IEEE Access
Multidisciplinary : Rapid Review : Open Access Journal

Received December 3, 2018, accepted January 6, 2019, date of publication January 11, 2019, date of current version February 4, 2019.

Digital Object Identifier 10.1109/ACCESS.2019.2892060

A Black-Box External Equivalent Method Using Tie-Line Power Mutation

WEI YAN[1], (Member, IEEE), CHAO ZHANG[1], (Student Member, IEEE), JUNJIE TANG[1], (Member, IEEE), WEI QIAN[1], SHIMING LI[2], AND QIAN WANG[3]

[1]State Key Laboratory of Power Transmission Equipment and System Security and New Technology, Chongqing University, Chongqing 400044, China
[2]Power Dispatching Control Center, Guangdong Grid Company, Guangzhou 510600, China
[3]State Grid Deyang Power Supply Company, Deyang 618000, China

Corresponding author: Chao Zhang (947804250@qq.com)

This work was supported in part by the National Natural Science Foundation of China under Project 51677012.

ABSTRACT A new black-box equivalent method by using tie-line power mutation is proposed in this paper, which is especially suitable for a two-port or three-port interconnected grid that does not know any information of the external network. The proposed method, based on the simplified Ward equivalent model of the external network, aims to addresses the following issues: unstable parameter estimation of the existing black-box-based static equivalence method and low accuracy or difficult implementation. Correspondingly, the main features of the proposed method, which makes it as a solution for these existing issues above, are summarized as follows: 1) it is easy to implement, as it only involves the associated measurements of boundary nodes in two adjacent time instants that are before and after a designed large disturbance of the internal grid; 2) it is robust to measurement errors, thus guaranteeing more accurate equivalent modeling; and 3) it can be applied to both the two-port and three-port equivalent networks. The simulation results of the New England 39-node test case and the actual electric grid case in Guangdong of China validate the effectiveness and performance of the proposed method.

INDEX TERMS Static equivalent method, black-box external network, parameter estimation, tie line, power mutation.

I. INTRODUCTION

Due to the technical issues or the commercial confidentiality requirements of the electricity market, there is no real-time data sharing among subsystems in the interconnected power grid. For this reason, subsystem dispatch centers often make an equivalence on the adjacent external power grid in terms of PQ or PV of boundary nodes, [1], [2], and then they perform independent state estimation (SE) based on the data of subsystems. When there is a meshed network connection between the adjacent external power grid and the internal power grid, PQ/PV equivalence of the boundary nodes often brings large errors into the online safety analysis, and directly it leads to a wrong decision-making for power grid operation, and even leading to a great potential security hazard. Therefore, to ensure the effectiveness of on-line security analysis of interconnected power grids, it is necessary to study black-box equivalence which does not depend on synchronous data sharing of external network and complete power flows of the whole system.

The existing static equivalent methods of external grid are divided into three categories. First, if the complete structure and power flow data of external grid is available, the equivalent parameters can be calculated and determined. This type of equivalent method is called as topological method. The main methods include Ward equivalent [3], [4], extended Ward equivalent [5], [6], sensitivity equivalent [7]–[9] and so on. Amongst them, the general Ward equivalent circuit model consists of the admittance between boundary nodes, grounding impedance and power injection which are connected to boundary nodes. The simplified Ward equivalent method ignores the Ward equivalence of grounding branches. Furthermore, an extended Ward equivalent method is formed by adding voltage-source equivalent branches to boundary nodes on the basis of simplified Ward model.

In the second category, the equivalent model and parameters of typical operation mode of external grid are known. The equivalent parameters of current operating mode are estimated through the multi-period data of boundary nodes and internal network. This is named as the gray-box equivalent method. In [10], a multi-port gray-box equivalent method is proposed. According to this method, the external power grid adopts a simplified Ward equivalent model, and the measurements in terms of power and voltage are collected in two adjacent time instants that before and after an operation of branch outage in the internal network. The method [10] requires that the current equivalent parameters are as close as the equivalent parameters for the typical operation mode. Another gray-box equivalent method is proposed in [11] the external network of which adopts the extended Ward equivalent model with the grounding impedance branches that are connected to boundary nodes, and the measured data are voltage and current vector for multiple snapshots. Moreover, the [11] method desires that the equivalent parameters of the maximum and minimum operation mode are provided, and hence the equivalent parameters of the current mode should fall between the two extreme conditions.

The third category does not rely on any information from the external grid. The parameters of current equivalent model can be determined by measurements from the multi-period data of the internal grid. This is defined as the black-box equivalent method. Reference [12] proposes a multi-port black-box equivalent method, in which, the external network adopts Ward equivalent model, and the measured data are power and voltage of multiple sampling snapshots before and after opening/close operation of internal network branches for several times. At least two operations are conducted for two boundary nodes, while the more boundary nodes entails that the more times branch opening/closing operation are required. In order to satisfy the equivalent condition, the equivalent parameters of external network are assumed to remain constant during the operation of internal network branch for multiple opening/closing. In [13], a two-stage, two-port black-box equivalent method is proposed: in the first stage, the external grid adopts the simplified Ward equivalent model; in the second stage, an improved Extended-Ward equivalent model is adopted without the power injection of boundary nodes. The measurements used for the method above includes power and voltage of the multiple measurement snapshots. The method proposed in [14] is similar to the one in [13], and the main difference between them is the application of diverse models in the second stage. In the equivalent model of [13], each boundary node corresponds to an equivalent power node, and the equivalent power nodes are merged into one. Hence, the modified Ward-PV equivalent model of [14] is formed in a further step.

