|.$$

III. EXISTENCE OF ANTI-PERIODIC SOLUTION

Here, we derive some adequate conditions of existence of anti-periodic solution of (1).

Describe the operator

$$\begin{cases} (\Phi x)(t) = ((\Phi_1 x)(t), (\Phi_2 x)(t), \cdots, (\Phi_n x)(t))^T, \\ (\Psi x)(t) = ((\Psi_1 x)(t), (\Psi_2 x)(t), \cdots, (\Psi_n x)(t))^T. \end{cases} \tag{8}$$

where

$$\begin{aligned} (\Phi_i x)(t) = \int_t^{t+\omega} H_i(t,s) \Bigg[&\sum_{j=1}^n d_{ij}(s) f_j(x_j(s)) \\ &+ \bigwedge_{j=1}^n \alpha_{ij}(s) g_j(x_j(s-\tau_{ij}(s))) \\ &+ \bigvee_{j=1}^n \beta_{ij}(s) g_j(x_j(s-\tau_{ij}(s))) \\ &+ E_i(s) \Bigg] ds, \end{aligned} \tag{9}$$

$$(\Psi_i x)(t) = \sum_{t_k\in[t,t+\omega]} H_i(t,t_k) I_{ik}(t_k, x_i(t_k)), \quad i = 1, 2, \cdots, n. \tag{10}$$

$H_i(t,s), i = 1, 2, \cdots, n$, are defined by (3), it is easy to get, for $i = 1, 2, \cdots, n$,

$$\frac{1}{1+\zeta_i} \leq |H_i(t,s)| \leq \frac{\zeta_i}{\zeta_i+1}, \quad s \in [t, t+\omega].$$

where $\zeta_i = e^{\int_0^\omega a_i(\theta)d\theta}$.

Theorem 3.1: Suppose that $(A1)-(A4)$ hold, if the next assumption is satisfied
(A5):

$$\omega\left[\sum_{i=1}^{n}(\Upsilon_i)^2\right]^{\frac{1}{2}}+\omega\left[\sum_{i=1}^{n}(\Upsilon_i')^2\right]^{\frac{1}{2}}+\sum_{k=1}^{q}\left[\sum_{i=1}^{n}\left(\frac{\zeta_i c_{ik}}{\zeta_i+1}\right)^2\right]^{\frac{1}{2}}<1, \tag{11}$$

where

$$\Upsilon_i=\frac{\zeta_i}{\zeta_i+1}\left[\sum_{j=1}^{n}(\overline{d}_{ij}\mu_j)^2\right]^{\frac{1}{2}},$$

$$\Upsilon_i'=\frac{\zeta_i}{\zeta_i+1}\left[\sum_{j=1}^{n}((\overline{\alpha}_{ij}+\overline{\beta}_{ij})\nu_j)^2\right]^{\frac{1}{2}},$$

then there exists a unique ω anti-periodic solution of system (1).

Proof: Let us state operator $F=\Phi+\Psi: X\to X$, where Φ, Ψ are defined by (8),(9) and (10). Calculating the norm of $\|(Fx)(t)\|$, then

$$\begin{aligned}
&\|(Fx)(t)\|\\
&=\sup_{t\in[0,\omega]}\left\{\sum_{i=1}^{n}\left|\int_t^{t+\omega}H_i(t,s)\left[\sum_{j=1}^{n}d_{ij}(s)f_j(x_j(s))\right.\right.\right.\\
&\quad+\bigwedge_{j=1}^{n}\alpha_{ij}(s)g_j(x_j(s-\tau_{ij}(s)))\\
&\quad\left.+\bigvee_{j=1}^{n}\beta_{ij}(s)g_j(x_j(s-\tau_{ij}(s)))+E_i(s)\right]ds\\
&\quad\left.\left.+\sum_{t_k\in[t,t+\omega]}H_i(t,t_k)I_{ik}(t_k,x_i(t_k))\right|^2\right\}^{\frac{1}{2}}\\
&=\sup_{t\in[0,\omega]}\left\{\sum_{i=1}^{n}\left|\int_t^{t+\omega}H_i(t,s)\left[\sum_{j=1}^{n}d_{ij}(s)f_j(x_j(s))\right.\right.\right.\\
&\quad-\sum_{j=1}^{n}d_{ij}(s)f_j(0)\\
&\quad+\bigwedge_{j=1}^{n}\alpha_{ij}(s)g_j(x_j(s-\tau_{ij}(s)))-\bigwedge_{j=1}^{n}\alpha_{ij}(s)g_j(0)\\
&\quad+\bigvee_{j=1}^{n}\beta_{ij}(s)g_j(x_j(s-\tau_{ij}(s)))-\bigvee_{j=1}^{n}\beta_{ij}(s)g_j(0)\\
&\quad\left.\left.\left.+E_i(s)\right]ds+\sum_{t_k\in[t,t+\omega]}H_i(t,t_k)I_{ik}(t_k,x_i(t_k))\right|^2\right\}^{\frac{1}{2}}
\end{aligned}$$

