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A Novel Fuzzy Approach for Combining Uncertain Conflict Evidences in the Dempster-Shafer Theory

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ABSTRACT Fusing conflict evidences is one of the fundamental needs to data fusion, but this task is challenging in the decision-making domain because of the fusion of ever-increasing uncertain data. In this paper, a novel fuzzy-based multi-sensor data fusion method is proposed for fusing high-conflict uncertain data and avoiding the counter-intuition problem. Our key idea is to introduce the fuzzy inference mechanism into the similarity measurement model to measure conflict degree between the evidences. On this basis, belief entropy is used to calculate the uncertainty of evidences, so as to express the relative importance of the evidences. The reliability of evidences can be obtained by the credibility which is gained through the above method, and the quantitative information volume is used to revise each credibility degree to get the final weight according to the evidence. The numerical experimental results demonstrate that the presented method is feasible and effective in dealing with conflicting evidences. In addition, the application of fault diagnosis is given to show that the proposed approach is effective and advantageous compared with state-of-the-art approaches.

INDEX TERMS Belief entropy, conflict evidence, DS evidence theory, fuzzy-based similarity measurement, multi-sensor data fusion.

I. INTRODUCTION

Multi-sensor data fusion is a technology that combines information from several sources to form a unified picture [1]. However, due to the complexity of the targets or objects, it is inevitable to judge correctly the results from the multi-source and multi-type sensors, and then the uncertainty, divergence, heterogeneity of the collected data and the insufficiency of the details can result in decision-making errors, especially, counter-intuitive problem exists for high-conflict evidences from various data sources [1]. Therefore, multi-sensor data fusion study with the accurate and robust performance under various evidence types and conflict cases is of great significance both in theory and in practice. Fusing conflict evidences may be a difficult task in the data fusion community, especially when the experts have different viewpoints about the same problem [1]. As a result, various multi-sensor data fusion methods and algorithms have been investigated in the past few decades, but none of the suggestions can be declared with absolute certainty [2]. Further, these approaches are also becoming more and more widely used in evidence fusion with practical applications, such as fault diagnosis [3], image processing [4], health monitoring [5], wireless sensor networks [6], the risk analysis [7], target tracking [8], and references therein [9], [10].

As we known, the Dempster-Shafer (*DS*) theory can manage uncertain information and offer a useful fusion tool for decision-making. Dempster put forward a theory in 1967 [11], which is the evidence theory, and then Shafer further studied the theory in 1976 [12]. However, the quality of combining is affected by conflicting information, especially when the sources of evidence are unreliable [2] and the combination of Dempster's rule would generate counterintuitive results as first highlighted by Zadeh [13]. Many scholars have done a lot of research for the above problem. Some of them believe that counterintuitive results are caused by *DS* combination rules, so they have improved the *DS* combination rules [14]–[16]. Others believe that counterintuitive results should be attributed to unreliable sources of evidence, so the sources of evidence are modified [17]–[19]. However, it is most important to accurately measure the conflict between evidences before choosing the fusion method. The conflict coefficient k is used for conflict measurement in the *DS* evidence theory, but this classical conflict coefficient has been proven not to make accurate measurements in high-conflict situations [13]. How to correctly measure the degree of conflict/similarity between evidences, and reduce the negative effects of conflict evidences in the final *DS* fusion, in order to improve the accuracy of the fusion results, and avoid the counter-intuitive results of the combination of *DS* combination rules?

In order to solve this problem, many methods of conflict measurement have been provided by many scholars [20]–[24]. The conflict rate was used to describe the conflict degree between two pieces of evidence in [20]. But the conflict rate can cause a counterintuitive result when the two pieces of evidence are equal. The conflict measurement method for conflict inconsistency measurement was proposed by Zhao *et al*. [23]. The conflict between each two pieces of evidence is classified and then the conflict coefficient is calculated and updated. Once the frame of discernment is too large, this method cannot be applied. Recently, Xiao [24] put forward to a conflict measurement method by introducing the Belief Jensen-Shannon divergence measure method. The effect of evidence itself on the weight was considered in this method. But using Belief Jensen-Shannon divergence for conflict measurement cannot reflect the change in the degree of conflict between evidences due to changes in the elements of the multi-subset focal elements. And, the novel dissimilarity measure method presented by Liu *et al*. [21] is used to describe the divergences of two aspects between two pieces of evidence, that is, the difference of beliefs and the difference of hypotheses. In [22], based on the improved probability transformation, a new similarity measure was proposed by integrating the fuzzy nearness and correlation coefficient with Hamacher T-conorm rule. Both Liu [21] and Ma and An [22]'s conflict measurement methods take into account multiple aspects of conflict measurement and are more conducive to comprehensive measurement of conflicts. However, there is no accurate model for describing the nonlinear relationship between multiple angles of conflict measurement. In addition, the existing evidence fusion method has some defects in the conflict data processing of the evidence sources of uncertainty, resulting in low accuracy of the fusion result. The defects are that the conflict measure between evidences is only considered when improving weights, and there are different methods of conflict measurement under different types of conflict situations. It is more complicated and does not consider the influence of the uncertainty of the evidence itself on the evidence source. Based on the above analysis, the current methods are not adequate to precisely delineate the divergence between two pieces of evidence.

In order to overcome the shortcomings of the existing methods of conflict measurement, the combination problem of high conflict evidences in the *DS* theory frame is considered and an novel and comprehensive conflict measurement model is constructed in this paper. In this way, the new proposed fusion method in this paper can comprehensively consider different types of conflicts between evidences, and measure the degree of conflict between evidences, then combine the uncertainty of the evidence itself to modify the evidence sources and obtain a reasonable evidence body for fusion. Inspired by the work of our previous results in [22], Xiao [24], and Sarabi-Jamab and Araabi [2], a new Fuzzy-based Similarity Measurements (*FSM*) is developed for measuring the differences and similarity between the bodies of the evidences, and also the dissimilarity among evidences can be obtained. And, a novel multi-sensor data fusion method is proposed by combining the above *FSM* with the belief entropy, which considers the uncertainty and conflict of evidences at the same time. In detailed, the novel dissimilarity measure is defined through integrating the fuzzy nearness and correlation coefficient and thus designing a novel fuzzy inference mechanism to show in detailed the relationship between them. Next, the information volume of the evidences is calculated by the belief entropy to express the uncertainties of the evidences. Furthermore, the credibility of the evidences is obtained to represent the reliability of the evidences, in which it is modified by the information volume of the evidences. When dealing with the body of evidences, some reasonable weights are assigned to each of evidences, and thus reasonable average evidence is arrived before using the *DS* combination rule. Therefore, the *DS* combination rules are used for fusion with these average evidences. At last, some numerical examples and an application to the fault diagnosis system are utilized to demonstrate the effectiveness, rationality and merit of the proposed approach.

For reader's convenience, Table 1 provided in this paper gives a summary of notations and their definitions in this paper.

The rest of this paper is organized as follows. Section 2 introduces the preliminaries and background. In Section 3, a new similarity measurement approach called as *FSM* is developed. A novel multi-sensor data fusion method based on the *FSM* and the belief entropy is put forward to in Section 4. Numerical examples are given to show the feasibility, robustness and effectiveness of the proposed approach in Section 5, while an application to fault diagnosis verified the superior in Section 6. A conclusion is drawn in Section 7.

II. PRELIMINARIES

A. DEMPSTER-SHAFER EVIDENCE THEORY

DS evidence [11], [25] theory is a systematic theory for dealing with uncertain information. It can flexibly and effectively model uncertain information without prior information. Its requirements are not as strict as Bayesian requirements. When the probability is confirmed, *DS* evidence theory can be transformed into Bayesian probability theory, so *DS* evidence theory is considered to be an extension of Bayesian theory. The *DS* evidence theory can express the uncertainty of the hypothesis by assigning different belief values to the hypotheses containing different elements through the basic

TABLE 1. Summary of notations and definitions.

Notation	Definition	Notation	Definition
Θ	The frame of discernment	θ_i	The ith incompatible focal element
2^θ	The power set composed of all the possible subsets of Θ	A_i or B_i	The ith subset of Θ
m_i	The ith basic belief assignment function	ϕ	The empty set
K	The conflict coefficient of DS evidence theory	$\mu(x)$	The membership function
$\mu_A(u)$	The u's membership to A	c	The mean value of the Gaussian function
m	The center value of the Trigonometric function	σ	The standard variance value of the Gaussian functions
f	The abscissa value of the left vertex in the lower part of the Trigonometric function	g	The abscissa value of the right vertex in the lower part of the Trigonometric function
a, b, c, d	The abscissas of the four vertices of the trapezoid	u_{im}	The mth characteristic indicator of the ith object
U_*	The characteristic indicator matrix	U/V	The domain
r_{ij}	The similarity between objects u_i and u_j	$R(u,v)$	The fuzzy relationship between u and v
y_{out}	The fuzzy inference output accurate value	Fn	The fuzzy nearness
E_d	The Deng entropy	$P_i(\theta_s)$	The transformed probability of sth hypothesis on the ith evidence
$\theta^{m_i}_{\max / \min}$	The hypothesis that the ith basic belief assignment function assigns the maximum/minimum value	Cor	The correlation coefficient
$\tilde{A}_{ij}$	The jth linguistic rule for the ith input parameter	$\tilde{B}_j$	The jth linguistic rule for the output parameter
$sim(m_i, m_j)$	The similarity between m_i and m_j	$Dism(m_i, m_j)$	The dissimilarity between m_i and m_j
Bel	The belief values matrix	pl	The plausibility belief values matrix
P	The transformed probability distribution matrix	Sim	The similarity matrix
α	The parameter of Example 5	d	The Jousselme *et al.*'s distance
$W(m_i)$	The weight of ith piece of evidence	$Sd(m_i)$	The total support degree of ith piece of evidence
$Cd(m_i)$	The credibility of ith piece of evidence	$Iv(m_i)$	The information volume of ith piece of evidence
$\tilde{I}v(m_i)$	The normalized form of $Iv(m_i)$	$RCd(m_i)$	The revised credibility degree of ith piece of evidence
DR	Dynamic Reliability	SR	Static Reliability

belief assignment function. In addition, the new evidences can be obtained by combining the evidences. The basic concepts are as follows.

Definition 1: The frame of discernment is $\Theta = \{\theta_1, \theta_2, \ldots, \theta_j, \ldots, \theta_n\}$, θ is incompatible focal element, the set of all the possible subsets of Θ is called a power set represented by 2^θ . There is Non-zero masses of A which is subset of Θ are named the focal elements. The m is a basic belief assignment (BBA) function, which maps from 2^θ to [0, 1], it meets the following requirements:

$$\begin{cases} m(\phi) = 0 \\ \sum\limits_{A\subset\Theta} m(A) = 1. \end{cases} \tag{1}$$

During the data fusion process, the level of belief in the final result are expressed by $Bel(A)$ and $Pl(A)$.

Definition 2: $Bel(A)$ is a mapping from set 2^Θ to [0, 1]. If A represents any subset of the frame of discernment 2^Θ, $Bel(A)$ indicates the belief degree of subset A, which is defined as follows:

$$Bel(A) = \sum_{B\subseteq A} m(B) \quad \forall A, B \subseteq \Theta, \tag{2}$$

when A is a single subset, ie $|A| = 1$, $Bel(A) = m(A)$.

Definition 3: $pl(A)$ is a mapping from set 2^Θ to [0, 1]. If A represents any subset of the frame of discernment 2^Θ, it is denoted as $A \subseteq \Theta$ and satisfies:

$$pl(A) = \sum_{A\cap B\neq\phi} m(B) \quad \forall A, B \subseteq \Theta. \tag{3}$$

The function $pl(A)$ is a plausibility function of A, indicating a non-false belief degree to subset A.

Definition 4: Suppose $m_1, m_2 \ldots, m_n$ be n $BBAs$ on Θ, then the Dempster's combination rule can be defined as:

$$m(A) = m_1 \oplus m_2 \oplus \ldots m_n = \begin{cases} \dfrac{\sum\limits_{\cap A_i = A} \prod\limits_{1<i\leq n} m_i(A_i)}{1-K} & A \neq \phi \\ 0 & A = \phi \end{cases} \tag{4}$$

where $K = \sum\limits_{\cap A_i=\phi} \prod\limits_{1<i\leq n} m_i(A_i)$, $0 < K < 1$. The degree of conflict which among sources of evidence is showed by the conflict coefficient named K. Notice that, the Dempster's combination rule is only practicable for the two $BBAs$ with the condition $K < 1$.

B. FUZZY THEORY

Fuzzy set theory [26], [27] is another theoretical inference scheme for dealing with imperfect data. It introduces the novel notion of partial set membership, which enables imprecise (rather than crisp) inference [28]. The related concepts are as follows.

