IEEE Access
Multidisciplinary : Rapid Review : Open Access Journal

Received January 5, 2019, accepted January 8, 2019, date of publication January 14, 2019, date of current version February 6, 2019.
Digital Object Identifier 10.1109/ACCESS.2019.2892844

An Aspiration-Based Approach for Qualitative Decision-Making With Complex Linguistic Expressions

HAI WANG[1], (Member, IEEE), CHAO XU[1], ZESHUI XU[2], (Fellow, IEEE), XIAO-JUN ZENG[3], (Member, IEEE), AND WITOLD PEDRYCZ[4,5,6], (Fellow, IEEE)
[1]School of Information Engineering, Nanjing Audit University, Nanjing 211815, China
[2]Business School, Sichuan University, Chengdu 610064, China
[3]School of Computer Science, The University of Manchester, Manchester M13 9PL, U.K.
[4]Department of Electrical and Computer Engineering, University of Alberta, Edmonton, AB T6R 2V4, Canada
[5]Department of Electrical and Computer Engineering, Faculty of Engineering, King Abdulaziz University, Jeddah 21589, Saudi Arabia
[6]Systems Research Institute, Polish Academy of Sciences, 00-656 Warsaw, Poland

Corresponding author: Zeshui Xu (xuzeshui@263.com)

This work was supported in part by the National Natural Science Foundation of China under Grant 71571123, Grant 71601092, and Grant 61640220, in part by the Key University Science Research Project of Jiangsu Province under Grant 16KJA520002 and Grant 18KJB413006, and in part by the Collaborative Innovation Center of Audit Information Engineering and Technology.

ABSTRACT Decisions are usually made based on not only the performances of alternatives but also are implied by how the performances satisfy the decision makers' aspiration levels. This paper presents a linguistic aspiration-based solution to qualitative decision-making (QDM) where the aspiration levels and performances can be expressed by complex linguistic expressions (CLEs) such as hesitant fuzzy linguistic term sets and linguistic terms with weakened hedges. The proposed approach can deal with complex problems, which involve multi-criteria, multi-groups of experts, and multi-granular linguistic information. Based on the conventional aspiration-based approaches, the value function is defined by the probability of a CLE achieving its linguistic aspiration level. The performance of the proposed QDM approach is then demonstrated by solving the problem regarding the provider evaluation and selection. The proposed approach extends the range of available natural linguistic expressions that can be considered and used by experts and emphasizes the role of linguistic aspiration levels in QDM.

INDEX TERMS Decision making, aspiration, hesitant fuzzy linguistic term sets, linguistic terms with hedges, auditing.

I. INTRODUCTION

Qualitative information, taking the form of subjective assessments, exists in almost everywhere of the real-world decision-making problems [1], [2]. Such subjective assessments, instead of exact numerical values, are convenient to represent information originating in complex, ill-defined and unstructured problems. For instance, an expert would like to use terms such as *low* and *high* to evaluate the capability of data analysis of a software. To handle subjective assessments in qualitative decision-making (QDM), the techniques for computing with words (CWW) [3] are required. The fuzzy linguistic approach, whose core concepts involve linguistic variables, might be the most commonly used one in CWW because it strengthens the feasibility, flexibility and reliability of decision models [4]. The values of a linguistic variable take the form of words and expressions constructed in natural or artificial languages [3]. A suitable operation of these values could narrow down possible gaps existing in human-computer communication. In classical models, such as the linguistic 2-tuple model [5] and virtual linguistic model [6], [7], the values of a linguistic variable come from a linguistic term set (LTS) defined by the syntactic and semantic rules. Based on certain classical models, the recent developments of QDM are two-fold, i.e., they concern the developments of linguistic representation models and the construction of novel QDM approaches.

The first aspect of these pursuits improves the existing models to meet the requirement implied by more complex circumstances. Because of the complexity of problems and different levels of granularities of individual knowledge and expertise, it is often not sufficient to express opinions based on a single predefined LTS. Two strategies have been adopted. The first strategy considers multi-granular linguistic models, which are usually built at various levels of information granularity [8], [9]. A collection of LTSs should be prepared for experts at first. An additional transformation phase is usually necessary to uniform the obtained multi-granular information. Different from some traditional multi-granular linguistic models, the personalized individual semantics model considers multi granularities from the aspect of individual semantic cognition and defines the semantics of terms for each individual [10], [11]. This strategy is convenient and suitable for the cases where different granules come from different sources or different individuals. However, the necessity of preparing multiple LTSs and transforming among different granularities make this strategy relatively complex. The second strategy extends simple linguistic terms to specific types of complex linguistic expressions (CLEs) [12]. Xu [13] introduced the uncertain linguistic terms (ULTs) for the case when a linguistic argument does not match any of the terms in the LTS but instead is located between two different terms. Hesitant fuzzy linguistic term sets (HFLTSs) [14], [15] are developed to fit the case when the experts are thinking of several terms at the same time and express the opinions in terms of comparative linguistic expressions. According to the syntactic rules, the range of CLEs represented by HFLTSs covers those of ULTs. Recently, Wang *et al.* [16] proposed a new type of CLEs, namely linguistic terms with weakened hedges (LTWHs), to express the uncertainty behind using single terms. Weakened hedges, such as *more or less* and *roughly*, are employed to modify a given generic term. These CLEs could be elicited by natural languages. There are also some other models of CLEs which focus on artificial languages, such as linguistic distributions [17] and probabilistic linguistic term sets [18]. Due to the complexity of applications, these two strategies could be mixed. For instance, multi-granular linguistic models have been developed in the setting of HFLTSs [19], [20].

The second aspect develops new concepts and algorithms for the QDM problems based on the aforementioned linguistic models. In multi-granular linguistic setting, the QDM processes were proposed on the basis of fuzzy sets of basic LTSs [8], fuzzy numbers [21], antonyms-based aggregation [22], and others. Some QDM approaches based on ULTs were proposed in [23]–[25]. Since its inception, QDM with HFLTSs has become a topic of interest . Some researchers presented solutions to related theoretical issues, such as order relations [26], operational laws [27], and fuzzy envelopes [28]. Furthermore, many contributions focus on multi-criteria QDM (MCQDM) and/or group QDM (GQDM) [29], [30] with HFLTSs.

When handling uncertain linguistic information in the QDM framework, there are at least two limitations in the current developments which constitute our motivations. First, linguistic expressions which can be used by experts are not flexible or diversified enough. Most of the existing studies assume that the linguistic expressions, with respect to a certain criterion, can take only one form of either single terms, ULTs or HFLTSs, and should be subjected to a specific syntax. This is mainly for the convenience of computational process. It is however, not a suitable or feasible choice to limit the types of linguistic expressions. For instance, some auditors are authorized to assess the degree of reliability of a Big Data based auditing platform (BDAP) by the LTS:

$$\begin{aligned} S^{(8)} = \{&s_0 : \textit{nothing}, s_1 : \textit{extremely low}, s_2 : \textit{very low}, \\ &s_3 : \textit{low}, s_4, \textit{medium}, s_5 : \textit{high}, s_6 : \textit{very high}, \\ &s_7 : \textit{extremely high}, s_8 : \textit{perfect}\} \end{aligned} \tag{1}$$

The evaluation could be expressed by different types of complex linguistic expressions such as:

(1) A HFLTS: *at least high*;

(2) A LTWH: *more or less high*.

In this case, two aspects should be further investigated. First, Wang *et al.* [16] only defined the computational model for LTWHs while the issue on how to apply them in practical QDM problems needs to be addressed. Second, the experts might make use of any types of linguistic expressions for evaluations according to their individual linguistic conventions. It is essential to enable computing with all the types of CLEs mentioned above in a single QDM problem, or even in the process of evaluating a single criterion. Therefore, the first motivation of this paper is to handle the QDM problems in which the experts can express their opinions in a more fulfilling, fruitful and flexible way. It would definitely facilitate the process of expressing evaluations without the limitation imposed by certain types of linguistic expressions.

Second, the utilities and aspiration levels of experts are not well taken into account. Most of the papers mentioned above simply substitute the experts' utilities with the evaluation values. Nevertheless, the utility function may not be regarded as an increasingly or decreasingly linear function with respect to the evaluation values. To the contrary, people usually make decisions based on the aspiration levels (or targets) instead of the performances of alternatives because of the limited cognitive resources and incomplete information. The concept of aspiration levels, which rests at the core concept of bounded rationality [31], plays a key role in multi-criteria decision-making (MCDM). Real-world decisions could plausibly be made by accepting the first encountered alternative which meets a sufficiently good target [32]. It is rational and essential to consider aspiration levels in QDM. Till now, there are a number of studies focusing on aspirations in the MCDM problems. Refer to Section II for more details. However, very few of them consider the QDM problems with complex linguistic information. Feng and Lai [33] presented an aspiration-based approach for heterogeneous information. However the linguistic information in that study can only take the form of either single terms (for a certain criterion) or ULTs (for another criterion). Thereby, the second motivation of this study is to introduce the idea of aspiration into the field of QDM where the decision information takes the form of multi-types of CLEs.

Based on the two arguments, this paper proposes a novel QDM approach established in linguistic setting. The originality of the proposed approach is implied by the following: (1) Two types of CLEs, i.e., HFLTSs (including ULTs), and LTWHs, are included simultaneously to represent uncertain linguistic information. Experts could express their opinions by means of these CLEs without any limitation. We focus on these CLEs because they frequently emerge in natural languages. (2) Experts' linguistic aspiration levels are considered. When making decisions, one looks for the alternative which satisfies the experts' aspirations to the highest extent. The experts' aspirations could be conveniently expressed by the two types of CLEs as well. (3) The focused QDM problem involves multi-criteria, multi-groups of experts and multi-granular linguistic information. Thus the proposed approach can be referred to as the M^3QDM one.

The reason of focusing multi-criteria, multi-groups and multi-granularities is driven by the application problem in Section V which considers a provider selection of BDAPs. Many scholars and auditors have shown their affirmative attitude about the productivity of big data in the audit domain [34]–[36]. In 2015, the Chinese government issued a new regulation to construct the mode of Big Data auditing, enhance the capability, efficiency and quality of auditing, ensure the implementation of full audit coverage in the Big Data era. Before the application of Big Data auditing, the selection of BDAP is vital to auditors. Due to the various uncertainty factors resulting from the sources of information, expertise of experts, qualitative nature of some criteria etc., evaluation and aspiration information might be represented by multi-granular CLEs. Because of the complexity of the underlying problem, multi-groups of experts are involved, each of which is going to evaluate a subset of criteria. Roughly speaking, data scientists, information system specialists, auditors, financial department are indispensable in the overall auditing process.

The paper focuses mainly on the development of the M^3QDM approach. Following the conventional utility approaches and aspiration-based approaches, we develop a common syntactic rule to represent the focused types of CLEs, and then define the value functions of CLEs based on their semantics. The approach is then illustrated by the problem of selecting a BDAP.

The paper is organized as follows: Section II recalls the recent advance of MCDM with aspiration levels. Three types of CLEs are reviewed in Section III. Then, Section IV describes the problem and offers a solution. The approach is then illustrated by solving the BDAP provider selection problem in Section V. Finally, Section VI presents a comparative analysis while Section VII concludes the paper.

II. MCDM WITH ASPIRATION LEVELS: A REVIEW

The concept of aspiration levels plays an important role in managerial decision-making. In the satisficing model [31], subjects seek an alternative or solution that meets aspiration levels, instead of maximizing the expected utility in the classical sense. In the conventional utility theory, the utility function is monotone with respect to the performance values. However, ample and substantial empirical evidence indicates that individual preferences cannot be described by the conventional concave or convex utility functions [32], [37], [38]. The satisficing heuristic works as follows: if a solution (or a small set of solutions) can be found to satisfy the stated aspiration levels, then it is accepted; otherwise, the aspiration levels should be relaxed. If too many solutions are admitted by the aspiration levels, then they should be tightened [39]. The consideration of aspiration levels would benefit to decrease the complexity of the problem in hand, because of the subject limitation of cognitive capabilities [40], [41].

Except for some specific concentration of decision-making with single criterion utility function [37], most of the existing studies contribute to MCDM. Among them, most studies link aspiration levels to probabilities where risk choices are involved, some link them to reference points, and others consider the fuzzy aspirations. The following is organized based on this taxonomy.

Stochastic MCDM, with aspiration levels, are usually solved by searching alternatives, which approach the aspiration levels at most. Frequently, this is implemented by the satisficing heuristic. The first interactive method, proposed in [42], selects the closest non-dominated alternative by obtaining feedback information and adjusting the aspiration levels. Thereafter, a number of solutions have been proposed based on the similar ideas [43]–[45]. When the size of alternatives is large, a quad tree-based method was developed [46]. Apart from the development of MCDM solutions, Wang and Zionts [47] considered the robustness of solutions derived by interactive models, where a solution is robust if many aspiration levels map to it. Tsetlin and Winkler [48] developed a theoretical model to consider uncertain dependent aspiration levels and performances. Another theoretical model [40] is devoted to combining expected aspiration-based utility with loss and gain probabilities. Recently, Fantozzi and Spizzichino [49] formally described the connections between aspiration-based utility and aggregation-based extensions of capacities. Besides, there are also endeavors which seek for the alternative with the greatest degree of approaching to aspiration levels by optimization models. Yun *et al.* [50] utilized the genetic algorithm and data envelopment analysis to list the Pareto optimal solutions located close to aspiration levels. Associated with a case study, Feng and Lai [33] developed a MCDM method with aspirations where the performances could be linguistic terms and ULTs. Instead of adjusting the experts' aspiration levels, an optimization model was built to seek for the collective alternative ranking that is agreed by at least half of the experts.

