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# A New Life Expectancy Assessment Method for Complex Systems With Multi-Characteristics: Case Study on Power-Shift Steering Transmission Control System

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**ABSTRACT** Life is an important monitoring index for prognostic and health management systems. This paper proposes a new life expectancy assessment method for high-value complex systems with multicharacteristics based on goal-oriented (GO) method. First, the new life expectancy assessment method is expounded in detail, and its process is formulated. Then, the control system of power-shift steering transmission for a heavy vehicle, which is a typical mechatronics control system with multi-characteristics, such as multi-state, multi-function, and correlations, is taken as an example to evaluate its life expectancy by this paper's method. In order to verify the advantages, feasibility, and rationality of the new life expectancy assessment method, the results are compared with the results by Monte Carlo method. All in all, this life expectancy assessment method not only improves the theory of GO method, so that GO method is applied to evaluate the life expectancy of complex systems with multi-characteristics first, but also provides a new approach for life expectancy assessment of complex systems, which can take into consideration various system characteristics and only use unit life data, so that it is low evaluation cost, higher estimating efficiency and accuracy, and a more stable assessment result.

**INDEX TERMS** Reliability assessment, complex systems, GO method, life expectancy, system characteristics, prognostic and health management.

#### **I. INTRODUCTION**

With the development of technology in electronics, communication and networking etc. rapidly in recent years, the systems applied in various engineering areas become more and more complex and integration because of complex structure, complex characteristics, complex working conditions, and so on. And their costs become more and more expensive. Quality and reliability are key attributes of economic success of a system because they result in an increase in productivity at little cost and vital for business growth and enhanced competitive

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position. Reliability assessment plays an important role in life cycle of systems, as follows:

- (1) In design phase, the reliability assessment can evaluate whether the system reliability meets the design requirements, so that it can provides product improvement suggestions effectively.
- (2) In production phase, the reliability assessment can judge whether the product is qualified, so that it contribute to controlling the product quality.
- (3) In usage phase, the reliability assessment can show the system reliability level, so that it can guidance the optimization maintenance and mission decisionmaking.

Life is an important monitoring index for Prognostic and Health Management systems. The Life Expectancy is a key reliability index of complex systems. The Life Expectancy assessment of complex systems is a quantitative estimation by using probabilistic method and reliability data. Because the cost of complex system is more and more valuableness, the test sample of system for evaluating its Life Expectancy is less or even none in the practical engineering. Thus, the accuracy of Life Expectancy assessment by using the test data of system itself is very low, and the assessment result cannot meet the engineering requirements. Until now, a large number of researchers focus on the Life Expectancy assessment by using small sample size and test data of unit. There are four kinds of system reliability assessment methods to evaluate the Life Expectancy [1], [2], which are exact methods, approximate methods, Bayesian methods, and Monte Carlo methods. While, the above methods have the following disadvantages:

- (1) The existing exact methods have high requirements for system structure, data type, etc., so it is not easy to establish modeling, and conduct computing so that it is difficult to apply in engineering.
- (2) The approximation methods adopt the data conversion among different data types, so the assessment result is relatively conservative. Moreover, the precision of assessment result is hard to control.
- (3) The Bayesian methods are conducted by the step-bystep conversion according to pyramid model, whose two key steps are calculating system prior moment and calculating system prior distribution, respectively. Because the above two key steps of Bayesian methods usually adopt the second-moment method and Markov Chain Monte Carlo simulation method, it is not easy to control the accuracy and bias of assessment results.
- (4) The Monte Carlo methods, which are most common Life Expectancy assessment method, use the sample technology to evaluate system Life Expectancy, so the simulation times affects the operation efficiency and accuracy of assessment result directly. Moreover, the assessment result is very unstable.
- (5) The information of system structure, function constitutes, and system characteristics for the reliability models of above assessment methods are ignored, so it is hard to obtain the accurate assessment result for a limited testing time and samples. Especially, the high value complex systems often operate the units' life test without system's life test.

