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Adaptive Prescribed Performance Fault Estimation and Accommodation for a Class of Stochastic Nonlinear Systems

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ABSTRACT This paper studies the problem of adaptive fault estimation and accommodation for a class of stochastic nonlinear systems with unknown time-varying faults. Different from existing fault estimation methods, a novel adaptive prescribed performance fault estimator is designed, which guarantees that the fault estimation error is confined within a pre-set region, and a better estimation accuracy is obtained. The designed fault-tolerant tracking controller is capable of attenuating the effect of faults through using back-stepping techniques. Furthermore, the proposed method guarantees that all the error signals of the closed-loop system be bounded in probability, the fault estimation error and output tracking error both converge to desired neighborhoods of origin in the sense of quadratic mean value. Finally, the simulation results are provided to verify the method proposed in this paper.

INDEX TERMS Adaptive fault estimation, fault-tolerant control, stochastic nonlinear systems, back-stepping technique.

I. INTRODUCTION

It is well known that fault detection (FD) and fault-tolerant control (FTC) have been an active research area over the past several decades, and it has been applied to flight control systems [1], [2], complex networks [3], [4], chemical processes [5], robot models [6], fuzzy methods [7]–[9], Markov jump systems [10]–[12]. Particularly, a fault-tolerant controller was designed based on adaptive fault estimation in [13] for nonlinear system with dead-zone nonlinearity, and for a class of stochastic nonlinear systems, fault estimation and fault tolerant control problems are considered in [14] and [15], respectively. Fuzzy adaptive fault-tolerant tracking controller was designed to compensate actuator failures in [16]. In [17] and [18], fault detection methods are proposed for switched control systems in finite frequency domain and full frequency domain, respectively. In [19], a fault estimation scheme was developed for a class of linear systems with Lipschitz nonlinearities and actuator failures. In addition, in [20] and [21], sensor failures were compensated by using adaptive mechanisms for output feedback systems.

On the other hand, stochastic disturbances often exist in practical systems, which may lead to severe performance deterioration or even instability of closed-loop systems [22]–[24]. Many significant control strategies have been proposed for stochastic nonlinear systems [25]–[34]. In [35], the tracking control object in the sense of quadratic mean value can be achieved for a class of stochastic nonlinear systems, and the proposed method can guarantee that all the closed-loop signals be bounded in probability. Liu *et al.* [36] studied the decentralized adaptive control strategy for a class of large-scale stochastic nonlinear systems based on output feedback. With unknown nonlinear functions being handled by fuzzy logic systems, the adaptive fuzzy control problem for a class of uncertain nonlinear stochastic systems was considered in [37]. Based on intermediate value theorem and neural network approximation technique, the adaptive tracking control scheme for a class of stochastic nonlinear systems with dead-zone nonlinearities was developed in [38]. Moreover, for non-Gaussian stochastic distributed control systems, a fault diagnosis and fault-tolerant control method was proposed by using Takagi-Sugeno fuzzy model in [39].

However, to the best knowledge of us, fault estimation and accommodation for uncertain nonlinear systems with stochastic disturbances have not been well investigated in the literature. The main difficulties are that 1) the design of observer-based fault estimation algorithms are quite difficult for uncertain nonlinear system; and 2) the exact information of stochastic disturbance is unknown, which may lead to severe deterioration in the accuracy of fault estimation. To deal with these issues, the prescribed performance constraint technique [40]–[43] is introduced in this paper to design fault estimation and accommodation schemes. Compared with existing methods, the main contributions of this paper are summarized as follows: 1) A novel adaptive prescribed performance fault estimator (APPFE) is designed to guarantee that fault estimation error is confined within a pre-set region for a class of stochastic nonlinear systems, which is more general than existing methods. 2) A fault-tolerant tracking control scheme is developed to compensate faults based on back-stepping technique, where disturbances are also attenuated effectively. 3) A prescribed performance constrained term is constructed such that the adaptive fault estimation law proposed in this paper can receive better estimation accuracy than exiting method. In addition, the proposed method guarantees that all the signals of the closed-loop system be bounded in probability, and the fault estimation error and output tracking error converge to desired neighborhoods of origin in the sense of quadratic mean value.

The organization of the rest of this paper are arranged as follows. In Section 2, the problem formulation and some related assumptions are given. In Section 3, APPFE is designed. In Section 4, a fault-tolerant tracking control scheme is proposed. Simulation results are provided in Section 5 to verify the efficiency of the approach developed in this paper. Section 6 concludes this paper.

II. PRELIMINARIES AND PROBLEM STATEMENT

A. STOCHASTIC NONLINEAR SYSTEM DESCRIPTIONS

Consider the following stochastic nonlinear system

$$\begin{aligned} dx_1 &= (x_2 + f_1(\bar{x}_1))dt + W_1^T(y)d\omega, \\ dx_2 &= (x_3 + f_2(\bar{x}_2))dt + W_2^T(y)d\omega, \\ &\vdots \\ dx_{n-1} &= (x_n + f_{n-1}(\bar{x}_{n-1}))dt + W_{n-1}^T(y)d\omega, \\ dx_n &= (\tau v + f_n(\bar{x}_n) + \theta(t))dt + W_n^T(y)d\omega, \\ y &= x_1 \end{aligned} \quad (1)$$

where $\bar{x}_i = [x_1, x_2, \cdots, x_i]^T \in \mathbb{R}^i, i = 1, 2, \cdots, n$ are system states, $v \in \mathbb{R}$ is control input, and $y \in \mathbb{R}$ denotes the output. ω is an $l-$ dimensional standard Brownian motion, which is defined on the complete probability space $(\Omega, \mathcal{F}, P)$. Ω denotes a sample space, $\mathcal{F}$ is a $\sigma-$field, and P represents a probability measure, respectively. τ is a known control gain, $f_j(\bar{x}_{j+1}) : \mathbb{R}^{j+1} \to \mathbb{R}, j = 1, 2, \cdots, n-1$, $W_i(\cdot) : \mathbb{R} \to \mathbb{R}^l$ stand for known smooth system functions with $f_i(0) = W_i(0) = 0, i = 1, 2, \cdots, n$, and $\theta(t)$ is an unknown time-varying fault function which needs to be estimated.

B. STOCHASTIC STABILITY

The following definitions and lemmas are essential for later development, consider the following stochastic system

$$dx = F(x)dt + H(x)d\omega \quad (2)$$

where x and ω are defined in (1), and $F(\cdot) : \mathbb{R}^n \to \mathbb{R}^n$ and $H(\cdot) : \mathbb{R}^n \to \mathbb{R}^{n\times r}$ are locally Lipschitz functions in x and satisfy $F(0) = 0$ and $H(0) = 0$, respectively.

Definition 1 [44]: For any given $V(x) \in C^2$, associated with the stochastic differential equation (2), define the differential operator $\mathcal{L}$ as follows

$$\mathcal{L}V = (\frac{\partial V}{\partial x})^T F(x) + \frac{1}{2}Tr\{H^T(x)\frac{\partial^2 V}{\partial x^2}H(x)\} \quad (3)$$

where $Tr(A)$ is the trace of A.

Definition 2 [44]: The solution process $\{x(t)|t \geq 0\}$ of stochastic system (2) is said to be bounded in probability, if $lim_{n\to\infty} \sup_{t\geq 0} P\{||x(t)|| > n\} = 0$, where $P\{A\}$ denotes the probability of event A.

Lemma 1 [45]: Consider the stochastic system (2). If there exists a positive definite, radially unbounded, twice continuously differentiable Lyapunov function $V(\cdot) : \mathbb{R}^n \to \mathbb{R}$, and constants $\lambda_1 > 0, \lambda_2 > 0$ such that the following inequality

$$\mathcal{L}V \leq -\lambda_1 V + \lambda_2 \quad (4)$$

then the system (2) has a unique solution almost surely and is bounded in probability.

Lemma 2 [48]: For $\forall(X, Y) \in \mathbb{R}^2$, the following inequality holds

$$XY \leq \frac{c_0^a}{a}|X|^a + \frac{1}{bc_0^b}|Y|^b \quad (5)$$

where $c_0 > 0, a > 1, b > 1$, and $\frac{1}{a} + \frac{1}{b} = 1$.

