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Robust Model Predictive Control for a Class of Discrete-Time Markovian Jump Linear Systems With Operation Mode Disordering

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ABSTRACT For a class of discrete-time Markovian jump linear systems subject to operation mode disordering, a robust model predictive control method can be proposed to solve this issue. A bijective mapping scheme between the original random process and a new random process is studied to cope with the problem of operation mode disordering. At each sampling time, the original "min–max" optimization problem is transformed into a convex optimization problem with linear matrix inequalities so that the complexity of solving the optimization problem can be greatly reduced. The sufficient stability condition of the Markovian jump linear systems can be achieved by using the Lyapunov stability theory. Moreover, a state feedback control law is obtained, which minimizes an infinite prediction horizon performance cost. Furthermore, the cases of uncertain and unknown transition probabilities are also considered in this paper. The simulation results show that the proposed method can guarantee the optimal control performance and the stability of Markovian jump linear systems.

INDEX TERMS Robust model predictive control, Markovian jump linear systems (MJLSs), operation mode disordering, linear matrix inequalities (LMIs).

I. INTRODUCTION

Markovian jump linear systems (MJLSs) are a particular class of hybrid systems that have become increasingly important because of their many applications and theoretical value [1], [2]. In practice, a great number of systems whose parameters or structures suddenly change can be described by a Markov mode, such as electrical systems, networked control systems [3], [4] and so on. The main characteristic of MJLSs is that the dynamic system change is both time trigger and event trigger. Discrete event-trigger changes are known as the modes of the system. The mode switching law can be described by a Markov chain [5]. In the past decades, MJLSs have been the focus of many studies, such as stability analysis [6]–[8], sampled-data control [9], [10], state estimation [11], neural network control [12], H_∞ filtering and control [13]–[15], Fault detection [16], fault-tolerant control [17] and sliding mode control [18]–[22].

On the other hand, model predictive control (MPC) is a very popular and practical control strategy. MPC has already been used in many applications, for example, in petrochemical processes, flotation processes, pharmaceutical processes, crystallization processes and networked control systems [23]–[27]. This method is a kind of closed-loop optimization control strategy based on a model such as the T-S model [28]–[30] and Hammerstein-Wiener model [31]–[33]. The goals of MPC are to predict the future dynamic behavior of the system, to achieve rolling optimization, and to provide feedback correction of the model error [34]–[42]. Moreover, for universal uncertainties and disturbances, a large number of results based on the robust model predictive control (RMPC) method have been found [43]–[47]. It is known that MJLSs are also treated as a class of uncertain systems, where the uncertainty conforms to certain statistical probabilities. The probability information can be reasonably included in designing a predictive controller, which can achieve better control performance and reduce conservativeness.

Recently, some important results from MJLSs have been obtained based on the MPC method. In [48], the one-step MPC scheme for uncertain discrete-time MJLSs whose transition probabilities were assumed to be convex sets was proposed. For discrete-time MJLSs with polytopic uncertainties, a novel multi-step mode-dependent MPC method was proposed and the mean-square stability of three cases was guaranteed [49]. Yang and Karimi *et al.* [50] studied a novel MPC method for a class of uncertain fuzzy MJLSs with partially unknown transition probabilities. Moreover, for discrete-time non-homogeneous MJLSs with time-varying transition probability matrices, a N-step off-line suboptimal MPC was proposed [51]. Reference [52] researched the stochastic model predictive control (SMPC) of nonlinear MJLSs. The terminal conditions of invariance and stability were used to fulfill the robustness constraint and guarantee mean-square stability. Chitraganti *et al.* [53] studied an one-step receding horizon control method for discrete-time state-dependent MJLSs subject to probabilistic state constraints and unbounded disturbances. On the other hand, for MJLSs subject to input/state constraints, an MPC method based on a periodic invariant set was designed [54]. Furthermore, a kind of discrete-time nonlinear Markovian jump system with non-homogeneous transition probabilities was developed by designing an MPC method [55]. Reference [56], for MJLSs with polytopic uncertainties in both system matrices and transition probability matrices, studied a robust distributed model predictive control (DMPC) strategy. At the same time, the stable receding-horizon scenario predictive control of constrained discrete-time MJLSs was also studied [57]. For the Markovian jump linear systems with bounded disturbance, Lu *et al.* [58] studied the constrained model predictive control method to achieve the disturbance rejection. For a class of constrained discrete-time Markovian nonlinear stochastic switching systems, Dombrovskii *et al.* [59] proposed an MPC method, which studied the dynamic investment portfolio selection problem in the presence of market frictions. Zhang *et al.* [60] researched that for saturating systems with packet dropouts, a distributed model predictive control strategy has been proposed.

We note that the MPC method is dependent on the operation modes of the system in all of the above studies. It is assumed that the system modes and operation modes of the controllers are synchronous and in the right sequence. However, in practice, operation mode disordering is universal. For example, in networked control systems, the data packets can choose multiple paths when being transmitted and can thus experience different time delays along these different paths. This can result in packets that were launched earlier reaching the target point later than packets that were sent later. In other words, the data packets arrive in the incorrect order. If the control signals of the MJLSs are transmitted by unreliable networks, disordered operation modes can easily occur. Moreover, this can also lead to instability of the control system and poor control performance. Therefore, it is very important to deal with the problem of MJLSs that have operation mode disordering [61]. Until now, to the best of our knowledge, discrete-time MJLSs with operation mode disordering have not been sufficiently studied, which motivates the study in this paper.

In this paper, the main contributions can be highlighted as follows: (1). For a class of discrete-time MJLSs subject to operation mode disordering, a novel RMPC method can be proposed to guarantee the stability and the optimal performance of system. (2). To better solve the issue of operation mode disordering in MJLSs, transition probability with uncertainty and incomplete information have also been researched, respectively. In this study, the discussed problems are more general than most of those in existing literatures. (3). From the perspective' technology, due to consider the transition probability with uncertainty and incomplete information, the complexity of the optimization problem can be greatly increased, which are also the challenge and innovation of this study.

The rest of the paper is arranged as follows. The problem description is given in section 2. Section 3 proposes the RMPC method for MJLSs with operation mode disordering. Section 4 contains the main results. Section 5 provides numerical examples. Section 6 gives a conclusion and our future works.

Notations: $\mathbb{R}^n$ represents n-dimensional Euclidean space. $x(k|k)$ denotes the measured state at sampling time k. A^{T} denotes the transpose of matrix A. $u(k+i|k)$ and $x(k+i|k)$ represent the control input and predicted state at step k, respectively. $\|x(k)\|^2$ represents the Euclidean norm of state vector $x(k)$. Ω refers to the sample space, $\mathcal{F}$ refers to the σ-algebras of the subsets of the sample space, and $\mathbb{P}$ refers to the probability measure on $\mathcal{F}$. $\mathscr{E}$ refers to the mathematical expectation. The symbol "$*$" denotes the symmetric parts of symmetric matrices. I represents the identity matrix of known compatible dimensions, and $(G)^\star = G + G^{\mathrm{T}}$.

II. PROBLEM FORMULATION

Consider a class of discrete-time MJLSs defined on a complete probability space $(\Omega, \mathcal{F}, \mathbb{P})$

$$x(k+1) = A_{\theta_1(k)}x(k) + B_{\theta_1(k)}u(k) \tag{1}$$

where $x(k) \in \mathbb{R}^{n_x}$ represents the system state, $u(k) \in \mathbb{R}^{n_u}$ represents the control input, and $\theta_1(k)$ represents the operation modes of the system. $A_{\theta_1(k)}$ and $B_{\theta_1(k)}$ represent the system matrices of known compatible dimensions. The original operation mode $\{\theta_1(k), k \in \mathbb{Z}\}$ is a Markov chain that takes values in the discrete finite set $\mathbb{M}_1 = \{1, 2, \ldots, N_1\}$. Therefore, the mode-dependent state feedback controller can be written as

$$u(k) = K_{\theta_1(k)}x(k) \tag{2}$$

However, $\theta_1(k)$ is transmitted through multiple channels in networked control system, which may suffer from operation mode disordering. Then, the corresponding controller can be described as follows

$$u(k) = K_{\theta_2(k)}x(k) \tag{3}$$

where $\{\theta_2(k), k \in \mathbb{Z}\}$ is another Markov chain that takes values in a discrete finite set $\mathbb{M}_2 = \{1, 2, \ldots, N_2\}$. To deal with operation mode disordering, the new operation modes, $\theta(k)$, of the system need to be obtained from the augmented vector $\{\theta_1(k), \theta_2(k)\}$. Thus we introduce a bijective mapping between $\theta(k)$ and $\{\theta_1(k), \theta_2(k)\}$. The bijective mapping is described as

