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Channel Constrained Multiple Selective Retransmissions for OFDM System: BER and Throughput Analysis

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ABSTRACT Hybrid automatic repeat request (HARQ) retransmission is an effective method to combat time-variant channel fading. Conventional HARQ methods initiate retransmission of failed packet without exploiting channel knowledge. When a packet fails, retransmission of data corresponding to high gain sub-carriers of an orthogonal frequency division multiplexing (OFDM) modulation is not necessary and deteriorates the throughput of the communication system. In this paper, we propose accumulated channel norm constrained selective retransmission (CNSR) at physical layer (PHY) in conjunction with HARQ method, which enhances throughput of the transceiver without compromising latency. When a packet fails, medium access control (MAC) layer requests retransmission of the failed packet. The proposed CNSR method initiates retransmission of data at PHY layer transmitted over low signal-to-noise ratio sub-carriers using outdated channel information. The transceiver pair continues retransmission of the failed packet until target channel norm of the sub-carriers is achieved or maximum number of retransmissions are completed. We maximize throughput by optimizing threshold on channel norm for OFDM modulation. We also present an upper bound on bit error rate and throughput analysis of the proposed method. The proposed method achieves higher throughput as compared with the existing Chase combining with selective repeat and conventional Chase combining schemes with and without forward error correction codes.

INDEX TERMS Hybrid ARQ, OFDM, retransmission, selective retransmission, throughput, LDPC, CCSR, cross layer.

I. INTRODUCTION

Contemporary communication systems [1], [2] employ advanced signal processing techniques such as precoding, beamforming and space-time block coding [3] in order to enhance spectral efficiency and throughput. Resource optimization techniques in [1] achieve higher spectral efficiency and guarantee quality of service (QoS). Despite the advanced signal processing technologies, due to dynamic nature of wireless channel, it is not viable to establish an error-free link in physical layer (PHY). In such scenarios, FEC codes are adopted to protect data from the potential channel errors [4]. In the event of poor channel realizations when average SNR is low and FEC codes fail to recover from channel errors, retransmission techniques such as ARQ and HARQ [5], [6] ameliorate the communication link by partial or full retransmission of unsuccessfully decoded packets. In ARQ method, transmitter retransmits failed packet when negative acknowledgment (NACK) is received. The ARQ retransmissions continue until packet is successfully received or maximum allowed retransmissions are reached. Note that HARQ integrates FEC and ARQ methods to recover from channel errors and contemporary communication standards adopted HARQ technology [1], [7].

Long term evolution (LTE) [1] employs two layers of retransmission. That is, ARQ and HARQ in radio link control (RLC) and medium access control (MAC), respectively. Quick retransmission when MAC packet fails by the HARQ process in MAC layer reduces overall latency of the system, whereas ARQ layer combats residual errors from HARQ by retransmission of the failed packet. Most of the failed packets have fewer erroneous bits and receiver can recover from errors by the retransmission of partial packet. Thus, unnecessary full retransmission in response to NACK is redundant and lowers the throughput.

Chase combining HARQ (CC-HARQ) and incremental redundancy HARQ (IR-HARQ) are two major types of HARQ method. When a data packet fails, IR-HARQ sends more parity bits to the receiver to correct errors instead of full retransmission in CC-HARQ method. Both IR-HARQ and CC-HARQ method do not exploit channel knowledge. IR-HARQ method achieves higher throughput as compared to CC-HARQ method in moderate and high SNR regimes. High spectral efficiency can be achieved by omitting unnecessary retransmission of reliable bits instead of full retransmission. Thanks to orthogonal frequency division multiplexing (OFDM), which allows marking of susceptible bits for selective retransmission by using SNR of sub-carriers as a measure of reliability of the decoded bits.

A. RELATED WORK

Error correction codes and ARQ has been the main focus of the existing work on HARQ [8], [9]. Throughput of capacity achieving FEC codes is optimized for Rayleigh fading channel in [10]. Performance analysis of HARQ over Rayleigh fading channel based on mutual information is provided in [11]. Work in [12]–[18] studies optimal power allocation of CC-HARQ. A fixed approach of partial retransmission without exploiting outdated channel information at PHY is investigated in [19]–[22]. Selective retransmission method in [23] embeds a retransmission sub-layer in PHY for single-input single-output OFDM (SISO-OFDM) system. For a MIMO-OFDM system, selective retransmission using channel condition number as sub-carrier quality measure is proposed in [24]. Significant throughput gain in [25] is observed for selective retransmission using orthogonal space-time block (OSTB) code [26]. In [27], selective retransmission is proposed in the presence of burst errors. Moreover, [28] proposes and analyzes the selective retransmission protocol for the parallel channel. Partial retransmission of the failed packet is considered in [29] to minimize the delay. Note that methods in [20], [23], and [26] consider one selective retransmission with target to achieve bit-error rate (BER) of full retransmission for OFDM modulation. These methods not only achieve the target BER, but also improves throughput of the system by omitting unnecessary retransmission of data symbols of the failed packet transmitted over the good quality OFDM sub-carriers.

Cross-layer selective Chase combining method in [30] considers multiple transmission of a failed packet with selective retransmission in PHY layer. Method in [30] requests selective retransmission of data symbols, which have poor SNR before decoding, which results in higher latency in high SNR regime. In addition to the selective retransmission, adaptive modulation and coding scheme is considered in [31] to improve throughput. Furthermore, work in [30] and [31] compares norm of each sub-carrier of the current transmission round with threshold τ for selective retransmission despite the fact that sum of channel norm of some sub-carriers is larger than the required channel norm to avoid packet failure.

B. MOTIVATION

This work builds on the selective retransmission method in [30]. In [30], receiver initiates selective retransmission before data decoding, which potentially lowers throughput and increases overall latency of the communication system. In this work, we propose multiple rounds of selective retransmission when a packet fails in PHY until MAC packet is successfully delivered to the receiver or maximum allowed selective transmission rounds are reached. Selective retransmission in [30] combines observations of previous retransmissions of a failed packet to enhance reliability of the decoded bits. Contrary to method in [30], in this work, we apply threshold τ on accumulated sub-carrier gains of OFDM signaling to mark the sub-carriers for selective retransmission. That is, a sub-carrier, which has accumulated sub-carrier norm larger than threshold $\left(\sum_{j=1}^{J} \|H_j(\ell)\|^2 \geq \tau\right)$ are omitted from selective retransmission. Note that J is a counter for the number of transmissions at MAC layer and $H_j(\ell)$ is channel gain of ℓ-th sub-carrier of an OFDM frame of the j-th retransmission. The proposed CNSR method requests retransmission of OFDM sub-carriers of the failed packet, which have low SNR instead of full retransmission. Furthermore, methods in [23], [24], and [26] did not optimize amount of information to be retransmitted selectively with objective to enhance spectral efficiency. The proposed CNSR method optimizes threshold on accumulated norm of gains of the OFDM sub-carriers of the failed packet for selective retransmission.