Thus, how to avoid above disadvantages of system reliability assessment methods to conduct Life Expectancy assessment of complex systems considering the system structure, function constitutes and system characteristics quickly, steadily and accurately has been a point of concern. Goal Oriented (GO) methodology [3], [4] is a system reliability analysis method by using unit reliability data according to GO model to conduct GO operation to obtain the quantitative

analysis result and qualitative analysis result. Moreover, GO method is especially suitable for reliability analysis of complex systems with complex correlations [5]–[9], multifunction [9], [10], multi-fault modes [11], [12], multistate [13], [14], closed-loop feedback link [15]–[17], and so on. Nowadays, GO method has been applied in reliability analysis of defense systems, water, oil and gas supply systems, manufacturing systems, transportation systems, power systems, etc., and it has become increasingly popular in recent years because of its advantages in terms of its ease of creating a model and of its representational and analysis power [3], as follows:

- (1) GO model is directly established according to system structure, working principle and function constitutes, and it can visually reflects the system characteristics. Moreover, GO model is easy to check, and can avoid the influence of human factors for reliability model.
- (2) Both quantitative analysis result and qualitative analysis result of system are obtained by multiple GO operations, which is easy to operate efficiently and quickly. Moreover, the reliability analysis results have highlevel consistency by different engineer(s). And it can avoid the complex analysis process and the influence of sampling.
- (3) The development space of GO methodology is larger, so that it can be improved to solve various kinds of practical engineering problems, such as safety analysis [18], reliability optimization allocation [19], [20], and so on.

In view of above advantages of GO method, this paper proposes a new system reliability assessment method for complex systems taking into consideration various system characteristics to evaluate system Life Expectancy firstly. This paper also aims to fill this gap. The main contributions of this study are as follows:

- (1) We propose a new Life Expectancy assessment method for complex systems based on GO method by using reliability data of unit and taking into consideration various system characteristics.
- (2) We formulate the process of this paper's Life Expectancy assessment method.
- (3) The power-shift steering transmission control system (PSSTCS) of a heavy vehicle is taken as a case study for the first time to evaluate its Life Expectancy by the new Life Expectancy assessment method.

The remainder of the paper is organized as follows. The Life Expectancy assessment method for complex system based on GO method is expounded in detail in Section 2. In Section 3, in order to illustrate the usage of the new Life Expectancy assessment method, a PSSTCS is taken as a case study. In order to verify the advantages, feasibility and rationality of the new Life Expectancy assessment method, the results are compared with the results by Monte Carlo methods in Section 4. Section 5 provides some conclusions on the findings of the research.

#### **II. LIFE EXPECTANCY ASSESSMENT METHOD FOR COMPLEX SYSTEMS WITH MULTI-CHARACTERISTICS BASED ON GO METHOD**

In this section, a new Life Expectancy assessment method for complex systems with multi-characteristics based on GO method is proposed by only using reliability data of unit in aspect of expounding its evaluating steps in detail, and formulating its process.

#### A. NEW LIFE EXPECTANCY ASSESSMENT METHOD

GO model can directly reflects the system structure, working principle, function constitutes and system characteristics, and GO operation can obtain the system minimum cutset and system reliability by taking into considering system characteristics. Thus, in view of advantages of GO model, qualitative analysis and quantitative analysis of GO method, the Life Expectancy of complex systems with multi-characteristics is obtained by using reliability data of selecting life test unit, whose evaluating steps are as follows:

1) CONDUCTING SYSTEM ANALYSIS OF COMPLEX SYSTEMS The system analysis is base of GO method, and it directly affects the establishing GO model and conducting GO operation. It mainly includes:

(i) To analyze system structure and system function constitutes according to the principle diagram, engineering drawing or function flowchart of system.

(ii) To determine system characteristics, such as complex correlations, multifunction, multi-state, and so on.

(iii) To define the success rule of system according to system analysis result.

#### 2) ESTABLISHING GO MODEL OF COMPLEX SYSTEMS

GO model is a key element of GO method. It is developed according to the system principle diagram, the engineering drawing, and the function flowchart directly. The qualitative analysis and quantitative analysis are conducted by GO operation according to GO model. GO operators and signal flows are fundamental elements of GO model. GO operator represents unit itself and logical relationship, and signal flow represents specific fluid flow or a logical process. Establishing GO model of complex systems has two steps, as follows:

(i) To select GO operator. GO operator has function GO operator, logical GO operator, and auxiliary GO operator, which are used to describe the unit itself, the logical relationship in system, and auxiliary operation. There are 17 basic GO operators [3] and 10 developed GO operators, as presented in Table I. For the non-system units, such as action signal, they are used to establish GO model of system, but such units do not affect the system Life Expectancy, so they are described by visual Type 5 operator whose reliability is 1.

(ii) To develop GO model. According to system analysis result, the GO model is developed through the signal flow to connect GO operator. And the GO operation is done along the direction of signal flow.