1) FUZZY SETS AND MEMBERSHIP FUNCTIONS

A fuzzy set $A \subseteq U$ is defined by the gradual membership function $\mu_A(u)$ in the interval [0, 1] as below:

$$\mu_A(u) \in [0, 1] \quad \forall u \in U. \tag{5}$$

The membership function μ_A maps each element u in U to a value $\mu_A(u)$ on [0, 1], indicating the degree to which the element belongs to A. The larger the value, the higher the degree of membership, and vice versa.

The membership function is the basis of fuzzy set and theory. The determination process is determined by experience and statistics. At present, there are three kinds of membership functions widely used, Gaussian membership function, triangular membership function and Trapezoidal membership function [29] .

1) Gaussian membership function

$$\mu(x) = e^{-\frac{(x-c)^2}{2\sigma^2}}, \tag{6}$$

where c and σ respectively represent the mean value and standard variance of the Gaussian membership function, and the Gaussian membership function has smooth and stable transition characteristics.

2) Triangle membership function

$$\mu(x) = \begin{cases} 0, & x \leq f \\ \dfrac{x-f}{m-f}, & f < x \leq m \\ \dfrac{g-x}{g-m}, & m < x \leq g \\ 0, & g < x \end{cases} \tag{7}$$

where the parameters f and g correspond to the abscissa value of the left and right vertices in the lower part of the triangle, and the parameter m corresponds to the apex abscissa value of the upper part of the triangle. The structure of the triangular membership function is simple and convenient to calculate.

3) Trapezoidal membership function

$$\mu(x) = \begin{cases} 0 & x \leq a \\ \dfrac{x-a}{b-a} & a < x \leq b \\ 1 & b < x \leq c \\ \dfrac{d-x}{d-c} & c < x \leq d \\ 0 & x > d \end{cases} \tag{8}$$

where the parameters b, c are the abscissas of the two vertices of the trapezoidal upper base, and the parameters a, d are the abscissas of the two vertices of the trapezoidal bottom.

2) FUZZY INFERENCE

The theory of fuzzy inference is established, using fuzzy set theory [27] as a basic description tool, by the expansion of mathematical logic based on general set theory basic description tool. In 1975, Zade [30] first proposed the synthetic rule of fuzzy inference and the rule of converting the conditional statement "if X is A, then Y is B" into a fuzzy relationship. Fuzzy control is to use the knowledge of fuzzy theory to imitate the thinking mode of the human brain, to identify and judge the fuzzy phenomenon to complete the control of the controlled object [31], which mainly solves four problems: fuzzy quantization processing, fuzzy control rules, fuzzy inference decision and anti-fuzzification processing.

- Fuzzy quantization process
  The process of fuzzy quantization processing is a process of making a precise quantity have fuzzy characteristics. The most basic way is to convert the input into fuzzy control by means of membership function.
- Fuzzy control rules
  The fuzzy control system is described by a series of linguistic rules, usually from experts'knowledge, and using the 'IF-THEN' language form [31]. This series of linguistic rules is called fuzzy control rules. There are four main methods for establishing fuzzy control rules, such as expert experience method, observation method, fuzzy model method and self-organization method.
- Fuzzy inference decision
  In fuzzy control, the fuzzy subset of control variables needs to be obtained through fuzzy inference decision. At present, the mature and widely used fuzzy inference method is Mamdani inference [31].

  Definition 5: There are two domain U and V, $\tilde{A}$, $\tilde{B}$ are two fuzzy subsets, assuming a fuzzy implication relationship: "If $\tilde{A}$ then $\tilde{B}$", denoted by $\tilde{A} \rightarrow \tilde{B}$, and $\tilde{A} \in U$, $\tilde{B} \in V$, then the fuzzy relationship of $\tilde{A} \rightarrow \tilde{B}$ on $U \times V$ can be expressed for:

  $$(\tilde{A} \rightarrow \tilde{B})(u, v) = R(u, v) \in U \times V \tag{9}$$

  where $R(u, v)$ represents the fuzzy relationship between u and v. The Mamdani inference [31] is defined as:

  $$R(u, v) = \tilde{A}(u) \wedge \tilde{B}(v). \tag{10}$$

- Anti-fuzzification process
  In the fuzzy system, the result obtained by fuzzy inference is given in the form of fuzzy set. It is the main task of defuzzification to take the exact value of the optimal representative in the fuzzy set obtained by inference. The most common method is the center of gravity method, which is defined as:

  $$y_{out} = \frac{\int_V y\mu_V(y)dy}{\int_V \mu_V(y)dy}, \tag{11}$$

  for discrete cases with m output quantization levels, the final output value should be:

  $$y_{out} = \frac{\sum_{k=1}^{m} y_k u_V(y_k)}{\sum_{k=1}^{m} u_V(y_k)}. \tag{12}$$

C. BELIEF ENTROPY

Belief entropy is an effective method for measuring uncertain information. It was first proposed by Deng [32], so it is also known as Deng entropy, which is a generalization of Shannon entropy [33], [34]. Deng entropy can be applied to evidence theory, and *BBA* represents uncertain information. The related concepts are as follows.

Let A_i be a hypothesis of the belief function m, $|A_i|$ is the cardinality of set A_i. Deng entropy E_d of set A_i is defined as follows:

$$E_d = -\sum_i m(A_i) log \frac{A_i}{2^{|A_i|}-1}. \tag{13}$$

When the belief value is only allocated to the single element, Deng entropy degenerates to Shannon entropy, i.e.,

$$E_d = -\sum_i m(A_i) log \frac{A_i}{2^{|A_i|}-1} = -\sum_i m(A_i) log\, m(A_i). \tag{14}$$

If the number of elements included in the hypothesis is greater, it means that the cardinality is larger, then according to the formula 13, the Deng entropy is larger, so it is known that the evidence contains more information. When an evidence has a large Deng entropy, indicating that other evidence supports it better, then the evidence should be assigned a larger weight to play its important role in final fusion.

III. FUZZY-BASED SIMILARITY MEASUREMENT

In this section, a new method based on Fuzzy-based Similarity Measurement (*FSM*) is proposed for conflict measurement by combining *DS* evidence theory and fuzzy inference mechanism. Notice that in order to comprehensively and accurately measure the degree of conflict/similarity in different types of conflicts between evidences, this paper proposes that the similarity between evidences is calculated from two dimensions, namely correlation coefficient (*Cor*) and fuzzy nearness (*Fn*) [22]. The fuzzy nearness and correlation coefficient are complementary feature for the similarity of evidences and they separately capture different aspects of the dissimilarity of *BBAs*, and the relationship between the above features is uncertain and strong non-linearity, thus a new fuzzy inference mechanism is firstly designed to measure the similarity of evidences.

A. CALCULATION OF FUZZY NEARNESS AND CORRELATION COEFFICIENT

In our previous studies [22], the fuzzy nearness and correlation coefficient have been proposed to measure the degree of conflict of conflict evidences. The fuzzy nearness is constructed based on the theory of fuzzy sets [26], which reflects the similarity between *BBAs* of evidence sources.

Definition 6: There is a sequence of k pieces of probability evidence $\{P_1, P_2, \cdots P_k\}$ rebuilt by the probabilistic transformation from $\{m_1, m_2, \cdots m_k\}$. The degree of similarity between two *BBAs* can be calculated by

$$Fn(m_i, m_j) = \frac{\sum_{s=1}^{n} (P_i(\theta_s) \wedge P_j(\theta_s))}{\sum_{s=1}^{n} (P_i(\theta_s) \vee P_j(\theta_s))} \quad i, j = 1, 2, 3 \ldots, k, \tag{15}$$

where $\wedge$ and $\vee$ are the operators for calculating the minimum and maximum, respectively. The fuzzy nearness satisfies $Fn(m_i, m_j) \in [0, 1]$. The probabilistic transformation [22] $P(\theta_i)$ is defined as

$$\begin{aligned} P(\theta_i) &= Bel(\theta_i) \\ &\quad + \frac{BEL \cdot Bel(\theta_i) + (1-BEL)pl(\theta_i)}{\sum_{\theta_i \in \Theta} BEL \cdot Bel(\theta_i) + (1-BEL)pl(\theta_i)} (1-BEL). \end{aligned} \tag{16}$$

where $BEL = \sum Bel(\theta_i)$.

It is worth noting that the belief function and the plausibility function can be used for non-single subsets, but non-single subsets are uncertain for decision making. Therefore, the probabilistic transformation formula (16) is required to assign the belief value on the non-single subset to the corresponding single subset. In the final decision, the belief level assigned on the single subset is the basis for decision making, so only the belief degree and plausibility on the single subset need to be calculated.

Example 1: Assume m_1 and m_2 over $\Theta = (\theta_1, \theta_2, \theta_3)$ are defined as

$$\begin{aligned} m_1 &: m_1(\theta_1) = 0.9, \quad m_1(\theta_2) = 0.1, m_1(\theta_3) = 0; \\ m_2 &: m_2(\theta_1) = 0, \quad m_2(\theta_2) = 0.1, m_2(\theta_3) = 0.9. \end{aligned}$$

According to (15) and (16), we get $Fn(m_1, m_2) = \frac{0+0.1+0}{0.9+0.1+0.9} = 0.0526$ and $Fn(m_1, m_1) = \frac{0.9+0.1+0}{0.9+0.1+0} = 1$, respectively, which means that the similarity between two highly conflict *BBAs* or two equal *BBAs* can be well reflected by the fuzzy nearness.

However, judging the degree of conflict between evidences only from the similarity of *BBAs* is unstable, which is also put forward by [22]. In order to measure the degree of conflict between evidences reasonably, a new conflict coefficient is proposed to reflect the difference between hypotheses strongly supported by evidence sources. The conflict coefficient proposed in this paper improves that the conflict coefficient in [21] does not reflect the difference between two non-conflicting sources of evidence.

Definition 7: Let θ_i be a hypothesis of the belief function $m_i (i = 1, 2, 3 \ldots k)$, the frame of discernment Θ contains n mutually exclusive and exhaustive hypotheses. The correlation coefficient is defined by

$$Cor(m_i, m_j) = \begin{cases} \dfrac{m_i(\theta_{\max}^{m_i}) + m_j(\theta_{\max}^{m_j})}{2}, & if\, m_i(\theta_{\max}^{m_i}) = m_j(\theta_{\max}^{m_j}) \\ \dfrac{m_i(\theta_{\min}^{m_i}) + m_j(\theta_{\min}^{m_j})}{2}, & if\, m_i(\theta_{\max}^{m_i}) \neq m_j(\theta_{\max}^{m_j}) \end{cases} \tag{17}$$

where $\theta_{\max}^{m_i} = \arg\max m_i(\theta), \theta_{\min}^{m_i} = \arg\min m_i(\theta)$. The *Cor* means the similarity degree of the hypothesis supported by the maximum probability of the evidence sources. The meaning of *Fn* is the similarity of the probability distribution structure between evidences. *Cor* and *Fn* respectively represent different aspects of the similarity between evidences. The correlation coefficient is defined according to the maximum value of the trust value of the *BBAs* for the single subset distribution, and it shows whether the assumptions supported by maximum belief values of the two evidences are the same. If they are the same, the similarity between the evidences is calculated by the average of the maximum mass values of the same hypothesis by two pieces of evidences. If they are different, the assumption that the two evidences are supported by maximum belief is different. It is concluded that the two evidences conflict with each other, and the conflict between the evidences is calculated by the average of the minimum mass values of the two evidences.

Example 2: Supposing that there is the frame $\Theta = \{\theta_1, \theta_2, \theta_3\}$, and let the following three independent *BBAs* over the same frame of discernment be as follows:

$$E_1 : m_1(\theta_1) = 0.5, \quad m_1(\theta_2) = 0.3, \; m_1(\theta_3) = m_1(\Theta) = 0.1;$$
$$E_2 : m_2(\theta_1) = 0.3, \quad m_2(\theta_2) = 0.5, \; m_2(\Theta) = 0.2;$$
$$E_3 : m_3(\theta_1) = 0.8, \quad m_3(\theta_3) = 0.2.$$

It is calculated that $Fn_{12} = 0.5576$, $Fn_{13} = 0.5000$. From the analysis of Example 2, it is known that E_1 and E_3 strongly support θ_1 and E_2 supports θ_2, so E_1 and E_3 are more similar than E_1 and E_2, while $Fn_{12} > Fn_{13}$, which does not accord with the intuitive results. However, One gets $Cor(m_1, m_2) = \frac{0.1+0}{2} = 0.05$, $Cor(m_1, m_3) = \frac{0.5+0.8}{2} = 0.65$, which indicates that the correlation coefficient can well reflect the similarity of the hypothesis supported by two evidences with maximum belief.

