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Fuzzy Fault Detection Filter Design for Nonlinear Distributed Parameter Systems

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ABSTRACT This paper is devoted to investigating the observer-based fault detection (FD) filters for nonlinear distributed processes described by hyperbolic partial differential equations (PDEs). To this end, the PDE systems are first approximated by the Takagi–Sugeno fuzzy models with spatiotemporal uncertainties. Then, the fuzzy FD filter is developed for the hyperbolic PDE systems to guarantee that the residual signal is robust against process inputs including disturbances. The dynamic threshold is designed to ensure the real-time detection of potential faults. It is worth mentioning that the distributed weighting factors are used to weigh the residual signal such that the overall fault detectability can be optimized.

INDEX TERMS Fault detection filter, distributed parameter nonlinear systems, fuzzy dynamic modeling.

I. INTRODUCTION

Over the past decades, observer-based fault detection (FD) and control for industrial processes has become one significant research subject. Numerous approaches have been investigated for linear systems, see for instance [1]–[5] and the references therein. Most recently, intensive research efforts have been made to the nonlinear FD and reliable control approaches [6]–[11]. However, the main focus of the existing results is on those systems that can be expressed by linear or ordinary differential equations (ODEs). Nevertheless, many industrial processes, such as heat conduction and transport-reaction processes, are inherently distributed in space and time, which can be generally described by nonlinear partial differential equations (PDEs) [12]–[14]. Roughly speaking, the PDE systems can be classified into elliptic, hyperbolic, parabolic and mixed type based on the spatial differential operator properties [15]. Owning to the infinite-dimensional nature of PDE processes, the fault detection methodologies for ODE processes can not be applied directly to the fault detection system design for PDE processes.

Over the past decades, considerable research efforts from both the application and academic fields have been made to the analysis and control design schemes for nonlinear processes based on Takagi-Sugeno fuzzy dynamic modeling technique [16]–[19]. As a result, the framework of the controller and filter design schemes for fuzzy systems have been very well established [20]–[22]. Moreover, some results on fuzzy observer-based FD approaches for nonlinear processes have been proposed in [23]–[29]. Inspired by these successful results, significant research efforts have been dedicated to the T-S fuzzy control design approaches for nonlinear PDE systems based on the fuzzy dynamic modeling technique [30]–[36]. Noting that the dominant dynamics of the parabolic PDE system can be approximated by an ODE system, the stabilization issues for H_∞ fuzzy control for nonlinear parabolic PDE systems are addressed via the low-dimensional approximations by applying the Galerkin's method [37], [38]. On the other hand, owing to the fact that the spatial differential operator (SDO) of hyperbolic PDE systems include eigenmodes of nearly the same amount of energy, the infinite-dimensional dynamics of the hyperbolic PDE systems should be taken into consideration in the controller design. To cope with this problem, the systematic stability/performance analysis for a class of nonlinear hyperbolic PDE systems are studied by applying the recursive linear matrix inequality algorithm [31], [32], [34]. So far, very limited attention has been dedicated to develop the fuzzy fault detection schemes for nonlinear hyperbolic PDE systems.

This paper is devoted to investigate the distributed observer-based fault detection filters for nonlinear hyperbolic PDE systems via fuzzy modeling technique. To be specific, the T-S fuzzy model of the nonlinear hyperbolic PDE system is established first. Then the fuzzy fault detection filter is developed for the PDE systems in terms of spatial differential linear matrix inequalities to attain the robustness against uncertainty and process inputs. A recursive algorithm is applied to solve the obtained spatial differential linear matrix inequalities. Specifically, the distributed weighting factors are used to weight the residual signal such that the overall fault detectability can be optimized. Moreover, a dynamic threshold which is a function of input variables is proposed.

This paper is organized as follows. In Section II, the preliminaries on hyperbolic PDE systems and the problem formulation is given. The design method of fuzzy distributed fault detection filter for PDE systems is investigated in Section III. Examples are given in Section IV to show the effectiveness of the proposed approaches.

Notation: The notations adopted in this paper is fairly standard. $\mathtt{Sym}\{M\}$ represents $M+M^T$. $\star$ denotes the symmetric elements in a symmetric matrix. The space-varying matrix $P(\Lambda)$ is said to be positive definite for each $\Lambda \in [k_1,\ k_2]$ if $P(\Lambda) > 0, \Lambda \in [k_1,\ k_2]$. $\mathcal{H}^{k_u} = \mathcal{L}_2([k_1,\ k_2]; \mathcal{R}^{k_u})$ represents the Hilbert space of k_u-dimensional square integrable vector $u(\Lambda, t) \in \mathcal{R}^{k_u}, \Lambda \in [k_1,\ k_2], t \geq 0$ with inner product and norm given by $< u_1(\Lambda, t), u_2(\Lambda, t) >= \int_{k_1}^{k_2} u_1^T(\Lambda, t)u_2(\Lambda, t)\, d\Lambda$, where $u_1(\Lambda, t), u_2(\Lambda, t) \in \mathcal{H}^{k_u}$.

II. PROCESS DESCRIPTION AND PROBLEM FORMULATION

Consider the following class of nonlinear hyperbolic PDE systems

$$\begin{aligned}\frac{\partial z(\Lambda,t)}{\partial t} &= \varphi(\Lambda)\frac{\partial z(\Lambda,t)}{\partial \Lambda} + f(z(\Lambda,t),\Lambda)\\ &\quad + g_2(z(\Lambda,t),\Lambda)w(\Lambda,t) + g_1(z(\Lambda,t),\Lambda)u(\Lambda,t)\\ y(\Lambda,t) &= h(z(\Lambda,t),\Lambda) + m_1(z(\Lambda,t),\Lambda)u(\Lambda,t)\\ &\quad + m_2(z(\Lambda,t),l)w(\Lambda,t) \qquad (1)\end{aligned}$$

where $z(\Lambda, t) \in \mathcal{R}^{k_z}, u(\Lambda, t) \in \mathcal{R}^{k_u}, y(\Lambda, t) \in \mathcal{R}^{k_y}$ represents the vector for the state, the input and the output, respectively; $w(\Lambda, t) \in \mathcal{R}^{k_w}$ indicates the disturbances; t denotes the time; $\Lambda \in [k_1,\ k_2]$ represents the position; $f(z(\Lambda, t), \Lambda)$, $g_1(z(\Lambda, t), \Lambda)$, $g_2(z(\Lambda, t), \Lambda)$, $h(z(\Lambda, t), \Lambda)$, $m_1(z(\Lambda, t), \Lambda)$, $\varphi(\Lambda)$, and $m_2(z(\Lambda, t), \Lambda)$ are continuously differentiable nonlinear functions.

It is assumed that the boundary conditions for the hyperbolic PDE system (1) in this paper is given by

$$M_1 z(k_1,t) + M_2 z(k_2,t) = l(t) \qquad (2)$$

where $l(t)$ is a continuous function of time. M_1 and M_2 are given real matrices of appropriate dimensions.

In this paper, the following T-S fuzzy system is adopted to describe the dynamics of the PDE system (1).

