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Sliding Mode Control for Uncertain T-S Fuzzy Singular Biological Economic System

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ABSTRACT In this paper, the design of sliding mode controller for a singular biological economic model with a stage structure and uncertain parameters is studied. First, a biological economic system with uncertain parameters for the invasion of alien species is modeled by a singular system. Then, according to the theory of the T-S fuzzy system, an uncertain T-S fuzzy singular model is established and the stability of the model at the positive equilibria is discussed. Furthermore, a fuzzy integral sliding surface and a sliding mode controller are designed for the T-S fuzzy singular system with an uncertain term. At the same time, the state trajectories can be driven onto the integral sliding surface in finite time and a sufficient condition about the global asymptotic stability of sliding motion is obtained. Finally, a simulation is given to verify the effectiveness of the obtained results.

INDEX TERMS The invasion of alien species, Positive equilibria, sliding mode controller, global asymptotic stability.

I. INTRODUCTION

Singular system has been widely investigated since it can describe various fields, to mention a few, information technology, electricity, physics, chemistry, biology and economy [1]–[5]. Compared with normal system, singular system has complicated feature, e.g., impulsiveness is one of the characteristics. It is worth noticing that singular system can model biodynamic systems [1]–[4]. With the development of the theory about T-S fuzzy system, [1] studied the control problem of a class of bioeconomic models with H_∞ performance. The problem of bifurcation and control for biological economic system has been considered in [2] and [3]. [2] studied a single species model with invasive alien species. [3] considered a class of singular biological economic model with stage structure. Their ideas originate from the economic theory in [5]: Net Economic Profit = Total Revenue − Total Cost. [6] studied the exotic biological economic model with alien species invasion. However, there are few research results on the biological model with stage structure and invasion of alien species, and we know that the growth of the population is affected by the external environment. Therefore, based on the above literature, we will establish a bio-economic model with stage structure and uncertain parameters.

On the other hand, T-S fuzzy model [6], which was proposed by Takagi and Sugeno in 1985, provides a method for the study of nonlinear systems. A series of IF-THEN rules can represent nonlinear system, and some fuzzy membership functions are used to show the local linear input-output relationship. In the past 20 years, T-S fuzzy system theory has been applied in various fields, such as circuit system, economy system and so on. So far, many achievements have been obtained about T-S fuzzy systems, such as stability analysis [7]–[9], robust control of uncertain fuzzy systems [10], [11]. According to the theory of sliding mode control for T-S fuzzy singular system, we construct the model as [12] and study the sliding mode control problem with stage structure. [13] has given the analysis and control for T-S fuzzy singular systems. herefore, we apply the T-S fuzzy sliding mode control to the bioeconomic system. This is a new attempt.

As we all know, sliding mode control uses discontinuous control to realize that the system trajectory can arrive a predetermined sliding mode surface, whose merits include fast response, robustness to parameters, such as uncertainty and external disturbances. There are many achievements in this field. The problem of sliding mode control was firstly proposed by Emelyanov in the 1970s [14]. Based on their research, sliding mode control has evolved into a nonlinear model. Now, the application of variable structure control includes information, aerospace, robotics, etc. [15]–[17]. [15] studies that variable structure system theory can design a model reference adaptive controller when only input and output measurements are available, [16] studies that based on the full state feedback technology of sliding mode control and the feedback linearization technology, a variable structure controller for spacecraft maneuver regulation is developed. [17] studies that the sliding mode control theory is used to design the controller. The tracking and data relay satellites are taken as examples, restrain the elastic vibrating of the flexible appendages effectively. Over the past few decades, there are many results related to fuzzy sliding mode control methods [18], such as backstepping based on sliding mode control [19], adaptive sliding mode control [20]. At present, although fuzzy sliding mode control is widely used, its application in bio-economic system is still in its infancy. For uncertain systems with stage structure, the research results are even fewer, which motivates our investigation.

According to the above mentioned references, we try to solve the problem about the stability and control for singular biological economic systems with uncertain terms. Firstly, a biological economic system of the alien species invasion with uncertain parameters has been constructed by singular systems. Based on T-S fuzzy system theory, the uncertain T-S fuzzy singular model is established and the stability of the system at the positive equilibria is discussed. Then, the fuzzy integral sliding surface and sliding mode controller are designed. By using the designed sliding mode controller, the state trajectories of the system can arrive the sliding surface in finite time. And a sufficient condition about global asymptotic stability of sliding motion is provided.

In this paper, uncertain parameters are added to the model in literature [2]–[6], which makes the biological system more suitable for the real life. The bio-economic system with stage structure is studied by T-S fuzzy sliding mode control, and a new sliding mode controller is designed to control the population density of alien species reasonably. This is also the main contribution of this paper.

Notations: $A \in R^{n\times n}$ is a $n \times n$ real matrix, the superscript T represents the transpose of vector or matrix, A^{-T} represents the inverse of transpose of matrix A, $\|x\|$ denotes the Eucliden norm of vector x and $\|A\|$ denotes the 2-norm of matrix A, I_n represents the $n \times n$ identity matrix and $0_{n\times m}$ represents the $n \times m$ zero matrix.

II. MODELING

Before 1990, the stage structure system has attracted some attention in [25] and [27]. However, until 1990, O. Aiello and H. Freedman established the well-known stage structure system of single-species organisms [28], marking the beginning of research in this field. Next, under the guidance of the work in literature [28], many scholars began to study the ranks of biodynamic systems with stage structures, and a series of research results were obtained in [29]–[31]. On the basis of the existing conclusions, we construct a generalized system with uncertainties of stage structural parameters for invasive alien species.