Most reference point-based methods are based on prospect theory where the value function divides outcomes into gains and losses. Fan *et al.* [51] proposed a prospect-theory-based MCDM solution where the performance values are either numeric values or interval numbers and the reference point is fixed by aspiration levels. In a similar contribution [52], three different types of aspirations are taken into account. The method proposed by Tan *et al.* [53] focuses on a stochastic MCDM problems. They model the psychological behavior of decision makers by a prospect stochastic dominance degree.

In the fuzzy environment, the aspiration level is neither a reference point nor a probabilistic distribution of choices, but a fuzzy set (like a linguistic term). The employment of fuzzy set theory enables decision makers to specify imprecise and vague aspiration levels. Prior work of this field can be found in [54] and [55]. Based on a bounded domain, their work presents solutions to obtain the probability of meeting fuzzy aspiration levels. The involved utility functions are monotonically increasing. Later, the fuzzy aspiration-oriented model proposed by Yan *et al.* [56] handles three types of fuzzy preferences by the formulation of three types of fuzzy targets: fuzzy min, fuzzy max and fuzzy equal. Due to the vagueness of evaluating aesthetics, performing Kansei evaluation by fuzzy sets is much more efficient than using numerical data. Thereby several contributions, which focus on the Kansei evaluation, develop the theory and methods related to fuzzy aspiration levels. Yan *et al.* [57] first introduced three types of fuzzy aspiration levels to Kansei evaluation. The model has been improved by including the linguistic 2-tuple approach in Yan *et al.* [58]. The aggregation strategy in these two papers is criticized and improved in another development [59] where both vagueness and variation are included in the proposed uncertain Kansei profile. In a more recent study, Yan *et al.* [60] employed both stochastic dominance and fuzzy targets in order to avoid the potential subjectivity of CWW techniques.

These current developments have delivered great contributions to MCDM with aspiration levels. The merits of three categories of investigation are prominent. The probability-based methods and fuzzy aspiration-based methods have the advantage to model uncertainties of representing aspiration levels, whereas the reference point-based methods pay more attention to model the psychological behavior of decision makers. The interactive methods seem to be a wonderful way to follow the idea of satisficing heuristic. Yet the optimization models can reduce the participation from the experts.

However, there are some limitations in the existing fuzzy aspiration-based methods. Only single terms and ULTs are available in the methods. This would limit their applicability to complex problems in which the experts may prefer to express their opinions by various types of linguistic expressions due to their language custom and the degrees of uncertainties. Moreover, multi-granular linguistic information is inevitable in complex problems because one LTS may not be suitable for the entire evaluation criteria. But this has not been considered in the existing methods. All these identified limitations and omits are the issues to be addressed in the following sections.

III. COMPLEX LINGUISTIC EXPRESSIONS AND THEIR SEMANTICS

This section is devoted to recalling and specifying some necessary preliminaries with regard to CLEs. We start from the basis of CWW models.

A. PRELIMINARIES

Linguistic variables can be considered to approximate the characteristic of phenomena that are too complex or too ill-defined to be described by numerical variables. Given a domain $U = [L, R]$, where L and R are real numbers, a linguistic variable can be defined by a syntactic rule to present the names of values and a semantic rule to identify the meaning of each value [3].

The set of linguistic values are collected by a LTS. A set of $\tau + 1$ linguistic terms can be denoted by:

$$S^{(\tau)} = \{s_\alpha | \alpha = 0, 1, \dots, \tau\} \tag{2}$$

For example, a LTS with 9 terms can be found in (1). The semantics of a term s_α is a fuzzy set defined in the domain U, usually represented by a trapezoidal fuzzy number (TraFN) (a, b, c, d), where

$$\mu_{s_\alpha}(x) = \begin{cases} (x-a)/(b-a), & \max\{L, a\} \le x < b \\ 1, & b \le x \le c \\ (d-x)/(d-c), & c < x \le \min\{d, R\} \\ 0, & otherwise \end{cases} \tag{3}$$

If $b = c$, then the TraFN reduces to a triangular fuzzy number (TriFN). As shown in many studies [8], [61], the domain U is usually assumed to be uniformly distributed or piecewise uniformly distributed. In such cases, the semantics of terms can be represented by TriFNs. Specifically, the semantics of terms can be generated as follows: (1) insert a set of $\tau - 1$ points, denoted by $x_1, x_2, \dots, x_{\tau-1}$, into the domain U; (2) let $x_0 = L$ and $x_\tau = R$; (3) insert two sets of points $\{x_{-i} = L - (R-L) \cdot i/\tau | i = 1, 2, \dots \varsigma + 1\}$ and $\{x_{\tau+j} = R + (R-L) \cdot j/\tau | j = 1, 2, \dots \varsigma + 1\}$ into the intervals $(-\infty, L)$ and $(R, +\infty)$, respectively; and (4) let all the points be ordered by $x_i < x_j \Leftrightarrow i < j$. Then for each $s_\alpha \in S^{(\tau)}$, the semantics can be denoted by $\mu_{s_\alpha}(x) = (x_{\alpha-1}, x_\alpha, x_{\alpha+1})$. Note that, in (3), the selected $2(\varsigma + 1)$ points are out of the domain U and thus could be called virtual points. They are selected so that the semantics of LTWHs could be represented by means of TriFNs, as can be seen in Theorem 1.

In the ordered structure model, the following is usually required:

(1) An total order: $s_\alpha \le s_\beta \Leftrightarrow \alpha \le \beta$;

(2) A negation operator: $Neg(s_\alpha) = s_\beta$, where $\beta = \tau - \alpha$.

In applications, experts may realize that it becomes difficult to select one term from the given LTS. Being different from the strategy in multi-granular linguistic decision-making, the experts may seek for richer linguistic expressions, which can be generated from the LTS, as supplementary values of the linguistic variable. This results in the so-called CLEs. In this sense, weakened hedges which weaken the intensity of an original linguistic term and represent the uncertain degree in a qualitative manner are frequently considered in natural languages. In a QDM problem, the set of weakened hedges which might be considered by the involved experts are denoted by [16]:

$$H^{(\varsigma)} = \{h_t | t = 1, 2, \ldots, \varsigma\} \tag{4}$$

In practice, $H^{(\varsigma)}$ is determined based on the linguistic convention of the involved experts. Specifically, the set of weakened hedges can be collected at first and then classified according to their weakening power. The hedges with the same or very similar weakening power are encoded by a h_t. For instance, one may think that *rather* and *roughly* have nearly the same power. As a result, the weakening power of distinct h_t is distinguishable. Finally, hedges are ordered such that h_j has more weakening power than h_i if and only if $i < j$.

B. SYNTACTIC RULES OF COMPLEX LINGUISTIC EXPRESSIONS

The following context-free grammar is presented to serve as a common syntactic rule to generate the existing types of CLEs based on the above preliminaries.

Definition 1: Let G_H be a context-free grammar, $S^{(\tau)}$ and $H^{(\varsigma)}$ be the LTS and weakened hedge set specified in (2) and (4), respectively. The elements of $G_H = (V_N, V_T, I, P)$ are defined as:

$$\begin{aligned}
V_N &= \{\langle\text{primary term}\rangle, \langle\text{unary relation}\rangle, \langle\text{binary relation}\rangle,\\
&\qquad \langle\text{conjunction}\rangle\},\\
V_T &= \{\text{lower than, greater than, at least, at most, between,}\\
&\qquad \text{and}, s_0, s_1, \ldots, s_\tau, h_1, h_2, \ldots, h_\varsigma\},\\
I &\in V_N,\\
P &= \{I ::= \langle\text{unary relation}\rangle\langle\text{primary term}\rangle | \langle\text{binary relation}\rangle\\
&\qquad\qquad \langle\text{primary term}\rangle\langle\text{conjunction}\rangle\langle\text{primary term}\rangle,\\
&\quad \langle\text{primary term}\rangle ::= s_0 | s_1 | \cdots | s_\tau,\\
&\quad \langle\text{unary relation}\rangle ::= \text{lower than}|\text{greater than}|\text{at least}|\\
&\qquad\qquad\qquad\qquad \text{at most}|h_1|h_2|\cdots|h_\varsigma,\\
&\quad \langle\text{binary relation}\rangle ::= \text{between},\\
&\quad \langle\text{conjunction}\rangle ::= \text{and}\}.
\end{aligned}$$

Similar to [14], there are some limitations in Definition 1. If $\langle$unary relation$\rangle$ = *lower than*, then the "*primary term*" cannot be s_0; if $\langle$unary relation$\rangle$ = *greater than*, then the "*primary term*" cannot be s_τ. Different from [14], (1) we introduce some weakened hedges to be the possible values of "*unary relation*" to include LTWHs in the common framework; and (2) single terms are not considered as CLEs. Thus, for the case "between s_α and s_β", we require $s_\alpha < s_\beta$. Generally, we state that a linguistic expression is called a CLE if it included at least two primary terms in a direct way (such as HFLTSs) or an indirect way (such as LTWHs).

The use of linguistic hedges is complicated than that of conjunctions in CLEs. This is because there are two different interpretations of hedges in psychology. A hedge with inclusive interpretation expresses the degree of uncertainty of using single terms in a qualitative manner, whereas a hedge with non-inclusive interpretation modifies a term to another [62]. The purpose of LTWHs is to model the uncertainty of using single terms. Thus we focus only on the hedges with inclusive interpretation. Accordingly, the CLEs generated by Definition 1 is syntactically right if the hedges included in (4) are considered to express the uncertainty of using single terms.

HFLTSs, which were developed for the situation where the experts are thinking of several terms at the same time [14], tend to list all original terms involved in a comparative linguistic expression. Generally, a HFLTS is an ordered finite subset of $S^{(\tau)}$, which can be generated by the following transformation function E_{G_H}:

$$\begin{aligned}
E_{G_H}(\text{at most } s_\beta) &= \{s_\alpha | s_\alpha \in S^{(\tau)}, s_\alpha \le s_\beta\}\\
E_{G_H}(\text{lower than } s_\beta) &= \{s_\alpha | s_\alpha \in S^{(\tau)}, s_\alpha < s_\beta\}\\
E_{G_H}(\text{at least } s_\alpha) &= \{s_\beta | s_\beta \in S^{(\tau)}, s_\alpha \le s_\beta\}\\
E_{G_H}(\text{greater than } s_\alpha) &= \{s_\beta | s_\beta \in S^{(\tau)}, s_\alpha < s_\beta\}\\
E_{G_H}(\text{between } s_\alpha \text{ and } s_\beta) &= \{s_\gamma | s_\gamma \in S^{(\tau)}, s_\alpha \le s_\gamma \le s_\beta,\\
&\qquad s_\alpha < s_\beta\}
\end{aligned}$$

Remark 1: Actually, the first type of CLEs is the ULTs proposed in [6] which model the expression "between s_α and s_β" generated by Definition 1. In this study, we considers ULTs as a special case of HFLTSs because: (1) the syntactic rule is included in that of HFLTSs; and (2) as will be seen in the next subsection, the semantics of ULTs coincides with that of HFLTSs if the latter is represented by envelopes.

HFLTSs represent the uncertainty by including more than one linguistic term. The envelope of a HFLTS is a ULT [14]. For the convenience of applications, some researchers, such as [63] and [64], suggested the use of the linguistic interval $[s_\alpha, s_\beta]$ instead of a linguistic set $\{s_\alpha, s_{\alpha+1}, \ldots, s_\beta\}$.

Being different from HFLTSs which emphasize their boundaries, LTWHs start from one original term that could possibly be the real value, and modify this term by a weakened hedge. A LTWH, denoted by a 2-tuple $\langle h_t, s_\alpha\rangle$, can be generated by the following transformation function E_{G_H}:

$$E_{G_H}(h_t, s_\alpha) = \langle h_t, s_\alpha\rangle$$

In many cases of evaluations, it is enough to consider the two most frequently used hedges h_1 = *more or less* and h_2 = *roughly*. In addition, if one does not have any doubt about the use of a single term, actually he/she is using the hedge *definitely* which has no weakened power. Therefore, we specify (4) as:

$$\bar{H}^{(2)} = \{h_0 : \textit{definitely}, h_1, \textit{more or less}, h_2 : \textit{roughly}\} \tag{5}$$

Accordingly, an original term $s_\alpha \in S^{(\tau)}$ is the same as the LTWH $\langle h_0, s_\alpha\rangle$. An original term is seen as a special case of LTWHs.

Remark 2: Different from the existing models of linguistic hedges, such as the powering hedges defined by Zadeh [3], LTWHs are proposed to model the uncertainty of using single linguistic terms which is implied by the weakened hedges. The transparent feature of LTWHs is that they focus on a new type of CLE, which is frequently considered by the experts under uncertain circumstances.

Example 1: Given $\bar{H}^{(2)}$ in (5) and $S^{(8)}$ in (1), some LTWHs could be: $\langle h_0, s_4\rangle$ = (*definitely*) medium; $\langle h_1, s_6\rangle$ = *more or less* very high; $\langle h_2, s_1\rangle$ = *roughly* extremely low.