$$\Psi : \mathbb{M}_1 \times \mathbb{M}_2 \rightarrow \mathbb{M} \tag{4}$$

where $\eta = \Psi(\eta_1, \eta_2)$ with $\eta \in \mathbb{M}$, $\eta_1 \in \mathbb{M}_1$ and $\eta_2 \in \mathbb{M}_2$, and

$$\Psi^{-1} : \mathbb{M} \rightarrow \mathbb{M}_1 \times \mathbb{M}_2 \tag{5}$$

where $(\eta_1, \eta_2) = \Psi^{-1}(\eta)$ with $\eta_1 = \Psi_1^{-1}(\eta) \in \mathbb{M}_1$ and $\eta_2 = \Psi_2^{-1}(\eta) \in \mathbb{M}_2$. It is assumed that $\{\theta(k), k \in \mathbb{Z}\}$ is a stationary ergodic Markov chain that takes values in $\mathbb{M} = \mathbb{M}_1 \times \mathbb{M}_2$, where $\mathbb{M} \in \{1, 2, \ldots, N\}$ and the element number $N = N_1 \times N_2$. Therefore, it can be shown that

$$x(k+1) = A_{\theta(k)}x(k) + B_{\theta(k)}u(k) \tag{6}$$

Furthermore, for the operation mode disordering, the non-fragile state feedback controller can be designed

$$u(k) = (K_{\theta(k)} + \Delta K_{\theta(k)})x(k) \tag{7}$$

satisfying

$$\Delta K^{\mathrm{T}}_{\theta(k)}\Delta K_{\theta(k)} \leq W_{\theta(k)} \tag{8}$$

where

$$\Delta K_{\theta(k)} = K_{\theta_1(k)} - K_{\theta(k)} \tag{9}$$

and $\Delta K_{\theta(k)}$ is the control gain fluctuation. $W_{\theta(k)}$ is a positive matrix, and we define $A_\eta = A_{\eta_1}$, $B_\eta = B_{\eta_1}$ and $K_\eta = K_{\eta_2}$ for any $\theta(k) = \eta \in \mathbb{M}$. The transition probability matrix of operation modes $\theta(k)$ is defined as $\prod \triangleq (\pi_{\eta\mu}) \in \mathbb{R}^{N\times N}$

$$\Pr\{\theta(k+1) = \mu|\theta(k) = \eta\} = \pi_{\eta\mu} \tag{10}$$

where $0 < \pi_{\eta\mu} < 1$ and $\sum_{\mu=1}^{N} \pi_{\eta\mu} = 1$ for any $\eta, \mu \in \mathbb{M}$.

Remark 1: It is worth mentioning that it is different in form from the traditional asynchronous controller. The traditional model uses two independent probability distributions, which do not directly deal with the operation mode disordering [62]. However, in this paper, the non-fragile method is used to cope with the asynchronous mode signals, and the way of the augmented Markov modes are also taken advantage of dealing with the asynchronous phenomenon, which can sufficiently consider the impact of mode disordering.

III. DESIGN OF RMPC FOR MJLS WITH OPERATION MODE DISORDERING

At each sampling time k, the aim of the RMPC scheme is to derive a state feedback control law such that the infinite horizon cost function (11) for MJLSs subject to operation mode disordering is minimized. Consider the following "min-max" performance index

$$\min_{u(k+\tau|k),\tau\geq 0} \max \quad J_\infty(k) \tag{11}$$

where

$$J_\infty(k) = \mathscr{E}\{\sum_{\tau=0}^{\infty}[x^{\mathrm{T}}(k+\tau|k)Q_{\theta(k+\tau)}x(k+\tau|k) + u^{\mathrm{T}}(k+\tau|k)R_{\theta(k+\tau)}u(k+\tau|k)]\}$$

and $Q_{\theta(k+\tau)}$ and $R_{\theta(k+\tau)}$ are positive symmetric weighting matrices of the states and inputs, respectively, and are of known appropriate dimension. Obviously, the closed-loop system is described as follows

$$x(k+\tau+1|k) = [A_{\theta(k+\tau)} + B_{\theta(k+\tau)}(K_{\theta(k+\tau)} + \Delta K_{\theta(k+\tau)})]x(k+\tau|k) \tag{12}$$

Definition 1: For any initial condition (x_0, θ_0), the closed-loop system (12) is stochastically stable if

$$\mathscr{E}\{\sum_{k=0}^{\infty} x^{\mathrm{T}}(k)x(k)|x_0, \theta_0\} < \infty$$

Lemma 1: *[43]:* Assume that positive symmetrical matrices $Q(x)$, $R(x)$ and $S(x)$ are affine functions of x. The following linear matrix inequality holds:

$$\begin{bmatrix} Q(x) & S(x) \\ S^{\mathrm{T}}(x) & R(x) \end{bmatrix} > 0 \tag{13}$$

It is equivalent to

$$R(x) > 0, Q(x) - S(x)R^{-1}(x)S^{\mathrm{T}}(x) > 0 \tag{14}$$

or

$$Q(x) > 0, R(x) - S^{\mathrm{T}}(x)Q^{-1}(x)S(x) > 0 \tag{15}$$

Lemma 2: *[63]:* Consider any positive definite symmetric matrix $P \in \mathbb{R}^n$ and given any vectors $X, Y \in \mathbb{R}^n$, the following inequality holds:

$$2X^{\mathrm{T}}PY \leq \varepsilon X^{\mathrm{T}}PX + \varepsilon^{-1}Y^{\mathrm{T}}PY, \quad \forall\varepsilon > 0 \tag{16}$$

Remark 2: The idea behind RMPC is to design the non-fragile state feedback controller (7) and substitute it into the optimization problem (11). At each sampling period, the state feedback control gain matrix is obtained by solving the optimization problem (11). The first control input of the obtained control sequence is then applied to the closed-loop system. At the next sampling period, the "min-max" optimization problem (11) can be again solved with the updated state information to obtain a new control input.

Remark 3: It is well known that one of the characteristics of MPC is a prediction model. In other words, the future behavior of the system can be predicted based on the prediction model at each sampling time. Then, at the time step $k+\tau$, the transition probability matrix of operation modes $\theta(k)$ is assumed to be

$$\Pr\{\theta(k+\tau+1) = l|\theta(k+\tau) = h\} = \pi_{hl} \tag{17}$$

where $0 < \pi_{hl} < 1$ and $\sum_{l=1}^{N} \pi_{hl} = 1$ for any $h, l \in \mathbb{M}$. In this paper, we write the following variables in a concise form: $A_{\theta(k+\tau)} = A_h$, $B_{\theta(k+\tau)} = B_h$, $K_{\theta(k+\tau)} = K_h$, $K_{\theta_1(k+\tau)} = K_{h1}$, $Q_{\theta(k+\tau)} = Q_h$, $R_{\theta(k+\tau)} = R_h$, and $W_{\theta(k+\tau)} = W_h$.

However, at each sampling time, it is difficult to solve the optimization problem (11) directly. Therefore, we need to derive an upper bound of the performance index. Corresponding to the closed-loop system (12), the following Lyapunov function is considered

$$V(x(k|k), \theta(k), k) = x^{\mathrm{T}}(k|k)P_{\theta(k)}x(k|k) \tag{18}$$

where $P_{\theta(k)} = P^{\mathrm{T}}_{\theta(k)} > 0$. At each sampling time k, the following robust stability condition should be satisfied

$$\begin{aligned}&\mathscr{E}[V(x(k+\tau+1|k))] - V(x(k+\tau|k))\\ &\quad \leq -\mathscr{E}[x^{\mathrm{T}}(k+\tau|k)Q_{\theta(k+\tau)}x(k+\tau|k)\\ &\qquad + u^{\mathrm{T}}(k+\tau|k)R_{\theta(k+\tau)}u(k+\tau|k)]\end{aligned} \tag{19}$$

Taking the expectation of both sides of (19) and summing from $\tau = 0$ to $\tau = \infty$, one obtains

$$\begin{aligned}&\sum_{\tau=0}^{\infty}\mathscr{E}[V(x(k+\tau+1|k)) - V(x(k+\tau|k))]\\ &\quad \leq -\sum_{\tau=0}^{\infty}\mathscr{E}[x^{\mathrm{T}}(k+\tau|k)Q_{\theta(k+\tau)}x(k+\tau|k)\\ &\qquad + u^{\mathrm{T}}(k+\tau|k)R_{\theta(k+\tau)}u(k+\tau|k)]\end{aligned} \tag{20}$$

It is assumed that the closed-loop system is asymptotically stable. Since $x(\infty|k) = 0$, it can be concluded that $V(x(\infty|k)) = 0$. Then,

$$\mathscr{E}\{\sum_{\tau=0}^{\infty}[V(x(k+\tau+1|k)) - V(x(k+\tau|k))]\} \leq -J_{\infty}(k) \tag{21}$$

The upper bound of the performance index can be derived as follows

$$J_{\infty}(k) \leq \mathscr{E}\{V[x(k|k)]\} \leq \gamma_1 \tag{22}$$

where γ_1 is a given positive scalar. The main results are presented in the following theorems.

