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Adaptive Neural-Network-Based Control for a Class of Nonlinear Systems With Unknown Output Disturbance and Time Delays

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ABSTRACT This paper pays close attention to the adaptive neural network tracking control. Aiming at a class of uncertain nonlinear systems with completely unknown output disturbance and unknown time delay, a corresponding robust control method is proposed based on the backstepping design technology. Neural network approximation is introduced as a very effective estimation technique for modeling uncertain partitions in the design process of virtual controller. The suitable Lyapunov–Krasovskii function is constructed, and by using the organic combination of Young's inequality, unknown time delays are compensated. Nussbaum function is used to handle unknown virtual control directions. A practical robust control method is proposed to deal with the controller singularity problems. *A priori* knowledge is not required for this method. In this method, all signals achieve semi-global uniform ultimate boundedness, and it is demonstrated that the tracking error eventually converges the region around the origin. The simulation results verify this method's feasibility and effectiveness.

INDEX TERMS Adaptive control, nonlinear systems, time-delay, output disturbance, neural networks.

I. INTRODUCTION

In recent years, a hot topic is adaptive control and many effective control strategies have been proposed, including adaptive backstepping design [2], [3], intelligent control [5], [6], sliding mode control [7], distributed control [8], [9] and more. And the adaptive neural network control has caused widespread concern. It has become an important part of adaptive control. The adaptive control method is a control method that can effectively deal with the uncertainty of the model. Adaptive neural control is a control method combining neural network and adaptive control, which can effectively deal with the nonlinear part of the system and the uncertainty of the model. And thus it has been extensively used. Highly uncertain nonlinear systems often use this method to control [10], [11], [13], [16]–[19]. Because of their general approximation performance, the basic concept is to use neural network to estimate the uncertain nonlinear function and then use the backstepping. The technique gradually constructs Lyapunov functions to design nonlinear systems, and lots of researches have been accomplished. In addition, there are several types of effective modeling and control methods. For example, the recurrent neural network method can effectively model time-varying matrices, and most recursive neural network models do not require offline learning in advance [34]. Zeroing neural dynamics is a systematic and effective method that has been officially promoted from CZNN (conventional Zhang neural network) since 2008. It has been widely used in neural network models and nonlinear optimization [35]. The Jacobian-matrix-adaption method is a conventional control method for finding the joint variable vector by first calculating the inverse or pseudo-inverse of the Jacobian matrix, which can conveniently handle the control system with redundancy [36].

For nonlinear dynamics systems, neural network candidate computing architecture shows that multi-layer neural networks may be ideal for real-time adaptive control. In [14] and [27], multilayer neural networks were untilized. The radial basis function network has its foundation in conventional approximation theory. It has the capability of universal approximation [24]. In [15] and [20], based on the above theories, unknown function can be approached by using radial basis functions for the approximation. In [12], an approximate adaptive backstepping method is proposed. In [1], [23], and [25], the above method is extended to adaptive neural control, in order to avoid possible controllers, adaptive control is achieved through backstepping techniques. For a system which is multi-input and multi-output, an output feedback tracking controller is proposed [21]. For unknown systems, neural networks approximate unknown functions. In [21], backstepping technology not only ensures that all signals are bounded, but also makes the error of tracking time-varying signals within a small range. In uncertain nonlinear systems, uncertain parameters, uncertain dynamics and external disturbances are also ubiquitous. For stabilization and performance recovery of nonlinear systems with unmodeled dynamic, a time-scale separation redesign is presented [37]. And in [38], it proposes two different robust redesign techniques based on time scale separation. In [39], a high-gain predictor is designed for output feedback control of nonlinear systems in the presence of input, output, and state delays. The actual design can be used for the decoupling backstep design because a new control function is proposed [4]. In [22], it has been investigated that MIMO stochastic nonlinear systems which have high-frequency gains. Utilizing the combination of Nussbaum gain and adaptive neural network, it can be sure that all signals are bounded.

One of main advantages of previous work is to ensure system's stability, because of the expectation of the adaptive law is in view of the Lyapunov stability theory. For the research of nonlinear systems, previous work has certain enlightenment. In [15] and [23], the system has an unknown time delay. In [22], the systems have an unknown smooth nonlinear function. And in [11], it has the unknown disturbance. In [27] and [32], the virtual control coefficients $\varphi_i = 1$. In [21], [29], and [33], the virtual control coefficients φ_i is an unknown constant. In [4], [26], [30], and [31], the virtual control coefficients was extended to time-varying. However, it hasn't been discussed that the nonlinear systems which have unknown time delays, unknown disturbance and unknown time-varying virtual control coefficient. And, the system output of the nonlinear system has an unknown time-varying disturbance. This type of nonlinear systems is widespread in reality. So it is necessary to study it at the present stage. In the current study, unknown time delays use Lyapunov-Krasovskii function for compensation. Nussbaum function is used to handle unknown virtual control directions. Practical robust control deals with controller singularity problems. This article has the following contributions: i) The continuous function $\kappa(\cdot)$ is introduced to avoid the possibility of controlling the saturation of the actuator. At the same time, the problem that the system output has unknown time-varying interference in the nonlinear system is solved. ii) The combination of the use of integral Lyapunov function and Nussbaum function is used to prevent the problem of controller singularity problem and solve the unknown virtual control direction problems in nonlinear systems. iii) Time delay τ_i is removed by the Lyapunov-Krasovskii functional and the organic combination of Young's inequality, which makes neural network parametrization. iv) Neural network approximation is introduced as a very effective estimation technique for modeling uncertain partitions in the design process of virtual controller. The smooth virtual control functions are provided by introducing of continuous even functions $q_i(\cdot)$. Because any degree of need can be distinguished by smooth virtual control functions, the practical control of backstepping design can be achieved.

The paper is structured in the following sections. The problem formulation and preparation are given in section 2. In section 3, the adaptive controller is designed and the system's stability of is ensured. This method's performance is reflected in the results of extensive simulation studies in section 4. In section 5, the work is summed up.

II. PROBLEM FORMULATION AND PRELIMINARIES

A. PLANT DYNAMICS

Consider a class of nonlinear SISO systems with time-delay.

$$\begin{cases} \dot{x}_i = \varphi_i(\bar{x}_i)x_{i+1} + f_i(\bar{x}_i) + \xi_i(\bar{x}_i(t-\tau_i)) \\ \qquad + \Lambda_i(x,t), & 1 \le i \le n-1, \\ \dot{x}_n = \varphi_n(\bar{x}_n)u + f_n(\bar{x}_n) + \xi_n(\bar{x}_n(t-\tau_n)) \\ \qquad + \Lambda_n(x,t), \\ x_i = \phi_i(t), \quad t \in [-\tau_{\max}, 0], & i = 1, \ldots, n, \\ y = x_1 + d(t), \end{cases} \tag{1}$$

where $\bar{x}_i = [x_1, x_2, \ldots, x_i]^T$, $x = [x_1, x_2, \ldots, x_n] \in R^n$ are state variables, $u \in R$ is system input and $y \in R$ is system output, $\varphi_i(\cdot), f_i(\cdot)$ and $\xi_i(\cdot)$ are smooth functions which are unknown, $\Lambda_i(x,t)$ is a disturbance and it is time-varying, τ_i are time delays which are unknown, $i = 1, \ldots, n$. $d(t)$ is an unknown disturbance, which is a bounded smoothing function, i.e. $|\dot{d}(t)| \le D_{\max}$. By the adaptive controller designed, signals are bounded and $y(t)$ meets the reference signal $y_d(t)$. $\bar{y}_d = [y_d, \dot{y}_d, \ldots, y_d^{(i)}]^T, i = 1, 2, \ldots, n-1$, and it is desired trajectory.

Assumption 1: Functions $\varphi_i(\bar{x}_i)$ are unknown, but $0 < \varphi_{i0} \le |\varphi_i(\bar{x}_i)| \le \bar{\varphi}_i(\bar{x}_i)$, $\forall \bar{x}_i \in R_i$, among them,$\varphi_{i0}$ is constants and smooth functions $\bar{\varphi}_i(\bar{x}_i)$ are known.

Assumption 2: $\bar{\varphi}_i(\bar{x}_i)$ meets the formula $l_i^- \le \bar{\varphi}_i(\bar{x}_i) \le l_i^+$ and $I_i := [l_i^-, l_i^+] \subset [\varphi_{i0}, +\infty)$.

Assumption 3: $\bar{x}_{di}$, $i = 2, \ldots, n$, are desired trajectory continuous vectors, and $\bar{x}_{di} \in \Omega_{di} \subset R^i$ with Ω_{di} known compact sets.

