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Finite-Time Lag Synchronization of Uncertain Complex Dynamical Networks With Disturbances via Sliding Mode Control

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ABSTRACT This paper investigates lag synchronization between two uncertain complex dynamical network with time-varying coupling delay, fully unknown parameters, and disturbances in finite-time. First, a nonsingular terminal sliding surface is proposed and its finite convergence is proved. Then, appropriate adaptive laws are derived to estimate the unknown parameters of the networks. Subsequently, based on the finite-time stability theory and adaptive laws, an adaptive sliding mode control is designed for achieving finite-time lag synchronization. In addition, the unknown bounded disturbances are also overcame by the proposed control and some corollaries are given. Finally, analytical results show that the states trajectory of the networks error converge to the sliding phase within finite-time. Furthermore, numerical simulation results demonstrate the applicability and the effectiveness of the designed method.

INDEX TERMS Complex dynamical networks, finite-time lag synchronization, sliding mode control, unknown parameters, external disturbances.

I. INTRODUCTION

Due to the pioneering works of Pecora and Carroll in the 1990s, synchronization of the dynamical systems has appeared as an active area of research in dynamical systems and nonlinear dynamics theory. Specially, synchronization of coupled chaotic dynamical systems attracted attention of many researchers because its evolution is sensitive to initial conditions. This property implies that the trajectories of chaotic systems intrinsically defy synchronization, even when the systems start from very slightly different initial conditions. Ever since then, theoretical researche and applications of chaos synchronization have been developed rapidly in recent years, and many researchers in various disciplines have devoted their great efforts to investigate network synchronization of coupled chaotic systems because of its wide applications in different fields as seen in fireflies flashing in unison and heart cells beating in rhythm [1]–[4]. As far as we all know, network synchronization cannot occur without added controller, which forces the state trajectories of the response network to follow the behavior of the state trajectories of the drive network dynamics. Thus, many control methods have been designed for achieving different synchronization schemes such as activation control, linear separation, linear coupling, adaptive control, etc [5]–[17].

In real-world complex networks, time delay unavoidably occurs commonly in many implementation of real systems, which is often regarded as the potential source of systems instability and performance deterioration. For instance, in the telephone communication system, the voice one hears on the receive side at time t is often the voice from the transmitter side at time $t - \tau$ [18]. Various types of time delays such as internal delay, coupling delay and other hybrid delays have been put forward [19]–[22]. In particularly, time delay is not always a constant but varied with time, which is called time-varying delay that has also received some attentions [23]–[25]. Thus, time-varying delay is requisite to take into the consideration when modeling networks. In addition, many uncertain factors have been recognized which demolish the network stability in engineering applications such as external disturbance, uncertain parameters etc. In advance, the values of these factors can be often not exactly known, thus the synchronization with uncertain factors has become a hot topic, and many works have been presented. In [26], robust outer synchronization problem between two nonlinear complex networks with parametric disturbances and mixed time-varying delays has been reached by linear matrix control. Ji *et al.* [27] have investigated lag synchronization of uncertain complex dynamical networks having constant delay coupling, uncertain parameters and disturbances by adaptive control. In [28], hybrid feedback controllers have been designed for projective lag synchronization with time-varying delay, mismatch terms and disturbances. Based on adaptive controller, projective lag synchronization of uncertain dynamical networks with time-varying delay, uncertain parameters and disturbance terms has been accomplished in [29]–[31]. The switched topologies as well as parametric uncertainties, on this basis, Yang *et al.* [32] further considered the asymptotic synchronization problem for a class of uncertain complex networks with hybrid switching and impulsive effects. Even though the increase attention on synchronization of complex networks, how to realize the robust synchronization in networks is still an open topic. In addition, a few of related results have been established for lag synchronization of uncertain complex dynamical networks with time-varying delay, which is one of the main motivation of this work.

Sliding mode control technique is an effective and robust method to deal with uncertainties which is the focus of our paper. Its process aims to switch the control law to force the states of the system from the initial states onto some predefined sliding mode surface, on which the system has desired performance such as stability, insensitivity to system parameter variations, disturbance rejection capability, and tracking ability. Therefore, this control method has potential applications in electrical engineering, aircraft, chemical reactors and so on [33]. In recent years, many results have been reported in chaos synchronization between two chaotic systems by using this technique. For example, Lin and Wang proposed observer-based decentralized fuzzy neural sliding mode control for interconnected unknown chaotic systems via network structure adaptation [34]. In [35] and [36], active sliding mode control was designed to study modified projective synchronization of chaotic systems with disturbances and fractional-order chaotic system with external noise respectively. Based on adaptive sliding mode control, synchronization problem between two different uncertain chaotic systems with external disturbances and unknown parameters was investigated in [37]. Chen *et al.* [38] investigated two classes of synchronization problems of multiple chaotic systems with disturbances by sliding mode control. Up to now, to the best of our knowledge, no results have been established for synchronization of complex dynamical networks involving the effects of uncertainties and external disturbances by sliding mode control, which is one of the focus of this work.

All results mentioned above have been focused on the asymptotic networks synchronization, which means the synchronization can only be achieved when time tends to infinity. In that case, the synchronization is not optimal, such as in digital telecommunication where the encoded message can not be recovered or sent and the information can suffer irreversible loss [39], [40]. Thus, reducing the time of synchronization for fast synchronization is crucial in real world application. For instance, in secure communication the finite-time synchronization can recover the transmitted signals in finite-time which increases the performance and the confidentiality greatly [41], [42]. In addition, network of robots will use accurate information to accomplish other tasks, when it reaches to the exact value of a quantity sensed in a finite-time [43]. Therefore, finite-time control techniques are desirable which not only demonstrate perfect performance in convergence time but also better disturbance rejection properties and robustness against uncertainties [44]–[48]. Due to their diverse advantages, many researchers have devoted themselves to develop the appropriate finite-time control methods [49]–[54]. For instance, Cheng *et al.* [55] have designed adaptive intermittent control for finite-time hybrid projective synchronization of rive-response complex networks with distributed-delay. In [56], finite-time synchronization of general complex networks with time-varying delays and hybrid couplings was studied by designing a simple discontinuous state feedback controller. Mei *et al.* [57] explore the finite-time synchronization in drive-response dynamical networks with non-delay coupling and the response networks has uncertain parameter by feedback control and an updated law. Periodically intermittent control was designed to achieve finite-time synchronization of complex dynamical networks with time-varying delay [58], [59]. Linear and adaptive error-feedback controllers have been proposed for finite-time lag synchronization of drive-response dynamical networks with unknown signal time-delays [60]. In [61], periodically intermittent control technique along with sliding mode control has been utilized for realizing finite-time lag synchronization of complex networks with time-varying delay, however it does not consider the parameter identification and external disturbance parameters.