In short, the aforementioned static equivalent methods of external network suffer from the issues individually as below:

1) The topology method needs to share the complete power flow of the external network, which is difficult to be implemented in the electricity market environment.
2) The existing gray-box equivalent method requires equivalent parameters from the typical mode and the current mode to be related, either identical or very close. In fact, the relationship between them is uncertain, and it is even possible to be completely unsatisfied with the requirements of the gray-box equivalent constraints, which may lead to large equivalent errors.
3) The black-box equivalent method of [12] requires at least two rounds of branch opening or closing operations, and the equivalent parameters are required to remain unchanged during multiple operations. However, the actual system's branch opening or closing operations cannot be conducted so frequently. If the adjacent multiple operations are separated for a long time, it is difficult to satisfy the assumption that the equivalent parameters are unchanged. The black-box equivalence methods in [13] and [14] are based on measurement information of consecutive periods, thus the marginal difference between each period of measurement data can be confused with measurement errors. So, it is difficult to ensure its accuracy since the result can also be significantly affected by random measurement errors (RME).

In order to overcome the problems mentioned above, a black-box external equivalent method using tie-line power mutation is proposed in this paper. In general, the external network adopts a simplified Ward model, and the least-squares model of equivalent parameters is a quadratic function. In more details, the features of the equivalent method proposed herein are introduced as follows:

- Basically, the proposed method only requires the measurement data of two measuring snapshots. The relevant data can be obtained by only a disturbance of the internal network, and hence the method is easier for implementation.
- In particular, the proposed method utilizes measurements between different measuring snapshots which are allowable to change greatly, thus effectively reducing the adverse impact of RME on the accuracy of estimated equivalent parameters.
- Furthermore, the proposed method includes two specific models of static equivalent of black box external network, which are applicable to two-port and three-port equivalent networks, respectively.

The remaining of this paper is organized as: method principles and port applications are introduced in Section II. The modified New England system and the actual system of Guangdong power grid in China are selected and designed in Section III to demonstrate the effectiveness of this method. Conclusions are drawn in Section IV.

II. BLACK-BOX EQUIVALENT METHOD USING TIE-LINE POWER MUTATION

A. SIMPLIFIED WARD MODEL

As shown in Fig. 1, the original network consists of external network, internal network, and tie lines. The simplified Ward model adopted in this paper is shown in Fig. 2, where the variable subscript i or j denotes the numbering of the boundary node. Three boundary nodes are considered in the figure, with numbers 1, 2, and 3, respectively. Subscript t is the sampling time of measurement snapshot. $\tilde{S}^{S}_{i,t}$ is defined as the power supply of the boundary node. $\tilde{S}^{L}_{i,t}$ and $\tilde{S}^{eq}_{i,t}$, separately,

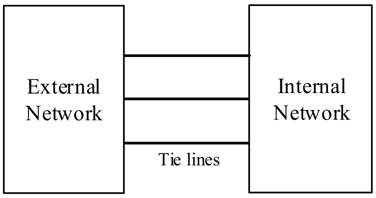

FIGURE 1. Original network before equivalence.

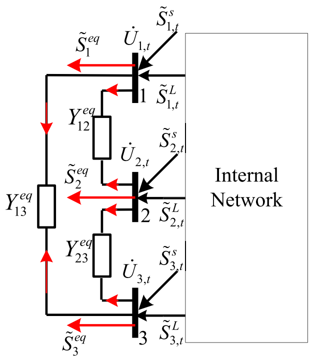

FIGURE 2. Simplified Ward equivalent network.

represent the tie-line power and equivalent load power of external network which are connected to the node i. $\dot{U}_{i,t}$ is the voltage phasor of the node i. The equivalent admittance of branch l_{i-j} is denoted by Y_{ij}^{eq}.

B. INCREMENTAL EQUATIONS OF TIE-LINE POWER

An estimated equation of the equivalent admittance parameters is established by means of corresponding measurements, which are obtained from the SE results of boundary nodes of two measurement snapshots before and after the designed large perturbation. At the moments t_1 and t_2 before and after the perturbation, the power balance equation of the boundary node in Fig. 2 can be expressed as follows:

$$\sum_{j\in i} \tilde{S}_{ij,t}^{eq} + \tilde{S}_{i,t}^{eq} = \tilde{S}_{i,t}^{s} + \tilde{S}_{i,t}^{L} t \in \{t_1, t_2\} \tag{1}$$

where $j \in i$ represents other boundary nodes connected to the boundary node i, and $\tilde{S}_{ij}^{eq}$ represents the equivalent power on the i side of l_{i-j}.

With an assumption on that the time interval between t_1 and t_2 is very short, e.g., 1–5 minutes, the network parameters as well as operating conditions of the external system between t_1 and t_2 can remain almost unchanged. The corresponding $\tilde{S}_{ij}^{eq}$, $\tilde{S}_{i,t}^{S}$ and $\tilde{S}_{i,t}^{eq}$ are approximately the same. Based on the above assumptions, the incremental equation of tie-line power can be obtained by eliminating $\tilde{S}_{i,t}^{eq}$ and $\tilde{S}_{i,t}^{S}$ which are located in (1) at t_1 and t_2.

$$\sum_{j\in i} \left(\tilde{S}_{ij,t_1}^{eq} - \tilde{S}_{ij,t_2}^{eq}\right) = \tilde{S}_{i,t_1}^{L} - \tilde{S}_{i,t_2}^{L} = \Delta\tilde{S}_{i}^{L} i = 1, 2, 3 \tag{2}$$

where $\Delta\tilde{S}_{i}^{L}$ represents the tie-line power increment between t_1 and t_2 boundary at node i.

The equation (2) expresses that the power increment of tie line connected to node i is identical to that of equivalent admittance branch located in the external network. In addition, the number of boundary nodes is the same as the number of complex power increment equations.