C. CONTRIBUTION

We propose a bandwidth efficient and low complexity CNSR method for OFDM signaling. The main contributions of this work are as follows:

- The proposed CNSR method initiates selective retransmission in the event of packet failure using outdated channel information and considers accumulated channel norm of each OFDM sub-carrier of the observations of the failed packet from multiple retransmission to select sub-carriers for selective retransmission. This approach avoids unnecessary selective retransmission rounds and reduces latency.
- We also present tight upper bound on BER and throughput analysis of our method. The comparison between analytical results and Monte Carlo results reveals that analytical and Monte Carlo simulation results have marginal gap.
- We optimize threshold on sub-carrier gains to select symbols for retransmission in order to enhance throughput. We also compare latency of the proposed method with [30] and conventional retransmission approaches.
- The simulation results demonstrates throughput gain of the proposed CNSR method as compared to the conventional retransmission methods and [30] for uncoded and coded transmission.

The rest of our manuscript is organized as follows. We present system model for channel norm constrained multiple selective retransmissions of OFDM modulation in Section II. We conclude our work in Section VII.

II. SYSTEM MODEL

We consider SISO-OFDM modulation over frequency selective channel $\mathbf{h}$ of order L with Rayleigh fading. The elements of channel gain vector $\mathbf{h}$ are independent and identically distributed (i.i.d.). Note that $h_i \sim \mathcal{N}(0, \sigma^2 = \frac{1}{L})$. The OFDM modulation converts a frequency selective channel vector $\mathbf{h}$ into N_s parallel flat fading sub-carriers $H(\ell) \in C$, where ℓ is index to the sub-carrier [23] and $\ell = 1, 2, \cdots, N_s$. We assume quasi-static channel which has coherent time longer than the duration of one OFDM symbol. The bit vector of length $N_s \log_2 M$ is mapped to M quadrature amplitude modulation (QAM) symbols vector $\mathbf{s} = [s(1), s(2), \cdots, s(N_s)]^T$. In the event of failed packet, instead of full retransmission, receiver requests retransmission of those information symbols that suffer from poor quality OFDM sub-channels with $|H(\ell)|^2 < \tau$. The transceiver continuously retransmits selective symbols belong to other poor quality sub-carriers until packet transmission is successful or constraint on channel norm of all sub-carriers is satisfied or maximum number of allowed retransmissions are reached. The vector model of observations for joint detection, which combines first observation and $J-1$ selective retransmissions (J transmissions) and joint detection are

$$\mathbf{y}(\ell) = \underbrace{[H_1(\ell)\, H_2(\ell)\, \ldots\, H_J(\ell)]^T}_{\mathcal{H}_J(\ell)} s(\ell) + \mathbf{w}(\ell),$$
$$\hat{s}(\ell) = s(\ell) + \tilde{u}(\ell), \quad (1)$$

respectively, where $\mathbf{w}$ represents AWGN noise, $\tilde{u}(\ell) = \frac{\mathcal{H}_J^H(\ell)w(\ell)}{\|\mathcal{H}_J(\ell)\|^2}$ with $\tilde{u}(\ell) \sim N\left(0, \frac{N_o}{2\|\mathcal{H}_J(\ell)\|^2}\right)$ and $\mathcal{H}_J(\ell)$ is a joint channel vector of J transmissions for the ℓ-th sub-carrier. Next, we present CNSR algorithm.

III. CHANNEL NORM CONSTRAINED SELECTIVE RETRANSMISSION

This section presents the proposed CNSR method in PHY layer. The proposed method achieves error-free communication by retransmitting information symbols selectively belonging to the OFDM sub-channels, which have accumulated channel norm of the J transmissions below threshold. That is, $\|\mathcal{H}_J(\ell)\|^2 < \tau$. In one ARQ round of the proposed CNSR method, MAC layer of the receiver requests selective retransmission of poor SNR sub-carriers which have $\|\mathcal{H}_J(\ell)\|^2 < \tau$ until μ iterations are met or CRC check is satisfied. For the J-th transmission or $(J-1)$-th selective retransmission, if needed at modulation layer, receiver requests retransmission of poor quality sub-carrier which have $\|\mathcal{H}_J(\ell)\|^2 = \sum_{j=1}^{J} \|H_j(\ell)\|^2 < \tau$ for $J = 1, \ldots, \mu - 1$, where μ is the number of allowed transmissions of a packet at PHY layer in an ARQ transmission round. Note that $J = 1$ represents first transmission of a packet and $\mathcal{T}_J$ is set of indices of sub-carriers with low accumulated gain up to the J-th selective retransmission.

The selectively retransmitted data symbols are appended with the new OFDM symbol. In order to serve request for selective retransmission, each OFDM symbol can carry new data symbols and poor quality data symbols of a failed packet. The set of indices of low SNR quality sub-carriers of $(J+1)$-th selective retransmission $\mathcal{T}_{J+1}$ is the subset of set of poor quality indices $\mathcal{T}_J$ of the J-th selective retransmission. That is, $\mathcal{T}_{J+1} \subseteq \mathcal{T}_J$. The receiver stops selective retransmission at PHY when $\|\mathcal{H}_J(\ell)\|^2 \geq \tau$ or $J = \mu$ or packet transmission is successful in an ARQ transmission round. For the J-th transmission of a failed packet, receiver jointly detects a data symbol by combining at most μ observations of an OFDM sub-carrier. We optimize threshold τ on the accumulative channel norm in order to maximize throughput of the communication system. The flow graph of CNSR selective retransmission method is as follows:

Algorithm 1 CNSR Protocol

1: Set transmission counter $J = 1$, of the packet index K.
2: **if** CRC of the J-th transmission is successful **then** $K = K + 1$ and go to 1
3: **if** allowed transmissions are reached $J = \mu$ **then** go to 6
4: Initiate selective retransmission for the OFDM sub-carriers with accumulative sub-carrier gain norm $\sum_{j=1}^{J} \|H_j(\ell)\|^2 < \tau$ and preserve observations for joint detection. Set $J = J + 1$
5: Perform joint detection, CRC check and go to 2
6: Discard observation, $K = K + 1$ and go to 1

We present bound on the BER of the proposed CNSR in the next section.

IV. BER ANALYSIS

Now, we provide BER analysis of the proposed CNSR approach, which allows maximum μ transmissions at MAC layer. The proposed method is different from the Chase combining method in the sense that instead of transmitting a failed packet in full, CNSR method retransmits data symbols belong to low SNR sub-carriers and have accumulated channel norm up to J transmissions $\|\mathcal{H}_J(\ell)\|^2 < \tau$. We derive BER bound for joint detection of J retransmissions of the proposed method. We optimize throughput as a function of threshold τ on accumulated channel norm of the OFDM sub-carriers.