$$\begin{aligned}
&\le\left[\sum_{i=1}^{n}\left(\frac{\zeta_i}{\zeta_i+1}\int_0^{\omega}\sum_{j=1}^{n}\overline{d}_{ij}|f_j(x_j(s))-f_j(0)|ds|\right)^2\right]^{\frac{1}{2}}\\
&\quad+\left[\sum_{i=1}^{n}\left(\frac{\zeta_i}{\zeta_i+1}\int_0^{\omega}\sum_{j=1}^{n}(\overline{\alpha}_{ij}+\overline{\beta}_{ij})\right.\right.\\
&\quad\times\left.\left.|g_j(x_j(s-\tau_{ij}(s)))-g_j(0)|ds|\right)^2\right]^{\frac{1}{2}}\\
&\quad+\omega\overline{E}\left[\sum_{i=1}^{n}\left(\frac{\zeta_i}{\zeta_i+1}\right)^2\right]^{\frac{1}{2}}\\
&\quad+\left[\sum_{i=1}^{n}\left(\frac{\zeta_i}{\zeta_i+1}\sum_{k=1}^{q}|I_{ik}(t_k,x_i(t_k))-I_{ik}(t_k,0)|\right)^2\right]^{\frac{1}{2}}\\
&\quad+\left[\sum_{i=1}^{n}\left(\frac{\zeta_i}{\zeta_i+1}\sum_{k=1}^{q}|I_{ik}(t_k,0)|\right)^2\right]^{\frac{1}{2}}\\
&\le\left[\sum_{i=1}^{n}\left(\frac{\zeta_i}{\zeta_i+1}\int_0^{\omega}\sum_{j=1}^{n}\overline{d}_{ij}\mu_j|x_j(s)|ds\right)^2\right]^{\frac{1}{2}}\\
&\quad+\left[\sum_{i=1}^{n}\left(\frac{\zeta_i}{\zeta_i+1}\int_0^{\omega}\sum_{j=1}^{n}(\overline{\alpha}_{ij}+\overline{\beta}_{ij})\nu_j\right.\right.\\
&\quad\times\left.\left.|x_j(s-\tau_{ij}(s))|ds\right)^2\right]^{\frac{1}{2}}\\
&\quad+\left[\sum_{i=1}^{n}\left(\frac{\zeta_i}{\zeta_i+1}\sum_{k=1}^{q}c_{ik}|x_i(t_k)|\right)^2\right]^{\frac{1}{2}}\\
&\quad+q\overline{I}\left[\sum_{i=1}^{n}\left(\frac{\zeta_i}{\zeta_i+1}\right)^2\right]^{\frac{1}{2}}
\end{aligned}$$

where $\overline{I}=\max_{1\le i\le n,k=1,2,\cdots,q}|\sup_{t\in[0,\omega]}I_{ik}(t,0)|$. Then by utilizing inequality

$$\sum_{i=1}^{n}a_ib_i\le\left(\sum_{i=1}^{n}a_i^2\right)^{\frac{1}{2}}\left(\sum_{i=1}^{n}b_i^2\right)^{\frac{1}{2}},$$

where $a_i\ge0$, $b_i\ge0$, it follows that

$$\begin{aligned}
&\|(Fx)(t)\|\\
&\le\left\{\sum_{i=1}^{n}\left[\frac{\zeta_i}{\zeta_i+1}\int_0^{\omega}\left(\sum_{j=1}^{n}(\overline{d}_{ij}\mu_j)^2\right)^{\frac{1}{2}}\right.\right.\\
&\quad\times\left.\left.\left(\sum_{j=1}^{n}|x_j(s)|^2\right)^{\frac{1}{2}}ds\right]^2\right\}^{\frac{1}{2}}
\end{aligned}$$

$$
\begin{aligned}
&+\left\{\sum_{i=1}^{n}\left[\frac{\zeta_i}{\zeta_i+1}\int_0^{\omega}\left(\sum_{j=1}^{n}((\overline{\alpha}_{ij}+\overline{\beta}_{ij})\nu_j)^2\right)^{\frac{1}{2}}\right.\right.\\
&\left.\left.\times\left(\sum_{j=1}^{n}|x_j(s-\tau_{ij}(s))|^2\right)^{\frac{1}{2}}ds\right]^2\right\}^{\frac{1}{2}}\\
&+\sum_{k=1}^{q}\left[\sum_{i=1}^{n}\left(\frac{\zeta_i c_{ik}}{\zeta_i+1}|x_i(t_k)|\right)^2\right]^{\frac{1}{2}}\\
&+q\bar{I}\left[\sum_{i=1}^{n}\left(\frac{\zeta_i}{\zeta_i+1}\right)^2\right]^{\frac{1}{2}}\\
\leq&\left\{\omega\left[\sum_{i=1}^{n}(\Upsilon_i)^2\right]^{\frac{1}{2}}+\omega\left[\sum_{i=1}^{n}(\Upsilon_i')^2\right]^{\frac{1}{2}}\right.\\
&\left.+\sum_{k=1}^{q}\left[\sum_{i=1}^{n}\left(\frac{\zeta_i c_{ik}}{\zeta_i+1}\right)^2\right]^{\frac{1}{2}}\right\}\rho\\
&+q\bar{I}\left[\sum_{i=1}^{n}\left(\frac{\zeta_i}{\zeta_i+1}\right)^2\right]^{\frac{1}{2}}\\
\leq&\ \rho
\end{aligned}
$$