The objects of this paper are as follows: 1) a APPFE is designed such that the fault estimation error is confined within a pre-set region; 2) the designed fault-tolerant tracking controller guarantees that all the closed-loop signals are bounded in probability; and 3) the system output tracking error converges to desired neighborhoods of origin in the sense of quadratic mean value. To achieve these control objectives, the following assumptions are necessary.

Assumption 1 The time-varying fault function $\theta(t)$ and its derivative $\dot{\theta}(t)$ satisfy that $\bar{\theta}_l^* \leq |\theta(t)| \leq \bar{\theta}_u^*$ and $|\dot{\theta}(t)| \leq \bar{\dot{\theta}}^*$ with $\bar{\theta}_l^*$ and $\bar{\theta}_u^*$ being two positive known constants, and $\bar{\dot{\theta}}^*$ being a positive unknown constant, respectively.

Assumption 2 The nonlinear function $f_i(\cdot)$ is assumed to be subject to Lipschitz for any $x_{i1}, x_{i2} \in \mathbb{R}^i$, i.e., there exist the Lipschitz constants L_i such that $|f_i(x_{i1}) - f_i(x_{i2})| \leq L_i\|x_{i1} - x_{i2}\|, i = 1, 2, \cdots, n$.

Assumption 3 [38]: For each $1 \leq i \leq n$, there exists a known nonnegative smooth function $\omega_i(\cdot) : \mathbb{R} \to \mathbb{R}$ with $\omega_i(0) = 0$ such that $||W_i(y)|| \leq \omega_i(y)$.

Remark 1 Assumption 1 is standard in the literature. A large number of fault signals and their derivatives are bounded, for slowing time varying faults [46] and actuator stuck faults [47], the bounds of faults can be known. Assumption 2 implies that nonlinear functions $f_i(\cdot), i = 1, 2, \cdots, n$ are subject to Lipschitz, and it is common for many practical systems. By constructing nonnegative smooth functions, Assumption 3 means that all the stochastic disturbances of nonlinear system (1) are bounded within an appropriate compact set.

III. ADAPTIVE PRESCRIBED PERFORMANCE FAULT ESTIMATOR DESIGN

In this section, to achieve the fault estimation and fault-tolerant tracking control objective, a novel APPFE is designed to obtain the estimates of both unknown faults and unmeasured states. Firstly, note that stochastic nonlinear system (1) can be rewritten as

$$\begin{aligned} dx &= (Ux + Ly + \sum_{i=1}^{n-1} B_i f_i(\bar{x}_i) \\ &\quad + B(\tau v + f_n(x) + \theta(t)))dt + W(x)dw \\ y &= Cx \end{aligned} \tag{6}$$

where $L = [l_1, l_2, \cdots, l_n]^T, B = [0, 0, \cdots, 1]^T, B_i = [0, \cdots, 0, \underbrace{1}_{ith}, 0, \cdots, 0]^T, i = 1, 2, \cdots, n-1, W(x) = [W_1(y), W_2(y), \cdots, W_n(y)]^T, C = [1, 0, \cdots, 0]$ and

$$U = \begin{bmatrix} -l_1 & 1 & 0 & \cdots & 0 \\ -l_2 & 0 & 1 & \cdots & 0 \\ & \ddots & & \ddots & \\ -l_{n-1} & 0 & 0 & \cdots & 1 \\ -l_n & 0 & 0 & \cdots & 0 \end{bmatrix} \tag{7}$$

By choosing appropriate parameters $l_i, i = 1, 2, \cdots, n$, U can be constructed as an Hurwitz matrix from the structure of U. Moreover, there exists a positive definite matrix $P = P^T > 0$ such that

$$U^T P + PU = -Q \tag{8}$$

with $Q = Q^T > 0$, where U, P, Q are matrices of appropriate dimensions.

Next, the APPFE is designed as follows

$$\begin{aligned} \dot{\hat{x}} &= U\hat{x} + Ly + \sum_{i=1}^{n-1} B_i f_i(\hat{\bar{x}}_i) \\ &\quad + B(\tau v + f_n(\hat{x}) + \hat{\theta}(t)) \\ \hat{y} &= C\hat{x} \end{aligned} \tag{9}$$

with fault estimation update law

$$\dot{\hat{\xi}} = -\kappa_0\hat{\xi} - \kappa_0(\tau v + f_n(\hat{x}) + \kappa_0\hat{x}_n) + N(\theta_\sigma) \tag{10}$$

where v is control input and $\xi = \theta - \kappa_0 x_n$, $\hat{\xi} = \hat{\theta} - \kappa_0\hat{x}_n$, $\tilde{\theta} = \theta - \hat{\theta}$ with $\hat{\theta}$ being the estimation of θ, and $\kappa_0 > 0$, l_i, $i = 1, 2, \cdots, n$ are corresponding design parameters. Especially, prescribed performance function $N(\theta_\sigma)$ is defined as

$$N(\theta_\sigma) = \frac{1}{2}\frac{\theta_\sigma}{\rho_\theta^2 - \theta_\sigma^2} + \tau_0\theta_\sigma - \kappa_0 l_n(y - \hat{x}_1) + \epsilon_0^{-1}\rho_\theta^2\theta_\sigma \tag{11}$$

with θ_σ being chosen as

$$\theta_\sigma = \begin{cases} \theta_{\sigma 0}, & \hat{\theta} \le \bar{\theta}_l^*, \\ \bar{\theta}_\sigma, & \bar{\theta}_l^* < \hat{\theta} < \bar{\theta}_u^*, \\ -\theta_{\sigma 0}, & \hat{\theta} \ge \bar{\theta}_u^* \end{cases} \tag{12}$$

where $\tau_0, \kappa_0, \epsilon_0, \theta_{\sigma 0}$ are positive constants, and ρ_θ is a positive adjustable parameter satisfying $|\tilde{\theta}| \le \theta_{\sigma 0} < \rho_\theta$, and $\bar{\theta}_\sigma = \bar{\theta}_u^* - \hat{\theta}$ satisfying $|\bar{\theta}_\sigma| \le \theta_{\sigma 0}^* < \rho_\theta$, where $\theta_{\sigma 0}^*$ is a positive constant. For convenience, denote $\theta_{\sigma m} = \max\{\theta_{\sigma 0}, \theta_{\sigma 0}^*\}$ and a prescribed set $\Omega_{\tilde{\theta}} := \{\tilde{\theta} | |\tilde{\theta}| < \rho_\theta\}$, which will be specified later.

Remark 2 It should be pointed out that the proposed adaptive fault estimation algorithm is different from existing results. By introducing a nonlinear prescribed performance term $N(\tilde{\theta})$, both the unmeasurable states and unknown fault signals are estimated by the APPFE in (9)-(12) with better estimation accuracy. In addition, the fault estimation error $\tilde{\theta}$ is confined in a pre-assigned set $\Omega_{\tilde{\theta}}$ by the driving signal v and the tuning signal θ_σ, respectively.

Based upon (6) and (9), the estimation error equation of the stochastic nonlinear system (1) can be obtained as

$$\begin{aligned} d\tilde{x} &= (U\tilde{x} + \sum_{i=1}^{n-1} B_i(f_i(\bar{x}_i) - f_i(\hat{\bar{x}}_i)) \\ &\quad + B(f_n(x) - \hat{f}_n(\hat{x}) + \tilde{\theta}(t)))dt + W(x)dw \end{aligned} \tag{13}$$

where $\tilde{x} = x - \hat{x}$ is the observer error vector. Furthermore, the following lemma is necessary for the stability analysis.

Lemma 3 Under Assumptions 1-3, given positive constants τ_0, κ_0, ϵ_i, $i = 0, 1, 2, 3, 4$, if the initial value of $\tilde{\theta}$ satisfies $\tilde{\theta}(0) \in \Omega_{\tilde{\theta}}$, fault estimator (9)-(12) guarantees that fault estimation error $\tilde{\theta}$ is uniformly ultimately bounded.