C. ESTIMATION EQUATIONS OF EQUIVALENT PARAMETERS FOR TWO-PORT NETWORK

There is an equivalent admittance branch that is distributed between two boundary nodes in the two-port network. Therefore, (2) can be expanded as follows:

$$\begin{cases} \Delta P_1^L = \left(\Delta U^2(1) - \Delta UU\cos(1,2)\right) g_{12}^{eq} \\ \qquad -\Delta UU \sin(1,2)\, b_{12}^{eq} \\ \Delta Q_1^L = -\Delta UU\sin(1,2)\, g_{12}^{eq} \\ \quad +\left(\Delta UU\cos(1,2) - \Delta U^2(1)\right) b_{12}^{eq} \\ \Delta P_2^L = \left(\Delta U^2(2) - \Delta UU\cos(2,1)\right) g_{12}^{eq} \\ \quad -\Delta UU\sin(2,1)\, b_{12}^{eq} \\ \Delta Q_2^L = -\Delta UU\sin(2,1)\, g_{12}^{eq} \\ \quad +\left(\Delta UU\cos(2,1) - \Delta U^2(2)\right) b_{12}^{eq} \end{cases} \tag{3}$$

where

$$\begin{aligned} \Delta UU\sin(i,j) &= U_{i,t_1}U_{j,t_1}\sin\theta_{ij,t_1} - U_{i,t_2}U_{j,t_2}\sin\theta_{ij,t_2} \\ \Delta UU\cos(i,j) &= U_{i,t_1}U_{j,t_1}\cos\theta_{ij,t_1} - U_{i,t_2}U_{j,t_2}\cos\theta_{ij,t_2} \\ \Delta U^2(i) &= U_{i,t_1}^2 - U_{i,t_2}^2 \end{aligned}$$

U and θ are the voltage amplitude and phase angle of boundary node, respectively. g_{12}^{eq} and b_{12}^{eq} represent the equivalent conductance and susceptance, respectively.

As shown in equation (3), there are four active and reactive incremental equations of tie lines associated with the two boundary nodes. The state variables include the voltage amplitude and phase angle of boundary nodes and the active and reactive power of tie lines. Parameter variables include equivalent conductance and susceptance. In the actual power grid, the state variables can be determined by the voltage vector of boundary nodes in the PMU measurements and the current vector of the associated tie lines, or through the real-time SE of the internal network [16], [17]. When the state variables are constant, equation (3) comprises linear equations of equivalent conductance and susceptance. Therefore, the two equivalent parameters can be solved by four linear equations.

D. ESTIMATION EQUATIONS OF EQUIVALENT PARAMETERS FOR THREE-PORT NETWORK

As another common and practical situation, there are three boundary nodes and three equivalent branches in the three-port network. Six equivalent parameters are distributed on three equivalent branches.

Following the idea of two-port network, (2) can be expanded into six equations, which are similar to (3). From the perspective of solving equations, mathematically, these six equations can uniquely determine six unknown variables. Unfortunately, errors occur either in the PMU measurements, SCADA measurements or equivalent model of external network; thus, (2) may not strictly be satisfied. If six parameters are solved directly based on the abovementioned six equations, the error is likely to be directly transferred to the calculated values of parameters, which results in the accuracy loss of the equivalent parameters.

In order to restrict the error transfer, the following treatments are considered. Since resistance of transmission network is far less than reactance, its equivalent conductance should be far less than equivalent susceptance. To improve the estimation accuracy of equivalent parameters, for the three-port network, the equivalent conductance is neglected and only the equivalent susceptance is considered [15]. Thus, in the three-port network condition, (2) can be revised as follows:

$$\begin{cases} \Delta P_1^L = -\Delta UU\sin(1,3)\, b_{13}^{eq} - \Delta UU\sin(1,2)\, b_{12}^{eq} \\ \quad \Delta Q_1^L = \left(\Delta UU\cos(1,3) - \Delta U^2(1)\right) b_{13}^{eq} \\ \qquad + \left(\Delta UU\cos(1,2) - \Delta U^2(1)\right) b_{12}^{eq} \\ \Delta P_2^L = -\Delta UU\sin(2,1)\, b_{12}^{eq} - \Delta UU\sin(2,3)\, b_{23}^{eq} \\ \quad \Delta Q_2^L = \left(\Delta UU\cos(2,1) - \Delta U^2(2)\right) b_{12}^{eq} \\ \qquad + \left(\Delta UU\cos(2,3) - \Delta U^2(2)\right) b_{23}^{eq} \\ \Delta P_3^L = -\Delta UU\sin(3,2)\, b_{23}^{eq} - \Delta UU\sin(3,1)\, b_{13}^{eq} \\ \quad \Delta Q_3^L = \left(\Delta UU\cos(3,2) - \Delta U^2(3)\right) b_{23}^{eq} \\ \qquad + \left(\Delta UU\cos(3,1) - \Delta U^2(3)\right) b_{13}^{eq} \end{cases} \tag{4}$$

The meanings of abbreviations $\Delta UU\sin(i,j)$, $\Delta UU\cos(i,j)$, and $\Delta U^2(i)$ have been explained in (3).

Obviously, when two measurement snapshots of boundary nodes voltage and tie-line power are given, (4) are also linear redundant equations for the equivalent susceptance.

E. THE LEAST SQUARES ESTIMATION OF EQUIVALENT PARAMETERS

In this paper, the least squares estimation is used to solve redundant linear equations involving equivalent parameters. (3) and (4) are written in matrix form as shown in (5).

$$\boldsymbol{Ay} = \boldsymbol{B} \tag{5}$$

where $\boldsymbol{A}$represents the coefficient matrix of estimation equations, which is determined by (3) and (4). $\boldsymbol{y}$ represents column vector of equivalent admittance. B represents column vector of power increment of tie lines. For (3), $y = \left[g_{12}^{eq}, b_{12}^{eq}\right]^T$, $B = \left[\Delta P_1^L, \Delta Q_1^L, \Delta P_2^L, \Delta Q_2^L\right]^T$. For (4), $y = \left[b_{12}^{eq}, b_{13}^{eq}, b_{23}^{eq}\right]^T$, $B = \left[\Delta P_1^L, \Delta Q_1^L, \Delta P_2^L, \Delta Q_2^L, \Delta P_3^L, \Delta Q_3^L\right]^T$.