B. FUZZY INFERENCE MECHANISM

In order to comprehensively and accurately measure the degree of conflict/similarity in different types of conflicts between evidences, this paper proposes that the similarity between evidences is calculated from two dimensions, namely *Cor* and *Fn*. Although the fuzzy nearness and correlation coefficient can represent the similarity between evidences to some extent, the relationship between them is nonlinear and complex, and there is currently no precise mathematical model that can be used to describe the relationship between the fuzzy nearness and the correlation coefficient. However, the fuzzy theory can simulate this fuzzy and uncertain nonlinear relationship very well. Therefore, a new Fuzzy-based Similarity Measurement method called *FSM* fuzzy inference model is proposed in this section to express the nonlinear relationship between the fuzzy nearness, the correlation coefficient and similarity. Among them, the fuzzy nearness (*Fn*) and correlation coefficient (*Cor*) of *BBAs* are taken as input variables, and the similarity (*sim*) between the evidences is taken as output variables, as shown in Fig. 1.

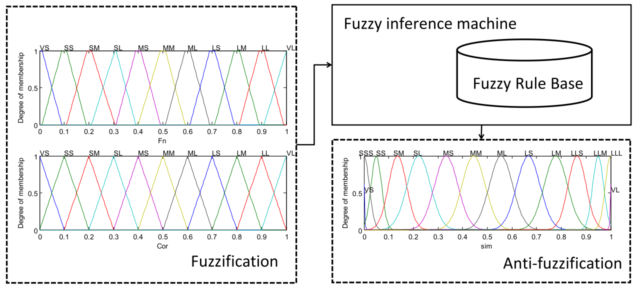

FIGURE 1. The FSM fuzzy inference model architecture.

The present *FSM* fuzzy inference model is mainly divided into three parts: fuzzification, fuzzy rules and inference, and anti-fuzzification. Specifically, firstly, the two factors (*Cor*, *Fn*) that measure the degree of conflict between evidences are fuzzified, and fuzzy inference rules are formulated according to experience and logical inference. Then, the fuzzy inference mechanism is used to perform fuzzy inference on the fuzzy set to which the similarity between conflict evidence belongs. Finally, the degree of similarity between the evidences is output by defuzzification.

Through this model, two aspects of measuring the similarity between evidences are considered, and the inference under different conflict types is considered comprehensively. Each rule represents inference in different types of conflict situations, and the condition part of each rule varies within the interval of the membership function of the corresponding fuzzy set, indicating that the degree of conflict of the conflict type corresponding to the rule varies within the range. Constructing such a rule base makes it easy to make fuzzy inference for conflicts of various types and levels without the need for classification considerations, which is convenient and efficient.

1) FUZZIFICATION

The process of fuzzification is the process of mapping the values in the range of input variables to the fuzzy subset of the corresponding membership functions. The fuzzy nearness *Fn* and the correlation coefficient *Cor* range from [0, 1]. The degree of similarity between the evidences is also in the range [0, 1]. In order to explain the relevant meaning of these variables, for example, the fuzzy nearness is large, the correlation coefficient is small, etc., and the linguistic variables are used to describe the input variable characteristics. According to the actual situation, the fuzzy nearness (*Fn*) and correlation coefficient (*Cor*) can set the following fuzzy subset, as shown in Table 2 and Table 3.

The fuzzy nearness (*Fn*) and correlation coefficient (*Cor*) are divided into 11 levels in the interval [0, 1], where 'VS' represents the value of *Fn* or *Cor* is very small, and 'VL' represents the value of *Fn* or *Cor* is very large. For the setting of the parameters of the membership function for each level, firstly, initialize the initial value by experience, and then the experiment is repeatedly demonstrated by

TABLE 2. The Trapezoidal membership function Parameters for fuzzy nearness.

Fuzzy sets	Abbreviation	Trapezoidal MF Parameters
Very Small	VS	$a = 0, b = 0, c = 0.01, d = 0.09$
Small-Small	SS	$a = 0.01, b = 0.09, c = 0.11, d = 0.192$
Small-Medium	SM	$a = 0.11, b = 0.192, c = 0.21, d = 0.3056$
Small-Large	SL	$a = 0.21, b = 0.3056, c = 0.31, d = 0.39$
Medium-Small	MS	$a = 0.31, b = 0.39, c = 0.41, d = 0.49$
Medium-Medium	MM	$a = 0.4, b = 0.49, c = 0.51, d = 0.59$
Medium-Large	ML	$a = 0.51, b = 0.59, c = 0.61, d = 0.69$
Large-Small	LS	$a = 0.61, b = 0.69, c = 0.71, d = 0.79$
Large-Medium	LM	$a = 0.71, b = 0.79, c = 0.81, d = 0.89$
Large-Large	LL	$a = 0.81, b = 0.89, c = 0.91, d = 1.00$
Very Large	VL	—

TABLE 3. The Trigonometric Membership Function Parameters for correlation coefficient.

Fuzzy sets	Abbreviation	Trigonometric MF Parameters
Very Small	VS	$f = 0, m = 0, g = 0.1$
Small-Small	SS	$f = 0, m = 0.1, g = 0.2$
Small-Medium	SM	$f = 0.1, m = 0.2, g = 0.3$
Small-Large	SL	$f = 0.2, m = 0.3, g = 0.4$
Medium-Small	MS	$f = 0.3, m = 0.4, g = 0.5$
Medium-Medium	MM	$f = 0.4, m = 0.5, g = 0.6$
Medium-Large	ML	$f = 0.5, m = 0.6, g = 0.7$
Large-Small	LS	$f = 0.6, m = 0.7, g = 0.8$
Large-Medium	LM	$f = 0.7, m = 0.8, g = 0.9$
Large-Large	LL	$f = 0.8, m = 0.9, g = 1.0$
Very Large	VL	$f = 0.9, m = 1.0, g = 1.0$

setting different conflict situations and types to obtain the best value. The optimal value is the best theoretical analysis result in different conflict situations. Finally, on the basis of testing data and experts' experience, the relevant parameters of the Trapezoidal membership function and Trigonometric membership function of each fuzzy set are set up, as shown in Table 2 and Table 3.

It is worth noting that in order to make *Fn* infinitely close to 1, the fuzzy inference output is also infinitely close to 1, and the *VL* fuzzy set is fuzzified using a triangular membership function. Since the *VL* fuzzy set corresponds to a value infinitely close to 1 in the numerical domain, the range of the mapping is extremely small, so a triangular membership function is used instead of a trapezoidal membership function with a large mapping range. The formula is as follows

$$\mu_{VL} = \begin{cases} \dfrac{x - 0.91}{1 - 0.91}, & 0.91 \leq x < 1 \\ 0, & others \end{cases} \tag{18}$$

The complexity of two-dimensional input variables with multiple fuzzy values is considered in this paper, and the output language set based on the input language set is further refined, as shown in Table 4.

Among them, according to (15) and (16), it can be analyzed that when $Fn = 0$, the probability of the two evidences after the transformation corresponds to the minimum probability sum of 0, indicating that the two evidences do not have the greatest probability to support the same proposition, and the minimum probability distribution in each evidence has at least one propositional distribution of 0, then according to (17), $Cor = 0$ is inferred, so $Sim = 0$ can be inferred. But when $Cor = 0$, it is not possible to infer that $Fn = 0$, and the value of *Sim* changes as the *Fn* value changes. Therefore, in the fuzzy subset of the output, a very small value set is defined to indicate that the interval of *Sim* tends to 0. Similarly, according to (15) and (16), it can be analyzed that when $Fn = 1$, it indicates that the transformed probability distributions are equal, so $Sim = 1$ is introduced, and according to (17), when $Cor = 1$, both evidences have the maximum probability. And the probability is 1 to support the same proposition, so it is inferred that $Sim = 1$. So the fuzzy subset of the output should define a very large value to indicate that the *Sim* interval tends to 1.

TABLE 4. The Gaussian Membership Function Parameters for output fuzzy sets.

Fuzzy sets	Abbreviation	Gaussian MF Parameters
Very Small	VS	$c = 0, \sigma = 0.003$
Small-Small-Small	SSS	$c = 8.674e - 19, \sigma = 0.02491$
Small-Small	SS	$c = 0.05, \sigma = 0.02491$
Small-Medium	SM	$c = 0.1326, \sigma = 0.0366$
Small-Large	SL	$c = 0.2222, \sigma = 0.04718$
Medium-Small	MS	$c = 0.3333, \sigma = 0.04718$
Medium-Medium	MM	$c = 0.4444, \sigma = 0.04718$
Medium-Large	ML	$c = 0.5555, \sigma = 0.04718$
Large-Small	LS	$c = 0.6666, \sigma = 0.04718$
Large-Medium	LM	$c = 0.7777, \sigma = 0.04718$
Large-Large-Small	LLS	$c = 0.8888, \sigma = 0.02491$
Large-Large-Medium	LLM	$c = 0.9555, \sigma = 0.02491$
Large-Large-Large	LLL	$c = 0.9899, \sigma = 0.02491$
Very Large	VL	$c = 1.000, \sigma = 0.003$

TABLE 5. Fuzzy rule.

sim		Fn										
		VS	SS	SM	SL	MS	MM	ML	LS	LM	LL	VL
Cor	VS	VS	SSS	SM	SL	LS	LLS	LLM	LLM	LLM	LLM	VL
	SS	SSS	SM	SM	SL	SL	ML	ML	LS	LM	LLS	VL
	SM	VS	SM	MS	MS	MM	MM	LS	LS	LM	LLS	VL
	SL	VS	SL	MS	MM	ML	LS	LM	LM	LLS	LLS	VL
	MS	VS	SL	MM	ML	LS	LS	LM	LM	LLS	LLM	VL
	MM	VS	ML	ML	LS	LS	LM	LM	LLS	LLS	LLM	VL
	ML	VS	ML	LS	LM	LM	LM	LLS	LLS	LLS	LLM	VL
	LS	VS	LS	LS	LM	LM	LLS	LLS	LLS	LLM	LLM	VL
	LM	VS	LM	LM	LLS	LLS	LLS	LLS	LLM	LLL	LLL	VL
	LL	VS	LLS	LLS	LLS	LLM	LLM	LLM	LLM	LLL	LLL	VL
	VL	VS	VL	VL	VL	VL	VL	VL	VL	VL	VL	VL

According to the above analysis, for output variables, the Gaussian membership function has a high approximation effect, so it is used as the membership function of fuzzy set, and in order to more accurately reflect the mapping distribution between the larger value and the smaller value, the distribution is refined at the two poles, as shown in Fig. 2.

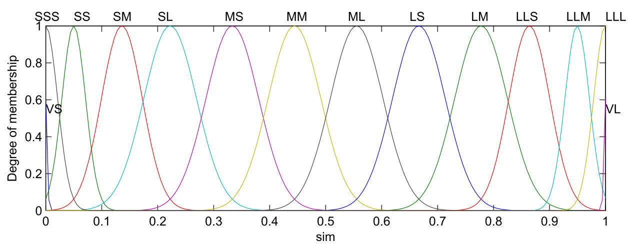

FIGURE 2. Membership function of output.

Similarly, the initial value is set according to the above analysis, and then the parameters are verified and modified by continuous experiment until the optimal parameters are obtained. On the basis of testing data and experts'experience, the relevant parameters of the Gaussian membership function of each fuzzy set are shown in the following Table 4.

2) FUZZY RULES AND INFERENCE

Fuzzy rules are described using a series of fuzzy languages and are established through theoretical analysis and experts'experience and practical experiences. This section adopts a combination of theory and experience to establish a fuzzy rule base. Firstly, the rough relationship between the fuzzy nearness Fn and the correlation coefficient Cor is analyzed in this paper, and the preliminary fuzzy rule base is established according to the expert'experience and logic analysis, and then the similarity between the evidences is tested and the preliminary fuzzy rule base is fine-tuned according to the test results. As the core of fuzzy system, fuzzy rule base is essentially composed of fuzzy 'IF-THEN' rule set, its format is as follows:

$$\text{If } Fn \text{ is } \tilde{A}_{1j} \text{ AND } Cor \text{ is } \tilde{A}_{2j} \text{ Then } sim \text{ is } \tilde{B}_j,$$

here $\tilde{A}_{ij}$ and $\tilde{B}_j$ are the fuzzy sets representing the jth linguistic rule for the ith input parameter and the output parameter sim, and the final fuzzy rules are shown in Table 5.

It is worth noting that, according to the analysis in the previous section, when $Fn = 0$, $Cor = 0$ and $sim = 0$ are obtained, so when Fn belongs to VS, the maximum value of Cor does not exceed SS. Therefore, the value of Cor that exceeds the SS set does not exist. In the set where the Cor value does not exist, the sim is represented by the VS fuzzy set, as shown in Table 5. When Cor belongs to VS, the size of the sim value depends only on the size of the Fn value. When Fn is large, the output sim value will also be very large. But when Cor or Fn does not belong to VS, which fuzzy set sim belongs to must consider the size of Fn, Cor at the same time.

After the establishment of the fuzzy rule base, the Mamdani fuzzy inference method is then used in this paper to reason and obtain the fuzzy set of the output.