Proof: Choose Lyapunov function

$$V_0 = \tilde{x}^T P\tilde{x} + \frac{1}{2}\log(\frac{\rho_\theta^2}{\rho_\theta^2 - \tilde{\theta}^2}) + \frac{1}{2}\kappa_0^{-1}\tilde{\xi}^2$$

for the error system (13), similar to the proof of [41, Lemma 1], we can obtain that the fault estimator (9)-(12) guarantee that the following inequality holds

$$\mathcal{L}V_0 \le -\tilde{x}^T(Q - \mu I)\tilde{x} - \chi\tilde{\xi}^2 - \frac{\tau_0\tilde{\theta}^2}{\rho_\theta^2 - \tilde{\theta}^2} + \nu(y) \tag{14}$$

hold, where $\mu = \epsilon_0\|P\|^2 + \sum_{i=1}^n L_i\|P\| + \frac{L_n^2}{2\epsilon_2} + \frac{\kappa_0}{2\epsilon_2} + \frac{l_n^2}{2\epsilon_4}$, $\chi = 1 - \frac{\kappa_0^{-1}}{2} - \frac{\epsilon_1}{2} - \frac{\epsilon_2}{2} - \frac{\kappa_0\epsilon_3}{2} - \frac{\kappa_0^{-1}\epsilon_4}{2}$ *and* $\nu(y) = \|P\|\sum_{i=1}^n \bar{W}_i^2\omega_i^2(y) + \frac{1}{2}\bar{\dot{\theta}}^{*2} + \epsilon_1^{-1}\bar{\theta}^{*2} + \frac{\kappa_0^{-1}}{2}(\tau_0\rho_\theta + \frac{1}{2\rho_\theta} + \frac{\rho_\theta^3}{\epsilon_0})^2$, *in addition, since* $\log(\frac{\rho_\theta^2}{\rho_\theta^2 - \tilde{\theta}^2}) < \frac{\tilde{\theta}^2}{\rho_\theta^2 - \tilde{\theta}^2}$ *where* $|\tilde{\theta}| < \rho_\theta$ *[41], we have that* $\tilde{\theta}$ *is uniformly ultimately bounded by adjusting* *parameters τ_0, κ_0, ϵ_i, $i = 0, 1, 2, 3, 4$ appropriately, this completes the proof.*

Remark 3 A novel adaptive fault estimation law (10) with a prescribed performance term (11) and a tuning function (12) is designed to obtain better fault estimation performance. Also, the proposed adaptive law (10) will degrade into (17) of [13] when the prescribed performance term $N(\theta_\sigma)$ becomes zero. This implies that (17) of [13] is a special case of (10) of this paper.

IV. FAULT-TOLERANT TRACKING CONTROLLER DESIGN

In this section, a fault-tolerant tracking control scheme will be developed for stochastic nonlinear system (1) based on back-stepping design method. Accordingly, the recursive design procedure contains n steps. From Step 1 to $n-1$, a virtual control is constructed at each step by Young's inequality and $It\hat{o}$ formula. In the end, the actual control v is designed at Step n. For this purpose, the controller design begins with the following transformations

$$\begin{aligned} e_1 &= y - y_d \\ e_i &= \hat{x}_i - \alpha_{i-1}, \quad i = 2, 3, \cdots, n \end{aligned} \tag{15}$$

where y_d is a reference signal of n-order differentiability, α_{i-1} is the virtual control function in Step $i-1$, and the actual controller v will be given in Step n. For simplicity, denote $\bar{y}_d^{(i)} = [y_d, \dot{y}_d, \cdots, y_d^{(i)}]^T, i = 1, 2, \cdots, n-1$. Next, the standard back-stepping method will be used to design fault-tolerant tracking controller.

Step 1: Invoking (1), (15) and $It\hat{o}$ formula, it can be obtained that

$$\begin{aligned} de_1 &= (x_2 + f_1(x_1) - \dot{y}_d)dt + W_1^T(y)dw \\ &= (\hat{x}_2 + \tilde{x}_2 + f_1(x_1) - \dot{y}_d)dt + W_1^T(y)dw \end{aligned} \tag{16}$$

Choose the Lyapunov function as follows

$$V_1 = V_0 + \frac{1}{4}e_1^4 \tag{17}$$

Based on (16), taking the differential operator of (17) yields

$$\begin{aligned} \mathcal{L}V_1 &= \mathcal{L}V_0 + e_1^3(\hat{x}_2 + \tilde{x}_2 + f_1(x_1) - \dot{y}_d) + \frac{3}{2}e_1^2||W_1(y)||^2 \\ &\le -\tilde{x}^T(Q - \mu I)\tilde{x} - \chi\tilde{\xi}^2 - \frac{\tau_0\tilde{\theta}^2}{\rho_\theta^2 - \tilde{\theta}^2} + \nu(y) + e_1^3\tilde{x}_2 \\ &\quad + e_1^3(\hat{x}_2 + f_1(x_1) - \dot{y}_d) + \frac{3}{2}e_1^2||W_1(y)||^2 \end{aligned} \tag{18}$$

By triangle inequalities $e_1^3\tilde{x}_2 \le \frac{1}{2}e_1^6 + \frac{1}{2}||x||^2$ and $e_1^2||W_1(y)||^2 \le \frac{1}{2}c_1^2 + \frac{1}{2c_1^2}e_1^4\omega_1^4(y)$, we have

$$\begin{aligned} \mathcal{L}V_1 &\le -\tilde{x}^T(Q - (\mu + \frac{1}{2})I)\tilde{x} - \chi\tilde{\xi}^2 - \frac{\tau_0\tilde{\theta}^2}{\rho_\theta^2 - \tilde{\theta}^2} + \nu(y) \\ &\quad + e_1^3(\hat{x}_2 + S_1(\hat{\bar{E}}_1)) - \frac{3}{4}e_1^4 + \frac{3}{4}c_1^2 \end{aligned} \tag{19}$$

where $c_1 > 0$ is a design parameter, and $S_1(\hat{\bar{E}}_1) = \frac{3}{4c_1^2}e_1\omega_1^4(y) + \frac{1}{2}e_1^3 + \frac{3}{4}e_1 - \dot{y}_d$ with $\hat{\bar{E}}_1$ being $\hat{\bar{E}}_1 = [x_1, y_d, \dot{y}_d]^T \in \mathbb{R}^3$.

Furthermore, it should be noted that $||P||\sum_{i=1}^n \omega_i^2(y) \le ||P||\sum_{i=1}^n \omega_i^{*2}$ with ω_i^* being an upper bound of $\omega_i(y)$ over the corresponding compact $\Omega_{\hat{\bar{E}}_1}$. Accordingly, setting $\nu^* = ||P||\sum_{i=1}^n \omega_i^{*2} + \frac{\kappa_0^{-1}}{2}(\tau_0\rho_\theta + \frac{1}{2\rho_\theta} + \frac{\rho_\theta^3}{\epsilon_0})^2$, and combining (15), (19) becomes

$$\begin{aligned} \mathcal{L}V_1 &\le -\tilde{x}^T(Q - (\mu + \frac{1}{2})I)\tilde{x} - \chi\tilde{\xi}^2 - \frac{\tau_0\tilde{\theta}^2}{\rho_\theta^2 - \tilde{\theta}^2} + \nu^* \\ &\quad + e_1^3e_2 + e_1^3(\alpha_1 + S_1(\hat{\bar{E}}_1)) - \frac{3}{4}e_1^4 + \frac{3}{4}c_1^2 \end{aligned} \tag{20}$$

By utilizing Lemma 2 with $p = \frac{3}{4}$, $q = 4$, one can get that

$$e_1^3e_2 \le \frac{3}{4}e_1^4 + \frac{1}{4}e_2^4 \tag{21}$$

Construct a virtual control function α_1 as follows

$$\alpha_1 = -k_1e_1 - S_1(\hat{\bar{E}}_1) \tag{22}$$

where k_1 is a positive design parameter.