It is clear that HFLTSs and LTWHs are not mutually substitutable. They models different types of CLEs which are both frequently emerged in natural languages. Mathematically, there are usually two ways to depict uncertainties of a variable. The first one is to determine the boundaries of the variable and then form an interval. The second one is to seek for a constant that could be the real value and then decide a radius to indicate the range of the variable. Obviously, HFLTSs model uncertainties of CLEs by the former manner and LTWHs model those by the latter manner.

C. SEMANTICS OF COMPLEX LINGUISTIC EXPRESSIONS

The semantics of HFLTSs introduced in [14] is based on the concept of envelopes. In this sense, given $\mu_{s_\alpha} = (x_{\alpha-1}, x_\alpha, x_{\alpha+1})$, $\mu_{s_\beta} = (x_{\beta-1}, x_\beta, x_{\beta+1})$, the semantics of HFLTS $[s_\alpha, s_\beta]$ is:

$$\mu_{[s_\alpha,s_\beta]}(x) = (x_{\alpha-1}, x_\alpha, x_\beta, x_{\beta+1}) \tag{6}$$

Another solution for the semantics was the fuzzy envelope proposed in [28]. The underlying idea of computing fuzzy envelopes is to assume that the important degrees of the terms in a HFLTS are generally different. This is not always implied by the comparative linguistic expressions. Some other definitions of semantics of HFLTSs were summarized in [15]. In this study, for the sake of computing aspiration-based value functions, the semantics of CLEs will be utilized. To ease the procedure, we consider the envelopes as the semantics of HFLTSs. Thus a HFLTS $\{s_\alpha, s_{\alpha+1}, \ldots, s_\beta\}$ is also written as $[s_\alpha, s_\beta]$. An example of envelopes of HFLTSs can be found in Fig. 1.

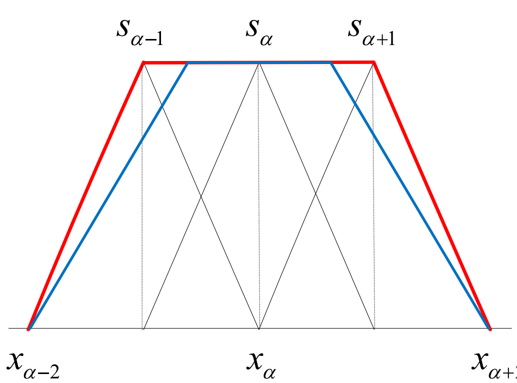

FIGURE 1. An example of the envelope (in red) and fuzzy envelope (in blue) of HFLTSs ($\{s_{\alpha-1}, s_\alpha, s_{\alpha+1}\}$).

The intuitive motivation of defining the semantics of LTWHs is that $x(\in U)$ is *more or less* s_α if x is similar to some $y(\in U)$ which are also s_α. To address this fact, the semantics of $\langle h_1, s_\alpha\rangle$ can be defined based upon the idea of upper approximation of rough fuzzy sets as follows [16]:

Definition 2: Let $S^{(\tau)}$ be a LTS defined in the domain U. For any $x \in U$, $\mu_{\langle h_1,s_\alpha\rangle}$ is defined by

$$\mu_{\langle h_1,s_\alpha\rangle}(x) = \sup_{y\in U} \mathcal{T}(sim(x,y), \mu_{s_\alpha}(y)) \tag{7}$$

where the function sim is a similarity measure defined in U and $\mathcal{T}$ is a triangular norm.

Remark 3: The similarity measure should be defined according to the distribution of the domain. If the domain is uniformly distributed, then similarity can be defined by $1-|x-y|/(R-L)$. Otherwise, the domain must be piecewise uniformly distributed due to the way we construct the semantics. Then a bijection can be employed to map the domain into a uniformly distributed one. See [16] for more details.

According to Definition 2, we have $\mu_{s_\alpha}(x) \le \mu_{\langle h_1,s_\alpha\rangle}(x)$ for any $x \in U$, which means that the inclusive relation holds, i.e., $s_\alpha \subseteq \langle h_1, s_\alpha\rangle$. Based on the linear (or piecewise linear) similarity measure in [16], the semantics of $\langle h_1, s_\alpha\rangle$ can be represented by a TriFN, $\mu_{\langle h_1,s_\alpha\rangle}(x) = (x_{\alpha-2}, x_\alpha, x_{\alpha+2})$. To compute the semantics of any LTWHs, Wang *et al.* [16] extended the conclusion of Definition 2 based on two premises: (1) Given $s_\alpha \in S^{(\tau)}$, $\langle h_1, s_\alpha\rangle \subseteq \langle h_2, s_\alpha\rangle \subseteq \cdots \subseteq \langle h_\varsigma, s_\alpha\rangle$ holds for any $h_t \in H^{(\varsigma)}$; and (2) the gap of weakening power between h_t and h_{t+1} are equal, where $h_t, h_{t+1} \in H^{(\varsigma)}$. These allow us to define the semantics of LTWHs recursively, i.e., $\langle h_{t+1}, s_\alpha\rangle = \langle h_1, \langle h_t, s_\alpha\rangle\rangle$. As a result, the semantics of a LTWH can be simply represented by a TriFN [16].

Theorem 1: Let $S^{(\tau)}$ be the LTS defined in U, $H^{(\varsigma)}$ be the weakened hedge set, and $\mathcal{T}(x,y) = min\{x,y\}$. For any $s_\alpha \in S^{(\tau)}$ and $h_t \in H^{(\varsigma)}$, the semantics of LTWH $\langle h_t, s_\alpha\rangle$ is

$$\mu_{\langle h_t,s_\alpha\rangle}(x) = (x_{\alpha-t-1}, x_\alpha, x_{\alpha+t+1}) \tag{8}$$

Example 2: The semantics of LTWHs in Example 1 are, as shown in Fig. 2,

more or less very high: $\mu_{\langle h_1,s_6\rangle}(x) = (0.5, 0.75, 1)$,
roughly extremely low: $\mu_{\langle h_2,s_1\rangle}(x) = (-0.25, 0.125, 0.5)$.

IV. THE PROPOSED APPROACH FOR MULTI-CRITERIA, MULTI-GROUPS AND MULTI-GRANULAR QUALITATIVE DECISION-MAKING

Based on the traditional framework of MCDM problems, this section develops the M^3QDM approach based on the preliminaries presented in the above section. The QDM problem is described in Section IV-A and then the framework of the M^3QDM approach is presented in Section IV-B. Finally, each procedure of the approach is specified in a subsection.

A. DESCRIPTION OF THE FOCUSED PROBLEMS

The traditional MCDM problems refer to the selection among a set of alternatives $A = \{a_i | i = 1, 2, \ldots, I\}$ with respect to the collection of criteria $C = \{c_j | j = 1, 2, \ldots, J\}$, associated with a weighting vector $\omega = (\omega_1, \omega_2, \ldots, \omega_J)^T$

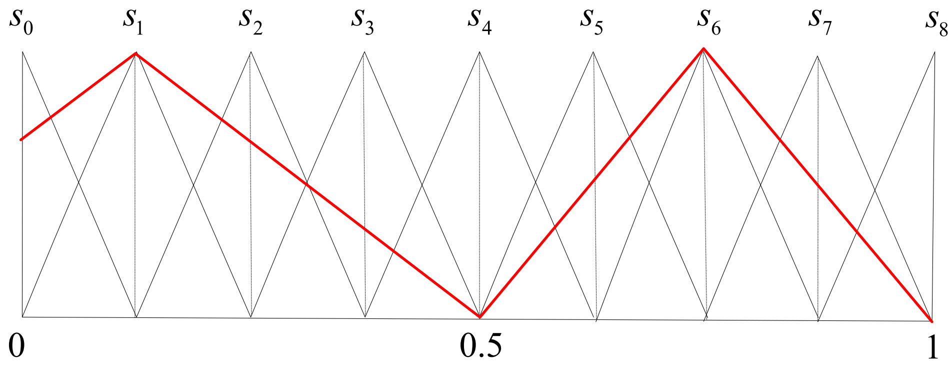

FIGURE 2. Examples of LTWHs ($\langle h_2, s_1\rangle$ and $\langle h_1, s_6\rangle$).

such that $\sum_{j=1}^{J}\omega_j = 1$ and $\omega_j \in [0,1]$ $(j = 1, 2, \ldots, J)$. However, due to the complexity of the problem, like the provider selection of a BDAP in this study, one individual cannot evaluate alternatives with respect to all the criteria. On the contrary, multiple experts are usually considered in order to avoid the arbitrary and biased opinions and enable a way to handle the uncertainty of evaluations (by measuring the group's consensus, for example).

Moreover, if the number of experts is large, the group can be divided into several small groups based on the knowledge and expertise of experts. Formally, a set of experts is divided into M groups, denoted here by $G = \{G_m | m = 1, 2, \ldots, M\}$. The experts in the mth group are collected by $G_m = \{e_{mn} | n = 1, 2, \ldots, \#G_m\}$, where e_{mn} denotes the nth expert in the mth group, $\#G_m$ is number of experts in the group. The experts in the same group are with the similar profession and expertise, and thus can be authorized to evaluate the same subset of criteria. Suppose that each criterion is evaluated by only a single group. Thus, the set C can be divided into M subsets, denoted by $C_m = \{c_{mp} | p = 1, 2, \ldots, \#C_m\}$, where $\#C_m$ is the number of criteria in the set C_m. Especially, the criteria in C_m are supposed to be evaluated by the group G_m. Furthermore, in a qualitative setting, the experts prefer to evaluate different criteria by distinct granularities of LTSs. In our focused problem, we assume that a set of Q context-free LTSs defined in the same domain $U = [L, R]$ are provided for the experts, which are denoted by $\{S^{(\tau_q)} | q = 1, 2, \ldots, Q\}$ with $S^{(\tau_q)} = \{s_\alpha^{(q)} | \alpha = 0, 1, \ldots, \tau_q\}$. The weakened hedge set is fixed by $\bar{H}^{(2)}$ in (5). The performance values, as well as linguistic aspiration levels, are allowed to be represented by the CLEs that can be generated by Definition 1. The structure of the problem can be seen in the top of Fig. 3. Assume that the matrix of performance values and the vector of linguistic aspiration levels, with respect to the criteria in C_m, provided by the expert e_{mn}, are listed in Table 1, where $n = 1, 2, \ldots, \#G_m$, $m = 1, 2, \ldots, M$.

The goal of the M^3QDM problem is to select the most desirable alternative(s) from A, according to the linguistic information provided by the experts.

TABLE 1. The linguistic aspiration levels and performance values provided by expert e_{mn}.

Criterion	c_{m1}	c_{m2}	$\cdots$	$c_{m,\#C_m}$
Aspiration	$ll_{m,n,1}^{(A)}$	$ll_{m,n,2}^{(A)}$	$\cdots$	$ll_{m,n,\#C_m}^{(A)}$
a_1	$ll_{11}^{(m,n)}$	$ll_{12}^{(m,n)}$	$\cdots$	$ll_{1,\#C_m}^{(m,n)}$
a_2	$ll_{21}^{(m,n)}$	$ll_{22}^{(m,n)}$	$\cdots$	$ll_{2,\#C_m}^{(m,n)}$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
a_I	$ll_{I1}^{(m,n)}$	$ll_{I2}^{(m,n)}$	$\cdots$	$ll_{I,\#C_m}^{(m,n)}$

B. FRAMEWORK OF THE PROPOSED M^3QDM APPROACH

As shown in Fig. 3, the proposed solution for the M^3QDM problem includes four procedures. The main aims and tasks are illustrated as follows:

Step 1: Derivation of value functions. The original linguistic information LM_{mn} taking the form of multi-granular CLEs generated by the set of LTSs, as shown in Table 1, is transformed to its corresponding value matrix VM_{mn}, where $n = 1, 2, \ldots, \#G_m$, $m = 1, 2, \ldots, M$, by the predefined value functions in Section IV-C.

Step 2: Groups' consensus checking. As the aspiration levels are considered in our approach, the consensus can be defined and checked by the fact that to what degree the alternatives meet the aspiration levels of the experts in a group. Specifically, each group's consensus is measured according to the experts' value functions rather than the performance values. In order that, an optimal model is formed to uncover if VM_{mn} $(n = 1, 2, \ldots, \#G_m)$ are acceptable by assigning proper weights to the group's members. Based on which, an interactive algorithm is then developed to improve the group's consensus.

Step 3: Fusion. For each G_m, if the matrices are acceptable, then the mth block of the overall value matrix, denoted by VM_m, i.e., the overall values of the alternative with respect to the criteria in C_m, can be obtained by a weighting averaging operator, where $m = 1, 2, \ldots, M$. Therefore, the overall value matrix VM is available after all the blocks of the groups are collected.