Constructing least-squares objective function as:

$$\boldsymbol{J} = (\boldsymbol{Ay} - \boldsymbol{B})^{\boldsymbol{T}} (\boldsymbol{Ay} - \boldsymbol{B}) \tag{6}$$

Evidently, (6) is a quadratic function, and the extreme condition equation is linear. So, $\boldsymbol{y}$ can be written as follows:

$$\boldsymbol{y} = \left(\boldsymbol{A}^{\boldsymbol{T}}\boldsymbol{A}\right)^{-1} \boldsymbol{A}^{\boldsymbol{T}}\boldsymbol{B} \tag{7}$$

By using (7), the estimated value of equivalent admittance parameters can be directly solved without any iteration, and there is no influence from initial gauss on the problem of nonlinear equations. In addition, once the equivalent admittance

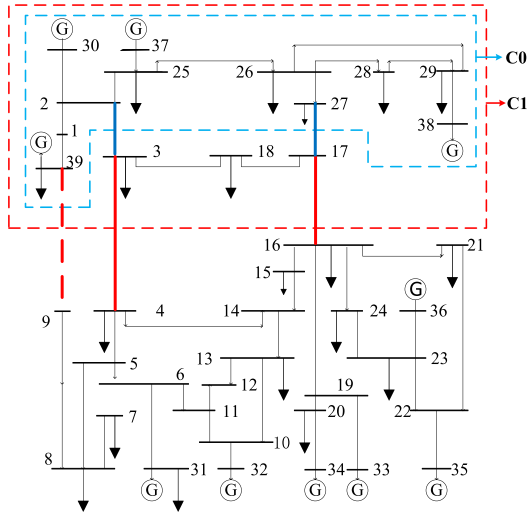

FIGURE 3. New England 39-node test system with different divisions.

parameters of the external network are determined, the equivalent load power can be further calculated based on power balance equations of boundary nodes [18]. Thus, the status and parameters of the external network are estimated.

III. CASE STUDY

A. BASIC DATA AND TEST CONDITION

1) STANDARD EXAMPLE

Test cases are built on the modified New England 39-node system [19], as shown in Fig. 3. The external systems are represented by the dotted box, with diverse colors representing two different cases.

- Different cases

C0 (by blue): Two-port interconnection system is formed after the original system disconnects l_{9-39}. l_{3-2} and l_{17-27} are, respectively, used as the internal network tie lines, while node 3 and node 17 are treated as the boundary nodes.

C1 (by red): Three-port interconnection system is built, in which l_{4-3}, l_{9-39}, and l_{16-17} are used as the internal network tie lines; nodes 4, 9, and 16 are the boundary nodes.

In order to maintain the state of the external power flow before and after the disturbance as much as possible, the generator nodes 30, 37 and 38 of the external network are treated as PQ buses.

- Simulated measuring snapshots

Step 1: Superpose Gaussian white noise on the solution of the improving system, and select the internal network data as the measurements.

Step 2: Execute SE of the internal network based on measurements, after the external grid matches well the PQ power sources of boundary nodes.

The standard deviation of measurement errors of voltage amplitude, power injection, and branch power flow are set to 0.004, 0.01 and 0.008, respectively [20].

- Different Methods

TABLE 1. Estimated results of equivalent parameters for the two-port network and the power increment of tie tines and external network equivalent load.

Method	RME	Calculation conditions	Equivalent parameters (p.u.)			Tie-line power increment (MVA)		Increments of equivalent load power in external network (MVA)	
			Y_{3-17}	Z_{3ex1}	Z_{17ex2}	$\Delta\tilde{S}_3^L$	$\Delta\tilde{S}_{17}^L$	$\Delta\tilde{S}_3^{eq}$	$\Delta\tilde{S}_{17}^{eq}$
M2			1.7871-j11.0753	0.0055+j0.0948	0.0681+j0.4586	–	–	–	–
M3	0	[0,0] [0,-0.5]	6.1308-j12.1505	-0.030+j0.0599 -0.283+j0.1912	-0.2798-j0.2416 -0.7216-j1.8653	-0.80+j0.52	0.80-j0.56	0.04-j0.28	-0.03+j0.35
	1	[0,0]	2.2658+j0.0406	-0.033+j0.0778	-0.3173-j0.0998	0.37+j1.47	-0.04-j0.43	–	–
M4	0	l_{16-17} out of service	2.6223-j11.296	–	–	-38.95+j9.10	39.80-j11.52	0.27-j1.66	-1.10+j4.22
	1	l_{16-17} out of service	3.0867-j9.8818	–	–	-38.14+j10.09	39.08-j12.32	–	–

M1: PQ equivalence [1].

M2: Extended Ward equivalence [5]. It belongs to the topological method and the model is similar to one used in this paper. Thus, the results obtained by M2 are regarded as the reference values.

M3: The improved Ward-PV [14]. The measurement snapshots of multi-period are generated with 0.5% step on the internal grid load.

M4: The proposed method in this paper.

2) ACTUAL EXAMPLE

C2: Guangdong actual power grid. There are 1926 lines of 220 kV and above voltage levels, as well as 1207 nodes (not including 220 kV load substation). Between Guangdong power grid (internal network) and Hong Kong power grid (external network), the electric magnetic ring network is connected through four 400 kV ac lines and six contact transformers. There are two boundary nodes on the Guangdong side, which are 400 kV buses located in Shenzhen station and Dayawan station. So, this makes Guangdong-Hong Kong power grid a two-port interconnection system.

B. EVALUATION INDEX OF EQUIVALENT ERROR

To reasonably analyze the influence of equivalent error on the power flow distribution, the relative error e_r and safety error e_s are introduced [9].

$$e_r = \left| \frac{x - x^{eq}}{x} \right| \times 100\% \tag{8}$$

$$e_s = \left| \frac{x - x^{eq}}{S_{base}} \right| \times 100\% \tag{9}$$

where x is derived from the entire network power flows without equivalence, and x^{eq} is the power flows of independent internal network considering the external equivalent models. S_{base} is set to 305 MVA in the New England 39-node test system. More specifically, e_{r_Vm}, e_{r_P}, and e_{r_Q} denote the maximum relative error of voltage amplitude, active power, and reactive power, respectively. e_{s_P} and e_{s_Q} represent the maximum safety error of active power and reactive power, respectively.