The following model proposed by [15] is introduced:

$$\begin{cases} \dot{x}_1(t) = rx_2(t) - a_1x_1(t) - bx_1(t) - c_1x_1^2(t) \\ \dot{x}_2(t) = bx_1(t) - a_2x_2(t) - c_2x_2^2(t). \end{cases} \tag{1}$$

where $x_1(t)$, $x_2(t)$ are, respectively, the density of the juveniles, adults of the fish population at time t. a_1 and a_2 are the death rates of the juveniles and adults of the fish population respectively, c_1 and c_2 represent the intraspecific competition intensity at different stages respectively.

According to the actual situation, we assume that the young population is not affected by artificial fishing, and the alien species are not subject to external conditions at the initial stage of invasion.

In 1954, the theory of the open or public fishery economic was established by Gordon [5]. When the harvested effort $E_1(t)$ is known, Sustainable Economic Profit = Sustainable Total Revenue − Sustainable Total Cost. Furthermore, with time t varying, harvested effort $E_1(t)$ switches, then the following equation can be obtained

$$E_1(t)(x(t)p - c) = m(t). \tag{2}$$

In summary, the following bioeconomic model is established,

$$\begin{aligned} \dot{x}_1(t) &= rx_2(t) - a_1x_1(t) - bx_1(t) - c_1x_1^2(t) \\ \dot{x}_2(t) &= bx_1(t) - a_2x_2(t) - E_1(t)x_2(t) - c_2x_2^2(t) \\ &\quad - \eta x_2(t)y(t) \\ \dot{y}(t) &= ay(t) - hP(t) \\ \dot{P}(t) &= \beta y(t) - \theta x_2(t) - wP(t) \\ 0 &= E_1(t)(x_2(t)p - c) - m(t). \end{aligned} \tag{3}$$

where $y(t)$ and $P(t)$ represent the density of the alien species and the harvested capability of alien species at time t, respectively. a denotes the intrinsic growth rate of the invasive species, η denotes the restriction rate for invasive species on adult fish populations, $-hP(t)$ represents the quantity of purification to the invasive species, $\beta y(t)$ represents the targeted purification of alien species, and θ represents the effect of purification on adults of the fish population. According to the actual situation, the impact of purification on juvenile populations is not considered. $\omega P(t)$ represents the cost of capture for alien species. In practical systems, since the system is affected by the external environment, the following model with uncertain parameters is proposed,

$$\begin{aligned} \dot{x}_1(t) &= (r + \Delta r(t))x_2(t) - a_1x_1(t) - (b + \Delta b(t))x_1(t) \\ &\quad - c_1x_1^2(t) \\ \dot{x}_2(t) &= (b + \Delta b(t))x_1(t) - a_2x_2(t) - E_1(t)x_2(t) \\ &\quad - c_2x_2^2(t) - \eta x_2(t)y(t) \\ \dot{y}(t) &= (a + \Delta a(t))y(t) - hP(t) \\ \dot{P}(t) &= (\beta + \Delta\beta(t))y(t) - \theta x_2(t) - wP(t) \\ 0 &= E_1(t)(x_2(t)p - c) - m(t). \end{aligned} \tag{4}$$

where $\Delta r(t)$, $\Delta b(t)$, $\Delta a(t)$, $\Delta\beta(t)$ are the uncertain part of parameters r, b, a, β. And the following conditions are established,

$$|\Delta r(t)| \le \sigma_1, \quad |\Delta b(t)| \le \sigma_2,$$
$$|\Delta a(t)| \le \sigma_3, \quad |\Delta \beta(t)| \le \sigma_4.$$

where $\sigma_1, \sigma_2, \sigma_3$ and σ_4 are nonnegative constants.

III. MAIN RESULT

A. THE EQUILIBRIUM POINT ANALYSIS AND T-S FUZZY LINEARIZATION

Let

$$\begin{aligned} r(t) &= r + \Delta r(t), b(t) = b + \Delta b(t) \\ a(t) &= a + \Delta a(t), \beta(t) = \beta + \Delta\beta(t). \end{aligned} \tag{5}$$

When $m(t) = 0$, two possible positive equilibrium points of the system (4) are obtained. Assume that the positive equilibrium point for the system is $p^*\left(x_1^*, x_2^*, y^*, P^*, E_1{}^*\right)$.

Theorem 1: The system (4) is stable at the positive equilibrium $p^*\left(x_1^*, x_2^*, y^*, P^*, E_1{}^*\right)$ when the following inequalities hold,

$$a_1 + b + 2c_1x_1^* > 0, \quad \omega - a > 0, \quad \beta h - a\omega > 0.$$

Proof: The Jacobian matrix of the system (4) is

$$J = \begin{bmatrix} \Theta_{11} & r & 0 & 0 & 0 \\ b & \Theta_{22} & -\eta x_2 & 0 & -x_2 \\ 0 & 0 & a & -h & 0 \\ 0 & -\theta & \beta & -\omega & 0 \\ 0 & pE_1 & 0 & 0 & x_2p - c \end{bmatrix}$$

where

$$\Theta_{11} = -a_1 - b - 2c_1x_1, \quad \Theta_{22} = -a_2 - E_1 - 2c_2x_2 - \eta y.$$