A. BER UPPER BOUND

Now, we derive upper bound on BER for the proposed CNSR method for joint detection up to μ transmissions in one ARQ round at PHY layer. Note that there are at most μ transmissions of a MAC packet in one ARQ round. In conventional MAC retransmission with Chase combining, transmitter transmits failed packet in full. The proposed CNSR method retransmits data symbols which correspond to the sub-carriers with accumulated channel norm below threshold τ. The ℓ-th sub-channel of OFDM modulation is a candidate for selective retransmission when a packet failure occurs after J transmissions if joint channel norm $\|\mathcal{H}_J(\ell)\|^2 < \tau$, where $J = 1, 2, \ldots, \mu - 1$. A MAC layer initiates $(J+1)$-th transmission of failed packet if joint channel norm of J transmissions $\|\mathcal{H}_J(\ell)\|^2 < \tau$. We denote probability that $\chi_J = \|\mathcal{H}_J(\ell)\|^2 < \tau$ by $m_J = \Pr(\chi_J < \tau)$, which is the probability that an OFDM sub-carrier will be marked for selective retransmission when packet failure occurs, where χ_J is a random variable with chi-2 distribution with degree of freedom $2J$. Thus, probability that a ℓ-th sub-carrier will be marked for retransmission after J transmissions is [32]

$$m_J = 1 - \frac{1}{(J-1)!}\Gamma\left(J, \frac{\tau}{2\sigma^2}\right), \tag{2}$$

where $\Gamma(n,x)$ is an incomplete gamma function defined as $\Gamma(n,x) = (n-1)!\exp(-x)\sum_{k=0}^{n-1}\frac{x^k}{k!}$. For probability of error of the joint detection at the end of J transmissions, we define events $\xi_1, \ldots, \xi_{J-1}, \xi_J$. The event ξ_j occurs if joint channel norm $\|\mathcal{H}_j(\ell)\|^2 \geq \tau$ given that $\|\mathcal{H}_{j-1}(\ell)\|^2 < \tau$ for $j = 1, \ldots, J-1$ and the retransmission of data symbol corresponding to ℓ-th sub-carrier stops. An event ξ_J occurs if $\|\mathcal{H}_{J-1}(\ell)\|^2 < \tau$ and the channel norm of J-th transmission $\|H_J(\ell)\|^2 \in R^+$. The norm $\chi_J = \|\mathcal{H}_J(\ell)\|^2$ has chi-2 distribution of degree of freedom $2J$. The following proposition provides insight of the BER upper bound on joint detection for J transmission rounds:

Proposition 1: The upper BER bound for an un-coded transceiver for joint decoding at the end of J transmission rounds for CNSR method, where $J = 1, 2, \ldots, \mu$ is

$$\begin{aligned} P_{e_J} \leq& \frac{c}{12}\left(\frac{\rho}{\sigma}\right)^2\frac{1}{(J-2)!}\Gamma\left(J-1, \frac{\tau}{2\sigma^2}\right) \\ &+\frac{c}{4}\left(\frac{\rho_1}{\sigma}\right)^2\frac{1}{(J-2)!}\Gamma\left(J-1, \frac{\tau}{2\sigma^2}\right) \\ &+\frac{c}{12}\left(\frac{\rho}{\sigma}\right)^{2J}\left(1-\frac{1}{(J-2)!}\Gamma\left(J-1, \frac{\tau}{2\rho^2}\right)\right) \\ &+\frac{c}{4}\left(\frac{\rho_1}{\sigma}\right)^{2J}\left(1-\frac{1}{(J-2)!}\Gamma\left(J-1, \frac{\tau}{2\rho_1^2}\right)\right), \end{aligned} \tag{3}$$

where $c = \frac{4(\sqrt{M}-1)}{\sqrt{M}}$ and $g = \frac{3\log_2(M)}{(M-1)}$ are modulation constants [33] for M-QAM constellation. Note that $\rho = \sqrt{\frac{\sigma^2 N_o}{g\sigma^2+N_o}}$ and $\rho_1 = \sqrt{\frac{\sigma^2 N_o}{g_1\sigma^2+N_o}}$.

Proof: The event ξ_j, where $j = 1, \ldots, J-1$ occurs, when $\|\mathcal{H}_j(\ell)\|^2 \geq \tau|_{\|\mathcal{H}_{j-1}(\ell)\|^2<\tau}$. That is, joint channel norm of the ℓ-th OFDM sub-carrier satisfies the threshold τ and will not be a candidate for the selective retransmission when a packet fails. Also the event ξ_J occurs if $\|\mathcal{H}_{J-1}(\ell)\|^2 < \tau$ and $\|\mathcal{H}_J(\ell)\|^2 \in R^+$. That is, $\|\mathcal{H}_J(\ell)\|^2 \in R^+|_{\|\mathcal{H}_{J-1}(\ell)\|^2<\tau}$. The transmission of a MAC packet is terminated if $\|\mathcal{H}_j(\ell)\|^2 \geq \tau$ or packet is successfully received for all $\ell = 1, \ldots, N$. The probability of error of joint detection after J transmissions is

$$P_{e_J} = \sum_{j=1}^{J} E_H\left[P_{e|\xi_j}\Pr(\xi_j)\right] \tag{4}$$

First, we evaluate $E_H\left[P_{e|\xi_j}\Pr(\xi_j)\right]$ for $j = 1, \ldots, J-1$, which is joint detection using j channel gains of the ℓ-th OFDM sub-carrier with $\|\mathcal{H}_j(\ell)\|^2 \geq \tau|_{\|\mathcal{H}_{j-1}(\ell)\|^2<\tau}$. Note that random variable $\chi_j = \|\mathcal{H}_j(\ell)\|^2$ has chi-2 distribution with degree of freedom $2j$ and channel gains of each transmission over ℓ-th sub-carriers are independent [34] and can be written in terms of Q-function [32] as,

$$\begin{aligned} E_H\left[P_{e|\xi_j}\Pr(\xi_j)\right] &= \Pr(\xi_j)E_{H|\xi_j}\left[c\,Q\left(\sqrt{g\frac{\chi_j}{N_0}}\right)\right] \\ &= \Pr(\xi_j)E_{H|\xi_j}\left[c\,Q\left(\sqrt{g\frac{\chi_x+\chi_z}{N_0}}\right)\right], \end{aligned}$$

where $\chi_x = \|\mathcal{H}_{j-1}(\ell)\|^2$ and $\chi_z = |H_j(\ell)|^2$ are chi-2 random variables of degree of freedom $2j-2$ and 2, respectively. We denote probability density function of random variable χ_x and χ_z by $f_X(x)$ and $f_{X_z}(z)$, respectively. Following the probability density function and cumulative distribution function of chi-2 random variable [32], [35] we have