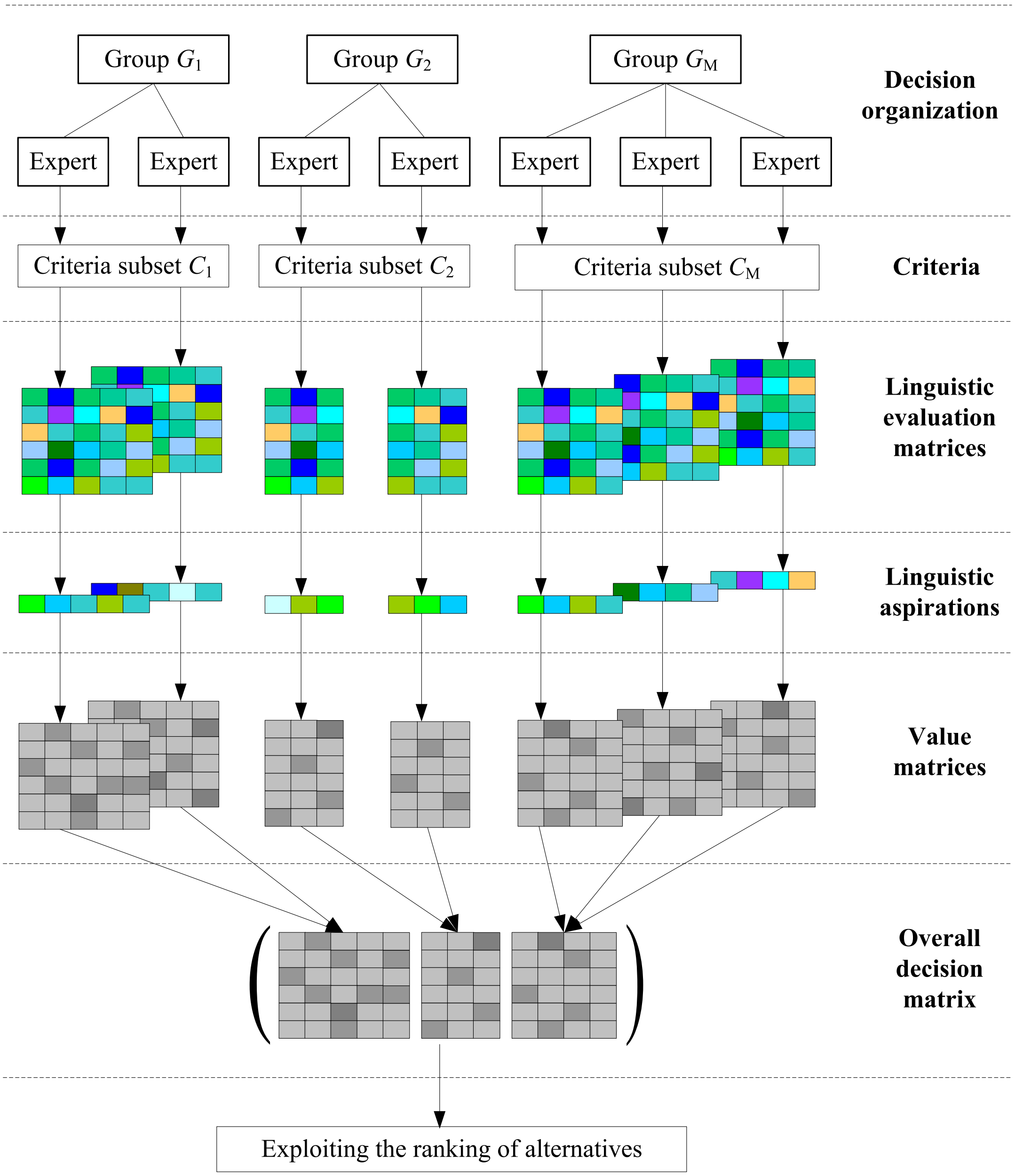

FIGURE 3. The framework of the proposed M^3QDM problems.

Step 4: Exploitation. A weighted aggregating operator can be adopted to exploit the ranking of alternatives based on the overall value matrix.

Note that the uniformation of multi-granular CLEs is implemented by computing their semantics. Because the set of LTSs are defined in the same domain, the multi-granular CLEs could be transferred to TraFNs in U, according to the preliminaries in Section III. Thus, the collected information is uniformed once the transformation in Step 1 is completed. For the convenience of description, in the rest of this section, a linguistic term $s_\alpha^{(q)}(\in S^{(\tau_q)})$ is simplified as s_α.

C. VALUE FUNCTIONS BASED ON LINGUISTIC ASPIRATION LEVELS

This subsection is devoted to defining the value functions with respect to three types of linguistic aspiration levels. Similar to other aspiration-based solutions in fuzzy circumstance, we consider fuzzy aspiration levels which are expressed by linguistic expressions and refer to them as linguistic aspiration levels. In the proposed approach, aspiration levels and performance values can be represented by the CLEs defined in Definition 1. For convenience, a CLE is denoted as ll in the sequel.

For the convenience of notations, let us consider an attribute X whose evaluation value x falls in the domain $[X_{min}, X_{max}]$. In the conventional utility theory, decisions are made based on the concept of expected utility [65]. Based on some axioms, the Von Neumann and Morgenstern utility function $u(x)$ is frequently considered. In the aspiration-based model, the value function is aspiration-oriented and depends only on whether the aspiration is achieved [66]. In this sense, a decision maker has only two different utility levels, that is,

$$u(x) = \begin{cases} 1, & \textit{the target is achieved} \\ 0, & \textit{otherwise} \end{cases} \tag{9}$$

Given an aspiration $t \in [X_{min}, X_{max}]$, assume that the attribute is montonically increasing and x is independent to t, then the value function of x is:

$$v(x) = 1 \cdot P(x \geq t) + 0 \cdot (1 - P(x \geq t)) = P(x \geq t) \tag{10}$$

where $P(x \geq t)$ is the probability of x meeting the aspiration t. For an uncertain aspiration T associated with its probability density function $p(t)$, $P(x \geq t)$ could be specified as:

$$P(x \geq t) = \int_{X_{min}}^{x} p(t)dx \tag{11}$$

In the QDM environment, the experts' aspiration levels are usually imprecise and vague, and could be expressed by linguistic expressions. In our study, the linguistic aspiration levels, denoted by $ll^{(A)}$, are represented by CLEs. In real-world QDM problems, based on a predefined LTS $S^{(\tau)}$ and weakened hedge set $H^{(\varsigma)}$, the linguistic aspiration levels could be the following three types (as listed in Table 2).

TABLE 2. Three types of linguistic aspiration levels and their representation.

Type	Notation	Representation	
		HFLTS	LTWH
benefit form	$ll^{(BA)}$	$[s_\beta, s_\tau]$	$\langle h_t, s_\tau \rangle$
cost form	$ll^{(CA)}$	$[s_0, s_\alpha]$	$\langle h_t, s_0 \rangle$
interval form	$ll^{(IA)}$	$[s_\alpha, s_\beta]$	$\langle h_t, s_\gamma \rangle$

(1) Linguistic aspiration levels with benefit form (denoted by $ll^{(BA)}$). They could take the forms of HFLTSs like "at least s_β" (denoted as $\{s_\beta, s_{\beta+1}, \ldots, s_\tau\}$ or $[s_\beta, s_\tau]$), "greater than $s_{\beta-1}$", and LTWHs like $\langle h_t, s_\tau \rangle$, where $s_0 < s_\beta \leq s_\tau$, $s_\beta \in S^{(\tau)}$, and $h_t \in H^{(\zeta)}$.

(2) Linguistic aspiration levels with cost form (denoted by $ll^{(CA)}$). Also, three different forms might be involved, which are HFLTSs like "at most s_α" (denoted as $\{s_0, s_1, \ldots, s_\alpha\}$ or $[s_0, s_\alpha]$), "lower than $s_{\alpha+1}$", and LTWHs like $\langle h_t, s_0 \rangle$, where $s_0 \leq s_\alpha < s_\tau$, $s_\alpha \in S^{(\tau)}$, and $h_t \in H^{(\zeta)}$.

(3) Linguistic aspiration levels with interval form (denoted by $ll^{(IA)}$). This type also includes three specific forms, i.e., HFLTSs like "between s_α and s_β" (denoted as $[s_\alpha, s_\beta]$), and LTWHs like $\langle h_t, s_\gamma \rangle$, where $s_0 < s_\alpha \leq s_\beta < s_\tau$, $s_\alpha, s_\gamma, s_\beta \in S^{(\tau)}$, and $h_t \in H^{(\zeta)}$.

It can be observed that we extend the existing concept of fuzzy aspirations by two aspects. On the one hand, the experts can express their aspiration levels by means of either HFLTSs or LTWHs. On the other hand, the two types of CLEs can be used arbitrarily by the experts according to the linguistic expression in mind.

Probability distributions are vital for expected utility models and aspiration-based models. Whereas it is sometimes not so straightforward to specify the probability distribution for uncertain aspirations. Fuzzy sets present a mathematical counterpart of probability distributions, i.e., possibility distributions, by means of membership functions [67]. For a fuzzy set ll defined in the domain U, the possibility distribution of $x(\in U)$ is represented by the membership function $\mu_{ll}(x)$. Based on the possibility/probability consistency principle [68], the conversion between possibility and probability can be achieved as follows [69]: Given x in a continuous domain U associated with its possibility distribution $\mu_{ll}(x)$, the derived probability distribution is:

$$p(x) = \mu_{ll}(x) / \int_U \mu_{ll}(x)dx \tag{12}$$

To make sure the definitions coincides with our intuition, we specify the value functions according the form of linguistic aspiration levels. For simplification, we use the notations $x \geq ll^{(BA)}$, $x \leq ll^{(CA)}$, and $x \in ll^{(IA)}$ to represent that $x(\in U)$ achieves the benefit, cost, and interval forms of aspirations and $ll \geq ll^{(BA)}$, $ll \leq ll^{(CA)}$, and $ll \in ll^{(IA)}$ to represent that a CLE ll achieves the three forms of aspirations, respectively.

1) VALUE FUNCTIONS OF LINGUISTIC ASPIRATION LEVELS WITH BENEFIT FORMS

Assume that the linguistic aspiration levels $ll^{(BA)}$ take the form of "at least s_β". Given $x \in U = [L, R]$, the probability of x achieving the aspiration, denoted as $P(x \geq ll^{(BA)})$, is

$$P(x \geq ll^{(BA)}) = \frac{\int_L^x \mu_{ll^{(BA)}}(t)dt}{\int_L^R \mu_{ll^{(BA)}}(t)dt} \tag{13}$$

then the value function of a given CLE ll is

$$\begin{aligned} v(ll) &= P(ll \geq ll^{(BA)}) = \int_L^R P(x \geq ll^{(BA)})p(x)dx \\ &= \int_L^R \frac{\int_L^x \mu_{ll^{(BA)}}(t)dt}{\int_L^R \mu_{ll^{(BA)}}(t)dt} \cdot \frac{\mu_{ll}(x)}{\int_L^R \mu_{ll}(t)dt} dx \end{aligned} \tag{14}$$

If the linguistic aspiration levels take the form of "greater than $s_{\beta-1}$", then the value function of ll is the same as that in (14).

If the linguistic aspiration levels take the form of $\langle h_t, s_\tau \rangle$, then for $x \in U$, the probability of x achieving $\langle h_t, s_\tau \rangle$ is:

$$P(x \geq \langle h_t, s_\tau \rangle) = \frac{\int_L^x \mu_{\langle h_t, s_\tau \rangle}(t)dt}{\int_L^R \mu_{\langle h_t, s_\tau \rangle}(t)dt} \tag{15}$$

Given the CLE ll, its value function is:

$$v(ll) = \int_L^R \frac{\int_L^x \mu_{\langle h_t, s_\tau \rangle}(t)dt}{\int_L^R \mu_{\langle h_t, s_\tau \rangle}(t)dt} \cdot \frac{\mu_{ll}(x)}{\int_L^R \mu_{ll}(t)dt} dx \tag{16}$$

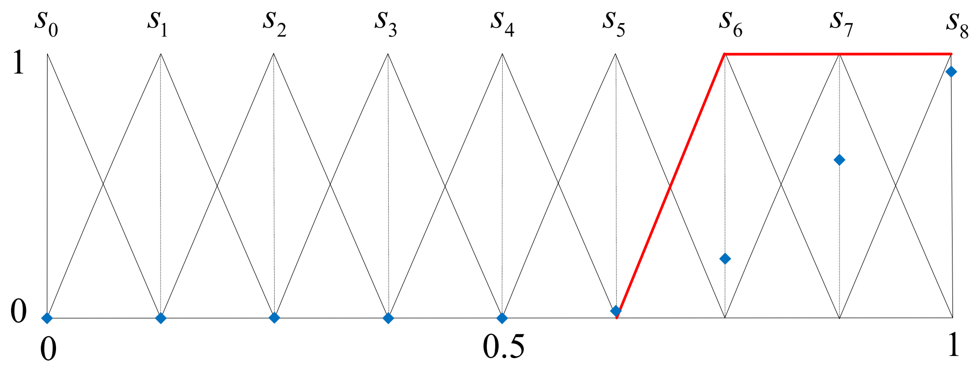

FIGURE 4. Values of $s_\alpha (\alpha = 0, 1, \ldots, 8)$ with respect to the linguistic aspiration level "at least s_6".

2) VALUE FUNCTIONS OF LINGUISTIC ASPIRATION LEVELS WITH COST FORMS

This case is similar to the case of benefit forms. Firstly, consider a linguistic aspiration level $ll^{(CA)}$ taking the form of "at most s_α", then for $x \in U$, the probability of x achieving $ll^{(CA)}$ is:

$$P(x \le ll^{(CA)}) = \frac{\int_x^R \mu_{ll^{(CA)}}(t)dt}{\int_L^R \mu_{ll^{(CA)}}(t)dt} \tag{17}$$

thus the value function of a given ll is:

$$\begin{aligned} v(ll) &= \int_L^R P(x \le ll^{(CA)})p(x)dx \\ &= \int_L^R \frac{\int_x^R \mu_{ll^{(CA)}}(t)dt}{\int_L^R \mu_{ll^{(CA)}}(t)dt} \cdot \frac{\mu_{ll}(x)}{\int_L^R \mu_{ll}(t)dt} dx \end{aligned} \tag{18}$$

If $ll^{(CA)}$ takes the form of "lower than $s_{\alpha+1}$", then the value function of ll is the same as that in (18).