IV. MAIN RESULT

Theorem 1: Consider the system (1), and let $x(k|k) = x(k)$ be the measured system state at each sampling time k. For given symmetric matrices $Q_h > 0$ and $R_h > 0$ and scalars $\gamma_1 > 0, \gamma_2 > 0$ and $\gamma_3 > 0$, there exists a non-fragile state feedback controller (7) such that the performance index is minimized. Then, the closed-loop system (12) can be stochastically stable if there exist matrices $X_h > 0$, $G > 0$, $Y_h > 0$, $Y_{h1} > 0$ and $W_h > 0$ satisfying

$$\min_{X_h, G, Y_h, Y_{h1}, W_h} \gamma_1 \tag{23}$$

subject to

$$\begin{bmatrix} -(G)^{\star}+X_h & \sqrt{2(\gamma_2+\gamma_3)}G^{\mathrm{T}} & G^{\mathrm{T}} & \sqrt{2}Y_h^{\mathrm{T}} & \sqrt{2\pi_{h1}}\bar{A}_h^{\mathrm{T}} & \cdots & \sqrt{2\pi_{hN}}\bar{A}_h^{\mathrm{T}}\\ * & -\overline{W}_h & 0 & 0 & 0 & \cdots & 0\\ * & * & -\overline{Q}_h & 0 & 0 & \cdots & 0\\ * & * & * & -\overline{R}_h & 0 & \cdots & 0\\ * & * & * & * & -X_1 & \cdots & 0\\ * & * & * & * & * & \ddots & \vdots\\ * & * & * & * & * & \cdots & -X_N \end{bmatrix} \leq 0 \tag{24}$$

$$\begin{bmatrix} -\gamma_2 I & \sqrt{\pi_{h1}}B_h^{\mathrm{T}} & \cdots & \sqrt{\pi_{hN}}B_h^{\mathrm{T}}\\ * & -X_1 & \cdots & 0\\ * & * & \ddots & 0\\ * & * & \cdots & -X_N \end{bmatrix} \leq 0 \tag{25}$$

$$\begin{bmatrix} -(G)^{\star}+\overline{W}_h & (Y_{h1}-Y_h)^{\mathrm{T}}\\ * & -I \end{bmatrix} \leq 0 \tag{26}$$

$$\begin{bmatrix} -\gamma_1 I & x^{\mathrm{T}}(k|k)\\ * & -X_\eta \end{bmatrix} \leq 0 \tag{27}$$

and

$$R_h \leq \gamma_3 I \tag{28}$$

where

$$\bar{A}_h = A_hG + B_hY_h,\ W_h^{-1} = \overline{W}_h,\ Q_h^{-1} = \overline{Q}_h,\ R_h^{-1} = \overline{R}_h.$$

Then, the gain K_{h1} of controller (2) and the gain K_h of controller (7) can be obtained as

$$K_{h1} = Y_{h1}G^{-1}$$

and

$$K_h = Y_hG^{-1}$$

Proof: For (22), applying to the Schur complement, (27) can be obtained. According to (19), it can be equal to the following inequality

$$\begin{aligned}&[A_h + B_h(K_h + \Delta K_h)]^{\mathrm{T}}\sum_{l=1}^{N}\pi_{hl}P_l[A_h + B_h(K_h + \Delta K_h)]\\ &\quad - P_h + Q_h + (K_h + \Delta K_h)^{\mathrm{T}}R_h(K_h + \Delta K_h) \leq 0\end{aligned} \tag{29}$$

Furthermore, it can also be derived that

$$(A_h+B_hK_h)^{\mathrm{T}}\sum_{l=1}^{N}\pi_{hl}P_l(A_h+B_hK_h)$$
$$+2(A_h+B_hK_h)^{\mathrm{T}}\sum_{l=1}^{N}\pi_{hl}P_lB_h\Delta K_h$$
$$+(B_h\Delta K_h)^{\mathrm{T}}\sum_{l=1}^{N}\pi_{hl}P_lB_h\Delta K_h-P_h+Q_h+K_h^{\mathrm{T}}R_hK_h$$
$$+2K_h^{\mathrm{T}}R_h\Delta K_h+\Delta K_h^{\mathrm{T}}R_h\Delta K_h\leq 0 \tag{30}$$

Moreover, it is concluded that

$$2(A_h+B_hK_h)^{\mathrm{T}}\sum_{l=1}^{N}\pi_{hl}P_lB_h\Delta K_h$$
$$\leq (A_h+B_hK_h)^{\mathrm{T}}\sum_{l=1}^{N}\pi_{hl}P_l(A_h+B_hK_h)$$
$$+(B_h\Delta K_h)^{\mathrm{T}}\sum_{l=1}^{N}\pi_{hl}P_lB_h\Delta K_h \tag{31}$$

and

$$2K_h^{\mathrm{T}}R_h\Delta K_h\leq \Delta K_h^{\mathrm{T}}R_h\Delta K_h+K_h^{\mathrm{T}}R_hK_h \tag{32}$$

By applying conditions (31) and (32) to inequality (30), (30) can be rewritten as

$$2(A_h+B_hK_h)^{\mathrm{T}}\sum_{l=1}^{N}\pi_{hl}P_l(A_h+B_hK_h)$$
$$+2(B_h\Delta K_h)^{\mathrm{T}}\sum_{l=1}^{N}\pi_{hl}P_lB_h\Delta K_h$$
$$-P_h+Q_h+2K_h^{\mathrm{T}}R_hK_h+2\Delta K_h^{\mathrm{T}}R_h\Delta K_h\leq 0 \tag{33}$$

We can see that

$$B_h^{\mathrm{T}}\sum_{l=1}^{N}\pi_{hl}P_lB_h\leq \gamma_2 I \tag{34}$$
$$R_h\leq \gamma_3 I \tag{35}$$

and

$$\Delta K_h^{\mathrm{T}}\Delta K_h\leq W_h \tag{36}$$

Therefore, (33) can be reformulated as

$$2(A_h+B_hK_h)^{\mathrm{T}}\sum_{l=1}^{N}\pi_{hl}P_l(A_h+B_hK_h)+2(\gamma_2+\gamma_3)W_h$$
$$-P_h+Q_h+2K_h^{\mathrm{T}}R_hK_h\leq 0 \tag{37}$$

Multiplying the right and left sides by $G>0$ and its transpose, respectively, and definition $P_h^{-1}=X_h$, $Y_h=K_hG$ and $Y_{h1}=K_{h1}G$. Then, (37) is also equivalent to

$$2(A_hG+B_hY_h)^{\mathrm{T}}\sum_{l=1}^{N}\pi_{hl}P_l(A_hG+B_hY_h)$$
$$+2(\gamma_2+\gamma_3)G^{\mathrm{T}}W_hG$$
$$-G^{\mathrm{T}}P_hG+G^{\mathrm{T}}Q_hG+2Y_h^{\mathrm{T}}R_hY_h\leq 0 \tag{38}$$

By applying the Schur complement lemma, (38) can be transformed into the inequality as follows

$$\begin{bmatrix} -G^{\mathrm{T}}P_hG & \sqrt{2(\gamma_2+\gamma_3)}G^{\mathrm{T}} & G^{\mathrm{T}} & \sqrt{2}Y_h^{\mathrm{T}} & \sqrt{2\pi_{h1}}\bar{A}_h^{\mathrm{T}} & \cdots & \sqrt{2\pi_{hN}}\bar{A}_h^{\mathrm{T}} \\ * & -W_h^{-1} & 0 & 0 & 0 & \cdots & 0 \\ * & * & -Q_h^{-1} & 0 & 0 & \cdots & 0 \\ * & * & * & -R_h^{-1} & 0 & \cdots & 0 \\ * & * & * & * & -X_1 & \cdots & 0 \\ * & * & * & * & * & \ddots & \vdots \\ * & * & * & * & * & \cdots & -X_N \end{bmatrix}\leq 0 \tag{39}$$