Substituting (21) and (22) into (20) leads to

$$\begin{aligned} \mathcal{L}V_1 &\le -\tilde{x}^T(Q - (\mu + \frac{1}{2})I)\tilde{x} - \chi\tilde{\xi}^2 - \frac{\tau_0\tilde{\theta}^2}{\rho_\theta^2 - \tilde{\theta}^2} + \nu^* \\ &\quad + \frac{1}{4}e_2^4 - k_1e_1^4 + \frac{3}{4}c_1^2 \end{aligned} \tag{23}$$

where the function $\frac{1}{4}e_2^4$ will be handled in the next step.

Step 2: Using $It\hat{o}$ formula for $e_2 = \hat{x}_2 - \alpha_1$ gives

$$\begin{aligned} de_2 &= (\hat{x}_3 + f_2(\hat{\bar{x}}_2) + l_2\tilde{x}_1)dt - \sum_{i=0}^{1}\frac{\partial\alpha_1}{\partial y_d^{(i)}}y_d^{(i+1)}dt \\ &\quad - \frac{1}{2}\frac{\partial^2\alpha_1}{\partial x_1^2}||W_1(y)||^2dt - \frac{\partial\alpha_1}{\partial x_1}dx_1 \\ &= (\hat{x}_3 + f_2(\hat{\bar{x}}_2) + l_2\tilde{x}_1 - \sum_{i=0}^{1}\frac{\partial\alpha_1}{\partial y_d^{(i)}}y_d^{(i+1)} \\ &\quad - \frac{\partial\alpha_1}{\partial x_1}(x_2 + f_1(x_1)) - \frac{1}{2}\frac{\partial^2\alpha_1}{\partial x_1^2}||W_1(y)||^2)dt \\ &\quad - \frac{\partial\alpha_1}{\partial x_1}W_1^T(y)dw \end{aligned} \tag{24}$$

Consider the following Lyapunov function

$$V_2 = V_1 + \frac{1}{4}e_2^4 \tag{25}$$

applying the differential operator to V_2 yields

$$\begin{aligned} \mathcal{L}V_2 &\le \mathcal{L}V_1 + e_2^3(\hat{x}_3 + f_2(\hat{\bar{x}}_2) + l_2\tilde{x}_1 - \sum_{i=0}^{1}\frac{\partial\alpha_1}{\partial y_d^{(i)}}y_d^{(i+1)} \\ &\quad - \frac{\partial\alpha_1}{\partial x_1}(\hat{x}_2 + f_1(x_1)) - \frac{1}{2}\frac{\partial^2\alpha_1}{\partial x_1^2}||W_1(y)||^2) \\ &\quad - \frac{\partial\alpha_1}{\partial x_1}e_2^3\tilde{x}_2 + \frac{3}{2}e_2^2|\frac{\partial\alpha_1}{\partial x_1}|^2||W_1(y)||^2 \end{aligned} \tag{26}$$

Noticing that $-\frac{\partial\alpha_1}{\partial x_1}e_2^3\tilde{x}_2 \leq \frac{1}{2}||\tilde{x}||^2 + \frac{1}{2}|\frac{\partial\alpha_1}{\partial x_1}|^2 e_2^6$ and $e_2^2|\frac{\partial\alpha_1}{\partial x_1}|^2||W_1(y)||^2 \leq \frac{1}{2}c_2^2 + \frac{1}{2c_2^2}e_2^4|\frac{\partial\alpha_1}{\partial x_1}|^4\omega_1^4(y)$ and using (15), (26) becomes

$$\begin{aligned}\mathcal{L}V_2 &\leq -\tilde{x}^T(Q-(\alpha+1)I)\tilde{x} - \chi\tilde{\xi}^2 - \frac{\tau_0\tilde{\theta}^2}{\rho_\theta^2-\tilde{\theta}^2} + \nu^* \\ &\quad - k_1e_1^4 + e_2^3(\hat{x}_3 + S_2(\hat{\bar{E}}_2)) - \frac{3}{4}e_2^4 + \frac{3}{4}c_2^2 + \frac{3}{4}c_1^2 \\ &= -\tilde{x}^T(Q-(\alpha+1)I)\tilde{x} - \chi\tilde{\xi}^2 - \frac{\tau_0\tilde{\theta}^2}{\rho_\theta^2-\tilde{\theta}^2} + \nu^* \\ &\quad - k_1e_1^4 + e_2^3e_3 + e_2^3(\alpha_2 + S_2(\hat{\bar{E}}_2)) - \frac{3}{4}e_2^4 \\ &\quad + \frac{3}{4}c_2^2 + \frac{3}{4}c_1^2 \end{aligned} \tag{27}$$

where $c_2 > 0$ is a design parameter, and $S_2(\hat{\bar{E}}_2) = \frac{3}{4c_2^2}e_2|\frac{\partial\alpha_1}{\partial x_1}|^4\omega_1^4(y) + \frac{1}{2}|\frac{\partial\alpha_1}{\partial x_1}|^2e_2^3 + f_2(\hat{\bar{x}}_2) + l_2\tilde{x}_1 - \frac{\partial\alpha_1}{\partial x_1}(\hat{x}_2 + f_1(x_1)) - \sum_{i=0}^{1}\frac{\partial\alpha_1}{\partial y_d^{(i)}}y_d^{(i+1)} + \frac{3}{4}e_2$ with $\hat{\bar{E}}_2$ being $\hat{\bar{E}}_2 = [x_1, \hat{x}_1, \hat{x}_2, y_d, \dot{y}_d, \ddot{y}_d]^T \in \mathbb{R}^6$.

Subsequently, by $e_2^3e_3 \leq \frac{3}{4}e_2^4 + \frac{1}{4}e_3^4$, the virtual control function α_2 can be chosen as

$$\alpha_2 = -k_2e_2 - S_2(\hat{\bar{E}}_2) \tag{28}$$

where k_2 is a positive design parameter. Substituting (28) into (27) yields

$$\begin{aligned}\mathcal{L}V_2 &\leq -\tilde{x}^T(Q-(\alpha+1)I)\tilde{x} - \chi\tilde{\xi}^2 - \frac{\tau_0\tilde{\theta}^2}{\rho_\theta^2-\tilde{\theta}^2} + \nu^* \\ &\quad - \sum_{i=1}^{2}k_ie_i^4 + \frac{1}{4}e_3^4 + \frac{3}{4}\sum_{i=1}^{2}c_i^2 \end{aligned} \tag{29}$$

with the function $\frac{1}{4}e_3^4$ will be handled in the next step.

Step $i(3 \leq i \leq n-1)$: Recursively, by invoking $It\hat{o}$ formula for $e_i = \hat{x}_i - \alpha_{i-1}$, we have

$$\begin{aligned} de_i &= (\hat{x}_{i+1} + f_i(\hat{\bar{x}}_i) + l_i\tilde{x}_1)dt - \sum_{j=0}^{i-1}\frac{\partial\alpha_{i-1}}{\partial y_d^{(j)}}y_d^{(j+1)}dt \\ &\quad - \sum_{j=2}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\hat{x}_j}(\hat{x}_{j+1} + f_j(\hat{\bar{x}}_j) + l_j\tilde{x}_1)dt \\ &\quad - \frac{1}{2}\frac{\partial^2\alpha_{i-1}}{\partial x_1^2}||W_1(y)||^2dt - \frac{\partial\alpha_{i-1}}{\partial x_1}dx_1 \\ &= (\hat{x}_{i+1} + f_i(\hat{\bar{x}}_i) + l_ie_1) - \sum_{j=0}^{i-1}\frac{\partial\alpha_{i-1}}{\partial y_d^{(j)}}y_d^{(j+1)} \\ &\quad - \frac{\partial\alpha_{i-1}}{\partial x_1}(x_2 + f_1(x_1)) - \frac{1}{2}\frac{\partial^2\alpha_{i-1}}{\partial x_1^2}||W_1(y)||^2 \\ &\quad - \sum_{j=2}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\hat{x}_j}(\hat{x}_{j+1} + f_j(\hat{\bar{x}}_j) + l_je_1))dt \\ &\quad - \frac{\partial\alpha_{i-1}}{\partial x_1}W_1^T(y)dw \end{aligned} \tag{30}$$