Secondly, if $ll^{(CA)}$ takes the form of $\langle h_t, s_0 \rangle$, then the value function of ll is:

$$v(ll) = \int_L^R \frac{\int_x^U \mu_{\langle h_t, s_0 \rangle}(t)dt}{\int_L^R \mu_{\langle h_t, s_0 \rangle}(t)dt} \cdot \frac{\mu_{ll}(x)}{\int_L^R \mu_{ll}(t)dt} dx \tag{19}$$

3) VALUE FUNCTIONS OF LINGUISTIC ASPIRATION LEVELS WITH INTERVAL FORMS

Assume that the linguistic aspiration level $ll^{(IA)}$ takes the form of "between s_α and s_β". Given $x \in U$, according to (6), if $x \in [x_\alpha, x_\beta]$, then the probability of x achieving $ll^{(IA)}$ is 1. Generally, we have:

$$P(x \in [s_\alpha, s_\beta]) = \mu_{[s_\alpha, s_\beta]}(x) \tag{20}$$

For the CLE ll, its value function is:

$$v(ll) = \int_L^R \mu_{[s_\alpha, s_\beta]}(x) \cdot \frac{\mu_{ll}(x)}{\int_L^R \mu_{ll}(t)dt} dx \tag{21}$$

If $ll^{(IA)}$ takes the form of $\langle h_t, s_\gamma \rangle$, similarly, we obtain the value function of ll as follows:

$$v(ll) = \int_L^R \mu_{\langle h_t, s_\gamma \rangle}(x) \cdot \frac{\mu_{ll}(x)}{\int_L^R \mu_{ll}(t)dt} dx \tag{22}$$

Based on the semantic assumption specified in Section III-C, the computation of the value functions is very easy. We illustrate the procedures by the following example:

Example 3: Given the LTS $S^{(8)}$ in (1) with $\mu_{s_\alpha}(x) = (x_{\alpha-1}, x_\alpha, x_{\alpha+1})$, $\alpha = 0, 1, \ldots, 8$, and $\bar{H}^{(2)}$ in (5), let $ll^{(BA)}$ ="at least s_6" and ll ="more or less s_6". Their semantics are $(5/8, 6/8, 1, 1)$ and $(1/4, 1/2, 3/4)$, respectively. According to (13), we have

$$P(x \ge ll^{(BA)}) = \begin{cases} 0, & x < 5/8 \\ 64/5\,(x - 5/8)^2, & 5/8 \le x \le 6/8 \\ (16x - 11)/5, & x > 6/8 \end{cases}$$

Moreover, we have $\int_0^1 \mu_{ll}(t)dt = (1 - 0.5)/2 = 0.25$ which can be seen in Fig. 2. Then the value function of ll is $v(ll) = 4\int_0^1 P(x \ge ll^{(BA)})\mu_{ll}(x)dx$, where

$$\begin{aligned} &\int_0^1 P(x \ge ll^{(BA)})\mu_{ll}(x)dx \\ &\quad = \int_{5/8}^{6/8} \frac{64}{5}\left(x - \frac{5}{8}\right)^2 \cdot \frac{x - 1/2}{0.25} dx \\ &\quad\quad + \int_{6/8}^{1} \frac{16x - 11}{5} \cdot \frac{1 - x}{0.25} dx \\ &\quad = 0.0656 \end{aligned}$$

Therefore, $v(ll) = 0.2624$.

Furthermore, Fig. 4 and Fig. 5 show the values of the nine linguistic terms in $S^{(8)}$ with respect to linguistic aspiration levels "at least s_6" and "more or less s_4", respectively.

D. REACHING GROUPS' CONSENSUS

According to the linguistic aspiration levels provided by e_{mn} and the aspiration-based value defined in Section IV-C, the linguistic matrix $LM_{mn} = (ll_{ip}^{(m,n)})_{I \times \#C_m}$ can be transformed to the corresponding value matrix $VM_{mn} = (u_{ip}^{(m,n)})_{I \times \#C_m}$, where $v_{ip}^{(m,n)}$ is the value of a_i with respect to c_{mp} according to e_{mn}, $i = 1, 2, \ldots, I$, $p = 1, 2, \ldots, \#C_m$, $n = 1, 2, \ldots, \#G_m$, $m = 1, 2, \ldots, M$.

Given $m \in \{1, 2, \ldots, M\}$, we consider the consensus of G_m. Suppose that the weighting vector of the experts in

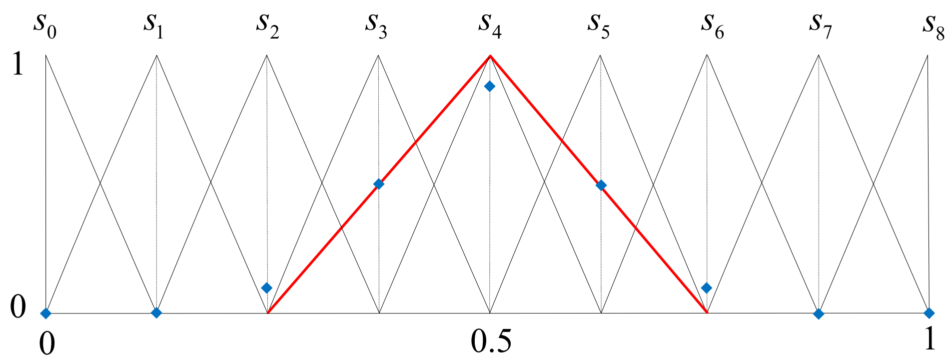

FIGURE 5. Values of s_α ($\alpha = 0, 1, \ldots, 8$) with respect to the linguistic aspiration level "more or less s_4".

G_m is $w = (w_1, w_2, \ldots, w_{\#G_m})^T$ such that $\sum_{n=1}^{\#G_m} w_n = 1$ and $w_n \in [0, 1](n = 1, 2, \ldots, \#G_m)$. Then the group's value matrix $VM_m = (v_{ip}^{(m)})_{I\times\#C_m}$ can be derived by synthesizing the $\#G_m$ value matrices of the group's members, where

$$v_{ip}^{(m)} = \sum_{n=1}^{\#G_m} w_n v_{ip}^{(m,n)}, \quad i = 1, 2, \ldots, I, \ p = 1, 2, \ldots, \#C_m. \tag{23}$$

Then the deviation $d(VM_{mn}, VM_m)$ between VM_{mn} and VM_m can be derived by:

$$d(VM_{mn}, VM_m) = \sum_{i=1}^{I}\sum_{p=1}^{\#C_m} \lambda_p (v_{ip}^{(m,n)} - v_{ip}^{(m)})^2 \tag{24}$$

where $n = 1, 2, \ldots, \#G_m$, λ_p is the weight of the criterion c_{mp} which can be computed by normalizing the weights of criteria in the set C_m as $\lambda_p = \omega_{mp} \Big/ \sum_{p=1}^{\#C_m} \omega_{mp}$. Consequently, the deviations among all the individual value matrices and the group's value matrix can be defined by:

$$g(w) = \sum_{n=1}^{\#G_m} d(VM_{mn}, VM_m) \tag{25}$$

Ideally, the deviation would approach 0 if the group's opinion is highly consensual. In our M^{3}QDM problem, the experts in each group are not assigned weights. To reduce the experts' work as much as possible, we intend to figure out whether there is at least one weighting vector, which results in an acceptable consensus degree of the group. Thus, the following model can be formed:

$$\begin{aligned} g(w) &= \min \sum_{n'=1}^{\#G_m}\sum_{i=1}^{I}\sum_{p=1}^{\#C_m} \lambda_p \Big(v_{ip}^{(m,n')} - \sum_{n=1}^{\#G_m} w_n v_{ip}^{(m,n)}\Big)^2 \\ s.t.\ &\begin{cases} \sum_{n=1}^{\#G_m} w_n = 1 \\ w_n \geq 0, n = 1, 2, \ldots, \#G_m \end{cases} \end{aligned} \tag{26}$$

By employing the technique of Lagrange multipliers, the solution to the problem reads as follows

$$w = G^{-1}r + \frac{1 - e^T G^{-1} r}{e^T G^{-1} e} G^{-1} e \tag{27}$$

where $e = (1, 1, \ldots, 1)^T$, the vector $r = (r_{n'})_{\#G_m \times 1}$ and the matrix $G = (G_{n,n'})_{\#G_m\times\#G_m}$ such that

$$r_{n'} = \sum_{i=1}^{I}\sum_{p=1}^{\#C_m}\sum_{n=1}^{\#G_m} \lambda_p v_{ip}^{(m,n)} v_{ip}^{(m,n')} \tag{28}$$

$$G_{n,n'} = \sum_{i=1}^{I}\sum_{p=1}^{\#C_m} \#G_m \lambda_p v_{ip}^{(m,n)} v_{ip}^{(m,n')} \tag{29}$$

The procedure of deriving the solution can be found in Appendix VII.

Based on (27), the optimal value of $g(w)$, denoted by $g^*(w)$, can be derived. The consensus index of the group G_m can be represented by:

$$CI_m = \frac{g^*(w)}{\#G_m I} \tag{30}$$

Then, a simple interactive algorithm can be employed to make CI_m reach its satisfactory level CI_m^*. Generally, the value can be assigned by the decision maker or the leader of the group. The smaller value implies the higher level of group consensus. As suggested by Xu [70], we let $CI_m^* = 0.5$ for any $m = 1, 2, \ldots, M$ in this paper. Given CI_m^* and the max number of interaction N_{max}, the interactive algorithm is depicted in Table 3. The algorithm is terminated once the group's consensus is acceptable or the number of interaction has reached N_{max}.

E. FUSION AND EXPLOITATION

The overall value matrix with respect to all the criteria can be formed after the consensus of all groups has been improved by the algorithm in Table 3.

For each $G_m(m = 1, 2, \ldots, M)$, $VM_m = (v_{ip}^{(m)})_{I\times\#C_m}$ can be calculated by (25). The value matrix VM_m is, actually, the mth block of the overall value matrix VM. Thus, VM is

TABLE 3. The interactive algorithm to improve the group's consensus.

Input:	Satisfactory consensus level CI_m^*, max number of interaction N_{max}, the value matrices $VM_{mn}(n = 1, 2, \ldots, \#G_m)$.
Output:	The improved value matrices $VM_{mn}(n = 1, 2, \ldots, \#G_m)$, the optimal weighting vector w.
Step 1:	Initiate $N = 1$, $VM_{mn}^{(N)} = UM_{mn}(n = 1, 2, \ldots, \#G_m)$.
Step 2:	Solve the model in (26), the optimal weight is denoted by $w^{(N)}$. Calculate $CI_m^{(N)}$ by (30).
Step 3:	If $CI_m^{(N)} > CI_m^*$ and $N < N_{max}$, then go to Step 4; else, go to Step 5.
Step 4:	Calculate VM_m by (25), return it to the experts in G_m. Let $N = N + 1$. Compute $VM_{mn}^{(N)}$ by using the linguistic information fed back by experts and the formula in Section IV-C, $n = 1, 2, \ldots, \#G_m$. Go to Step 2.
Step 5:	$VM_{mn} = VM_{mn}^{(N)}(n = 1, 2, \ldots, \#G_m)$, $w = w^{(N)}$.

obtained immediately as:

$$VM = (v_{ij})_{I\times J} = \begin{pmatrix} VM_1 & VM_2 & \cdots & VM_M \end{pmatrix} \quad (31)$$

The overall value of each alternative a_i, denoted by u_i, can be derived by (31) and the weighting vector ω is as follows:

$$v_i = \sum\nolimits_{j=1}^{J} \omega_j v_{ij} \quad (32)$$

where $i = 1, 2, \ldots, I$. Then the alternatives can be ranked by the overall values.

V. AN APPLICATION OF PROVIDER SELECTION

A case study regarding BDAP provider selection is presented to demonstrate the proposed M^{3}QDM approach. In order to focus on the illustration of the approach, we consider only a simple case of the selection problem. Also note that our major focus is the provider selection rather than the details of Big Data auditing.

A. DESCRIPTION OF THE PROVIDER SELECTION PROBLEM

Based on the drivers of the use of Big Data in the audit process [71]–[73], proper platforms and infrastructures should be implemented so that the Big Data techniques can be adopted. Especially, the new regulation released by Chinese government, mentioned in Section I, also delivers intensive requirement of designing BDAPs for certain industries. Selecting appropriate BDAPs would be significant for implementing the full audit coverage in China.

The determination of the criteria of BDAP provider selection is quite different from that of traditional decision support systems (DSSs). For instance, when developing an enterprise resource planning systems, most of the necessary techniques are common knowledge for all the potential providers. When facing Big Data, however, current techniques for almost all aspects of Big Data processing are scattered in different companies and institutions, and are far from meeting the ideal requirements [74]. This would result in the difficulty and uncertainty regarding assessing the quality of providers with respect to Big Data techniques-related criteria. As can be seen in Table 4, the selected criteria are classified into two classes. The first class focuses on the ability of processing big data and making informative decisions. Two subsets of criteria, namely Data curation (C_1) and Auditing decision support (C_2), are involved. Data curation refers to the ability of capturing, cleaning, aggregating, identifying, and protecting data. It prepares high-quality data for data analysis tools. Auditing decision support focuses on the techniques and technologies involved in the BDAP that are effective enough to support Big Data-based auditing decisions. The criteria in the second class are frequently considered in the provider selection of traditional information systems [75]–[77]. These criteria are classified into four subsets, namely, Service quality (C_3), Integration (C_4), Economics (C_5), and Professionalism (C_6). Consequently, the proposed hierarchical model contains 6 groups of 24 criteria.