To the best of our knowledge, we have not come across any theoretical results considering the problem of lag synchronization between two uncertain dynamical networks with time-varying delay coupling and external disturbances in finite-time based on sliding mode control method. In the light of the above discussion, we design sliding mode control together with adaptive control and update laws to realize lag synchronization of uncertain dynamical networks with time-varying delay coupling, fully uncertain parameters and external disturbances within finite-time. The proposed finite-time control method is based on novel nonsingular terminal sliding mode control technique. We construct a suitable novel nonsingular terminal sliding surface which can ensure the error state $e(t) = 0$ in finite-time, where the sliding surface that presented in [34]–[38] just satisfy the asymptotically stability of error state $e(t) \to 0$ as $t \to \infty$. Based on adaptive laws and finite-time control, adaptive update laws are designed to force the error trajectories onto sliding surface in finite-time and remain on it forever. In addition, the unknown bounded disturbances are overcome and the uncertain parameters are identified. Numerical simulations results are given to prove the efficiency of the designed control.

The paper is organized as follows: Section 2 introduce the network model and some necessary preliminaries are given. The main results are given and novel criteria are derived in Section 3. Section 4 presented examples and their simulations. Finally, the conclusions are drawn in Section 5.

II. MODEL DESCRIPTION

A general complex dynamical network model is consisting of N linearly coupled nodes with uncertain parameters and external disturbance can be described as follows:

$$\dot{x}_i(t) = f_i(x_i(t)) + F_i(x_i(t))\alpha_i + \sum_{j=1}^{N} a_{ij}\Gamma x_j(t-\tau(t)) + \omega_i(t), \tag{1}$$

where $x_i(t) = (x_{i1}(t), x_{i2}(t), \ldots, x_{in}(t))^T \in \mathbf{R}^n$ denotes the state vector of the ith node, $f_i : \mathbf{R}^n \longrightarrow \mathbf{R}^n$ and $F_i : \mathbf{R}^n \longrightarrow \mathbf{R}^{n\times N}$ are the continuous nonlinear function matrices. The α_i's are the unknown constant parameter vector, $\omega_i(t)$'s are external disturbances and Γ is the inner coupling matrix. The coupling time-varying delay $\tau(t)$ is a bounded and continuously differentiable function, which means there exist positive constants τ and ϵ satisfying $0 \leq \tau(t) \leq \tau$, $\dot{\tau}(t) \leq \epsilon < 1$. Also $A = (a_{ij}) \in \mathbf{R}^{N\times N}$ is the coupling configuration matrix, where the diagonal elements defined as

$$a_{ii} = -\sum_{j=1, j\neq i}^{N} a_{ij} \quad i = 1, 2, \ldots, N.$$

The initial states satisfies: $x_i(t) = \varphi_{1i}(t) \in \mathcal{C}([-\tau, 0], \mathbf{R}^n)$.

Then the controlled network can be described by

$$\dot{y}_i(t) = g_i(y_i(t)) + G_i(y_i(t))\beta_i + \sum_{j=1}^{N} a_{ij}\Gamma y_j(t-\tau(t)) + \mu_i(t) + u_i(t), \tag{2}$$

with initial condition $y_i(t) = \varphi_{2i}(t) \in \mathcal{C}([-\tau, 0], \mathbf{R}^n)$. Where $y_i(t) = (y_{i1}(t), y_{i2}(t), \ldots, y_{in}(t))^T \in \mathbf{R}^n$ is response state of the ith node, $g_i : \mathbf{R}^n \longrightarrow \mathbf{R}^n$ and $G_i : \mathbf{R}^n \longrightarrow \mathbf{R}^{n\times N}$ are the continuous nonlinear function matrices. The β_i's are the unknown constant parameter vector, $\mu_i(t)$'s are external disturbances and $u_i \in \mathbf{R}^n$ is the control input.

Definition 1: The drive network (1) and response network models (2) are said to achieve lag synchronization in finite-time for a given time delay $\theta > 0$, if there exists a constant $T > 0$ such that

$$\lim_{t\longrightarrow T} \|e_i(t)\| = \lim_{t\longrightarrow T} \|y_i(t) - x_i(t-\theta)\| = 0,$$

and

$$\|e_i(t)\| \equiv 0 \quad for \quad t > T,$$

where T depends on the initial state vector value $e_i(t) = \varphi_{2i}(t) - \varphi_{1i}(t-\theta)$ for $t \in [-\bar{\tau}, 0]$, $\bar{\tau} = \max\{\tau, \theta\}$.

The error dynamics can be obtained as:

$$\dot{e}_i(t) = g_i(y_i(t)) + G_i(y_i(t))\beta_i + \sum_{j=1}^{N} a_{ij}\Gamma e_j(t-\tau(t)) + u_i(t) + \rho_i(t) - \Big(f_i(x_i(t-\theta_i) + F(x_i(t-\theta))\alpha_i\Big). \tag{3}$$

Remark 2: For special case when $\theta = 0$, the complete synchronization can be achieved. Thus, our results have greater applicability.

In order to obtain the main results, we use the following assumptions and lemmas.

Assumption 3 [57]: The unknown parameters $\alpha_i(t)$ and $\beta_i(t)$ are bounded for any positive constants such that

$$\|\alpha_i(t)\| \leq \ell, \|\beta_i(t)\| \leq \varphi$$

Assumption 4 [27]: The time-varying disturbances $\omega_i(t)$ and $\mu_i(t)$ are bounded for any non-negative constants such that

$$\|\omega_i(t)\| \leq \Phi, \|\mu_i(t)\| \leq \Psi$$

Note let $\theta > 0$, $\rho_i(t) = \mu_i(t) - \omega_i(t-\theta)$, $\rho_i = sup|\rho_i(t)|$. From Assumption 4, we obtain that $\rho_i(t)$ is bounded.

Assumption 5: There exist a large positive constant $\bar{\rho}_i$ such that

$$\rho_i < \bar{\rho}_i$$

Lemma 6 [50]: Assume that a continuous and positive-definite function $V(t)$ satisfies the following inequality:

$$\dot{V}(t) \leq -kV^{\eta}(t), \quad t \geq t_0, \; V(t_0) \geq 0,$$

where $k > 0$, $0 < \eta < 1$. Then, for any given t_0 the following holds:

$$V^{1-\eta}(t) \leq V^{1-\eta}(t_0) - k(1-\eta)(t-t_0), \quad t_0 \leq t \leq T_1$$
$$V(t) \equiv 0, \quad \forall t \geq T_1$$

with T given by

$$T_1 \leq \frac{V^{1-\eta}(t_0)}{k(1-\eta)}$$

Lemma 7 [49]: For $a_1, a_2, \ldots, a_n \in \mathbf{R}^n$, $0 < r < 1$, $0 < q < 2$, then

$$(|a_1| + |a_2| + \ldots + |a_n|)^r \leq |a_1|^r + |a_2|^r + \ldots + |a_n|^r$$
$$(|a_1|^q + |a_2|^q + \ldots + |a_n|^q)^{\frac{1}{q}} \leq (|a_1|^r + |a_2|^r + \ldots + |a_n|^r)^{\frac{1}{r}}$$

Lemma 8 [57]: Suppose $a_1, a_2, \ldots, a_n \in \mathbf{R}^n$, $0 < q < 2$, then the following inequality holds

$$\|a_1\|^q + \|a_2\|^q + \ldots + \|a_n\|^q \geq (\|a_1\|^2 + \|a_2\|^2 + \ldots + \|a_n\|^2)^{\frac{q}{2}}$$

Lemma 9 [27]: For any vector $x, y \in \mathbf{R}^n$ and positive definite matrix $Q \in \mathbf{R}^{n\times n}$, the following matrix inequality holds:

$$2x^T y \leq x^T Qx + y^T Q^{-1} y.$$

III. CONTROLLER DESIGN FOR FINITE-TIME LAG SYNCHRONIZATION

In this section, we pay our attention to design an adaptive control and sliding mode control to ensure the error dynamics (3) converge to zero within a limited time. First, we need to select an appropriate sliding surface to realize the sliding mode motion, which is proposed as following:

$$s_i(t) = C_i e_i(t) + \int_0^t sign(e_i(\sigma))|e_i(\sigma)|^{\eta} d\sigma, \tag{4}$$

where $C_i > 0$ and $0 < \eta < 1$.