Plant Rule $\Re^i$: *IF* $\theta_1(\Lambda, t)$ is $\mathbb{N}_1^i$ and $\theta_2(\Lambda, t)$ is $\mathbb{N}_2^i$ and $\cdots$ and $\theta_d(\Lambda, t)$ is $\mathbb{N}_d^i$, *THEN*

$$\begin{aligned}\frac{\partial z(\Lambda,t)}{\partial t} &= \varphi(\Lambda)\frac{\partial z(\Lambda,t)}{\partial \Lambda} + (A_i(\Lambda)+\Delta A_i(\Lambda,t))z(\Lambda,t)\\ &\quad + (B_i(\Lambda)+\Delta B_i(\Lambda,t))u(\Lambda,t)\\ &\quad + (E_i(\Lambda)+\Delta E_i(\Lambda,t))w(\Lambda,t)\\ y(\Lambda,t) &= (C_i(\Lambda)+\Delta C_i(\Lambda,t))z(\Lambda,t)\\ &\quad + (D_i(\Lambda)+\Delta D_i(\Lambda,t))u(\Lambda,t)\\ &\quad + (F_i(\Lambda)+\Delta F_i(\Lambda,t))w(\Lambda,t),\ i\in\{1,2,\cdots,\nu\} \qquad (3)\end{aligned}$$

where $A_i(\Lambda), B_i(\Lambda), C_i(\Lambda), D_i(\Lambda), E_i(\Lambda), F_i(\Lambda)$ represent system matrices for the ith local model obtained by linearization around operation points; ν represents the number of the fuzzy rules; $\mathbb{N}_j^i(i = 1, \cdots, \nu)$ represents the fuzzy sets; $\theta(\Lambda, t) = \left[\theta_1(\Lambda, t) \cdots \theta_d(\Lambda, t)\right]$ denotes the premise variables for the fuzzy systems; $\Re^i$ indicates the ith fuzzy inference rule; $\Delta A_i(\Lambda, t)$, $\Delta B_i(\Lambda, t)$, $\Delta C_i(\Lambda, t)$, $\Delta D_i(\Lambda, t)$, $\Delta E_i(\Lambda, t)$, $\Delta F_i(\Lambda, t)$ denote the spatiotemporal uncertainties of the following form

$$\begin{aligned}&\begin{bmatrix}\Delta A_i(\Lambda,t) & \Delta B_i(\Lambda,t) & \Delta E_i(\Lambda,t)\\ \Delta C_i(\Lambda,t) & \Delta D_i(\Lambda,t) & \Delta F_i(\Lambda,t)\end{bmatrix}\\ &= \begin{bmatrix}T_{1i}(\Lambda)\\ T_{2i}(\Lambda)\end{bmatrix}\Delta_i(\Lambda,t)\begin{bmatrix}V_{1i}(\Lambda) & V_{2i}(\Lambda) & V_{3i}(\Lambda)\end{bmatrix} \qquad (4)\end{aligned}$$

with $\Delta_i(\Lambda, t)$ as the time-varying uncertainties bounded by

$$\Delta_i^T(\Lambda,t)\Delta_i(\Lambda,t) \leq \delta_\Delta \mathrm{I}. \qquad (5)$$

$\delta_\Delta > 0$ is a constant, $T_{1i}(\Lambda)$, $T_{2i}(\Lambda)$, $V_{1i}(\Lambda)$, $V_{2i}(\Lambda)$, $V_{3i}(\Lambda)$ are known matrices of appropriate dimensions.

Let $\mu_i(\theta(\Lambda, t)), i = 1, \cdots, \nu$ represent the fuzzy membership function for the inferred fuzzy set $\mathbb{N}^i := \prod_{j=1}^{d} \mathbb{N}_j^i$

$$\begin{aligned}&\mu_i(\theta(\Lambda,t))\\ &= \frac{\prod_{j=1}^{d}\upsilon_{ij}(\theta_j(\Lambda,t))}{\sum_{i=1}^{\nu}\prod_{j=1}^{d}\upsilon_{ij}(\theta_j(\Lambda,t))} \geq 0, \quad \sum_{i=1}^{\nu}\mu_i(\theta(\Lambda,t)) = 1 \qquad (6)\end{aligned}$$

where $\upsilon_{ij}(\theta_j(\Lambda, t)) \geq 0$ is the grade of membership function of $\theta_j(\Lambda, t)$ in $\mathbb{N}_j^i$. Notice that the fuzzy membership function adopted here is also distributed in space. Throughout of the paper, we denote μ_i as $\mu_i(\theta(\Lambda, t))$ for ease of presentation.

With the aid of standard fuzzy inference approach, the fuzzy PDE system can be described in the following form

$$\begin{aligned}\frac{\partial z(\Lambda,t)}{\partial t} &= \varphi(\Lambda)\frac{\partial z(\Lambda,t)}{\partial \Lambda} + (\mathcal{A}(\mu,\Lambda)+\Delta\mathcal{A}(\mu,\Lambda))z(\Lambda,t)\\ &\quad + (\mathcal{B}(\mu,\Lambda)+\Delta\mathcal{B}(\mu,\Lambda))u(\Lambda,t)\\ &\quad + (\mathcal{E}(\mu,\Lambda)+\Delta\mathcal{E}(\mu,\Lambda))u(\Lambda,t)\end{aligned}$$

$$\begin{aligned} y(\Lambda,t) &= (\mathcal{C}(\mu,\Lambda)+\Delta\mathcal{C}(\mu,\Lambda))z(\Lambda,t) \\ &\quad +(\mathcal{D}(\mu,\Lambda)+\Delta\mathcal{D}(\mu,\Lambda))u(\Lambda,t) \\ &\quad +(\mathcal{F}(\mu,\Lambda)+\Delta\mathcal{F}(\mu,\Lambda))w(\Lambda,t) \end{aligned} \tag{7}$$

where

$$\begin{aligned} \mathcal{A}(\mu,\Lambda) &= \sum_{i=1}^{\nu}\mu_i A_i(\Lambda), \quad \mathcal{B}(\mu,\Lambda) = \sum_{i=1}^{\nu}\mu_i B_i(\Lambda) \\ \mathcal{E}(\mu,\Lambda) &= \sum_{i=1}^{\nu}\mu_i E_i(\Lambda), \quad \mathcal{C}(\mu,\Lambda) = \sum_{i=1}^{\nu}\mu_i C_i(\Lambda) \\ \mathcal{D}(\mu,\Lambda) &= \sum_{i=1}^{\nu}\mu_i D_i(\Lambda), \quad \mathcal{F}(\mu,\Lambda) = \sum_{i=1}^{\nu}\mu_i F_i(\Lambda) \\ \Delta\mathcal{A}(\mu,\Lambda) &= \sum_{i=1}^{\nu}\mu_i \Delta A_i(\Lambda), \quad \Delta\mathcal{B}(\mu,\Lambda) = \sum_{i=1}^{\nu}\mu_i \Delta B_i(\Lambda) \\ \Delta\mathcal{E}(\mu,\Lambda) &= \sum_{i=1}^{\nu}\mu_i \Delta E_i(\Lambda), \quad \Delta\mathcal{C}(\mu,\Lambda) = \sum_{i=1}^{\nu}\mu_i \Delta C_i(\Lambda) \\ \Delta\mathcal{D}(\mu,\Lambda) &= \sum_{i=1}^{\nu}\mu_i \Delta D_i(\Lambda), \quad \Delta\mathcal{F}(\mu,\Lambda) = \sum_{i=1}^{\nu}\mu_i \Delta F_i(\Lambda). \end{aligned}$$

For fault detection purpose, the following distributed fuzzy residual generator is adopted.

Residual Generator Rule $\Re^i$: *IF* $\theta_1(\Lambda,t)$ is $\mathbb{N}_1^i$ and $\theta_2(\Lambda,t)$ is $\mathbb{N}_2^i$ and $\cdots$ and $\theta_d(\Lambda,t)$ is $\mathbb{N}_d^i$, *THEN*

$$\begin{aligned} \frac{\partial\hat{z}(\Lambda,t)}{\partial t} &= \varphi(\Lambda)\frac{\partial\hat{z}(\Lambda,t)}{\partial\Lambda} + A_i(\Lambda)\hat{z}(\Lambda,t) + B_i(\Lambda)u(\Lambda,t) \\ &\quad + L_i(\Lambda)(y(\Lambda,t)-\hat{y}(\Lambda,t)) \\ \hat{y}(\Lambda,t) &= C_i(\Lambda)\hat{z}(\Lambda,t) + D_i(\Lambda)u(\Lambda,t) \\ r(\Lambda,t) &= \omega_i(\Lambda)\left(y(\Lambda,t)-\hat{y}(\Lambda,t)\right), \quad i\in\{1,2,\cdots,\nu\} \end{aligned} \tag{8}$$

where $\hat{z}(\Lambda,t) \in \mathcal{R}^{k_z}$ denotes the estimate of the state; $\hat{y}(\Lambda,t) \in \mathcal{R}^{k_y}$ represents the estimate of the output; $\omega_i(\Lambda)$ represents the weighting factor of each local model; $L_i(\Lambda)$ denotes the gain matrix for the fuzzy residual generator of the ith model; $r(\Lambda,t) \in \mathcal{R}^{k_y}$ indicates the residual signal.