Then, the Lyapunov function can be chosen as follows

$$V_i = V_{i-1} + \frac{1}{4}e_i^4 \tag{31}$$

From (31), taking the differential operator of V_i yields

$$\begin{aligned}\mathcal{L}V_i &\leq \mathcal{L}V_{i-1} + e_i^3(\hat{x}_{i+1} + f_i(\hat{\bar{x}}_i) + l_i\tilde{x}_1 - \sum_{j=0}^{i-1}\frac{\partial\alpha_i}{\partial y_d^{(j)}}y_d^{(j+1)} \\ &\quad - \frac{\partial\alpha_{i-1}}{\partial x_1}(\hat{x}_2 + f_1(x_1)) - \frac{1}{2}\frac{\partial^2\alpha_{i-1}}{\partial x_1^2}||W_1(y)||^2 \\ &\quad - \sum_{j=2}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\hat{x}_j}(\hat{x}_{j+1} + f_j(\hat{\bar{x}}_j) + l_je_1)) - \frac{\partial\alpha_{i-1}}{\partial x_1}e_i^3\tilde{x}_2 \\ &\quad + \frac{3}{2}e_2^2|\frac{\partial\alpha_{i-1}}{\partial x_1}|^2||W_1(y)||^2 \end{aligned} \tag{32}$$

Applying mathematical induction procedures and noting the fact that $-\frac{\partial\alpha_{i-1}}{\partial x_1}e_i^3\tilde{x}_2 \leq \frac{1}{2}||\tilde{x}||^2 + \frac{1}{2}|\frac{\partial\alpha_{i-1}}{\partial x_1}|^2e_i^6$ and $e_i^2|\frac{\partial\alpha_{i-1}}{\partial x_1}|^2||W_1(y)||^2 \leq \frac{1}{2}c_i^2 + \frac{1}{2c_i^2}e_i^4|\frac{\partial\alpha_{i-1}}{\partial x_1}|^4\omega_1^4(y)$, (32) becomes

$$\begin{aligned}\mathcal{L}V_i &\leq -\tilde{x}^T(Q-(\alpha+\frac{i}{2})I)\tilde{x} - \chi\tilde{\xi}^2 - \frac{\tau_0\tilde{\theta}^2}{\rho_\theta^2-\tilde{\theta}^2} + \nu^* \\ &\quad - \sum_{j=1}^{i-1}k_je_j^4 + e_i^3(\hat{x}_{i+1} + S_i(\hat{\bar{E}}_i)) - \frac{3}{4}e_i^4 + \frac{3}{4}\sum_{j=1}^{i}c_j^2 \\ &\leq -\tilde{x}^T(Q-(\alpha+\frac{i}{2})I)\tilde{x} - \chi\tilde{\xi}^2 - \frac{\tau_0\tilde{\theta}^2}{\rho_\theta^2-\tilde{\theta}^2} + \nu^* \\ &\quad - \sum_{j=1}^{i-1}k_je_j^4 + e_i^3e_{i+1} + e_i^3(\alpha_i + S_i(\hat{\bar{E}}_i)) \\ &\quad - \frac{3}{4}e_i^4 + \frac{3}{4}\sum_{j=1}^{i}c_j^2 \end{aligned} \tag{33}$$

where $c_i > 0$ is a design parameter, and $S_i(\hat{\bar{E}}_i) = \frac{3}{4c_i^2}e_i|\frac{\partial\alpha_{i-1}}{\partial x_1}|^4\omega_1^4(y) + \frac{1}{2}|\frac{\partial\alpha_{i-1}}{\partial x_1}|^2e_i^3 + f_i(\hat{\bar{x}}_i) + l_i\tilde{x}_1 - \frac{\partial\alpha_{i-1}}{\partial x_1}(\hat{x}_2 + f_1(x_1)) - \sum_{j=0}^{i-1}\frac{\partial\alpha_{i-1}}{\partial y_d^{(j)}}y_d^{(j+1)} - \sum_{j=2}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\hat{x}_j}(\hat{x}_{j+1} + f_j(\hat{\bar{x}}_j) + l_je_1) + \frac{3}{4}e_i$ with $\hat{\bar{E}}_i$ being $\hat{\bar{E}}_i = [x_1, \hat{\bar{x}}_i, \bar{y}_d^{(i)}]^T \in \mathbb{R}^{2i+2}$.

By using the Young's inequality $e_i^3e_{i+1} \leq \frac{3}{4}e_i^4 + \frac{1}{4}e_{i+1}^4$, the virtual control function α_i is chosen as follows

$$\alpha_i = -k_ie_i - S_i(\hat{\bar{E}}_i) \tag{34}$$

with k_i is a positive design parameter. Substituting (34) into (33) yields

$$\begin{aligned}\mathcal{L}V_i &\leq -\tilde{x}^T(Q-(\alpha+\frac{i}{2})I)\tilde{x} - \chi\tilde{\xi}^2 - \frac{\tau_0\tilde{\theta}^2}{\rho_\theta^2-\tilde{\theta}^2} + \nu^* \\ &\quad - \sum_{j=1}^{i}k_je_j^4 + \frac{1}{4}e_{i+1}^4 + \frac{3}{4}\sum_{j=1}^{i}c_j^2 \end{aligned} \tag{35}$$

where $\frac{1}{4}e_{i+1}^4$ will be handled in the next step.

Step n: The actual control input v will be obtained in final step. Invoking $It\hat{o}$ formula for $e_n = \hat{x}_n - \alpha_{n-1}$ yields

$$de_n = (\tau v + f_n(\hat{\bar{x}}_n) + l_n\tilde{x}_1 - \sum_{j=0}^{n-1}\frac{\partial\alpha_{n-1}}{\partial y_d^{(j)}}y_d^{(j+1)} - \sum_{j=2}^{i-1}\frac{\partial\alpha_{n-1}}{\partial\hat{x}_j}(\hat{x}_{j+1} + f_j(\hat{\bar{x}}_j) + l_je_1) - \frac{\partial\alpha_{n-1}}{\partial x_1}e_2 - \frac{\partial\alpha_{n-1}}{\partial x_1}(\hat{x}_2 + f_1(x_1)) - \frac{1}{2}\frac{\partial^2\alpha_{n-1}}{\partial x_1^2}||W_1(y)||^2)dt - \frac{\partial\alpha_{n-1}}{\partial x_1}W_1^T(y)dw \quad (36)$$

Then, choose the Lyapunov function as follows

$$V_n = V_{n-1} + \frac{1}{4}e_n^4 \quad (37)$$

From (37), the differential operator of V_n is

$$\mathcal{L}V_n \leq \mathcal{L}V_{n-1} + e_n^3(\tau v + f_n(\hat{\bar{x}}_n) + l_n\tilde{x}_1 - \sum_{j=0}^{n-1}\frac{\partial\alpha_{n-1}}{\partial y_d^{(j)}}y_d^{(j+1)} - \frac{\partial\alpha_{n-1}}{\partial x_1}(\hat{x}_2 + f_1(x_1)) - \frac{1}{2}\frac{\partial^2\alpha_{n-1}}{\partial x_1^2}||W_1(y)||^2 - \sum_{j=2}^{n-1}\frac{\partial\alpha_{n-1}}{\partial\hat{x}_j}(\hat{x}_{j+1} + f_j(\hat{\bar{x}}_j) + l_j\tilde{x}_1)) - \frac{\partial\alpha_{n-1}}{\partial x_1}e_n^3\tilde{x}_2 + \frac{3}{2}e_n^2|\frac{\partial\alpha_{n-1}}{\partial x_1}|^2||W_1(y)||^2 \quad (38)$$