Then the characteristic polynomials at P^* is

$$\begin{aligned} &\det(\lambda E - J_P)|_{P^*} \\ &= \begin{vmatrix} \Theta_{11}^* & -r & 0 & 0 & 0 \\ -b & \Theta_{22}^* & \eta x_2{}^* & 0 & x_2{}^* \\ 0 & 0 & \lambda - a & h & 0 \\ 0 & \theta & -\beta & \lambda + \omega & 0 \\ 0 & -pE_1{}^* & 0 & 0 & \lambda + c - x_2{}^*p \end{vmatrix} \\ &= x_2^*pE_1^*\Theta_{11}^*\left((\lambda - a)(\lambda + \omega) + \beta h\right) \\ &= x_2^*pE_1^*\Theta_{11}^*\left(\lambda^2 + (\omega - a)\lambda + \beta h - a\omega\right) = 0. \end{aligned}$$

where

$$\begin{aligned} \Theta_{11}^* &= \lambda + a_1 + b + 2c_1x_1{}^* \\ \Theta_{22}^* &= \lambda + a_2 + E_1{}^* + 2c_2x_2{}^* + \eta y^*. \end{aligned}$$

Next, the following conditions will be established,

$$\Theta_{11}^* > 0, \quad \omega - a > 0, \quad \beta h - a\omega > 0.$$

Then based on the Routh-Hurwitz theorem [22], the system (4) is stable at p^*.

Furthermore, a control is applied to system (4) to ensure that the system is stable at equilibrium point. The model is built as follows:

$$\begin{aligned} \dot{x}_1(t) &= (r + \Delta r(t))x_2(t) - a_1x_1(t) - (b + \Delta b(t))x_1(t) \\ &\quad - c_1x_1^2(t) \\ \dot{x}_2(t) &= (b + \Delta b(t))x_1(t) - a_2x_2(t) - E_1(t)x_2(t) \\ &\quad - c_2x_2^2(t) - \eta x_2(t)y(t) \\ \dot{y}(t) &= (a + \Delta a(t))y(t) - hP(t) \\ \dot{P}(t) &= (\beta + \Delta\beta(t))y(t) - \theta x_2(t) - wP(t) + u(t) \\ 0 &= E_1(t)(x_2(t)p - c) - m(t). \end{aligned} \tag{6}$$

Assuming that $p^*\left(x_1^*, x_2^*, y^*, P^*, E_1{}^*\right)$ is the positive equilibrium point of system, the coordinate transformation is applied as follows,

$$\begin{aligned} &\zeta_1(t) = x_1(t) - x_1^*, \zeta_2(t) = x_2(t) - x_2^*, \\ &\zeta_3(t) = x_3(t) - x_3^*, \zeta_4(t) = x_4(t) - x_4^*, \\ &\zeta_5(t) = E_1(t) - E_1^*. \end{aligned} \tag{7}$$

Then system (6) can be transformed into the following form,

$$\begin{aligned} \dot{\zeta}_1(t) &= r(t)\left(\zeta_2(t) + x_2^*\right) - a_1\left(\zeta_1(t) + x_1^*\right) \\ &\quad - b(t)\left(\zeta_1(t) + x_1^*\right) - c_1\left(\zeta_1(t) + x_1^*\right)^2 \\ \dot{\zeta}_2(t) &= b(t)\left(\zeta_1(t) + x_1^*\right) - a_2\left(\zeta_2(t) + x_2^*\right) \\ &\quad - \left(\zeta_5(t) + E_1{}^*\right)\left(\zeta_2(t) + x_2^*\right) \\ &\quad - c_2\left(\zeta_2(t) + x_2^*\right)^2 \\ &\quad - \eta\left(\zeta_2(t) + x_2^*\right)\left(\zeta_3(t) + y^*\right) \\ \dot{\zeta}_3(t) &= a(t)\left(\zeta_3(t) + y^*\right) - h\left(\zeta_4(t) + P^*\right) \\ \dot{\zeta}_4(t) &= \beta(t)\left(\zeta_3(t) + y^*\right) - \theta\left(\zeta_2(t) + x_2^*\right) \\ &\quad - \omega\left(\zeta_4(t) + P^*\right) + u(t) \\ 0 &= \left(\zeta_5(t) + E_1{}^*\right)\left(\left(\zeta_2(t) + x_2^*\right)p - c\right) - m(t). \end{aligned} \tag{8}$$

For convenience, system (8) will be turned into the following form,

$$\begin{aligned} E\dot{\zeta}(t) &= \begin{bmatrix} \Xi_{11} & r(t) & 0 & 0 & 0 \\ b(t) & \Xi_{22} & -\eta x_2^* & 0 & -\left(\zeta_2 + x_2^*\right) \\ 0 & 0 & a(t) & -h & 0 \\ 0 & -\theta & \beta(t) & -\omega & 0 \\ 0 & pE_1{}^* & 0 & 0 & p\left(\zeta_2 + x_2^*\right) - c \end{bmatrix} \zeta(t) \\ &\quad + \begin{bmatrix} r(t)x_2^* - a_1x_1^* - b(t)x_1^* - c_1x_1^{*2} \\ b(t)x_1^* - a_2x_2^* - E_1{}^*x_2^* - c_2x_2^{*2} - \eta x_2^*y^* \\ a(t)y^* - hP^* \\ \beta(t)y^* - \theta x_2^* - \omega P^* + u(t) \\ \left(px_2^* - c\right)E_1{}^* - m(t) \end{bmatrix} \end{aligned} \tag{9}$$

where

$$\Xi_{11} = -b(t) - a_1 - 2c_1\zeta_1 - 2c_1x_1^*$$
$$\Xi_{22} = -a_2 - E_1{}^* - 2c_2\zeta_2 - 2c_2x_2^* - \eta\zeta_3 - \eta y^*.$$

$$\zeta(t) = \begin{bmatrix} \zeta_1(t) \\ \zeta_2(t) \\ \zeta_3(t) \\ \zeta_4(t) \\ \zeta_5(t) \end{bmatrix}, \quad E = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (10)$$