Remark 1: In the case where the above assumptions are met, the unknown $\varphi_i(\bar{x}_i)$ are positive or negative. So we consider $\varphi_i(\bar{x}_i) > 0$. Furthermore φ_{i0}, l_i^- and l_i^+ are only for analysis, it isn't necessarily to know their true value.

Assumption 4: Unknown functions $\xi_i(\bar{x}_i(t))$ and known positive smooth functions $\beta_i(\cdot)$ satisfy the inequality $|\xi_i(\bar{x}_i(t))| \leq \beta_i(\bar{x}_i(t))$.

Assumption 5: For $1 \leq i \leq n$, positive constant p_i^* and nonnegative smooth function Ψ_i satisfy $\forall (t, x) \in R_+ \times R^n$ $|\Lambda_i(x, t)| \leq p_i^* \Psi_i(\bar{x}_i)$.

Remark 2: The unknown time delays have a upper bound $\tau_{\max}$ i.e. $\tau_i \leq \tau_{\max}$, $i = 1, 2, \ldots, n$. The differential equation (1) can describe many practical physical processes. For example, the cold rolling mills [2]. And most recycling processes inherit the delay through their state equations.

Lemma 1: Let $N(\cdot)$ be an Nussbaum-type function which are smooth and functions $V(\cdot)$, $\zeta(\cdot)$ are smooth [22]. And $V(t) \geq 0$, $\forall t \in [0, t_f)$. If

$$V(t) \leq C_0 + e^{-C_1 t} \int_0^t (g(\cdot)N(\zeta) + 1)\dot{\zeta} e^{C_1 \tau} d\tau, \forall t \in [0, t_f)$$

where C_0 and C_1 represents constant and $C_1 > 0$, and $g(\cdot)$ is a bounded and time-varying parameter, and then $V(t)$, $\zeta(t)$ and $\int_0^t g(\cdot)N(\zeta)\dot{\zeta} d\tau$ are bounded on $[0, t_f)$.

Lemma 2: When $\epsilon > 0$, and for any $\vartheta \in R$, there is [28]

$$0 \leq |\vartheta| - \vartheta \tanh\left(\frac{\vartheta}{\epsilon}\right) \leq \lambda\epsilon,$$

where $\lambda = e^{-(\lambda+1)}$, i.e. $\lambda = 0.2785$.

Lemma 3: Even function $q_i(x) : R \to R$ [1]

$$q_i(x) = \begin{cases} 1, \quad |x| \geq \nu_{ai} + \nu_{bi} \\ c_{qi} \int_{\nu_{ai}}^{x} [(\frac{\nu_{bi}}{2})^2 - (\sigma - \nu_{ai} - \frac{\nu_{bi}}{2})^2]^{n-i} d\sigma, \\ \qquad \nu_{ai} < x < \nu_{ai} + \nu_{bi} \\ c_{qi} \int_{x}^{-\nu_{ai}} [(\frac{\nu_{bi}}{2})^2 - (\sigma + \nu_{ai} + \frac{\nu_{bi}}{2})^2]^{n-i} d\sigma, \\ \qquad -(\nu_{ai} + \nu_{bi}) < x < -\nu_{ai} \\ 0, \quad |x| \leq \nu_{ai} \end{cases}$$

where

$$c_{qi} = \frac{[2(n-i)+1]!}{\nu_{bi}^{2(n-i)+1}[(n-i)!]^2},$$

$\nu_{ai}, \nu_{bi} > 0, (i = 1, 2, \cdots, n)$.

Lemma 4: Even function

$$\kappa(a) = \frac{a^2 \cosh(a)}{1 + a^2}, \quad \forall a \in R$$

is continuous, and monotonically increasing.

B. RBFNN APPROXIMATION

Function $\xi(Z) : R^q \to R$ uses the following RBFNN for approximation in the paper.

$$\xi_{nn}(Z, W) = W^T S(Z), \tag{3}$$

where $Z \in \Omega \subset R^q$, $W = [\omega_1, \omega_2, \ldots, \omega_l]^T \in R^l$, and $S(Z) = [s_1(Z), \ldots, s_l(Z)]^T$, Gaussian functions is selected for $s_i(Z)$,

$$s_i(Z) = exp\left[\frac{-(Z - \eta_i)^T (Z - \eta_i)}{\varpi_i^2}\right], \quad i = 1, 2, \ldots, l.$$

where ϖ is the width and $\eta_i = [\eta_1, \eta_2, \ldots, \eta_q]^T$. And network (3) meets the following formula.

$$\xi(Z) = \xi_{nn}(Z, W^*) + \varepsilon(Z), \quad \forall \in \Omega_Z,$$

where the NN approximation error is $|\varepsilon(Z)| \leq \varepsilon^*$. Ideal weights is W^* and it makes for all $Z \in \Omega_Z$, $|\varepsilon| \leq \varepsilon^*$, where constant $\varepsilon^* > 0$. In addition, W^* is bounded on the Ω_Z, where $\|W^*\| \leq \omega_m$, ω_m is a positive constant.

Obviously, W^* needs to use functions to approximate, because W^* is usually unknown. On the basis of the discussion in [28]:

$$W^* = arg \min_{(W)} \left[\sup_{Z \in \Omega_Z} |\xi_{nn}(Z, W) - h(Z)|\right].$$

In design, $\hat{W}$ is used to estimate W^*, and the estimation error is represented by $\tilde{W} = \hat{W} - W^*$.

III. ADAPTIVE CONTROL DESIGN AND STABILITY ANALYSIS

There are n steps in the process. At every step, the appropriate Lyapunov function $V_i(t)$ is used to develop $\alpha_i(t)$. The following coordinate changes are used to design control laws and adaptive laws:

$$z_1 = y - y_d, \; z_i = x_i - \alpha_{i-1}, \quad i = 2, \ldots, n,$$

where $u(t)$ is used to stabilize the system, it is designed in the last step, and $\alpha_i(t)$ is present in the intermediate step. The definition of a compact set is as follows

$$\begin{aligned} \Omega_{z_i} &:= z_i \in \Omega_{Z_i} \mid |z_i| \leq c_{z_i}, \\ \Omega_{z_i}^I &:= z_i \in \Omega_{Z_i} \mid c_{z_i} < |z_i| \leq c_{z_i} + c_{z_i}^\epsilon, \\ \Omega_{z_i}^O &:= z_i \in \Omega_{Z_i} \mid |z_i| \geq c_{z_i} + c_{z_i}^\epsilon, \end{aligned}$$

with Ω_{z_i} being a compact set, $\Omega_{Z_i} = \Omega_{z_i} \cup \Omega_{z_i}^I \cup \Omega_{z_i}^O \cup \Omega_{di}$, and c_{z_i}, $c_{z_i}^\epsilon > 0$. For conciseness of notation, function $V_{z_i}(t)$, $V_{U_i}(t)$, and $V_i(t)$ are as follows:

$$V_{z_i}(t) = \frac{1}{2} z_i^2(t), \tag{4}$$

$$V_{U_i}(t) = \frac{1}{2} \sum_{j=1}^{i} \int_{t-\tau_{max}}^{t} U_j(\bar{x}_j(\tau)) d\tau, \tag{5}$$

$$V_{\omega_i}(t) = \frac{1}{2} \tilde{W}_i^T \Gamma_i^{-1} \tilde{W}_i, \tag{6}$$

$$V_{b_i}(t) = \frac{1}{2\gamma_{b_i}} \tilde{b}_i^2, \tag{7}$$

$$V_i(t) = V_{z_i}(t) + V_{U_i}(t) + V_{\omega_i}(t) + V_{b_i}(t), \tag{8}$$

where positive function $U_j(\bar{x}_i(t)) = \beta_j^2(\bar{x}_j(t))$. The unknown functions $Q_i(Z_i)$ will be approximated by NNs as

$$Q_i(Z_i) = W_1^{*T} S(Z_i) + \varepsilon_i(Z_i); \quad \forall Z_i \in \Omega_{z_i}^O, \tag{9}$$

where

$$\begin{aligned} Q_1(Z_1) &= f_1(x_1) + \frac{1}{2z_1}\beta_1^2(x_1) + 2D_{\max}^2 - \dot{y}_d, \\ Q_i(Z_i) &= f_i(\bar{x}_i) + \frac{1}{2z_i}\sum_{j=1}^{i}\beta_j^2(\bar{x}_j) \\ &\quad - \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}(\varphi_j(\bar{x}_j)x_{j+1} + f_j(\bar{x}_j)) \\ &\quad + \frac{1}{2}z_i\sum_{j=1}^{i-1}(\frac{\partial\alpha_{i-1}}{\partial x_j})^2 - w_{i-1}, \end{aligned} \tag{10}$$

