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# Distributed LT Codes With Improved Error Floor Performance

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**ABSTRACT** Distributed Luby transform (DLT) codes achieve a significant performance improvement in multi-source relay networks compared with the individual Luby transform (LT) code designed for each source. Since the DLT codes preserve the properties of the LT codes, the error floor is dominated by the low variable-node degrees in the bipartite graph, developed for iterative message passing decoding. Therefore, we analyze the error floor of the DLT codes for a multi-source, single relay, and a single destination network over additive white Gaussian noise (AWGN) channels. We modify the encoding process at the sources and propose a new relay combining scheme at the relay. The encoding process at the sources and the combining scheme at the relay are coordinated with the aim of improving the error floor performance of the proposed DLT codes. We derive a lower bound of the bit error probability of the DLT codes over AWGN channels. Furthermore, we optimize the relay degree distribution in terms of the overhead by using the framework of extrinsic information transfer chart analysis. The numerical results confirm that the proposed DLT coding scheme significantly outperforms the conventional scheme, especially in the error floor region.

**INDEX TERMS** DLT codes, EXIT chart, error floor, multi-source relay networks, relay combining.

## I. INTRODUCTION

Next generation communication networks mainly focus on the multi-point paradigms [1]. In many applications, the information from the source may be delivered to the destination with the aid of other nodes between them. These intermediate nodes, in many cases, are referred to as relays. Cooperative relay communication technique is one of the emerging technologies towards building upcoming networks [2]. Consequently, it has been intensively studied to improve the spectrum efficiency and transmission reliability of the communication networks [3]–[9]. For the two-hop relay networks, the reliability on both the source-to-relay and relay-to-destination links should be guaranteed. Furthermore, for such networks, the end-to-end transmission latency has to be reduced in delay-critical applications.

Fixed-rate codes alone cannot provide satisfactory performance in terms of reliability, throughput, and delay in the applications where the quality of received signal is not known to the transmitter. The hybrid automatic repeat request (HARQ) schemes, which combine fixed-rate codes

and error control mechanism, can offer better performance trade-off [10], [11]. Nevertheless, the frequent feedback messages in these schemes would cause heavy end-to-end latency, especially when the channel is in poor condition. To overcome the aforementioned shortcoming, fountain codes were proposed to improve the transmission reliability and reduce the end-to-end latency. In contrast to the fixed-rate codes, their realized rate is adapted on-the-fly, which is desirable in many practical applications [12]–[14]. Fountain codes are inherently rateless and capable of generating infinite number of encoded bits [12]. The benefits of fountain codes in terms of efficiency, reliability, and robustness over fixed-rate codes were also demonstrated in [15]–[18] for additive white Gaussian noise (AWGN) and block fading channels. Recently, the online property of fountain codes has been considered to reduce the encoding overhead [19], [20]. The benefits of fountain codes over rate-compatible codes are: 1) Rate-compatible codes exhibit high encoding complexity as the encoder generates all parity bits irrespective of the required number of parity bits for the successful decoding;

2) The decoding complexity of rate-compatible codes is much higher than that of LT codes; 3) The performance of rate-compatible codes is very sensitive to the design of mother code.

As the first realization of fountain codes, the Luby transform (LT) codes [21] exhibit excellent performance for transmission over the binary erasure channel (BEC). However, the encoding and decoding complexities grow very fast with increasing information block length. The structure of an LT code is similar to an irregular non-systematic low-density generator matrix (LDGM) code [22]. Therefore, it has poor minimum-distance property, resulting in a relatively high error floor. The high error floor performance of the LT codes over binary symmetric channel (BSC) and AWGN channels was demonstrated in [23]. Nevertheless, the introduction of Raptor code [24], [25], a serial concatenation of a precoder and an LT code, could ensure linear complexity and lower error floor. Similarly, in order to enhance the error floor performance of the LDGM-like codes, a serially-concatenated structure of the LDGM codes was further studied in [26]. Batched sparse (BATS) codes which belong to the class of Raptor codes have been well designed for distributed fog computing [27].