The sufficient condition for existence of sliding mode that

$$s_i(t) = \dot{s}_i(t) = 0$$

Therefore, we can obtain the sliding mode dynamics as

$$\dot{e}_i(t) = -\frac{1}{C_i} sign(e_i(t))|e_i(t)|^{\eta} \quad i = 1, 2, \ldots, N. \tag{5}$$

Theorem 10: The sliding mode dynamics (5) is finite-time stable and their states trajectory converge to the equilibriums $e_i(t) = 0$ in finite-time T_1 given by

$$T_1 \le \frac{(1/2\sum_{i=1}^{N} e_i^2(0))^{\frac{1-\eta}{2}}}{(1-\eta)2^{\frac{\eta-1}{2}} c_m}, \tag{6}$$

where $e(0)$ is the is the initial values of $e(t)$ and $c_m = min\{\frac{1}{C_i}\}$.

Proof: Choosing the following Lyapunov function as

$$V(t) = \frac{1}{2}\sum_{i=1}^{N} e_i^2(t)$$

Taking the derivative, we obtain

$$\begin{aligned}\dot{V}(t) &= \sum_{i=1}^{N} e_i^T(t)\dot{e}_i(t)\\ &= \sum_{i=1}^{N} e_i^T(t)\Big(-\frac{1}{C_i} sign(e_i(t))|e_i(t)|^{\eta}\Big)\end{aligned}$$

Using $sign(e_i(t)) = \frac{|e_i(t)|}{e_i(t)}$, we get

$$\dot{V}(t) = \sum_{i=1}^{N} -\frac{1}{C_i}|e_i(t)|^{\eta+1} \le -c_m \sum_{i=1}^{N} |e_i(t)|^{\eta+1},$$

By Lemma 8, one can obtain

$$\dot{V}(t) \le -2^{\frac{\eta+1}{2}} c_m \sum_{i=1}^{N}\Big(\frac{1}{2}e_i^2(t)\Big)^{\frac{\eta+1}{2}} = -2^{\frac{\eta+1}{2}} c_m V^{\frac{\eta+1}{2}},$$

Applying Lemma 6, we conclude that the error dynamics converge to zero within finite-time T_1. Hence, the proof is completed.

Remark 11: In [34]–[38], the authors studied different synchronization schemes by sliding mode control, where the proposed sliding surfaces just satisfy the asymptotically stability ($e(t) \to 0$ as $t \to \infty$). By comparison, we construct a suitable novel nonsingular terminal sliding surface which can ensure the error state $e(t) = 0$ within finite-time. Therefore, our results are more reasonable and have a greater applicability.

Second, we need to design adaptive control to ensure the trajectories of the error dynamics converge to the sliding surface $s_i(t) = 0$ in finite-time and remain on it forever. The finite-time adaptive sliding mode control is proposed as follows:

$$\begin{aligned}u_i(t) = &-\frac{1}{C_i} sign(e_i(t))|e_i(t)|^{\eta} - (K_i + \hat{\rho}_i) sign(s_i(t))\\ &-\gamma\Delta\frac{sign(s_i(t))}{C_i|s_i(t)|} - p_i e_i(t) - G_i(y_i(t))\hat{\beta}_i(t)\\ &+ f_i(x_i(t-\theta)) + F_i(x_i(t-\theta))\hat{\alpha}_i(t) - g_i(y_i(t))\\ &-\xi\Big(r\int_{t-\tau(t)}^{t} e_i^T(\chi)e_i(\chi)d\chi\Big)^{\frac{1+\eta}{2}}\frac{s_i(t)}{C_i\|s_i(t)\|^2},\end{aligned} \tag{7}$$

The appropriate adaptive update laws defined as:

$$\dot{\hat{\alpha}}_i(t) = -F_i^T(x_i(t-\tau))\Lambda, \tag{8}$$

$$\dot{\hat{\beta}}_i(t) = G_i^T(y_i(t))\Lambda, \tag{9}$$

$$\dot{\hat{\rho}}_i(t) = C_i|s_i|. \tag{10}$$

where $\Delta = \|\hat{\alpha}_i\| + \ell + \|\hat{\beta}_i\| + \varphi + \|\hat{\rho}_i\| + \bar{\rho}_i$. $C_i, K_i, p_i, r > 0$. $\hat{\rho}_i$ are the estimated value of the upper bound ρ_i, $\hat{\alpha}_i(t)$ and $\hat{\beta}_i(t)$ are the estimated parameters of , $\alpha_i(t), \beta_i(t)$, respectively. $\Lambda = [C_1 s_1, C_2 s_2, \ldots, C_N s_N]^T$.

Theorem 12: Using the controller (7) and the adaptive laws (8)- (10), then the trajectory of the error dynamics (3) will converge to the sliding surface $s_i(t) = 0$ within finite-time T_2, and remain on it forever if the following conditions hold:

1) $\frac{1}{2}(r - I) \le 0$,
2) $\frac{1}{2}(I - r(1-\tau)) \le 0$,
3) $\frac{1}{2}\Lambda^2(\lambda_{min}(QQ^T) - pp^T) \le 0$,

where the sliding mode reaching time T_2 given by

$$T_2 \le 2^{\frac{1-\eta}{2}}\frac{V^{\frac{1-\eta}{2}}(0)}{\varepsilon(1-\eta)},$$

$$\begin{aligned}V(0) = \frac{1}{2}\sum_{i=1}^{N}\Big(&s_i^2(0) + \|\tilde{\alpha}_i(0)\|^2 + \|\tilde{\beta}_i(0)\|^2 + \|\tilde{\rho}_i(0)\|^2\\ &+ r\int_{-\tau(t)}^{0} e_i^T(\chi)e_i(\chi)d\chi\Big),\end{aligned} \tag{11}$$

where $\varepsilon = min\{\nu, \gamma, \xi\}$, $\nu = min\{C_1K_1, C_2K_2, \ldots, C_NK_N\}$. $\tilde{\alpha}_i(0) = \hat{\alpha}_i(0) - \alpha_i$, $\tilde{\beta}_i(0) = \hat{\beta}_i(0) - \beta_i(0)$, $\tilde{\rho}_i(0) = \hat{\rho}_i(0) - \rho_i$ and $\lambda_{min}(Q^TQ)$ represent the minimal eigenvalues of Q^TQ.