$$\begin{aligned} &E_H\left[\Pr(\xi_j)P_{e|\xi_j}\right] \\ &= c\Pr(\xi_j)E_{H|\xi_j}\left[Q\left(\sqrt{g\frac{\chi_x+\chi_z}{N_0}}\right)\right] \\ &\leq \frac{c}{12}\int_0^\tau \exp(-g\frac{x}{2N_0})f_X(x)\int_{\tau-x}^\infty \exp(-g\frac{z}{2N_0})f_{X_z}(z)dzdx \\ &\quad+\frac{c}{4}\int_0^\tau \exp(-g\frac{2x}{3N_0})f_X(x)\int_{\tau-x}^\infty \exp(-g\frac{2z}{3N_0})f_{X_z}(z)dzdx \\ &= \frac{c}{12}\int_0^\tau \exp\left(-g\frac{x}{2N_0}\right)\frac{x^{(j-2)}}{(j-2)!(2\sigma^2)^{(j-1)}}\exp\left(-\frac{x}{2\sigma^2}\right)dx. \\ &\quad\times\int_{\tau-x}^\infty \exp\left(-g\frac{z}{2N_0}\right)\frac{1}{2\sigma^2}\exp\left(-\frac{z}{2\sigma^2}\right)dz \\ &\quad+\frac{c}{4}\int_0^\tau \exp\left(-g\frac{2x}{3N_0}\right)\frac{x^{(j-2)}}{(j-2)!(2\sigma^2)^{(j-1)}}\exp\left(-\frac{x}{2\sigma^2}\right). \\ &\quad\times\int_{\tau-x}^\infty \exp\left(-g\frac{2z}{3N_0}\right)\frac{1}{2\sigma^2}\exp\left(-\frac{z}{2\sigma^2}\right)dzdx \end{aligned} \tag{5}$$

Now, let $\rho = \sqrt{\frac{\sigma^2 N_o}{g\sigma^2+N_o}}$, $\rho_1 = \sqrt{\frac{\sigma^2 N_o}{g_1\sigma^2+N_o}}$ and $g_1 = \frac{4g}{3}$, then

$$\begin{aligned} &E_H\left[\Pr(\xi_j)P_{e|\xi_j}\right] \\ &\leq \frac{c}{12}\int_0^\tau \frac{x^{(j-2)}}{(j-2)!(2\sigma^2)^{(j-1)}} \\ &\quad\times\exp\left(-g\frac{x}{2\rho^2}\right)\cdot\frac{1}{2\sigma^2}\int_{\tau-x}^\infty \exp\left(-\frac{z}{2\rho^2}\right)dzdx \\ &\quad+\frac{c}{4}\int_0^\tau \frac{x^{(j-2)}}{(j-2)!(2\sigma^2)^{(j-1)}}\exp\left(-x/2\rho_1^2\right) \\ &\quad\cdot\frac{1}{2\sigma^2}\int_{\tau-x}^\infty \exp\left(-\frac{z}{2\rho_1^2}\right)dzdx \end{aligned} \tag{6}$$

$$
\begin{aligned}
&E_H\left[\Pr(\xi_j)P_{e|\xi_j}\right]\\
&\quad\leq \frac{c}{12}\int_0^{\tau}\frac{x^{(j-2)}}{(j-2)!(2\sigma^2)^{(j-1)}}\\
&\qquad\times\exp\left(-\frac{x}{2\rho^2}\right)\cdot\left(\frac{\rho}{\sigma}\right)^2\exp\left(\frac{-\tau}{2\rho^2}\right)\exp\left(\frac{x}{2\rho^2}\right)dx\\
&\qquad+\frac{c}{4}\int_0^{\tau}\frac{x^{(j-2)}}{(j-2)!(2\sigma^2)^{(j-1)}}\exp\left(-g\frac{x}{2\rho_1^2}\right)dx\\
&\qquad\times\left(\frac{\rho_1}{\sigma}\right)^2\exp\left(\frac{-\tau}{2\rho_1^2}\right)\exp\left(\frac{x}{2\rho_1^2}\right)dx, \qquad (7)
\end{aligned}
$$

We can simplify (7) as

$$
\begin{aligned}
&E_H\left[\Pr(\xi_j)P_{e|\xi_j}\right]\\
&\quad\leq \frac{c}{12}\left(\frac{\rho}{\sigma}\right)^2\frac{1}{(j-1)!}\left(\frac{\tau}{2\sigma^2}\right)^{(j-1)}\\
&\qquad\times\exp\left(\frac{-\tau}{2\rho^2}\right)+\frac{c}{4}\left(\frac{\rho_1}{\sigma}\right)^2\frac{1}{(j-1)!}\left(\frac{\tau}{2\sigma^2}\right)^{(j-1)}\exp\left(\frac{-\tau}{2\rho_1^2}\right), \qquad (8)
\end{aligned}
$$

where $j=1,\ldots,J-1$. The event ξ_J occurs if $\|\mathcal{H}_{J-1}(\ell)\|^2<\tau$. For event ξ_J, the joint channel $\|\mathcal{H}_J(\ell)\|^2$ has chi-2 distribution of order $2J$. Note that $\|\mathcal{H}_J(\ell)\|^2\in R^+$ with $\|\mathcal{H}_{J-1}(\ell)\|^2<\tau$ and channel gain of the last transmission $|H_J(\ell)|^2\in R^+$.

$$
\begin{aligned}
E_H\left[P_{e|\xi_J}\Pr(\xi_J)\right]&=\Pr(\xi_J)E_{H|\xi_J}\left[c\,Q\left(\sqrt{g\frac{\chi_J}{N_0}}\right)\right]\\
&=\Pr(\xi_J)E_{H|\xi_J}\left[c\,Q\left(\sqrt{g\frac{\chi_x+\chi_z}{N_0}}\right)\right],
\end{aligned}
$$

where $\chi_x=\|\mathcal{H}_{J-1}(\ell)\|^2$ and $\chi_z=|H_J(\ell)|^2$ are chi-2 random variables of order $2J-2$ and 2, respectively. We have

$$
\begin{aligned}
&E_H\left[\Pr(\xi_J)P_{e|\xi_J}\right]\\
&\quad=cE_{H|\xi_J}\left[\Pr(\xi_J)Q\left(\sqrt{g\frac{\chi_x+\chi_z}{N_0}}\right)\right]\\
&\quad\leq\frac{c}{12}\int_0^{\tau}\exp(-g\frac{x}{2N_0})f_X(x)\cdot\int_0^{\infty}\exp(-g\frac{z}{2N_0})\\
&\qquad\times f_{X_z}(z)dz\,dx+\frac{c}{4}\int_0^{\tau}\exp(-g\frac{4x}{3.2N_0})f_X(x)\\
&\qquad\cdot\int_0^{\infty}\exp(-g\frac{4z}{3.2N_0})f_{X_z}(z)dz\,dx \qquad (9)
\end{aligned}
$$

$$
\begin{aligned}
&E_H\left[\Pr(\xi_J)P_{e|\xi_J}\right]\\
&\quad\leq\frac{c}{12}\int_0^{\tau}\exp\left(-g\frac{x}{2N_0}\right)\\
&\qquad\times\frac{x^{(J-2)}}{(J-2)!(2\sigma^2)^{(J-1)}}\exp\left(-\frac{x}{2\sigma^2}\right)\frac{1}{2\sigma^2}\exp\left(-\frac{x_z}{2\sigma^2}\right)\\
&\qquad\times dz\,dx+\frac{c}{4}\int_0^{\infty}\exp\left(-g\frac{z}{2N_0}\right)\int_0^{\tau}\exp\left(-g\frac{4x}{3.2N_0}\right)\\
&\qquad\frac{x^{(J-2)}}{(J-2)!(2\sigma^2)^{(J-1)}}\exp\left(-\frac{x}{2\sigma^2}\right). \qquad (10)\\
&\int_0^{\infty}\exp\left(-g\frac{4z}{3.2N_0}\right)\frac{1}{2\sigma^2}\exp\left(-\frac{x_z}{2\sigma^2}\right)dz\,dx
\end{aligned}
$$

$$
\begin{aligned}
&=\frac{c}{12}\left(\frac{\rho}{\sigma}\right)^{2J}\left(1-\frac{1}{(J-2)!}\Gamma\left(J-1,\frac{\tau}{2\rho^2}\right)\right)\\
&\quad+\frac{c}{4}\left(\frac{\rho_1}{\sigma}\right)^{2J}\left(1-\frac{1}{(J-2)!}\Gamma\left(J-1,\frac{\tau}{2\rho_1^2}\right)\right), \qquad (11)
\end{aligned}
$$