#### **TABLE 1.** Description of frequently-used GO operators.



#### 3) SELECTING LIFE TEST UNIT OF COMPLEX SYSTEMS

The new Life Expectancy assessment method is conducted by using reliability data of unit. Thus, selecting life test unit is one key step of evaluating Life Expectancy for complex system, which can be obtained by multiple GO operations. It has three steps, as follows:

(i) To obtain the minimum cut sets of system by qualitative analysis of GO method. And the minimum cut sets of system can provides the guidance for selecting life test unit. Its steps are as follows:

- *a)* To obtain the one-order minimum cut set of system. Setting the reliability of a function GO operator in GO model is 0, and the reliabilities of other GO operators are kept constant; in this case, if system reliability is 0 by GO operation, this GO operator will be a one-order minimum cut set, as shown in Fig. 1(A).
- *b)* To obtain the two-order minimum cut set of system. Setting the reliabilities of two function GO operators in GO model are 0 except one-order minimum cut set, and the reliabilities of other GO operators are kept constant; in this case, if system reliability is 0 by GO operation, the two GO operators will be a two-order minimum cut set, as shown in Fig. 1(B).
- *c)* To obtain the higher-order minimum cut sets of system in above same way.

(ii) To classify the units according to unit type, i.e. the units belonging to the same unit type as a classification.

(iii) To determine the life test unit according to the following rules:

- *a)* To conduct life test for all classifications of units if conditions allow, such as enough testing expenses and testing facilities.
- *b)* To conduct life test for part classifications of units belonging the minimum cut sets of system if condition does not allow.



**FIGURE 1.** Process of One-order minimum cut set and Two-order minimum cut set.

#### 4) COLLECTING DATA OF LIFE TEST UNIT FOR COMPLEX **SYSTEMS**

Collecting data of life test unit is another key step of evaluating Life Expectancy for complex system, which is of crucial importance, and it includes:

(i) To determine test type of life test unit. There are five test types, i.e. complete samples life test, fixed time censoring with replacement life test, non-substitute time tac-tail life test, fixed number censoring with replacement life test, and non-substitute number tac-tail life test. The principles of determining test type of life test unit are as follows:

- *a)* To conduct complete samples life tests if condition allow, such as enough testing expenses and testing facilities.
- *b)* To conduct fixed failure number test and fixed time test if condition does not allow.

(ii) To conduct unit test according to test type of life test unit.

(iii) To collect test data. According to test type of life test unit, the life data of each sample should be collected.

#### 5) EVALUATING FAILURE RATE OF UNIT FOR COMPLEX **SYSTEMS**

The failure rate of unit is used to calculating the reliability of unit at different time point. There are two kinds of units, which are non-life test unit and life test unit, respectively. For non-life test unit, it needs not to evaluate its failure rate, and its failure rate is set 0. For life test unit, according to the test type of life test unit to evaluate its failure rate, the corresponding steps are as follows:

(i) To select evaluation method of failure rate for life test unit. For the large sample of life test unit, the evaluating method by using test data can be selected. Because the Bayes method can evaluate the failure rate of life test unit by making the most of its prior information, and using a small number of its test data. Thus, for the small sample of life test unit, it needs to select the Bayes method.

(ii) To evaluate the basic failure rate of life test unit. The steps of the evaluating method by using test data are as follows:

- *a)* To determine the total time test according to selecting data of life test unit.
- *b)* To develop the likelihood function according to test data of test sample.
- *c)* To obtain the point estimation of basic failure rate for life test unit by solving the likelihood equation, which is obtained by using logarithm derivation for likelihood function.

The steps of the Bayes method are as follows:

- *a)* To determine the total time test according to selecting data of life test unit.
- *b)* To develop the likelihood function according to test data of test sample, as shown in Eq. [\(1\)](#page-3-0).

<span id="page-3-0"></span>
$$
L(X | \lambda) = L(x_1, x_2, \cdots, x_n; \lambda)
$$
 (1)

where,  $X = (x_1, x_2, \dots, x_n)$  is the test data of test sample,  $x_i$  is the test data of *ith* test sample,  $\lambda$  is the basic failure rate of life test unit,  $L(X | \lambda)$  is the likelihood function.

- *c*) To determine the prior distribution  $\pi(\lambda)$  of the basic failure rate for life test unit.
- *d*) To determine the posteriori distribution  $\pi(\lambda | X)$  of the basic failure rate for life test unit, as shown in Eq. [\(2\)](#page-3-1).

<span id="page-3-1"></span>
$$
\pi(\lambda \mid X) = \frac{h(X, \lambda)}{m(X)} = \frac{L(X \mid \lambda)\pi(\lambda)}{\int_{\Theta} L(X \mid \lambda)\pi(\lambda)d\lambda} \tag{2}
$$

where,  $m(X)$  is the marginal density function of X,  $h(X, \lambda)$  is the joint distribution of *X* and  $\lambda$ .