where

$$
\begin{aligned}
\rho &\geq \frac{q\bar{I}\left[\sum_{i=1}^{n}\left(\frac{\zeta_i}{\zeta_i+1}\right)^2\right]^{\frac{1}{2}}}{1-L}>0\\
L &= \omega\left[\sum_{i=1}^{n}(\Upsilon_i)^2\right]^{\frac{1}{2}}+\omega\left[\sum_{i=1}^{n}(\Upsilon_i')^2\right]^{\frac{1}{2}}\\
&\quad+\sum_{k=1}^{q}\left[\sum_{i=1}^{n}\left(\frac{\zeta_i c_{ik}}{\zeta_i+1}\right)^2\right]^{\frac{1}{2}}
\end{aligned}
$$

Thus $FB_\rho \subset B_\rho$, here $B_\rho = \{x \in X : \|x\| \leq \rho\}$. Now, for $x, y \in X$, from the above method, we have

$$
\begin{aligned}
&\|Fx-Fy\|\\
=&\sup_{t\in[0,\omega]}\left\{\sum_{i=1}^{n}\left|\int_t^{t+\omega}H_i(t,s)\left[\sum_{j=1}^{n}d_{ij}(s)f_j(x(s))\right.\right.\right.\\
&-\sum_{j=1}^{n}d_{ij}(s)f_j(y_j(s))\\
&+\bigwedge_{j=1}^{n}\alpha_{ij}(s)g_j(x_j(s-\tau_{ij}(s)))\\
&-\bigwedge_{j=1}^{n}\alpha_{ij}(s)g_j(y_j(s-\tau_{ij}(s)))\\
&+\bigvee_{j=1}^{n}\beta_{ij}(s)g_j(x_j(s-\tau_{ij}(s)))\\
&\left.-\bigvee_{j=1}^{n}\beta_{ij}(s)g_j(y_j(s-\tau_{ij}(s)))\right]ds\\
&+\sum_{t_k\in[t,t+\omega]}H_i(t,t_k)\\
&\left.\times(I_{ik}(t_k,x_i(t_k))-I_{ik}(t_k,y_i(t_k)))|^2\right\}^{\frac{1}{2}}\\
\leq&\left\{\omega\left[\sum_{i=1}^{n}(\Upsilon_i)^2\right]^{\frac{1}{2}}+\omega\left[\sum_{i=1}^{n}(\Upsilon_i')^2\right]^{\frac{1}{2}}\right.\\
&\left.+\sum_{k=1}^{q}\left[\sum_{i=1}^{n}\left(\frac{\zeta_i c_{ik}}{\zeta_i+1}\right)^2\right]^{\frac{1}{2}}\right\}\|x-y\|
\end{aligned}
$$

Noting assumption (A5), it is clear that F is a contraction mapping. Thus system (1) possesses a unique ω anti-periodic solution.

Theorem 3.2: Assume that (A1)-(A4) hold, if the next assumption is satisfied

(A6)

$$
\sum_{k=1}^{q}\left[\sum_{i=1}^{n}\left(\frac{\zeta_i c_{ik}}{\zeta_i+1}\right)^2\right]^{\frac{1}{2}}<1, \tag{12}
$$

then system (1) has at least ω anti-periodic solution.

Proof: We define the operator Φ, Ψ as (8), (9) and (10). Choosing

$$
\begin{aligned}
\rho &\geq \frac{n\omega\bar{d}M_f+n\omega(\overline{\alpha}+\overline{\beta})M_g+\omega\overline{E}+q\bar{I}}{1-\sum_{k=1}^{q}\left[\sum_{i=1}^{n}\left(\frac{\zeta_i c_{ik}}{\zeta_i+1}\right)^2\right]^{\frac{1}{2}}}\\
&\quad\times\left[\sum_{i=1}^{n}\left(\frac{\zeta_i}{\zeta_i+1}\right)^2\right]^{\frac{1}{2}}\\
&>0
\end{aligned} \tag{13}
$$