Let

$$z_1(t) = -b(t) - a_1 - 2c_1\zeta_1(t) - 2c_1x_1^*,$$
$$z_2(t) = -a_2 - E_1^* - 2c_2\zeta_2(t) - 2c_2x_2^* - \eta\zeta_3(t) - \eta y^*.$$

Therefore

$$\max z_1(t) = -b(t) - a_1 - 2c_1\zeta_1{}^{\min}(t) - 2c_1x_1^*,$$
$$\min z_1(t) = -b(t) - a_1 - 2c_1\zeta_1{}^{\max}(t) - 2c_1x_1^*,$$
$$\max z_2(t) = -a_2 - E_1^* - 2c_2\zeta_2{}^{\min}(t) - 2c_2x_2^* - \eta\zeta_3{}^{\min}(t) - \eta y^*,$$
$$\min z_2(t) = -a_2 - E_1^* - 2c_2\zeta_2{}^{\max}(t) - 2c_2x_2^* - \eta\zeta_3{}^{\max}(t) - \eta y^*,$$
$$\max z_3(t) = \zeta_2{}^{\max}(t) + x_2^*, \min z_3(t) = \zeta_2{}^{\min}(t) + x_2^*.$$

Using the maximum and minimum values, $z_1(t)$, $z_2(t)$ and $z_3(t)$ can be expressed as

$$z_1(t) = M_{11}(z_1(t)) \max z_1(t) + M_{12}(z_1(t)) \min z_1(t),$$
$$z_2(t) = M_{21}(z_2(t)) \max z_2(t) + M_{22}(z_2(t)) \min z_2(t),$$
$$z_3(t) = M_{31}(z_3(t)) \max z_3(t) + M_{32}(z_3(t)) \min z_3(t).$$

where $M_{i1} + M_{i2} = 1, i = 1, 2, 3$, and M_{ij} represents the membership function. According to the principle of T-S blur, the system (9) can be written as

$$E\dot{\zeta} = \sum_{i=1}^{8} \lambda_i(z(t))(A_i(t)\zeta(t) + B_i(t)U(t)). \quad (11)$$

where $A_i(t)$ and $B_i(t)$ represent $A_i(t) = A_i + \Delta A_i(t)$ and $B_i(t) = B_i + \Delta B_i(t)$ respectively, $z(t) = [z_1(t)\ z_2(t)\ z_3(t)]^T$, $U(t) = [1\ 1\ 1\ u(t)\ 1]^T$ □

B. THE DESIGN OF SLIDING MODE CONTROLLER

In this section, we design the fuzzy integral switching function as follows:

$$s(t) = S_\zeta[\zeta(t) - \zeta(0)] + S_U[U(t) - U(0)] - \int_0^t \sum_{i=1}^{8} \lambda_i(z(\tau)) S_\xi(A_i\zeta(\tau) + B_iU(\tau))\, d\tau - \int_0^t \sum_{i=1}^{8} \lambda_i(z(\tau)) S_U(C_i\zeta(\tau) + D_iU(\tau))\, d\tau. \quad (12)$$

where $S_\zeta \in R^{m\times n}, S_U \in R^{m\times m}, C_i \in R^{m\times n}, D_i \in R^{m\times m}$, matrix $\begin{bmatrix} A_i & B_i \\ C_i & D_i \end{bmatrix}$ is Hurwitz.

Firstly, we give the fuzzy sliding mode controller as follows:

Model rule R_i : IF $z_1(t)$ is N_{i1}, $z_2(t)$ is N_{i2} and $z_p(t)$ is N_{ip}, THEN

$$\dot{U}(t) = S_U^{-1}S_\zeta[A_i\zeta(t) + B_iU(t)] + C_i\zeta(t) + D_iU(t) - \eta_1(t) S_U^{-1} sgn(s(t)), \quad i = 1, 2, \ldots, 8. \quad (13)$$

where $\eta_1(t) = \delta + \varepsilon \|S_\zeta\| \left\| [\zeta^T(t) U^T(t)]^T \right\|$, ε and δ are positive constants. In addition, ε satisfies the assumption: $\|\Delta A_i, \Delta B_i\| \le \varepsilon$.

The controller (13) can be equivalent to the following form,

$$\dot{U}(t) = \sum_{i=1}^{8} \lambda_i(z(t))\{C_i\zeta(t) + D_iU(t) - \eta_1(t) S_U^{-1} sgn(s(t)) + S_U^{-1}S_\zeta(A_i\zeta(t) + B_iU(t))\}. \quad (14)$$

Theorem 2: For system (8), a sliding mode controller is designed as (14) such that the state trajectories can reach the sliding mode surface and maintain it in finite time.

Proof: The following quadratic Lyapunov function is taken into consideration,

$$V(t) = s^T(t) E^T Qs(t). \quad (15)$$

where Q is a nonsingular matrix and $E^TQ = Q^TE \ge 0$.