C. TEST RESULTS AND ANALYSIS

1) TEST RESULTS AND ANALYSIS FOR C0

To compare the proposed method with other methods, Table 1 shows the results of M2-M4 for C0. 0 in the second column indicates that RME is not considered, and 1 indicates the opposite. [0,0] and [0,−0.5] in the third column represent the initial values of M3's equivalent admittance. l_{16-17} out of service in M4 refers to the disconnection of l_{16-17}. The parameters of the external network for nodes 3 and 17 are divided two categories. Connecting both 3 and 17 simultaneously: Y_{3-17} represents the equivalent admittance. Connecting either 3 or 17 separately: Z_{3ex1} and Z_{17ex2} represent the equivalent impedance of power supply branches; $\Delta\tilde{S}_3^L$ and $\Delta\tilde{S}_{17}^L$ are the power increment of tie lines; the increments of equivalent load power are represented by $\Delta\tilde{S}_3^{eq}$ and $\Delta\tilde{S}_{17}^{eq}$, which can be obtained with the true values of full-network power flows.

a) Influence of initial values on the equivalent impedance in M3

The equivalent impedance results (Z_{3ex1} and Z_{17ex2}) of M3 in Table 1 indicate that when the initial values change from [0,0] to [0,−0.5], the reactance of Z_{17ex2} changes from −0.2416 to −1.8653, in which two values are very different to each other. By comparing Z_{3ex1} and Z_{17ex2} of M2 and M3, it can be seen that they have great differences in numerical value. The resistance in Z_{3ex1} and Z_{17ex2} are both negative. Obviously, the selection on the initial values has a significant influence on the estimation of M3's equivalent impedance.

b) Accuracy analysis of equivalent admittance when RME = 0

In Table 1, Y_{3-17} using M2–M4 are 1.7871-j11.0753, 6.1308-j12.1505, 2.6223-j11.296, respectively. The data indicates that when the results of M2 is regarded as the reference values, the equivalent admittance accuracy of M4 is significantly higher than that of M3.

The last four columns of Table 1 can help in analyzing the accuracy of Y_{3-17}. One observation is that $\Delta\tilde{S}_3^{eq}$ of M3 and M4 are not equal to zero. It should be noted that although the disturbance (e.g. l_{16-17} is out of service) occurred in the inner network, $\tilde{S}_3^{eq}$ is still affected on account of variation of

TABLE 2. Results of M4 in different disturbance.

Out of service	The relative error of equivalent susceptance	$\Delta\tilde{S}_3^L$
$l_{10\text{-}13}$	14.66%	-5.86+j4.74
$l_{13\text{-}14}$	10.14%	-8.45+j5.53
$l_{17\text{-}18}$	0.84%	84.42+j5.24
$load_{23}$	1.05%	19.78-j2.77
gen_{36}	1.46%	-18.08+j5.16

external power flow by disturbance. However, both M3 and M4 assume that $\tilde{S}_3^{eq}$ remained constant in estimating Y_{3-17}. The results as shown in Table 1 also indicate that this assumption is not strictly true, which in turn affects the estimation accuracy of equivalent admittance. Therefore, compared with the results of M2, errors are found in the equivalent admittance of M3 and M4.

As depicted in Table 1, $\Delta\tilde{S}_3^{eq}$ in M3 is 0.04 − j0.28 MVA, which is smaller than that 0.27 − j1.66 MVA in M4. Apparently, the increment of equivalent load power in M3 is closed to 0. Moreover, $\Delta\tilde{S}_3^L$ in M3 and M4 are −0.80 + j0.52 MVA and 38.95+j9.10 MVA. In terms of the ratio of $\Delta\tilde{S}_3^{eq}$ to $\Delta\tilde{S}_3^L$, the ratio of active and reactive power in M4 is 140 times and 5 times, respectively, which is much higher than that 20 times and 2 times in M3. It can be concluded that the accuracy of external equivalent parameters majorly depends on the tie-line power change. The greater the increment of tie-line power, particularly the larger proportion of increment of tie-line power to the increment of the equivalent load power, the higher the accuracy of the equivalent admittance. This is the reason why accuracy of M4 in equivalent admittance is higher than that of M3.

Table 2 gives the relative error of equivalent reactance under different tie-line powers. It can be seen from Table 2 that the equivalent parameters in M4 can be also obtained from generator tripping or load shedding. The increments of tie-line power caused by different disturbance is different actually, which leads to different accuracy of equivalent parameters. According to the general conclusion limited to this test case, if the active power of a single tie line reaches more than 10 MW, the relative error of equivalent reactance is less than 2% compared with M2's reference value. Through the data in Table 2 and further simulation analysis, 10MW can be applied to the empirical index for selecting disturbance in M4.

c) Accuracy analysis of equivalent admittance when RME = 1

Y_{3-17} offered by M3 and M4 are 2.2658 + j0.0406 and 3.0867 − j9.8818 when RME = 1. Compared with M2's reference, Y_{3-17} of M3 has lost their physical significance since the value of equivalent susceptance is positive. Although the estimation error of M4 is also increased (compared with RME = 0), it still maintains a high accuracy. This indicates that M4 has a strong tolerance to RME, while M3 is poor in this aspect.

The above conclusion can be explained by the influence of RME on the tie-line power increment. When RME = 0 is adjusted to RME = 1, $\Delta\tilde{S}_3^L$ in M3 changes from −0.80 + j0.52 MVA to 0.37+j1.47 MVA, and the amplitude increment is almost 60%; $\Delta\tilde{S}_3^L$ produced by M4 is from −38.95 + j9.10 MVA to −38.14 + j10.09 MVA, and the change of amplitude is less than 2%. Consequently, M4's tie-line power increment is much larger than that of RME, whereas that of M3 is very small. In other words, RME has little influence on the tie-line power increment of M4, but it has a great influence on M3. Thus, it leads to a more robust performance of M4.