with

$$\begin{aligned} Z_1(t) &= [x_1, y_d, \dot{y}_d]^T \subset \Omega_{z_1}^O, \\ Z_i(t) &= \left[\bar{x}_i, \alpha_{i-1}, \frac{\partial\alpha_{i-1}}{\partial x_1}, \frac{\partial\alpha_{i-1}}{\partial x_2}, \ldots, \frac{\partial\alpha_{i-1}}{\partial x_{i-1}}, \omega_{i-1}\right]^T \\ &\quad \in \Omega_{z_i}^O, \quad 2 \le i \le n, \\ w_{i-1} &= \frac{\partial\alpha_{i-1}}{\partial\zeta_{i-1}}\dot{\zeta}_{i-1} + \frac{\partial\alpha_{i-1}}{\partial\bar{x}_{di}}\dot{\bar{x}}_{di} + \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\hat{W}_j}\dot{\hat{W}}_j \\ &\quad + \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\hat{b}_j}\dot{\hat{b}}_j + \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial k_j}\dot{k}_j. \end{aligned} \tag{11}$$

The practical adaptive control is proposed, for $i = 1, \ldots, n$

$$\begin{aligned} \alpha_i &= q_i(z_i)N(\zeta_i)(k_i z_i + \hat{W}_i^T S(Z_i) \\ &\quad + \hat{b}_i\bar{\Psi}_i(\bar{x}_i)\tanh\left[\frac{z_i\bar{\Psi}_i(\bar{x}_i)}{\epsilon_i}\right]), \end{aligned} \tag{12}$$

$$\begin{aligned} \dot{\zeta}_i &= k_i(t)z_i^2 + \hat{W}_i^T S(Z_i)z_i \\ &\quad + \hat{b}_i z_i\bar{\Psi}_i(\bar{x}_i)\tanh\left[\frac{z_i\bar{\Psi}_i(\bar{x}_i)}{\epsilon_i}\right], \end{aligned} \tag{13}$$

$$\dot{\hat{b}}_i = \gamma_{bi}(z_i\bar{\Psi}_i(\bar{x}_i)\tanh\left[\frac{z_i\bar{\Psi}_i(\bar{x}_i)}{\epsilon_i}\right] - \sigma_{bi}\hat{b}_i), \tag{14}$$

$$\dot{\hat{W}}_i = \Gamma_i(S(Z_i)z_i - \sigma_{wi}\hat{W}_i), \tag{15}$$

$$k_i(t) = \frac{3}{4} + k_{i0} + k_{i1}(t), \tag{16}$$

where

$$k_{i1} = \frac{\varepsilon_{i0}\kappa(z_i)}{2z_i^2}\sum_{j=1}^{i}\int_{t-\tau_{\max}}^{t} U_j(\bar{x}_j(\tau))d\tau, \tag{17}$$

$k_{i0} > 0$, $\epsilon_i > 0$, matrix $\Gamma_1 = \Gamma_1^T > 0$, $\varepsilon_{i0} > 0$ is a constant, $\sigma_{w_i}, \sigma_{b_i}$ are small constants for σ-modification introduced into the system. When $i = n$, $\alpha_n = u(t)$.

Remark 3: If we let

$$k_{i1} = \frac{\varepsilon_{i0}}{2z_i^2}\sum_{j=1}^{i}\int_{t-\tau_{max}}^{t} U_j(\bar{x}_j(\tau))d\tau$$

as in [4], it is will found that if c_{z_i} is chosen to be very small. We introduce the function $\kappa(\cdot)$ into k_{i1} because it can effectively avoid the saturation of the execution controller when $k_{i1}(t)$ takes a very large value.

Step 1:

$$\begin{aligned} \dot{z}_1(t) &= \varphi_1(x_1(t))[z_2(t) + \alpha_1(t)] + f_1(x_1(t)) \\ &\quad + \Lambda_1(x,t) + \xi_1(x_1(t-\tau_1)) - \dot{y}_d(t). \end{aligned} \tag{18}$$

Consider the difference of V_1, noting (18), we have

$$\begin{aligned} \dot{V}_1 &= z_1 z_2\varphi_1(x_1) + z_1[\varphi_1(x_1)\alpha_1(t) + f_1(x_1) \\ &\quad + \xi_1(x_1(t-\tau_1)) + \Lambda_1(x,t) + \dot{d}(t) - \dot{y}_d(t)] \\ &\quad + \frac{1}{2}U_1(x_1) - \frac{1}{2}U_1(x_1(t-\tau_1)) \\ &\quad + \tilde{W}_1^T\Gamma_1^{-1}\dot{\hat{W}}_1 + \frac{1}{\gamma_{b_1}}\tilde{b}_1\dot{\hat{b}}_1. \end{aligned} \tag{19}$$

Applying the inequalities

$$\begin{aligned} z_1\dot{d}(t) &\le \frac{1}{8}z_1^2 + 2\dot{d}^2(t), \\ z_1 z_2\varphi_1(x_1) &\le \frac{1}{8}z_1^2 + 2z_2^2\varphi_1^2(x_1), \\ z_1\xi_1(x_1(t-\tau_1)) &\le \frac{1}{2}z_1^2 + \frac{1}{2}\xi_1^2(x_1(t-\tau_1)), \end{aligned}$$

and Assumption 4, then (19) becomes

$$\begin{aligned} \dot{V}_1(t) &\le \frac{3}{4}z_1^2 + z_1\varphi_1(x_1)\alpha_1 + z_1 Q_1(Z_1) + z_1\Lambda_1(x,t) \\ &\quad + \tilde{W}_1^T\Gamma_1^{-1}\dot{\hat{W}}_1 + \frac{1}{\gamma_{b_1}}\tilde{b}_1\dot{\hat{b}}_1 + \varphi_1^2(x_1)z_2^2. \end{aligned}$$

Note that (9) and the inequalities

$$\begin{aligned} z_1\varepsilon_1 + z_1\Lambda_1(x,t) &\le |z_1|\varepsilon^* + |z_1|p_1^*\Psi_1(x_1) \\ &\le |z_1|b_1^*\bar{\Psi}_1(x_1), \end{aligned}$$

where

$$b_1^* = \max\{\varepsilon_1^*, p_1^*\}, \quad \bar{\Psi}_1(x_1) = 1 + \Psi_1(x_1)$$

we have

$$\begin{aligned} \dot{V}_1(t) &\le \frac{3}{4}z_1^2 + z_1\varphi_1(x_1)\alpha_1 + z_1 W_1^T S(Z_1) + b_1^*|z_1|\bar{\Psi}_1(x_1) \\ &\quad + \tilde{W}_1^T\Gamma^{-1}\dot{\hat{W}}_1 + \frac{1}{\gamma_{b_1}}\tilde{b}_1\dot{\hat{b}}_1 + 2\varphi_1^2(x_1)z_2^2. \end{aligned} \tag{20}$$

Adding and subtracting

$$k_1 z_1^2 + z_1\hat{W}_1^T S(Z_1) + z_1\hat{b}_1\bar{\Psi}_1(x_1)\tanh\left[\frac{z_1\bar{\Psi}_1(x_1)}{\epsilon_1}\right].$$

We can get

$$\begin{aligned} \dot{V}_1(t) &\le -k_{10}z_1^2 + \varphi_1(x_1)q_1(z_1)N(\zeta_1)\dot{\zeta}_1 + \dot{\zeta}_1 \\ &\quad + b_1^*|z_1|\bar{\Psi}_1(x_1) - b_1^* z_1\bar{\Psi}_1(x_1)\tanh\left[\frac{z_1\bar{\Psi}_1(x_1)}{\epsilon_1}\right] \\ &\quad - k_{11}z_1^2 - \sigma_{\omega_1}\tilde{W}_1^T\hat{W}_1 - \sigma_{b_1}\tilde{b}_1^T\hat{b}_1 \\ &\quad + 2z_2^2\varphi_1^2(x_1). \end{aligned} \tag{21}$$