3) ANTI-FUZZIFICATION

The final step of the fuzzy inference mechanism is anti-fuzzification. In this paper, the centroid method is used to find the abscissa of the position of the center of gravity in the graph of the membership function corresponding to the fuzzy set, which is taken as the most representative and accurate value to be sim.

4) THE DISSIMILARITY OF THE EVIDENCES

Since the similarity is inversely proportional to the degree of conflict, in order to more intuitively compare with other existing methods, here the variable $Dism$ is defined in this paper and is denoted by

$$Dism(m_i, m_j) = 1 - sim(m_i, m_j), \quad 1 \leq i, j \leq k. \quad (19)$$

There are some properties for the FSM model based on the above fuzzy method:

1) $Dism(m_1, m_2)$ is symmetric and always well defined;
2) $Dism(m_1, m_2)$, is bounded, $0 \leq Dism(m_1, m_2) \leq 1$
3) Its square root, $\sqrt{Dism(m_1, m_2)}$ verifies the triangle inequality.

In order to illustrate the effectiveness of the new method of conflict measurement, some examples are shown in the following.

In Example 2, it can be noticed that m_1 and m_3 have relatively large belief values to support the object θ_1, where $m_1(\theta_1) = 0.5$ and $m_3(\theta_1) = 0.8$, whereas m_2 supports the proposition θ_2 with the maximal belief values. Therefore, the conflict between m_1 and m_2 should be larger than the one between m_1 and m_3 according to the intuition. Then, the specific calculation processes of similarity based on *FSM* fuzzy inference model are listed as follows:

$$Bel = \begin{bmatrix} m_1(\theta_1) & m_1(\theta_2) & m_1(\theta_3) \\ m_2(\theta_1) & m_2(\theta_2) & m_2(\theta_3) \\ m_3(\theta_1) & m_3(\theta_2) & m_3(\theta_3) \end{bmatrix} = \begin{bmatrix} 0.5 & 0.3 & 0.1 \\ 0.3 & 0.5 & 0 \\ 0.8 & 0 & 0.2 \end{bmatrix};$$

$$pl = \begin{bmatrix} pl_1(\theta_1) & pl_1(\theta_2) & pl_1(\theta_3) \\ pl_2(\theta_1) & pl_2(\theta_2) & pl_2(\theta_3) \\ pl_3(\theta_1) & pl_3(\theta_2) & pl_3(\theta_3) \end{bmatrix} = \begin{bmatrix} 0.6 & 0.4 & 0.2 \\ 0.5 & 0.7 & 0.2 \\ 0.8 & 0 & 0.2 \end{bmatrix}.$$

According to (16), the transformed probability distribution matrix is obtained as

$$P = \begin{bmatrix} 0.5548 & 0.3333 & 0.1118 \\ 0.3739 & 0.6174 & 0.0087 \\ 0.8000 & 0 & 0.2000 \end{bmatrix}.$$

The fuzzy nearness *Fn* matrix and the correlation coefficient *Cor* matrix can be obtained by (15) and (17),

$$Fn = \begin{bmatrix} 1.0000 & 0.5576 & 0.5000 \\ 0.5576 & 1.0000 & 0.2366 \\ 0.5000 & 0.2366 & 1.0000 \end{bmatrix},$$

$$Cor = \begin{bmatrix} 0.5000 & 0.0500 & 0.6500 \\ 0.0500 & 0.5000 & 0 \\ 0.6500 & 0 & 0.8000 \end{bmatrix}.$$

Based on the fuzzy inference mechanism, intelligently calculate the following *Sim* matrix

$$Sim = \begin{bmatrix} 1.0000 & 0.7221 & 0.8119 \\ 0.7221 & 1.0000 & 0.1695 \\ 0.8119 & 0.1695 & 1.0000 \end{bmatrix}.$$

One gets

$$Dism(m_1, m_2) = 1 - sim(m_1, m_2) = 1 - 0.7221 = 0.2779;$$
$$Dism(m_1, m_3) = 1 - sim(m_1, m_3) = 1 - 0.8119 = 0.1881.$$

For this example, the results obtained by different measurement methods are shown in Table 6 where *D* represent the dissimilarity between *BBAs*.

TABLE 6. The dissimilarity measurement for different methods.

	D(m_1,m_2)	D(m_1, m_3)
difBetP [35]	0.2334	0.3333
DistP [21]	0.2334	0.3834
BJS [24]	0.0987	0.2375
Our method	0.2779	0.1881

From Table 6, it's worthy of mentioning that the counter-intuitive example cannot be completed by the existing ones, see *difBetP* [35], *DistP* [21], and *BJS* [24], while the present approach can work. Specifically, the dissimilarity between m_1 and m_3 is larger than that between m_1 and m_2 according to the distance measures *difBetP* [35], *DistP* [21], and *BJS* [24] which are counterintuitive. The new dissimilarity $Dism(m_1, m_2) > Dism(m_1, m_3)$ shows m_1 and m_3 are more similar than m_1 and m_2, which is in line with the intuitive judgment. This indicates that a new method based on fuzzy inference mechanism to measure dissimilarity between evidences can correctly measure conflicts.

At the same time, this example verifies that when m_1 and m_1 have the same *BBAs*, the similarity between m_1 and m_1 tends to 1, which is consistent with the intuitive results. Besides, from the above results, it can be see that the similarity between m_1 and m_2 $sim(m_1, m_2)$ is equal to the similarity between m_2 and m_1 $sim(m_2, m_1)$, then the $Dism(m_1, m_2) = Dism(m_2, m_1)$ can be obtained, the symmetric property of similarity based on *FSM* is verified in this example.

Example 3: Let the frame of discernment $\Theta = \{a, b, c, d, e, f, g, h\}$, and there are two *BBAs*:

$$\begin{aligned} m_1 : m_1(a) &= 0.25, \quad m_1(b) = 0.25, \\ m_1(c) &= 0.25, \quad m_1(d) = 0.25; \\ m_2 : m_2(e) &= 0.25, \quad m_2(f) = 0.25, \\ m_2(g) &= 0.25, \quad m_2(h) = 0.25. \end{aligned}$$

As shown in Example 3, the evidence m_1 and the evidence m_2 are completely conflicting. The specific calculation processes of similarity based on *FSM* are given as follows:

$$Bel = \begin{bmatrix} 0.25 & 0.25 & 0.25 & 0.25 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.25 & 0.25 & 0.25 & 0.25 \end{bmatrix},$$

the probability distribution of a single subset has been given, so no probability needs to be transformed. The fuzzy nearness *Fn* matrix and the correlation coefficient *Cor* matrix can be obtained by (15) and (17).

$$\begin{aligned} Fn_{12} &= Fn_{21} \\ &= \frac{0+0+0+0+0+0+0+0}{0.25+0.25+0.25+0.25+0.25+0.25+0.25+0.25} \\ &= 0; \end{aligned}$$

$$Cor_{12} = Cor_{21} = \frac{0+0}{2} = 0;$$

$$Fn = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad Cor = \begin{bmatrix} 0.25 & 0 \\ 0 & 0.25 \end{bmatrix}.$$

Based on the fuzzy inference mechanism, intelligently calculate the following *Sim* matrix

$$Sim = \begin{bmatrix} 1.000 & 0.000 \\ 0.000 & 1.000 \end{bmatrix},$$

It can be seen from the results that $sim(m_1, m_2) = 0$, so the degree of conflict between the m_1 and m_2 is $Dism(m_1, m_2) = 1$, which is in line with the intuitive results.

Example 4: Supposing that there are three *BBAs* m_1, m_2 and m_3 in the frame of discernment $\Theta = \{A, B\}$ which is complete, and the three *BBAs* are given as follows:

$$m_1 : m_1(A) = 0.99, \quad m_1(B) = 0.01;$$
$$m_2 : m_2(A) = 0.90, \quad m_2(B) = 0.10;$$
$$m_3 : m_3(A) = 0.01, \quad m_3(B) = 0.99.$$

As shown in Example 4, it can be seen that m_1 and m_2 have great belief values to support the object A, where $m_1(A) = 0.99$ and $m_2(A) = 0.90$. On the contrary, m_3 has a great belief value to support the object B, where $m_3(B) = 0.99$. So cognitively, the similarity between m_1 and m_2 should be greater than m_1 and m_3. The similarity based on *FSM* is calculated below:

$$Bel = \begin{bmatrix} 0.99 & 0.01 \\ 0.90 & 0.1 \\ 0.01 & 0.99 \end{bmatrix},$$

the probability distribution of a single subset has been given, so no probability needs to be transformed. The fuzzy nearness *Fn* matrix and the correlation coefficient *Cor* matrix can be obtained by (15) and (17).

$$Fn = \begin{bmatrix} 1.0000 & 0.8349 & 0.0101 \\ 0.8349 & 1.0000 & 0.0582 \\ 0.0101 & 0.0582 & 1.0000 \end{bmatrix},$$
$$Cor = \begin{bmatrix} 0.9900 & 0.9450 & 0.0100 \\ 0.9450 & 0.9000 & 0.0550 \\ 0.0100 & 0.0550 & 0.9900 \end{bmatrix}.$$

Based on the fuzzy inference mechanism, intelligently calculate the following *Sim* matrix:

$$Sim = \begin{bmatrix} 1.0000 & 0.9747 & 0.0163 \\ 0.9747 & 1.0000 & 0.1065 \\ 0.0163 & 0.1065 & 1.0000 \end{bmatrix}.$$

After that, their corresponding square root values can be calculated as follows:

$$\sqrt{Dism(m_1, m_2)} = \sqrt{1 - 0.9747} = 0.1591;$$
$$\sqrt{Dism(m_2, m_3)} = \sqrt{1 - 0.1065} = 0.9453;$$
$$\sqrt{Dism(m_1, m_3)} = \sqrt{1 - 0.0163} = 0.9918.$$

It can be noticed that $\sqrt{Dism(m_1, m_2)} + \sqrt{Dism(m_2, m_3)} = 1.1044$, so that $\sqrt{Dism(m_1, m_3)} < \sqrt{Dism(m_1, m_2)} + \sqrt{Dism(m_2, m_3)}$ which satisfies the triangle inequality property of *FSM* model.

Example 5: Supposing that there are two *BBAs* m_1 and m_2 in the frame of discernment $\Theta = \{A, B\}$ which is complete, and the two *BBAs* are given as follows:

$$m_1 : m_1(A) = \alpha, \quad m_1(B) = 1 - \alpha;$$
$$m_2 : m_2(A) = 0.9999, \quad m_2(B) = 0.0001.$$

As shown in Example 5, Target A is supported by m_2 with the maximum belief value, where $m_2(A) = 0.9999$. When the parameter α changes from 0 to 1, the variation of dissimilarity based on *FSM* between m_1 and m_2 is depicted in Fig. 3.

Obviously, when α tends to 1, the dissimilarity based on *FSM* between m_1 and m_2 is going to 0. As can be seen

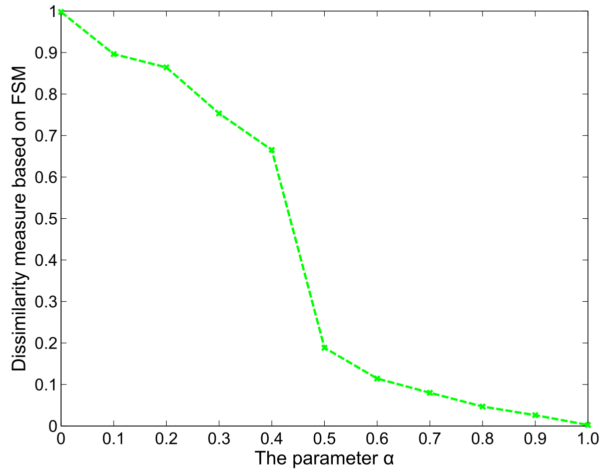

FIGURE 3. An example of FSM with changing parameter α.

from Fig. 3, the phenomenon intuitively can be explained where m_1 and m_2 are almost the same at this time, that is, the object A as the target was supported by m_1 and m_2 with a great belief value.

Conversely, when α approaches 0, the dissimilarity between m_1 and m_2 becomes 1, indicating that m_1 and m_2 are completely different. Specifically, m_1 strongly supports object B as the target, while m_2 strongly supports object A as the target.

In a word, the bounded property of the similarity based on *FSM* [0, 1] is verified in this example.

Example 6: Let Θ be a frame of discernment with 20 elements. Here, 1, 2, etc. are used to denote element 1, element 2, etc. in the frame of discernment. Two *BBAs* are defined as follows [36]:

$$m_1 : m_1(\{2, 3, 4\}) = 0.05, \quad m_1(\{7\}) = 0.05,$$
$$m_1(\Theta) = 0.1, \quad m_1(\{A\}) = 0.8;$$
$$m_2 : m_2(\{1, 2, 3, 4, 5\}) = 1.$$

Here, A that has 20 cases is the subset of Θ. It starts from Case 1 ($A = \{1\}$), and ends at Case 20 ($A = \{1, 2, \ldots, 20\}$).