*e)* To obtain the point estimation of basic failure rate for life test unit by calculating the expectation of  $\pi(\lambda | X)$ , as shown in Eq. [\(3\)](#page-3-2).

<span id="page-3-2"></span>
$$
\widehat{\lambda} = E(\lambda | X)
$$
 (3)

where,  $\widehat{\lambda}$  is the point estimation of basic failure rate for life test unit,  $E(\lambda|X)$  is the expectation of  $\pi(\lambda|X)$ .

(iii) To evaluate the failure rate of life test unit by Eq. [\(4\)](#page-4-0).

<span id="page-4-0"></span>
$$
a = \lambda \cdot \alpha_W \cdot \pi_Q \cdot \pi_E \tag{4}
$$

where, *a* is the point estimation of failure rate for life test unit;  $\alpha_W$  is the correction factor of life test unit and time dependent functions, and it is equal to specific value of average lifetime and rated operating time of unit;  $\pi$ <sup>*Q*</sup> is the quality rank coefficient of life test unit;  $\pi_E$  is the environmental influence coefficient of life test unit. For life test unit obeying exponential distribution,  $\alpha_W$ ,  $\pi_Q$  and  $\pi_E$  are 1, respectively; For life test unit obeying non exponential distribution,  $\alpha_W$ ,  $\pi_Q$ and  $\pi_E$  should be determined according to the experimental data and historical data of life test unit, respectively.

#### 6) EVALUATING LIFE EXPECTANCY OF COMPLEX SYSTEMS

The Life Expectancy of system is evaluated by using test data of unit based on GO method. The evaluating steps are as follows:

(i) To determine the time points for fitting system reliability function. In order to fit system reliability function accurately, the time point should be throughout the whole system life cycle for Life Expectancy assessment as much as possible.

(ii) To calculate the unit reliability at different time points by using the failure rate of unit according to Eq. [\(5\)](#page-4-1).

<span id="page-4-1"></span>
$$
R(t) = e^{-at} \tag{5}
$$

where,  $R(t)$  is the reliability of unit at time  $t$ .

(iii) To calculate the system reliability at different time points based on the quantitative analysis of GO method by using the unit reliability. For the GO model without shared signals, the quantitative analysis of GO method adopts the direct algorithm [21]. And for the GO model with shared signals, the quantitative analysis of GO method adopts the exact algorithm with shared signals [22], otherwise the system reliability at different time points will have a large bias [16].

(iv) To evaluate system Life Expectancy by Eq. [\(6\)](#page-4-2).

<span id="page-4-2"></span>
$$
MTTF = \int_0^{T_0} R(t)dt
$$
 (6)

where,  $R(t)$  is the system reliability function  $T_0$  is the upper limitation of system estimation life,.

#### B. PROCESS OF THE NEW LIFE EXPECTANCY ASSESSMENT METHOD

The process of the new Life Expectancy assessment method for complex systems with multi-characteristics based on GO method is the operation standard in order to guide the Life Expectancy assessment accurately. In this study, we formulate the new Life Expectancy assessment method proposed in this paper according to Section II-A, as shown in Fig. 2.

#### **III. EXAMPLE**

The power-shift steering transmission control system (PSSTCS) is a typical mechatronics control system with multi-characteristics, such as multi-state, multi-function, one system for multi-conditions, and correlations. Usually, in real



#### 1) TO ANALYZE STRUCTURE AND FUNCTION CONSTITUTES OF PSSTCS

The PSSTCS is composed of a hydraulic oil supply system, an integration pump-motor system, a fan control system, an electronic control system, and hydraulic control system. And the hydraulic oil supply system consist of a fill oil and constant pressure system of pressure oil tank, a pump group, and a fill oil system of transmission control and fan control.



**FIGURE 2.** Process of new life expectancy assessment method for complex system based on GO method.

project, because the value of PSSTCS is more than one million dollars, the life-test of system for PSSTCS is rarely conducted. Thus, taking the power-shift steering transmission control system (PSSTCS) as a case study, it is used to illustrate the usage method, advantages, feasibility and reasonability of the new Life Expectancy assessment method for complex system based on GO method. In order to present this example and compare with Monte Carlo method conveniently, we assume the following.

- (1) The oil tubes and interfaces of the PSSTCS are not considered.
- (2) In Eq. [\(4\)](#page-4-0),  $\alpha_W$  is set 1.
- (3) The testing expenses and testing facilities cannot support all units to conduct life test.
- (4) In this case, the Life Expectancy of PSSTCS operating 500 hours, which is an overhaul period, is evaluated.