For $x, y \in B_\rho = \{x \in X : \|x\| \leq \rho\}$, we get

$$
\begin{aligned}
&\|(\Phi x)(t)+(\Psi y)(t)\|\\
=&\sup_{t\in[0,\omega]}\left\{\sum_{i=1}^{n}\left|\int_t^{t+\omega}H_i(t,s)\left[\sum_{j=1}^{n}d_{ij}(s)f_j(x_j(s))\right.\right.\right.\\
&+\bigwedge_{j=1}^{n}\alpha_{ij}(s)g_j(x_j(s-\tau_{ij}(s)))\\
&\left.+\bigvee_{j=1}^{n}\beta_{ij}(s)g_j(x_j(s-\tau_{ij}(s)))+E_i(s)\right]ds\\
&\left.\left.+\sum_{t_k\in[t,t+\omega]}H_i(t,t_k)I_{ik}(t_k,y_i(t_k))\right|^2\right\}^{\frac{1}{2}}
\end{aligned}
$$

$$= \sup_{t\in[0,\omega]} \Bigg\{ \sum_{i=1}^{n} \Bigg| \int_t^{t+\omega} H_i(t,s) \times \Bigg[\sum_{j=1}^{n} d_{ij}(s) f_j(x_j(s)) - \sum_{j=1}^{n} d_{ij}(s) f_j(0) + \bigwedge_{j=1}^{n} \alpha_{ij}(s) g_j(x_j(s-\tau_{ij}(s))) - \bigwedge_{j=1}^{n} \alpha_{ij}(s) g_j(0) + \bigvee_{j=1}^{n} \beta_{ij}(s) g_j(x_j(s-\tau_{ij}(s))) - \bigvee_{j=1}^{n} \beta_{ij}(s) g_j(0) + E_i(s) \Bigg] ds + \sum_{t_k\in[t,t+\omega]} H_i(t,t_k) I_{ik}(t_k, y_i(t_k)) \Bigg|^2 \Bigg\}^{\frac{1}{2}}$$

$$\leq \Bigg[\sum_{i=1}^{n} \Bigg(\frac{\zeta_i}{\zeta_i+1} \int_0^{\omega} \sum_{j=1}^{n} \overline{d}_{ij} |f_j(x_j(s))| ds| \Bigg)^2 \Bigg]^{\frac{1}{2}} + \Bigg[\sum_{i=1}^{n} \Bigg(\frac{\zeta_i}{\zeta_i+1} \int_0^{\omega} \sum_{j=1}^{n} (\overline{\alpha}_{ij} + \overline{\beta}_{ij}) \times |g_j(x_j(s-\tau_{ij}(s)))| ds| \Bigg)^2 \Bigg]^{\frac{1}{2}} + \omega \overline{E} \Bigg[\sum_{i=1}^{n} \Bigg(\frac{\zeta_i}{\zeta_i+1} \Bigg)^2 \Bigg]^{\frac{1}{2}} + \Bigg[\sum_{i=1}^{n} \Bigg(\frac{\zeta_i}{\zeta_i+1} \sum_{k=1}^{q} |I_{ik}(t_k, y_i(t_k)) - I_{ik}(t_k,0)| \Bigg)^2 \Bigg]^{\frac{1}{2}} + \Bigg[\sum_{i=1}^{n} \Bigg(\frac{\zeta_i}{\zeta_i+1} \sum_{k=1}^{q} |I_{ik}(t_k,0)| \Bigg)^2 \Bigg]^{\frac{1}{2}}$$

$$\leq \Bigg\{ \sum_{k=1}^{q} \Bigg[\sum_{i=1}^{n} \Bigg(\frac{\zeta_i c_{ik}}{\zeta_i+1} \Bigg)^2 \Bigg]^{\frac{1}{2}} \Bigg\} \rho + (n\omega \overline{d} M_f + n\omega(\overline{\alpha}+\overline{\beta}) M_g + \omega \overline{E} + q\overline{I}) \times \Bigg[\sum_{i=1}^{n} \Bigg(\frac{\zeta_i}{\zeta_i+1} \Bigg)^2 \Bigg]^{\frac{1}{2}} \leq \rho$$

Therefore, $\Phi x + \Psi y \in B_\rho$. Since $f_j(\cdot), g_j(\cdot), j = 1, 2, \cdots, n$, are continuous. Thus the operator Φ is continuous. For $x \in B_\rho$, then

$$\|\Phi x\| \leq (n\omega \overline{d} M_f + n\omega(\overline{\alpha}+\overline{\beta}) M_g + \omega \overline{E}) \Bigg[\sum_{i=1}^{n} \Bigg(\frac{\zeta_i}{\zeta_i+1} \Bigg)^2 \Bigg]^{\frac{1}{2}} \tag{14}$$

Therefore Φ is uniformly bounded on B_ρ. Next, let us show the compactness of the operator Φ. For $t_1, t_2 \in [0, \omega]$, we have

$$\|(\Phi x)(t_1) - (\Phi x)(t_2)\| \leq \Bigg[\sum_{i=1}^{n} \Bigg| \int_0^{\omega} |H_i(t_1,s) - H_i(t_2,s)| \Bigg[\sum_{j=1}^{n} d_{ij}(s) f_j(x_j(s)) + \bigwedge_{j=1}^{n} \alpha_{ij}(s) g_j(x_j(s-\tau_{ij}(s))) + \bigvee_{j=1}^{n} \alpha_{ij}(s) g_j(x_j(s-\tau_{ij}(s))) + E_i(s) \Bigg] ds \Bigg|^2 \Bigg]^{\frac{1}{2}}$$