By completing the squares

$$-\sigma_{\omega_1}\tilde{W}_1^T\hat{W}_1 = \frac{1}{2}\sigma_{\omega_1}\|W_1^*\|^2 - \frac{1}{2}\sigma_{\omega_1}\|\tilde{W}_1\|^2,$$
$$-\sigma_{b_1}\tilde{b}_1^T\hat{b}_1 = \frac{1}{2}\sigma_{b_1}b_1^{*2} - \frac{1}{2}\sigma_{b_1}\tilde{b}_1^2,$$

and using Lemma 3, equation (21) can be further written as

$$\begin{aligned}\dot{V}_1 \le & -k_{10}z_1^2 - \varepsilon_{10}\kappa(c_{z_1})V_{U_1} - \frac{1}{2}\sigma_{\omega_1}\|\tilde{W}_1\|^2 \\ & - \frac{1}{2}\sigma_{b_1}\tilde{b}_1^2 + [\varphi_1(x_1)q_1(z_1)N(\zeta_1)+1]\dot{\zeta}_1 \\ & + 0.2785b_1^*\epsilon_1 + \frac{1}{2}\sigma_{\omega_1}\|W_1^*\|^2 + \frac{1}{2}\sigma_{b_1}b_1^{*2} \\ & + 2z_2^2\varphi_1^2(x_1).\end{aligned}$$

This yields

$$\dot{V}_1 \le -C_{11}V_1 + C_{12} + [\varphi_1(x_1)q_1(z_1)N(\zeta_1)+1]\dot{\zeta}_1 + 2\varphi_1^2(x_1)z_2^2, \tag{22}$$

where $C_{11} > 0$, $C_{12} > 0$ are defined as

$$C_{11} = \min\left\{2k_{10}, \varepsilon_{10}\kappa(c_{z_i}), \frac{\sigma_{\omega_1}}{\gamma_{\max}(\Gamma_1^{-1})}, \sigma_{b1}\gamma_{b_1}\right\},$$
$$C_{12} = 0.2785b_1^*\epsilon_1 + \frac{1}{2}\sigma_{\omega_1}\|W_1^*\|^2 + \frac{1}{2}\sigma_{b_1}b_1^{*2}.$$

Let $\rho_1 = C_{12}/C_{11}$, upon multiplication of (22) by $e^{C_{11}t}$, then integrating it over $[0, t]$, we get

$$\begin{aligned}V_1(t) \le & \rho_1 + [V_1(0)-\rho_1]e^{-C_{11}t} \\ & + e^{-C_{11}t}\int_0^t [\varphi_1(x_1)q_1(z_1)N(\zeta_1)+1]\dot{\zeta}_1 e^{C_{11}t}d\tau \\ & + e^{-C_{11}t}\int_0^t \varphi_1^2(x_1)z_2^2 e^{C_{11}\tau}d\tau, \\ \le & \rho_1 + V_1(0) \\ & + e^{-C_{11}t}\int_0^t [\varphi_1(x_1)q_1(z_1)N(\zeta_1)+1]\dot{\zeta}_1 e^{C_{11}t}d\tau \\ & + 2e^{-C_{11}t}\int_0^t \varphi_1^2(x_1)z_2^2 e^{C_{11}\tau}d\tau.\end{aligned} \tag{23}$$

Noting Assumption 2, we have inequality

$$\begin{aligned}& e^{-C_{11}t}\int_0^t \varphi_1^2(x_1)z_2^2 e^{C_{11}\tau}d\tau \\ & \le e^{-C_{11}t}\int_0^t \bar{\varphi}_1^2(x_1)z_2^2 e^{C_{11}\tau}d\tau, \\ & \le e^{-C_{11}t}l_1^{+2}\sup_{\tau\in(0,t)}[z_2^2(t)]\int_0^t e^{C_{11}\tau}d\tau, \\ & \le \frac{1}{C_{11}}l_1^{+2}\sup_{\tau\in(0,t)}[z_2^2(t)].\end{aligned} \tag{24}$$

Next to the stability analysis:

a) Region 1: $z_1 \in \Omega_{z_1}^O \bigcup \Omega_{z_1}^I$. Noting (23)(24) and Assumption 1 and 2, we known that if z_2 are bounded, we can regard $\varphi_1(x_1)q_1(z_1)$ in (23) as $g(\cdot)$, which take a value in interval $I = [\varphi_{10}q_1(c_{z_1}),\ l_1^+]$, with $0 \notin I$. According to the Lemma 1, $V_1(t)$, z_1, x_1, ζ_1, $\hat{W}_1$ and $\hat{b}_1$ are bounded. In the next steps, we will dealt with z_2.

b) Region 2: $z_1 \in \Omega_{z_1}$. In this area, $|z_1| \le c_{z_1}$ and $x_1 = z_1 + y_d$ are already bounded. Consider $V_{z_1}(t)$ and $V_{U_1}(t)$, they all bound. Now, we consider $V_{\omega_1}(t)$, and $V_{b_1}(t)$.Their time derivation along (15)(14)respectively, are

$$\dot{V}_{w_1}(t) = \tilde{W}_1^T[S(Z)z_1 - \sigma_{\omega_1}\hat{W}_1], \tag{25}$$
$$\dot{V}_{b_1}(t) = \tilde{b}_1\left(z_1\bar{\Psi}_1(x_1)\tanh\left[\frac{z_1\bar{\Psi}_1(x_1)}{\epsilon_1}\right] - \sigma_{b_1}\hat{b}_1\right). \tag{26}$$

Applying the inequalities

$$\tilde{W}_1^T S(Z_1)z_1 \le \frac{k_{\omega_1}}{2}\|\tilde{W}_1\|^2 + \frac{1}{2k_{\omega_1}}S^T(Z_1)S(Z_1)z_1^2, \tag{27}$$
$$-\sigma_{\omega_1}\tilde{W}_1^T\hat{W}_1 \le -\frac{1}{2}\sigma_{\omega_1}\|\tilde{W}_1\|^2 + \frac{1}{2}\sigma_{\omega_1}\|W_1^*\|^2, \tag{28}$$
$$\begin{aligned}-\sigma_{b_1}\tilde{b}_1\hat{b}_1 &= -\sigma_{b_1}\tilde{b}_1^2 - \sigma_{b_1}(\tilde{b}_1b_1^*), \\ &\le -\sigma_{b_1}\tilde{b}_1^2 + \frac{\sigma_{b_1}}{2}\tilde{b}_1^2 + \frac{\sigma_{b_1}}{2}b_1^{*2}, \\ &= \frac{\sigma_{b_1}}{2}b_1^{*2} - \frac{\sigma_{b_1}}{2}\tilde{b}_1^2,\end{aligned} \tag{29}$$

and

$$\begin{aligned}&\tilde{b}_1z_1\bar{\Psi}_1(x_1)\tanh\left[\frac{z_1\bar{\Psi}_1(x_1)}{\epsilon_1}\right] \\ &\le \frac{k_{b_1}}{2}\tilde{b}_1^2 + \frac{1}{2k_{b_1}}z_1^2\bar{\Psi}_1^2(x_1)\tanh^2\left[\frac{z_1\bar{\Psi}_1(x_1)}{\epsilon_1}\right].\end{aligned} \tag{30}$$

Therefore, noting (27)(28), we have

$$\begin{aligned}\dot{V}_{w_1}(t) \le & -\frac{1}{2}(\sigma_{\omega_1} - k_{\omega_1})\|\tilde{W}_1\|^2 \\ & + \frac{1}{2k_{\omega_1}}S^T(Z_1)S(Z_1)z_1^2 + \frac{1}{2}\sigma_{\omega_1}\|W_1^*\|^2.\end{aligned} \tag{31}$$

choose k_{ω_1} such that $\sigma_{\omega_1}^* := \sigma_{\omega_1} - k_{\omega_1} > 0$, and let

$$C_{w_1} := \frac{1}{2}\sigma_{\omega_1}^*/\lambda_{\max}(\Gamma_1^{-1}),$$
$$\lambda_{w_1} = \sup_{z_1\in\Omega_{z_1}}\{1/k_{\omega_1}S^T(Z_1)S(Z_1)z_1^2 + 1/2\sigma_{\omega_1}\|W_1^*\|\},$$

and $\rho_{\omega_1} := \lambda_{w_1}/C_{w_1}$, it follows from (31) that

$$V_{\omega_1} \le [V_{\omega_1}(0) - \rho_{\omega_1}]e^{-c_{w_1}t} + \rho_{\omega_1}, \tag{32}$$
$$\le V_{\omega_1}(0) + \rho_{\omega_1}. \tag{33}$$