Proof: Constructing the following Lyapunov function candidate:

$$V(t) = V_1(t) + V_2(t), \tag{12}$$

where

$$V_1 = \frac{1}{2}\sum_{i=1}^{N} s_i^2(t) + \frac{1}{2}r\sum_{i=1}^{N}\int_{t-\tau(t)}^{t} e_i^T(\chi)e_i(\chi)\mathrm{d}s,$$
$$V_2 = \frac{1}{2}\sum_{i=1}^{N}(\hat{\alpha}_i(t)-\alpha_i(t))^2 + \frac{1}{2}\sum_{i=1}^{N}(\hat{\beta}_i(t)-\beta_i(t))^2 + \frac{1}{2}\sum_{i=1}^{N}(\hat{\rho}_i(t)-\bar{\rho}_i)^2.$$

Then, the derivative of $V_1(t)$ along the error dynamics (3) can be derived as follows:

$$\begin{aligned}
\dot{V}_1(t) &= \sum_{i=1}^{N} s_i(t)\dot{s}_i(t) + \frac{1}{2}r\sum_{i=1}^{N} e_i^T(t)e_i(t) - \frac{1}{2}r\sum_{i=1}^{N} e_i^T(t-\tau(t))e_i(t-\tau(t))(1-\dot{\tau}(t))\\
&= \sum_{i=1}^{N} s_i^T(t)\Big[C_i\dot{e}_i(t) + sign(e_i(t))|e_i(t)|^{\eta}\Big] - \frac{1}{2}r\sum_{i=1}^{N} e_i^T(t-\tau(t))e_i(t-\tau(t))(1-\dot{\tau}(t)) + \frac{1}{2}r\sum_{i=1}^{N} e_i^T(t)e_i(t)\\
&= \sum_{i=1}^{N} s_i^T(t)\Big[C_i\Big(g_i(y_i(t)) + G_i(y_i(t))\beta_i(t) + \sum_{j=1}^{N} a_{ij}\Gamma e_j(t-\tau(t)) + u_i(t) + \rho_i(t)\\
&\quad - f_i(x_i(t-\theta)) - F(x_i(t-\theta))\alpha_i(t)\Big) + sign(e_i(t))|e_i(t)|^{\eta}\Big] + \frac{1}{2}r\sum_{i=1}^{N} e_i^T(t)e_i(t)\\
&\quad - \frac{1}{2}r\sum_{i=1}^{N} e_i^T(t-\tau(t))e_i(t-\tau(t))(1-\dot{\tau}(t)).
\end{aligned}$$

Apply of the control function (7), we obtain

$$\begin{aligned}
\dot{V}_1(t) &= \sum_{i=1}^{N} s_i(t)C_i\Big(F(x_i(t-\theta))(\hat{\alpha}_i(t)-\alpha_i(t)) - G_i(y_i(t))(\hat{\beta}_i(t)-\beta_i(t)) - p_ie_i(t)\\
&\quad - \xi\Big(r\int_{t-\tau(t)}^{t} e_i^T(\chi)e_i(\chi)d\chi\Big)^{\frac{1+\eta}{2}}\frac{s_i(t)}{C_i||s_i(t)||^2} + \sum_{j=1}^{N} a_{ij}\Gamma e_j(t-\tau(t))\Big)\\
&\quad - \sum_{i=1}^{N} C_iK_i|s_i(t)| + \sum_{i=1}^{N} s_i(t)C_i\rho_i(t) - \sum_{i=1}^{N} C_i|s_i(t)|\hat{\rho}_i) + \frac{1}{2}r\sum_{i=1}^{N} e_i^T(t)e_i(t)\\
&\quad - \frac{1}{2}r\sum_{i=1}^{N} e_i^T(t-\tau(t))e_i(t-\tau(t))(1-\dot{\tau}(t)) - \gamma\Delta.
\end{aligned}$$

Using the fact

$$s_i(t)C_i\rho_i(t) \le |s_i(t)C_i\rho_i(t)| = C_i|s_i(t)||\rho_i(t)| \le C_i|s_i(t)|\rho_i$$

let $e(t) = \Big(e_1^T(t), e_2^T(t), \ldots, e_N^T(t)\Big)^T$, $e(t-\tau(t)) = \Big(e_1^T(t-\tau(t)), e_2^T(t-\tau(t)), \ldots, e_N^T(t-\tau(t))\Big)^T \in \mathbf{R}^{nN}$, and $Q = (A\otimes\Gamma)$, we obtain

$$\begin{aligned}
\dot{V}_1(t) &\le \sum_{i=1}^{N}\Big((\hat{\alpha}_i(t)-\alpha_i(t))^T F_i^T(x_i(t-\theta))\Lambda - (\hat{\beta}_i(t)-\beta_i(t))^T G_i^T(y_i(t))\Lambda\Big) - e^T(t)p^T\Lambda\\
&\quad + e^T(t-\tau(t))Q^T\Lambda - \xi(r\int_{t-\tau(t)}^{t} e^T(\chi)e(\chi)d\chi\Big)^{\frac{1+\eta}{2}}\\
&\quad - \nu\sum_{i=1}^{N}|s_i(t)| + \sum_{i=1}^{N} C_i|s_i(t)|(\rho_i-\hat{\rho}_i) + \frac{1}{2}re^T(t)e(t)\\
&\quad - \frac{1}{2}re^T(t-\tau(t))e(t-\tau(t))(1-\dot{\tau}(t)) - \gamma\Delta.
\end{aligned}$$

Using Lemma 9, we get

$$\begin{aligned}
\dot{V}_1(t) &\le \sum_{i=1}^{N}\Big((\hat{\alpha}_i(t)-\alpha_i(t))^T F_i^T(x_i(t-\theta))\Lambda - (\hat{\beta}_i(t)-\beta_i(t))^T G_i^T(y_i(t))\Lambda\Big)\\
&\quad - \xi(r\int_{t-\tau(t)}^{t} e^T(\chi)e(\chi)d\chi\Big)^{\frac{1+\eta}{2}} - \nu\sum_{i=1}^{N}|s_i(t)|\\
&\quad + \sum_{i=1}^{N} C_i|s_i(t)|(\rho_i-\hat{\rho}_i) + \frac{1}{2}e^T(t)(r-I)e(t)\\
&\quad + \frac{1}{2}e^T(t-\tau)(I-r(1-\tau))e(t-\tau(t))\\
&\quad + \frac{1}{2}\Lambda^2(\lambda_{max}(QQ^T) - pp^T) - \gamma\Delta
\end{aligned} \tag{13}$$

The derivative of $V_2(t)$ can be calculate as

$$\dot{V}_2(t) = \sum_{i=1}^{N}(\hat{\alpha}_i(t)-\alpha_i(t))^T\dot{\hat{\alpha}}_i(t) + \sum_{i=1}^{N}(\hat{\beta}_i(t)-\beta_i(t))^T\dot{\hat{\beta}}_i(t) + \sum_{i=1}^{N}(\hat{\rho}_i(t)-\bar{\rho}_i)^T\dot{\hat{\rho}}_i(t)$$

Inserting the adaptation laws, we have

$$\begin{aligned}\dot{V}_2(t) &= \sum_{i=1}^{N}(\alpha_i(t)-\hat{\alpha}_i(t))^T F_i^T(x_i(t-\tau))\Lambda \\ &\quad + \sum_{i=1}^{N}(\hat{\beta}_i(t)-\beta_i(t))^T G_i^T(y_i(t))\Lambda \\ &\quad + \sum_{i=1}^{N}(\hat{\rho}_i(t)-\bar{\rho}_i)^T C_i|s_i| \end{aligned} \tag{14}$$