$$\leq \sum_{i=1}^{n} \frac{1}{\zeta_i+1} \int_0^{\omega} \Big| e^{\int_{t_1}^{s} a_i(\theta)d\theta} - e^{\int_{t_2}^{s} a_i(\theta)d\theta} \Big| \times \Bigg[\sum_{j=1}^{n} \overline{d}_{ij} M_f + \bigwedge_{j=1}^{n} \overline{\alpha}_{ij} M_g + \bigvee_{j=1}^{n} \overline{\beta}_{ij} M_g + \overline{E} \Bigg] ds$$

$$\leq |t_1 - t_2| \Bigg[\sum_{j=1}^{n} \overline{d}_{ij} M_f + \bigwedge_{j=1}^{n} \overline{\alpha}_{ij} M_g + \bigvee_{j=1}^{n} \overline{\beta}_{ij} M_g + \overline{E} \Bigg] \omega a^+ \sum_{i=1}^{n} \frac{\zeta_i}{\zeta_i+1}$$

$$\leq |t_1 - t_2| [n\overline{d} M_f + n(\overline{\alpha}+\overline{\beta}) M_g + \overline{E}] \omega a^+ \sum_{i=1}^{n} \frac{\zeta_i}{\zeta_i+1}$$

Consequently, by means of Arzela-Ascoli theorem, Φ is compact on B_ρ. By presumption (A6), it is clear that Ψ is contraction mapping. Utilizing Lemma 2.2, framework (1) has at least ω anti-periodic solution.

IV. GLOBAL EXPONENTIAL STABILITY OF ANTI-PERIODIC SOLUTIONS

Assume that $x^*(t) = (x_1^*(t), , \cdots, x_n^*(t))^T$ is an ω anti-periodic solutions of framework (1). In this segment, we will develop some appropriate Lyapunov functional to demonstrate the global exponential stability of this anti-periodic solution.

Theorem 4.1: Suppose that assumptions $(A1)-(A5)$ hold. In the event that the accompanying suppositions are fulfilled $(A7)$ there is $\gamma, \overline{c}_{ik} \geq 0$, $i = 1, 2, \cdots, n$, $k = 1, 2, \cdots$, such that, for $u, v \in R$,

$$|u + I_{ik}(t,u) - v - I_{ik}(t,v)| \leq \overline{c}_{ik}|u - v|, \quad t \in [0,\omega], \tag{15}$$

and for $k = 1, 2, \cdots$,

$$2 \ln c_k \leq \gamma(t_k - t_{k-1}). \tag{16}$$

$(A8)$ there exist $\gamma_i > 0$ and $\delta_{ij}, \eta_{ij}, \vartheta_{ij}, \xi_{ij} \in R$, $i = 1, 2, \cdots, n$, such that

$$-\Theta_1 + c\Theta_2 + \gamma = 0 \tag{17}$$

where

$$\Theta_1 = \min_{1\le i\le n}\left\{2a_i^- - \sum_{j=1}^{n}(\overline{d}_{ij})^{2\delta_{ij}}\mu_j^{2\eta_{ij}} - \sum_{j=1}^{n}\frac{\gamma_j}{\gamma_i}(\overline{d}_{ji})^{2(1-\delta_{ij})}\mu_j^{2(1-\eta_{ij})} - \sum_{j=1}^{n}(\overline{\alpha}_{ij}+\overline{\beta}_{ij})^{2\vartheta_{ij}}\nu_j^{2\xi_{ij}}\right\}.$$

$$\Theta_2 = \max_{1\le i\le n}\left\{\sum_{j=1}^{n}\frac{\gamma_j}{\gamma_i}(\overline{\alpha}_{ji}+\overline{\beta}_{ji})^{2(1-\vartheta_{ij})}\mu_j^{2(1-\xi_{ij})}\right\},$$

$$c_k = \max_{1\le i\le n}\{\overline{c}_{ik}\},$$

$$c = \max_{1\le k<+\infty}\left\{e^{\gamma(t_k-t_{k-1})}, \frac{1}{e^{\gamma(t_k-t_{k-1})}}\right\},$$

then ω anti-periodic solution of system (1) is globally exponentially stable with convergence rate $\lambda/2$, and λ is a unique positive solution of $\lambda - \Theta_1 + c\Theta_2 e^{\lambda\tau} + \gamma = 0$.