Noting (30)(29), (26) can be written as

$$\begin{aligned}\dot{V}_{b_1}(t) \le & -\frac{1}{2}(\sigma_{b_1} - k_{b_1})\tilde{b}_1^2 \\ & + \frac{1}{2k_{b_1}}z_1^2\bar{\Psi}_1^2(x_1)\tanh^2\left[\frac{z_1\bar{\Psi}_1(x_1)}{\epsilon_1}\right] \\ & + \frac{\sigma_{b_1}}{2}b_1^*.\end{aligned} \tag{34}$$

In (34), note that $0 < \tanh(\cdot) < 1$, (34) can be written as

$$\dot{V}_{b_1}(t) \le -\frac{k_{b_1}}{2\gamma_{b_1}}\tilde{b}_1^2 + \frac{1}{2k_{b_1}}z_1^2\bar{\Psi}_1(x_1) + \frac{\sigma_{b_1}}{2}b_1{}^*. \tag{35}$$

choose k_{b_1} such that $\sigma_{b_1}^* := \sigma_{b_1} - k_{b_1} > 0$, and let

$$C_{b_1} = \frac{1}{2}\sigma_{b_1}^*,\ \lambda_{b_1} := \sup_{z_1 \in \Omega_{z_1}} \{\frac{1}{2k_{b_1}} z_1^2 \bar{\Psi}_1^2(x_1) + \frac{\sigma_{b_1}}{2} b_1^{*2}\}$$

and $\rho_{b_1} := \lambda_{b_1}/C_{b_1}$, it follows from (35) that

$$V_{b_1} \le [V_{b_1}(0) - \rho_{b_1}]e^{-C_{b_1}t} + \rho_{b_1}, \quad (36)$$
$$\le V_{b_1}(0) + \rho_{b_1}. \quad (37)$$

From (33)(37), we can conclude that $V_{\omega_1}(t)$, V_{b_1} and $\tilde{W}_1$, $\tilde{b}_1$ are bounded. According to the (5), $V_1(t)$ is bounded for $z_1 \in \Omega_{z_1}$ because $V_{z_1}(t)$, V_{U_1}, $\tilde{W}_1$ and $\tilde{b}_1$ are bounded.

Step i $(2 \le n \le n-1)$**:** The recursion process is similar for step $i = 2, \ldots, n-1$.

$$\begin{aligned}\dot{V}_i(t) &= z_i z_{i+1}\varphi_i(\bar{x}_i) + z_i[\varphi_i(x_i)\alpha_i(t) + f_i(x_i) \\ &\quad + \xi_i(x_i(t-\tau_i)) + \Lambda_i(x,t) - \dot{\alpha}_{i-1}] \\ &\quad + \frac{1}{2}\sum_{j=1}^{i} U_j(\bar{x}_j(t)) - \frac{1}{2}\sum_{j=1}^{i} U_j(\bar{x}_j(t-\tau_j)) \\ &\quad + \tilde{W}_i^T \Gamma_1^{-1}\dot{\tilde{W}}_i + \frac{1}{\gamma_{b_i}}\tilde{b}_i\dot{\tilde{b}}_i.\end{aligned}$$

Since α_i is a function of $\bar{x}_{i-1}, \zeta_{i-1}, \bar{x}_{d_i}, \hat{W}_1, \ldots, \hat{W}_{i-1}$, $\hat{b}_1, \ldots, \hat{b}_{i-1}, k_1, \ldots, k_{i-1}$, and $\dot{\alpha}_{i-1}$ can be expressed as

$$\begin{aligned}\dot{\alpha}_{i-1} &= \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}\dot{x}_j + w_{i-1}(t), \\ &= \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}[\varphi_j(\bar{x}_j)x_{j+1} + f_j(\bar{x}_j) + \xi_j(\bar{x}_j(t-\tau_j)) \\ &\quad + \Lambda_j(x,t)] + w_{i-1}(t),\end{aligned} \quad (38)$$

where

$$\begin{aligned}w_{i-1}(t) &= \frac{\partial\alpha_{i-1}}{\partial\zeta_{i-1}}\dot{\zeta}_{i-1} + \frac{\partial\alpha_{i-1}}{\partial\,\bar{x}_{d_i}}\dot{\bar{x}}_{d_i} + \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\hat{W}_j}\dot{\hat{W}}_j \\ &\quad + \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\hat{b}_j}\dot{\hat{b}}_j + \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial k_j}\dot{k}_j.\end{aligned}$$

All previous subsystems will experience an unknown time delay due to the $\dot{\bar{x}}_{i-1}$ and $\dot{\alpha}_{i-1}$ required in the recursive backstep design and should be compensated for in this step. And Lyapunov-krosovskii function (5) can compensate for the unknown time delay τ_i, and $\tau_{i-1}, \ldots, \tau_1$.

Applying Assumption 4, and noting (1)(4)(38)

$$\begin{aligned}\dot{V}_{z_i} &= z_i z_{i+1}\varphi_i(\bar{x}_i) + z_i[\varphi_i(\bar{x}_i)\alpha_i + f_i(\bar{x}_i)] \\ &\quad + z_i\Lambda_i(x,t) + z_i\xi_i(\bar{x}_i(t-\tau_i)) \\ &\quad - z_i\left(\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}[\varphi_i(\bar{x}_j)x_{j+1} + f_j(\bar{x}_j)] + w_{i-1}\right) \\ &\quad - z_i\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}\xi_j(\bar{x}_j(t-\tau_j)) \\ &\quad - z_i\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}\Lambda_j(x,t).\end{aligned}$$

Noting (10) and using the inequalities

$$\begin{aligned}z_i z_{i+1}\varphi_i(\bar{x}_i) &\le \frac{1}{4}z_i^2 + z_{i+1}^2\varphi_i^2(\bar{x}_i), \\ z_i\xi_i(\bar{x}_i(t-\tau_i)) & \\ &\le \frac{1}{2}z_i^2 + \frac{1}{2}\xi_i^2(\bar{x}_i(t-\tau_i)), \\ &\le \frac{1}{2}z_i^2 + \frac{1}{2}\beta_i^2(\bar{x}_i(t-\tau_i)),\end{aligned} \quad (39)$$

$$\begin{aligned}&- z_i\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}\xi_j(\bar{x}_j(t-\tau_j)) \\ &\le \sum_{j=1}^{i-1}|z_i\frac{\partial\alpha_{i-1}}{\partial x_j}||\xi_j(\bar{x}_j(t-\tau_j))|, \\ &\le \sum_{j=1}^{i-1}\left[\frac{1}{2}z_i^2\left(\frac{\partial\alpha_{i-1}}{\partial x_j}\right)^2 + \frac{1}{2}\xi_j^2(\bar{x}_j(t-\tau_j))\right], \\ &= \frac{1}{2}z_i^2\sum_{j=1}^{i-1}(\frac{\partial\alpha_{i-1}}{\partial x_j})^2 + \frac{1}{2}\sum_{j=1}^{i-1}\beta_j^2(\bar{x}_j(t-\tau_j)\end{aligned} \quad (40)$$

We have

$$\begin{aligned}\dot{V}_{z_i} &\le \frac{3}{4}z_i^2 + z_i\varphi_i(\bar{x}_i)\alpha_i + z_iQ_i + z_i\Lambda_i(x,t) \\ &\quad - z_i\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}\Lambda_j(x,t) + \frac{1}{2}\sum_{j=1}^{i}\beta_j^2(\bar{x}_j(t-\tau_j)) \\ &\quad - \frac{1}{2}\sum_{j=1}^{i}\beta_j^2(\bar{x}_j) + z_{i+1}^2\varphi_i^2(\bar{x}_i).\end{aligned} \quad (41)$$

Noting Assumption 5, we have

$$z_i\Lambda_i(x,t) \le |z_i|p_i^*\Psi_i(\bar{x}_i), \quad (42)$$

$$-z_i\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}\Lambda_j(x,t) \le |z_i|\sum_{j=1}^{i-1}|\frac{\partial\alpha_{i-1}}{\partial x_j}p_j^*\Psi_j(\bar{x}_j). \quad (43)$$

Noting (4)-(8), (41)-(43), we have

$$\begin{aligned}\dot{V}_i &\le \frac{3}{4}z_i^2 + z_i\varphi_i(\bar{x}_i)\alpha_i + z_iW_i^*S(Z_i) + z_i\varepsilon_i \\ &\quad + |z_i|p_i^*\Psi_i(\bar{x}_i) + |z_i|\sum_{j=1}^{i-1}|\frac{\partial\alpha_{i-1}}{\partial x_j}|p_j^*\Psi_j(\bar{x}_j) \\ &\quad + \frac{1}{2}\sum_{j=1}^{i}\beta_j^2(\bar{x}_j(t-\tau_j)) - \frac{1}{2}\sum_{j=1}^{i}\beta_j^2(\bar{x}_j) \\ &\quad + \frac{1}{2}\sum_{j=1}^{i}U_j(\bar{x}_j(t)) - \frac{1}{2}\sum_{j=1}^{i}U_j(\bar{x}_j(t-\tau_j)) \\ &\quad + \tilde{W}_i^T\Gamma_i^{-1}\dot{\tilde{W}}_i + \frac{1}{\gamma_{b_i}}\tilde{b}_i\dot{\tilde{b}}_i + z_{i+1}^2\varphi_i^2(\bar{x}_j).\end{aligned} \quad (44)$$