TABLE 4. A summary on the hierarchy and criteria of the provider selection model.

Criterion in level 1	Criterion in level 2	Weight	Type
Data curation (C_1)	Data consistency	0.048	benefit
	Data Integrity	0.037	benefit
	Data identification	0.043	benefit
	Data aggregation	0.032	benefit
	Data confidentiality	0.053	benefit
Auditing decision support (C_2)	Various data analysis	0.058	benefit
	Real-time data analysis	0.052	benefit
	Admissible data analysis	0.058	benefit
	Data visualization	0.064	benefit
Service quality (C_3)	System update	0.036	benefit
	Maintain service	0.037	benefit
	Training	0.044	benefit
	System reliability	0.027	benefit
	Specialization	0.038	benefit
Integration (C_4)	Compatibility	0.025	benefit
	Links/Connection	0.023	benefit
	Flexible	0.021	benefit
	Customization	0.027	benefit
Economics (C_5)	Price	0.031	cost
	Setup cost	0.061	cost
	Maintain cost	0.042	cost
Professionalism (C_6)	Reputation	0.053	interval
	Audit-related experience	0.058	interval
	Big data-related experience	0.064	benefit

Note: The weights of 24 criteria are derived by an extended version of analytical hierarchical process where the entries of each judgement matrix take the form of LTWHs.

In our case study, three groups of experts are invited to evaluate three providers denoted by a_1, a_2 and a_3. The group G_1 is formed by Big Data experts and data scientists; G_2 includes the auditors and the experts who specialize in DSS; and the experts of G_3 come from the financial department. The criteria in C_1 and C_2 are evaluated by the experts in G_1; C_3, C_4 and C_6 are evaluated by G_2; finally is evaluated by G_3. Moreover, there are three experts in each group.

In the evaluation process, three context-free LTSs, denoted by $S^{(4)}$, $S^{(6)}$ and $S^{(8)}$, are available. The semantics is shown in Fig. 6. Associated with the set of linguistic hedges in (5),

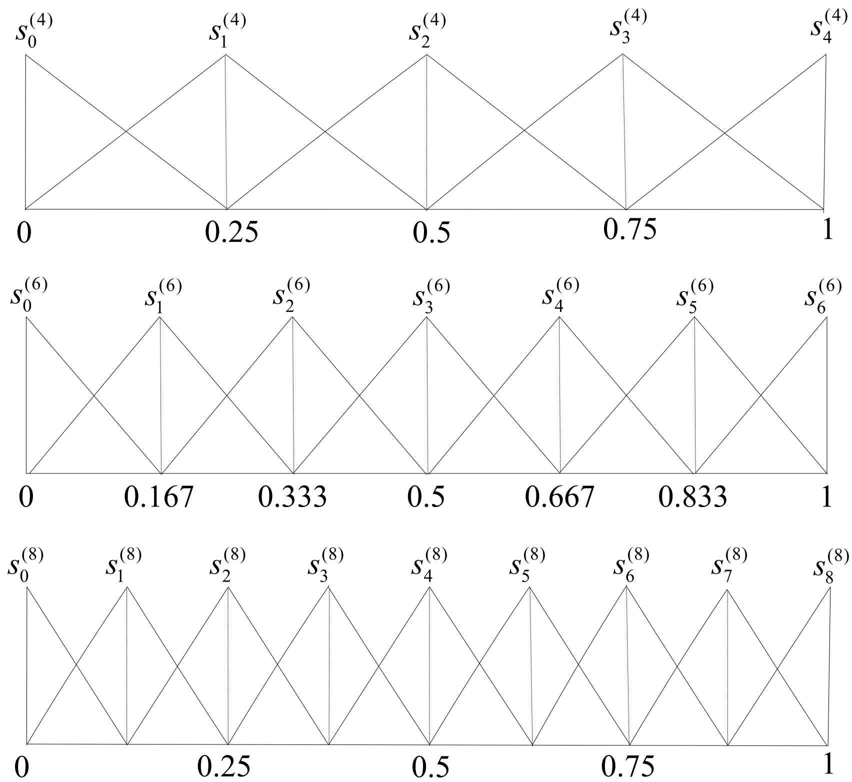

FIGURE 6. The sets of multi-granularity LTSs for provider selction.

TABLE 5. Evaluation information and linguistic aspiration levels of alternatives with respect to C_1.

Expert	Criterion	Aspiration	a_1	a_2	a_3
e_{11}	c_{11}	at least $s_3^{(4)}$	definitely $s_3^{(4)}$	between $s_2^{(4)}$ and $s_3^{(4)}$	more or less $s_3^{(4)}$
	c_{12}	at least $s_4^{(6)}$	between $s_4^{(6)}$ and $s_5^{(6)}$	definitely $s_4^{(6)}$	more or less $s_5^{(6)}$
	c_{13}	at least $s_3^{(6)}$	between $s_4^{(6)}$ and $s_5^{(6)}$	more or less $s_4^{(6)}$	roughly $s_5^{(6)}$
	c_{14}	at least $s_2^{(4)}$	definitely $s_3^{(4)}$	at least $s_3^{(4)}$	definitely $s_3^{(4)}$
	c_{15}	at least $s_7^{(8)}$	between $s_6^{(8)}$ and $s_7^{(8)}$	definitely $s_6^{(8)}$	more or less $s_6^{(8)}$
e_{12}	c_{11}	at least $s_4^{(6)}$	more or less $s_4^{(6)}$	definitely $s_4^{(6)}$	between $s_3^{(6)}$ and $s_4^{(6)}$
	c_{12}	at least $s_2^{(4)}$	between $s_2^{(4)}$ and $s_3^{(4)}$	more or less $s_3^{(4)}$	definitely $s_3^{(4)}$
	c_{13}	more or less $s_6^{(6)}$	definitely $s_4^{(6)}$	between $s_4^{(6)}$ and $s_5^{(6)}$	more or less $s_5^{(6)}$
	c_{14}	at least $s_4^{(6)}$	between $s_3^{(6)}$ and $s_4^{(6)}$	at least $s_5^{(6)}$	definitely $s_5^{(6)}$
	c_{15}	more or less $s_6^{(6)}$	more or less $s_4^{(4)}$	definitely $s_3^{(4)}$	between $s_2^{(4)}$ and $s_3^{(4)}$
e_{13}	c_{11}	at least $s_4^{(6)}$	between $s_3^{(6)}$ and $s_5^{(6)}$	between $s_4^{(6)}$ and $s_5^{(6)}$	between $s_3^{(6)}$ and $s_4^{(6)}$
	c_{12}	at least $s_3^{(6)}$	definitely $s_3^{(6)}$	more or less $s_4^{(6)}$	between $s_3^{(6)}$ and $s_5^{(6)}$
	c_{13}	at least $s_3^{(4)}$	between $s_2^{(4)}$ and $s_3^{(4)}$	definitely $s_3^{(4)}$	more or less $s_2^{(4)}$
	c_{14}	at least $s_2^{(4)}$	definitely $s_2^{(4)}$	more or less $s_2^{(4)}$	between $s_1^{(4)}$ and $s_3^{(4)}$
	c_{15}	more or less $s_8^{(8)}$	more or less $s_5^{(8)}$	between $s_5^{(8)}$ and $s_6^{(8)}$	between $s_6^{(8)}$ and $s_7^{(8)}$

the experts are allowed to express their aspiration levels and evaluation values by means of ULTs, HFLTS, or LTWHs.

B. SOLVING THE PROBLEM BY THE PROPOSED APPROACH

It is enough to illustrate the process in Section IV-B by the provided information with respect to one subset of criteria. The collected linguistic information with respect to C_1 is listed in Table 5.

Step 1: Three value matrices, as shown in Table 6, can be derived by using the value functions defined in Section IV-C.

Step 2: To address the group consensus, we obtain the optimal weights of the three experts $w = (0.33, 0.33, 0.33)^T$ according to (27). Therefore, we have $CI_1 = 0.047$ when using (30). Thus, the group consensus is acceptable.

Step 3: Associated with the optimal weights, the three value matrices can be fused to a group value matrix

TABLE 6. Three value matrices of experts in G_1 with respect to C_1.

Expert	Alter-native	c_{11}	c_{12}	c_{13}	c_{14}	c_{15}
e_{11}	a_1	0.3396	0.4084	0.5715	0.5513	0.1945
	a_2	0.1945	0.2452	0.4295	0.6890	0.0822
	a_3	0.3396	0.5438	0.6722	0.5513	0.0822
e_{12}	a_1	0.2452	0.4084	0.0579	0.1167	0.3705
	a_2	0.2452	0.5513	0.1563	0.6890	0.1727
	a_3	0.1167	0.5513	0.2811	0.5438	0.0738
e_{13}	a_1	0.2778	0.1694	0.1945	0.2371	0.0012
	a_2	0.4084	0.4295	0.3396	0.2371	0.0104
	a_3	0.1167	0.4326	0.0618	0.2778	0.1563

as follows:

$$\begin{pmatrix} 0.2875 & 0.3287 & 0.2746 & 0.3017 & 0.1887 \\ 0.2827 & 0.4087 & 0.3085 & 0.5384 & 0.0884 \\ 0.1910 & 0.5092 & 0.3384 & 0.4576 & 0.1041 \end{pmatrix}$$

This matrix serves as the first five columns of the overall value matrix with respect to all criteria. Repeating the above process for all six subsets of criteria, the overall value matrix can be derived.

Step 4: The weighted averaging values of three alternatives are $v_1 = 0.2863$, $v_2 = 0.2837$, and $v_3 = 0.3158$. Thereafter, a_3 is the best alternative.

For the purpose of comparison in the coming section, if only the criteria in C_1 are considered, then $v_1 = 0.2696$, $v_2 = 0.2999$, and $v_3 = 0.2945$. Accordingly, a_2 is the best alternative.

VI. COMPARISONS AND FURTHER DISCUSSIONS

We will analyze the proposed M^{3}QDM approach by comparing it with some similar techniques. Without loss of generality, we will conduct the comparisons by using the linguistic information with respect to the criteria in C_1, i.e., the data in Table 5.

A. COMPARATIVE ANALYSIS

We begin with comparing it with two multi-granular QDM approaches proposed by Herrera *et al.* [8] and Fan and Liu [78]. They are chosen for comparison because their essential procedures are based on the semantics of linguistic terms. However, they cannot be compared straightforward since they cannot handle multi-types of CLEs or multi-groups. Therefore, we only use the information collected by the first group, i.e., the information listed in Table 5, to avoid multi-groups, and then transform the CLEs into their semantics, i.e., TraFNs. In this manner, the two approaches are comparable.

In [8], a basic LTS whose granularity is fine enough is employed. A linguistic expression is then transformed into a fuzzy set on the basic LTS, according to its semantics. Here, $S^{(8)}$ plays the role of basic LTS. For instance, the CLE $\langle h_1, s_6^{(8)}\rangle$, as shown in Fig. 2, can be represented as $\{(s_4^{(8)}, 0.33), (s_5^{(8)}, 0.66), (s_6^{(8)}, 1), (s_7^{(8)}, 0.66), (s_8^{(8)}, 0.33)\}$, where the number in each 2-tuple represents the membership degree. To obtain the collective performance of each alternative, an aggregating operator should be relied on. To make the approach comparable, we extend it by aggregating the group's opinion so that it is suitable for GDM. If the weighted averaging operator is considered, then according to their proposed ranking exploitation method, we get $a_1 < a_3 < a_2$.

The approach in [78] handles both simple linguistic terms and ULTs by means of TraFNs. In order to figure out the collective performance matrix, the traditional trapezoidal fuzzy weighted averaging operator is utilized. Associated with the weighting information, the derived matrix is:

$$\begin{pmatrix} (0.39, 0.64, 0.75, 1.0) & (0.36, 0.56, 0.69, 0.89) & (0.42, 0.61, 0.75, 0.94) & (0.33, 0.58, 0.64, 0.86) & (0.50, 0.79, 0.83, 1.0) \\ (0.42, 0.61, 0.75, 0.94) & (0.36, 0.69, 0.69, 1.0) & (0.44, 0.69, 0.75, 1.0) & (0.39, 0.76, 0.83, 1.0) & (0.54, 0.71, 0.75, 0.92) \\ (0.31, 0.58, 0.69, 0.97) & (0.44, 0.69, 0.81, 1.0) & (0.28, 0.72, 0.72, 1.0) & (0.22, 0.69, 0.86, 1.0) & (0.46, 0.67, 0.79, 1.0) \end{pmatrix}$$

Then the classical TOPSIS is considered, where the positive and negative ideal TraFNs are (1, 1, 1, 1) and (0, 0, 0, 0), respectively, and the Minkowski distance measure between two TraFNs is used and its parameter is fixed by 2. Finally, the ranking derived by closeness coefficients is $a_3 < a_1 < a_2$.

It can be seen that the rankings of alternatives with respect to the criteria in C_1 are different. This can be concluded that the consideration of aspiration levels in linguistic setting would greatly influence the final decision.