Proof: Let $x^*(t) = (x_1^*(t), x_2^*(t), \cdots, x_n^*(t))^T$ be an ω anti-periodic solution of (1), and $x(t) = (x_1(t), x_2(t), \cdots, x_n(t))^T$ is a solution of (1). Set $y(t) = x(t) - x^*(t)$. Then, for $k = 1, 2, \cdots, i = 1, 2, \cdots, n$,

$$\begin{cases} y_i'(t) = -a_i(t)(x_i(t) - x_i^*(t)) \\ \qquad + \sum_{j=1}^{n} d_{ij}(t)[f_j(x_j(t)) - f_j(x_j^*(t))] \\ \qquad + \bigwedge_{j=1}^{n}\alpha_{ij}(t)g_j(x_j(t-\tau_{ij}(t))) \\ \qquad - \bigwedge_{j=1}^{n}\alpha_{ij}(t)g_j(x_j^*(t-\tau_{ij}(t))) \\ \qquad + \bigvee_{j=1}^{n}\beta_{ij}(t)g_j(x_j(t-\tau_{ij}(t))) \\ \qquad - \bigvee_{j=1}^{n}\beta_{ij}(t)g_j(x_j^*(t-\tau_{ij}(t))), \quad t\ge 0,\ t\ne t_k \\ y_i(t_k^+) = x_i(t_k) - x_i^*(t_k) + I_{ik}(t_k, x_i(t_k)) \\ \qquad - I_{ik}(t_k, x_i^*(t_k)). \end{cases} \tag{18}$$

Considering the next function

$$V(t) = \sum_{i=1}^{n}\gamma_i|y_i(t)|^2. \tag{19}$$

Computing the upper right derivative of $V(t)$, for $t \ne t_k$,

$$\begin{aligned} D^+V(t) &= \sum_{i=1}^{n}2\gamma_i D^+|y_i(t)| \\ &\le \sum_{i=1}^{n}-2\gamma_i a_i(t)|y_i(t)||y_i(t)| \\ &\quad + \sum_{i=1}^{n}2\gamma_i\sum_{j=1}^{n}|d_{ij}(t)||y_i(t)||f_j(x_j(t)) - f_j(x_j^*(t))| \\ &\quad + \sum_{i=1}^{n}2\gamma_i\sum_{j=1}^{n}(|\alpha_{ij}(t)| + |\beta_{ij}(t)|)|y_i(t)| \\ &\quad \times|g_j(x_j(t-\tau_{ij}(t))) - g_j(x_j^*(t-\tau_{ij}(t)))| \\ &\le \sum_{i=1}^{n}-2\gamma_i a_i^-|y_i(t)|^2 + \sum_{i=1}^{n}2\gamma_i\sum_{j=1}^{n}\overline{d}_{ij}|y_i(t)|\mu_j|y_j(t)| \\ &\quad + \sum_{i=1}^{n}2\gamma_i\sum_{j=1}^{n}(\overline{\alpha}_{ij}+\overline{\beta}_{ij})|y_i(t)|\nu_j|y_j(t-\tau_{ij}(t))| \end{aligned} \tag{20}$$

Using inequality $ab \le \frac{1}{2}a^2 + \frac{1}{2}b^2$, we have

$$\begin{aligned} &\sum_{j=1}^{n}\overline{d}_{ij}|y_i(t)|\mu_j|y_j(t)| \\ &= \sum_{j=1}^{n}[(\overline{d}_{ij})^{\delta_{ij}}\mu_j^{\eta_{ij}}|y_i(t)|][(\overline{d}_{ij})^{1-\delta_{ij}}\mu_j^{1-\eta_{ij}}|y_j(t)|] \\ &\le \sum_{j=1}^{n}\Big[\frac{1}{2}(\overline{d}_{ij})^{2\delta_{ij}}\mu_j^{2\eta_{ij}}|y_i(t)|^2 \\ &\quad + \frac{1}{2}(\overline{d}_{ij})^{2(1-\delta_{ij})}\mu_j^{2(1-\eta_{ij})}|y_j(t)|^2\Big] \end{aligned} \tag{21}$$

and

$$\begin{aligned} &\sum_{j=1}^{n}(\overline{\alpha}_{ij}+\overline{\beta}_{ij})|y_i(t)|\nu_j|y_j(t-\tau_{ij}(t))| \\ &\le \sum_{j=1}^{n}\Big[\frac{1}{2}(\overline{\alpha}_{ij}+\overline{\beta}_{ij})^{2\vartheta_{ij}}\nu_j^{2\xi_{ij}}|y_i(t)|^2 \\ &\quad + \frac{1}{2}(\overline{\alpha}_{ij}+\overline{\beta}_{ij})^{2(1-\vartheta_{ij})}\mu_j^{2(1-\xi_{ij})} \\ &\quad \times|y_j(t-\tau_{ij}(t))|^2\Big] \end{aligned} \tag{22}$$