TABLE 3. Branch outage error results for different methods of two-port network.

Outage branch	Error Index /%	Methods			
		M1	M2	M3	M4
$l_{5\text{-}6}$	e_{r_Vm}	0.083	0.049	0.226	0.051
	e_{r_P}	6.655	2.597	60.349	2.654
	e_{r_Q}	17.029	9.285	191.736	9.785
	e_{s_P}	1.127	0.149	10.198	0.168
	e_{s_Q}	1.560	0.851	3.967	0.897
$l_{6\text{-}11}$	e_{r_Vm}	0.081	0.052	0.215	0.057
	e_{r_P}	10.383	0.332	43.156	0.671
	e_{r_Q}	14.902	8.797	260.569	10.132
	e_{s_P}	2.483	0.181	10.305	0.183
	e_{s_Q}	1.322	0.778	3.958	0.899
$l_{10\text{-}11}$	e_{r_Vm}	0.067	0.042	0.184	0.045
	e_{r_P}	8.459	0.454	51.708	0.685
	e_{r_Q}	13.419	7.470	156.096	8.391
	e_{s_P}	1.656	0.133	10.109	0.134
	e_{s_Q}	1.131	0.630	3.949	0.707

d) Influence of two-port external equivalence on power flow distribution

To test performance of different equivalent model in the static security analysis, the equivalent error listed in Table 3 is calculated by M1–M4. M3 selects the equivalent parameters that is generated under the [0,0] and RME = 0.

As shown in Table 3, the security errors of power are far less than relative errors. Taking Line 6-11 as an example, the e_{r_Q} is 260.569% in M3, but the corresponding e_{s_Q} is 3.958%. It also reveals that although absolute errors and the safety errors are small, the relative errors may be large due to small branch power. Hence, adopting a security error is more appropriate to measure the influence of external equivalence on power flow of the internal network.

The errors in Table 3 of different methods are arranged from small to large: M2, M4, M1, M3. Specifically, the e_s and the e_{r_Vm} using M1–M4 are 2.483%, 0.851%, 10.305%, 0.899% and 0.083%, 0.052%, 0.226%, 0.057%, respectively. The results indicate that the accuracy of M2 is the highest in the four methods, and the equivalent accuracy of M4 is higher than that other two black-box equivalent methods. Moreover, the maximum error of M2 and M4 is less than 1%, which fully meets the application requirements of practical engineering. However, there are errors in the actual application of M3.

2) TEST RESULTS AND ANALYSIS FOR C1

a) Accuracy analysis of equivalent parameters when RME = 1

TABLE 4. Equivalent parameter estimation results of different methods in three-port network.

Method	Calculation condition	Y(p.u.)			Z(p.u.)		
		$Y_{4\text{-}9}$	$Y_{4\text{-}16}$	$Y_{9\text{-}16}$	Z_{4ex1}	Z_{9ex2}	Z_{16ex3}
M2	-	0.1948-j4.324	1.4803-j18.7153	0.3991-j4.4832	0.0095+j0.1798	0.0010+j0.0250	0.0167+j0.1723
M3	[0,0]	-6.8142-j1.9488	-0.5035-j16.6937	4.4486-j6.1200	-0.1290+j01759	-1.1582-j0.2566	-0.0068-j0.0772
M4_A	$l_{6\text{-}7}$	-14.3871-j96.8732	846.1538+j230.7692	3.3014+j90.789	–	–	–
M4_B	$l_{6\text{-}7}$	-j4.5372	-j19.2678	-j4.931	–	–	–

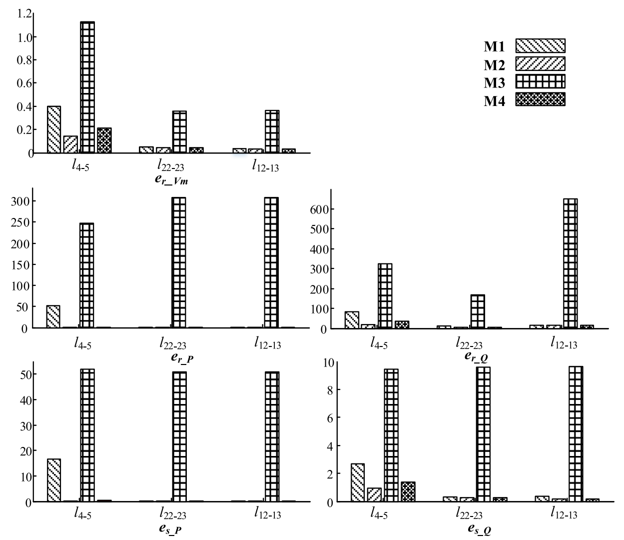

FIGURE 4. Branch outage error results for different methods of three-port network.

Similar to the test results of the two-port network, M3 is still affected by the initial values and measurement errors in the three-port network. To avoid repetition, the equivalent parameters given in Table 4 are only the results at RME = 1. In addition, M4_A and M4_B represent the scheme in which M4 considers and does not consider conductance in the three-port network, respectively.

Table 4 presents that the equivalent parameters of M3 and M4_B deviate significantly from the reference value of M2. Simultaneously, they are fundamentally different in the positive and negative aspects of the complex number, while the maximum relative error of the equivalent susceptance of M4_B is only 9.99% (= (4.931-4.4832)/4.4832×100%). It can be verified that equivalent parameters are completely unavailable when considering conductance. Without considering conductance, the accuracy of equivalent parameters is greatly improved.

b) Influence of three-port external equivalence on power flow distribution

Similar to the simulation at *d*) in C0, FIGURE 4 gives the results of evaluation index that come from the calculation with different methods. The ordering of different methods shown in FIGURE 4 are consistent with the two-port interconnection system. The conclusion is similar with C1, and the accuracy of M4 is still higher than that of M1's and M3's.