By using the inequalities $-\frac{\partial\alpha_{n-1}}{\partial x_1}e_n^3\tilde{x}_2 \leq \frac{1}{2}||\tilde{x}||^2 + \frac{1}{2}|\frac{\partial\alpha_{n-1}}{\partial x_1}|^2e_n^6$ and $e_n^2|\frac{\partial\alpha_{n-1}}{\partial x_1}|^2||W_1(y)||^2 \leq \frac{1}{2}c_n^2 + \frac{1}{2c_n^2}e_n^4|\frac{\partial\alpha_{n-1}}{\partial x_1}|^4\omega_1^4(y)$, we have

$$\mathcal{L}V_n \leq -\tilde{x}^T(Q - (\alpha + \frac{n}{2})I)\tilde{x} - \chi\tilde{\xi}^2 - \frac{\tau_0\tilde{\theta}^2}{\rho_\theta^2 - \tilde{\theta}^2} + \nu^* - \sum_{j=1}^{n-1}k_je_j^4 + e_n^3(\tau v + S_n(\hat{\bar{E}}_n) + \hat{\theta}) + \frac{3}{4}\sum_{j=1}^{n}c_j^2 \quad (39)$$

where $c_n > 0$ is a design parameter, and $S_n(\hat{\bar{E}}_n) = \frac{3}{4c_n^2}e_n|\frac{\partial\alpha_{n-1}}{\partial x_1}|^4\alpha_1^4(y) + \frac{1}{2}|\frac{\partial\omega_{n-1}}{\partial x_1}|^2e_n^3 + f_n(\hat{\bar{x}}_n) + l_n\tilde{x}_1 - \sum_{j=0}^{n-1}\frac{\partial\alpha_n}{\partial y_d^{(j)}}y_d^{(j+1)} - \frac{\partial\alpha_{n-1}}{\partial x_1}(\hat{x}_2 + f_1(x_1)) - \frac{1}{2}\frac{\partial^2\alpha_{n-1}}{\partial x_1^2}||W_1(y)||^2 - \sum_{j=2}^{n-1}\frac{\partial\alpha_{n-1}}{\partial\hat{x}_j}(\hat{x}_{j+1} + f_j(\hat{\bar{x}}_j) + l_j\tilde{x}_1)$ with $\hat{\bar{E}}_n$ being $\hat{\bar{E}}_n = [x_1, \hat{\bar{x}}_n, \bar{y}_d^{(n)}]^T \in \mathbb{R}^{2n+2}$.

Based on (39), the actual controller can be designed as

$$v = \tau^{-1}(-k_ne_n - S_n(\hat{\bar{E}}_n) - \hat{\theta}) \quad (40)$$

Substituting (40) into (39) leads to

$$\mathcal{L}V_n \leq -\tilde{x}^T(Q - (\alpha + \frac{n}{2})I)\tilde{x} - \chi\tilde{\xi}^2 - \frac{\tau_0\tilde{\theta}^2}{\rho_\theta^2 - \tilde{\theta}^2} + \nu^* - \sum_{j=1}^{n}k_je_j^4 + \frac{3}{4}\sum_{j=1}^{n}c_j^2 \quad (41)$$

Meanwhile, from the definitions of V_0 and V_n, it is easy to see that

$$V_n = \tilde{x}^TP\tilde{x} + \frac{1}{2}\log(\frac{\rho_\theta^2}{\rho_\theta^2 - \tilde{\theta}^2}) + \frac{1}{2}\kappa_0^{-1}\tilde{\xi}^2 + \frac{1}{4}\sum_{i=1}^{n}e_i^4 \quad (42)$$

As proved in [42], the following inequality holds

$$\log(\frac{\rho_\theta^2}{\rho_\theta^2 - \tilde{\theta}^2}) \leq \frac{\tilde{\theta}^2}{\rho_\theta^2 - \tilde{\theta}^2} \quad (43)$$

It follows from (42) and (43) that

$$\mathcal{L}V_n \leq -\lambda_{\min}(Q - (\alpha + \frac{n}{2})I)||\tilde{x}||^2 - \chi\tilde{\xi}^2 - \frac{\tau_0\tilde{\theta}^2}{\rho_\theta^2 - \tilde{\theta}^2} + \nu^* - \sum_{j=1}^{n}k_je_j^4 + \frac{3}{4}\sum_{j=1}^{n}c_j^2 \leq -\lambda_1V_n + \lambda_2 \quad (44)$$

where $\lambda_1 = \min\{\frac{\lambda_{\min}(Q-(\alpha+\frac{n}{2})I)}{\lambda_{\max}(P)}, 2\chi\kappa_0, 2\tau_0, 4k_j|j = 1, 2, \cdots, n\}$ and $\lambda_2 = \frac{3}{4}\sum_{j=1}^{n}c_j^2 + \nu^*$.

So far, the fault-tolerant tracking control design has been completed via back-stepping technique, and the main result is given in the following theorem.

Theorem 1 Consider the stochastic nonlinear system described by (1) with unknown fault function, adaptive fault estimator (9)-(12), and fault-tolerant controller (40). Under Assumptions 1-3, the proposed fault estimation and tracking control method can guarantee that the error of fault estimation is confined in a pre-assigned set, all the closed-loop signals are uniformly bounded in probability, and error signals $\tilde{x}, e_j, j = 1, 2, \cdots, n, \tilde{\xi}$ *remain in the compact sets* $\Lambda_{\tilde{x}}, \Lambda_e, \Lambda_{\tilde{\xi}}$ *and* $\Lambda_{\tilde{\theta}}$ *in the sense that*

$$\Lambda_{\tilde{x}} = \{\tilde{x}|E[\|\tilde{x}\|^2] \leq \sqrt{V_n^*/\lambda_{\min}(P)}\}$$
$$\Lambda_e = \{e_i|E[\sum_{i=1}^{n}|e_i|^4] \leq 4V_n^*, 1 \leq i \leq n\}$$
$$\Lambda_{\tilde{\xi}} = \{\tilde{\xi}||E[|\tilde{\xi}|^2]| \leq 2\kappa_0V_n^*\}$$
$$\Lambda_{\tilde{\theta}} = \{\tilde{\theta}|E[|\tilde{\theta}|] \leq \rho_\theta\sqrt{1 - e^{-\frac{2\lambda_2}{\lambda_1}}}\}, \quad (45)$$

where $V_n^* = V_n|_{t=0} + \frac{\lambda_2}{\lambda_1}$.

Proof: By the above analysis in (44), it can be concluded that $\mathcal{L}V_n \leq -\lambda_1V_n + \lambda_2$. Furthermore, following similar proof of [48, Th.4.1] and [35, eqs. (61)–(63)], we have

$$E[V(t)] \leq V_n|_{t=0} + \frac{\lambda_2}{\lambda_1} \quad (46)$$

where $V_n^* = V_n|_{t=0} = \tilde{x}^T(0)P\tilde{x}(0) + \frac{1}{2}\log(\frac{\rho_\theta^2}{\rho_\theta^2-\tilde{\theta}^2(0)}) + \frac{1}{2}\kappa_0^{-1}\tilde{\xi}^2(0) + \frac{1}{4}\sum_{i=1}^{n}e_i^4(0)$. In addition, considering the following inequality

$$\log(\frac{\rho_\theta^2}{\rho_\theta^2 - \tilde{\theta}^2}) \leq 2(V_n^* - \frac{\lambda_2}{\lambda_1})e^{-\lambda_1 t} + \frac{2\lambda_2}{\lambda_1} \quad (47)$$

Then, taking exponentials on both sides of (47) results in

$$|\tilde{\theta}| \leq \rho_\theta \sqrt{1 - e^{-2(V_n^* - \frac{\lambda_2}{\lambda_1})e^{-\lambda_1 t} - \frac{2\lambda_2}{\lambda_1}}} \tag{48}$$

If $V_n^* = \frac{\lambda_2}{\lambda_1}$, then $|\theta| \leq \rho_\theta \sqrt{1 - e^{-\frac{2\lambda_2}{\lambda_1}}}$. If $V_n^* \neq \frac{\lambda_2}{\lambda_1}$, we conclude that for any given $U > \rho_\theta \sqrt{1 - e^{-\frac{2\lambda_2}{\lambda_1}}}$, the inequality $|\tilde{\theta}| \leq \rho_\theta \sqrt{1 - e^{-\frac{2\lambda_2}{\lambda_1}}}$ holds as $t \to \infty$. Moreover, it follows from (46)-(48) that (45) holds, which implies that all the closed-loop signals $\tilde{x}, \tilde{\xi}, \tilde{\theta}, e_i, i = 1, 2, \cdots, n$ are uniformly bounded in probability. This completes the proof.