Substituting (21) and (22) into (20), we have, for $t \ne t_k$,

$$\begin{aligned} &D^+V(t) \\ &\le \sum_{i=1}^{n}\gamma_i\Big\{-2a_i^-|y_i(t)| + \sum_{j=1}^{n}\Big[(\overline{d}_{ij})^{2\delta_{ij}}\mu_j^{2\eta_{ij}}|y_i(t)|^2 \\ &\quad + (\overline{d}_{ij})^{2(1-\delta_{ij})}\mu_j^{2(1-\eta_{ij})}|y_j(t)|^2\Big] \\ &\quad + \sum_{j=1}^{n}\Big[(\overline{\alpha}_{ij}+\overline{\beta}_{ij})^{2\vartheta_{ij}}\nu_j^{2\xi_{ij}}|y_i(t)|^2 \\ &\quad + (\overline{\alpha}_{ij}+\overline{\beta}_{ij})^{2(1-\vartheta_{ij})}\mu_j^{2(1-\xi_{ij})}|y_j(t-\tau_{ij}(t))|^2\Big]\Big\} \\ &= \sum_{i=1}^{n}\gamma_i\Big\{\Big[-2a_i^- + \sum_{j=1}^{n}(\overline{d}_{ij})^{2\delta_{ij}}\mu_j^{2\eta_{ij}} \\ &\quad + \sum_{j=1}^{n}\frac{\gamma_j}{\gamma_i}(\overline{d}_{ji})^{2(1-\delta_{ij})}\mu_j^{2(1-\eta_{ij})} \\ &\quad + \sum_{j=1}^{n}(\overline{\alpha}_{ij}+\overline{\beta}_{ij})^{2\vartheta_{ij}}\nu_j^{2\xi_{ij}}\Big]|y_i(t)|^2 \\ &\quad + \sum_{j=1}^{n}\frac{\gamma_j}{\gamma_i}(\overline{\alpha}_{ji}+\overline{\beta}_{ji})^{2(1-\vartheta_{ij})}\mu_j^{2(1-\xi_{ij})} \\ &\quad \times|y_j(t-\tau_{ij}(t))|^2\Big\} \\ &\le -\Theta_1 V(t) + \Theta_2\overline{V}(t) \end{aligned} \tag{23}$$

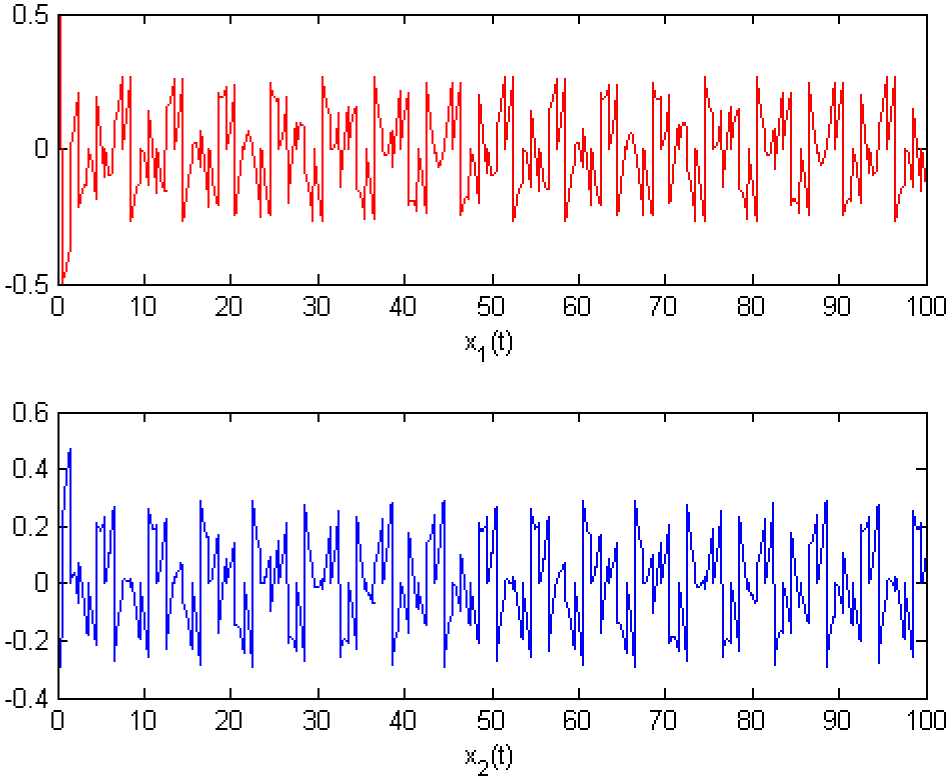

FIGURE 1. Numerical solution $x(t)=(x_1(t),x_2(t))^T$ of frameworks (27) for initial conditions $\varphi(s)=(0.5,-0.4)^T, s\in[-2,0]$.

where $V(t)=\sup_{t-\tau\le\eta\le t}V(\eta)$. From (A6), we have

$$V(t_k^+)=\sum_{i=1}^{n}\gamma_i|y_i(t_k^+)|^2\le\sum_{i=1}^{n}\gamma_i\bar{c}_{ik}^2|y_i(t_k)|^2<c_k^2V(t_k). \quad (24)$$

By Lemma 2.3, there is $c>1$ satisfying

$$V(t)\le c(\sup_{-\tau\le t\le 0}V(t))e^{-\lambda t}. \quad (25)$$

Thus

$$\|x(t)-x^*(t)\|\le\left(\frac{c\max_{1\le i\le n}(\gamma_i)}{\min_{1\le i\le n}(\gamma_i)}\right)^{\frac{1}{2}}\|\varphi-\varphi^*\|e^{-\lambda t/2}. \quad (26)$$

The proof of Theorem 4.1 is completed.