TABLE 5. Active power and related information of the tie tine in guangdong–hong kong power grid.

Operation Ways	Num.	Sampling Time	P_{act1} (MW)	P_{act2} (MW)	P_{sum} (MW)	$P_{H\text{-}S}$ (MW)
Loop Closing	T1	11:54:48	251	1262	1513	0
	T2	11:59:48	407	1084	1491	-222
	D1	–	-156	178	22	222
Loop Opening	T3	12:29:48	470	1085	1555	-219
	T4	12:34:48	314	1201	1515	0
	D2	–	156	-116	40	219

3) APPLICATION RESULTS ANALYSIS FOR C2

The proposed method has been also well applied in the actual power grid in Guangdong. Table 5 collects the active power of tie lines before and after twice L_{H-S} (abbreviation for actual line name) loop closing/opening operation. T1 to T2 correspond to the loop closing, while T3 to T4 correspond to the loop opening. P_{act1} and P_{act2} represent the active power of actual tie line 1 and 2, respectively. P_{sum} is the sum of active power of tie lines. P_{H-S} is the power of L_{H-S}. D1 represents the active power difference between T1 and T2, while D2 represents the difference of active power between T3 and T4.

The following investigations can be drawn from Table 5:

➢ The time interval between T1 and T2 or T3 and T4 is 5 minutes, and the time interval between T1 and T3 is 30 minutes. P_{sum} between T1 and T2 or T3 and T4 are 22 MW and 40 MW respectively, which are 1.45% and 2.57% compared with P_{sum} of T1 and T3, respectively. These observations indicate that the time interval are relatively short between before and after operation, and the active power of different measurement snapshots are nearly unchanged. Hence, the assumption on the external static equivalence is valid.

➢ P_{act1} of D1 is 156MW, which is 10.13% of P_{sum} (1513 MW) at T1, and it is 70.27% of P_{H-S} (222 MW) at T2. It shows that the L_{H-S} is suitable as loop closing line, due to that L_{H-S} has a great influence on the transmission power of tie lines. Furthermore, there is a large change in the transmission power before and after the loop closing, which is conducive to the accurate estimation of the equivalent parameters.

The main steps for C2 simulation are executed as follows.

Step 1: Calculate equivalent admittance Y_{act} between boundary nodes based on the data of T1 and T2.

TABLE 6. The equivalent error of loop opening simulation in guangdong power grid.

Device Name	M1(MVA)		M4(MVA)	
	P_ε	Q_ε	P_ε	Q_ε
$L_{H\text{-}H}$	109.23	7.27	2.44	1.10
T_{t1_SZ}	52.30	14.09	16.38	7.21
T_{t2_SZ}	52.30	14.09	16.38	7.21
T_{t3_SZ}	52.45	13.68	16.25	13.64
T_{t1_DY}	56.35	4.07	0.11	5.48
T_{t2_DY}	56.35	4.07	0.11	5.48

Step 2: Calculate equivalent load power $\tilde{S}_{act}^{eq}$ of the simplified Ward model based on Y_{act} and the data of T3. Then, Y_{act}, $\tilde{S}_{act}^{eq}$ and the data of T3 make up the stitching data (SD).

Step 3: Simulate the loop opening of L_{H-S} based on the SD. Then, the power flow calculation is carried out to obtain the simulated power flow data (SPFD) of T4.

Step 4: Compare SPFD with the data of T4.

In Table 6, the results of the active and reactive power deviation (P_ε and Q_ε) of the six devices are given, and the absolute values of the power deviation are ranked in the top six in the whole network. Devices include a line (start with 'L') and five transformers (start with 'T'). Unlike M4, M1 uses the state estimation data instead of SD.

From the data in Table 6, the maximum P_ε of M4 is 16.38 MW, and the maximum complex power deviation is 21.22 MVA ($P_\varepsilon = 16.25$ MW, $Q_\varepsilon = 13.64$ Mvar). If the safety standard of 500kV line is selected (e.g. 1082 MVA), the maximum safety error of P_ε and complex power are 1.50% and 1.96%, respectively. As for M1, the maximum P_ε and complex power deviation are 109.23 MW and 109.47 MVA, respectively, whilst the corresponding safety error are 10.10% and 10.12% respectively. The results show that M4 has high equivalence accuracy and meets the requirements of practical engineering. On the contrary, M1 brings the gross equivalence error and safety hazard.

IV. CONCLUSIONS

This paper proposed a black-box external equivalent method using tie-line power mutation. Its effectiveness has been demonstrated by the New England 39-node case and the actual grid case in Guangdong of China, and the following detailed conclusions are reached:

1) The estimation of equivalent parameters often requires the measurement data of multiple periods. If variation between the measured states at different periods is insufficient, measurement errors will seriously affect the estimation results of equivalent parameters and even leads to a complete failure.

2) In order to improve equivalence accuracy during measurement changes in multiple measurement snapshots, on the one hand, it is necessary to ensure that the power flow state of the external network is basically unchanged, that is, the sum of transmission power of tie lines is basically unchanged. On the other hand, it is also necessary to ensure a large change in the transmission power of a single tie line.

3) Different disturbances will result in different increments of tie-line power, which also leads to different accuracy of equivalent parameters, and an empirical standard of 10 MW is given by this paper to select an acceptable disturbance for the New England 39-node case.

REFERENCES

[1] J. V. Milanovic, K. Yamashita, S. M. Villanueva, S. Ž. Djokic, and L. M. Korunović, "International industry practice on power system load modeling," *IEEE Trans. Power Syst.*, vol. 28, no. 3, pp. 3038–3046, Aug. 2013.