In this example, the new method of conflict measurement is compared with classical conflict coefficient (K) [11], Jousselme *et al.*'s distance (d) [36], pignistic probability distance (*difBetP*) [37], Ma and An's dissimilarity measure (*DisSim*) [22], Zhao *et al.*'s new conflict coefficient (k_{new}) [23], and Xiao's Belief divergence measure (*BJS*) [24]. The results are shown in Fig. 4.

As can be seen from Fig. 4, K is always equal to 0.05 regardless of how subset A changes. *BJS* are equal to 1 except when $A = \{1, 2, 3, 4, 5\}$. Although method [24] considers the impact of evidence uncertainty on the source of evidence, it cannot effectively measure conflicts between evidences. The result shows that neither of two methods above can express the conflict between the evidences well. In Fig. 4, the d, *difBetP*, *DisSim*, *knew* and *Dism* have the same curve trend. At first, the curve decreases with the increase of the elements contained in A. When $A = \{1, 2, 3, 4, 5\}$, both

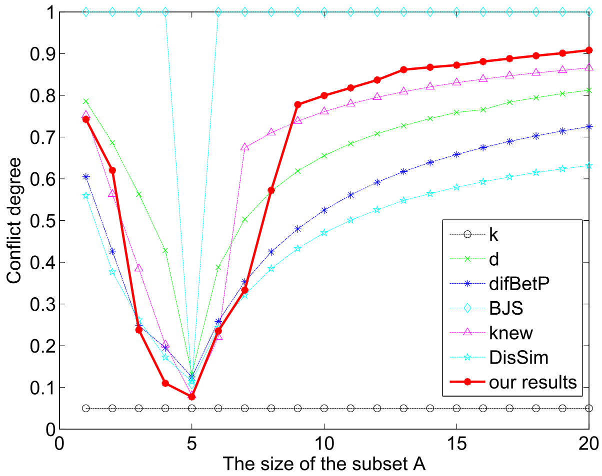

FIGURE 4. **The comparisons of different methods.**

two pieces of evidences support proposition A with the maximum transformation probability, which indicates that the conflict between the evidences reaches the minimum and the curve drops to the lowest. After that, with the increase of elements in A, the degree of conflict between the two evidences increases gradually, and the curve shows an upward trend, which accords with intuitive judgment. And the higher the curve is, the more effective the conflict identification is, the higher the conflict degree is measured, and the smaller the weight will be given in the later discount processing, so as to reduce the negative impact of high conflict evidence. As can be seen from Fig. 4, the curve of the conflict measurement model presented in this paper falls to the lowest point when the conflict is minimal compared with that of other methods, while the curve in this paper is higher than that of the existing methods when the conflict increases gradually, which shows that the new conflict measurement method proposed in this paper can accurately and effectively measure the conflict between evidences.

At the same time, when the elements of the A set are increasing, the change of the belief value assigned to each element will become smaller and smaller, and the rate of change of the conflict between the evidences will gradually decrease. It can be seen from the Fig. 4 that the curve of the method proposed in this paper has a smaller and smaller change rate in the later stage, especially when the element size of the A set is greater than 10, the curve of the proposed method is more stable than the curve of other methods, and the curve is also the highest, indicating that the method proposed in this paper can effectively measure the high conflict between evidences.

In this experiment, the change of parameter A is used to simulate the change of one piece of evidence uncertainty and the change of type and degree of conflict between two evidences, so as to verify the Fuzzy Similarity Measurement model proposed in this paper can effectively measure conflicts in different types and degrees of conflict situations. Fig. 4 shows that the conflict measurement method proposed in this paper cover the existing conflict measurement method.

IV. THE PROPOSED METHOD

In this section, a new multi-sensor conflict data fusion method is proposed, which is based on *FSM* and belief entropy. It includes the following parts. Firstly, the mass of multi-subset focal element is transformed into a single subset probability distribution through the probability transformation formula, and the fuzzy nearness and correlation coefficient are obtained. Then the *FSM* model is designed to measure the differences and similarities between the evidences; The credibility derived from the similarity measure is then obtained to represent the believablity of the evidences. The greater the evidence is supported by other evidences, the greater the similarity between it and other evidences, the less conflict it has with other evidences, and the larger weight should be assigned to the evidence. On the contrary, when other evidences rarely supports this evidence, it is considered that the conflict between the evidence and other evidences is relatively large, and the similarity is smaller, so the evidence should have a smaller weight. Next, the belief entropy is used to calculate the information volume of evidences to denote the uncertainty of the evidences. Thereafter, the information volume of the evidences is utilized to modify the credibility of the evidences, and the final results obtained are normalized as the final weights of the evidences. Finally, the weights are used to adjust the evidence sources, and the average evidence is obtained and then merged by the Dempster's combination rule. The flow chart of the presented method is shown in Fig. 5.

A. THE CREDIBILITY DEGREE OF THE EVIDENCES

The degree of support between the evidences can be used to judge evidence which differs greatly from other evidences. If one piece of evidence is strongly supported by other evidences, it means more similar to other evidences, and the degree of support will be greater. Conversely, if the support obtained from other evidences is small, it indicates that there is a conflict between the evidence and other evidences, and the similarity between the evidences must be small. Therefore, the degree of support between evidences can be expressed by similarity coefficients or conflicting coefficients. As we known, the most typical method is that the collision coefficient K in [11] was used to quantify the degree of support, and any other definitions of disagreement were obtained, such as, Zhao *et al.*'s new conflict coefficient (k_{new}) [23], and Xiao's belief divergence measure (*BJS*) [24], etc. However, these conflict coefficient adopted the method of distance measurement between evidences, in which only one aspect of the conflict between evidences has been considered. It's worthy of mentioning that the conflict situation cannot be comprehensively described to show the credibility of the evidences. In our previous study [22], the authors considered the various aspects of similarity measurement between evidences, that is, the correlation coefficient and the fuzzy nearness, which are combined to measure the similarity and conflict between the evidences. On this basis, a new similarity

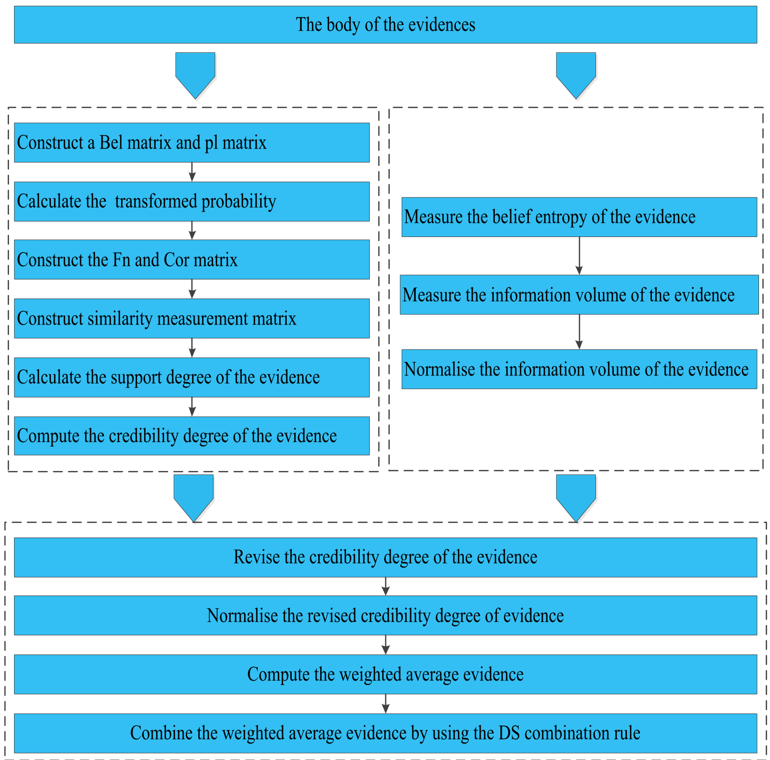

FIGURE 5. The flowchart of the proposed method.

measurement method based on fuzzy inference mechanism is first developed in this paper. People's knowledge and experience are combined by this method to describe the relationship of correlation coefficient, fuzzy nearness and similarity. Therefore, the degree of support between the evidences is expressed by the relationship of correlation coefficient and fuzzy nearness, and similarity based on the FSM, as shown below:

Supposing that there are k *BBAs* $m_i(i = 1, 2, \ldots, k)$ in the frame of discernment $\Theta = \{\theta_1, \theta_2, \ldots, \theta_n\}$, by making use of probabilistic transformation (16), the probability distribution calculation process is as follows:

Constructing a *Bel* matrix according to (2)

$$Bel = \begin{bmatrix} m_1(\theta_1) & \cdots & m_1(\theta_j) & \cdots & m_1(\theta_n) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ m_i(\theta_1) & \cdots & m_i(\theta_j) & \cdots & m_i(\theta_n) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ m_k(\theta_1) & \cdots & m_k(\theta_j) & \cdots & m_k(\theta_n) \end{bmatrix},$$

constructing a *pl* matrix according to (3)

$$pl = \begin{bmatrix} pl_1(\theta_1) & \cdots & pl_1(\theta_j) & \cdots & pl_1(\theta_n) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ pl_i(\theta_1) & \cdots & pl_i(\theta_j) & \cdots & pl_i(\theta_n) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ pl_k(\theta_1) & \cdots & pl_k(\theta_j) & \cdots & pl_k(\theta_n) \end{bmatrix},$$

then the probability distribution matrix is calculated by (16)

$$P = \begin{bmatrix} P_1(\theta_1) & \cdots & P_1(\theta_j) & \cdots & P_1(\theta_n) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ P_i(\theta_1) & \cdots & P_i(\theta_j) & \cdots & P_i(\theta_n) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ P_k(\theta_1) & \cdots & P_k(\theta_j) & \cdots & P_k(\theta_n) \end{bmatrix}.$$

Constructing a *Fn* matrix according to (15)

$$Fn = \begin{bmatrix} fn_{11} & \cdots & fn_{1j} & \cdots & fn_{1k} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ fn_{i1} & \cdots & fn_{ij} & \cdots & fn_{ik} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ fn_{k1} & \cdots & fn_{kj} & \cdots & fn_{kk} \end{bmatrix},$$

constructing a *Cor* matrix according to (17)

$$Cor = \begin{bmatrix} cor_{11} & \cdots & cor_{1j} & \cdots & cor_{1k} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ cor_{i1} & \cdots & cor_{ij} & \cdots & cor_{ik} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ cor_{k1} & \cdots & cor_{kj} & \cdots & cor_{kk} \end{bmatrix},$$

then the evidence similarity can be represented with matrix *Sim* according to *FMS* model

$$Sim = \begin{cases} sim(m_i, m_j), & i \neq j \\ 1, & i = j. \end{cases} \tag{20}$$

Definition 8: The total support degree of ith piece of evidence is $Sd(m_i)$, defined as

$$Sd(m_i) = \sum_{s=1,s\neq i}^{k} sim(m_i, m_s). \tag{21}$$

Definition 9: The credibility of evidence can be calculated by the following formula:

$$Cd(m_i) = \frac{Sd(m_i)}{\sum_{s=1}^{k} Sd(m_s)} 1 \leq i \leq k. \tag{22}$$

B. THE INFORMATION VOLUME OF THE EVIDENCES

In this subsection, the information volume of the evidences is calculated by considering the uncertainty of the evidence itself, on the basis of the degree of conflict between evidences. This consideration is based on the fact that the uncertainty of the evidences may show/judge the importance of the evidences. Note that the greater the uncertainty of the evidences, the lower the importance of the evidences, and the corresponding weight of the distribution is relatively small. On the contrary, if the uncertainty of the evidences is small, the amount of information is large and the weights assigned are relatively large. Therefore, in order to consider the amount of information carried by the evidence itself, the method [24] is introduced in this paper to calculate the total amount of information in the evidences.

Definition 10: The uncertainty of the evidence m_i is calculated by the information volume $Iv(m_i)$ and is defined as:

$$Iv(m_i) = e^{E_d} = e^{-\sum_i m(A_i)\log(\frac{m(A_i)}{2^{|A_i|}-1})}, \quad 1 \leq i \leq k, \tag{23}$$

where the belief entropy E_d of the evidence is calculated by (13).

Definition 11: The information volume of the evidence m_i is normalized as below,

$$\tilde{I}v(m_i) = \frac{Iv(m_i)}{\sum_{s=1}^{k} Iv(m_s)}, \quad 1 \leq i \leq k. \tag{24}$$

C. THE WEIGHT OF EVIDENCE

The weight of evidence is obtained by the uncertainty, information volume, and credibility of the evidences, in order to assign the corresponding effect/degree of each evidence in the integration process. It's easy seen that when the uncertainty of evidences is smaller, that is, the larger the amount of information, the greater the credibility of the evidences, therefore it can be shown that this evidence is very credible and thus the weight assigned at this evidence should be larger. On the contrary, if the credibility of evidences is small and the amount of information is small, it indicates that there is high-conflict between this evidence and other evidences, then a small weight value should be assigned to this evidence to reduce its negative effects during the evidences fusion process.