Remark 4.1: The global exponential stability of FCNNs is vital dynamical behavior. Time delays and impulsive effective regularly cause framework instability or oscillatory. Unmistakably the results got are connected with the time delay and impulses for advocating globally exponentially stability of ω anti-periodic solution of system (1).

Theorem 4.2: Suppose that assumptions $(A1)-(A4)$ and $(A6)-(A8)$ hold. Then ω anti-periodic solution of system (1) is globally exponentially stable with convergence rate $\lambda/2$.

V. AN ILLUSTRATIVE EXAMPLE

In this segment, a precedent is given to prove adequacy of results acquired.

Example 5.1: Consider the accompanying FCNNs with time-varying delay and impulses.

$$\begin{cases} x_i'(t)=-a_i(t)x_i(t)+\sum_{j=1}^{2}d_{ij}(t)f_j(x_j(t)) \\ \qquad +\bigwedge_{j=1}^{2}\alpha_{ij}(t)g_j(x_j(t-\tau_{ij}(t))) \\ \qquad +\bigvee_{j=1}^{2}\beta_{ij}(t)g_j(x_j(t-\tau_{ij}(t))) \\ \qquad +E_i(t),\quad t\ne\frac{k\pi}{2},\ k=1,2,\cdots, \\ \Delta x_i(t_k))=-\frac{2}{3}x_i(t_k),\quad t=t_k=\frac{k\pi}{2},\ i=1,2, \end{cases} \quad (27)$$

where $a_1(t)=a_2(t)=\frac{1}{8}, f_j(x)=g_j(x)=\arctan x (j=1,2)$.

$$(d_{ij}(t))_{2\times2}=\begin{pmatrix}1/4 & 1/8\\ 1/6 & 1/3\end{pmatrix},$$

$$(\alpha_{ij}(t))_{2\times2}=\begin{pmatrix}1/8 & 1/6\\ 1/6 & 1/8\end{pmatrix},$$

$$(\beta_{ij}(t))_{2\times2}=\begin{pmatrix}1/16 & 1/4\\ 1/4 & 1/16\end{pmatrix},$$

$$(E_i(t))_{2\times1}=\begin{pmatrix}1/4\sin t\\ 1/3\cos t\end{pmatrix}.$$

impulsive functions $I_{1k}(t,x)=I_{2k}(t,x)=-\frac{2}{3}x$, impulsive points $t_k=\frac{k\pi}{2}$, $\tau_{11}(t)=\tau_{21}(t)=|\sin(2\pi t)|$, $\tau_{12}(t)=\tau_{22}(t)=|\cos(2\pi t)|$, then, we can easily check that $u=v=\frac{\pi}{2}$, $c_{1k}=c_{2k}=\frac{2}{3}$, $\bar{c}_{1k}=\bar{c}_{2k}=\frac{1}{3}$, $c_k=\frac{1}{3}$, $c=1$, $\gamma_1=\gamma_2=e^{\frac{\pi}{8}}$, Taking $\delta_{ij}=\eta_{ij}=\vartheta_{ij}=\xi_{ij}=\frac{1}{2}(i=1,2)$, $\frac{2\ln c_k}{t_k-t_{k-1}}\le-1.39=\gamma$.

It is easy to conclude that assumptions (A6) and (A8) hold. Using Theorem 3.2 and Theorem 4.2, framework (27) has at least one π anti-periodic solution which is globally exponentially stable (see Figure 1).

VI. DISCUSSION

Anti-periodic solution of neural networks can be applied to describe the dynamical behavior of neural networks and they play an important role in designing the neural networks. Existence of anti-periodic solution for fuzzy cellular neural networks, namely system (1), has been established by Krasnoselskiı́'s fixed point theorem and contraction principle. It has been shown that the main result is that if assumption (A5) holds true, then system (1) has a unique ω anti-periodic solution; if assumption (A6) is fulfilled, then system has at least ω anti-periodic solution. In fact it is easy to find that assumption (A5) can imply assumption (A6). This indicates that the existence of anti-periodic solution for system (1) is satisfied with the assumption (A6) besides assumptions (A1)-(A4). Nevertheless, the existence and uniqueness of anti-periodic solution for system (1) holds true under the assumption (A5) besides assumption (A1)-(A4). This provides objective criteria for designing the fuzzy cellular neural systems.

As is all known that it is a classical method to prove global stability of delay differential system applying Lyapunov functional. However it is very difficult to construct appropriate Lyapunov functional to obtain the result according to the conditions of system. In this paper, we apply differential inequality and construct a simple and suitable Lyapunov functional which differ from [25]–[34] to obtain our result. In these sense, the paper has novelty of techniques. Notice that global stability results of system (1) are independent of initial conditions of system (1). This shows that for system (1), the parameters of system, delay and impulsive operators satisfy the assumptions (A1)-(A5), (A7),(A8) (Theorem 4.1) or (A1)-(A4), (A6)-(A8) (Theorem 4.2), the solution of system (1) converges to anti-periodic solution eventually.

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