In two-hop transmission, independent rateless codes were proposed for each hop to ensure the overall reliability [15]. The performance of a multi-source relay network can be further improved by combining the incoming data at the relay. Consequently, decoding at the destination will be performed over a larger graph, thus, exploiting the larger block length advantage of graph based codes. Furthermore, low decoding complexity is required at the relay. Therefore, designing rateless codes for the entire relay network is required. A technique that makes reasonable trade-off between the complexity and performance is to use the distributed LT (DLT) codes; they require lower encoding and decoding complexities compared to random linear network coding. In the DLT codes, the sources encode their information using the LT encoding [21] and transmit the encoded bits to the relay. The relay then combines the incoming packets efficiently and further transmits them to the destination. The DLT codes have been introduced in [28] to obtain a decoding graph identical to the original LT code. Further improvements for the DLT codes have been identified in [14], [30], and [31]. An overview and subsequent development of DLT coding schemes can be found in [31], and references therein. However, the links in all the aforementioned DLT coding approaches are assumed to be BECs, where a hard decision is performed at the relays. In contrast, it is very challenging to design the DLT coding scheme for the multi-source relay network over AWGN channels. This is due to the unreliability of symbol-wise hard decisions at the relay. Therefore, it is of great significance to investigate the DLT codes in the multi-source relay network for the transmission over AWGN channels.

In this paper, we consider a network having multiple sources communicating with the destination via a single relay.

All links are modeled as AWGN channels. The data is continuously transmitted from the sources to the relay and onwards to the destination. The transmission will continue unless a feedback message is sent from the destination to the relay and subsequently to the sources. This process demonstrates the properties of the rateless codes in the relay networks. We design the DLT codes by proposing a new encoding scheme at the sources. However, the overall performance of the multi-source relay network is influenced by both the encoding at the sources and relay combining technique. Therefore, we also propose a relay combining scheme to ensure the connectivity of all information packets from all the sources to the destination. Consequently, the error floor of the proposed DLT codes is significantly reduced compared to the existing DLT coding schemes. In addition to the error floor improvement, we optimize the relay degree distribution by using the extrinsic information transfer (EXIT) chart [32] framework to minimize the overhead of the network. Numerical results confirm the error floor performance improvement achieved by the proposed DLT codes compared to the existing ones.

The remaining part of the paper is organized as follows. In Section II, we describe the fundamentals of the DLT codes, their convergence property represented by the EXIT chart, and our system model. The proposed encoding process at the sources and relay combining scheme are detailed in Section III. The design framework based on the EXIT chart for the optimization of the relay degree distribution is developed in Section IV. Numerical examples are provided in Section V. Finally, conclusions are drawn in Section VI.

## II. PRELIMINARIES

### A. DLT CODES AND EXIT CHART

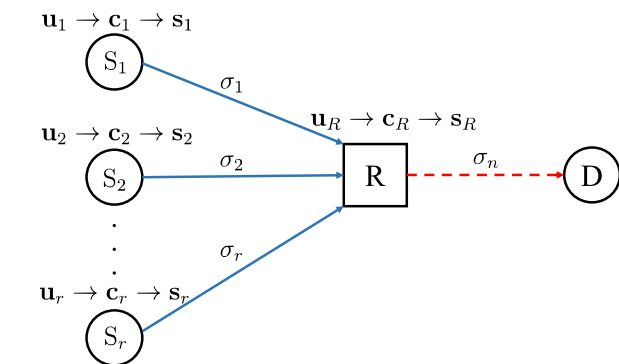
It has been well known that in multi-source relay networks, the DLT codes exhibit excellent performance compared to designing an individual LT code for each source [14], [28]–[30]. A DLT code is obtained by decomposing the encoding of a standard LT code [21] to allow for the applications where data is collected from multiple sources. For the transmission in such applications, each source encodes its own information bit sequence by using an LT encoder [21]. The encoded symbols are transmitted from each source to the relay at a dedicated time slot. Once all the sources have completed their transmissions, the relay combines the estimated information bits of the sources according to a pre-determined relay degree distribution to form relay-coded bits. Finally, the relay transmits the relay-coded bits to the destination. Iterative decoding at the destination is performed by using the sum-product message passing algorithm [25].

To track the asymptotic performance, we use the EXIT chart [32]–[34] to investigate the convergence of iterative decoders. The mutual information evolves by exchanging the extrinsic information between the constituent decoders. For the EXIT chart analysis of the LT codes, we divide

the LT decoder into an LT check-node decoder (CND) and an LT variable-node decoder (VND), exchanging updated likelihood messages with each other. We optimize the relay combining scheme in terms of the overhead by using a linear program based on the EXIT chart analysis.