Likewise, the global fuzzy residual generator can be expressed as follows

$$\begin{aligned} \frac{\partial\hat{z}(\Lambda,t)}{\partial t} &= \varphi(\Lambda)\frac{\partial\hat{z}(\Lambda,t)}{\partial\Lambda} + \mathcal{A}(\mu,\Lambda)\hat{z}(\Lambda,t) + \mathcal{B}(\mu,\Lambda) \\ &\quad \times u(\Lambda,t) + \mathcal{L}(\mu,\Lambda)(y(\Lambda,t)-\hat{y}(\Lambda,t)) \\ \hat{y}(\Lambda,t) &= \mathcal{C}(\mu,\Lambda)\hat{z}(\Lambda,t) + \mathcal{D}(\mu,\Lambda)u(\Lambda,t) \\ r(\Lambda,t) &= \omega(\Lambda,\mu)\left(y(\Lambda,t)-\hat{y}(\Lambda,t)\right) \end{aligned} \tag{9}$$

where

$$\mathcal{L}(\mu,\Lambda) = \sum_{i=1}^{\nu}\mu_i L_i(\Lambda), \quad \omega(\mu,\Lambda) = \sum_{i=1}^{\nu}\mu_i\omega_i(\Lambda).$$

Remark 1: It is noteworthy that the weighting factors $\omega(\Lambda,\mu)$ are introduced to circumvent the conservatism of standard fuzzy approaches which generally handle the overall fuzzy systems in a uniform manner.

In this paper, we are devoted to investigate the $\mathcal{L}_2$ type of observer-based FD approach for nonlinear hyperbolic PDE processes (1). For our purpose, the residual generator will be first designed such that

$$\begin{aligned} \int_0^{\tau}\varphi_1(\|r(\Lambda,t)\|)dt &\le \int_0^{\tau}\varphi_2(\|u(\Lambda,t)\|)dt \\ &\quad + \int_0^{\tau}\varphi_3(\|w(\Lambda,t)\|)dt \\ &\quad + \int_{k_1}^{k_2}\rho_o(z(\Lambda,0),\hat{z}(\Lambda,0))d\Lambda \end{aligned} \tag{10}$$

where $\varphi_1(\cdot) \in \mathcal{K}, \varphi_2(\cdot) \in \mathcal{K}_\infty, \varphi_3(\cdot) \in \mathcal{K}_\infty$ and $\rho_o(\cdot) \ge 0$ is a positive constant with respect to given $z(\Lambda,0), \hat{z}(\Lambda,0)$.

Together with the following dynamic (adaptive) threshold and evaluation function

$$\begin{aligned} J(r) &= \int_0^{T}\varphi_1(\|r(\Lambda,t)\|)dt \\ J_{\text{th}} &= \int_0^{T}\varphi_2(\|u(\Lambda,t)\|)dt + \int_0^{T}\varphi_3(\|w(\Lambda,t)\|)dt + \bar{\delta}_0 \\ \bar{\rho}_0 &= \sup_{z(\Lambda,0),\hat{z}(\Lambda,0)}\int_{k_1}^{k_2}\rho_o(z(\Lambda,0),\hat{z}(\Lambda,0))d\Lambda \end{aligned} \tag{11}$$

the following decision logic will promise reliable fault detection system

$$\begin{cases} J(r) > J_{\text{th}} \Longrightarrow \text{faulty} \\ J(r) \le J_{\text{th}} \Longrightarrow \text{fault-free.} \end{cases} \tag{12}$$

Remark 2: It is important to note that $\int_0^T\varphi_1(\|r(\Lambda,t)\|)dt$ is a general form of the $\mathcal{L}_2$-norm of the signal $r(\Lambda,t)$ [39].

III. DISTRIBUTED FAULT DETECTION SYSTEM DESIGN

In this section, the distributed fuzzy FD approach will be investigated via distributed Lyapunov function.

A. DISTRIBUTED FAULT DETECTION FILTER DESIGN

By defining $e(\Lambda,t) = z(\Lambda,t) - \hat{z}(\Lambda,t)$, $\eta(\Lambda,t) = [\, e^T(\Lambda,t) \;\; z(\Lambda,t)\,]^T$ and $\xi(\Lambda,t) = [\, u^T(\Lambda,t) \;\; w^T(\Lambda,t)\,]^T$, we have

$$\begin{aligned} \frac{\partial\eta(\Lambda,t)}{\partial t} &= \bar{\varphi}(\Lambda)\frac{\partial\eta(\Lambda,t)}{\partial\Lambda} + (\bar{\mathcal{A}}(\mu,\Lambda)+\Delta\bar{\mathcal{A}}(\mu,\Lambda))\eta(\Lambda,t) \\ &\quad + (\bar{\mathcal{B}}(\mu,\Lambda)+\Delta\bar{\mathcal{B}}(\mu,\Lambda))\xi(\Lambda,t) \\ r(\Lambda,t) &= (\bar{\mathcal{C}}(\mu,\Lambda)+\Delta\bar{\mathcal{C}}(\mu,\Lambda))\eta(\Lambda,t) + (\bar{\mathcal{D}}(\mu,\Lambda) \\ &\quad + \Delta\bar{\mathcal{D}}(\mu,\Lambda))\xi(\Lambda,t) \end{aligned} \tag{13}$$

where $\bar{\varphi}(\Lambda)$, $\bar{\mathcal{D}}(\mu,\Lambda)$, $\Delta\bar{\mathcal{D}}(\mu,\Lambda)$, $\bar{\mathcal{A}}(\mu,\Lambda)$, $\bar{\mathcal{B}}(\mu,\Lambda)$, $\Delta\bar{\mathcal{A}}(\mu,\Lambda)$, $\Delta\bar{\mathcal{B}}(\mu,\Lambda)$, $\Delta\bar{\mathcal{C}}(\mu,\Lambda)$, and $\bar{\mathcal{C}}(\mu,\Lambda)$ are shown at the top of the next page.

In what follows, the main results for the distributed fuzzy fault detection filter are summarized in the following theorem.

Theorem 1: Consider the hyperbolic PDE systems (1) and the distributed fuzzy residual generators (8), if there exist matrices

$$P(\Lambda) = \begin{bmatrix} P_1(\Lambda) & 0 \\ 0 & P_2(\Lambda) \end{bmatrix} > 0 \tag{14}$$