Definition 12: The revised credibility degree $RCd(m_i)$ of ith piece of evidence is defined as

$$RCd(m_i) = Cd(m_i) \times \tilde{I}v(m_i), \quad 1 \leq i \leq k. \tag{25}$$

The high degree of credibility indicates that the evidence is highly supported by other evidences. In other words, the evidence is highly similar to other evidences, and the low amount of information indicates that the evidence is highly uncertain. The revised credibility should be reduced, because the uncertainty of the evidence indicates that the evidence is unreliable, even if it is similar to other evidence, it is unreliable in the evidence itself. Therefore, the overall credibility of the evidence should be reduced. Similarly, when the uncertainty of the piece of evidence is small, that is, when the reliability is high and the amount of information is large, but the similarity between it and other evidences is low, that is, the credibility is low. Then the overall credibility of the evidence is relatively low. These properties can all be reflected in equation (25).

Definition 13: The revised credibility degree is normalized as the final weight of evidence m_i and defined as

$$W(m_i) = \frac{RCd(m_i)}{\sum_{s=1}^{k} RCd(m_s)}, \quad 1 \leq i \leq k. \tag{26}$$

D. THE WEIGHTED AVERAGE EVIDENCE

The weighted average evidence (WAE) is constructed to weight the original evidence to obtain the average evidence, as the method of [24]. Notice that the average evidence can greatly reduce the negative impact of conflict evidence, and can make full use of the information of credible evidence to make the final fusion result optimal.

Definition 14: The weighted average evidence $WAE(m)$ is defined as follows:

$$WAE(m) = \sum_{i=1}^{k} W(m_i) \times m_i, \quad i \leq i \leq k. \tag{27}$$

Definition 15: Suppose the frame of discernment $\Theta = \{\theta_1, \theta_2, \cdots \theta_n\}$, and there are k pieces of evidence $m_i (i = 1, 2 \cdots, k)$. The combination of the weighted average evidence $WAE(m)$ by (4), and defined as

$$m(\theta_i) = \underbrace{WAE(\theta_i) \oplus WAE(\theta_i) \cdots \oplus WAE(\theta_i)}_{k}, \quad 1 \leq i \leq n. \tag{28}$$

V. EXPERIMENT

In this section, a numerical example will be enumerated to demonstrate the feasibility, robustness and effectiveness of the proposed method.

A. FEASIBILITY

1) PROBLEM STATEMENT

In this part, the classic example of Zadeh is used in this paper to illustrate that the proposed method can effectively solve the counterintuitive problem of using *DS* combination rules to fuse the high-conflict evidence.

As shown in example 1, m_1 supports θ_1 with a probability of 0.9, while m_2 supports θ_3 with a probability of 0.9. It can be seen that the two evidences support different hypotheses with the highest belief value, so the two pieces of evidence are highly conflicting.

2) IMPLEMENTATION BASED ON THE PROPOSED METHOD

Step 1: Constructing a *Bel* matrix according to (2)

$$Bel = \begin{bmatrix} 0.9 & 0.1 & 0 \\ 0 & 0.1 & 0.9 \end{bmatrix}$$

Step 2: Since the evidence sources give a basic allocation of a single subset, there is no need to perform a probability transition on the basic belief assignment of the evidence sources.

$$P = \begin{bmatrix} 0.9 & 0.1 & 0 \\ 0 & 0.1 & 0.9 \end{bmatrix}$$

Step 3: Constructing a *Fn* matrix according to (15)

$$Fn = \begin{bmatrix} 1.0000 & 0.0526 \\ 0.0526 & 1.0000 \end{bmatrix}$$

Step 4: Constructing a *Cor* matrix according to (17)

$$Cor = \begin{bmatrix} 0.9 & 0 \\ 0 & 0.9 \end{bmatrix}$$

Step 5: Constructing the similarity measurement matrix $Sim_{k\times k}$ according to *FMS* model as

$$Sim = \begin{bmatrix} 1.0000 & 0.0208 \\ 0.0208 & 1.0000 \end{bmatrix}$$

Step 6: Calculate the support degree of the evidence m_i as below:

$$Sd = \begin{bmatrix} 0.0208 & 0.0208 \end{bmatrix}$$

Step 7: Calculate the credibility degree of the evidence m_i as below:

$$Cd = \begin{bmatrix} 0.5 & 0.5 \end{bmatrix}$$

Step 8: Measure the information volume of the evidence m_i as below:

$$Iv = \begin{bmatrix} 1.5984 & 1.5984 \end{bmatrix}$$

Step 9: Normalise the information volume of the evidence m_i as follows:

$$\tilde{I}v = \begin{bmatrix} 0.5 & 0.5 \end{bmatrix}$$

Step 10: Revise the credibility degree of the evidence m_i based on the information volume of the evidence as below:

$$RCd = \begin{bmatrix} 0.25 & 0.25 \end{bmatrix}$$

Step 11: Normalise the revised credibility degree of the evidence m_i as below:

$$w = \begin{bmatrix} 0.5 & 0.5 \end{bmatrix}$$

Step 12: Compute the weighted average evidence as follows:

$$m(\theta_1) = 0.45, m(\theta_2) = 0.1, m(\theta_3) = 0.45$$

Step 13: Combine the weighted average evidence via the Dempster's rule, and the fusing results are shown in Table 7.

TABLE 7. Combination results of m_1 and m_2 for Example 1.

Methods	$m(\theta_1)$	$m(\theta_2)$	$m(\theta_3)$	$m(\Theta)$
DS [11]	0	1	0	0
Our results	0.4880	0.0241	0.4880	0

3) DISCUSSION

As can be seen from Table 7, the *DS* combination rule assigns a 100% belief value to θ_2, which is clearly a counter-intuitive result. The method proposed in this paper can balance the conflicts and assign the θ_1 and θ_3 with the same belief value of 0.4480, which this is not enough to make a decision. Moreover, when the new evidence source m_3 supporting θ_1 is collected, the results of the fusion are shown in the Table 8. It can be seen from Table 8 that the method proposed in this paper fuses the newly added evidence and supports θ_1 with a belief value of 0.9869, which is in line with the intuitive result.

As can be seen from this part, if a piece of evidence is correct and the other evidences are wrong, then the results obtained after the fusion are not enough to make a decision. This set of evidences is either invalid or requires more evidences to prove the event. In this example, a new evidence is added and re-fused to obtain reliable results. The event supported by the highest belief value may be the final decision result. To obtain this result correctly, other evidences are needed to support this event. So if under normal circumstances, the other evidences with the greatest belief support the same event, the *DS* combination rule can be used to fuse the evidences to obtain a belief value higher than the evidence sources to support the event. Finally, the event can be determined with certainty as the final decision result. However, the fact is that evidences with noise cannot support this event with the maximum belief value, even the lowest belief supports the event. Therefore, it is necessary to reduce the negative influence of the noise evidences in the process of *DS* combination rule fusion. Otherwise, the belief value of the correct result will be reduced in the final fusion result, even lower than the threshold of the judgment. Then the counterintuitive result appears. So, it is an advantage that maximizes the belief value of the same event supported by most evidences after *DS* combination, and can make decisions more reliably. It can be seen from Table 8 that the method proposed in this paper not only solves the problem of using the *DS* combination rule to fuse the high conflict evidences to produce counter-intuitive results, but also achieves this advantage most effectively.

TABLE 8. The new evidence source m_3 and combined results.

	$m(\theta_1)$	$m(\theta_2)$	$m(\theta_3)$	$m(\Theta)$
m_3	0.9	0.1	0	0
DS [11]	0	1	0	0
Ma [22]	0.6772	0.0716	0.1301	0.1217
Our results	0.9869	0.0127	0.0004	0

The previous work [22] was to measure the conflict between evidences by using Hamacher T-conorm rule combined with fuzzy nearness and correlation coefficient. But the complex relationship between fuzzy nearness, correlation coefficient and the conflict between evidences is not enough to express a detailed expression with a functional relationship. In order to better eliminate the negative effects of various conflicting uncertain data appearing in practical application fusion, the fuzzy inference mechanism is proposed to model the fuzzy relationship of the fuzzy nearness, the correlation coefficient and the conflict degree between evidences in this paper. At the same time, in order to further reduce the uncertainty of the evidence source, the Deng Entropy theory [32] is used in this paper to measure the information entropy of evidences to express the relative importance of evidence sources. The two factors that influence the quality of evidence fusion results are considered simultaneously in this paper, that is, the degree of conflict between evidence and the uncertainty of the evidence itself. Combining these two factors further reduces the impact of uncertain evidence and high-conflict evidence on the evidence fusion process. The experimental results show that the proposed method improves the reliability and accuracy of the fusion results.

B. ROBUSTNESS

1) PROBLEM STATEMENT

In practical applications, the perception and inference of the sensor to the surrounding environment is often unstable, resulting in the belief function collected by the sensor changing within a certain range. Therefore, the robustness of the fusion method directly affects the fusion result.

Example 7 (Employed From [22]): There are three simple $BBAs$ in the frame of discernment $\Theta = \{A, B, C\}$ as shown in Table 9.

TABLE 9. Four sources of evidence in Example 7.

	$m(A)$	$m(B)$	$m(C)$
m_1	0	0.9	0.1
m_2	0.6	0.25	0.15
m_{3a}	0.75	0.15	0.1
m_{3b}	0.7	0.2	0.1

2) IMPLEMENTATION BASED ON THE PROPOSED METHOD

Step 1: Constructing a Bel matrix according to (2)

$$Bel_{123a} = \begin{bmatrix} 0 & 0.9 & 0.1 \\ 0.6 & 0.25 & 0.15 \\ 0.75 & 0.15 & 0.1 \end{bmatrix};$$

$$Bel_{123b} = \begin{bmatrix} 0 & 0.9 & 0.1 \\ 0.6 & 0.25 & 0.15 \\ 0.7 & 0.2 & 0.1 \end{bmatrix}.$$

Step 2: The probability distribution of a single subset has been given,so no probability needs to be transformed. The Fn matrix and the Cor matrix can be obtained by (15) and (17).

$$Fn_{123a} = \begin{bmatrix} 1.0000 & 0.2121 & 0.1429 \\ 0.2121 & 1.0000 & 0.7391 \\ 0.1429 & 0.7391 & 1.0000 \end{bmatrix};$$

$$Cor_{123a} = \begin{bmatrix} 0.9000 & 0 & 0 \\ 0 & 0.6000 & 0.6750 \\ 0 & 0.6750 & 0.7500 \end{bmatrix};$$

$$Fn_{123b} = \begin{bmatrix} 1.0000 & 0.2121 & 0.1765 \\ 0.2121 & 1.0000 & 0.8182 \\ 0.1765 & 0.8182 & 1.0000 \end{bmatrix};$$

$$Cor_{123b} = \begin{bmatrix} 0.9000 & 0 & 0 \\ 0 & 0.6000 & 0.6500 \\ 0 & 0.6500 & 0.7000 \end{bmatrix}.$$

Step 3: Constructing the similarity measurement matrix $Sim_{k\times k}$ according to FMS model as

$$Sim_{123a} = \begin{bmatrix} 1.0000 & 0.1376 & 0.0966 \\ 0.1376 & 1.0000 & 0.9134 \\ 0.0966 & 0.9134 & 1.0000 \end{bmatrix};$$

$$Sim_{123b} = \begin{bmatrix} 1.0000 & 0.1376 & 0.1217 \\ 0.1376 & 1.0000 & 0.9202 \\ 0.1217 & 0.9202 & 1.0000 \end{bmatrix}.$$

Step 4: Calculate the support degree and credibility degree of the evidence m_i according to (21) and (22).

$$Sd_{123a} = \begin{bmatrix} 0.2342 & 1.0510 & 1.0099 \end{bmatrix};$$
$$Cd_{123a} = \begin{bmatrix} 0.1020 & 0.4579 & 0.4400 \end{bmatrix};$$
$$Sd_{123b} = \begin{bmatrix} 0.2593 & 1.0579 & 1.0419 \end{bmatrix}$$
$$Cd_{123b} = \begin{bmatrix} 0.1099 & 0.4484 & 0.4417 \end{bmatrix}$$

Step 5: Measure the information volume of the evidence m_i and normalize it according to (23) and (24) as follows:

$$Iv_{123a} = \begin{bmatrix} 1.5984 & 3.8679 & 2.8691 \end{bmatrix};$$
$$\tilde{I}v_{123a} = \begin{bmatrix} 0.1918 & 0.4640 & 0.3442 \end{bmatrix};$$
$$Iv_{123b} = \begin{bmatrix} 1.5984 & 3.8679 & 3.1797 \end{bmatrix};$$
$$\tilde{I}v_{123b} = \begin{bmatrix} 0.1849 & 0.4474 & 0.3678 \end{bmatrix},$$

Step 6: Revise the credibility degree of the evidence m_i based on the information volume of the evidence and normalize it according to (25) and (26) as below:

$$RCrd_{123a} = \begin{bmatrix} 0.0196 & 0.2125 & 0.1515 \end{bmatrix};$$
$$W_{123a} = \begin{bmatrix} 0.0510 & 0.5541 & 0.3949 \end{bmatrix};$$
$$RCrd_{123b} = \begin{bmatrix} 0.0203 & 0.2006 & 0.1624 \end{bmatrix};$$
$$W_{123b} = \begin{bmatrix} 0.0530 & 0.5233 & 0.4237 \end{bmatrix},$$

TABLE 10. Combination results of different methods in Example 7.