Combining (13), (14) and using the conditions (1-3), we have

$$\begin{aligned}\dot{V}(t) &\le -\nu\sum_{i=1}^{N}|s_i(t)| - \gamma\Delta \\ &\quad -\xi\Big(r\int_{t-\tau(t)}^{t} e^T(\chi)e(\chi)d\chi\Big)^{\frac{1+\eta}{2}} \\ &\le -\nu\sum_{i=1}^{N}|s_i(t)| - \gamma\Big(\|\hat{\alpha}_i(t)\| + \ell + \|\hat{\beta}_i(t)\| \\ &\quad +\varphi + \|\hat{\rho}_i(t)\| + \bar{\rho}_i\Big) \\ &\quad -\xi\Big(r\int_{t-\tau(t)}^{t} e^T(\chi)e(\chi)d\chi\Big)^{\frac{1+\eta}{2}} \\ &\le -\nu\sum_{i=1}^{N}|s_i(t)| - \gamma\Big(\|\hat{\alpha}_i(t)-\alpha_i\| + \|\hat{\beta}_i(t)-\beta_i\| \\ &\quad +\|\hat{\rho}_i(t)-\rho_i\|\Big) - \xi\Big(r\int_{t-\tau(t)}^{t} e^T(\chi)e(\chi)d\chi\Big)^{\frac{1+\eta}{2}} \end{aligned}$$

By Lemma 7 and Lemma 8, we get

$$\begin{aligned}\dot{V}(t) &\le -\varepsilon\Big(\sum_{i=1}^{N}|s_i(t)| + \|\hat{\alpha}_i-\alpha_i\| + \|\hat{\beta}_i-\beta_i\| \\ &\quad +\|\hat{\rho}_i-\rho_i\| + (r\int_{t-\tau(t)}^{t} e^T(\chi)e(\chi)d\chi)^{\frac{1+\eta}{2}}\Big) \\ &\le -2^{\frac{1+\eta}{2}}\varepsilon\Big(\frac{1}{2}\sum_{i=1}^{N}s_i^2(t) + \frac{1}{2}\|\hat{\alpha}_i-\alpha_i\|^2 \\ &\quad +\frac{1}{2}\|\hat{\beta}_i-\beta_i\|^2 + \frac{1}{2}\|\hat{\rho}_i-\rho_i\|^2 \\ &\quad +\frac{1}{2}(r\int_{t-\tau(t)}^{t} e^T(\chi)e(\chi)d\chi\Big)^{\frac{1+\eta}{2}} \\ &\le -2^{(1+\eta)/2}\varepsilon V^{(1+\eta)/2}(t)\end{aligned}$$

According to Lemma 6, the states of error $e_i(t)$ converge to the sliding surface $s_i(t)=0$ in finite-time T_2. This completes the proof.

Remark 13: According to the Theorem 10 and 12, the sliding mode control (7) with adaptive laws (8)- (10) can lead the drive networks and the response networks to lag synchronization within finite-time $T \le T_1 + T_2$.

Remark 14: According to the previous discussion, the convergence times T_1, T_2 and the controller u_i are dependent on the control gains $C_i, K_i, P_i, \xi, \gamma$. On the one hand, T_1 is proportional to the value of C_i, which means a smaller C_i leads to shorter convergence times T_1. On the other hand, the sliding mode reaching times T_2 is inversely proportional to $\varepsilon = \min\{\nu, \gamma, \xi\} = \min\{C_1K_1, C_2K_2, \ldots, C_NK_N, \gamma, \xi\}$. At the same time, the controller u_i is proportional to $\frac{1}{C_i}, K_i, p_i, \xi, \gamma$. Based on these relationships, the appropriate control gains should be selected according to the specific designer requirements.

Remark 15: We investigate the robust lag synchronization problem of uncertain complex networks in finite-time with time-varying delay, fully unknown parameters, and external disturbances. The robustness of the proposed control is mainly reflected in the resistance to time-varying delay, unknown parameters and external parametric disturbances.

Remark 16: In this paper, we explore lag synchronization with time-varying delay coupling, uncertain parameters and disturbances by utilizing adaptive control along with sliding mode control in finite-time whilst the results in [27] and [29]–[31] discussed lag synchronization problems in long time for drive-response dynamical network with delay coupling, uncertain parameters and disturbances by adaptive control.

Corollary 17: When $\omega_i(t) = \mu_i(t) = 0$, then lag synchronization can be realized in finite-time T_2 under the following sliding mode controller:

$$\begin{aligned}u_i(t) &= -\frac{1}{C_i}sign(e_i(t))|e_i(t)|^{\eta} - K_i sign(s_i(t)) \\ &\quad -\gamma\Big(\|\hat{\alpha}_i\| + \ell + \|\hat{\beta}_i\| + \varphi\Big)\frac{sign(s_i(t))}{C_i|s_i(t)|} \\ &\quad -\xi\Big(r\int_{t-\tau(t)}^{t} e_i^T(\chi)e_i(\chi)d\chi\Big)^{\frac{1+\eta}{2}}\frac{s_i(t)}{C_i||s_i(t)||^2} \\ &\quad -p_ie_i(t) + f_i(x_i(t-\theta)) + F_i(x_i(t-\theta))\hat{\alpha}_i(t) \\ &\quad -g_i(y_i(t)) - G_i(y_i(t))\hat{\beta}_i(t), \end{aligned} \tag{15}$$

where T_2 given by

$$T_2 \le 2^{(1+\eta)/2}\varepsilon V^{(1+\eta)/2}(0) \tag{16}$$

Corollary 18: Suppose that $\tau(t) = 0$, then lag synchronization can be achieve in finite-time under Assumptions 3,4 and 5, if the controller designed as

$$\begin{aligned}u_i(t) &= -\frac{1}{C_i}sign(e_i(t))|e_i(t)|^{\eta} - (K_i + \hat{\rho}_i)sign(s_i(t)) \\ &\quad -\gamma\Delta\frac{sign(s_i(t))}{C_i|s_i(t)|} + f_i(x_i(t-\theta)) - g_i(y_i(t)) \\ &\quad +F_i(x_i(t-\theta))\hat{\alpha}_i(t) - G_i(y_i(t))\hat{\beta}_i(t) \\ &\quad -\sum_{j=1}^{N}a_{ij}(t)\Gamma e_j(t), \end{aligned} \tag{17}$$

where $V(0) = \frac{1}{2}\sum_{i=1}^{N}s_i^2(0) + \frac{1}{2}\sum_{i=1}^{N}(\hat{\alpha}_i(0)-\alpha_i)^2 + \frac{1}{2}\sum_{i=1}^{N}(\hat{\beta}_i(0)-\beta_i))^2 + \frac{1}{2}\sum_{i=1}^{N}(\hat{\rho}_i(0)-\bar{\rho}_i)^2$.

Remark 19: When the model does not contain unknown parameters and external disturbances, then finite-time lag synchronization of networks was discussed in [60] and [61] by utilizing adaptive error-feedback control and intermittent sliding mode control respectively.

Remark 20: When the network model has no external disturbances and the propagation delay coupling $\tau(t) = 0$, then finite-time synchronization in drive-response complex dynamical network was discussed in [57] by adaptive control.