$$\begin{aligned}
\bar{\varphi}(\Lambda) &= \begin{bmatrix} \varphi(\Lambda) & 0 \\ 0 & \varphi(\Lambda) \end{bmatrix} \quad \bar{\mathcal{D}}(\mu,\Lambda) = \begin{bmatrix} 0 & \mathcal{F}(\mu,\Lambda) \end{bmatrix} \quad \Delta\bar{\mathcal{D}}(\mu,\Lambda) = \begin{bmatrix} \Delta\mathcal{D}(\mu,\Lambda) & \Delta\mathcal{F}(\mu,\Lambda) \end{bmatrix} \\
\bar{\mathcal{A}}(\mu,\Lambda) &= \begin{bmatrix} \mathcal{A}(\mu,\Lambda) - \mathcal{L}(\mu,\Lambda)\mathcal{C}(\mu,\Lambda) & 0 \\ 0 & \mathcal{A}(\mu,\Lambda) \end{bmatrix} \quad \bar{\mathcal{B}}(\mu,\Lambda) = \begin{bmatrix} 0 & \mathcal{E}(\mu,\Lambda) - \mathcal{L}(\mu,\Lambda)\mathcal{F}(\mu,\Lambda) \\ \mathcal{B}(\mu,\Lambda) & \mathcal{E}(\mu,\Lambda) \end{bmatrix} \\
\Delta\bar{\mathcal{A}}(\mu,\Lambda) &= \begin{bmatrix} 0 & \Delta\mathcal{A}(\mu,\Lambda) - \mathcal{L}(\mu,\Lambda)\Delta\mathcal{C}(\mu,\Lambda) \\ 0 & \Delta\mathcal{A}(\mu,\Lambda) \end{bmatrix} \\
\Delta\bar{\mathcal{B}}(\mu,\Lambda) &= \begin{bmatrix} \Delta\mathcal{B}(\mu,\Lambda) - \mathcal{L}(\mu,\Lambda)\Delta\mathcal{D}(\mu,\Lambda) & \Delta\mathcal{E}(\mu,\Lambda) - \mathcal{L}(\mu,\Lambda)\Delta\mathcal{F}(\mu,\Lambda) \\ \Delta\mathcal{B}(\mu,\Lambda) & \Delta\mathcal{E}(\mu,\Lambda) \end{bmatrix} \\
\Delta\bar{\mathcal{C}}(\mu,\Lambda) &= \begin{bmatrix} 0 & \Delta\mathcal{C}(\mu,\Lambda) \end{bmatrix}, \ \bar{\mathcal{C}}(\mu,\Lambda) = \begin{bmatrix} \mathcal{C}(\mu,\Lambda) & 0 \end{bmatrix}. \\
\Theta_i(\Lambda) &= \begin{bmatrix} \Omega_{ii}(\Lambda) & \star \\ \bar{T}_i^T(\Lambda)\Pi_{ij}^T(\Lambda) & -\vartheta_{ii}\mathrm{I} \end{bmatrix}, \quad \Pi_{ij}(\Lambda) = \begin{bmatrix} P(\Lambda)\bar{L}_j(\Lambda) \\ 0 \\ \bar{Z}_i(\Lambda) \end{bmatrix} \\
\bar{\Theta}_{ij}(\Lambda) &= \begin{bmatrix} \Omega_{ij}(\Lambda)+\Omega_{ji}(\Lambda) & \star & \star \\ \bar{T}_i^T(\Lambda)\Pi_{ij}^T(\Lambda) & -\vartheta_{ij}\mathrm{I} & \star \\ \bar{T}_j^T(\Lambda)\Pi_{ji}^T(\Lambda) & 0 & -\vartheta_{ji}\mathrm{I} \end{bmatrix} \quad \Omega_{ij}(\Lambda) = \begin{bmatrix} \Gamma_{ij}^{(1)}(\Lambda) & \star & \star \\ \Gamma_{ij}^{(2)}(\Lambda) & \Gamma_i^{(3)}(\Lambda) & \star \\ \omega_i(\Lambda)\bar{C}_i(\Lambda) & \omega_i(\Lambda)\bar{D}_i(\Lambda) & -\mathrm{I} \end{bmatrix} \\
\Gamma_{ij}^{(1)}(\Lambda) &= \Psi(\Lambda) + \begin{bmatrix} \mathrm{Sym}\{P_1(\Lambda)A_i(\Lambda) - Q_j(\Lambda)C_i(\Lambda)\} & \star \\ 0 & \mathrm{Sym}\{P_2(\Lambda)A_i(\Lambda)\} + \vartheta_{ij}V_{1i}^T(\Lambda)V_{1i}(\Lambda) \end{bmatrix} \\
\Gamma_{ij}^{(2)}(\Lambda) &= \begin{bmatrix} 0 & B_i^T(\Lambda)P_2^T(\Lambda) + \vartheta_{ij}V_{2i}^T(\Lambda)V_{1i}(\Lambda) \\ E_i^T(\Lambda)P_1^T(\Lambda) - F_i^T(\Lambda)Q_j^T(\Lambda) & E_i^T(\Lambda)P_2^T(\Lambda) + \vartheta_{ij}V_{3i}^T(\Lambda)V_{1i}(\Lambda) \end{bmatrix} \\
\Gamma_i^{(3)}(\Lambda) &= \begin{bmatrix} -\mathrm{I} + \vartheta_{ij}V_{2i}^T(\Lambda)V_{2i}(\Lambda) & \star \\ \vartheta_{ij}V_{3i}^T(\Lambda)V_{2i}(\Lambda) & -\mathrm{I} + \vartheta_{ij}V_{3i}^T(\Lambda)V_{3i}(\Lambda) \end{bmatrix} \\
\bar{T}_i(\Lambda) &= \begin{bmatrix} \delta_\Delta T_{1i}(\Lambda) \\ \delta_\Delta T_{2i}(\Lambda) \end{bmatrix}, \quad \bar{L}_j(\Lambda) = \begin{bmatrix} \mathrm{I} & -L_j(\Lambda) \\ \mathrm{I} & 0 \end{bmatrix} \quad \bar{C}_i(\Lambda) = \begin{bmatrix} C_i(\Lambda) & 0 \end{bmatrix}, \ \bar{D}_i(\Lambda) = \begin{bmatrix} 0 & F_i(\Lambda) \end{bmatrix} \\
\bar{Z}_i(\Lambda) &= \begin{bmatrix} 0 & \omega_i(\Lambda)\mathrm{I} \end{bmatrix} \quad \Psi(\Lambda) = (\delta(\Lambda - k_2) - \delta(\Lambda - k_1))P(\Lambda) - \frac{\partial}{\partial\Lambda}(P(\Lambda)\bar{\varphi}(\Lambda))
\end{aligned} \tag{17}$$

$Q_i(\Lambda)$, $1 \le i \le \nu$ and constant $\vartheta_{ij} > 0$, $1 \le i < j \le \nu$, $\omega_i(\Lambda) > 0$, $1 \le i \le \nu$, such that

$$\Theta_i(\Lambda) < 0, \quad 1 \le i \le \nu \tag{15}$$

$$\bar{\Theta}_{ij}(\Lambda) < 0, \quad 1 \le i < j \le \nu \tag{16}$$

where (17), as shown at the top of this page, then, we have $L_i(\Lambda) = (P_1(\Lambda))^{-1}Q_i(\Lambda)$ and

$$\int_0^T ||r(\cdot,t)||_2^2 dt \le \int_0^T \varphi_2(\|u(\Lambda,t)\|)dt + \rho_0 + \int_0^T \varphi_3(\|w(\Lambda,t)\|)dt \tag{18}$$

where

$$\begin{aligned}
\varphi_2(\|u(\Lambda,t)\|) &= \int_{k_1}^{k_2} u^T(\Lambda,t)u(\Lambda,t)d\Lambda \\
\varphi_2(\|w(\Lambda,t)\|) &= \int_{k_1}^{k_2} w^T(\Lambda,t)w(\Lambda,t)d\Lambda \\
\rho_0 &= \int_{k_1}^{k_2} \eta^T(\Lambda,0)P(\Lambda)\eta(\Lambda,0)d\Lambda.
\end{aligned} \tag{19}$$

Proof: Consider the following type of Lyapunov function

$$V(t) = \int_{k_1}^{k_2} \eta^T(\Lambda,t)P(\Lambda)\eta(\Lambda,t)d\Lambda \tag{20}$$

where $P(\Lambda) > 0$ is defined on the interval $[k_1\ k_2]$.