The function constitutes and the structure principle drawing of PSSTCS are shown in Fig. 3 and Fig. 4, respectively.



**FIGURE 3.** Function constitutes of PSSTCS.



**FIGURE 4.** Structure principle drawing of PSSTCS.

#### 2) TO DETERMINE SYSTEM CHARACTERISTICS OF PSSTCS

According to engineering practice, the PSSTCS is a complex system with multi-characteristics, which are standby correlation, multi-state, multi-function and one system for multi-conditions. The system characteristics of PSSTCS are analyzed, as follows:

(i) To analyze the standby correlation structure of PSSTCS

When LF2 group is obstructed and pressure between input and output becomes more than 0.5 mega Pascal, oil will be injected into pressure oil tank via LF2B. LF3 and LF3B are same like LF2 group and LF2B. Thus, the LF2 group and LF2B, LF3 and LF3B are standby correlation structures.

(ii) To analyze the multi-state unit of PSSTCS

Pump group often output oil with unstable oil pressure, and RV1 and RV2 are constant pressure valve of hydraulic control system and integration pump-motor system, respectively. They can turn unstable oil pressure of system into the goal oil pressure, so that they make system normal operating and improve the system reliability.

(iii) To analyze the multi-function and one system for multi-conditions of PSSTCS

The PSSTCS can adjust the speed of left and right fans at the same time by the fan control system, control left and right turnings by the integration pump-motor system, and achieve various speed thresholds under electronic control condition and manual emergency condition by the hydraulic control system, respectively. The hydraulic control system achieves Gear 1L, 1, 2, 3, 4, R1 and R2 under electronic control condition, and achieves Gear 1, 3 and R1 under manual emergency condition. The shifting strategy of the hydraulic control system is presented in Tab. II. In Tab.1, E1, E2 and E3 are clutches, and F1, F2 and F3 are brakes. Different combination of clutches and brakes achieve different gears.

**TABLE 2.** Shifting strategy of hydraulic control system for PSSTCS.

Gear	E <sub>3</sub>	E2	E1	F1	F2	F3
1L						
C	$\times$					
3						
4						
R <sub>1</sub>						
R <sub>2</sub>						

#### 3) TO DEFINE THE SUCCESS RULE OF PSSTCS

According to above analysis of PSSTCS, the success rule can be defined as that system can achieve the control of the steering, speed changing, fan driving, and lubricating.

#### B. ESTABLISHING GO MODEL OF PSSTCS

#### 1) TO SELECT GO OPERATOR IN GO MODEL OF PSSTCS

According the system analysis result and the types of GO operator, the function GO operators, logical GO operators and auxiliary GO operator are selected to describe the units itself, logical relationships, auxiliary GO operation in PSSTCS, respectively, as presented in Tab. III and Tab. IV.

#### 2) TO DEVELOP GO MODEL OF PSSTCS

The GO model of PSSTCS is developed by using the signal flows to connect above GO operators, as shown in Appendix I. The type and numbering of GO operator are represented by the first number and the second number in GO operator, respectively. The numbering of signal flow is represented by the number on signal flow. And the signal flow 124 is the system output of PSSTCS.

#### C. SELECTING LIFE TEST UNIT OF PSSTCS

#### 1) TO OBTAIN THE MINIMUM CUT SETS OF PSSTCS

The minimum cut sets of PSSTCS are obtained according to Section II-A, as presented in Tab. V. And they are used to provide the guidance for selecting life test unit.

#### 2) TO CLASSIFY THE UNITS

Because the testing expenses and testing facilities cannot support all units to conduct life test, the life test units are





selected in minimum cut sets of PSSTCS. And the units of minimum cut sets for PSSTCS are classified according to unit type, as presented in Tab. VI.

#### 3) TO DETERMINE THE LIFE TEST UNIT

 $\overline{\phantom{0}}$ 

In this case, although the testing expenses and testing facilities cannot support all units to conduct life test, they can support 10 samples for each classification presented in Tab. VI to conduct life test.

#### D. COLLECTING DATA OF LIFE TEST UNIT FOR PSSTCS

1) TO DETERMINE TEST TYPE OF LIFE TEST UNIT

In this case, all life test units are conduct to the complete samples life tests.

#### 2) TO CONDUCT UNIT TEST

According to complete samples life tests, 10 samples of each classification are conduct unit test until failures of them.

#### **TABLE 3.** Function GO operator in GO model of PSSTCS. **TABLE 3.** (Continued.) Function GO operator in GO model of PSSTCS.