Remark 21: In the existing literature, there are some results concerning finite-time synchronization [55], [56], [58]–[61]. However, to the best of our knowledge, there is no result dealing with finite-time lag synchronization of networks with time-varying delay coupling, unknown parameters, and external disturbances. In this paper, we show another kind of effective control strategy for finite-time synchronization of complex dynamic network. Obviously, our proposed control can be extended to investigate finite-time synchronization of general uncertain complex dynamical networks, chaotic and hyperchaotic systems.

Remark 22: The control input in u_i contains the factor $\frac{sign(s_i(t))}{|s_i(t)|}$, which may cause undesirable chattering. The effective way in the application of sliding mode control to overcome this problem is to replace the factor by

$$\frac{sign(s_i(t))}{|s_i(t)| + \delta},$$

where δ is a small positive constant.

IV. NUMERICAL ANALYSIS

In this section, two numerical examples are given to demonstrate the efficiency of the proposed control method obtained in the previous section. In each examples, we consider the networks one with external disturbances and one without external disturbances. In principle, the proposed control can be used to investigate finite-time synchronization for general complex networks. That means our proposed control strategy has no restrictions on the number of the network node. We choose 5 nodes for ease of illustration and computation.

Example 23 (Synchronization With Delay Coupling): In this example, the Chen chaotic system is considered as the node of the drive dynamics, which is given by

$$\dot{x} = \begin{pmatrix} 0 \\ -x_1x_3 \\ x_1x_2 \end{pmatrix} + \begin{pmatrix} (x_2 - x_1) & 0 & 0 \\ -x_1 & x_1 + x_2 & 0 \\ 0 & 0 & -x_3 \end{pmatrix} \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix}$$

Take the Lü chaotic system as the node of the response dynamics which is described by the following:

$$\dot{y} = \begin{pmatrix} 0 \\ -y_1y_3 \\ y_1y_2 \end{pmatrix} + \begin{pmatrix} (y_2 - y_1) & 0 & 0 \\ 0 & y_2 & 0 \\ 0 & 0 & -y_3 \end{pmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix}$$

where the unknown parameter vector $\alpha_i = [\alpha_1\ \alpha_2\ \alpha_3]^T = [35\ 28\ 3]^T$, $\beta_i = [\beta_1\ \beta_2\ \beta_3]^T = [36\ 20\ 3]^T$. We take the propagation delay as $\tau(t) = 1 + 0.5sin(t), \theta = 1$.

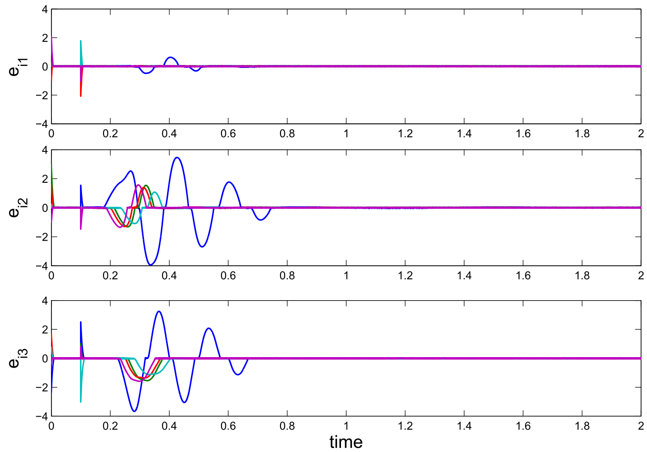

FIGURE 1. Time evolution of the lag synchronization error with time-varying coupling delay and external disturbances.

The inner coupling matrix Γ as the identity matrix and the outer coupling matrix is given by

$$A = \begin{pmatrix} -3 & 1 & 1 & 0 & 1 \\ 1 & -4 & 1 & 1 & 1 \\ 1 & 0 & -2 & 1 & 0 \\ 0 & 1 & 1 & -4 & 1 \\ 1 & 0 & 1 & 1 & -3 \end{pmatrix}$$

Case 1: When the models have external disturbances, which are chosen as $\omega_i(t) = [2\sin(t)\cos(t)\ \sin(2t)\ 2\sin(t)]$, $\mu_i(t) = [\cos(2t)\ \sin(t)\ \cos(t)]$. The error initial conditions are $e_1(0) = (-2, -1.86, -1.04)^T, e_2(0) = (-1.50, -0.30, 0.55)^T$, $e_3(0) = (-1.22, -1.84, 1.82)^T, e_4(0) = (0.36, -0.66, -3.16)^T, e_5(0) = (-0.22, -0.50, -2.034)^T$. For the sake of simplicity of computation, the control gains are chosen as $C_i = 2, K_i = 80, p_i = 180, \eta = 0.1, \gamma = 0.5, \xi = 1$. The bound vectors are chosen as $\bar{\alpha} = \bar{\beta} = 75, \rho = 18$. By simple calculating (6), (11), we have $T \leq T_1 + T_2 = 105.29$ which meets the estimated upper bound we proposed.

Using the controller (7) and adaptive update laws with $\delta = 0.1$, the lag synchronization error is revealed in Fig. 1. It is showed that the lag synchronization error converge to zero before T. The time evolution of unknown parameter of $\hat{\alpha}$ and $\hat{\beta}$ are presented in Fig. 2 and Fig. 3, respectively. It observed that the the identified parameters $\tilde{\alpha}$ and $\tilde{\beta}$ converge to their real values, which means that the unknown parameters are successfully estimated. These results illustrate the effectiveness of the designed control (7) with adaptive law (8)-(10) for uncertain complex dynamical networks with time varying coupling delayed and external disturbances.

Case 2: When the networks do not have external disturbances ($\omega_i(t) = \mu_i(t) = 0$), the error initial conditions are $e_1(0) = (-1.91, 1.28, -0.25)^T, e_2(0) = (-0.67, -1.97, -0.52)^T, e_3(0) = (-0.74, 2.71, 0.61)^T$, $e_4(0) = (-2.17, 2.11, -0.67)^T$, $e_5(0) = (1.06, 2.48, -1.17)^T$. The initial of unknown parameters $\alpha_i(0) = \beta_i(0) = 1$.

According to Corollary 17, the synchronization error converge to the origin in finite-time before $T = 104.6$, which

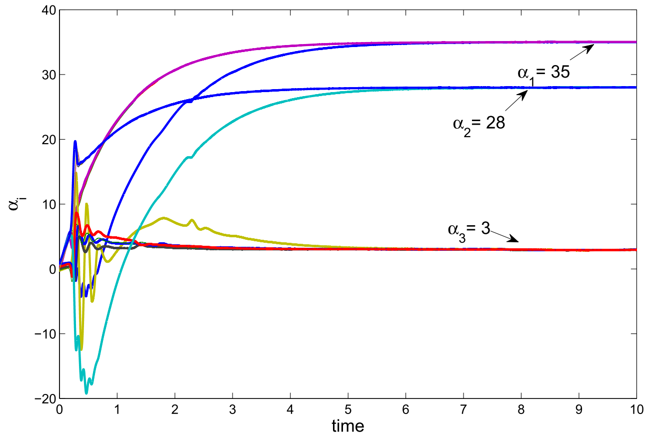

FIGURE 2. The estimated unknown parameter of $\hat{\alpha}$ when the networks have time-varying delay coupling and disturbances.