As for the nonlinear term $-G^{\mathrm{T}}P_hG$, it can be shown that

$$-G^{\mathrm{T}}P_hG\leq -(G)^{\star}+X_h \tag{40}$$

Therefore, (24) can be directly obtained. Similarly, taking into account (34), (25) can also be guaranteed. On the other hand, by substituting (9) into (36) and multiplying the right and left sides by G and its transpose, respectively, we have

$$-G^{\mathrm{T}}W_hG+(Y_{h1}-Y_h)^{\mathrm{T}}(Y_{h1}-Y_h)\leq 0 \tag{41}$$

As for the nonlinear term $-G^{\mathrm{T}}W_hG$, we use the fact that

$$-G^{\mathrm{T}}W_hG\leq -(G)^{\star}+W_h^{-1} \tag{42}$$

Finally, it is easy to deduce the inequality (26). Therefore, from (19), when $\tau=0$, one can get

$$\begin{aligned}&\Delta V(x(k),\theta(k),k)\\&\quad=\mathscr{E}[V(x(k+1))]-V(x(k))\\&\quad=x^{\mathrm{T}}(k)\Lambda x(k)\leq -\lambda_{\min}(-\Lambda)x^{\mathrm{T}}(k)x(k)\leq -\rho x^{\mathrm{T}}(k)x(k)\end{aligned} \tag{43}$$

where

$$\Lambda=[A_\eta+B_\eta(K_\eta+\Delta K_\eta)]^{\mathrm{T}}\sum_{\mu=1}^{N}\pi_{\eta\mu}P_\mu \times[A_\eta+B_\eta(K_\eta+\Delta K_\eta)]-P_\eta$$

and $\lambda_{\min}(-\Lambda)$ denotes the minimal eigenvalue of $(-\Lambda)$ and $\rho=\inf\{\lambda_{\min}(-\Lambda)\}$, for any $\eta,\mu\in\mathbb{M}$. Taking the expectation of both sides of (43) and summing from $k=0$ to $k=\infty$,

$$\mathscr{E}\{\sum_{k=0}^{\infty}\Delta V(x(k),\theta(k),k)\}=\mathscr{E}[V(x(\infty))]-V(x(0))$$
$$\leq -\rho\mathscr{E}\{\sum_{k=0}^{\infty}x^{\mathrm{T}}(k)x(k)\} \tag{44}$$

then the following inequality holds

$$\begin{aligned}\mathscr{E}\{\sum_{k=0}^{\infty} x^{\mathrm{T}}(k)x(k)\} &\leq \frac{1}{\rho}\{V(x(0)) - \mathscr{E}[V(x(\infty))]\} \\ &\leq \frac{1}{\rho}V(x(0)) \end{aligned} \tag{45}$$

which implies

$$\mathscr{E}\{\sum_{k=0}^{\infty} x^{\mathrm{T}}(k)x(k)|x_0, \theta_0\} \leq \frac{1}{\rho}V(x(0)) < \infty \tag{46}$$

From Definition 1, it can be got that the system is stochastically stable. This completes the proof.

From the above results, we can see that π_{hl} plays an important role in designing the RMPC, guaranteeing the robust stability and the optimal control performance of the closed-loop system. However, in practice, π_{hl} cannot be exactly obtained. Therefore, it is essential that the RMPC takes into account cases in which the transition probabilities cannot be determined with certainty. It follows that

$$\pi_{hl} = \tilde{\pi}_{hl} + \Delta\tilde{\pi}_{hl}, \quad \tilde{\pi}_{hl} \in [0, 1] \tag{47}$$

where $\tilde{\pi}_{hl}$ denotes the estimation of π_{hl} in accordance with (10). $\Delta\tilde{\pi}_{hl} \in [-\xi_{hl}, \xi_{hl}]$, where $\xi_{hl} \in [0, 1]$ denotes the admissible uncertainty. Therefore, the following theorem can be obtained.

Theorem 2: Consider the system (1), and let $x(k|k) = x(k)$ be the measured system state at each sampling time k. For given symmetric matrices $Q_h > 0$ and $R_h > 0$ and scalars $\gamma_1 > 0$, $\gamma_2 > 0$, $\gamma_3 > 0$, $\gamma_4 > 0$ and $\xi_{hl} > 0$, there exists a non-fragile state feedback controller (7) such that the performance index is minimized. Then the closed-loop system (12) is stochastically stable if there exist matrices $X_h > 0, G > 0, Y_h > 0, Y_{h1} > 0, M_h > 0$ and $W_h > 0$ satisfying

$$\begin{bmatrix} \Theta & \sqrt{2(\gamma_3-\gamma_2)}G^{\mathrm{T}} & G^{\mathrm{T}} & \sqrt{2}Y_h^{\mathrm{T}} & \sqrt{2\tilde{\pi}_{h1}}\bar{A}_h^{\mathrm{T}} & \cdots & \sqrt{2\tilde{\pi}_{hN}}\bar{A}_h^{\mathrm{T}} & \sqrt{2\xi_{h1}}\bar{A}_h^{\mathrm{T}} & \cdots & \sqrt{2\xi_{hN}}\bar{A}_h^{\mathrm{T}} \\ * & -\overline{W}_h & 0 & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \\ * & * & -\overline{Q}_h & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \\ * & * & * & -\overline{R}_h & 0 & \cdots & 0 & 0 & \cdots & 0 \\ * & * & * & * & -X_1 & \cdots & 0 & 0 & \cdots & 0 \\ * & * & * & * & * & \ddots & 0 & 0 & \cdots & 0 \\ * & * & * & * & * & * & -X_N & 0 & \cdots & 0 \\ * & * & * & * & * & * & * & -\overline{M}_h & \cdots & 0 \\ * & * & * & * & * & * & * & * & \ddots & 0 \\ * & * & * & * & * & * & * & * & * & -\overline{M}_h \end{bmatrix} \leq 0 \tag{48}$$

$$\begin{bmatrix} -\gamma_3 I + R_h & \sqrt{\tilde{\pi}_{h1}}B_h^{\mathrm{T}} & \cdots & \sqrt{\tilde{\pi}_{hN}}B_h^{\mathrm{T}} & \sqrt{\xi_{h1}}B_h^{\mathrm{T}} & \cdots & \sqrt{\xi_{hN}}B_h^{\mathrm{T}} \\ * & -X_1 & \cdots & 0 & 0 & \cdots & 0 \\ * & * & \ddots & 0 & 0 & \cdots & 0 \\ * & * & * & -X_N & 0 & \cdots & 0 \\ * & * & * & * & -\overline{M}_h & \cdots & 0 \\ * & * & * & * & * & \ddots & 0 \\ * & * & * & * & * & * & -\overline{M}_h \end{bmatrix} \leq 0 \tag{49}$$

$$\begin{bmatrix} -\gamma_2 I & \sqrt{\xi_{h1}}B_h^{\mathrm{T}} & \cdots & \sqrt{\xi_{hN}}B_h^{\mathrm{T}} \\ * & -X_1 & \cdots & 0 \\ * & * & \ddots & 0 \\ * & * & \cdots & -X_N \end{bmatrix} \leq 0 \tag{50}$$

$$\begin{bmatrix} -\gamma_4 I & \sqrt{\xi_{h1}}\bar{A}_h^{\mathrm{T}} & \cdots & \sqrt{\xi_{hN}}\bar{A}_h^{\mathrm{T}} \\ * & -X_1 & \cdots & 0 \\ * & * & \ddots & 0 \\ * & * & \cdots & -X_N \end{bmatrix} \leq 0 \tag{51}$$

and

$$\begin{bmatrix} -(I)^{\star} + \overline{M}_h & I^{\mathrm{T}} \\ * & -X_l \end{bmatrix} < 0 \tag{52}$$

where

$$\begin{aligned} \bar{A}_h &= A_h G + B_h Y_h, \quad \overline{W}_h = W_h^{-1}, \ \overline{M}_h = M_h^{-1} \\ \Theta &= -(G)^{\star} + X_h - 2\gamma_4 I, \quad Q_h^{-1} = \overline{Q}_h, \ R_h^{-1} = \overline{R}_h. \end{aligned}$$

Proof: Taking into account the proof of Theorem 1, it is obvious that condition (29) should be affected by the uncertainty (47) such that

$$\begin{aligned} &[A_h + B_h(K_h + \Delta K_h)]^{\mathrm{T}} \sum_{l=1}^{N} (\tilde{\pi}_{hl} + \Delta\tilde{\pi}_{hl}) P_l [A_h + B_h(K_h + \Delta K_h)] \\ &\quad - P_h + Q_h + (K_h + \Delta K_h)^{\mathrm{T}} R_h (K_h + \Delta K_h) \leq 0 \end{aligned} \tag{53}$$