It is evident that (18) holds provided that the following inequality is feasible

$$\dot{V}(t) + ||r(\cdot,t)||_2^2 - ||u(\cdot,t)||_2^2 - ||w(\cdot,t)||_2^2 < 0. \tag{21}$$

It follows directly from (13) and (20) that the left-hand-side (LHS) of (21) is equivalent to

$$\begin{aligned}
\mathrm{LHS}(21) &= \int_{k_1}^{k_2} \eta^T(\Lambda,t)P(\Lambda)\frac{\partial\eta(\Lambda,t)}{\partial t}d\Lambda \\
&\quad + \int_{k_1}^{k_2} \frac{\partial\eta^T(\Lambda,t)}{\partial t}P(\Lambda)\eta(\Lambda,t)d\Lambda \\
&\quad - \int_{k_1}^{k_2} \xi^T(\Lambda,t)\xi(\Lambda,t)d\Lambda \\
&\quad + (\omega(\Lambda,\mu))^2 \int_{k_1}^{k_2} \begin{bmatrix} z^T(\Lambda,t) & \xi^T(\Lambda,t) \end{bmatrix} \\
&\quad \times \begin{bmatrix} (\bar{\mathcal{C}}(\mu,\Lambda) + \Delta\bar{\mathcal{C}}(\mu,\Lambda))^T \\ (\bar{\mathcal{D}}(\mu,\Lambda) + \Delta\bar{\mathcal{D}}(\mu,\Lambda))^T \end{bmatrix} (\star) d\Lambda.
\end{aligned} \tag{22}$$

It is noted that

$$\begin{aligned}&\int_{k_1}^{k_2} \eta^T(\Lambda,t)P(\Lambda)\bar{\varphi}(\Lambda)\frac{\partial \eta(\Lambda,t)}{\partial \Lambda}d\Lambda \\ &\quad + \int_{k_1}^{k_2} \frac{\partial \eta^T(\Lambda,t)}{\partial \Lambda}P(\Lambda)\bar{\varphi}(\Lambda)\eta(\Lambda,t)d\Lambda \\ &= \eta^T(\Lambda,t)P(\Lambda)\bar{\varphi}(\Lambda)\eta(\Lambda,t)|_{\Lambda=k_1}^{\Lambda=k_2} \\ &\quad - \int_{k_1}^{k_2} \eta^T(\Lambda,t)\frac{\partial}{\partial \Lambda}\left(P(\Lambda)\bar{\varphi}(\Lambda)\right)\eta(\Lambda,t)d\Lambda. \quad (23)\end{aligned}$$

By introducing the following Dirac delta function

$$\delta(\Lambda) = \begin{cases} \infty, & \Lambda = 0 \\ 0, & \Lambda \neq 0 \end{cases}$$

$$\int_{-\infty}^{\infty} \delta(\Lambda)d\Lambda = 1 \quad (24)$$

one has that

$$\begin{aligned}\text{LHS}(23) &= -\int_{k_1}^{k_2} \eta^T(\Lambda,t)\frac{\partial}{\partial \Lambda}\left(P(\Lambda)\bar{\varphi}(\Lambda)\right)\eta(\Lambda,t)d\Lambda \\ &\quad + \int_{k_1}^{k_2} (\delta(\Lambda-k_2)-\delta(\Lambda-k_1))\eta^T(\Lambda,t) \\ &\quad \times P(\Lambda)\bar{\varphi}(\Lambda)\eta(\Lambda,t)d\Lambda. \quad (25)\end{aligned}$$

Therefore, we have

$$\begin{aligned}\text{LHS}(21) &= 2\int_{k_1}^{k_2} \eta^T(\Lambda,t)P(\Lambda)(\bar{\mathcal{B}}(\mu,\Lambda) \\ &\quad + \Delta\bar{\mathcal{B}}(\mu,\Lambda))\xi(\Lambda,t)d\Lambda \\ &\quad + \int_{k_1}^{k_2} \eta^T(\Lambda,t)\text{Sym}\{P(\Lambda)(\bar{\mathcal{A}}(\mu,\Lambda) \\ &\quad + \Delta\bar{\mathcal{A}}(\mu,\Lambda))\}\eta(\Lambda,t)d\Lambda \\ &\quad + (\omega(\Lambda,\mu))^2\int_{k_1}^{k_2} \begin{bmatrix} z^T(\Lambda,t) & \xi^T(\Lambda,t)\end{bmatrix} \\ &\quad \times \begin{bmatrix} (\bar{\mathcal{C}}(\mu,\Lambda)+\Delta\bar{\mathcal{C}}(\mu,\Lambda))^T \\ (\bar{\mathcal{D}}(\mu,\Lambda)+\Delta\bar{\mathcal{D}}(\mu,\Lambda))^T\end{bmatrix}(\star)d\Lambda \\ &\quad - \int_{k_1}^{k_2} \xi^T(\Lambda,t)\xi(\Lambda,t)d\Lambda + \int_{k_1}^{k_2} \eta^T(\Lambda,t) \\ &\quad \times \Psi(\Lambda)\eta(\Lambda,t)d\Lambda \quad (26)\end{aligned}$$

where

$$\Psi(\Lambda) = (\delta(\Lambda-k_2)-\delta(\Lambda-k_1))P(\Lambda) - \frac{\partial}{\partial \Lambda}\left(P(\Lambda)\bar{\varphi}(\Lambda)\right). \quad (27)$$

It is evident that by Schur complement, (21) holds provided the condition (28), is feasible as shown at the bottom of this page.

By expanding the fuzzy basis function of (28), one has that

$$\sum_{i=1}^{\nu}\sum_{j=1}^{\nu} \mu_i\mu_j \Xi_{ij}(\Lambda) < 0 \quad (29)$$

where

$$\Xi_{ij}(\Lambda) = \Omega_{ij}(\Lambda) + \text{sym}\left\{\Pi_j(\Lambda)\Delta_i(\Lambda)\right\}$$

$$\Omega_{ij}(\Lambda) = \begin{bmatrix} \Psi(\Lambda)+\text{Sym}\{P(\Lambda)\bar{A}_{ij}(\Lambda)\} & \star & \star \\ (P(\Lambda)\bar{B}_{ij}(\Lambda))^T & -\text{I} & \star \\ \omega_i(\Lambda)\bar{C}_i(\Lambda) & \omega_i(\Lambda)\bar{D}_i(\Lambda) & -\text{I}\end{bmatrix}$$

$$\Delta_i(\Lambda) = \begin{bmatrix} \begin{bmatrix} 0 & \Delta A_i(\Lambda)\end{bmatrix} & \begin{bmatrix}\Delta B_i(\Lambda) & \Delta E_i(\Lambda)\end{bmatrix} & 0 \\ \begin{bmatrix} 0 & \Delta C_i(\Lambda)\end{bmatrix} & \begin{bmatrix}\Delta D_i(\Lambda) & \Delta F_i(\Lambda)\end{bmatrix} & 0\end{bmatrix}$$

$$\Pi_{ij}(\Lambda) = \begin{bmatrix} P(\Lambda)\bar{L}_j(\Lambda) \\ 0 \\ \bar{Z}_i(\Lambda)\end{bmatrix}, \quad \bar{Z}(\Lambda) = \begin{bmatrix} 0 & \omega_i(\Lambda)\text{I}\end{bmatrix}$$

$$\bar{L}_j(\Lambda) = \begin{bmatrix} \text{I} & -L_j(\Lambda) \\ \text{I} & 0\end{bmatrix}, \quad \bar{D}_i(\Lambda) = \begin{bmatrix} 0 & F_i(\Lambda)\end{bmatrix}$$

$$\bar{A}_{ij}(\Lambda) = \begin{bmatrix} A_i(\Lambda)-L_j(\Lambda)C_i(\Lambda) & 0 \\ 0 & A_i(\Lambda)\end{bmatrix}$$

$$\bar{C}_i(\Lambda) = \begin{bmatrix} C_i(\Lambda) & 0\end{bmatrix}$$

$$\bar{B}_{ij}(\Lambda) = \begin{bmatrix} 0 & E_i(\Lambda)-L_j(\Lambda)F_i(\Lambda) \\ B_i(\Lambda) & E_i(\Lambda)\end{bmatrix}.$$