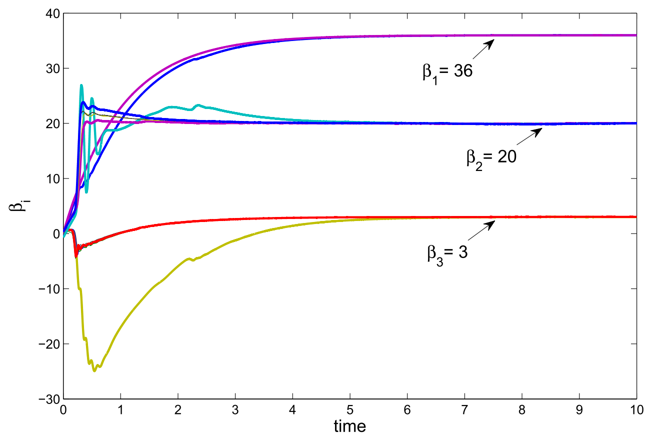

FIGURE 3. The estimated unknown parameter of $\hat{\beta}$ when the networks have time-varying delay coupling and disturbances.

is depicted in Fig. 4. That means the lag synchronization is realized. The estimated parameters of networks nodes are successfully estimated which shown in Fig. 5 and Fig. 6 respectively. These results prove the efficiency of our designed control (15) with adaptive law (8)-(10) for uncertain complex dynamical networks with time-varying delay coupling.

Example 24 (Synchronization With Non-Delay Coupling): Take the Chen chaotic system as the node drive network. The following Lorenz chaotic system selected as the node of response network:

$$\dot{y} = \begin{pmatrix} 0 \\ -y_1y_3 - y_2 \\ y_1y_2 \end{pmatrix} + \begin{pmatrix} (y_2 - y_1) & 0 & 0 \\ 0 & y_1 & 0 \\ 0 & 0 & -y_3 \end{pmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix}$$

where the unknown parameter vector $\beta_i = [\beta_1\ \beta_2\ \beta_3]^T = [10\ 28\ 8/3]^T$. The propagation delay and outer coupling matrix are chosen as in the previous example. For the sake of simplicity of computation, the control gains are chosen as $C_i = 1, K_i = 100, p_i = 100, \eta = 0.1, \gamma = 1$. The bound vectors are chosen as $\bar{\alpha} = \bar{\beta} = 75$, $\rho = 15$.

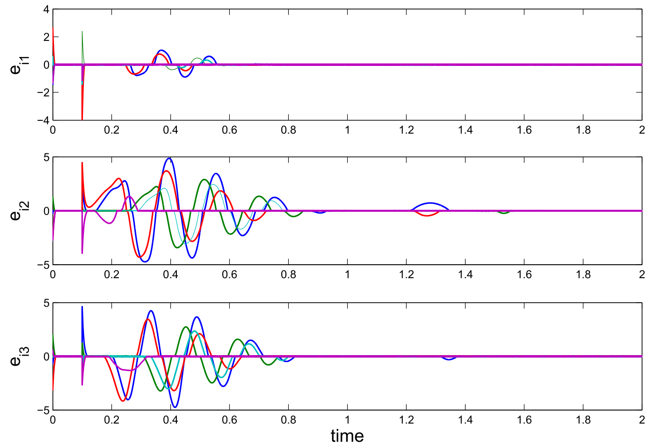

FIGURE 4. Time evolution of the lag synchronization error with time-varying delay coupling.

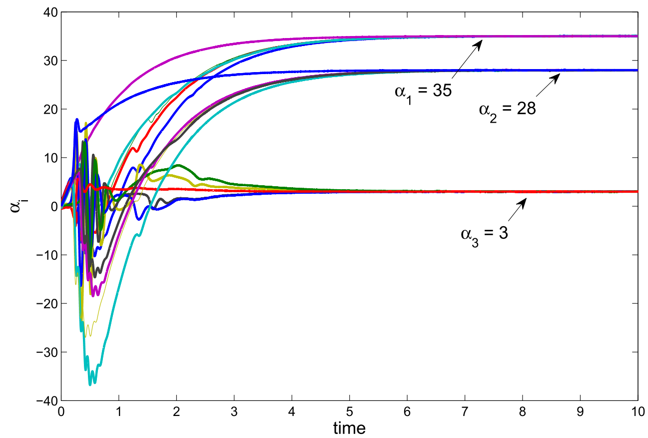

FIGURE 5. The estimated unknown parameter of $\hat{\alpha}$ when the networks have time-varying delay coupling.

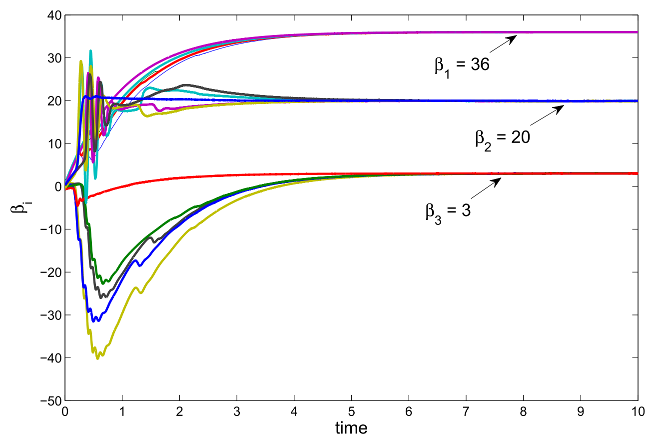

FIGURE 6. The estimated unknown parameter of $\hat{\beta}$ when the networks have time-varying delay coupling.

Case 1: When the models have external disturbances which choose as in the previous example, the error initial conditions are $e_1(0) = (-0.12, 0.78, = -0.50)^T, e_2(0) = (1.70, -0.16, -0.45)^T$, $e_3(0) = (-0.02, -2.42, -2)^T$, $e_4(0) = (-1.88, 0.74, 0.37)^T, e_5(0) = (1.27, -0.1, 0.89)^T$.

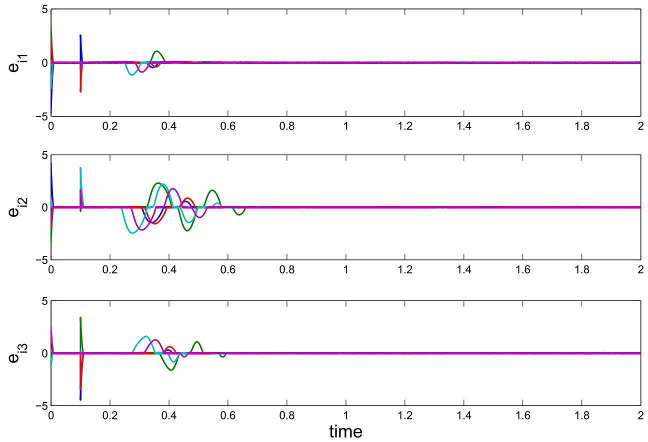

FIGURE 7. Time evolution of the lag synchronization error with non-delay coupling and external disturbances.