We rewrite (4) as follows:

$$
P_{e_J}=\textstyle\sum_{j=1}^{J-1}E_H\left[P_{e|\xi_j}\Pr(\xi_j)\right]+E_H\left[P_{e|\xi_J}\Pr(\xi_J)\right] \qquad (12)
$$

Now, by substituting (11) and (8) in (12), we have

$$
\begin{aligned}
P_{e_J}\leq&\frac{c}{12}\left(\frac{\rho}{\sigma}\right)^2\frac{1}{(J-2)!}\Gamma\left(J-1,\frac{\tau}{2\sigma^2}\right)\\
&+\frac{c}{4}\left(\frac{\rho_1}{\sigma}\right)^2\frac{1}{(J-2)!}\Gamma\left(J-1,\frac{\tau}{2\sigma^2}\right)\\
&+\frac{c}{12}\left(\frac{\rho}{\sigma}\right)^{2J}\left(1-\frac{1}{(J-2)!}\Gamma\left(J-1,\frac{\tau}{2\rho^2}\right)\right)\\
&+\frac{c}{4}\left(\frac{\rho_1}{\sigma}\right)^{2J}\left(1-\frac{1}{(J-2)!}\Gamma\left(J-1,\frac{\tau}{2\rho_1^2}\right)\right),
\end{aligned}
\qquad (13)
$$

□

Next, we propose throughput analysis for proposed method in detail.

V. THROUGHPUT ANALYSIS

For throughput analysis of CNSR method, we consider maximum μ transmissions ($\mu-1$ selective retransmissions) at MAC layer in one round of non-truncated ARQ. In each retransmission of MAC layer under CNSR, transmitter only retransmits information symbols transmitted over poor quality sub-carriers which have accumulated sub-carrier norm smaller than threshold τ $\left(\|\mathcal{H}_J(\ell)\|^2=\sum_{j=1}^{J}|H_j(\ell)|^2<\tau\right)$. At the end of the J-th transmission, where $J=1,\ldots,\mu$, the receiver combines saved observations of the first full transmission and subsequent selective retransmissions up to the J-th transmission for joint decoding. When CRC fails after μ transmissions ($\mu-1$ selective retransmissions), ARQ layer requests new transmission round of the failed packet to the peer ARQ layer.

For throughput analysis, we define normalized throughput η as the ratio of error-free information bits received k to the total number of bits transmitted n ($\eta=\frac{k}{n}$) as in [30]. Let P_{e_J} be the BER of the joint detection of J-th transmission at MAC layer in (3) for un-coded transmission. Note that P_{e_J} for coded transmission can be modeled by exponential functions [36]. Thus, the presented throughput analysis is valid for coded and un-coded transmission. We assume that each bit in the packet is independent from the other bits due to longer interleaver. Thus the probability of receiving a correct packet p_{c_J} of length L_f with BER P_{e_J} is $p_{c_J}=(1-P_{e_J})^{L_f}$. The probability of a packet to fail after joint detection is $p_{\epsilon_J}=1-p_{c_J}$. Due to joint detection, BER $P_{e_J}>P_{e_{J+1}}$ and probability of a good packet $p_{c_J}<p_{c_{J+1}}$. The first transmission of a packet at MAC layer consists of k information bits. If a packet fails in the first transmission,

the average number of m_1 information symbols have norm of OFDM sub-carriers gains $\|\mathcal{H}_1(\ell)\|^2 < \tau$. At the end of J selective retransmission, receiver requests retransmission of m_J information symbols, where $m_J = \Pr(\|\mathcal{H}_J\|^2 < \tau)$. Note that $m_1 > m_2 > \ldots > m_J > \ldots > m_\mu$, where $J = 1, 2, \ldots, \mu$.

The probability of packet failure after μ transmission rounds is given by $\alpha = \prod_{j=1}^{\mu} p_{\epsilon_j} = p_{\epsilon_1}.p_{\epsilon_2}\ldots p_{\epsilon_\mu}$. The packet failure probability after q transmission rounds under joint detection of at most μ transmission rounds at MAC layer is as follows:

$$p_{\epsilon_q} = \left(p_{\epsilon_1}.p_{\epsilon_2}\ldots p_{\epsilon_\mu}\right)^{\gamma}\prod_{j=1}^{J}p_{\epsilon_j} = \alpha^{\gamma}\prod_{j=1}^{J}p_{\epsilon_j} \tag{14}$$

where $\gamma = \lfloor\frac{q-1}{\mu}\rfloor$, $J = [(q-1) \mod \mu]$. Also, $\gamma = 0, 1, \ldots, \infty$, $J = 1, 2, \ldots, \mu$. The receiver jointly detects at most μ packets and discards received signal when a packet is successfully decoded or failure occurs after every μ-th packet. The probability of a packet being error free at $\mu+1$-th transmission is p_{c_1}. The probability that a packet is successfully decoded at the q-th transmission of a corrupt packet is $p_{\epsilon q}$. The following proposition presents throughput of the proposed CNSR method, which combines μ packets at MAC layer for joint detection:

Proposition 2: Throughput of CNSR method with μ transmission rounds is

$$\eta_\mu = \frac{(1-\alpha)^2}{\sum_{J=1}^{\mu}\prod_{j=1}^{J}p_{\epsilon_{J-j}}p_{c_J}\left(\sum_{i=0}^{J-1}m_i + \sum_{\hat{i}=J}^{\mu-1}m_{\hat{i}}\alpha\right)}, \tag{15}$$

where $\alpha = \prod_{J=1}^{\mu} p_{\epsilon_J}$, $p_{\epsilon_0} = 1$ and $m_0 = 1$.