To address the uncertainty, (53) can be changed such that

$$[A_h + B_h(K_h + \Delta K_h)]^{\mathrm{T}} \times [\sum_{l=1}^{N}(\tilde{\pi}_{hl} + \Delta\tilde{\pi}_{hl})P_l + \sum_{l=1}^{N}\xi_{hl}P_l - \sum_{l=1}^{N}\xi_{hl}P_l - \sum_{l=1}^{N}(\Delta\tilde{\pi}_{hl} + \xi_{hl})M_h + \sum_{l=1}^{N}\xi_{hl}M_h][A_h + B_h(K_h + \Delta K_h)] - P_h + Q_h + (K_h + \Delta K_h)^{\mathrm{T}} R_h (K_h + \Delta K_h) \leq 0 \quad (54)$$

Furthermore, it can be concluded that

$$[A_h + B_h(K_h + \Delta K_h)]^{\mathrm{T}}[\sum_{l=1}^{N}(\tilde{\pi}_{hl} - \xi_{hl})P_l + \sum_{l=1}^{N}(\Delta\tilde{\pi}_{hl} + \xi_{hl}) \times (P_l - M_h) + \sum_{l=1}^{N}\xi_{hl}M_h][A_h + B_h(K_h + \Delta K_h)] - P_h + Q_h + (K_h + \Delta K_h)^{\mathrm{T}} R_h (K_h + \Delta K_h) \leq 0 \quad (55)$$

It can also be guaranteed that

$$[A_h + B_h(K_h + \Delta K_h)]^{\mathrm{T}}[\sum_{l=1}^{N}(\tilde{\pi}_{hl} - \xi_{hl})P_l + \sum_{l=1}^{N}\xi_{hl}M_h] [A_h + B_h(K_h + \Delta K_h)] - P_h + Q_h + (K_h + \Delta K_h)^{\mathrm{T}} R_h (K_h + \Delta K_h) \leq 0 \quad (56)$$

and

$$\sum_{l=1}^{N}(\Delta\tilde{\pi}_{hl} + \xi_{hl})(P_l - M_h) < 0 \quad (57)$$

As for (56), it is equivalent to

$$[(A_h + B_h K_h) + B_h \Delta K_h]^{\mathrm{T}}(\sum_{l=1}^{N}\tilde{\pi}_{hl}P_l - \sum_{l=1}^{N}\xi_{hl}P_l + \sum_{l=1}^{N}\xi_{hl}M_h) [(A_h + B_h K_h) + B_h \Delta K_h] - P_h + Q_h + (K_h + \Delta K_h)^{\mathrm{T}} R_h (K_h + \Delta K_h) \leq 0 \quad (58)$$

It can also be shown that

$$(A_h + B_h K_h)^{\mathrm{T}}(\sum_{l=1}^{N}\tilde{\pi}_{hl}P_l - \sum_{l=1}^{N}\xi_{hl}P_l + \sum_{l=1}^{N}\xi_{hl}M_h)(A_h + B_h K_h) + 2(A_h + B_h K_h)^{\mathrm{T}}(\sum_{l=1}^{N}\tilde{\pi}_{hl}P_l - \sum_{l=1}^{N}\xi_{hl}P_l + \sum_{l=1}^{N}\xi_{hl}M_h)B_h\Delta K_h + (B_h \Delta K_h)^{\mathrm{T}}(\sum_{l=1}^{N}\tilde{\pi}_{hl}P_l - \sum_{l=1}^{N}\xi_{hl}P_l + \sum_{l=1}^{N}\xi_{hl}M_h)B_h\Delta K_h - P_h + Q_h + K_h^{\mathrm{T}} R_h K_h + 2K_h^{\mathrm{T}} R_h \Delta K_h + \Delta K_h^{\mathrm{T}} R_h \Delta K_h \leq 0 \quad (59)$$

Similar to the (31) and (32), the following inequality can be obtained that

$$2(A_h + B_h K_h)^{\mathrm{T}} \sum_{l=1}^{N}\tilde{\pi}_{hl}P_l(A_h + B_h K_h) + 2(A_h + B_h K_h)^{\mathrm{T}} \sum_{l=1}^{N}\xi_{hl}M_h(A_h + B_h K_h) - 2(A_h + B_h K_h)^{\mathrm{T}} \sum_{l=1}^{N}\xi_{hl}P_l(A_h + B_h K_h) - 2\Delta K_h^{\mathrm{T}} B_h^{\mathrm{T}} \sum_{l=1}^{N}\xi_{hl}P_l B_h \Delta K_h + 2\Delta K_h^{\mathrm{T}}(R_h + B_h^{\mathrm{T}}\sum_{l=1}^{N}\tilde{\pi}_{hl}P_l B_h + B_h^{\mathrm{T}}\sum_{l=1}^{N}\xi_{hl}M_h B_h)\Delta K_h - P_h + Q_h + 2K_h^{\mathrm{T}} R_h K_h \leq 0 \quad (60)$$

where

$$B_h^{\mathrm{T}} \sum_{l=1}^{N}\xi_{hl}P_l B_h \leq \gamma_2 I \quad (61)$$

and

$$R_h + B_h^{\mathrm{T}}\sum_{l=1}^{N}\tilde{\pi}_{hl}P_l B_h + B_h^{\mathrm{T}}\sum_{l=1}^{N}\xi_{hl}M_h B_h \leq \gamma_3 I \quad (62)$$

Applying the Schur complement, (50) and (49) can be derived from (61) and (62), respectively. Moreover, (60) can be expressed as follows

$$2(A_h + B_h K_h)^{\mathrm{T}} \sum_{l=1}^{N}\tilde{\pi}_{hl}P_l(A_h + B_h K_h) + 2(A_h + B_h K_h)^{\mathrm{T}} \sum_{l=1}^{N}\xi_{hl}M_h(A_h + B_h K_h) - 2(A_h + B_h K_h)^{\mathrm{T}} \sum_{l=1}^{N}\xi_{hl}P_l(A_h + B_h K_h) + 2(\gamma_3 - \gamma_2)W_h - P_h + Q_h + 2K_h^{\mathrm{T}} R_h K_h \leq 0 \quad (63)$$

Multiplying the right and left sides by G and its transpose, respectively, (63) is equivalent to

$$2(A_h G + B_h Y_h)^{\mathrm{T}} \sum_{l=1}^{N}\tilde{\pi}_{hl}P_l(A_h G + B_h Y_h) + 2(A_h G + B_h Y_h)^{\mathrm{T}} \sum_{l=1}^{N}\xi_{hl}M_h(A_h G + B_h Y_h) - 2(A_h G + B_h Y_h)^{\mathrm{T}} \sum_{l=1}^{N}\xi_{hl}P_l(A_h G + B_h Y_h) + 2(\gamma_3 - \gamma_2)G^{\mathrm{T}} W_h G - G^{\mathrm{T}} P_h G + G^{\mathrm{T}} Q_h G + 2Y_h^{\mathrm{T}} R_h Y_h \leq 0 \quad (64)$$

where

$$(A_h G + B_h Y_h)^{\mathrm{T}} \sum_{l=1}^{N}\xi_{hl}P_l(A_h G + B_h Y_h) \leq \gamma_4 I \quad (65)$$

Similarly, (51) can be derived very obvious. Then, (64) can also be rewritten as

$$\begin{aligned} &2(A_hG+B_hY_h)^{\mathrm{T}}\sum_{l=1}^{N}\tilde{\pi}_{hl}P_l(A_hG+B_hY_h)\\ &\quad+2(A_hG+B_hY_h)^{\mathrm{T}}\sum_{l=1}^{N}\xi_{hl}M_h(A_hG+B_hY_h)\\ &\quad-2\gamma_4I+2(\gamma_3-\gamma_2)G^{\mathrm{T}}W_hG-G^{\mathrm{T}}P_hG+G^{\mathrm{T}}Q_hG\\ &\quad+2Y_h^{\mathrm{T}}R_hY_h\leq 0 \end{aligned} \tag{66}$$

By using the Schur complement, inequality (66) can be converted to the inequality (48). As for (57), it holds if and only if the following inequality is satisfied

$$P_l-M_h<0 \tag{67}$$

Similarly, (67) can be written as follows

$$\begin{bmatrix} -M_h & I^{\mathrm{T}} \\ * & -X_l \end{bmatrix}<0 \tag{68}$$

where the following inequality should also be considered

$$-I^{\mathrm{T}}M_hI<-(I)^{\star}+\overline{M}_h \tag{69}$$

Then, (52) can be obtained, and the proof is complete.