**TABLE 5.** Minimum cut sets of PSSTCS.







#### **TABLE 6.** Classifications of minimum cut sets for PSSTCS.



#### 3) TO COLLECT TEST DATA

According to the complete samples life tests, the life data of each sample for all classifications are collected, as presented in Appendix II.

#### E. EVALUATING FAILURE RATE OF UNIT FOR PSSTCS

For non-life test units, their failure rates are set 0. And for life test units, their failure rates are evaluated, as follows:

#### 1) TO SELECT EVALUATION METHOD FOR EVALUATING BASIC FAILURE RATE OF LIFE TEST UNIT

Because the samples of each life test units are very small, whose number is 10, the Bayes method is adopted to evaluate the basic failure rates of life test units in this case.

#### 2) TO EVALUATE THE BASIC FAILURE RATE OF LIFE TEST UNIT

According to Section II-A, the deducing processes of the point estimation formula of the basic failure rate for life test unit are as follows:

(i) To determine the total time test according to selecting data of life test unit, as Eq. [\(7\)](#page-8-0).

<span id="page-8-0"></span>
$$
T_j = \sum_{i=1}^{10} t_{ji}
$$
 (7)

where,  $T_j$  is the total time of *jth* classification,  $j =$ 1, 2,  $\dots$ , 28;  $t_{ji}$  is the test date of *ith* life test unit for *jth* classification,  $i = 1, 2, \dots, 10$ .

(ii) To develop the likelihood function, as shown in Eq. [\(8\)](#page-8-1).

<span id="page-8-1"></span>
$$
L(T_j|\lambda) = \prod_{i=1}^{10} P(t_{ji}|\lambda)
$$
  
=  $\lambda^{10} \exp\left\{-\lambda \sum_{i=1}^{10} t_{ji}\right\} = \lambda^{10} \exp\left\{-\lambda T_j\right\}$  (8)

where,  $L(T_j|\lambda)$  is the likelihood function.

(iii) In this case,  $\pi(\lambda)$  is adopted the Jeffery's prior distribution.

(iv) To determine the posteriori distribution  $f\left(\lambda \middle| T_j\right)$  of the basic failure rate for life test unit, as shown in Eq. [\(9\)](#page-8-2). It obeys the Gamma distribution with two parameters  $\Gamma\left(\lambda \middle| 10, T_j\right)$ .

<span id="page-8-2"></span>
$$
f\left(\lambda \left|T_j\right.\right) = \frac{L(T_j \left|\lambda\right) \pi(\lambda)}{\int_{\Theta} L(T_j \left|\lambda\right) \pi(\lambda) d\lambda} = \frac{T_j^{10} \lambda^9 e^{-\lambda T_j}}{\Gamma(10)}\tag{9}
$$

(v) To obtain the point estimation of the basic failure rate for life test unit by calculating the expectation of  $f(\lambda | T_j)$ , as shown in Eq. [\(10\)](#page-8-3).

<span id="page-8-3"></span>
$$
\hat{\lambda} = E(\lambda | T_j) = \int_0^{+\infty} \lambda \cdot \frac{T_j^{10}}{\Gamma(10)} \lambda^{10-1} e^{-T_j \lambda} d\lambda \n= \int_0^{+\infty} \frac{T_j^{10}}{\Gamma(10)} \cdot \frac{\Gamma(10+1)}{T_j^{10+1}} \n\cdot \frac{T_j^{10+1}}{\Gamma(10+1)} \lambda^{10+1-1} e^{-T_j \lambda} d\lambda \n= \frac{T_j^{10}}{\Gamma(10)} \cdot \frac{\Gamma(11)}{T_j^{11}} = \frac{10\Gamma(10)}{\Gamma(10) T_j} = \frac{10}{T_j}
$$
\n(10)

where,  $E(\lambda | T_j)$  is the expectation of  $f(\lambda | T_j)$ .



**TABLE 8.** Reference value of  $\pi_{0}$ .



**TABLE 9.** Reference value of  $\pi_{\bm{E}}$ .



3) TO EVALUATE THE FAILURE RATE OF LIFE TEST UNIT BY EQ. [\(4\)](#page-4-0), AS PRESENTED IN TAB. VII. π*<sup>Q</sup>* AND π*<sup>E</sup>* REFER TO TAB. VIII AND TAB. VIIII, RESPECTIVELY [29].