Remark 4 From Theorem 1, it can seen that adaptive fault estimation and fault-tolerant tracking control scheme is developed for stochastic nonlinear system based on back-stepping method, and the designed APPFE and fault-tolerant controller can guarantee that all the closed-loop signals are bounded in probability, and the errors of fault estimation and output tracking can converge to a desired neighborhood of origin.

V. SIMULATION STUDIES

In this section, two numerical examples are studied to demonstrate the advantages of the proposed adaptive fault estimation and accommodation scheme.

A. EXAMPLE 1

The following stochastic nonlinear system is first studied:

$$\begin{aligned} dx_1 &= (x_2 + 0.2x_1^2\cos(x_1))dt + 0.1x_1\cos(x_2)dw \\ dx_2 &= (\tau v - \frac{0.3\sin(x_1^2 x_2)}{1 + x_2^2} + \theta)dt \\ &\quad + (0.2 - 0.1x_2^4\sin(x_1))dw \\ y &= x_1 \end{aligned} \tag{49}$$

for $\tau = 1$, and the reference signal is $y_d = 0.2\sin(2t)$. Additionally, the faulty function is designed by

$$\theta(t) = \begin{cases} 0.2, & t < 30, \\ 0.5, & 30 \leq t \leq 50 \end{cases} \tag{50}$$

The simulation parameters and the initial values are chosen as $l_1 = 10, l_2 = 20, k_1 = 6, k_2 = 5, c_1 = c_2 = 5, \rho_\theta = 0.03, \theta_0 = 0.01, \kappa_0 = 5, \tau_0 = 0.8, \epsilon_0 = 0.6$ and $x(0) = [-0.1, 0.1]^T, \hat{x}(0) = [0, 0]^T, \hat{\xi}(0) = 0$, respectively. Fig. 1 shows the fault signal and its estimation by the adaptive observer designed in this paper, and Fig. 2 shows the reference output y_d and the system output y, from which it can be seen that satisfactory fault estimation and output tracking performance are obtained. Fig. 3 and Fig. 4 show the state estimation results.

B. EXAMPLE 2

In this subsection, a single-link robot system [13] with the corresponding stochastic nonlinear model is given by

$$\begin{aligned} dx_1 &= (x_2 + 0.1x_1\cos(2x_1))dt - x_1^3\sin(x_1)dw \\ dx_2 &= (\tau v - \frac{1}{2}mgl\sin(x_1) + \theta)dt \\ &\quad + (x_2^2\cos(x_1 + 0.5))dw \\ y &= x_1 \end{aligned} \tag{51}$$

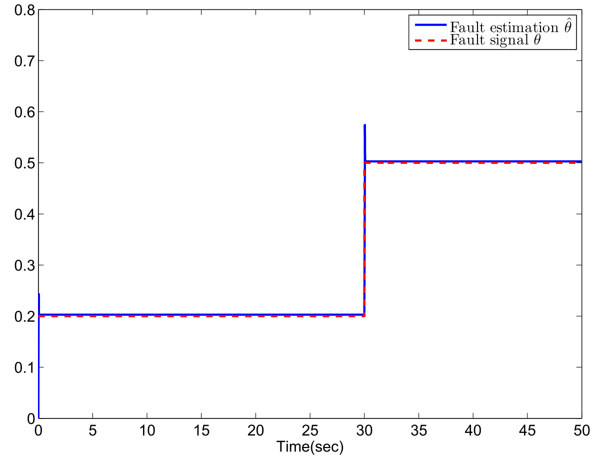

FIGURE 1. Respond curves of fault signal θ and is estimations $\hat{\theta}$ in Example 1.

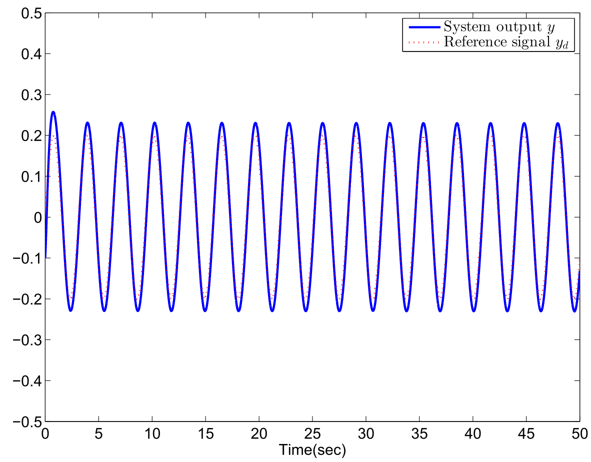

FIGURE 2. System output y and the reference signal y_d in Example 1.

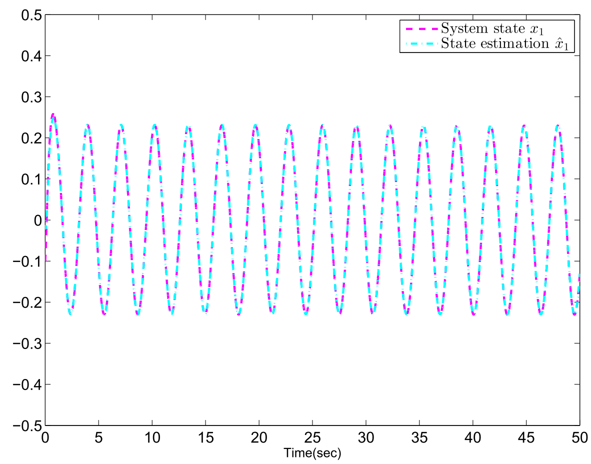

FIGURE 3. System state x_1 and its estimation $\hat{x}_1$ in Example 1.

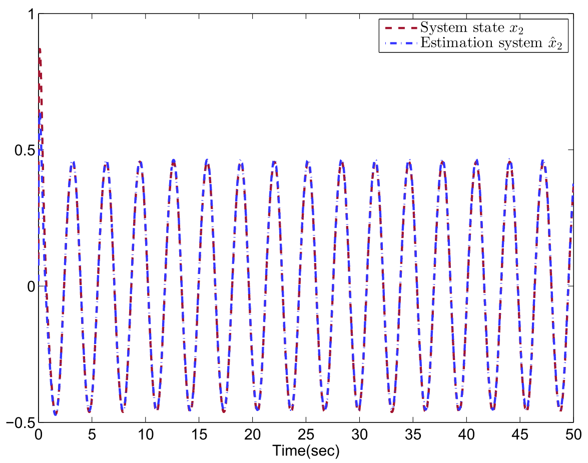

FIGURE 4. State x_2 and its estimation $\hat{x}_2$ in Example 1.

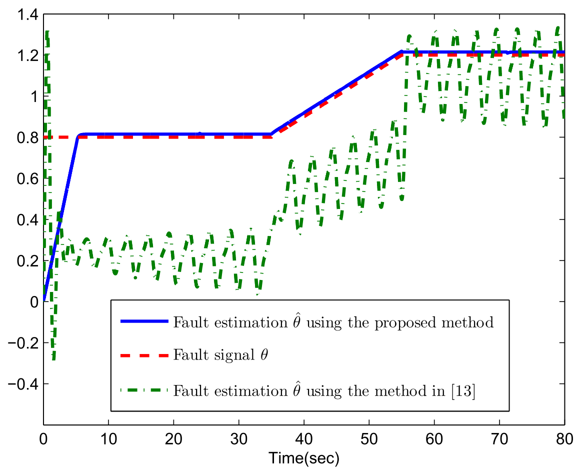

FIGURE 5. Respond curves of fault signal θ and is estimations $\hat{\theta}$.