Methods	$m_{123a}(A)$	$m_{123b}(A)$	$m_{123a}(B)$	$m_{123b}(B)$	$m_{123a}(C)$	$m_{123b}(C)$
DS [11]	0.0000	0.0000	0.9574	0.9677	0.0426	0.0323
Murphy [17]	0.5235	0.4674	0.4674	0.5235	0.0091	0.0091
Ma [22]	0.8837	0.8513	0.0931	0.1159	0.0232	0.0327
Our results	0.9375	0.9183	0.0546	0.0736	0.0079	0.0081

Step 7: Compute the weighted average evidence as follows:

$$WAE_{123a}(A) = 0.6286; \quad WAE_{123a}(B) = 0.2437;$$
$$WAE_{123a}(C) = 0.1277;$$
$$WAE_{123b}(A) = 0.6106; \quad WAE_{123b}(B) = 0.2633;$$
$$WAE_{123b}(C) = 0.1262;$$

Step 8: Combine the weighted average evidence via the Dempster's rule of combination, and the fusing results are shown in Table 10.

3) DISCUSSION

It can be seen from Table 9 that m_2, m_{3a} and m_{3b} are assigned to the maximum belief value of A, whereas the maximum belief of m_1 supports B, and it can be seen that m_1 is relatively unreliable compared to m_2, m_{3a} and m_{3b}. From the fusion results in Table 10, m_{123a}, m_{123b} are very similar. The DS rule gives an unreasonable result that the maximum belief value supports B. In the fusion method of Murphy, m_{123a} thinks that the most likely result is A, but the result given by m_{123b} is B. It can be seen that for the Murphy's method, as long as the evidence has a small change, the fusion result will change, which indicates that the method is poor in robustness. Our method supports A with the maximum belief value in both fusion results. It can be seen that the small difference of evidences has little effect on the fusion result, which proves that the proposed method can provide the most effective results once applied to dissimilar measures and determine weighting factors. Even in the case of high conflicts, the proposed method is perfectly robust.

C. EFFECTIVENESS

1) PROBLEM STATEMENT

Example 8: Consider a multi-sensor-based target recognition problem associated with the sensor reports that are collected from five different types of sensors. These sensor reports which are modeled as the $BBAs$ are given in Table 11 from [22], where the frame of discernment Θ that consists of three potential objects is given by $\Theta = \{A, B, C\}$.

2) IMPLEMENTATION BASED ON THE PROPOSED METHOD

Step 1: Constructing a Bel matrix according to (2)

$$Bel = \begin{bmatrix} 0.8 & 0.1 & 0 \\ 0.5 & 0.2 & 0.1 \\ 0 & 0.9 & 0.1 \\ 0.5 & 0.1 & 0.1 \\ 0.6 & 0.1 & 0 \end{bmatrix}$$

TABLE 11. The BBAs for a multi-sensor-based target recognition.

	A	B	C	$\{A,B\}$	$\{B,C\}$	Θ
m_1	0.8	0.1	0	0	0	0.1
m_2	0.5	0.2	0.1	0.2	0	0
m_3	0	0.9	0.1	0	0	0
m_4	0.5	0.1	0.1	0.1	0	0.2
m_5	0.6	0.1	0	0	0.1	0.2

Step 2: Constructing a pl matrix according to (3)

$$pl = \begin{bmatrix} 0.9 & 0.2 & 0.1 \\ 0.7 & 0.4 & 0.1 \\ 0 & 0.9 & 0.1 \\ 0.8 & 0.4 & 0.3 \\ 0.8 & 0.4 & 0.3 \end{bmatrix}$$

Step 3: The probability distribution matrix P is calculated by (16)

$$P = \begin{bmatrix} 0.8871 & 0.1118 & 0.0011 \\ 0.6227 & 0.2545 & 0.1227 \\ 0 & 0.9000 & 0.1000 \\ 0.6883 & 0.1606 & 0.1511 \\ 0.8106 & 0.1606 & 0.0287 \end{bmatrix}$$

Step 4: Constructing a Fn matrix according to (15)

$$Fn = \begin{bmatrix} 1.0000 & 0.5818 & 0.0598 & 0.6683 & 0.8579 \\ 0.5818 & 1.0000 & 0.2155 & 0.8283 & 0.6836 \\ 0.0598 & 0.2155 & 1.0000 & 0.1498 & 0.1046 \\ 0.6683 & 0.8283 & 0.1498 & 1.0000 & 0.7820 \\ 0.8579 & 0.6836 & 0.1046 & 0.7820 & 1.0000 \end{bmatrix}$$

Step 5: Constructing a Cor matrix according to (17)

$$Cor = \begin{bmatrix} 0.8000 & 0.6500 & 0 & 0.6500 & 0.7000 \\ 0.6500 & 0.5000 & 0.0500 & 0.5000 & 0.5500 \\ 0 & 0.0500 & 0.9000 & 0.0500 & 0 \\ 0.6500 & 0.5000 & 0.0500 & 0.5000 & 0.5500 \\ 0.7000 & 0.5500 & 0 & 0.5500 & 0.6000 \end{bmatrix}$$

Step 6: Constructing the similarity measurement matrix $Sim_{k\times k}$ according to FMS model as

$$Sim = \begin{bmatrix} 1.0000 & 0.8477 & 0.0198 & 0.8887 & 0.9537 \\ 0.8477 & 1.0000 & 0.1477 & 0.9051 & 0.8535 \\ 0.0198 & 0.1477 & 1.0000 & 0.1032 & 0.0169 \\ 0.8887 & 0.9051 & 0.1032 & 1.0000 & 0.8887 \\ 0.9537 & 0.8535 & 0.0169 & 0.8887 & 1.0000 \end{bmatrix}$$

Step 7: Calculate the support degree of the evidence m_i as below:

$$Sd_1 = 2.7099,$$
$$Sd_2 = 2.7539,$$
$$Sd_3 = 0.2876,$$
$$Sd_4 = 2.7858,$$
$$Sd_5 = 2.7128.$$

Step 8: Calculate the credibility degree of the evidence m_i as below:

$$Cd_1 = 0.2409,$$
$$Cd_2 = 0.2448,$$
$$Cd_3 = 0.0256,$$
$$Cd_4 = 0.2476,$$
$$Cd_5 = 0.2411.$$

Step 9: Measure the information volume of the evidence m_i as below:

$$Iv_1 = 3.3290,$$
$$Iv_2 = 7.9881,$$
$$Iv_3 = 1.5984,$$
$$Iv_4 = 14.5987,$$
$$Iv_5 = 9.8840.$$

Step 10: Normalise the information volume of the evidence m_i as follows:

$$\tilde{I}v_1 = 0.0890,$$
$$\tilde{I}v_2 = 0.2136,$$
$$\tilde{I}v_3 = 0.0427,$$
$$\tilde{I}v_4 = 0.3904,$$
$$\tilde{I}v_5 = 0.2643.$$

Step 11: Revise the credibility degree of the evidence m_i based on the information volume of the evidence as below:

$$RCd_1 = 0.0214,$$
$$RCd_2 = 0.0523,$$
$$RCd_3 = 0.0011,$$
$$RCd_4 = 0.0967,$$
$$RCd_5 = 0.0637.$$

Step 12: Normalise the revised credibility degree of the evidence m_i as below:

$$W_1 = 0.0912,$$
$$W_2 = 0.2223,$$
$$W_3 = 0.0046,$$
$$W_4 = 0.4110,$$
$$W_5 = 0.2709.$$

Step 13: Compute the weighted average evidence as follows:

$$m(\{A\}) = 0.5521,$$
$$m(\{B\}) = 0.1259,$$
$$m(\{C\}) = 0.0638,$$
$$m(\{A, B\}) = 0.0856,$$
$$m(\{B, C\}) = 0.0271,$$
$$m(\Theta) = 0.1455.$$

Step 14: Combine the weighted average evidence via the Dempster's rule of combination with 4 times, and the fusing results are shown in Table 12 and Fig. 6.

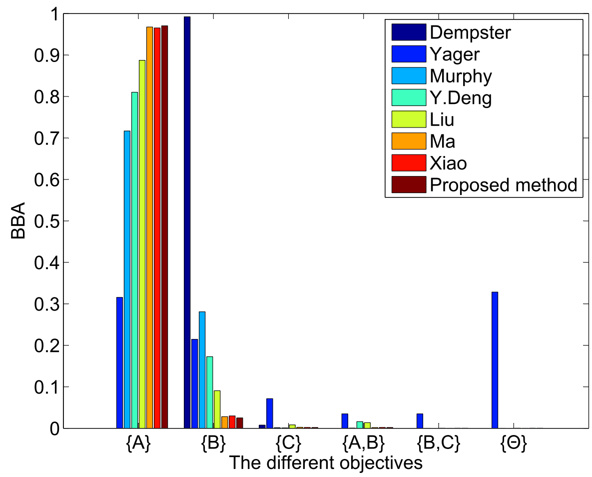

FIGURE 6. The comparison of *BBAs* generated by different methods in Example 8.

3) DISCUSSION

From Example 8, it can be noticed that the evidence m_3 is different from the propositions strongly supported by other evidences. Specifically, the evidence m_3 strongly supports B, and other evidences strongly support A, so it can be seen that the evidence m_3 is highly conflicting with other evidences. The fusion was carried out by different combination methods, and the results are shown in Table 12. The comparison of the *BBA* of A after evidence fusion by different methods is shown in Fig. 7.

As can be seen from Table 12, the counterintuitive results generated by *DS* combination rules which support object B as the target, but other evidences strongly support object A as the target. Whereas, Yager's method [14], Murphy's method [17], Deng *et al.*'s method [18], Liu *et al.*'s method [21], Ma and An's method [22], Xiao's method [24] and the proposed method recognize A as the target, which is in line with the intuitive judgment. Additionally, it is more efficient for the proposed method to handle the conflicting evidences with the highest belief (97.05%) as shown in Fig. 7. The reason is that the proposed method considers two aspects of the similarity measure, namely fuzzy nearness Fn and correlation

TABLE 12. Combination results of different methods of the target recognition system.

Method	A	B	C	$\{A, B\}$	$\{B, C\}$	Θ	Target
DS [11]	0	0.9922	0.0078	0	0	0	B
Yager [14]	0.3157	0.2146	0.0715	0.0349	0.0348	0.3285	A
Murphy [17]	0.7169	0.2809	0.0014	0.0006	0.0001	0.0001	A
Y.Deng [18]	0.8102	0.1727	0.0010	0.0161	0	0	A
Liu [21]	0.8873	0.0905	0.0082	0.0136	0.0002	0.0002	A
Ma [22]	0.9675	0.0280	0.0025	0.0018	0.0001	0.0001	A
Xiao [24]	0.9654	0.0299	0.0021	0.0021	0.0003	0.0002	A
Our results	0.9705	0.0251	0.0019	0.0020	0.0003	0.0002	A

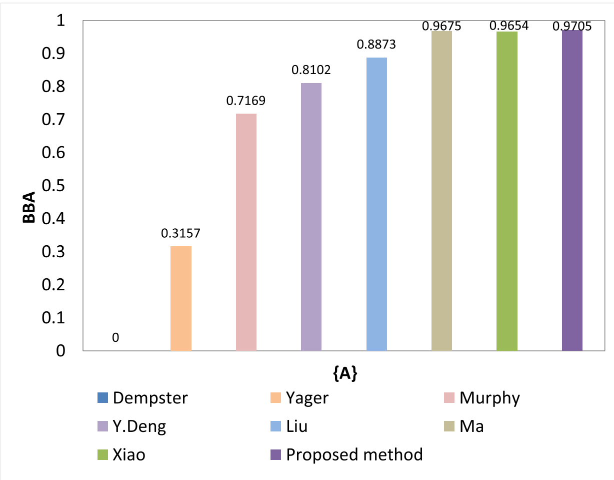

FIGURE 7. ***BBAs*** **for target** ***A*****.**

coefficient *Cor*, and designs a fuzzy inference mechanism based on experts' experience to measure the degree of conflict in all kinds of conflict types. The core is the construction of rule base. Each rule simulates the inference of different types of conflict situations. As long as a reasonable and comprehensive rule base is constructed, the measurement of conflict degree in different conflict situations can be solved. Compared with the existing methods [22], [24], the proposed *FSM* model can effectively measure the degree of conflict under more conflict types. For example, in Example 2 and Example 6, the method in [24] cannot effectively measure the degree of conflict between two evidences, but the proposed *FSM* model can effectively measure the degree of conflict between two evidences. As can be seen from Fig. 7, the conflict measurement model proposed in this paper can measure more conflict situations without losing or even improving the accuracy of fusion. Only when the conflict degree of evidence is effectively measured and discounted can the fusion accuracy be improved. Compared with [22], another advantage of this paper is to consider the uncertain information of the evidence sources, which further reduces the impact of unreliable evidences on fusion results. The weight obtained after conflict measurement is modified by belief entropy, which further improves the quality of the revised evidence sources and the accuracy of fusion results after *DS* fusion.