In addition, it is necessary that another general case be considered, that in which the transition probability matrix is partially unknown. In other words, the transition probability matrix must be divided into unknown and known parts. For example, for the closed-loop system in (12) with four operation modes, the transition probability matrix can be described as

$$\prod=\begin{bmatrix} ? & \pi_{12} & ? & \pi_{14}\\ \pi_{21} & \pi_{22} & ? & \pi_{24}\\ \pi_{31} & \pi_{32} & ? & ?\\ ? & ? & \pi_{43} & \pi_{44} \end{bmatrix} \tag{70}$$

where "?" denotes the unknown elements. For any $h\in\mathbb{M}$ and $l\in\mathbb{M}$, we define

$$\mathbb{M}^h=\mathbb{M}_k^h\bigcup\overline{\mathbb{M}}_k^h \tag{71}$$

where

$$\begin{aligned} \mathbb{M}_k^h&=\{l:\pi_{hl}\ \text{is known},\ l\in\mathbb{M}\},\\ \overline{\mathbb{M}}_k^h&=\{l:\pi_{hl}\ \text{is unknown},\ l\in\mathbb{M}\} \end{aligned} \tag{72}$$

Furthermore, $\mathbb{M}_k^h=\{\pi_{h1},\cdots,\pi_{hm}\}$, where $\{\pi_{hi},\ i\in 1,\cdots,m,\forall\ 1\leq m\leq N\}$ refers to the known element in row h and column i. $\overline{\mathbb{M}}_k^h=\{\overline{\pi}_{h1},\cdots,\overline{\pi}_{h(N-m)}\}$, where $\{\overline{\pi}_{hj},j\in 1,\cdots,N-m\}$ refers to the unknown element in row h and column j. Similar to the proof of Theorem 2, the following theorem can also be obtained.

Theorem 3: Consider the system (1), and let $x(k|k)=x(k)$ be the measured system state at each sampling time k. For given symmetric matrices $Q_h>0$ and $R_h>0$ and scalars $\gamma_1>0,\gamma_2>0,\gamma_3>0$ and $\gamma_4>0$, there exists a non-fragile state feedback controller (7) such that the performance index is minimized. Then, the closed-loop system (12) is stochastically stable if there exist matrices $X_h>0,G>0,Y_h>0,Y_{h1}>0,W_h>0$ and $V_h>0$ satisfying

$$\begin{bmatrix} -(G)^{\star}+X_h-2\gamma_4I & \sqrt{2(\gamma_3-\gamma_2)}G^{\mathrm{T}} & G^{\mathrm{T}} & \sqrt{2}Y_h^{\mathrm{T}} & \sqrt{2\pi_{h1}}\bar{A}_h^{\mathrm{T}} & \cdots & \sqrt{2\pi_{hm}}\bar{A}_h^{\mathrm{T}} & \sqrt{2}\bar{A}_h^{\mathrm{T}}\\ * & -\overline{W}_h & 0 & 0 & 0 & \cdots & 0 & 0\\ * & * & -\overline{Q}_h & 0 & 0 & \cdots & 0 & 0\\ * & * & * & -\overline{R}_h & 0 & \cdots & 0 & 0\\ * & * & * & * & -X_1 & \cdots & 0 & 0\\ * & * & * & * & * & \ddots & 0 & 0\\ * & * & * & * & * & * & -X_m & 0\\ * & * & * & * & * & * & * & -\overline{V}_h \end{bmatrix}\leq 0 \tag{73}$$

$$\begin{bmatrix} -\gamma_3I+R_h & \sqrt{\pi_{h1}}B_h^{\mathrm{T}} & \cdots & \sqrt{\pi_{hm}}B_h^{\mathrm{T}} & B_h^{\mathrm{T}}\\ * & -X_1 & \cdots & 0 & 0\\ * & * & \ddots & 0 & 0\\ * & * & * & -X_m & 0\\ * & * & * & * & -\overline{V}_h \end{bmatrix}\leq 0 \tag{74}$$

$$\begin{bmatrix} -\gamma_2I & \sqrt{\pi_{h1}}B_h^{\mathrm{T}} & \cdots & \sqrt{\pi_{hm}}B_h^{\mathrm{T}}\\ * & -\overline{V}_h & \cdots & 0\\ * & * & \ddots & 0\\ * & * & * & -\overline{V}_h \end{bmatrix}\leq 0 \tag{75}$$

$$\begin{bmatrix} -\gamma_4I & \sqrt{\pi_{h1}}\bar{A}_h^{\mathrm{T}} & \cdots & \sqrt{\pi_{hm}}\bar{A}_h^{\mathrm{T}}\\ * & -\overline{V}_h & \cdots & 0\\ * & * & \ddots & 0\\ * & * & * & -\overline{V}_h \end{bmatrix}\leq 0 \tag{76}$$

and

$$\begin{bmatrix} -(I)^{\star}+\overline{V}_h & I^{\mathrm{T}}\\ * & -X_l \end{bmatrix}<0 \tag{77}$$

where

$$\bar{A}_h=A_hG+B_hY_h,\ \overline{V}_h=V_h^{-1},\quad \overline{W}_h=W_h^{-1}\quad Q_h^{-1}=\overline{Q}_h,\quad R_h^{-1}=\overline{R}_h.$$

Proof: Taking into account the proof of Theorem 1, it can be seen that the following term in condition (29) will be affected in the case of partially unknown transition probabilities

$$\begin{aligned} &[A_h+B_h(K_h+\Delta K_h)]^{\mathrm{T}}\sum_{l\in\mathbb{M}^h}\pi_{hl}P_l[A_h+B_h(K_h+\Delta K_h)]\\ &\quad-P_h+Q_h+(K_h+\Delta K_h)^{\mathrm{T}}R_h(K_h+\Delta K_h)\leq 0 \end{aligned} \tag{78}$$

According to (71), one obtains

$$[A_h + B_h(K_h + \Delta K_h)]^{\mathrm{T}}(\sum_{l\in\mathbb{M}_k^h} \pi_{hl}P_l + \sum_{l\in\overline{\mathbb{M}}_k^h} \pi_{hl}P_l) \times [A_h + B_h(K_h + \Delta K_h)] - P_h + Q_h + (K_h + \Delta K_h)^{\mathrm{T}} R_h (K_h + \Delta K_h) \leq 0 \quad (79)$$

which is also equivalent to

$$[A_h + B_h(K_h + \Delta K_h)]^{\mathrm{T}}[\sum_{l\in\mathbb{M}_k^h} \pi_{hl}P_l + \sum_{l\in\overline{\mathbb{M}}_k^h} \pi_{hl}(P_l - V_h) + \bar{\alpha}_h V_h][A_h + B_h(K_h + \Delta K_h)] - P_h + Q_h + (K_h + \Delta K_h)^{\mathrm{T}} R_h (K_h + \Delta K_h) \leq 0 \quad (80)$$

where

$$\bar{\alpha}_h = \sum_{l\in\overline{\mathbb{M}}_k^h} \pi_{hl} = 1 - \sum_{l\in\mathbb{M}_k^h} \pi_{hl}$$

If inequality (80) is to be satisfied, we need to have

$$[A_h + B_h(K_h + \Delta K_h)]^{\mathrm{T}}[\sum_{l\in\mathbb{M}_k^h} \pi_{hl}P_l + \bar{\alpha}_h V_h] \times [A_h + B_h(K_h + \Delta K_h)] - P_h + Q_h + (K_h + \Delta K_h)^{\mathrm{T}} R_h (K_h + \Delta K_h) \leq 0 \quad (81)$$

and

$$\sum_{l\in\overline{\mathbb{M}}_k^h} \pi_{hl}(P_l - V_h) < 0 \quad (82)$$

As for (81), it can be shown to be equivalent to

$$[(A_h + B_hK_h) + B_h\Delta K_h]^{\mathrm{T}}[\sum_{l\in\mathbb{M}_k^h} \pi_{hl}P_l + V_h - \sum_{l\in\mathbb{M}_k^h} \pi_{hl}V_h][(A_h + B_hK_h) + B_h\Delta K_h] - P_h + Q_h + (K_h + \Delta K_h)^{\mathrm{T}} R_h (K_h + \Delta K_h) \leq 0 \quad (83)$$