**B. SYSTEM MODEL**

We consider a network with  $r$  sources communicating to a common destination via a single relay without direct links between the sources and the destination,<sup>1</sup> as shown in Fig. 1. The transmission round is divided into two phases. In the first one, denoted by source-to-relay phase, the sources transmit their messages to the relay, represented by the solid lines in Fig. 1. Similarly, in the second phase, the relay transmits its received message to the destination, referred to as relay-to-destination phase. The relay-to-destination phase is represented by the dashed line in Fig. 1. Thus, each source-to-relay phase is followed by the relay-to-destination phase to complete one transmission round. The first phase consists of  $r$  time slots such that source  $S_i$  transmits at time slot  $i$  for  $i = 1, 2, \dots, r$ . Each source encodes its information block of  $K_i$  bits  $\mathbf{u}_i = (u_{i,1}, u_{i,2}, \dots, \dots, u_{i,K_i})$  by following a pre-determined degree distribution  $\Omega(x)$ , where  $i = 1, 2, \dots, r$  and  $K = \sum_{i=1}^r K_i$ . The check-node degree distribution is given by  $\Omega(x) = \sum_{j=1}^J \Omega_j x^j$ , where  $\Omega_j$  is the fraction of choosing degree  $j$  from the node perspective and  $J$  is the maximum degree of the check node. The  $l$ -th coded bit from the source  $S_i$  is denoted by  $c_{i,l}$  and is generated by first selecting a degree  $j$  from  $\Omega(x)$ , for  $j = 1, 2, \dots, J$ . Then  $j$  information bits are randomly selected and subsequently combined using modulo-2 binary summation. Each generated coded bit is then modulated using binary phase-shift keying (BPSK). Finally, the coded modulated symbols (denoted by  $s_i$  in Fig. 1) from source  $S_i$  is transmitted to the relay with energy  $E_{s_i}$  per symbol over an AWGN channel at the corresponding time slot. The whole signaling chain at source  $S_i$  can be summarized by  $\mathbf{u}_i \rightarrow \mathbf{c}_i \rightarrow \mathbf{s}_i$ , as illustrated in Fig. 1.



**FIGURE 1. System model of the multi-source, single relay, and single destination network.**

<sup>1</sup>In general, if there exist direct links between the sources and the destination, better performance in terms of bit error rate (BER) and frame error rate (FER) can be achieved because of the increased diversity.

Each source communicates with the relay at the dedicated time slot. The relay has  $r$  buffers such that one buffer is reserved for each source. The received signal at the relay from source  $S_i$  is given by  $\mathbf{z}_i = \sqrt{E_{s_i}}\mathbf{s}_i + \mathbf{w}_i$ , where  $\mathbf{w}_i$  is the white Gaussian noise vector with element-wise variance  $\sigma_i^2 = N_{0i}/2$  ( $N_{0i}$  is the two-sided power spectral density of the AWGN channel), for  $i = 1, 2, \dots, r$ . The signal-to-noise (SNR) per symbol between source  $S_i$  and the relay is defined as  $\text{SNR}_i = E_{s_i}/N_{0i}$ , for  $i = 1, 2, \dots, r$ . As soon as the relay receives the data transmitted by the sources, it calculates the log-likelihood ratio (LLR) of each incoming symbol. Based on the calculated LLR values, hard decision is made on the received signals and stored in the corresponding buffer  $B_i$ , for  $i = 1, 2, \dots, r$ . The total number of bits to generate a relay-coded bit is determined by the relay degree distribution. The relay degree distribution is represented by  $\Gamma(x) = \sum_{d=1}^r \Gamma_d x^d$ , where  $\Gamma_d$  is the fraction of the relay-coded bits generated by combining  $d$  bits at the relay. The relay combines the estimated symbols (denoted by  $\mathbf{u}_R$  in Fig. 1) into  $\mathbf{c}_R$ , modulates (again with BPSK) them into  $\mathbf{s}_R$ , and transmits them to the destination with energy  $E_R$  per symbol.