First of all, the derivative of $s(t)$ is obtained. Then by substituting controller (14) into it, we can obtain

$$\begin{aligned}
E\dot{s}(t) &= S_\zeta E\dot{\zeta}(t) - E\sum_{i=1}^{8} \lambda_i(z(t)) S_\zeta(A_i\zeta(t) + B_iU(t)) \\
&\quad + ES_u\dot{U}(t) - E\sum_{i=1}^{8} \lambda_i(z(t)) S_u(C_i\zeta(t) + D_iU(t)) \\
&= \sum_{i=1}^{8} \lambda_i(z(t)) S_\zeta(A_i(t)\zeta(t) + B_i(t)U(t)) \\
&\quad - E\sum_{i=1}^{8} \lambda_i(z(t)) S_\zeta(A_i\zeta(t) + B_iU(t)) \\
&\quad + ES_U \left\{ \begin{array}{l} \sum_{i=1}^{8} \lambda_i(z(t))(C_i\zeta(t) + D_iU(t)) \\ + S_U^{-1}S_\zeta \sum_{i=1}^{8} \lambda_i(z(t))[A_i\zeta(t) + B_iU(t)] \\ -\eta_1(t) S_U^{-1} sgn(s(t)) \end{array} \right\} \\
&\quad - E\sum_{i=1}^{8} \lambda_i(z(t)) S_U(C_i\zeta(t) + D_iU(t)) \\
&= \sum_{i=1}^{8} \{\lambda_i(z(t))S_\zeta(A_i(t)\zeta(t) + B_i(t)U(t)) \\
&\quad - E\eta_1(t)sgn(s(t))\}.
\end{aligned}$$

$$\begin{aligned}\dot{V}(t) &= 2s^T(t)Q^T E\dot{s}(t)\\ &= 2s^T(t)Q^T\sum_{i=1}^{8}\lambda_i(z(t))S_\zeta(A_i(t)\zeta(t)\\ &\quad + B_i(t)U(t)) - E\eta_1(t)sgn(s(t))\\ &\le 2\|s(t)\|\,\varepsilon\,\|S_\zeta\|\;\|[\zeta(t),U(t)]\|\\ &\quad - 2\|E\|\,\eta_1(t)\,\|s(t)\|. \end{aligned} \tag{16}$$

According to (15) and (16), we can obtain

$$\dot{V}(t) \le -2\delta\|s(t)\| \le -2\bar{\delta}\sqrt{V(t)}. \tag{17}$$

where $\bar{\delta} = \frac{\delta}{\sqrt{\lambda_{\min}(E^TQ)}}$.

Then for any given t_0, $V(t)$ satisfies

$$\begin{cases}\sqrt{V}(t) \le \sqrt{V}(t_0) - 2\delta\left(1-\frac{1}{2}\right)(t-t_0), & t_0 \le t \le t_s\\ V(t) = 0, & t > t_s\end{cases}$$

where the settling time $t_s = t_0 + \frac{\sqrt{V(t_0)}}{\delta}$.

From (17), $V(t)$ will converge to zero in the finite time, that is, the state trajectories will arrive the sliding surface in finite time. The proof is completed. □

C. STABILITY OF SLIDING MOTION

For the biological economic system, how the state of the population to change with time needs to be considered. Hence, the global asymptotic stability of the system is investigated. In this section, we will give a condition about global asymptotic stability of sliding motion.

In order to simplify the calculation, the following form is given,

$$\begin{aligned}&\chi(t) = [\xi^T(t), U^T(t)]^T, \quad G_i(t) = [A_i(t), B_i(t)],\\ &S = [S_\xi, S_U],\\ &K_i = [C_i + S_U^{-1}S_\xi A_i, D_i + S_U^{-1}S_\xi B_i], \quad R_1 = [I_n, 0_{n\times m}]^T,\\ &R_2 = [0_{n\times m}, I_m].\end{aligned}$$

Then equation (12) can be written as follows,

$$\begin{aligned}&s(t)\\ &= S\left\{\chi(t) - \chi(0) - \int_0^t \sum_{i=1}^{8}\lambda_i(\tau)[R_1G_i(t) + R_2K_i]\chi(\tau)\right\}d\tau.\end{aligned} \tag{18}$$

The following system is given,

$$\begin{aligned}E\dot{\zeta}(t) &= \sum_{i=1}^{8}\lambda_i(z(t))(A_i(t)\zeta(t) + B_i(t)U(t))\\ \dot{U}(t) &= \sum_{i=1}^{8}\lambda_i(z(t))(C_i\zeta(t) + D_iU(t))\\ &\quad + S_U^{-1}S_\xi\sum_{i=1}^{8}\lambda_i(z(t))(A_i\zeta(t) + B_iU(t)).\end{aligned} \tag{19}$$

System (19) is equivalent to

$$\bar{E}\dot{\chi}(t) = \sum_{i=1}^{8}\lambda_i(z(t))[R_1G_i + R_2K_i]\chi(t). \tag{20}$$

where

$$\bar{E} = \begin{bmatrix} E & 0_{n\times m}\\ 0_{m\times n} & I_m\end{bmatrix}$$

Obviously, for the term

$$\chi(0) + \int_0^t \sum_{i=1}^{8}\lambda_i(z(t))[R_1G_i + R_2K_i]\chi(\tau)d\tau.$$

of sliding surface (18), (20) is the closed-loop control system.