Furthermore, (83) can be obtained as

$$(A_h + B_hK_h)^{\mathrm{T}}(\sum_{l\in\mathbb{M}_k^h} \pi_{hl}P_l + V_h - \sum_{l\in\mathbb{M}_k^h} \pi_{hl}V_h)(A_h + B_hK_h) + 2(A_h + B_hK_h)^{\mathrm{T}}(\sum_{l\in\mathbb{M}_k^h} \pi_{hl}P_l + V_h - \sum_{l\in\mathbb{M}_k^h} \pi_{hl}V_h)B_h\Delta K_h + (B_h\Delta K_h)^{\mathrm{T}}(\sum_{l\in\mathbb{M}_k^h} \pi_{hl}P_l + V_h - \sum_{l\in\mathbb{M}_k^h} \pi_{hl}V_h)B_h\Delta K_h - P_h + Q_h + K_h^{\mathrm{T}}R_hK_h + 2K_h^{\mathrm{T}}R_h\Delta K_h + \Delta K_h^{\mathrm{T}}R_h\Delta K_h \leq 0 \quad (84)$$

Similar to (31) and (32), it can be rewritten as follows:

$$2(A_h + B_hK_h)^{\mathrm{T}} \sum_{l\in\mathbb{M}_k^h} \pi_{hl}P_l(A_h + B_hK_h) - 2(A_h + B_hK_h)^{\mathrm{T}} \sum_{l\in\mathbb{M}_k^h} \pi_{hl}V_h(A_h + B_hK_h) + 2(A_h + B_hK_h)^{\mathrm{T}}V_h(A_h + B_hK_h) + 2\Delta K_h^{\mathrm{T}}(R_h + B_h^{\mathrm{T}} \sum_{l\in\mathbb{M}_k^h} \pi_{hl}P_lB_h + B_h^{\mathrm{T}}V_hB_h)\Delta K_h - 2\Delta K_h^{\mathrm{T}}B_h^{\mathrm{T}} \sum_{l\in\mathbb{M}_k^h} \pi_{hl}V_hB_h\Delta K_h - P_h + Q_h + 2K_h^{\mathrm{T}}R_hK_h \leq 0 \quad (85)$$

To satisfy the inequality, the following conditions should be guaranteed

$$B_h^{\mathrm{T}} \sum_{l\in\mathbb{M}_k^h} \pi_{hl}V_hB_h \leq \gamma_2 I \quad (86)$$

and

$$R_h + B_h^{\mathrm{T}} \sum_{l\in\mathbb{M}_k^h} \pi_{hl}P_lB_h + B_h^{\mathrm{T}}V_hB_h \leq \gamma_3 I \quad (87)$$

Applying to the Schur complement lemma, (74) and (75) can be obtained. Then, (85) can be written as

$$2(A_h + B_hK_h)^{\mathrm{T}} \sum_{l\in\mathbb{M}_k^h} \pi_{hl}P_l(A_h + B_hK_h) + 2(A_h + B_hK_h)^{\mathrm{T}}V_h(A_h + B_hK_h) - 2(A_h + B_hK_h)^{\mathrm{T}} \sum_{l\in\mathbb{M}_k^h} \pi_{hl}V_h(A_h + B_hK_h) + 2(\gamma_3 - \gamma_2)W_h - P_h + Q_h + 2K_h^{\mathrm{T}}R_hK_h \leq 0 \quad (88)$$

Multiplying the right and left sides by G and its transpose, respectively, we have

$$2(A_hG + B_hY_h)^{\mathrm{T}} \sum_{l\in\mathbb{M}_k^h} \pi_{hl}P_l(A_hG + B_hY_h) + 2(A_hG + B_hY_h)^{\mathrm{T}}V_h(A_hG + B_hY_h) - 2(A_hG + B_hY_h)^{\mathrm{T}} \sum_{l\in\mathbb{M}_k^h} \pi_{hl}V_h(A_hG + B_hY_h) + 2(\gamma_3 - \gamma_2)G^{\mathrm{T}}W_hG - G^{\mathrm{T}}P_hG + G^{\mathrm{T}}Q_hG + 2Y_h^{\mathrm{T}}R_hY_h \leq 0 \quad (89)$$

where

$$(A_hG + B_hY_h)^{\mathrm{T}} \sum_{l\in\mathbb{M}_k^h} \pi_{hl}V_h(A_hG + B_hY_h) \leq \gamma_4 I \quad (90)$$

Inequality (90) can be converted to (76). Finally,

$$2(A_hG + B_hY_h)^{\mathrm{T}} \sum_{l\in\mathbb{M}_k^h} \pi_{hl}P_l(A_hG + B_hY_h) + 2(A_hG + B_hY_h)^{\mathrm{T}}V_h(A_hG + B_hY_h) - 2\gamma_4 I + 2(\gamma_3 - \gamma_2)G^{\mathrm{T}}W_hG - G^{\mathrm{T}}P_hG + G^{\mathrm{T}}Q_hG + 2Y_h^{\mathrm{T}}R_hY_h \leq 0 \quad (91)$$

Moreover, (73) can be obtained from (91). As for (82), it holds if and only if the following inequality is satisfied

$$P_l - V_h < 0 \quad (92)$$

This is equivalent to

$$\begin{bmatrix} -V_h & I^T \\ * & -X_l \end{bmatrix} < 0 \tag{93}$$

Furthermore, it should also be guaranteed that

$$-I^T V_h I < -(I)^\star + \overline{V}_h \tag{94}$$

Similarly, (77) can also be derived, and the proof is complete.

Lemma 3: The feasible solutions of optimization problem at time k are also feasible solutions for all time instants $t > k$. Therefore, at time k, if the optimization problem is feasible, for all time instants $t > k$, it is also feasible.

Proof: Assume that the optimization problem has feasible solution at time k. Note that (27) is the mere constraint dependent on the states of the system, then, for all future system states $x(k+\tau), \tau \geq 1$, it only needs to guarantee that (27) is feasible. At time k, when the optimization problem is feasible, $x^T(k+1|k)X_\eta^{-1}x(k+1|k) < \gamma_1$ holds. Moreover, at time $k+1$, for the measured state $x(k+1|k+1) = x(k+1)$, it can be obtained very easy that $x^T(k+1|k+1)X_\eta^{-1}x(k+1|k+1) < \gamma_1$ holds. In other words, the feasibility can be guaranteed based on the above analysis. Similarly, this argument can be continued for time $k+2$, $k+3, \cdots$.

Theorem 4: Consider a class of discrete-time MJLSs subject to operation mode disordering. According to the proof of lemma 1, the closed-loop system (12) is asymptotically stable with the state feedback gain matrix as $K_{h1} = Y_{h1}G^{-1}$ and $K_h = Y_hG^{-1}$.

Proof: Based on the proof of feasibility, at time $k+1$, we can construct the same feasible solution as that at time k. At time k, assume that the optimal solution is expressed as $\bigcup_k^* = \{X_h, G, Y_h, Y_{h1}, W_h, \gamma_1^*(k)\}$. Then, at time $k+1$, the feasible solution $\bigcup_{k+1} = \{X_h, G, Y_h, Y_{h1}, W_h, \gamma_1(k+1)\}$ can be constructed, which is the optimal solution at the time k. Moreover, it is easy to see that $\bigcup_{k+1}$ can satisfy the optimization problem. Therefore, based on the optimal theory, we can get $\gamma_1^*(k+1) \leq \gamma_1(k+1) = \gamma_1^*(k)$.

On the other hand, the following Lyapunov function $V(x(k|k), \theta(k), k) = x^T(k|k)P_{\theta(k)}x(k|k)$ need to be established, where $P_{\theta(k)}$ is the optimal solution of optimization problem at time k. Furthermore, based on the proof of feasibility, one has $x^T(k+1|k+1)P_{\theta(k+1)}x(k+1|k+1) \leq x^T(k+1|k+1)P_{\theta(k)}x(k+1|k+1)$. This is because $P_{\theta(k+1)}$ is optimal, but $P_{\theta(k)}$ is only feasible at time $k+1$. According to (19), when $\tau = 0$, one can get $x^T(k+1|k)P_{\theta(k)}x(k+1|k) \leq x^T(k|k)P_{\theta(k)}x(k|k)$. Meanwhile, it has $x^T(k+1|k+1)P_{\theta(k)}x(k+1|k+1) \leq x^T(k+1|k)P_{\theta(k)}x(k+1|k)$. So, the following inequality can be obtained $x^T(k+1|k+1)P_{\theta(k+1)}x(k+1|k+1) \leq x^T(k|k)P_{\theta(k)}x(k|k)$. The Lyapunov function is strictly decreasing for the closed-loop system. Therefore, the closed-loop system is asymptotically stable. The proof is completed.

TABLE 1. The bijective mapping relations between (η_1, η_2) and η.