Thanks to their semantics-based computational strategy, these two approaches can solve multi-granular linguistic GDM problems. But the advantages of the proposed approach is prominent: Firstly, we take the linguistic aspiration levels of experts into account. Secondly, the uncertain decision information, including the performances and aspirations, is elicited by natural languages taking the form of several types of CLEs.

As has been reviewed in Section II, only the aspiration-based approach proposed in [33] could deal with a type of CLEs, specifically, ULTs. But it is not semantics-based approach. Thus it cannot handle multi types of CLEs, nor multi-granular linguistic information. Accordingly, it is hard to compare the existing aspiration-based approaches with the proposed approach.

B. FURTHER DISCUSSIONS

Although it is hard to compare with other techniques through a direct way, we can analyze their characteristics to illustrate the strengths and weaknesses of the proposed approach. Table 7 lists some features of some similar techniques of QDM. We discuss the techniques from the following aspects.

TABLE 7. Features of the comparative QDM approaches.

Reference	Basic CWW model	Type of CLEs	Multi-granular LTSs	Aspiration
Xu [13]	Virtual term model	ULT	N/A	N/A
Rodríguez et al. [29]	linguistic 2-tuple model	HFLTS	N/A	N/A
Wang et al. [16]	Semantics-based model	LTWH	Quasi linguistic hierarchy	N/A
Feng and Lai [33]	Ordered structure model	Single term & ULT	N/A	Single term & ULT
Yan et al. [57]	Semantics-based model	Single term	N/A	Single term
Chanyachatchawan et al. [79]	Ordered structure model	Single term	N/A	Single term
Yan et al. [60]	Ordered structure model	Single term	N/A	Single term
Herrera et al. [8]	Ordered structure model	N/A	Any	N/A
Herrera and Martínez [80]	linguistic 2-tuple model	N/A	Linguistic hierarchy	N/A
Espinilla et al. [61]	linguistic 2-tuple model	N/A	Extended linguistic hierarchy	N/A
Meng and Chen [20]	Ordered structure model	N/A	Any	N/A
The proposed approach	Semantics-based model	ULT, HFLTS, LTWH	Any	ULT, HFLTS, LTWH

1) REGARDING THE INVOLVED TYPES OF CLES

The consideration of CLEs in QDM is being a hot topic in recent a few years [12]. Most of the existing contributions assume that the linguistic information takes the form of a certain type of CLEs. For instance, the aggregation-based QDM approach in [13] considers only ULTs. HFLTSs have been focused in many studies. The approach in [16] assumes that the information is represented by LTWH. In these cases, the experts have to express their opinions by means of a special type of linguistic expressions, no matter the type coincides with their linguistic conventions or not. Feng and Lai [33] improved the cases by considering both single terms and ULTs. The proposed approach could handle all CLEs defined by Definition 1.

2) REGARDING ASPIRATION-BASED APPROACHES

Some of the contributions listed in Table 7 considered the uncertain aspirations as a criterion of decision-making. In [33], aspirations of qualitative criteria could be represented by single terms and ULTs. Similar to this paper, three types of linguistic aspiration levels were considered in [57] and represented by single terms. Subintervals of the domain, derived from the α-cut of semantics of terms, are utilized to compute the value functions. The linguistic aspiration levels in [60] and [79] are also single terms. Others in Table 7 did not consider the aspirations of experts. The proposed approach not only considers three types of uncertain aspirations but also enable them to be represented by CLEs. Therefore, in qualitative setting, the manner of expressing uncertainties is more flexible.

3) REGARDING MULTI-GRANULAR QDM APPROACHES

The approach in [8] was based on a so-called basic LTS to unify multi-granular linguistic information. The appraoches in [61] and [80] are based on a linguistic hierarchy (where $\tau_{q+1} = 2\tau_q$) and an extended hierarchy (where $\tau_{q+1} > \tau_q$), respectively. In a multi-granular QDM approach based on HFLTSs [20], the distance between a performance value and its ideal value with respect to a criterion is used and thus the unification phase is not necessary. Compared with these studies, the proposed approach generalizes the linguistic information by multi-types of CLEs. Instead of multi-granular unification, we compute the value functions of the performance values.

The strengths are summarized as the following points:

(1) The range of linguistic expressions is extended. According to Definition 1, more CLEs are involved. Based on the proposed semantics-based approach, we can deal with these CLEs by the same framework. The focused CLEs include most of the natural way to express uncertainties in linguistic setting. The experts are permitted to use either HFLTSs or LTWHs to express both linguistic aspiration levels or performance values. Hence, the experts can concentrate on the evaluation rather than stating their opinion by a fixed simple grammar.

(2) The processing of multi-granular linguistic information is very easy. For the convenience of evaluation, a set of LTSs are defined on the same domain. During the assessment, the experts can select the LTSs according to their preference and/or the acquisitus knowledge. When handling this multi-granular linguistic information, the proposed approach defines value functions by means of the probability of a performance value achieving its aspiration. This makes the linguistic information be operated as easy as usual.

(3) The manner of handling information provided by several groups of experts is straightforward and easy to carry out. In the focused problem, the whole evaluation work is divided into several parts and each part is undertaken by a group. The intersection of evaluation work of any two groups is empty. This manner can be regarded as a special case of the multi-groups decision-making framework defined in [81]. Roughly speaking, this manner decomposes a complex problem into several exclusive pieces and each of which could be solved by a group of experts.

In sum, the most prominent feature of the M^{3}QDM approach is that it considers multi-criteria, multi-groups of experts, and multi-granular linguistic aspiration levels and evaluation values simultaneously.

The M^{3}QDM approach suffers some weaknesses which could be improved in the future. Firstly, the group consensus reaching algorithm relies on the interactions with the experts. This might decrease the efficiency of the decision-making process. Secondly, the set of multi-granular LTSs are assumed to be defined in the same domain. This might trigger off obstacles for application when uncertainties come from different sources, such as the case in [20].

VII. CONCLUSION

This paper has been motivated by the problem of BDAP provider selection. The M^3QDM approach is necessary because it is quite natural that multi-criteria and multi-groups of experts are involved and multi-granular linguistic information, taking the form of CLEs, is inevitable in the focused problem. Moreover, linguistic aspiration levels have also been considered in the approach. The semantics of CLEs is utilized to aggregate the linguistic information with distinct granularities and obtain the value functions with respect to linguistic aspiration levels. The approach has been identified by a case study. Based on the completed study, we can draw the following conclusions:

(1) The proposed model enlarges the range of values that can be assigned to a linguistic variable. Linguistic expressions, taking the form of HFLTSs and LTWHs, are available to represent opinions under uncertainties. The use of CLEs increases the flexibility of modeling uncertainties.

(2) The consideration of aspiration levels in linguistic setting could greatly influence the final decisions. In real world problems, therefore, it is worthwhile to mine the aspiration levels of the experts.

The current study suffers some limitations as well. These result in the following future work:

(1) The proposed group consensus reaching algorithm is not efficient enough. As can be seen in the case study, the size of the entire group is large, thus the interactions with the experts could be time-consuming. It would be interesting to develop a more efficient algorithm. For instance, an automatic approach, which revises the most inconsistent opinions based on the collected information, could be developed to enhance the group consensus.

(2) The proposed solution heavily depends on the semantics of CLEs. It would be also interesting if other approaches can be developed based on the ordered structure of the LTSs. In this case, some other theoretical issues, such as the order relations of the set of CLEs, should be addressed at first.

APPENDIX
SOLUTION OF THE PROBLEM IN EQ. (26)

The Lagrange function is constructed as:

$$L(w,\xi)=\sum_{n'=1}^{\#G_m}\sum_{i=1}^{I}\sum_{p=1}^{\#C_m}\lambda_p\cdot(u_{ip}^{(m,n')}-\sum_{n=1}^{\#G_m}w_n\cdot u_{ip}^{(m,n)})^2 -2\xi\sum_{n=1}^{\#G_m}w_n-1 \tag{33}$$

Differentiating (33) with respect to w_n and letting these partial derivatives be equal to 0, we have

$$\frac{\partial L}{\partial w_n}=\sum_{i=1}^{I}\sum_{p=1}^{\#C_m}\sum_{n'=1}^{\#G_m}\lambda_p(\#G_m w_{n'}-1)u_{ip}^{(m,n)}u_{ip}^{(m,n')}-\xi=0 \tag{34}$$

where $n = 1, 2, \ldots, \#G_m$. Using the denotation defined in (28) and (29), the following tight form can be obtained:

$$Gw-r-\xi e=0 \tag{35}$$

It is obvious that G is positive definite and is non-negative. Associated with $e^T w = 1$, the parameter ξ can be solved at first:

$$\xi=\frac{1-e^TG^{-1}r}{e^TG^{-1}r} \tag{36}$$

Then the weighting vector w can be derived by combining (35) and (36), as shown in (27).

REFERENCES

[1] O. Despic and S. P. Simonovic, "Aggregation operators for soft decision making in water resources," *Fuzzy Sets Syst.*, vol. 115, no. 1, pp. 11–33, 2000.
[2] Z. Gong, H. Zhang, J. Forrest, L. Li, and X. Xu, "Two consensus models based on the minimum cost and maximum return regarding either all individuals or one individual," *Eur. J. Oper. Res.*, vol. 240, no. 1, pp. 183–192, 2015.
[3] L. A. Zadeh, "The concept of a linguistic variable and its application to approximate reasoning—I," *Inf. Sci.*, vol. 8, no. 3, pp. 199–249, 1975.
[4] F. Herrera and E. Herrera-Viedma, "Linguistic decision analysis: Steps for solving decision problems under linguistic information," *Fuzzy Sets Syst.*, vol. 115, no. 1, pp. 67–82, 2000.
[5] F. Herrera and L. Martínez, "A 2-tuple fuzzy linguistic representation model for computing with words," *IEEE Trans. Fuzzy Syst.*, vol. 8, no. 6, pp. 746–752, Dec. 2000.
[6] Z. Xu, "A method based on linguistic aggregation operators for group decision making with linguistic preference relations," *Inf. Sci.*, vol. 166, nos. 1–4, pp. 19–30, 2004.
[7] Z. Xu and H. Wang, "On the syntax and semantics of virtual linguistic terms for information fusion in decision making," *Inf. Fusion*, vol. 34, pp. 43–48, Mar. 2017.
[8] F. Herrera, E. Herrera-Viedma, and L. Martínez, "A fusion approach for managing multi-granularity linguistic term sets in decision making," *Fuzzy Sets Syst.*, vol. 114, no. 1, pp. 43–58, 2000.
[9] Z. Chen and D. Ben-Arieh, "On the fusion of multi-granularity linguistic label sets in group decision making," *Comput. Ind. Eng.*, vol. 51, no. 3, pp. 526–541, 2006.
[10] C.-C. Li, Y. Dong, F. Herrera, E. Herrera-Viedma, and L. Martínez, "Personalized individual semantics in computing with words for supporting linguistic group decision making. An application on consensus reaching," *Inf. Fusion*, vol. 33, pp. 29–40, Jan. 2017.
[11] C.-C. Li, R. M. Rodríguez, L. Martínez, Y. C. Dong, and F. Herrera, "Personalized individual semantics based on consistency in hesitant linguistic group decision making with comparative linguistic expressions," *Knowl.-Based Syst.*, vol. 145, pp. 156–165, Apr. 2018.
[12] H. Wang, Z. Xu, and X.-J. Zeng, "Modeling complex linguistic expressions in qualitative decision making: An overview," *Knowl.-Based Syst.*, vol. 144, pp. 174–187, Mar. 2018.
[13] Z. Xu, "Uncertain linguistic aggregation operators based approach to multiple attribute group decision making under uncertain linguistic environment," *Inf. Sci.*, vol. 168, nos. 1–4, pp. 171–184, Dec. 2004.
[14] R. M. Rodríguez, L. Martínez, and F. Herrera, "Hesitant fuzzy linguistic term sets for decision making," *IEEE Trans. Fuzzy Syst.*, vol. 20, no. 1, pp. 109–119, Feb. 2012.
[15] H. Wang, Z. Xu, and X.-J. Zeng, "Hesitant fuzzy linguistic term sets for linguistic decision making: Current developments, issues and challenges," *Inf. Fusion*, vol. 43, pp. 1–12, Sep. 2018.
[16] H. Wang, Z. Xu, and X.-J. Zeng, "Linguistic terms with weakened hedges: A model for qualitative decision making under uncertainty," *Inf. Sci.*, vols. 433–434, pp. 37–54, Apr. 2018.
[17] G. Zhang, Y. Dong, and Y. Xu, "Consistency and consensus measures for linguistic preference relations based on distribution assessments," *Inf. Fusion*, vol. 17, pp. 46–55, May 2014.
[18] Q. Pang, H. Wang, and Z. Xu, "Probabilistic linguistic term sets in multi-attribute group decision making," *Inf. Sci.*, vol. 369, pp. 128–143, Nov. 2016.

[19] H. Wei, Y. Sun, X. Hui, and Z. Che, "Hesitant fuzzy linguistic group DEMATEL method with multi-granular evaluation scales," *Int. J. Fuzzy Syst.*, vol. 20, no. 7, pp. 2187–2201, 2017.

[20] F. Y. Meng and X. Chen, "A hesitant fuzzy linguistic multi-granularity decision making model based on distance measures," *J. Intell. Fuzzy Syst.*, vol. 28, no. 4, pp. 1519–1531, 2015.