Theorem 3: The fuzzy singular system (20) is asymptotically stable if there exist nonsingular matrices $X \in R^{(n+m)\times(n+m)}$ and $X_1 \in R^{n\times n}$ which is the previous $n\times n$ block matrix of X, two full column rank matrices V and $U \in R^{n\times(n-r)}$, the matrix $S \in R^{(n-r)\times(n-r)}$ and positive scalar $\varepsilon_i (i = 1, 2, \ldots, l)$ satisfying the following conditions (21)-(22) and the controller gains are $K_i = [C_i + S_U^{-1}S_\xi A_i, D_i + S_U^{-1}S_\xi B_i] = W_iX^{-1}$,

$$\begin{bmatrix} R_1G_iX + R_2W_i + (R_1G_iX + R_2W_i)^T & X^T\\ X & -\varepsilon_i^{-1}I\end{bmatrix} < 0 \tag{21}$$

$$A_i(X_1E^T + VSU^T) + (X_1E^T + VSU^T)A_i^T < 0. \tag{22}$$

Proof: According to the Lemma in [24], $X_1E^T = EX_1^T \ge 0$, $X_1A_i^T + A_iX_1^T < 0$ and (22) are equivalent. Suppose that the quadratic Lyapunov function is $V(\chi(t)) = \chi^T(t)\bar{E}^TX^{-1}\chi(t)$. For any $\chi(t) \ne 0$, we can obtain

$$\begin{aligned}\dot{V}(\chi(t)) &= 2\chi^T(t)\bar{E}^TX^{-1}\dot{\chi}(t)\\ &= 2\chi^T(t)X^{-T}\bar{E}\dot{\chi}(t)\\ &= 2\chi^T(t)\sum_{i=1}^{8}\lambda_i(z(t))X^{-T}[R_1G_i + R_2K_i]\chi(t)\\ &= 2\chi^T(t)\sum_{i=1}^{8}\lambda_i(z(t))[X^{-T}R_1G_iXX^{-1}\\ &\quad + X^{-T}R_2W_iX^{-1}]\chi(t).\end{aligned} \tag{23}$$

$\dot{V}(\chi(t)) < 0$ holds if and only if

$$\begin{aligned}&[X^{-T}R_1G_iXX^{-1} + X^{-T}R_2W_iX^{-1}]\\ &\quad + [X^{-T}R_1G_iXX^{-1} + X^{-T}R_2W_iX^{-1}]^T < 0.\end{aligned} \tag{24}$$

Therefore, there exists positive scalar ε_i such that

$$\begin{aligned}&[X^{-T}R_1G_iXX^{-1} + X^{-T}R_2W_iX^{-1}]\\ &\quad + [X^{-T}R_1G_iXX^{-1} + X^{-T}R_2W_iX^{-1}]^T < 0 < -\varepsilon_iI.\end{aligned}$$

Pre- and post-multiplying the above inequality by X^T and X, respectively, the following inequality can be obtained

$$R_1G_iX + R_2W_i + (R_1G_iX + R_2W_i)^T < -\varepsilon_iX^TX \tag{25}$$

According to the Schur complement, the inequality (25) is equivalent to

$$\begin{bmatrix} R_1G_iX+R_2W_i+(R_1G_iX+R_2W_i)^T & X^T \\ X & -\varepsilon_i^{-1}I \end{bmatrix} < 0 \qquad (26)$$

From (21), system (20) is asymptotically stable.

For the uncertain fuzzy singular system (11), if the state trajectories can arrive and maintain the sliding surface, it must satisfy the following condition,

$$\begin{aligned} \dot{s}(t) = & \sum_{i=1}^{8} \lambda_i(z(t))S_\zeta(A_i\zeta(t)+B_iU(t)) \\ & - \sum_{i=1}^{8} \lambda_i(z(t))S_\zeta(A_i\zeta(t)+B_iU(t)) + S_U\,\dot{u}(t) \\ & - \sum_{i=1}^{8} \lambda_i(z(t))S_U(C_i\zeta(t)+D_iU(t)) = 0. \qquad (27) \end{aligned}$$

Based on (27), we can obtain the following equation,

$$\begin{aligned} \dot{U}(t) = & \sum_{i=1}^{8} \lambda_i(z(t))[C_i\zeta(t)+D_iU(t) \\ & - S_U^{-1}S_\zeta[\Delta A_i(t)\zeta(t)+\Delta B_i(t)U(t)]]. \qquad (28) \end{aligned}$$

For the sliding mode controller in this paper, the controller (28) is equivalent to the control law (14). The normal system (sliding motion) is composed by uncertain T-S fuzzy singular system (11), sliding surface (12) and feedback control law (28), and its specific forms are as follows:

$$\begin{aligned} E\dot{\zeta} = & \sum_{i=1}^{8} \lambda_i(z(t))(A_i(t)\zeta(t)+B_i(t)U(t)) \\ \dot{U}(t) = & \sum_{i=1}^{8} \lambda_i(z(t))[C_i\zeta(t)+D_iU(t) \\ & - S_U^{-1}S_\zeta[\Delta A_i(t)\zeta(t)+\Delta B_i(t)U(t)]]. \qquad (29) \end{aligned}$$

□

IV. SIMULATION EXAMPLE

In recent years, the ecological environment has attracted more and more attention. People are also aware of the importance of protecting the balance of the ecological environment. In some news reports, we see that the invasive alien species is gradually increasing. It is a pressing task to solve these problems.

Crayfish originated in North America, and the scientific name is freshwater lobster, also known as "shrimp." As we all know, in the past few years, crayfish had flooded the United States, and the California government allocated 800,000 dollars to eliminate 20,000 crayfish. In May of this year, crayfish thrives in the German capital and takes a large number of streets in Berlin. The rapid propagation of crayfish will greatly harm the ecosystem of Berlin.

FIGURE 1. Crayfish infestation.