As a result, the following inequality implies (29)

$$\Xi_{ii}(\Lambda) < 0, \quad 1 \le i \le \nu \quad (30)$$

$$\Xi_{ij}(\Lambda) + \Xi_{ji}(\Lambda) < 0, \quad 1 \le i < j \le \nu. \quad (31)$$

Noting that for any positive constant ϑ_{ij}, it holds that

$$\begin{aligned}&\text{sym}\left\{\Pi_j(\Lambda)\Delta_i(\Lambda)\right\} \\ &\le \vartheta_{ij}^{-1}\delta_\Delta^2\Pi_j(\Lambda)\begin{bmatrix} T_{1i}(\Lambda) \\ T_{2i}(\Lambda)\end{bmatrix}\begin{bmatrix} T_{1i}(\Lambda) \\ T_{2i}(\Lambda)\end{bmatrix}^T \Pi_j^T(\Lambda) \\ &\quad + \vartheta_{ij}\begin{bmatrix}\begin{bmatrix} 0 & V_{1i}(\Lambda)\end{bmatrix} & \begin{bmatrix} V_{2i}(\Lambda) & V_{3i}(\Lambda)\end{bmatrix} & 0\end{bmatrix}^T(\star). \quad (32)\end{aligned}$$

By defining $Q_i(\Lambda) = P_1(\Lambda)L_i(\Lambda)$, $1 \le i \le \nu$ and applying Schur complement, it is evident that (30)-(31) hold provided that the inequalities (15)-(16) are feasible, which completes the proof. ■

Remark 3: It is important to point out that the Dirac delta function $\delta(\Lambda - k_1)$, $\delta(\Lambda - k_2)$ are adopted here to deal with the boundary condition at the boundary points $\Lambda = k_1$ and $\Lambda = k_2$ [15], [31]. Due to the distinguished feature of the Dirac delta function, it has been widely adopted in the design scheme of point-wise controller for distributed parameter systems.

It is noteworthy that for optimizing the fault detectability of the FD system, the weighting factor $\omega_i(\Lambda)$, $i = 1, \cdots, \nu$ should be maximized. To this end, the following algorithm is proposed in order to determine $L_i(\Lambda)$, $\omega_i(\Lambda)$ for the residual

$$\begin{bmatrix} \Psi(\Lambda)+\text{Sym}\left\{P(\Lambda)\left(\bar{\mathcal{A}}(\mu,\Lambda)+\Delta\bar{\mathcal{A}}(\mu,\Lambda)\right)\right\} & \star & \star \\ (\bar{\mathcal{B}}(\mu,\Lambda)+\Delta\bar{\mathcal{B}}(\mu,\Lambda))^T P^T(\Lambda) & -\text{I} & \star \\ \omega(\Lambda,\mu)\left(\bar{\mathcal{C}}(\mu,\Lambda)+\Delta\bar{\mathcal{C}}(\mu,\Lambda)\right) & \omega(\Lambda,\mu)\left(\bar{\mathcal{D}}(\mu,\Lambda)+\Delta\bar{\mathcal{D}}(\mu,\Lambda)\right) & -\text{I}\end{bmatrix} < 0. \quad (28)$$

generator

$$\max_{\omega_i(\Lambda)} \sum_{i=1}^{\nu} \int_{k_1}^{k_2} (\omega_i(\Lambda)) \quad \text{subject to } (15)-(16). \tag{33}$$

With the residual generator, by setting

$$\begin{aligned} J(r) &= \int_0^T ||r(\cdot,t)||_2^2 dt \\ J_{\text{th}} &= \int_0^T ||u(\cdot,t)||_2^2 dt + \int_0^T ||w(\cdot,t)||_2^2 dt + \bar{\gamma}_0 \\ \bar{\gamma}_0 &= \sup_{z(\Lambda,0),\hat{z}(\Lambda,0)} \int_{k_1}^{k_2} \eta^T(\Lambda,0)P(\Lambda)\eta(\Lambda,0)d\Lambda \end{aligned} \tag{34}$$

the $\mathcal{L}_2$-type of fuzzy FD systems is obtained for the nonlinear hyperbolic PDE processes.

B. THE RECURSIVE ALGORITHM

It is worth mentioning that the conditions given in (15)-(16) are space-dependent linear matrix inequalities (SDLMIs). This requires solving infinity number of linear matrix inequalities (LMIs), which limits its application. To deal with this issue, we apply the following recursive algorithm:

- discretizing the position space $\Lambda \in [k_1, k_2]$ as

$$\begin{aligned} \Lambda_l &= \Lambda_{l-1} + \alpha, \quad 1 \le l \le Z \\ \alpha &= \frac{k_2 - k_1}{Z-1} \end{aligned} \tag{35}$$

- iteratively solving the following linear matrix inequalities

$$\Theta_i(\Lambda_l) < 0, \quad 1 \le i \le \nu \tag{36}$$
$$\bar{\Theta}_{ij}(\Lambda_l) < 0, \quad 1 \le i < j \le \nu \tag{37}$$

where $\frac{\partial}{\partial \Lambda}(P(\Lambda)\bar{\varphi}(\Lambda))$ is replaced by

$$\begin{aligned} &(P(\Lambda_l) - P(\Lambda_{l-1}))\,\bar{\varphi}(\Lambda_l)/\alpha \\ &\quad + P(\Lambda_l)\left(\bar{\varphi}(\Lambda_l) - \bar{\varphi}(\Lambda_{l-1})\right)/\alpha. \end{aligned} \tag{38}$$

with $P(\Lambda_0)$ given as a priori. Then, the weighting factors and the gain matrix for the residual generators can be obtained by $L_i(\Lambda_l) = (P_1(\Lambda_l))^{-1} Q_i(\Lambda_l)$ and $\omega_i(\Lambda_l)$, $1 \le i \le \nu$, $1 \le l \le Z$, respectively.

Remark 4: It is worth mentioning that the above iterative algorithm can be considered as the approximation of the solution to the SDLMIs. With a relatively large integer Z, the LMIs (36)-(37) obtained from the iterative algorithm can well approximate the SDLMIs (15)-(16).

IV. ILLUSTRATIVE EXAMPLE

Consider the following fuzzy hyperbolic PDE system (3) with the following system parameters

$$A_1(m) = \begin{bmatrix} 0.8\sin(2m) - 0.9 & 0.1 \\ -0.1 & 0.5\cos(2m) - 2 \end{bmatrix}$$

$$A_2(m) = \begin{bmatrix} 0.8\sin(2m) - 1.6 & 0.1 \\ -0.1 & 0.5\cos(2m) - 2 \end{bmatrix}$$

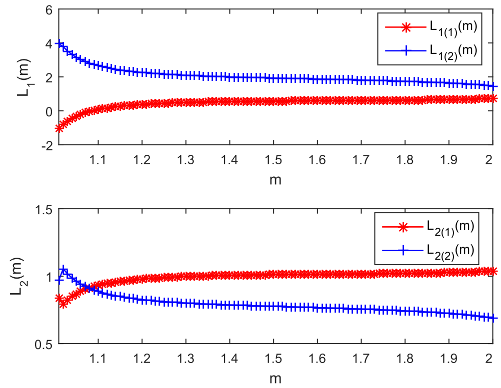

FIGURE 1. $L_1(m)$, $L_2(m)$ **for residual generator.**

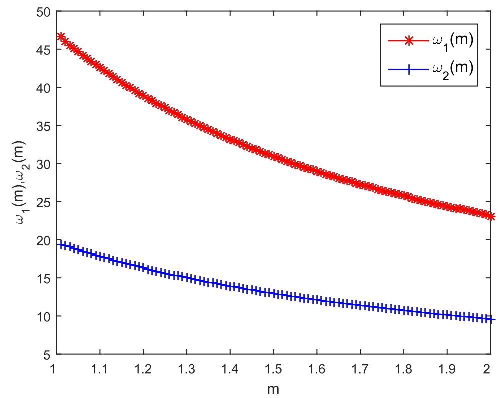

FIGURE 2. $\omega_1(m)$, $\omega_2(m)$ **for residual generator.**

$$C_1(m) = C_2(m) = \begin{bmatrix} 0 & 1 \end{bmatrix}, \quad B_1(m) = B_2(m) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$E_1(m) = \begin{bmatrix} 0.001m \\ 0.002m \end{bmatrix}, \quad E_2(m) = \begin{bmatrix} 0.005m \\ 0.003m \end{bmatrix}$$

$$F_1(m) = 0.002m, \quad F_2(m) = 0.005m$$

and m as the distribution parameter. In addition, the model uncertainties satisfies (4) with

$$T_{11}(m) = \begin{bmatrix} 0.001 \\ 0.002 \end{bmatrix}, \quad T_{21}(m) = 0.001, V_{31}(m) = 0.03 + m$$

$$V_{11}(m) = \begin{bmatrix} 0.01 + m & 0 \end{bmatrix}, \quad V_{21}(m) = 0.02 + m.$$