#### F. EVALUATING LIFE EXPECTANCY OF PSSTCS

#### 1) TO DETERMINE THE TIME POINTS FOR FITTING SYSTEM RELIABILITY FUNCTION OF PSSTCS

In this case, the 500 hours are averagely divided into 100 time points in order to fitting system reliability function of PSSTCS greatly.

#### 2) TO CALCULATE THE UNIT RELIABILITY AT DIFFERENT TIME POINTS

According to Eq. [\(5\)](#page-4-1), the reliabilities of units can be obtained by using their failure rates, which for the non-life test units and the life test units are 0 and the data presented in Tab. VII, respectively.

#### 3) TO CALCULATE THE RELIABILITY OF PSSTCS AT DIFFERENT TIME POINTS

The signal flow 1, 4, 7, 8, 15, 19, 20, 25, 29, 46, 59, 69, 70, 71, 73 and 115 are shared signals in Fig. 4, so the exact algorithm with shared signals is adopted to calculate the system reliability. The reliabilities of PSSTCS at different time points are presented in Tab. X.

#### G. TO EVALUATE LIFE EXPECTANCY OF PSSTCS

The system reliability function is fitted by using the data of Tab. X, as Eq.  $(11)$ . And according to Eq.  $(6)$ , the Life

**TABLE 10.** Reliability of PSSTCS at different time points.

NO.		NO.		
(time point)	System reliability	(time point)	System reliability	
1	0.9676	26	0.2651	
$\overline{c}$	0.9341	27	0.2488	
$\overline{\mathbf{3}}$	0.8999	28	0.2334	
4	0.8652	29	0.2188	
5	0.8304	30	0.2051	
6	0.7956	31	0.1921	
$\overline{7}$	0.7611	32	0.1798	
8	0.7270	33	0.1682	
9	0.6934	34	0.1574	
10	0.6605	35	0.1471	
11	0.6283	36	0.1375	
12	0.5970	37	0.1284	
13	0.5666	38	0.1199	
14	0.5372	39	0.1119	
15	0.5088	40	0.1045	
16	0.4815	41	0.0974	
17	0.4552	42	0.0909	
18	0.4299	43	0.0847	
19	0.4057	44	0.0789	
20	0.3826	45	0.0735	
21	0.3605	46	0.0685	
22	0.3395	47	0.0638	
23	0.3194	48	0.0593	
24	0.3004	49	0.0552	
25	0.2823	50	0.0514	
51	0.0478	76	0.0073	
52	0.0444	77	0.0068	
53	0.0413	78	0.0063	
54	0.0384	79	0.0058	
55	0.0357	80	0.0054	
56	0.0331	81	0.0050	
57	0.0308	82	0.0046	
58	0.0286	83	0.0043	
59	0.0265	84	0.0040	
60	0.0246	85	0.0037	
61	0.0229	86	0.0034	
62	0.0212	87	0.0031	
63	0.0197	88	0.0029	
64	0.0183	89	0.0027	
65	0.0169	90	0.0025	
66	0.0157	91	0.0023	
67	0.0146	92	0.0021	
68	0.0135	93	0.0020	
69	0.0125	94	0.0018	
70	0.0116	95	0.0017	
71	0.0108	96	0.0015	
72	0.0100	97	0.0014	
73	0.0092	98	0.0013	
74	0.0085	99	0.0012	
75	0.0079	100	0.0011	

Expectancy of PSSTCS can be obtained, as Eq. [\(12\)](#page-9-0).

<span id="page-9-0"></span>
$$
R(t) = e^{-0.01009t}, 0 \le t \le 500 \text{ hours}
$$
(11)  
MTTF =  $\int_0^{500} R(t)dt = \int_0^{500} e^{-0.01009t}dt = 98.4696 \text{ hours}$  (12)

#### **IV. RESULTS AND ANALYSIS**

Monte Carlo method is widely used to evaluate system Life Expectancy. Thus, in order to verify the advantages, feasibility and rationality of the new Life Expectancy assessment method for complex systems proposed in this paper, its result is compared with the results by Monte Carlo methods simulating 100,000 times and 1,000,000 times. And this paper's Life Expectancy assessment method and Monte Carlo method simulating 1,000,000 times operate 3 times. In addition to evaluating the failure rates of units, the Life Expectancy assessment steps by Monte Carlo method are mainly

- (1) To establishing Monte Carlo Simulation model according to the logical relationship of units and systems.
- (2) To calculate the unit reliability at the *ith* time point by using the data of Tab. VII and Eq.  $(5)$ ,  $i =$  $1, 2, \cdots, 100.$
- (3) To generate random number according to unit reliability at the *ith* time point.
- (4) To simulate a given number of simulation times according to Monte Carlo simulation model to obtain system reliability at the *ith* time point.
- (5) To repeat step [\(2\)](#page-3-1) and step [\(4\)](#page-4-0), the system reliabilities at all-time points are obtained.
- (6) To fit system reliability function by using the data of step [\(5\)](#page-4-1).
- (7) To evaluate system Life Expectancy by Eq. [\(6\)](#page-4-2)

Above all, the Life Expectancies of PSSTCS by Monte Carlo methods and this paper's method are presented in Tab. XI and Tab. XII.