Note that (44) and the inequalities

$$\begin{aligned} &z_i\varepsilon_i + |z_i|p_i^*\Psi_i(\bar{x}_i) + |z_i|\sum_{j=1}^{i-1} p_j^*|\frac{\partial\alpha_{i-1}}{\partial x_j}|\Psi_j(\bar{x}_j) \\ &\leq |z_i|\left(\varepsilon_i^* + p_i^*\Psi_i(\bar{x}_i) + \sum_{j=1}^{i-1} p_j^*|\frac{\partial\alpha_{i-1}}{\partial x_j}|\Psi_j(\bar{x}_j)\right), \\ &\leq b_i^*|z_i|\bar{\Psi}_i(\bar{x}_i), \end{aligned}$$

where

$$b_i^* = \max\{\varepsilon_i^*, p_1^*, p_2^*, \ldots, p_i^*\}$$

and

$$\bar{\Psi}_i(\bar{x}_i) \geq 1 + \Psi_i(\bar{x}_i) + \sum_{j=1}^{i-1} |\frac{\partial\alpha_{i-1}}{\partial x_j}|\Psi_j(\bar{x}_j)$$

is a smooth positive function.

$$\bar{\Psi}_i = 1 + \Psi_i + \sum_{j=1}^{i-1}\left(\frac{1}{4}\left(\frac{\partial\alpha_{i-1}}{\partial x_j}\right)^2 + 1\right)\Psi_j.$$

Thus, (44) can be rewritten as

$$\begin{aligned} \dot{V}_i \leq \frac{3}{4}z_i^2 + z_i\varphi_i(\bar{x}_i)\alpha_i + z_iW_i^*S(Z_i) + b_i^*|z_i|\bar{\Psi}_i(\bar{x}_i) \\ + \tilde{W}_i^T\Gamma_i^{-1}\dot{\hat{W}}_i + \frac{1}{\gamma_{b_i}}\tilde{b}_i\dot{\hat{b}}_i + z_{i+1}^2\varphi_i^2(\bar{x}_j). \end{aligned}$$

Noting (12)-(17) , we also obtain

$$\begin{aligned} \dot{V}_i \leq& -k_{i0}z_i^2 - \varepsilon_{i0}\kappa(z_{c_i})V_{U_i} - \frac{1}{2}\sigma_{w_i}\|\tilde{W}_i\|^2 - \frac{1}{2}\sigma_{b_i}\tilde{b}_i^2 \\ &+ [\varphi_i(\bar{x}_i)q_i(z_i)N(\zeta_i) + 1]\dot{\zeta}_i + 0.2785b_i^*\epsilon_i\frac{1}{2}\sigma_{w_i}\|W_i^*\|^2 \\ &+ \frac{1}{2}\sigma_{b_i}b_i^{*2} + \varphi_i^2(\bar{x}_i)z_{i+1}^2. \end{aligned}$$

Similarly this yields

$$\dot{V}_i \leq -C_{i1}V_i + C_{i2} + [\varphi_i(\bar{x}_i)q_i(z_i)N(\zeta_i) + 1]\dot{\zeta}_i + \varphi_i^2(\bar{x}_i)z_{i+1}^2, \quad (45)$$

where the constants $C_{i1} > 0$ and $C_{i2} > 0$

$$\begin{aligned} C_{i1} &= \min\left\{2k_{i0}, \varepsilon_{i0}\kappa(z_{c_i}), \frac{\sigma_{w_i}}{\gamma_{\max}(\Gamma_i^{-1})}, \sigma_{bi}\gamma_{b_i}\right\}, \\ C_{i2} &= 0.2785b_i^*\epsilon_i + \frac{1}{2}\sigma_{w_i}\|W_i^*\|^2 + \frac{1}{2}\sigma_{b_i}b_i^{*2}. \end{aligned}$$

Let $\rho_i = C_{i2}/C_{i1}$, upon multiplication of (45) by $e^{C_{i1}t}$, we get

$$\begin{aligned} V_i(t) \leq& \rho_i + [V_i(0) - \rho_i]e^{-C_{i1}t} \\ &+ e^{-C_{i1}t}\int_0^t [\varphi_i(\bar{x}_i)q_i(z_i)N(\zeta_i) + 1]\dot{\zeta}_ie^{C_{i1}t}d\tau \\ &+ e^{-C_{i1}t}\int_0^t \varphi_i^2(x_i)z_{i+1}^2e^{C_{i1}\tau}d\tau, \quad (46) \\ \leq& \rho_i + V_i(0) + e^{-C_{i1}t}\int_0^t [\varphi_i(\bar{x}_i)q_i(z_i)N(\zeta_i) + 1] \\ &\times\dot{\zeta}_ie^{C_{i1}t}d\tau + e^{-C_{i1}t}\int_0^t \varphi_i^2(x_i)z_{i+1}^2e^{C_{i1}\tau}d\tau. \quad (47) \end{aligned}$$

Noting Assumption 2, we have inequality

$$e^{-C_{i1}t}\int_0^t \varphi_i^2(x_i)z_{i+1}^2e^{C_{i1}\tau}d\tau \leq \frac{1}{C_{i1}}l_i^{+2}\sup_{\tau\in(0,t)}[z_{i+1}^2(\tau)]. \quad (48)$$

The stability analysis is next.

a) Region 1: $z_i \in \Omega_{z_i}^O \bigcup \Omega_{z_i}^I$. Noting (47)(48), we known that if z_{i+1} is bounded, we can regard $\varphi_i(x_i)q_i(z_i)$ in (23) as $g(\cdot)$, which is evaluated in $I = [\varphi_{i0}q_i(c_{z_i}), l_i^+]$, with $0 \notin I$. According to the Lemma 1, $V_i(t)$, z_i, x_i, ζ_i, $\hat{W}_i$ and $\hat{b}_i$ are bounded.

The processing of z_{i+1} will take place in the following steps.

b) Region 2: $z_i \in \Omega_{z_1}$. $z_i, z_{i-1}, \ldots, z_1$ are bounded, so that $x_i, x_{i_1}, \ldots, x_1$ are bounded as well. The boundedness analysis process for $\hat{W}_i$ and $\hat{b}_i$ are similar to the process performed in *Region 2* of *Step 1*, similar (32)-(33),(36)-(37), we have

$$\begin{aligned} V_{\omega_i} &\leq [V_{\omega_i}(0) - \rho_{\omega_i}]e^{-c_{w_i}t} + \rho_{\omega_i}, & (49) \\ &\leq V_{\omega_i}(0) + \rho_{\omega_i}, & (50) \\ V_{b_i} &\leq [V_{b_i}(0) - \rho_{b_i}]e^{-C_{b_i}t} + \rho_{b_i}, & (51) \\ &\leq V_{b_i}(0) + \rho_{b_i}, & (52) \end{aligned}$$

where

$$\begin{aligned} \rho_{b_i} &:= \lambda_{b_i}/C_{b_i}, \\ \sigma_{w_i}^* &:= \sigma_{w_i} - k_{w_i} > 0, \\ C_{w_i} &:= \frac{1}{2}\sigma_{w_i}^*/\lambda_{\max}(\Gamma_i^{-1}), \\ \lambda_{w_i} &:= \sup_{z_i\in\Omega_{z_i}}\{\frac{1}{k_{w_i}}S^T(Z_i)S(Z_i)z_i^2 + \frac{1}{2}\sigma_{w_i}\|W_i^*\|\}, \\ \rho_{\omega_i} &:= \lambda_{w_i}/C_{w_i}, \\ \sigma_{b_1}^* &:= \sigma_{b_1} - k_{b_1} > 0, \\ C_{b_1} &:= \frac{1}{2}\sigma_{b_1}^*, \\ \lambda_{b_1} &:= \sup_{z_1\in\Omega_{z_1}}\{\frac{1}{2k_{b_1}}z_1^2\bar{\Psi}_1^2(x_1) + \frac{\sigma_{b_1}}{2}b_1^{*2}\}, \\ \rho_{b_1} &:= \lambda_{b_1}/C_{b_1}. \end{aligned}$$

From (50)(52), we can deduce that V_{ω_i}, V_{b_i} are bounded, and therefore, $\tilde{W}_i$, $\tilde{b}_i$ are bounded. According to the (8), $V_i(t)$ is bounded for $z_i \in \Omega_{z_i}$ $(i = 2\ldots, n-1)$ because $V_{z_i}(t)$, $V_{U_1}(t)$, $\tilde{W}_i(t)$ and $\tilde{b}_i(t)$ are bounded.