where $x_1 = q$ and $x_2 = \dot{q}$ are the angle position and angle velocity, and the system parameters are chosen as $m = 1\ kg, M = 0.5kg\,m^2, g = 9.8m/s^2, l = 1m, \tau = \frac{1}{M}$. The reference signal is assumed to be $y_d = \cos(t) + 0.5\sin(t)$, and to verify the method proposed in this paper, fault θ is assumed to be

$$\theta(t) = \begin{cases} 0.8, & t < 35, \\ 0.02t + 0.1, & 35 \le t < 55, \\ 1.8, & 55 \le t \le 80 \end{cases} \tag{52}$$

In addition, the parameters of fault estimator (9)-(10) and fault tolerant controller (40) are chosen as $l_1 = 8, l_2 = 15, k_1 = 2.2, k_2 = 2.2, c_1 = 1, c_2 = 1, \rho_\theta = 0.05, \theta_0 = 0.03, \kappa_0 = 5, \tau_0 = 1, \epsilon_0 = 0.5$, and the initial values are selected as $x(0) = [0, 0.5]^T, \hat{x}(0) = [0, 0]^T, \hat{\xi}(0) = 0$. Figs. 5-6 show the fault estimation results by using the adaptive fault estimator designed in this paper and the non-prescribed performance fault estimation method proposed in [13] without considering the dead-zone effect, from which it can be seen that the fault estimation method proposed in this paper receives better result. The output y and the reference y_d

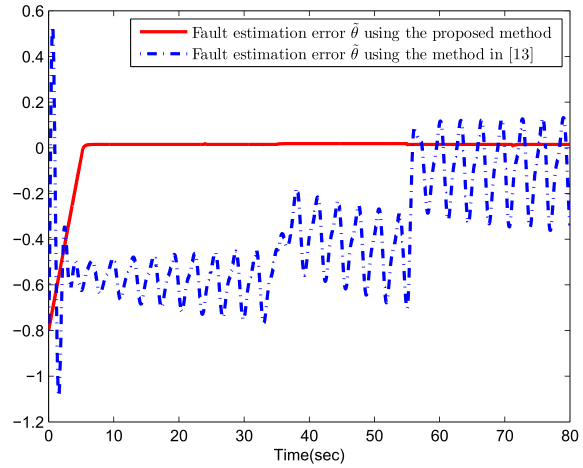

FIGURE 6. Respond curves of fault estimation errors $\tilde{\theta}$.

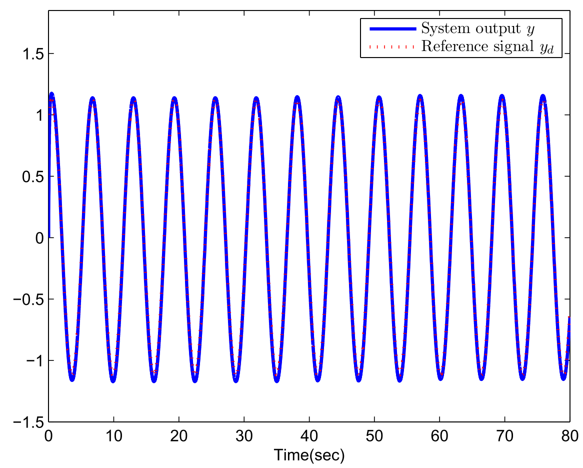

FIGURE 7. System output y and the reference signal y_d.

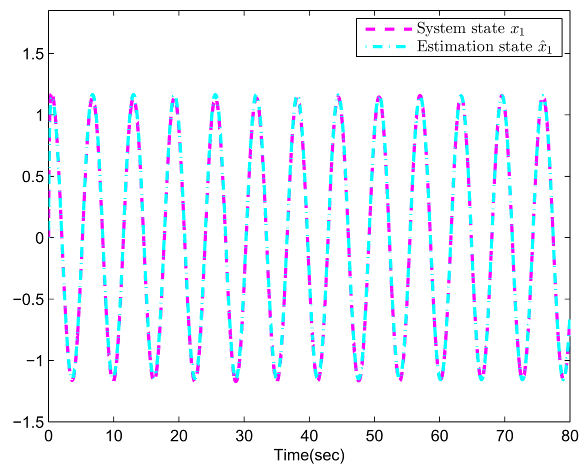

FIGURE 8. System state x_1 and its estimation $\hat{x}_1$.

are shown in Fig. 7, which shows that the output tracking performance is satisfactory and the system output tracking error can converge to a desired neighborhood of origin. State x_1, x_2 and their estimation $\hat{x}_1, \hat{x}_2$ are shown in Figs. 8 and 9, which show that state estimation performance is also satisfactory.

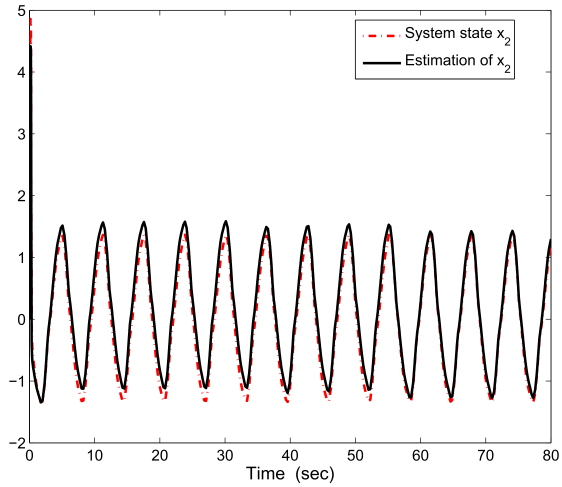

FIGURE 9. State x_2 and its estimation $\hat{x}_2$.

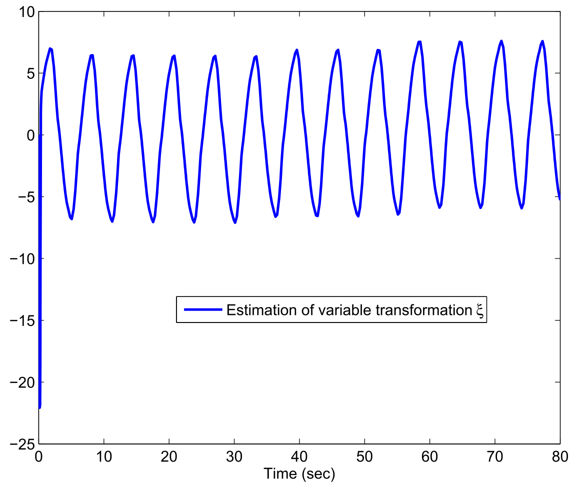

FIGURE 10. Response curve of $\hat{\xi}$.

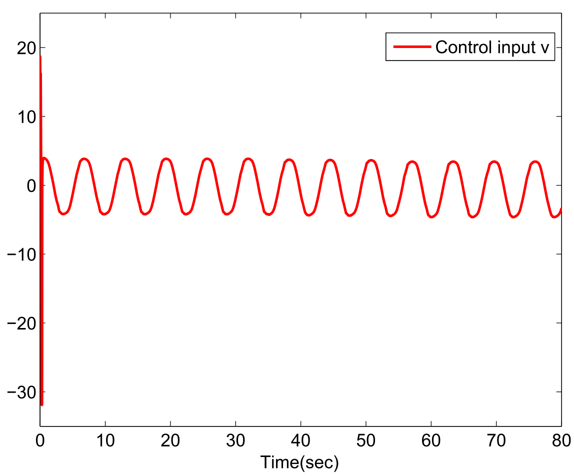

FIGURE 11. Response curve of the control signal $v(t)$.

In addition, the boundedness of fault estimation signal $\hat{\xi}$ and the designed control u are demonstrated in Figs. 10 and 11, respectively.

VI. CONCLUSION

This paper is concerned with the adaptive fault estimation and output tracking control problem for a class of stochastic nonlinear systems. By introducing a prescribed performance term and a switching tuning function, a novel adaptive fault estimator is designed which can receive better estimation accuracy. Moreover, the corresponding fault-tolerant tracking controller is constructed via back-stepping method. The proposed control scheme guarantees that all the closed-loop signals are uniformly bounded in probability, and the system output tracking error can converge to a small neighborhood in the sense of mean quadratic value via stochastic Lyapunov-based analysis. Finally, a numerical example is provided to verify the efficiency of the method proposed.

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