The received signal at the destination is given by

$$\mathbf{y}_D = \sqrt{E_R}\mathbf{s}_R + \mathbf{n}_D \tag{1}$$

where the dimension of  $\mathbf{s}_R$  is  $N$  and  $\mathbf{n}_D$  is the AWGN noise with zero mean and variance  $\sigma_n^2$  per each entry. The received SNR at the destination can be determined as  $\text{SNR}_D = E_R/N_{0D}$ , where  $N_{0D} = 2\sigma_n^2$ . The destination calculates the channel LLR vector  $\mathbf{l}_{\text{ch}}$  as

$$l_{\text{ch},j} = \ln \left( \frac{p(y_{D,j} | s_{R,j} = +1)}{p(y_{D,j} | s_{R,j} = -1)} \right), \quad j = 1, 2, \dots, N. \tag{2}$$

Decoding is then performed over a graph containing information bits from all the sources and relay-coded bits using the message passing algorithm as detailed in [25]. The variable nodes and the check nodes in the decoding graph represent the information bits of the sources and the relay-coded bits, respectively. We measure the BER and FER performances of the DLT codes against the commonly-used parameter called overhead given by  $\epsilon = 1/R = N/K$ , where  $R$  is the realized rate. The overhead represents the number of relay-coded bits needed to recover the information bits from all the sources, normalized by  $K$ . The overhead determines that how many relay-coded bits are needed to recover all the information bits at the destination.

**III. PROPOSED RELAY COMBINING SCHEME**

**A. MODIFIED SOURCE ENCODING AND RELAY COMBINING**

In the conventional DLT codes [29], the information bits are selected randomly during the encoding process at the sources. Similarly, the relay randomly selects the estimated bits from the sources. Therefore, the variable-node degrees formed in the decoding graph at the destination are binomially distributed. Subsequently, the variable-node degree distribution,

denoted by  $\Lambda(x)$ , is asymptotically Poisson distributed [25]

$$\Lambda(x) = \sum_{i=0}^I \Lambda_i x^i \approx e^{\mu(x-1)}, \quad (3)$$

where  $\Lambda_i$  is the fraction of variable nodes having degree  $i$  and  $I$  is the maximum variable-node degree. From (3), the fraction of the variable nodes not connected to any check nodes is  $e^{-\mu}$ , where  $\mu$  is the average variable-node degree at the instance of decoding. These variable nodes cannot be recovered during the decoding process at the destination. Furthermore, due to the binomial distribution, there exists low variable-node degrees. Thus, the conventional DLT codes exhibit high error floor similar to the original LT codes [35].

The transmission of the conventional DLT codes [29] over noisy channels is described as follows. Each source encodes the information bit sequence by a pre-determined degree distribution  $\Omega(x)$ . After encoding, the coded bits are transmitted to the relay at the dedicated time slots. In typical applications, the relay has limited power resources, and, thus, complicated decoding may not be possible at the relay. Therefore, simple symbol detection is preferred at the relay in such applications. Furthermore, complicated decoding scheme at the relay would incur significant latency in the overall transmission. Therefore, we set  $\Omega(x) = x$  to avoid complicated decoding procedure at the relay in the proposed scheme. In other words, every coded symbol transmitted from the sources is connected exactly to only one information symbol. Subsequently, the relay performs simple detection based on the received LLR values of the source-coded symbols. As a result, the received LLR values at the relay are sufficient to provide the hard decisions of the information bit of the sources. The LLR values of the received source-coded bits from source  $S_j$  at the relay can be determined as

$$l_{R,j} = \ln \left( \frac{p(z_{i,j}|s_{i,j} = +1)}{p(z_{i,j}|s_{i,j} = -1)} \right), \quad j = 1, 2, \dots, K_j. \quad (4)$$

At the relay, the received symbols from sources are detected based on the received channel LLR values with a pre-determined threshold  $\xi$ . If the absolute value of the LLR is greater than the absolute value of  $\xi$ , it is assumed that the relay node can decode the corresponding information bit correctly, expressed as

$$\begin{cases} \hat{s}_{i,j} = s_{i,j}, & \text{if } |l_{R,j}| > |\xi|, \\ \hat{s}_{i,j} \neq s_{i,j}, & \text{otherwise.} \end{cases} \quad (5)$$

The error-free decoding assumption is reasonable in the high SNR regime.

The relay stores the correctly decoded symbols from source  $S_i$  in buffer  $B_i$  for  $i = 1, 2, \dots, r$ . If the absolute value of LLR value is below  $\xi$ , the received symbol is discarded<sup>2</sup> and a retransmission is activated. Even though joint decoding for the multiple retransmissions can bring

<sup>2</sup>In other words, no storage in buffers is performed for this received symbol.

significant performance improvement, we only focus on single-transmission decoding for the sake of computational simplicity.