First of all, we choose the following parameters according to the actual situation,

$$\begin{aligned} & r=0.05, \quad a_1=0.01, \quad b=0.02, \quad c_1=0.01, \\ & a_2=0.01, \quad c_2=0.01, \quad \eta=0.02, \quad a=0.06, \\ & \beta=0.04, \quad c=1, \quad h=0.001, \quad \theta=0.01, \\ & w=0.02, \quad p=4. \end{aligned}$$

$0.5115-0.408cos(0.02288t)-0.2688sin(0.02288t)$ is used to denote the periodic parameter. Because the average value of r, b, a, β and uncertainty item Δr, Δb, Δa, $\Delta\beta$ are used to express the parameter, Δr, Δb, Δa and $\Delta\beta$ are the Fourier function, the specific expression are as follows:

$$\begin{aligned} \Delta r = & r(0.5115-0.408cos0.02288t \\ & -0.2688sin(0.02288t)), \\ \Delta b = & b(0.5115-0.408cos(0.02288t) \\ & -0.2688sin(0.02288t)), \\ \Delta a = & a(0.5115-0.408cos(0.02288t) \\ & -0.2688sin(0.02288t)), \\ \Delta\beta = & \beta(0.5115-0.408cos(0.02288t) \\ & -0.2688sin(0.02288t)). \end{aligned}$$

Therefore, we obtain

$$\begin{aligned} & |\Delta r(t)| \le r = 0.05, \quad |\Delta b| \le b = 0.02, \\ & |\Delta a| \le a = 0.06, \quad |\Delta\beta| \le \beta = 0.04. \end{aligned}$$

Then the following system is obtained,

$$\begin{aligned} \dot{x}_1(t) = & (0.05+0.05\Delta r(t))x_2(t) - 0.01x_1(t) \\ & -(0.02+0.02\Delta b(t))x_1(t) - 0.01x_1^2(t) \\ \dot{x}_2(t) = & (0.02+0.02\Delta b(t))x_1(t) - 0.02x_2(t) \\ & - E_1(t)x_2(t) - 0.01x_2^2(t) - 0.02x_2(t)y(t) \\ \dot{y}(t) = & (0.06+0.06\Delta a(t))y(t) - 0.001P(t) \\ \dot{P}(t) = & (0.04+0.04\Delta\beta(t))y(t) - 0.01x_2(t) \\ & -0.02P(t)+u(t) \\ 0 = & E_1(t)(x_2(t)4-1) - m(t). \qquad (30) \end{aligned}$$

where the units of $x_1(t)$, $x_2(t)$ and $y(t)$ are 10 pieces/m^3, the units of $P(t)$ and $E_1(t)$ are 10,000 pieces.

To be more reasonable, the following inequalities are established,

$$0 \leq x_1 \leq 250, \quad 0 \leq x_2 \leq 250, \quad 0 \leq y \leq 250,$$
$$0 \leq P \leq 250, \quad 0 \leq E_1 \leq 250.$$

when the economic profit $m = 0$ and $u(t) = 0$, the model (30) has the following positive equilibrium point,

$$P^* (0.3708, 0.25, 0.0893, 0.0536, 0.0975).$$

According to the linear transformation, $\zeta_1, \zeta_2, \zeta_3, \zeta_4$ and ζ_5 need to satisfy the following inequalities,

$$-0.3708 \leq \zeta_1 \leq 249.6292, \quad -0.25 \leq \zeta_2 \leq 249.75,$$
$$-0.0893 \leq \zeta_3 \leq 249.9107, \quad -0.0536 \leq \zeta_4 \leq 249.9464,$$
$$-0.0975 \leq \zeta_5 \leq 249.9025.$$

System (30) can be written as follows:

$$E\dot{\zeta}(t) = \begin{bmatrix} \Omega_{11} & r(t) & 0 & 0 & 0 \\ b(t) & \Omega_{22} & -0.005 & 0 & \Omega_{25} \\ 0 & 0 & a(t)p & -0.001 & 0 \\ 0 & -0.01 & \beta(t) & -0.02 & 0 \\ 0 & 0.39 & 0 & 0 & \Omega_{55} \end{bmatrix} \zeta(t) + \begin{bmatrix} 0.25r(t) - 0.3708b(t) - 0.0051 \\ 0.3708b(t) - 0.0279 \\ 0.0893a(t) - 0.0001 \\ 0.0893\beta(t) - 0.0201 + u(t) \\ -m(t) \end{bmatrix} \quad (31)$$

where

$$\Omega_{11} = -b(t) - 0.02\zeta_1 - 0.0174,$$
$$\Omega_{22} = -0.114286 - 0.02\zeta_2 - 0.02\zeta_3,$$
$$\Omega_{55} = 4(\zeta_2 + 0.25) - 1, \quad \Omega_{25} = -(\zeta_2 + 0.25).$$

Calculating the maximum and minimum values, $z_1(t)$, $z_2(t)$ and $z_3(t)$ are obtained as follows:

$$\max z_1(t) = -0.021, \quad \min z_1(t) = -5.02,$$
$$\max z_2(t) = -0.3475, \quad \min z_2(t) = -10.3475,$$
$$\max z_3(t) = 250, \quad \min z_3(t) = 0,$$
$$\max z_4(t) = 999, \quad \min z_4(t) = -1.$$

According to the above process, the following model is obtained

$$E\dot{\zeta} = \sum_{i=1}^{8} \lambda_i(z(t)) (A_i(t)\zeta(t) + B_i(t)U(t)). \quad (32)$$