[21] Z. Zhang and C. Guo, "A method for multi-granularity uncertain linguistic group decision making with incomplete weight information," *Knowl.-Based Syst.*, vol. 26, pp. 111–119, Feb. 2012.

[22] V. Torra, "Aggregation of linguistic labels when semantics is based on antonyms," *Int. J. Intell. Syst.*, vol. 16, no. 4, pp. 513–524, 2001.

[23] Z. Xu, "An approach based on the uncertain LOWG and induced uncertain LOWG operators to group decision making with uncertain multiplicative linguistic preference relations," *Decis. Support Syst.*, vol. 41, no. 2, pp. 488–499, 2006.

[24] Z. Xu, "Induced uncertain linguistic OWA operators applied to group decision making," *Inf. Fusion*, vol. 7, no. 2, pp. 231–238, 2006.

[25] Z. Wu and Y. Chen, "The maximizing deviation method for group multiple attribute decision making under linguistic environment," *Fuzzy Sets Syst.*, vol. 158, no. 14, pp. 1608–1617, 2007.

[26] H. Wang and Z. S. Xu, "Total orders of extended hesitant fuzzy linguistic term sets: Definitions, generations and applications," *Knowl.-Based Syst.*, vol. 107, pp. 142–154, Sep. 2016.

[27] C. P. Wei, N. Zhao, and X. J. Tang, "Operators and comparisons of hesitant fuzzy linguistic term sets," *IEEE Trans. Fuzzy Syst.*, vol. 22, no. 3, pp. 575–585, Jun. 2014.

[28] H. B. Liu and R. M. Rodríguez, "A fuzzy envelope for hesitant fuzzy linguistic term set and its application to multicriteria decision making," *Inf. Sci.*, vol. 258, pp. 220–238, Feb. 2014.

[29] R. M. Rodríguez, L. Martínez, and F. Herrera, "A group decision making model dealing with comparative linguistic expressions based on hesitant fuzzy linguistic term sets," *Inf. Sci.*, vol. 241, pp. 28–42, Aug. 2013.

[30] Z. Wu and J. Xu, "Possibility distribution-based approach for MAGDM with hesitant fuzzy linguistic information," *IEEE Trans. Cybern.*, vol. 46, no. 3, pp. 694–705, Mar. 2016.

[31] H. A. Simon, "Rational choice and the structure of the environment," *Psychol. Rev.*, vol. 63, no. 2, p. 129, 1956.

[32] D. B. Brown, E. Di Giorgi, and M. Sim, "Aspirational preferences and their representation by risk measures," *Manage. Sci.*, vol. 58, no. 11, pp. 2095–2113, 2012.

[33] B. Feng and F. Lai, "Multi-attribute group decision making with aspirations: A case study," *Omega*, vol. 44, pp. 136–147, Apr. 2014.

[34] K. Yoon, L. Hoogduin, and L. Zhang, "Big data as complementary audit evidence," *Accounting Horizons*, vol. 29, no. 2, pp. 431–438, 2015.

[35] J. Zhang, X. Yang, and D. Appelbaum, "Toward effective big data analysis in continuous auditing," *Accounting Horizons*, vol. 29, no. 2, pp. 469–476, 2015.

[36] M. Cao, R. Chychyla, and T. Stewart, "Big data analytics in financial statement audits," *Accounting Horizons*, vol. 29, no. 2, pp. 423–429, 2015.

[37] R. F. Bordley and C. W. Kirkwood, "Multiattribute preference analysis with performance targets," *Oper. Res.*, vol. 52, no. 6, pp. 823–835, 2004.

[38] J. W. Payne, "It is whether you win or lose: The importance of the overall probabilities of winning or losing in risky choice," *J. Risk Uncertainty*, vol. 30, no. 1, pp. 5–19, 2005.

[39] M. Köksalan, J. Wallenius, and S. Zionts, "An early history of multiple criteria decision making," *J. Multi-Criteria Decis. Anal.*, vol. 20, nos. 1–2, pp. 87–94, 2013.

[40] E. Diecidue and J. van de Ven, "Aspiration level, probability of success and failure, and expected utility," *Int. Econ. Rev.*, vol. 49, no. 2, pp. 683–700, 2008.

[41] E. Brandstätter, G. Gigerenzer, and R. Hertwig, "The priority heuristic: Making choices without trade-offs," *Psychol. Rev.*, vol. 113, no. 2, pp. 409–432, 2006.

[42] V. Lotfi, T. J. Stewart, and S. Zionts, "An aspiration-level interactive model for multiple criteria decision making," *Comput. Oper. Res.*, vol. 19, no. 7, pp. 671–681, 1992.

[43] M. Nowak, "Interactive approach in multicriteria analysis based on stochastic dominance," *Control Cybern.*, vol. 33, no. 3, pp. 463–476, 2004.

[44] M. Nowak, "INSDECM—An interactive procedure for stochastic multicriteria decision problems," *Eur. J. Oper. Res.*, vol. 175, no. 3, pp. 1413–1430, 2006.

[45] M. Nowak, "Aspiration level approach in stochastic MCDM problems," *Eur. J. Oper. Res.*, vol. 177, no. 3, pp. 1626–1640, 2007.

[46] M. Sun and R. E. Steuer, "InterQuad: An interactive quad tree based procedure for solving the discrete alternative multiple criteria problem," *Eur. J. Oper. Res.*, vol. 89, no. 3, pp. 462–472, 1996.

[47] J. Wang and S. Zionts, "The aspiration level interactive method (AIM) reconsidered: Robustness of solutions," *Eur. J. Oper. Res.*, vol. 175, no. 2, pp. 948–958, 2006.

[48] I. Tsetlin and R. L. Winkler, "Decision making with multiattribute performance targets: The impact of changes in performance and target distributions," *Oper. Res.*, vol. 55, no. 2, pp. 226–233, 2007.

[49] F. Fantozzi and F. Spizzichino, "Multi-attribute target-based utilities and extensions of fuzzy measures," *Fuzzy Sets Syst.*, vol. 259, pp. 29–43, Jan. 2015.

[50] Y. B. Yun, H. Nakayama, and M. Arakawa, "Multiple criteria decision making with generalized DEA and an aspiration level method," *Eur. J. Oper. Res.*, vol. 158, no. 3, pp. 697–706, 2004.

[51] Z.-P. Fan, X. Zhang, F.-D. Chen, and Y. Liu, "Multiple attribute decision making considering aspiration-levels: A method based on prospect theory," *Comput. Ind. Eng.*, vol. 65, no. 2, pp. 341–350, 2013.

[52] Z.-P. Fan, X. Zhang, Y.-R. Zhao, and F.-D. Chen, "Multiple attribute decision making with multiple formats of attribute aspirations: A method based on prospect theory," *Int. J. Inf. Technol. Decis. Making*, vol. 12, no. 4, pp. 711–727, 2013.

[53] C. Tan, W. H. Ip, and X. Chen, "Stochastic multiple criteria decision making with aspiration level based on prospect stochastic dominance," *Knowl.-Based Syst.*, vol. 70, pp. 231–241, Nov. 2014.

[54] V.-N. Huynh, Y. Nakamori, M. Ryoke, and T.-B. Ho, "Decision making under uncertainty with fuzzy targets," *Fuzzy Optim. Decis. Making*, vol. 6, no. 3, pp. 255–278, 2007.

[55] V.-N. Huynh, Y. Nakamori, and J. Lawry, "A probability-based approach to comparison of fuzzy numbers and applications to target-oriented decision making," *IEEE Trans. Fuzzy Syst.*, vol. 16, no. 2, pp. 371–387, Apr. 2008.

[56] H.-B. Yan, V.-N. Huynh, T. Ma, and Y. Nakamori, "Non-additive multi-attribute fuzzy target-oriented decision analysis," *Inf. Sci.*, vol. 240, pp. 21–44, Aug. 2013.

[57] H.-B. Yan, V.-N. Huynh, T. Murai, and Y. Nakamori, "Kansei evaluation based on prioritized multi-attribute fuzzy target-oriented decision analysis," *Inf. Sci.*, vol. 178, no. 21, pp. 4080–4093, 2008.

[58] H.-B. Yan, V.-N. Huynh, and Y. Nakamori, "A group nonadditive multiattribute consumer-oriented Kansei evaluation model with an application to traditional crafts," *Ann. Oper. Res.*, vol. 195, no. 1, pp. 325–354, 2012.

[59] V.-N. Huynh, H. Yan, and Y. Nakamori, "A target-based decision-making approach to consumer-oriented evaluation model for japanese traditional crafts," *IEEE Trans. Eng. Manage.*, vol. 57, no. 4, pp. 575–588, Nov. 2010.

[60] H.-B. Yan, T. Ma, S. Sriboonchitta, and V.-N. Huynh, "A stochastic dominance based approach to consumer-oriented Kansei evaluation with multiple priorities," *Ann. Oper. Res.*, vol. 256, no. 2, pp. 329–357, 2017.

[61] M. Espinilla, J. Liu, and L. Martínez, "An extended hierarchical linguistic model for decision-making problems," *Comput. Intell.*, vol. 27, no. 3, pp. 489–512, 2011.

[62] E. E. Kerre and M. de Cock, "Linguistic modifiers: An overview," in *Fuzzy Logic and Soft Computing*, vol. 9. 1999, pp. 69–85.

[63] E. Falcó, J. L. García-Lapresta, and L. Roselló, "Allowing agents to be imprecise: A proposal using multiple linguistic terms," *Inf. Sci.*, vol. 258, no. 3, pp. 249–265, 2014.

[64] J. Montserrat-Adell, N. Agell, M. Sánchez, F. Prats, and F. J. Ruiz, "Modeling group assessments by means of hesitant fuzzy linguistic term sets," *J. Appl. Logic*, vol. 23, pp. 40–50, Sep. 2017.

[65] R. C. Green and S. Srivastava, "Expected utility maximization and demand behavior," *J. Econ. Theory*, vol. 38, no. 2, pp. 313–323, 1986.

[66] R. Bordley and M. Licalzi, "Decision analysis using targets instead of utility functions," *Decis. Econ. Finance*, vol. 23, no. 1, pp. 53–74, 2000.

[67] L. A. Zadeh, "Fuzzy sets as a basis for a theory of possibility," *Fuzzy Sets Syst.*, vol. 1, no. 1, pp. 3–28, 1978.

[68] L. A. Zadeh, "Fuzzy sets," *Inf. Control*, vol. 8, no. 3, pp. 338–353, Jun. 1965.

[69] R. R. Yager, "OWA aggregation over a continuous interval argument with applications to decision making," *IEEE Trans. Syst., Man, Cybern. B, Cybern.*, vol. 34, no. 5, pp. 1952–1963, Oct. 2004.

[70] Z. Xu, "An automatic approach to reaching consensus in multiple attribute group decision making," *Comput. Ind. Eng.*, vol. 56, no. 4, pp. 1369–1374, 2009.

[71] M. G. Alles, "Drivers of the use and facilitators and obstacles of the evolution of big data by the audit profession," *Accounting Horizons*, vol. 29, no. 2, pp. 439–449, 2015.

[72] J. D. Warren, Jr., K. C. Moffitt, and P. Byrnes, ''How big data will change accounting,'' *Accounting Horizons*, vol. 29, no. 2, pp. 397–407, 2015.

[73] M. A. Vasarhelyi, A. Kogan, and B. M. Tuttle, ''Big data in accounting: An overview,'' *Accounting Horizons*, vol. 29, no. 2, pp. 381–396, 2015.

[74] H. Wang, Z. Xu, H. Fujita, and S. Liu, ''Towards felicitous decision making: An overview on challenges and trends of big data,'' *Inf. Sci.*, vol. 367, pp. 747–765, Nov. 2016.

[75] C. Low and Y. H. Chen, ''Criteria for the evaluation of a cloud-based hospital information system outsourcing provider,'' *J. Med. Syst.*, vol. 36, no. 6, pp. 3543–3553, 2012.

[76] G. Yucel, S. Cebi, B. Hoege, and A. F. Ozok, ''A fuzzy risk assessment model for hospital information system implementation,'' *Expert Syst. Appl.*, vol. 39, no. 1, pp. 1211–1218, 2012.

[77] A. Goscinski and M. Brock, ''Toward dynamic and attribute based publication, discovery and selection for cloud computing,'' *Future Gener. Comput. Syst.*, vol. 26, no. 7, pp. 947–970, 2010.

[78] Z.-P. Fan and Y. Liu, ''A method for group decision-making based on multi-granularity uncertain linguistic information,'' *Expert Syst. Appl.*, vol. 37, no. 5, pp. 4000–4008, 2010.

[79] S. Chanyachatchawan, H.-B. Yan, S. Sriboonchitta, and V.-N. Huynh, ''A linguistic representation based approach to modelling Kansei data and its application to consumer-oriented evaluation of traditional products,'' *Knowl.-Based Syst.*, vol. 138, pp. 124–133, Dec. 2017.

[80] F. Herrera and L. Martínez, ''A model based on linguistic 2-tuples for dealing with multigranular hierarchical linguistic contexts in multi-expert decision-making,'' *IEEE Trans. Syst., Man, Cybern. B, Cybern.*, vol. 31, no. 2, pp. 227–234, Apr. 2001.

[81] H. Wang and Z. S. Xu, ''Multi-groups decision making using intuitionistic-valued hesitant fuzzy information,'' *Int. J. Comput. Intell. Syst.*, vol. 9, no. 3, pp. 468–482, 2016.

Authors' photographs and biographies not available at the time of publication.