#### **TABLE 11.** Comparison I: assessment result and evaluation efficiency.



#### **TABLE 12.** Comparison II: Stability.



According to Tab. XI, the Life Expectancy of PSSTCS by this paper's method is in close proximity to the result by Monte Carlo method simulating 1,000,000 times, so it indicates that the new Life Expectancy assessment method proposed in this paper is feasible and reasonable. Furthermore, the Monte Carlo simulation model does not connect the system structure, function constitutes, and system characteristics closely, and it is not easy to check. And the Monte Carlo method does not take into consideration the system characteristics of PSSTCS, which can improve the system reliability, so that the Life Expectancies of PSSTCS by Monte Carlo methods are less than the result of this paper's method. Moreover, the operation time of this paper's method is much less than the operation times of Monte Carlo method simulating 100,000 times and 1,000,000 times. Thus, it shows that this paper's method has obvious advantages in aspects of establishing reliability model, evaluation accuracy and operation efficiency.

According to Tab. XII, the Life Expectancies of PSSTCS by Monte Carlo method at different operation number are





fluctuant because of the influence of sampling, but the Life Expectancies by this paper's method at different operation number are same. Clearly, it indicates that this paper's method can avoid the complex sampling process and the influence of sampling, so that it can obtain a stable and unique assessment result.

The process of this paper's method shows that it has some obvious advantageous, as follows:

- (1) Only the reliability data of unit is used to evaluate the system Life Expectancy, so that it can save evaluation cost.
- (2) GO model is closely linked to system structure, function constitute and system characteristics, so it is easy to check for complex systems with multicharacteristics. And GO operation can avoid the influence of sampling effectively.

(3) The evaluation process of this paper's method is easy to operate.

#### **V. CONCLUSION**

This study proposes a new Life Expectancy for complex systems with multi-characteristics based on GO method. First, the new Life Expectancy assessment method is expounded in detail from conducting system analysis of complex systems, establishing GO model of complex systems, selecting life test unit of complex systems, collecting data of life test unit for complex systems, evaluating failure rate of unit for complex systems, and evaluating Life Expectancy of complex systems. On this base, its process is formulated. Then, the control system of power-shift steering transmission for a heavy vehicle, which is a typical mechatronics control system with multi-characteristics, such as multi-state, multi-function,

#### **TABLE 13.** Test data of PSSTCS.



one system for multi-conditions, and standby correlation, is taken as an example to evaluate its Life Expectancy by this paper's method. Finally, in order to verify the advantages, feasibility and rationality of the new Life Expectancy assessment method, the results are compared with the results by Monte Carlo method. Moreover, compared with the evaluation results by Monte Carlo methods, it shows that this paper's method has the following outstanding advantages:

- (1) This paper's method can take into consideration various system characteristics to evaluate Life Expectancy of complex systems, so that its evaluation accuracy is higher.
- (2) This paper's method uses the GO model as system reliability model, so that it can connect the system structure, function constitutes, and system characteristics directly and closely, and it is easy to check.
- (3) This paper's method uses the GO operation to obtain the system reliability at different time points, so that it can avoids the influence of sampling for result to obtain the stable evaluation result, but it also has higher efficiency.
- (4) This paper's method uses only the reliability data of unit to evaluate the system Life Expectancy, so that it can save evaluation cost.
- (5) This paper' method has a brief and clear assessment process, and it is easy to make program to operate. Moreover, the Life Expectancy assessment results have high-level consistency by different engineer(s).

All in all, this Life Expectancy assessment method not only improves the theory of GO method, so that GO method is applied to evaluate the Life Expectancy of complex systems with multi-characteristics firstly; but it also can overcome the disadvantages of the existing system Life Expectancy evaluation methods efficiently. Furthermore, this paper provides a new approach for Life Expectancy assessment of complex systems, which can takes into consideration various system characteristics, and only uses unit life data, so that it is low cost, higher estimating efficiency and accuracy, and a more stable assessment result.

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#### **APPENDIX I**

See Fig. 5.

### **APPENDIX II**

See Table 13.

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