[2] S. Deckmann, A. Pizzolante, A. Monticelli, B. Stott, and O. Alsac, "Studies on power system load flow equivalencing," *IEEE Trans. Power App. Syst.*, vol. PAS-99, no. 6, pp. 2301–2310, Nov. 1980.

[3] R. A. M. van Amerongen and H. P. van Meeteren "A generalised ward equivalent for security analysis," *IEEE Trans. Power App. Syst.*, vol. PAS-101, no. 6, pp. 1519–1526, Jun. 1982.

[4] J. Srivani and K. S. Swarup, "Power system static security assessment and evaluation using external system equivalents," *Int. J. Elect. Power Energy Syst.*, vol. 30, no. 2, pp. 83–92, Feb. 2008.

[5] A. Monticelli, S. Deckmann, A. Garcia, and B. Stott, "Real-time external equivalents for static security analysis," *IEEE Trans. Power App. Syst.*, vols. PAS-98, no. 2, pp. 498–508, Mar. 1979.

[6] F. F. Wu and A. Monticelli, "Critical review of external network modelling for online security analysis," *Int. J. Elect. Power Energy Syst.*, vol. 5, no. 4, pp. 222–235, Oct. 1983.

[7] R. D. Shultz, M. Muslu, and R. D. Smith, "A new method in calculating line sensitivities for power system equivalencing," *IEEE Trans. Power Syst.*, vol. 9, no. 3 pp. 1465–1470, Aug. 1994.

[8] J. Yu, J. Liu, W. Li, R. Xu, W. Yan, and X. Zhao, "Limit preserving equivalent method of interconnected power systems based on transfer capability consistency," *IET Gener., Transmiss. Distrib.*, vol. 10, no. 14, pp. 3547–3554, Oct. 2016.

[9] J. Yu, M. Zhang, and W. Li, "Static equivalent method based on component particularity representation and sensitivity consistency," *IEEE Trans. Power Syst.*, vol. 29, no. 5, pp. 2400–2408, Sep. 2014.

[10] A. S. Debs, "Estimation of external network equivalents from internal system data," *IEEE Trans. Power App. Syst.*, vol. 94, no. 2, pp. 273–279, Mar. 1975.

[11] J. Yu, W, Dai, W. Li, X. Liu, and J. Liu, "Optimal reactive power flow of interconnected power system based on static equivalent method using border PMU measurements," *IEEE Trans. Power Syst.*, vol. 28, no. 3, pp. 421–429, Jan. 2018.

[12] G. Contaxis and A. S. Debs, "Identification of external system equivalents for steady-state security assessment," *IEEE Trans. Power App. Syst.*, vol. PAS-97, no. 2, pp. 409–414, Mar. 1978.

[13] Y. Wei *et al.*, "Parameter estimation for two-port external network static equivalent model on the basis of internal network measured information," *Proc. CSEE*, vol. 31, no. 13, pp. 101–106, 2011.

[14] S. Xu and S. Miao, "Calculation of TTC for multi-area power systems based on improved ward-PV equivalents," *IET Gener., Transmiss. Distrib.*, vol. 11, no. 4, pp. 987–994, Mar. 2017.

[15] G. Wijeweera, U. D. Annakkage, W. Zhang, and A. D. Rajapakse, "Derivation of an equivalent circuit for real-time security assessment," *IET Gener., Transmiss. Distrib.*, vol. 10, no. 8, pp. 1913–1920, May 2016.

[16] R. Emami and A. Abur, "External system line outage identification using phasor measurement units," *IEEE Trans. Power Syst.*, vol. 28, no. 2, pp. 1035–1040, May 2013.

[17] M. B. Do Coutto Filho, J. C. S. de Souza, and M. A. R. Guimaraens, "Enhanced bad data processing by phasor-aided state estimation," *IEEE Trans. Power Syst.*, vol. 29, no. 5, pp. 2200–2209, Sep. 2014.

[18] E. W. S. Ângelos and E. N. Asada, "Improving state estimation with real-time external equivalents," *IEEE Trans. Power Syst.*, vol. 31, no. 2, pp. 1289–1296, Mar. 2016.

[19] S. Chakrabarti and E. Kyriakides, "Optimal placement of phasor measurement units for power system observability," *IEEE Trans. Power Syst.*, vol. 23, no. 3, pp. 1433–1440, Aug. 2008.

[20] L. Zhao and A. Abur, "Multi area state estimation using synchronized phasor measurements," *IEEE Trans. Power Syst.*, vol. 20, no. 2, pp. 611–617, May 2005.

WEI YAN (M'09) received the Ph.D. degree in electrical engineering from Chongqing University, Chongqing, China, in 1999, where he is currently a Professor with the School of Electrical Engineering. His research interest includes optimal operation and control in power systems.

CHAO ZHANG (S'18) is currently pursuing the M.S. degree with the School of Electrical Engineering, Chongqing University, Chongqing, China. His research interests include power system state estimation and external network equivalence.

JUNJIE TANG (M'14) received the Ph.D. degree in electrical engineering from the Institute for Automation of Complex Power Systems, E.ON Energy Research Center, RWTH Aachen University, Aachen, Germany, in 2014. He is currently an Associate Professor with the State Key Laboratory of Power Transmission Equipment and System Security and New Technology, Power and Energy Reliability Research Center, School of Electrical Engineering, Chongqing University, Chongqing, China.

His research interests include power system uncertainty quantification analysis, power system security risk assessment, and optimal management.

WEI QIAN is currently pursuing the M.S. degree with the School of Electrical Engineering, Chongqing University, Chongqing, China. His research interest includes bad data identification in the power system and running schedule.

SHIMING LI is currently an Engineer with the Power Dispatching Control Center, Guangdong Grid Company, Guangzhou, China. His research interest includes power system optimization operation and control.

QIAN WANG received the M.S. degree in electrical engineering from Chongqing University, Chongqing, China, in 2017. She is currently with the State Grid Deyang Power Supply Company, Deyang, China. Her research interests include power system static equivalence and state estimation.