As an initialization, first few rounds of source-to-relay phase are performed so that the estimated bits are stored in the buffers to be used in the relay-coded bits. After initialization, the source-to-relay phase and relay-to-destination phase can operate simultaneously. The relay combines the estimated stored bits using the relay degree distribution  $\Gamma(x)$ , which is optimized in the following section. First, the relay selects degree  $d$  from  $\Gamma(x)$  and then combines the stored bits from the buffers to form a relay-coded bit. When buffer  $B_i$  is chosen, a bit is randomly selected from the available stored bits in conventional DLT codes. In contrast to the conventional DLT codes, we modify the encoding process at the sources such that the information bits are selected in a sequential manner to form the coded bits at sources. The sequential selection ensures the connectivity of each information bit at the sources. Therefore, the coded symbols select the information symbols in a sequential manner with  $\Omega(x) = x$ . Accordingly, the modified encoding process at the sources enables the estimated hard decision information bits stored in the buffer in a sequential manner at the relay. This procedure also allows to keep the same degree of variable nodes at the relay. When buffer  $B_i$  is selected, the stored bit at first position is used to generate the relay-coded bits. Subsequently, the used buffer bit is discarded to allow more buffer memory for the incoming bits. Hence, the same degree of variable nodes are preserved at the destination. The developed encoding graph of the proposed DLT code is illustrated in Fig. 2. The detailed process in Fig. 2 is further elaborated in the following example.

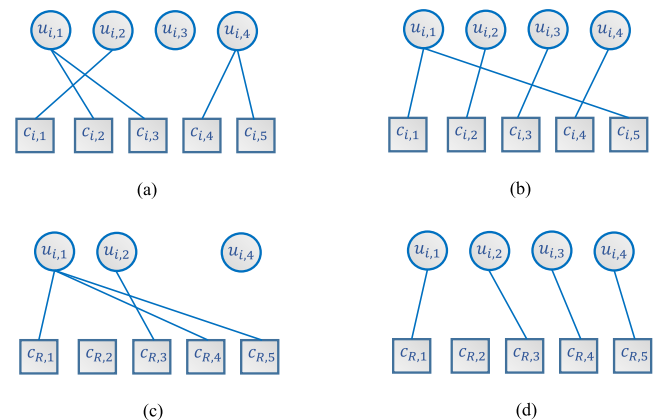


FIGURE 2. Conventional and proposed encoding graphs at source  $S_i$  and the relay.

*Example 1:* We make a comparison on the developed encoding graphs at source  $S_i$  and relay between the conventional DLT and the proposed one. The source-coded bit  $c_{i,j}$  from source  $S_j$  is produced using the LT encoding in [21] for  $j = 1, 2, \dots, 5$ , as shown in Fig. 2(a). Due to random selection of information bits, the information bit  $u_{i,3}$  is not connected while information bits  $u_{i,1}$  and  $u_{i,4}$  are selected twice.

In contrast, the proposed encoding scheme ensures that each information symbol is selected. The selection of the proposed encoding scheme is shown in Fig. 2(b). In the conventional scheme, the information bit  $u_{i,3}$  cannot be detected at the relay as it is not connected to any check node. Therefore, only  $u_{i,1}$ ,  $u_{i,2}$ , and  $u_{i,4}$  would be available at the relay for the relay combining, as shown in Fig. 2(c). In contrast, all the information symbols are connected to the coded symbols in the proposed encoding scheme, and the complete information sequence can be detected at the relay, as shown in Fig. 2(d).

For the relay combining, we further assume that the relay-coded bit  $c_{R,2}$  is formed without involving any information bits from source  $S_i$ . In the conventional relay combining, the relay randomly selects the information bits, and some of them may not be selected. For instance, the information bit  $u_{i,4}$  is correctly detected at the relay, but it is not selected by the relay-coded bits. The resulting code graph with the conventional relay combining is shown in Fig. 2(c). Therefore, information bits  $u_{i,3}$  and  $u_{i,4}$  cannot be recovered at the destination by using the conventional source encoding and relay combining. However, using the proposed sequential relay combining scheme, all the information bits are used in forming the relay-coded bits. Fig. 2(d) demonstrates that each variable node is connected to the relay-coded bits.