Step n: we have

$$z_n = \varphi_n(\bar{x}_n)u + f_n(\bar{x}_n) + \xi_n(\bar{x}_n(t-\tau_n)) + \Lambda_n(x,t) - \dot{\alpha}_{n-1}.$$

The time derivative of $V_n(t)$ is

$$\begin{aligned} \dot{V}_n \leq& \frac{1}{2}z_n^2 + z_n\varphi_n(\bar{x}_n)\alpha_n + z_nW_n^*S(Z_n) + z_n\varepsilon_n \\ &+ |z_n|p_n^*\Psi_n(\bar{x}_n) + |z_n|\sum_{j=1}^{n-1}|\frac{\partial\alpha_{n-1}}{\partial x_j}|p_j^*\Psi_j(\bar{x}_j) \\ &+ \frac{1}{2}\sum_{j=1}^{n}\beta_j^2(\bar{x}_j(t-\tau_j)) - \frac{1}{2}\sum_{j=1}^{n}\beta_j^2(\bar{x}_j) \end{aligned}$$

$$+\frac{1}{2}\sum_{j=1}^{n}U_j(\bar{x}_j(t))-\frac{1}{2}\sum_{j=1}^{n}U_j(\bar{x}_j(t-\tau_j))$$
$$+\tilde{W}_n^T\Gamma_n^{-1}\dot{\hat{W}}_n+\frac{1}{\gamma_{b_n}}\tilde{b}_n\dot{\hat{b}}_n.$$

Similarly, we have

$$\dot{V}_n\leq\frac{1}{2}z_n^2+z_n\varphi_n(\bar{x}_n)\alpha_n+z_nW_n^*S(Z_n)$$
$$+b_n^*|z_n|\bar{\Psi}_n(\bar{x}_n)+\tilde{W}_n^T\Gamma_n^{-1}\dot{\hat{W}}_n+\frac{1}{\gamma_{b_n}}\tilde{b}_n\dot{\hat{b}}_n,$$

where

$$b_n^*=\max\{\varepsilon_n^*,p_1^*,p_2^*,\ldots,p_n^*\},$$
$$\bar{\Psi}_n(\bar{x}_n)\geq 1+\Psi_n(\bar{x}_n)+\sum_{j=1}^{n-1}|\frac{\partial\alpha_{n-1}}{\partial x_j}|\Psi_j(\bar{x}_j),$$
$$\bar{\Psi}_n=1+\Psi_n+\sum_{j=1}^{n-1}\left(\frac{1}{4}\left(\frac{\partial\alpha_{n-1}}{\partial x_j}\right)^2+1\right)\Psi_j,$$

consider the control given by (12)-(17), similar previous steps, we have

$$\dot{V}_n\leq-(k_{n0}+\frac{1}{4})z_n^2-\varepsilon_{n0}\kappa_{z_{c_n}}V_{U_n}-\frac{1}{2}\sigma_{w_n}\|\tilde{W}_n\|^2$$
$$-\frac{1}{2}\sigma_{b_n}\tilde{b}_n^2+[\varphi_n(\bar{x}_n)q_n(z_n)N(\zeta_n)+1]\dot{\zeta}_n$$
$$+0.2785b_n^*\epsilon_n+\frac{1}{2}\sigma_{w_n}\|W_n^*\|^2+\frac{1}{2}\sigma_{b_n}b_n^{*2}.$$

Similarly this yields

$$\dot{V}_n\leq-C_{n1}V_n+C_{n2}+[\varphi_n(\bar{x}_n)q_n(z_n)N(\zeta_n)+1]\dot{\zeta}_n,\quad(53)$$

where the constants $C_{n1}>0$ and $C_{n2}>0$

$$C_{n1}=\min\left\{2k_{n0}+\frac{1}{2},\varepsilon_{n0}\kappa(z_{c_i}),\frac{\sigma_{w_n}}{\gamma_{max}(\Gamma_n^{-1})},\sigma_{bn}\gamma_{b_n}\right\},$$
$$C_{n2}=0.2785b_n^*\epsilon_n+\frac{1}{2}\sigma_{w_n}\|W_n^*\|^2+\frac{1}{2}\sigma_{b_n}b_n^{*2}.$$

Let $\rho_n=C_{n2}/C_{n1}$, upon multiplication of (53) by $e^{C_{n1}t}$, then we can get result of integrating it over $[0,t]$

$$V_n(t)\leq\rho_n+[V_n(0)-\rho_n]e^{-C_{n1}t}+e^{-C_{n1}t}\int_0^t[\varphi_n(\bar{x}_n)q_n(z_n)\times N(\zeta_n)+1]\dot{\zeta}_ne^{C_{n1}t}d\tau,\quad(54)$$
$$\leq\rho_n+V_n(0)+e^{-C_{n1}t}\int_0^t[\varphi_n(\bar{x}_n)q_n(z_n)\times N(\zeta_n)+1]\dot{\zeta}_ne^{C_{n1}t}d\tau.\quad(55)$$

Stability analysis is performed in two regions, similar to the previous steps.

a) For $z_n\in\Omega_{z_n}^O\bigcup\Omega_{z_n}^I$, the $u(t)$ is invoked, Similar to the discussion in *Region 1* of *step 1*, we can regard $\varphi_n(x_n)q_n(z_n)$ in (55) as $g(\cdot)$, which is evaluated in $I=[\varphi_{n0}q_n(c_{z_n}),\ l_n^+]$. In addition $z_n(t)$, $\hat{W}_n(t)$, and $\hat{b}_n(t)$ are bounded because we can deduce $\zeta_n(t)$ and $V_n(t)$ from (55) and Lemma 1. From the boundedness of $z_n(t)$, we conclude

$$e^{-C_{n-1,1}t}\int_0^t\varphi_{n-1}^2(x_{n-1})z_{n-1}^2e^{C_{n-1,1}\tau}d\tau,$$

which is bounded at *step* $n-1$. According Lemma 1, $V_i(t)$, $z_i(t)$, $\zeta_i(t)$, $\hat{W}_i(t)$, $\hat{b}_i(t)$, and $x_i(t)$ are bounded.

b) For $z_n\in\Omega_{z_n}$, z_n is bounded, so $z_{n-1},\ldots,z_1$ and $x_i,x_{i-1},\ldots,x_1$ are all bounded.The boundedness analysis process for $\hat{W}_n$ and $\hat{b}_n$ are similar to the process performed in Region 2 of *step* i.

Theorem 1: Consider the nonlinear control system (1), the control laws (12) and adaptive laws (13)(14)(15). Under Assumptions 1-5 and some bounded conditions, the system (1) has the following properties.

1) All signals in system (1) are SGUUB, and

$$Z=[Z_1^T,\ldots,Z_n^T]^T$$

remains in the compact set

$$\Omega_Z:=\Omega_{Z_1}\bigcup\cdots\bigcup\Omega_{Z_n},$$

which is specified as

$$\Omega_Z=\Big\{Z\mid\sum_{i=1}^{n}z_i^2\leq A_0,\sum_{i=1}^{n}\|\tilde{W}_i\|^2\leq A_1\sum_{i=1}^{n}\tilde{b}_i^2\leq A_2,$$
$$\bar{x}_{di}\in\Omega_{di},i=2,\ldots,n.\Big\}\quad(56)$$

2) All signal in system 1 will converge to a collection.

$$\Omega_S=\Big\{Z\mid\sum_{i=1}^{n}z_i^2\leq A_0^*,\sum_{i=1}^{n}\|\tilde{W}_i\|^2\leq A_1^*,\sum_{i=1}^{n}\tilde{b}_i^2\leq A_2^*,$$
$$\bar{x}_{di}\in\Omega_{di},i=2,\ldots,n.\Big\}$$

where $A_0,A_0^*,A_1,A_1^*,A_2,A_2^*>0$ are constants.