Therefore, the new conflict evidence fusion method proposed in this paper can not only conveniently and comprehensively measure the degree of conflict under various types of conflict, but also reduce the impact of uncertainty of evidences on the sources of evidence. Combining these two aspects, the *DS* combination rules can be effectively used to fuse high-conflict evidences in this paper.

VI. APPLICATION

In this section, a case of fault diagnosis of machines in [24] is listed to study the advantages of the proposed method in the application field and compare it with related methods.

A. PROBLEM STATEMENT

Suppose the machine has three types of faults, and these faults form the frame of discernment $\Theta = \{F_1, F_2, F_3\}$. The three sensors distributed in different locations S_1, S_2 and S_3 are given to collect reports separately, and *BBA* is used to model the collected reports. As shown in Table 13, where $m_1(\cdot)$, $m_2(\cdot)$ and $m_3(\cdot)$ represent *BBAs* reported from sensors S_1, S_2 and S_3, respectively.

TABLE 13. The collected sensor reports modeled as ***BBAs*** **in the fault diagnosis problem.**

BBA	$\{F_1\}$	$\{F_2\}$	$\{F_2, F_3\}$	Θ
$S_1 : m_1(\cdot)$	0.60	0.10	0.10	0.20
$S_2 : m_2(\cdot)$	0.05	0.80	0.05	0.10
$S_3 : m_3(\cdot)$	0.70	0.10	0.10	0.10

B. FAULT DIAGNOSIS BASED ON THE PROPOSED METHOD

Step 1: Constructing a *Bel* matrix according to (2)

$$Bel = \begin{bmatrix} 0.6 & 0.1 & 0 \\ 0.05 & 0.8 & 0 \\ 0.7 & 0.1 & 0 \end{bmatrix}$$

Step 2: Constructing a *pl* matrix according to (3)

$$pl = \begin{bmatrix} 0.8 & 0.4 & 0.3 \\ 0.15 & 0.95 & 0.15 \\ 0.8 & 0.3 & 0.2 \end{bmatrix}$$

Step 3: The probability distribution matrix P is calculated by (16).

$$P = \begin{bmatrix} 0.8106 & 0.1606 & 0.0287 \\ 0.0607 & 0.9356 & 0.0037 \\ 0.8600 & 0.1311 & 0.0089 \end{bmatrix}$$

Step 4: Constructing a Fn matrix according to (15).

$$Fn = \begin{bmatrix} 1.0000 & 0.1268 & 0.9059 \\ 0.1268 & 1.0000 & 0.1084 \\ 0.9059 & 0.1084 & 1.0000 \end{bmatrix}$$

Step 5: Constructing a Cor matrix according to (17).

$$Cor = \begin{bmatrix} 0.6000 & 0 & 0.6500 \\ 0 & 0.8000 & 0 \\ 0.6500 & 0 & 0.7000 \end{bmatrix}$$

Step 6: Constructing the similarity measurement matrix $Sim_{k\times k}$ according to FMS model as

$$Sim = \begin{bmatrix} 1.0000 & 0.0770 & 0.9535 \\ 0.0770 & 1.0000 & 0.0169 \\ 0.9535 & 0.0169 & 1.0000 \end{bmatrix}$$

Step 7: Calculate the support degree of the evidence m_i as below:

$$\begin{aligned} Sd_1 &= 1.0305, \\ Sd_2 &= 0.0939, \\ Sd_3 &= 0.9704. \end{aligned}$$

Step 8: Calculate the credibility degree of the evidence m_i as below:

$$\begin{aligned} Cd_1 &= 0.4919, \\ Cd_2 &= 0.0448, \\ Cd_3 &= 0.4633. \end{aligned}$$

Step 9: Measure the information volume of the evidence m_i as below:

$$\begin{aligned} Iv_1 &= 9.8840, \\ Iv_2 &= 3.9825, \\ Iv_3 &= 6.0256. \end{aligned}$$

Step 10: Normalise the information volume of the evidence m_i as follows:

$$\begin{aligned} \tilde{I}v_1 &= 0.4969, \\ \tilde{I}v_2 &= 0.2002, \\ \tilde{I}v_3 &= 0.3029. \end{aligned}$$

Step 11: Revise the credibility degree of the evidence m_i based on the information volume of the evidence as below:

$$\begin{aligned} w(DR)_1 &= RCd_1 = 0.2444, \\ w(DR)_2 &= RCd_2 = 0.0090, \\ w(DR)_3 &= RCd_3 = 0.1403. \end{aligned}$$

Step 12: Table 14 from [24] gives the parameters required for fault diagnosis applications, namely the sufficiency index $\mu(m)$ and importance index $\nu(m)$ of the evidences, which can be used to calculate the static reliability by the following formula.

TABLE 14. Parameters in the fault diagnosis application.

Evidence	m_1	m_2	m_3
Sufficiency index $\mu(m)$	1.00	0.60	1.00
Importance index $\nu(m)$	1.00	0.34	1.00

$$\begin{aligned} &w(SR)_i = \mu_i \times \nu_i, \quad 1 \le i \le k. \\ &w(SR)_1 = 1.0000, w(SR)_2 = 0.2040, w(SR)_3 = 1.0000. \quad (29) \end{aligned}$$

Step 13: Compute the final weight of the evidence m_i on basis of the static reliability and the dynamic reliability of the evidences as follows:

$$\begin{aligned} w_1 &= w(DR)_1 \times w(SR)_1 = 0.2444, \\ w_2 &= w(DR)_2 \times w(SR)_2 = 0.0018, \\ w_3 &= w(DR)_3 \times w(SR)_3 = 0.1403. \end{aligned}$$

Step 14: Normalise the final weight of the evidence m_i as below:

$$\begin{aligned} W_1 &= 0.6323, \\ W_2 &= 0.0047, \\ W_3 &= 0.3630. \end{aligned}$$

Step 15: Compute the weighted average evidence as follows:

$$\begin{aligned} m(\{F_1\}) &= 0.6337, \\ m(\{F_2\}) &= 0.1033, \\ m(\{F_2, F_3\}) &= 0.0998, \\ m(\{\Theta\}) &= 0.1632. \end{aligned}$$

Step 16: Combine the weighted average evidence via the Dempster's rule of combination with 2 times, and the fusing results are shown in Table 15 and Fig. 8.

TABLE 15. Fusion results in terms of different combination rules for fault diagnosis.

Method	$\{F_1\}$	$\{F_2\}$	$\{F_2, F_3\}$	Θ	Target
Dempster [11]	0.4519	0.5048	0.0336	0.0096	F_2
Fan and Zuo's method [38]	0.8119	0.1096	0.0526	0.0259	F_1
Yuan *et al.* [39]	0.8948	0.0739	0.0241	0.0072	F_1
Xiao [24]	0.8973	0.0688	0.0254	0.0080	F_1
Ma [22]	0.8245	0.0877	0.0553	0.0325	F_1
Proposed method	0.9108	0.0562	0.0251	0.0079	F_1

C. DISCUSSION

As can be seen from Table 15, the proposed method diagnoses the fault type as F_1, which is consistent with the results of the method diagnosis of Ma and An's method [22], Xiao's method [24], Fan and Zuo's method [38] as well as Yuan *et al.*'s method [39]. Therefore, when there is high conflict evidence m_2 in the evidence group, the above methods can deal with the conflict evidence well. However, the Dempster's rule of combination method [11] cannot effectively deal with the conflict evidence, resulting in counter-intuitive results, that is,

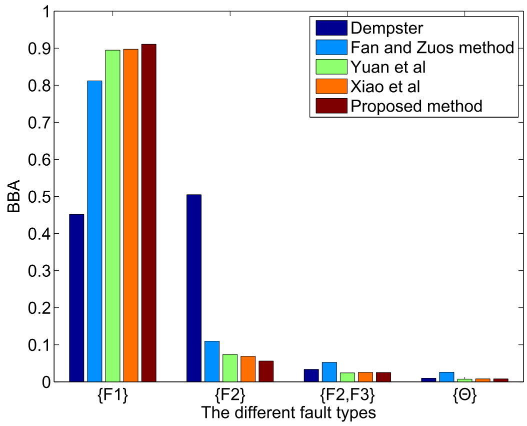

FIGURE 8. The comparison of BBAs generated by different methods for fault diagnosis.

the recognition fault is $m(F_2)$. Additionally, the proposed method has the highest belief degree on fault type $m(F_1)$ (91.08%) which is higher than Ma and An's method [22], Xiao's method [24], Fan and Zuo's method [38] and Yuan *et al*.'s method [39] as shown in Fig. 9. This is because that the distance measure of the proposed method is based on the proposed Fuzzy-based Similarity Measurement (*FSM*), and the *FSM* considers the multi-faceted (*Fn*, *Cor*) measure of similarity between evidences, and more effectively expresses the degree of conflict between evidences. In terms of conflict measurement, the *FSM* overcomes the singularity and incompleteness of the Hamacher T-conorm rule which is used to express the complex relationship between *Fn* and *Cor* in our previous work [22]. By making fuzzy rules, the relationship between *Fn* and *Cor* is divided and described in detail, so different conflict situations and conflicts are effectively measured. In addition, it can be seen from the analysis in the third section that the *FSM* is more effective than the method of Xiao [24] making use of Belief Jensen-Shannon divergence measure and the method of Yuan *et al.* [39] making use of Jousselme's distance function. At the same time, it also makes use of belief entropy to measure the information volume of the evidences and effectively express the uncertainty of evidences, the combination of these two aspects corrects the credibility of the evidences and minimizes the negative impact of the conflict evidences in the process of fusion. It is for this reason that the proposed method is effective and superior in dealing with high-conflict evidence fusion.

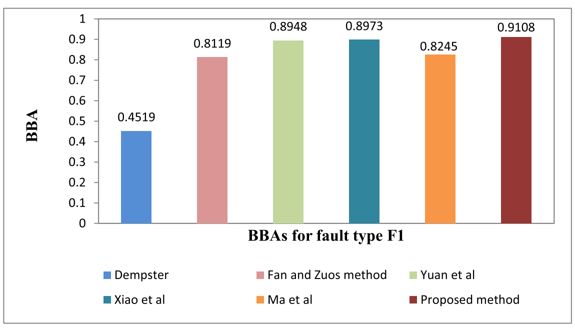

FIGURE 9. BBAs for target $F1$.

VII. CONCLUSION

When Dempster's rule is used to combine evidences, all sources of evidence are considered reliable. The counter-intuitive results occur, especially when there are some high levels of conflict among the evidences. This paper focused on the fuzzy-based similarity measurement and belief entropy approaches to handle conflict before the fusion step. This paper investigated a new fuzzy inference mechanism approach to manage different origins of conflict in the information elicited from multiple sources of evidence in the Dempster-Shafer framework.

In this paper, by considering both of the similarity/dissimilarity degree between the evidences and the effect of the uncertainty of evidences on the weight, a novel method for multi-sensor data fusion based on the presented fuzzy-based similarity measurement and the belief entropy was proposed. The proposed method consisted of three main procedures. Firstly, a new fuzzy-based similarity measurement was proposed for measuring the similarity between the bodies of the evidences, and then the credibility can be calculated based on the similarity to represent the reliability of the evidences. Secondly, the information volume of the evidences was obtained according to the trust entropy, thereby indicating the uncertainty of the evidences. Thirdly, based on the above processes, the final weights of each piece of evidences were computed, and the evidence sources were processed according to the weights, and therefore the average evidence was obtained and further fused by Dempster's combination rule. Finally, a numerical example was used to demonstrate the effectiveness of the proposed method for conflict data fusion problems and to show merit than other existing methods. In addition, an example of fault diagnosis was also listed to illustrate that the proposed method is more accurate in determining these real world faults.

Considering this fuzzy-based approach helps avoid the emerge of counter-intuitive behavior and biased judgment during the combination of conflict evidences, it would be interesting to generalize this approach to adaptive technology and other uncertainty theories, such as rough set, granular computing and imprecise probabilities. It would also be interesting to test this approach on real word sensor data fusion.

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