### B. LOWER BOUNDS FOR DLT CODES

The graph developed for the message passing decoding of the DLT codes is similar to that of the original LT codes [21]. Therefore, for the lower bound on the bit error probability (BEP), we follow the strategy outlined in [35]. The decoder at the destination is divided into sub-decoders each corresponding to one variable-node degree. The check-node degree distribution at the destination is determined as  $\Gamma(\Omega(x))$  [29]. The variable-node degrees are Poisson distributed and the average variable-node degree depends on  $\Gamma(\Omega(x))$  and  $N$ . It can readily be shown that the received LLRs are symmetric Gaussian distributed asymptotically with mean  $m = 2/\sigma_n^2$  and variance  $\sigma^2 = 4/\sigma_n^2$  for high SNR values. Thus, the final LLR of a sub-decoder with variable-node degree  $i$  is Gaussian distributed having mean  $m_i = 2i/\sigma_n^2$  and variance  $\tau_i^2 = 4i/\sigma_n^2$  [35]. Thus the BEP of the sub-decoder with variable-node degree  $i$  is determined by [35]

$$P_{e,i} = \frac{1}{\sqrt{2\pi\tau_i}} \int_{-\infty}^0 e^{-(x-m_i)^2/2\tau_i^2} dx = Q\left(\frac{m_i}{\tau_i}\right), \tag{6}$$

where the  $Q(\cdot)$  function is the tail probability of a standard normal distribution. Averaging over all the sub-decoders, the lower bound on the BEP of the conventional DLT is

$$P_{CDLT} \geq \frac{\Lambda_0}{2} + \sum_{i=1}^I \Lambda_i Q\left(\frac{m_i}{\tau_i}\right), \tag{7}$$

where the first term corresponds to the information bits not connected to any check nodes. Therefore, on average

the decoder estimates the unconnected information bits with error probability of 1/2.

In the proposed scheme, the variable-node degree distribution  $\Psi(x)$  at the destination has higher minimum variable-node degree. The distribution  $\Psi(x)$  consists of two degrees determined by  $\mu$  and detailed in [31]. The lower bound of the BEP of the proposed DLT code can be determined as

$$P_{PDLT} \geq \sum_{i=d_v-1}^{d_v} \Psi_i Q\left(\frac{m_i}{\tau_i}\right), \tag{8}$$

where  $d_v = \lceil \mu \rceil$  [31].

In the aforementioned analysis, we assumed high SNR values for transmission over source-to-relay channels to avoid any detection error at the relay. Thus, the transmitted data from the sources to the relay are correctly detected based on the LLR threshold. However, if the SNRs of source-to-relay links are very low, the transmitted data symbols from the sources cannot be detected correctly. The variable-node degrees at the destination are still binomially distributed irrespective of the SNR values when using the conventional encoding and relay combining. However, the low SNRs between sources and relay will disturb the regularized variable-node degrees formed by the proposed encoding and relay combining at the decoding graph. The variable-node degrees at the destination will be more dispersed for lower SNR values between the sources and the relay. Despite of the pattern disturbance, the minimum degree of the variable-node degree distribution of the proposed scheme is higher than that of its conventional counterpart. The corresponding variable-node degree distribution at the destination can be determined by using the approach in [36].

The proposed relay combining scheme can be further improved if feedback messages are available between the sources and the relay. After  $K$  transmission rounds, the relay informs the corresponding sources about the information bits not correctly recovered at the relay. Hence, from transmission round  $K + 1$  onward, each source selects only these information bits indicated by the relay during the encoding process.

### IV. OPTIMIZATION OF RELAY DEGREE DISTRIBUTION

We proposed a new relay combining scheme to reduce the error floor of the proposed DLT codes. As the proposed scheme modifies only the variable-node degree distribution at the decoding graph, we can optimize the relay combining scheme to provide higher throughput and lower error floor simultaneously. However, the maximum degree of the relay degree distribution is constrained by the number of sources in the network.

We use the EXIT chart to optimize the relay degree distribution for information combining at the relay. Instead of using node-perspective degree distributions, edge-perspective degree distributions are used in the EXIT analysis. Therefore, the corresponding edge-perspective

degree distributions are given as [37]

$$\psi(x) = \Psi'(x)/\Psi'(1), \quad (9)$$

$$\gamma(x) = \Gamma'(x)/\Gamma'(1). \quad (10)$$

where  $f'(x)$  is first derivative of  $f(x)$  with respect to  $x$ . Here, our objective is to minimize the overhead which is equivalent to maximizing the realized rate of the network. We consider the total number of edges in the decoding graph