In this study, $1 \le m \le 2$ represents the position. $\Delta_i(m,t)$ is chosen as $0.01\sin(mt)$ which is bounded by $\delta_\Delta = 0.01$. We first assume the input is set as 0 and the disturbance as $0.1\sin(t)$. The fuzzy membership function are chosen as

$$\mu_1(y(m,t)) = \begin{cases} 0, & \text{if } y(m,t) < -2 \\ 0.5\sin\left(\frac{y(m,t)\pi}{2}\right) + 0.5, & \text{if } -2 \le y(m,t) \le 2 \\ 1, & \text{if } y(m,t) > 2 \end{cases}$$

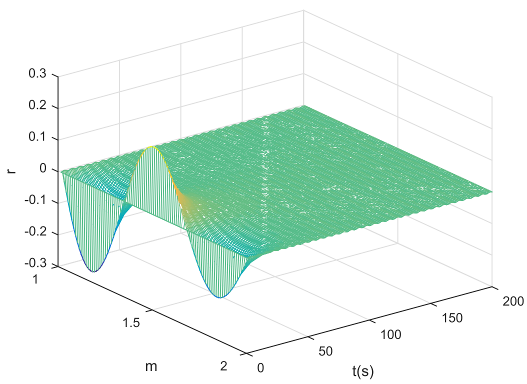

FIGURE 3. Residual signal for fault-free case.

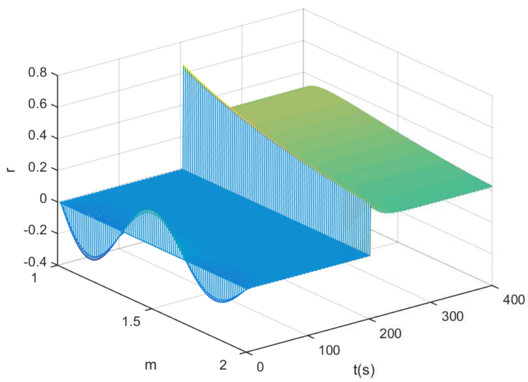

FIGURE 4. Residual signal for faulty case.

$$\mu_2(y(m,t)) = \begin{cases} 1, & \text{if } y(m,t) < -2 \\ -0.5\sin\left(\dfrac{y(m,t)\pi}{2}\right) + 0.5, & \text{if } -2 \le y(m,t) \le 2 \\ 0, & \text{if } y(m,t) > 2. \end{cases}$$

To apply the proposed results, the position space $m \in [1, 2]$ is first discretized as

$$m_{j+1} = m_j + \frac{1}{99}, \quad 1 \le j \le 99$$

with $m_1 = 1$. By applying

$$\max_{\omega_i(m_j)} \sum_{i=1}^{2}\sum_{j=1}^{100}(\omega_i(m_j)) \quad \text{subject to (15)} - \text{(16)} \qquad (39)$$

the gain matrices

$$L_1(m) = \begin{bmatrix} L_{1(1)}(m) \\ L_{1(2)}(m) \end{bmatrix}, \quad L_2(m) = \begin{bmatrix} L_{2(1)}(m) \\ L_{2(2)}(m) \end{bmatrix}$$

and the weighting factors $\omega_1(m)$, $\omega_2(m)$ are given in Fig. 1 and Fig. 2, respectively. By running the residual generator (9), the residual signal in the fault-free situation is shown in Fig. 3.

For demonstration purpose, an offset 0.02 is simulated on the measurement $y(m,t)$ from 200 s. The associated residual signal is shown in Fig. 4. The corresponding fault detection performance is given in Fig. 5.

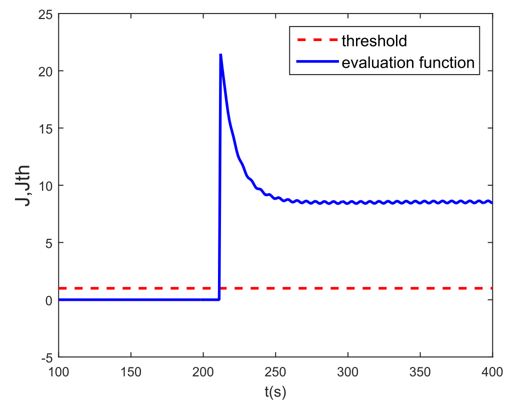

FIGURE 5. The fault detection performance.

V. CONCLUSIONS AND FUTURE WORK

The distributed fuzzy fault detection filter is investigated for nonlinear hyperbolic PDE systems in this paper. For our purpose, the T-S fuzzy model for the nonlinear PDE system is established first. Then, the distributed fault detection filter is investigated in a way that the residual is robust against process uncertainty and input. Then the recursive algorithm is applied for the solution of the design condition proposed for the PDE systems. The future work is dedicated to the sampled-data and point-wise based fault detection approaches for nonlinear PDE processes.

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REFERENCES

[1] S. X. Ding, *Model-Based Fault Diagnosis Techniques: Design Schemes, Algorithms, and Tools*, 2nd ed. London, U.K.: Springer-Verlag, 2013.
[2] J. Chen and R. J. Patton, *Robust Model-Based Fault Diagnosis for Dynamic Systems*. Norwell, MA, USA: Kluwer, 1999.
[3] M. Blanke, M. Kinnaert, J. Lunze, and M. Staroswiecki, *Diagnosis and Fault-Tolerant Control*, 2nd ed. Berlin, Germany: Springer, 2006.
[4] G. Sun, L. Wu, Z. Kuang, Z. Ma, and J. Liu, "Practical tracking control of linear motor via fractional-order sliding mode," *Automatica*, vol. 94, pp. 221–235, Aug. 2018.
[5] J. Liu, W. Luo, X. Yang, and L. Wu, "Robust model-based fault diagnosis for PEM fuel cell air-feed system," *IEEE Trans. Ind. Electron.*, vol. 63, no. 5, pp. 3261–3270, May 2016.
[6] S. Yin, H. Gao, J. Qiu, and O. Kaynak, "Adaptive fault-tolerant control for nonlinear system with unknown control directions based on fuzzy approximation," *IEEE Trans. Syst., Man, Cybern. Syst.*, vol. 47, no. 8, pp. 1909–1918, Aug. 2017, doi: 10.1109/TSMC.2016.2564921.
[7] H. Dong, Z. Wang, S. X. Ding, and H. Gao, "Finite-horizon reliable control with randomly occurring uncertainties and nonlinearities subject to output quantization," *Automatica*, vol. 52, pp. 355–362, Feb. 2015.
[8] B. Jiang and F. N. Chowdhury, "Parameter fault detection and estimation of a class of nonlinear systems using observers," *J. Franklin Inst.*, vol. 342, no. 7, pp. 725–736, 2005.
[9] C. De Persis and A. Isidori, "A geometric approach to nonlinear fault detection and isolation," *IEEE Trans. Autom. Control*, vol. 46, no. 6, pp. 853–865, Jun. 2001.
[10] P. Kabore and H. Wang, "Design of fault diagnosis filters and fault-tolerant control for a class of nonlinear systems," *IEEE Trans. Autom. Control*, vol. 46, no. 11, pp. 1805–1810, Nov. 2001.