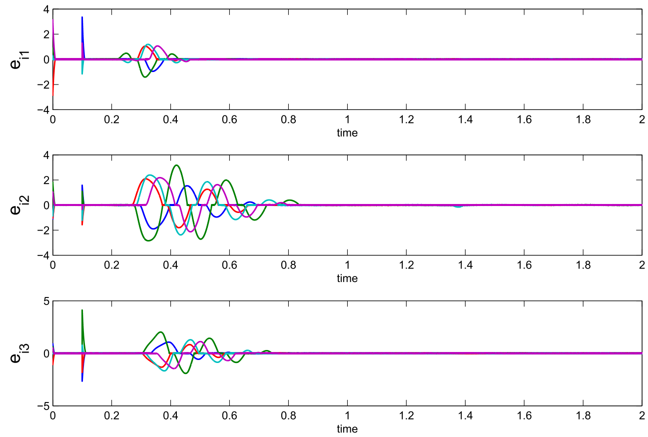

FIGURE 10. Time evolution of the lag synchronization error with non-delay coupling.

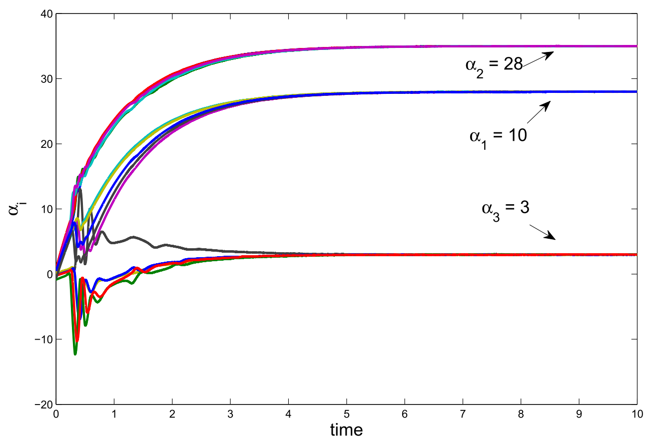

FIGURE 8. The estimated unknown parameter of $\hat{\alpha}$ when the networks have non-delay coupling and external disturbances.

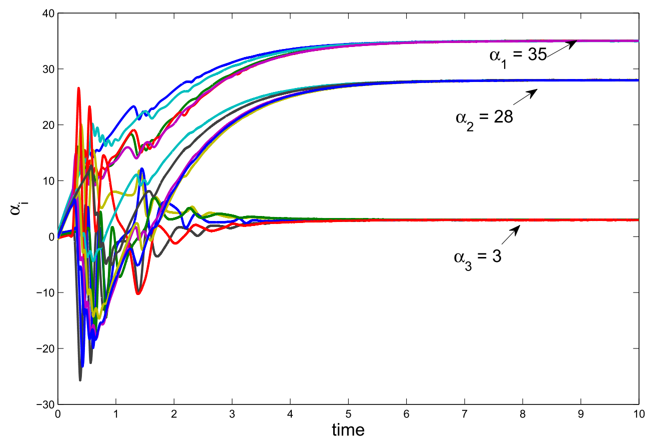

FIGURE 11. The estimated unknown parameter of $\hat{\alpha}$ when the networks have non-delay coupling.

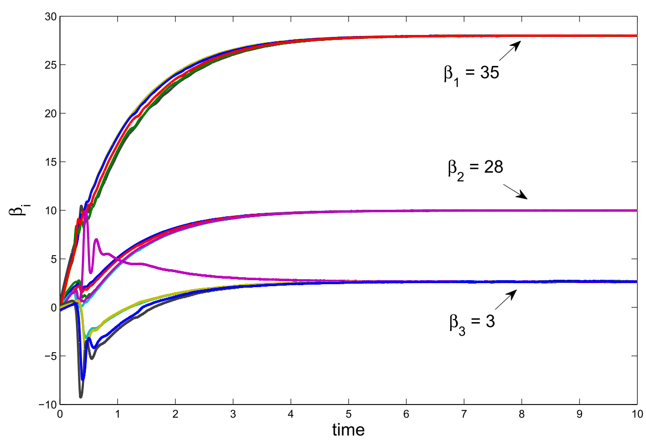

FIGURE 9. The estimated unknown parameter of $\hat{\beta}$ when the networks have non-delay coupling and external disturbances.

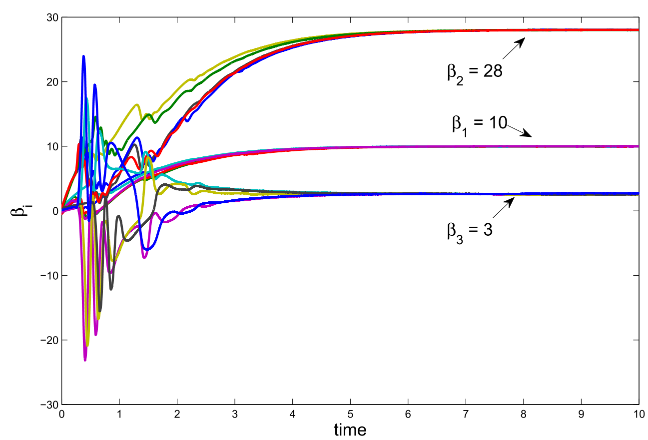

FIGURE 12. The estimated unknown parameter of $\hat{\beta}$ when the networks have non-delay coupling.

The estimation initial values of unknown parameter are chosen randomly in $[-1, 1]$.

In this numerical results, the time evolution of the synchronization errors are depicted in Fig. 7. This figure displays the error states converge to zero before finite-time $T \leq T_1+T_2 = 87.996$, which meets the estimated upper bound we proposed. The estimation values of unknown parameter of $\hat{\alpha}$ and $\hat{\beta}$ are presented in Fig. 8 and Fig. 9, respectively which converge to their real values. These results show that the required synchronization has been achieved with our proposed control.

Case 2: When the external disturbances ($\omega_i(t) = \mu_i(t) = 0$), the error initial conditions are $e_1(0) = (0.75, -0.55, -2.69)^T$, $e_2(0) = (-0.84, -0.80, -1.61)^T$, $e_3(0) = (1.90, 1.15, 2.26)^T$, $e_4(0) = (-2.38, -0.54, 1.38)^T$, $e_5(0) = (-1.26, 0.25, 1.80)^T$. The initial of unknown parameters are chosen randomly in $[-1, 1]$.

The synchronization error is illustrated in Figure 10, showing that the lag synchronization between the drive and response networks is achieved in finite-time before $T \leq 88.44$. Fig. 11 and Fig. 12 show the estimated parameters of nodes are successfully estimated. These results prove the efficiency of our designed control for uncertain complex dynamical networks with non-delay coupling.

V. CONCLUSION

In this paper, we have explored the problem of lag synchronization in complex dynamical networks with time varying delayed coupling, fully unknown parameters and external disturbances within finite-time. Based on the Lyapunov stability theory, sliding mode control and finite-time control, finite-time adaptive sliding mode control and appropriate adaptive laws were designed to realize lag synchronization. Furthermore, the unknown parameters were identified and many criteria were obtained. In addition, the unknown bounded disturbances were overcame by the proposed controllers. Finally, numerical simulation results have been showed the efficiency of the proposed method.

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