A_i can be calculated as

$$B_i = \begin{bmatrix} 0.000016 \\ -0.020484 \\ 0.005258 \\ -0.016528 \\ 0 \end{bmatrix}, \quad M_{11} = \frac{z_1 + 0.021}{4.999},$$

$$M_{12} = \frac{-0.021 - z_1}{4.999}, \quad M_{21} = \frac{z_2 + 10.3475}{10},$$

$$M_{22} = \frac{-0.3475 - z_2}{10}, \quad M_{31} = \frac{z_3}{250}, \quad M_{32} = \frac{250 - z_3}{250},$$

$$M_{41} = \frac{z_4 + 1}{1000}, \quad M_{42} = \frac{999 - z_4}{1000}.$$

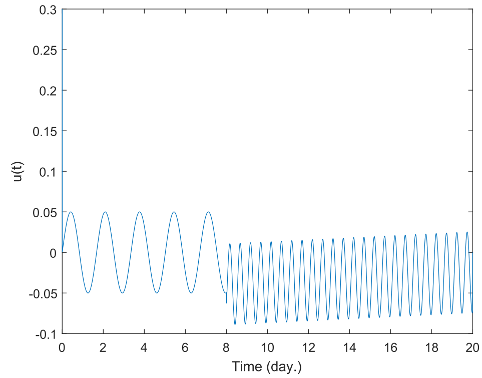

FIGURE 2. Sliding mode controller.

$\begin{bmatrix} A_i & B_i \\ C_i & D_i \end{bmatrix}$ is Hurwitz, C_i, D_i can be calculated as follows:

$$C_i = [13.2077 \quad -26.6971 \quad -44.1383 \quad -1.6628 \quad -2.2637], \quad D_i = -2.6939.$$

The following parameters can be obtained by using LMI toolbox

$$S_\xi^T = \begin{bmatrix} 408.6009 \\ -279.0836 \\ 781.0507 \\ -118.5229 \\ -231.3324 \end{bmatrix}, \quad S_U = 1,$$

$$K_1^T = \begin{bmatrix} -0.9546 \\ -0.6904 \\ -0.6208 \\ -0.0734 \\ -0.8242 \\ -0.3727 \end{bmatrix}, \quad K_2^T = \begin{bmatrix} -0.3975 \\ -78.5299 \\ -23.3536 \\ 7.1048 \\ -4.9480 \\ -4.7253 \end{bmatrix},$$

$$K_3^T = \begin{bmatrix} -55.2068 \\ 0 \\ -851.4481 \\ 86.4481 \\ -22.5404 \\ 3.0587 \end{bmatrix}, \quad K_4^T = \begin{bmatrix} -0.646 \\ 1.919 \\ -112.508 \\ 1.6514 \\ -14.6141 \\ -10.3991 \end{bmatrix}.$$

The sliding mode controller, the sliding mode surface and the state trajectories of the closed-loop system are shown in Fig.2, Fig.3 and Fig.4, respectively.

From the simulation, we can see that the reservoir ecosystem can be stabilized by using the sliding mode controller designed in this paper. In other words, the ecosystem will keep the balance of nature and it will remain benign. Whether in the United States or Germany, because the local environment is very suitable for the survival of crayfish, they multiply in the river, devour eggs and cubs from other animals, which can cause local bio-chain breaks and fatal damage to local reproduction. The data shows that Californian clams in Southern California have been threatened with extinction by crayfish. In this case, we need to artificially intervene in the number of crayfish. Therefore, a controller is added

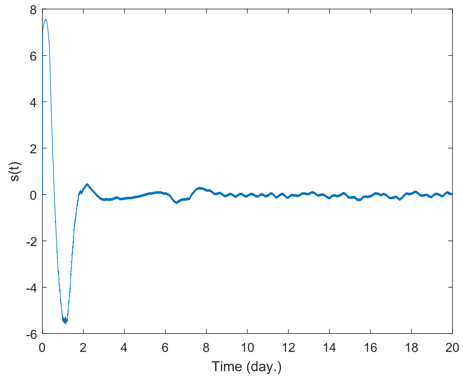

FIGURE 3. Sliding mode surface.

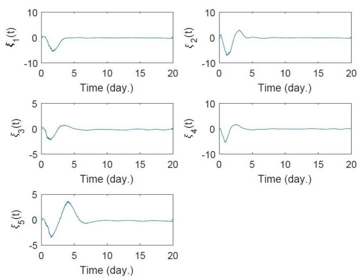

FIGURE 4. State trajectories of the closed-loop system.

to the fourth differential equations of (30). By sliding mode control, a controller is designed to effectively control the effort to capture of crayfish. It ensures the stability of the whole ecosystem.

V. CONCLUSIONS

The design of sliding mode controller for T-S fuzzy singular systems with stage structure parameter uncertainties for alien species invasion is developed. Combined with purification capacity and sliding mode control, the stress on environmental resources and ecosystems with stage structure is relieved. Firstly, on the basis of theory about T-S fuzzy system, the uncertain T-S fuzzy singular model is established, and at the positive equilibria, the stability of the system is discussed. Then, the fuzzy integral sliding surface and sliding mode controller are designed. At the same time, the state trajectories can arrive the integral sliding surface in finite time. Then sufficient conditions are obtained, which guarantees the admissibility of the sliding mode dynamics. Finally, the effectiveness of the obtained results is illustrated by a practical example. This paper has demonstrated that sliding mode controller can improve the performance of the system significantly.

From the perspective of biological, the sliding mode control approach that we proposed can availably control the population density of invasive alien species and guarantee the coexistence of local species and alien species in a certain range. Therefore, our research results have a profound effect on the introduction of exotic species.

Now, we still need to invest human and financial resources to manage the invasion of alien species. In the future, we are going to study how to ąřturn waste into treasureąś and hope that foreign invasive species will bring harm to the local area, but not the interests.

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