η_1	η_2	η
1	1	1
2	1	2
1	2	3
2	2	4

V. NUMERICAL EXAMPLE

The following numerical example can be used to illustrate the effectiveness of the proposed control method. Consider a discrete-time MJLS with two modes.

Mode 1:

$$A_1 = \begin{bmatrix} -0.2 & 0.25 \\ -0.1 & -0.16 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix}$$

Mode 2:

$$A_2 = \begin{bmatrix} 0.1 & 0.15 \\ -0.2 & -0.1 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0.17 \\ -0.1 \end{bmatrix}$$

Without loss of generality, the transition probability matrix of the Markov chain is given by

$$\Pr = \begin{bmatrix} 0.2 & 0.8 \\ 0.4 & 0.6 \end{bmatrix}$$

The weighting matrices are chosen as $Q_h = diag\{0.5, 0.5\}$, $R_h = 6$, and $h \in \{1, 2, 3, 4\}$. The initial values are $x_0 = \begin{bmatrix} 0.2 \; 0.13 \end{bmatrix}^T$, $\gamma_1 = 2$, $\gamma_2 = 1$ and $\gamma_3 = 0.2$. In this example, for the sake of proving the effectiveness of proposed method, [43] can be considered. The following three cases should be shown: Considering mode disordering and applying the conventional RMPC method in [43], the simulation results are as follows

In this example, the operation modes of $\theta_1(k)$ take values in $\mathbb{M}_1 = [1, 2]$. When $\theta_1(k)$ is subject to mode disordering, the process is described as another Markov process $\{\theta_2(k), \mathbb{M}_2 = [1, 2]\}$. Therefore, according to the proposed method, a bijective mapping is defined as $\Psi(\eta) = \eta_1 + 2(\eta_2 - 1)$. The system corresponding to the operation modes after bijective mapping is shown in Table 1.

The following transition probability matrix can be obtained

$$\Pr = \begin{bmatrix} 0.1 & 0.2 & 0.3 & 0.4 \\ 0.3 & 0.1 & 0.1 & 0.5 \\ 0.1 & 0.4 & 0.2 & 0.3 \\ 0.2 & 0.3 & 0.2 & 0.3 \end{bmatrix}.$$

The simulation figures are shown in Figures 1-4. From Figure 2, we can see that the closed-loop system with operation mode disordering cannot guarantee better control performance using the conventional RMPC method in [43]. Although the asymptotic stability of the closed-loop system can also be achieved by the conventional RMPC method, the system needs a long time to achieve it. Furthermore, from Figure 4, it is obvious that the asymptotic stability of the closed-loop system with operation mode disordering based on the proposed control method is achieved approximately four-times faster than with the conventional RMPC method.

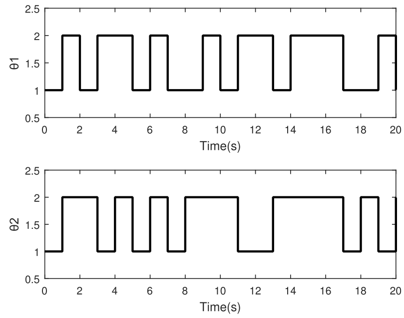

FIGURE 1. The original operation modes, θ_1, and disordered operation modes, θ_2, of the system.

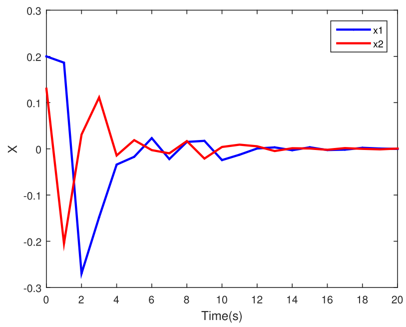

FIGURE 2. The state response of closed-loop system with mode disordering using the [43] method.

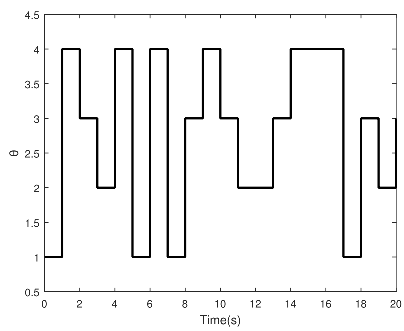

FIGURE 3. The operation modes, $\theta(k)$, of the system after bijective mapping.

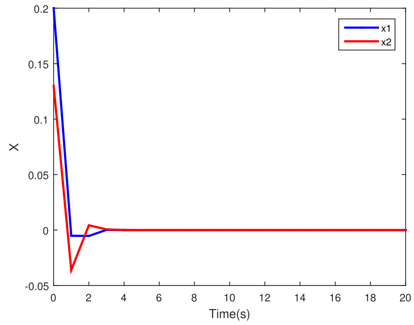

FIGURE 4. The state response of closed-loop system with mode disordering using the proposed method.

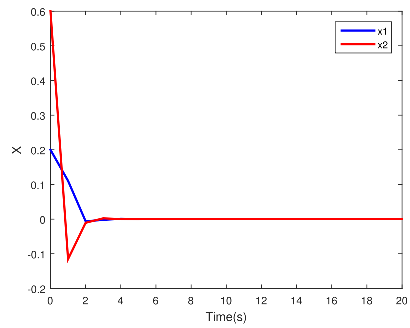

FIGURE 5. The state response of closed-loop system with mode disordering and the case of uncertainty in the transition probability matrix using the proposed method.

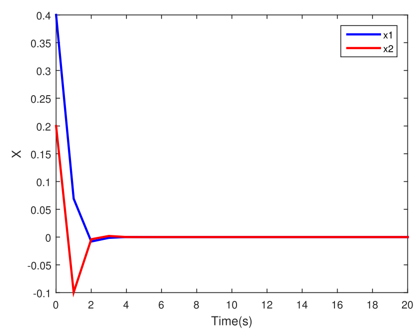

FIGURE 6. The state response of closed-loop system with mode disordering and the case of unknown transition probability using the proposed method.

In other words, it is very important that the case of operation mode disordering is considered in designing the control system, which can greatly reduce the impact of the operation mode disordering.

Considering the case of uncertainty in the transition probability matrix and applying the proposed control method in Theorem 2, we obtain the weighting matrices: $Q_h = diag\{0.5, 0.5\}$, $R_h = 6$, $h \in \{1, 2, 3, 4\}$. The initial values are $x_0 = \begin{bmatrix} 0.2 \ 0.6 \end{bmatrix}^{\mathrm{T}}$, $\gamma_1 = 2$, $\gamma_2 = 1$, $\gamma_3 = 10$ and $\gamma_4 = 1$. Additionally, $\xi_{hl} = 0.6$, where $h, l \in \{1, 2, 3, 4\}$. The simulation results are shown in Figure 5.

Considering the case of unknown transition probability matrix and applying the proposed control method in Theorem 3 of this paper. The weighting matrices are chosen as: $Q_h = diag\{0.5, 0.5\}$, $R_h = 6$, $h \in \{1, 2, 3, 4\}$. The initial values are $x_0 = \begin{bmatrix} 0.4 \ 0.2 \end{bmatrix}^{\mathrm{T}}$, $\gamma_1 = 2$, $\gamma_2 = 1$, $\gamma_3 = 10$ and $\gamma_4 = 1$. The simulation results are shown in Figure 6.

According to the simulation results shown in Figure 5 and Figure 6, when the transition probability matrix of the system with operation mode disordering is subjected to uncertainty or incomplete information, the closed-loop system can still achieve asymptotic stability very fast by using the proposed control method. It is worth mentioning that due to considering uncertainty and unknown cases, the conservativeness can be greatly reduced. All in all, the proposed control method can achieve better control performance, which can also guarantee the feasibility and strong robust stability.

VI. CONCLUSION

In this paper, for a class of discrete-time MJLSs subject to operation mode disordering, a RMPC method has been studied. To deal with operation mode disordering, a bijective mapping scheme between the original random process and a new random process has been introduced. At each sampling time, the complex "min-max" optimization problem is transformed into a convex optimization problem with LMIs, greatly reducing the complexity of solving the optimization problem. Furthermore, the conservativeness of the system has been reduced because probability information has been included in designing the predictive controller. Moreover, the cases of uncertain and unknown transition probabilities have also been considered. In all of these cases, the stochastic stability of the closed-loop system has been guaranteed. The simulation results illustrate that the proposed method is both feasible and very effective. The future work is that when Markovian jump linear systems subject to actuator and sensor faults, the robustness and stability of system will be researched.

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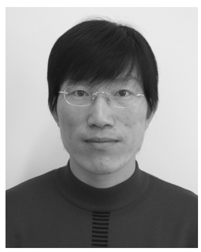

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