$$K\mu = N\beta, \quad (11)$$

where  $\beta = \Gamma'(\Omega(x))|_{x=1}$  is the average check-node degree at the destination. By rearranging, we obtain

$$\varepsilon = N/K = \mu/\beta. \quad (12)$$

As  $\Omega(x) = x$ , thus  $\Gamma'(\Omega(x))|_{x=1} = \Gamma'(x)|_{x=1} = \sum_{i=1}^r i\Gamma_i$ . Converting the node-perspective relay degrees into edge-perspective ones as

$$\beta = \frac{\sum_i i\gamma_i/i}{\sum_j \gamma_j/j} = \frac{1}{\sum_j \gamma_j/j},$$

and substituting the value of  $\beta$  into (12), we get

$$\varepsilon = \mu \sum_j \gamma_j/j. \quad (13)$$

Minimizing  $\sum \gamma_j/j$  will maximize the realized rate  $R$  and subsequently minimize the overhead. We optimize the relay degree distribution in terms of maximizing the realized rate. The convergence of the LT decoder can be analyzed by capturing the mutual information exchange between the constituent decoders. The EXIT curve for the VND of the DLT decoder can be determined as [32]

$$I_{E,VND} \approx \sum_i \psi_i J \left( \sqrt{(i-1)(J^{-1}(I_{A,VND}))^2} \right). \quad (14)$$

Here,  $I_{A,VND}$  is *a priori* information of the VND while operators  $J(\cdot)$  and  $J^{-1}(\cdot)$  are approximated as in [32]. Likewise, the EXIT curve for the CND of the DLT decoder is approximated by

$$I_{E,CND} \approx \sum_j \gamma_j \left( 1 - J \left( \sqrt{(j-1)\sigma_C^2 + \sigma^2} \right) \right), \quad (15)$$

where  $\sigma_C^2$  is the variance of the *a priori* information  $I_{A,CND}$  (equal to  $I_{E,VND}$ ). The optimization of the relay degree distribution can be transformed into a linear program as follows:

$$\begin{aligned} & \min \sum_j \gamma_j/j \\ & \text{s.t. } \gamma_j \geq 0, \\ & \sum_j \gamma_j = 1, \\ & I_{E,CND}(I_{A,CND}) > I_{E,VND}^{-1}(I_{A,VND}). \end{aligned}$$

Here, the third constraint ensures that the two EXIT curves described in (14) and (15) do not intersect each other, thus, guaranteeing open tunnel for the convergence.

We obtain the edge-perspective relay degree distribution by running the linear program for symmetric<sup>3</sup> source-to-relay links of SNR = 10 dB and  $r = 10$ . The corresponding node-perspective relay degree distribution is given by

$$\Gamma(x) = 0.0058x + 0.5544x^2 + 0.1464x^3 + 0.2934x^{10}. \quad (16)$$

### V. NUMERICAL EXAMPLES

In this section, we provide numerical results for  $r = 10$ ,  $\xi = 4$ , and the relay degree distribution detailed in (16). The performance is evaluated in terms of the BER and FER for different overhead values. For reference, we also provide the corresponding lower bounds given in (7) and (8) for the conventional and proposed DLT coding schemes, respectively. Fig. 3 shows the performance comparison between the proposed DLT and the conventional DLT [28]. The performance of uncoded relay system where the relay only forwards the incoming bits to the destination is also shown in Fig. 3. We set the SNR of all the source-to-relay links to  $\text{SNR}_1 = \text{SNR}_2 = \dots = \text{SNR}_{10} = 10$  dB and the SNR of relay-to-destination link to 3 dB. Fig. 3 clearly demonstrates the improved error floor performance of the proposed DLT coding scheme over the conventional one.

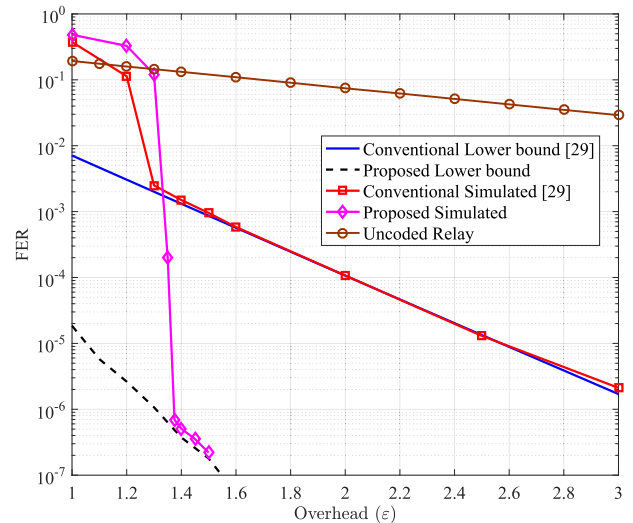


FIGURE 3. Performance comparison of the conventional and proposed DLT coding schemes for high source-to-relay link SNRs.