Proof: The average number of information bits transmitted successfully deliver k data bits for our CNSR approach are [37]

$$\begin{aligned}
n_\mu &= kp_{c_1} + k(1+m_1)p_{\epsilon_1}p_{c_2} + k(1+m_1+m_2)p_{\epsilon_1}p_{\epsilon_2}p_{c_3}\\
&\quad + \ldots + k(1+m_1+\ldots+m_{\mu-1})p_{\epsilon_1}p_{\epsilon_2}p_{\epsilon_3}\ldots p_{\epsilon_{\mu-1}}p_{c_\mu}\\
&\quad + k(2+m_1+\ldots+m_{\mu-1})p_{\epsilon_1}p_{\epsilon_2}p_{\epsilon_3}\ldots p_{\epsilon_\mu}p_{c_1}\\
&\quad + k(2+2m_1+m_2+\ldots+m_{\mu-1})p_{\epsilon_1}^2p_{\epsilon_2}p_{\epsilon_3}\ldots p_{\epsilon_\mu}p_{c_2}\\
&\quad + (2+2m_1+2m_2+m_3+\ldots+m_{\mu-1})\\
&\quad \times kp_{\epsilon_1}^2p_{\epsilon_2}^2p_{\epsilon_3}\ldots p_{\epsilon_\mu}p_{c_3}+\ldots\\
&\quad + k(2+2m_1+\ldots+2m_{\mu-1})p_{\epsilon_1}^2p_{\epsilon_2}^2p_{\epsilon_3}^2\ldots p_{\epsilon_{\mu-1}}p_{c_\mu}\\
&\quad + \ldots + k((\gamma+1)+\gamma m_1+\ldots+\gamma m_{\mu-1})\\
&\quad \times p_{\epsilon_1}^{\gamma}p_{\epsilon_2}^{\gamma}\ldots p_{\epsilon_{\mu-1}}^{\gamma}p_{c_1} + k((\gamma+1)(1+m_1)+\gamma m_2\\
&\quad + \ldots + \gamma m_{\mu-1})p_{\epsilon_1}^{\gamma+1}p_{\epsilon_2}^{\gamma}\ldots p_{\epsilon_{\mu-1}}^{\gamma}p_{c_2}\\
&\quad + ((\gamma+1)+(\gamma+1)m_1+(\gamma+1)m_2\\
&\quad + \gamma m_3+\ldots+\gamma m_{\mu-1})kp_{\epsilon_1}^{\gamma+1}p_{\epsilon_2}^{\gamma+1}p_{\epsilon_3}^{\gamma}\ldots p_{\epsilon_{\mu-1}}^{\gamma}p_{c_3}\\
&\quad + \ldots + ((\gamma+1)m_1+(\gamma+1)m_2+\ldots\\
&\quad + (\gamma+1)m_{\mu-1})kp_{\epsilon_1}^{\gamma+1}p_{\epsilon_2}^{\gamma+1}\ldots p_{\epsilon_{\mu-1}}^{\gamma+1}p_{c_\mu}+\ldots
\end{aligned} \tag{16}$$

By rearranging (16) and writing in the form of geometric series, we have

$$\begin{aligned}
n_\mu &= b_1\Big((1+2\alpha+3\alpha^2+4\alpha^3+\ldots)+(m_1+\ldots &(17)\\
&\quad +m_{\mu-1})\alpha(1+2\alpha+3\alpha^2+4\alpha^3+\ldots)\Big)+b_2\Big((1+m_1)\\
&\quad (1+2\alpha+3\alpha^2+4\alpha^3+\ldots)+(m_2+\ldots+m_{\mu-1})\alpha &(18)\\
&\quad (1+2\alpha+3\alpha^2+4\alpha^3+\ldots)\Big)+\ldots+b_J\Big((1+\ldots\\
&\quad +m_{J-1})(1+2\alpha+3\alpha^2+\ldots)+(m_J+\ldots+m_{\mu-1})\alpha &(19)\\
&\quad (1+2\alpha+3\alpha^2+4\alpha^3+\ldots)\Big)+\ldots+b_\mu\Big((1\\
&\quad +m_1+\ldots+m_{\mu-1})(1+2\alpha+3\alpha^2+4\alpha^3+5\alpha^4+\ldots)\Big) &(20)
\end{aligned}$$

where $b_1 = kp_{c_1}$, $b_2 = kp_{\epsilon_1}p_{c_2}$ and $b_J = kp_{\epsilon_1}p_{\epsilon_2}\ldots p_{\epsilon_{J-1}}p_{c_J} = k\prod_{j=1}^{J}p_{\epsilon_{J-j}}p_{c_J}$. Note that (20) has μ summation series and J-th summation series can be written and simplified as follows:

$$\begin{aligned}
n_{J,\mu} &= b_J\Big((1+\ldots+m_{J-1})(1+2\alpha+3\alpha^2+\ldots)\\
&\qquad +(m_J+\ldots+m_{\mu-1})\alpha(1+2\alpha+3\alpha^2+\ldots)\Big)\\
&= b_J\frac{(1+\ldots+m_{J-1})+(m_J+\ldots+m_{\mu-1})\alpha}{(1-\alpha)^2}\\
&= \frac{b_J}{(1-\alpha)^2}\left(\sum_{i=0}^{J-1}m_i+\sum_{\hat{i}=J}^{\mu-1}m_{\hat{i}}\alpha\right)
\end{aligned}$$

substituting b_J in (21), we have

$$n_{J,\mu} = \frac{k}{(1-\alpha)^2}\prod_{j=1}^{J}p_{\epsilon_{J-j}}p_{c_J}\left(\sum_{i=0}^{J-1}m_i+\sum_{\hat{i}=J}^{\mu-1}m_{\hat{i}}\alpha\right). \tag{21}$$

The average number n_μ of transmitted bits required to successfully deliver k bits at the $\mu-$th MAC round of the proposed method is

$$\begin{aligned}
n_\mu = \sum_{J=1}^{\mu}n_{J,\mu} &= \frac{k}{(1-\alpha)^2}\sum_{J=1}^{\mu}\prod_{j=1}^{J}p_{\epsilon_{J-j}}p_{c_J}\\
&\qquad\times\left(\sum_{i=0}^{J-1}m_i+\sum_{\hat{i}=J}^{\mu-1}m_{\hat{i}}\alpha\right).
\end{aligned}$$

Throughput of the CNSR is

$$\eta_\mu = \frac{k}{n_\mu} = \frac{(1-\alpha)^2}{\sum_{J=1}^{\mu}\prod_{j=1}^{J}p_{\epsilon_{J-j}}p_{c_J}\left(\sum_{i=0}^{J-1}m_i+\sum_{\hat{i}=J}^{\mu-1}m_{\hat{i}}\alpha\right)}, \tag{22}$$

□

Next, we present throughput and BER results for the proposed scheme.