[11] L. Li, S. X. Ding, J. Qiu, K. Peng, and Y. Yang, "An optimal fault detection approach for piecewise affine systems via diagnostic observers," *Automatica*, vol. 85, pp. 256–263, Nov. 2017.

[12] N. H. El-Farra, A. Armaou, and P. D. Christofides, "Analysis and control of parabolic PDE systems with input constraints," *Automatica*, vol. 39, pp. 715–725, Apr. 2003.

[13] S. Ghantasala and N. H. El-Farra, "Robust actuator fault isolation and management in constrained uncertain parabolic PDE systems," *Automatica*, vol. 45, no. 10, pp. 2368–2373, 2009.

[14] J. Alvarez-Ramirez, H. Puebla, and J. A. Ochoa-Tapia, "Linear boundary control for a class of nonlinear PDE processes," *Syst. Control Lett.*, vol. 44, no. 5, pp. 395–403, 2001.

[15] W. H. Ray, *Advanced Process Control*. New York, NY, USA: McGraw-Hill, 1981.

[16] T. Takagi and M. Sugeno, "Fuzzy identification of systems and its applications to modeling and control," *IEEE Trans. Syst., Man, Cybern. B, Cybern.*, vol. SMCB-15, no. 1, pp. 116–132, Jan. 1985.

[17] S.-G. Cao, N. W. Rees, and G. Feng, "Analysis and design for a class of complex control systems Part I: Fuzzy modelling and identification," *Automatica*, vol. 33, no. 6, pp. 1017–1028, 1997.

[18] L. Li, S. X. Ding, J. Qiu, and Y. Yang, "Real-time fault detection approach for nonlinear systems and its asynchronous T–S fuzzy observer-based implementation," *IEEE Trans. Cybern.*, vol. 47, no. 2, pp. 283–294, Feb. 2017.

[19] Y. Zhao, J. Lam, and H. Gao, "Fault detection for fuzzy systems with intermittent measurements," *IEEE Trans. Fuzzy Syst.*, vol. 17, no. 2, pp. 398–410, Apr. 2009.

[20] G. Feng, *Analysis and Synthesis of Fuzzy Control Systems: A Model-Based Approach*. Boca Raton, FL, USA: CRC Press, 2010.

[21] Z. Lendek, T. Guerra, R. Babuska, and B. D. Schutter, *Stability Analysis and Nonlinear Observer Design Using Takagi-Sugeno Fuzzy Models*. Dordrecht, The Netherlands: Springer-Verlag, 2010.

[22] Y. Zhao, J. Wang, F. Yan, and Y. Shen, "Adaptive sliding mode fault-tolerant control for type-2 fuzzy systems with distributed delays," *Inf. Sci.*, vol. 473, pp. 227–238, Jan. 2019.

[23] H. Dong, Z. Wang, J. Lam, and H. Gao, "Fuzzy-model-based robust fault detection with stochastic mixed time delays and successive packet dropouts," *IEEE Trans. Syst., Man, Cybern. B, Cybern.*, vol. 42, no. 2, pp. 365–376, Apr. 2012.

[24] M. Chadli, A. Abdo, and S. X. Ding, "H_-/H_∞ fault detection filter design for discrete-time Takagi-Sugeno fuzzy systems," *Automatica*, vol. 49, no. 7, pp. 1996–2005, 2013.

[25] S. Tong, B. Huo, and Y. Li, "Observer-based adaptive decentralized fuzzy fault-tolerant control of nonlinear large-scale systems with actuator failures," *IEEE Trans. Fuzzy Syst.*, vol. 22, no. 1, pp. 1–15, Feb. 2014.

[26] H. Schulte, M. Zajac, and P. Gerland, "Takagi-Sugeno sliding mode observer design for fault diagnosis in pitch control systems of wind turbines," in *Proc. 8th IFAC Symp. Fault Detection, Supervision Saf. Tech. Process.*, Mexico, 2012, pp. 546–551.

[27] R. J. Patton, J. Chen, and J. Chen, "A study on neuro-fuzzy systems for fault diagnosis," *Int. J. Syst. Sci.*, vol. 31, no. 11, pp. 1441–1448, 2000.

[28] C. J. Lopez-Toribio, R. J. Patton, and S. Daley, "Takagi–Sugeno fuzzy fault-tolerant control of an induction motor," *Neural Comput. Appl.*, vol. 9, no. 1, pp. 19–28, 2000.

[29] L. Li, M. Chadli, S. X. Ding, J. Qiu, and Y. Yang, "Diagnostic observer design for T–S fuzzy systems: Application to real-time-weighted fault-detection approach," *IEEE Trans. Fuzzy Syst.*, vol. 26, no. 2, pp. 805–816, Apr. 2018.

[30] H.-N. Wu and H.-X. Li, "H_∞ fuzzy observer-based control for a class of nonlinear distributed parameter systems with control constraints," *IEEE Trans. Fuzzy Syst.*, vol. 16, no. 2, pp. 502–516, Apr. 2008.

[31] J. Qiu, S. X. Ding, H. Gao, and S. Yin, "Fuzzy-model-based reliable static output feedback H_∞ control of nonlinear hyperbolic PDE systems," *IEEE Trans. Fuzzy Syst.*, vol. 24, no. 2, pp. 388–400, Apr. 2016.

[32] H.-N. Wu, J.-W. Wang, and H.-X. Li, "Design of distributed H_∞ fuzzy controllers with constraint for nonlinear hyperbolic PDE systems," *Automatica*, vol. 48, no. 10, pp. 2535–2543, 2012.

[33] E. Fridman and A. Blighovsky, "Robust sampled-data control of a class of semilinear parabolic systems," *Automatica*, vol. 48, no. 5, pp. 826–836, 2012.

[34] J.-W. Wang, H.-N. Wu, and H.-X. Li, "Distributed fuzzy control design of nonlinear hyperbolic PDE systems with application to nonisothermal plug-flow reactor," *IEEE Trans. Fuzzy Syst.*, vol. 19, no. 3, pp. 514–526, Jun. 2011.

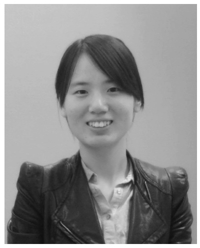
[35] M. A. Demetriou, "Guidance of mobile actuator-plus-sensor networks for improved control and estimation of distributed parameter systems," *IEEE Trans. Autom. Control*, vol. 55, no. 7, pp. 1570–1584, Jul. 2010.

[36] J.-W. Wang and H.-N. Wu, "Lyapunov-based design of locally collocated controllers for semi-linear parabolic PDE systems," *J. Franklin Inst.*, vol. 351, no. 1, pp. 429–441, 2014.

[37] K. Yuan, H. X. Li, and J. Cao, "Robust stabilization of the distributed parameter system with time delay via fuzzy control," *IEEE Trans. Fuzzy Syst.*, vol. 16, no. 3, pp. 567–584, Jun. 2008.

[38] B.-S. Chen and Y.-T. Chang, "Fuzzy state-space modeling and robust observer-based control design for nonlinear partial differential systems," *IEEE Trans. Fuzzy Syst.*, vol. 17, no. 5, pp. 1025–1043, Oct. 2009.

[39] L. Li, S. X. Ding, J. Qiu, Y. Yang, and D. Xu, "Fuzzy observer-based fault detection design approach for nonlinear processes," *IEEE Trans. Syst., Man, Cybern. Syst.*, vol. 47, no. 8, pp. 1941–1952, Aug. 2017.



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