To observe the consequences of lowering the source-to-relay SNRs, we reduce the source-to-relay link SNRs to 3 dB while all other parameters are the same as in Fig. 3. The corresponding performance comparison is shown in Fig. 4. The gap between the lower bounds and numerical results increases when lowering the SNR values between the sources and the relay as the relay fails to generate the relay-coded bits at some time slots with small absolute values of the received channel LLR. As evident from Fig. 4, the proposed

<sup>3</sup>For the asymmetric source-to-relay links, the optimal edge-perspective relay degree distribution can be obtained in the same way.

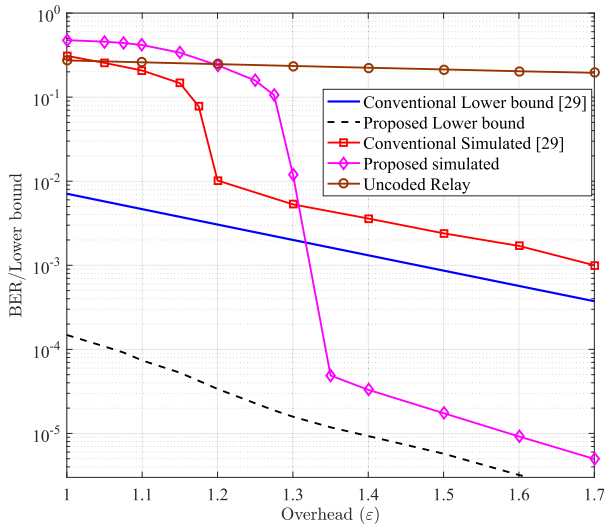


FIGURE 4. Proposed DLT coding scheme versus conventional DLT coding scheme for lossy symmetric source-to-relay channels.

DLT coding scheme has improved error floor performance compared to its conventional counterpart.

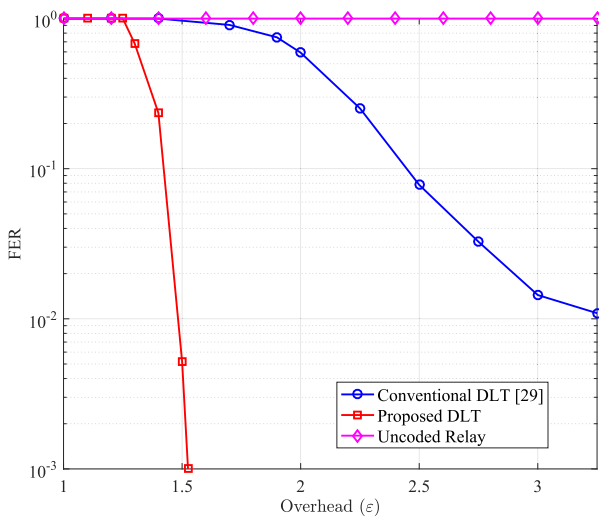


FIGURE 5. FER performance comparison of the conventional and proposed DLT codes.

Fig. 5 compares FER performance of the proposed DLT coding scheme with the conventional DLT coding and the uncoded relay scheme. We can clearly observe the significantly improved FER performance of our proposed DLT coding compared to its conventional counterpart.

### VI. CONCLUSIONS

We have analyzed the error floor of the proposed DLT codes over AWGN channels for a cooperative relay network with multiple sources. We have modified the encoding process at the sources and proposed a new relay combining scheme to improve the error floor performance of the DLT codes.

Subsequently, we have derived the lower bounds for the error floor of the DLT codes over AWGN channels. Furthermore, we have optimized the relay degree distribution in terms of overhead by using the EXIT chart design framework. Numerical examples have been provided to confirm the performance improvement of the proposed DLT codes over its conventional counterpart regarding error floor.

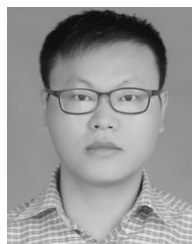
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