TABLE 1. $\frac{E_b}{N_o}$ **(dB) vs τ_o for $\mu = 2, 3$ and 4**

$\mu \backslash \frac{E_b}{N_0}(dB)$	0	4	8	12	16	20	24	28	32
2	3	2.506	0.795	0.379	0.174	0.0652	0.0301	0.0097	0.0019
3	3	2.406	1.359	0.403	0.165	0.0524	0.0195	0.009	0.0076
4	3	2.635	1.123	0.3508	0.1253	0.0506	0.018	0.006	0.0021

VI. SIMULATION

Now, we provide BER and throughput results of the proposed CNSR method. We provide throughput and BER comparison of CNSR method with conventional Chase combining (CC) method and Chase combining using selective retransmission (CCSR) [30]. In simulation setup, we consider packet size $L_f = 1024$ bits transmitted over 10-tap Rayleigh fading channel. We assume quasi-static channel for one OFDM symbol duration. Owing to the large retransmission delay as compared to channel coherence time, we assume that channel realizations of information symbols for the first transmission and subsequent retransmission (if CRC failure occurs) are independent. For CNSR method with HARQ scheme, we use half-rate LDPC (324,648) code with interleaver. We use 4-QAM modulation under OFDM signaling for BER and throughput comparison of the proposed CNSR method with conventional CC method and CCSR method. We denote CNSR, CCSR and CC methods without FEC by CNSR-ARQ, CCSR-ARQ and CC-ARQ. Also, we denote CNSR, CCSR and CC methods with FEC by CNSR-HARQ, CCSR-HARQ and CC-HARQ. The simulation results reveal throughput gain of CNSR as compared to conventional CC and CCSR methods. We also provide comparison of analytical and Monte Carlo throughput performances for the proposed method. Note that in throughput results for CNSR method, we use optimal threshold τ_o vector on channel norm provided in Table 1. The top row of the Table1 represents $\frac{E_b}{N_0}$(dB) and first column represent number of transmissions μ. The table provides optimal threshold τ_0 for the combination of transmissions μ and $\frac{E_b}{N_0}$(dB). Moreover, optimization of normalized throughput of CNSR method presented in Section V deals with multiple selective retransmissions at MAC layer. We optimize throughput using (3) and (22) because lower bound on throughput is a function of channel SNR, probability of error and frame length. The throughput optimization is an unconstrained and non-convex optimization problem that searches for optimal τ off-line for each SNR point with the objective to maximize throughput η. The offline computed τ vector is used in simulations to evaluate the performance of the proposed method. For simulation, we define normalized throughput for communication system [5] as $\eta = \frac{k}{n}$, where n is the number of bits transmitted for error-free delivery of k information bits.

First, Figure 1 presents accuracy of throughput analysis of the proposed CNSR method in (15) for MAC transmission rounds $\mu = 2$, 3, and 4. In this figure, we compare throughput from Monte Carlo simulation η_m with throughput from analysis η_a in (15). The marginal gap between the two curves demonstrates the accuracy of throughput analysis. This also reflects the upper bound on BER in (3) is also tight as throughput is a function of BER. In fact, marginal gap between the analytical and simulation results is due to approximation of Q-function in (5). Approximation of Q-function in (5) provides upper bound on BER. This upper bound is used for the throughput analysis. Thus, marginally higher analytical BER than simulation BER results into marginally low analytical throughput than simulation throughput as shown in Figure 1. In Figure 1, we use optimal threshold τ_o of the proposed CNSR method from Table 1 without FEC. Figure 2 provides BER comparison of CNSR-ARQ with conventional CC-ARQ and CCSR-ARQ methods. It is important to note that in CCSR-ARQ method with $\mu = 1$, there is first full transmission followed by

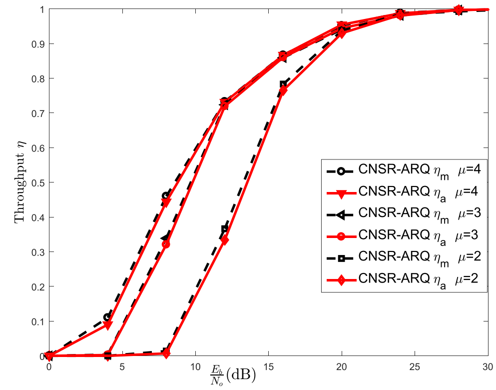

FIGURE 1. Comparison of analytical and simulation throughput of CNSR method using optimal threshold $\tau = \tau_o$ and transmission rounds $\mu = 2, 3$ and 4 for uncoded OFDM signaling.

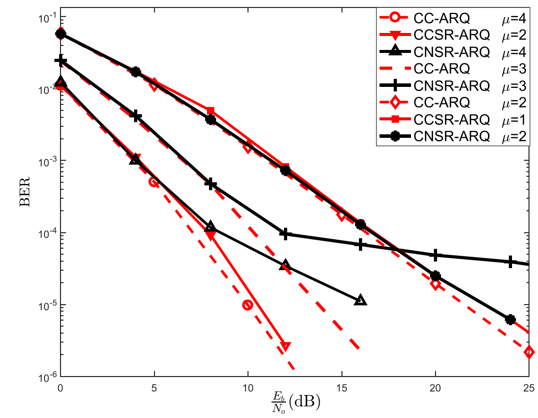

FIGURE 2. BER comparison of CC-ARQ ($\mu = 2, 3$ and 4), CCSR-ARQ ($\mu = 1$ and 2) and CNSR-ARQ ($\mu = 2, 3$ and 4) with optimal threshold τ_o.

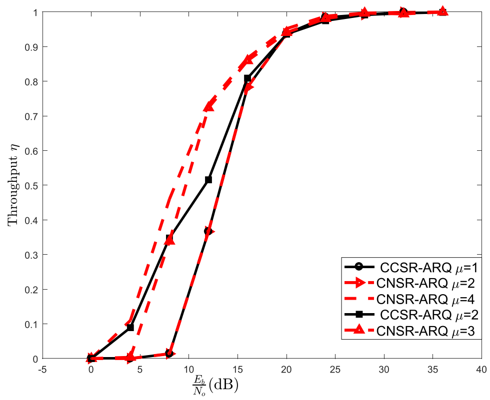

FIGURE 3. Throughput comparison of CCSR-ARQ ($\mu = 1$ and 2) and CNSR-ARQ ($\mu = 2, 3$ and 4) with optimal threshold τ_o (without FEC).

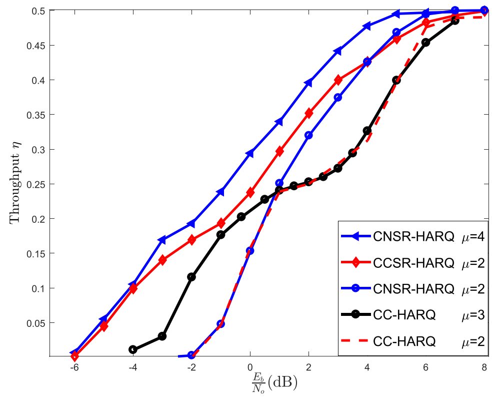

FIGURE 4. Monte Carlo throughput comparison of coded CNSR-HARQ ($\mu = 2$ and 4), CC-HARQ ($\mu = 2$ and 3) and CCSR-HARQ ($\mu = 2$).

selective retransmission in PHY prior to decoding. Thus, each OFDM sub-carrier can encounter two independent channel realizations which results into diversity order 2 for large threshold τ_o. However, CNSR-ARQ method for $\mu = 1$ initiates second transmission of a packet only in the event of CRC failure. As a consequence, each OFDM sub-carrier in CNSR-ARQ for $\mu = 1$ encounters one channel realization resulting into diversity order 1. In general, for μ rounds of CCSR-ARQ method, each sub-carrier may encounter 2μ channel realizations whereas, in CNSR-ARQ method, each OFDM sub-carrier can encounter at most μ channel realizations. Note that for CC-ARQ method with μ rounds, each sub-carrier encounters exact μ channel realizations. Thus, for a fair comparison, we compare CNSR-ARQ and CC-ARQ methods for $\mu = 2j$ with CCSR-ARQ method for $\mu = j$. BER and throughput performance of CCSR-ARQ for $\mu = 1$ and CNSR-ARQ for $\mu = 2$ is similar. Owing to the fact that CCSR-ARQ with $\mu = 1$ and CNSR-ARQ with $\mu = 2$ have similar optimal threshold on channel norm which results into almost same amount of information retransmitted in response to selective retransmission request. For CCSR-ARQ method with $\mu = 2$, each information symbol encounters at least two independent channel realization. In case of CNSR-ARQ with $\mu = 4$, information symbols which have $|H_1(\ell)|^2 \geq \tau_o$ are transmitted only once resulting into lower diversity. Figure 2 reveals that in moderate and high SNR regime, CNSR-ARQ has poor BER performance as compared to CCSR-ARQ and CC-ARQ method. This is due to the fact that CNSR method optimizes threshold τ_o to maximize throughput instead of BER improvement, by avoiding unnecessary retransmission results in lower diversity order in high SNR regime. Thus, with the objective of maximizing throughput η, poor BER performance of CNSR-ARQ in moderate and high SNR regimes achieves higher throughput as compared to CCSR-ARQ and CC-ARQ transmission schemes.

Figure 3 compares throughput of the proposed CNSR-ARQ method presented in Figure 1 with the existing CCSR-ARQ and CC-ARQ methods. Throughput of CNSR-ARQ for $\mu = 2$ is similar to CCSR-ARQ for $\mu = 1$. Typical transmission rounds of failed packet in contemporary standards such as LTE of a failed packet in MAC layer are $\mu = 4$. For fair comparison $\mu = 2j$ of CNSR should be compared with $\mu = j$ due to the fact that CCSR avails one more retransmission round prior to decoding. Throughput comparison in Figure 3 shows that CNSR-ARQ for $\mu = 4$ performs better than CCSR-ARQ method for $\mu = 2$ when SNR > 5dB. Note that allowing large number of transmissions of a failed packet does not achieve significant throughput gain for CNSR-ARQ and CCSR-ARQ methods in moderate and high SNR regime. This is due to the fact that in this SNR regime, probability that a packet is corrupted after three or more transmissions under joint detection is very low.

Now, we provide throughput comparison of the proposed CNSR-HARQ method as compared to CC-HARQ and CCSR-HARQ methods with LDPC code(324,648) of half rate in Figure 4. We consider 4-QAM constellation with $N_s = 972$ OFDM sub-carriers. It is clear from Figure 4 that performance of CNSR-HARQ with $\mu = 4$ and CCSR-HARQ with $\mu = 2$ are similar up to $\frac{E_b}{N_0} = -4$dB. For $\frac{E_b}{N_0} > -4$dB, CNSR-HARQ outperforms CCSR-HARQ. Furthermore, Figure also delineates that CNSR-HARQ for $\mu = 3$ surpasses the CCSR-HARQ for $\mu = 2$ in $\frac{E_b}{N_0} > -2$dB regime. Note that one transmission of CCSR-HARQ consists of a full transmission and a selective retransmission prior to decoding. For fair comparison, we consider $\mu = 2j$ and $\mu = j$ for CNSR-HARQ and CCSR-HARQ, respectively. For $\mu = 1$ under CCSR-HARQ and $\mu = 2$ under CNSR-HARQ, each symbol gets one chance of retransmission which results into similar throughput. CCSR-HARQ with $\mu = 2$ consists of two full transmissions and two selective retransmissions whereas in CNSR-HARQ with $\mu = 3$ and $\mu = 4$, there is one full transmission followed by two and three selective retransmissions, respectively. The optimal threshold τ_o converge to large values in low SNR regime to maximize throughput.

Figure 5 provides comparison of average delay D of CNSR-ARQ method with conventional CC-ARQ and CCSR-ARQ methods for transmission rounds $\mu = 4$, $\mu = 4$ and $\mu = 2$, respectively. For fair comparison, CCSR-ARQ for $\mu = 2$ is compared with CNSR-ARQ and CC-ARQ for $\mu = 4$. We consider one transmission as a delay unit for

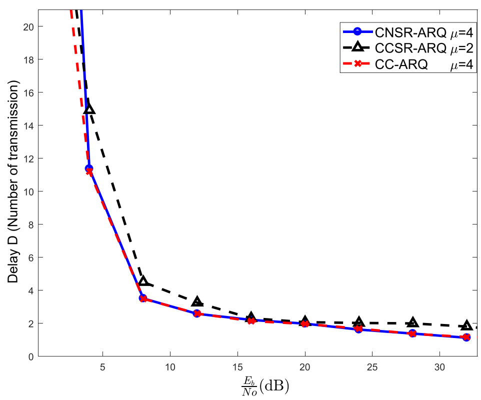

FIGURE 5. Delay comparison of CNSR, CCSR and conventional CC for uncoded transmission.

packet transmission. The simulation results in Figure 5 reveal that in low SNR regime ($\frac{E_b}{N_o} < 3$dB), CNSR-ARQ encounters larger delay as compared to CCSR-ARQ and CC-ARQ due to the fact that in the event of packet failure, CCSR-ARQ and CC-ARQ methods retransmit full failed packet in each round, whereas CNSR-ARQ method transmits full packet only once during μ retransmissions if needed. In low SNR regime, partial retransmission of a failed packet is less effective to recover from errors as compared to full retransmission, which leads to smaller delay for CCSR-ARQ an CC-ARQ methods. In high SNR regime, CC-ARQ and CNSR-ARQ encounter similar delay. Note that partial retransmission of a failed packet is more effective. When SNR is high, the CCSR-ARQ method has marginally large delay, which is measured as the number of transmissions to successfully deliver data. The large delay is attributed to the fact that receiver of CCSR-ARQ method initiates selective retransmission before decoding. Consequently, CCSR-ARQ method encounters delay of one packet even in the case of selective retransmission of one information symbol. The simulations results demonstrate that the proposed CNSR-ARQ method achieves higher throughput as compared to CCSR-ARQ and conventional CC-ARQ methods without increasing latency of the system.

VII. CONCLUSION

In this work, we presented channel norm constrained selective retransmission method that retransmits information symbols transmitted over the OFDM sub-carrier which have accumulated channel norm below threshold in the event of a packet failure. The proposed scheme optimizes threshold τ_o with focus to maximize throughput without compromising latency. We also presented BER and throughput analysis of the proposed approach. The simulation results demonstrated that proposed method outperforms conventional CC and previously proposed CCSR method with and without FEC.

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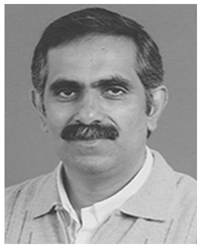

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