Proof:

$$V(t)=\sum_{i=1}^{n}V_{z_i}(t)+V_{U_i}(t)+V_{\omega_i}+V_{b_i}(t),\quad(57)$$

where $V_{z_i}(t)$, $V_{U_i}(t)$, V_{ω_i}, $V_{b_i}(t)$ are defined in (4)(5)(6) and (7), respectively. There are three cases.

Case 1: All $z_i\in\Omega_{z_i}^O\bigcup\Omega_{z_i}^I$, $i=1,\ldots,n$; At case 1, all the control effort are involved, we have

$$e^{-C_{n1}t}\int_0^t[\varphi_n(\bar{x}_n)q_n(z_n)N(\zeta_n)+1]\dot{\zeta}_ne^{C_{n1}\tau}d\tau$$

is bounded, letting d_{n0} be the upper bound of it, $d_n=\rho_n+V_n(0)$ and $\mu_{n1}=d_n+d_{n0}$ in (55), noting (8) we have

$$V_n(t)\leq\mu_{n1},\quad(58)$$
$$z_n^2\leq 2\mu_{n1},\|\tilde{W}_n\|^2\leq\frac{2\mu_{n1}}{\lambda_{\min}(\Gamma_n^{-1})},\tilde{b}_n^2\leq 2\gamma_{bn}\mu_{n1}.\quad(59)$$

It follows form (58) that $V_n(t)$ is bounded. Therefore, z_n, $\hat{W}_n$ and $\hat{b}_n$ are bounded. And let

$$d_{i0} = e^{-C_{i1}t}\int_0^t [\varphi_i(\bar{x}_i)q_i(z_i)N(\zeta_i)+1]\dot{\zeta}_n e^{C_{i1}\tau}d\tau.$$

Noting (48), we have

$$V_i(t) \le \rho_i + V_i(0) + d_{i0} + \frac{2}{C_{i1}}(l_i^+)^2\mu_{i+1,1}.$$

Let

$$d_i = \rho_i + V_i(0) + \frac{2}{C_{i1}}(l_i^+)^2\mu_{i+1,1},\ \ \mu_{i1} = d_{i0} + d_i.$$

Noting (13), we have

$$V_i(t) \le \mu_{i1},$$
$$z_i^2 \le 2\mu_{i1},\ \|\tilde{W}_i\|^2 \le \frac{2\mu_{i1}}{\lambda_{\min}(\Gamma_i^{-1})},\ \tilde{b}_i^2 \le 2\gamma_{bi}\mu_{i1}. \quad (60)$$

Furthermore, we noting (54), we rewrite it as

$$V_n \le \mu_{n1}^* + [V_n(0) - \rho_n]e^{-C_{n1}t},$$

where $\mu_{n1}^* = d_n^* + d_{n0},\ d_n^* = \rho_n$. As $t \mapsto \infty$, $V_n \le \mu_{n1}^*$. Hence, according to the V_n in (8), when $t \mapsto \infty$, we can get the following inequalities:

$$V_n(t) \le \mu_{n1}^*,$$
$$z_n^2 \le 2\mu_{n1}^*,\ \|\tilde{W}_n\|^2 \le \frac{2\mu_{n1}^*}{\lambda_{\min}(\Gamma_n^{-1})},\ \tilde{b}_n^2 \le 2\gamma_{bn}\mu_{n1}^*. \quad (61)$$

For z_i and $\hat{W}_i$, we can deduce a similar conclusion as follows:

$$V_i(t) \le \mu_{i1}^*,$$
$$z_i^2 \le 2\mu_{i1}^*,\ \|\tilde{W}_i\|^2 \le \frac{2\mu_{i1}^*}{\lambda_{\min}(\Gamma_i^{-1})},\ \tilde{b}_i^2 \le 2\gamma_{bi}\mu_{i1}^*. \quad (62)$$

Thus, noting (59)(60) we have

$$\sum_{i=1}^n z_i^2 \le 2\sum_{i=1}^n \mu_{i1}, \quad (63)$$
$$\sum_{i=1}^n \|\tilde{W}_i\|^2 \le \sum_{i=1}^n \frac{2\mu_{i1}}{\lambda_{\min}(\Gamma_i^{-1})},$$
$$\sum_{i=1}^n \tilde{b}_i^2 \le 2\sum_{i=1}^n \gamma_{bi}\mu_{i1}.$$

Furthermore, noting (61)(62), we have

$$\sum_{i=1}^n \|z_i\|^2 \le 2\sum_{i=1}^n \mu_{i1}^*, \quad (64)$$
$$\sum_{i=1}^n \|\tilde{W}_i\|^2 \le \sum_{i=1}^n \frac{2\mu_{i1}^*}{\lambda_{\min}(\Gamma_i^{-1})},$$
$$\sum_{i=1}^n \tilde{b}_i^2 \le 2\sum_{i=1}^n \gamma_{bi}\mu_{i1}^*,$$

with $\mu_{i1}^* = d_i^* + d_{i0}$ and

$$d_i^* = \rho_i + \frac{2}{C_{i1}}l_i^{+2}\mu_{i+1}.$$

As $t \mapsto \infty$, from(61)(62), we have

$$\lim_{t\to\infty}\|z\| \le \sqrt{2\sum_{i=1}^n \mu_{i1}^*}.$$

The analysis is for $z_i \in \Omega_{z_I}^O \bigcup \Omega_{z_i}^I$ i.e. $|z_i| \ge c_{z_i}$, $i = 1, 2, \ldots, n$. Let

$$z_{\min} \triangleq \sqrt{\sum_{i=1}^n c_{z_i}^2}$$

and

$$z_{\max} \triangleq \sqrt{2\sum_{i=1}^n \mu_i^*}.$$

First, if $z_{\max} \ge z_{\min}$, z starting at $\Omega_{z_i}^O \bigcup \Omega_{z_i}^I$, but when z converges to a boundary smaller than $\Omega_{z_i}^O \bigcup \Omega_{z_i}^I$, i.e. $z_{\min}$, the situation reveals $z_{\max} \le z_{\min}$. when a difference control is applied it falls into another compact set, the only properties is $\lim_{t\to\infty}\|z\| \le z_{\min}$. Hence, we can get

$$\lim_{t\to\infty}\|z\| \le \max\left\{\sqrt{2\sum_{i=1}^n \mu_{i1}^*},\ \sqrt{\sum_{i=1}^n c_{z_i}^2}\right\}$$

Case 2: All $z_i \in \Omega_{z_i}$, $i = 1, \ldots, n$. In the situation, z_i's are bounded. All control $\alpha_i(t) = 0, (i = 1, 2, \ldots, n)$, from the previous analysis, noting (6)(7)(33)(37)(50)(52), we letting $\mu_{\omega_1} = V_{\omega_i}(0) + \rho_{\omega_i},\ \mu_{b_i} = V_{b_i}(0) + \rho_{b_i}$, we have

$$\|\tilde{W}_i\|^2 \le \frac{2\mu_{\omega_1}}{\lambda_{\min}(\Gamma_i^{-1})},\ \ \tilde{b}_i^2 \le 2\gamma_{b_i}\mu_{b_i}. \quad (65)$$

Furthermore, note that (32)(36)(49)(51). As $t \to \infty$, we have $V_{\omega_i} \le \rho_{\omega_i}$, $V_{b_i} \le \rho_{b_i}$. Therefore, we can deduce the following inequalities

$$\|\tilde{W}_i\|^2 \le \frac{2\rho_{\omega_i}}{\lambda_{\min}(\Gamma_i^{-1})},\ \ \tilde{b}_i^2 \le 2\gamma_{b_i}\rho_{b_i}. \quad (66)$$

Thus, noting (65), we have

$$\sum_{i=1}^n z_i^2 \le 2\sum_{j=1}^n c_{z_i}^2,$$
$$\sum_{i=1}^n \|\tilde{W}_i\|^2 \le \sum_{i=1}^n \frac{2\mu_{\omega_1}}{\lambda_{\min}(\Gamma_i^{-1})},$$
$$\sum_{i=1}^n \tilde{b}_i^2 \le 2\sum_{i=1}^n \gamma_{bi}\mu_{bi}.$$

Furthermore, noting (66), when $t \to \infty$ we have

$$\sum_{i=1}^n z_i^2 \le 2\sum_{j=1}^n c_{z_i}^2,$$
$$\sum_{i=1}^n \|\tilde{W}_i\|^2 \le \sum_{i=1}^n \frac{2\rho_{\omega_i}}{\lambda_{\min}(\Gamma_i^{-1})},$$
$$\sum_{i=1}^n \tilde{b}_i^2 \le 2\sum_{i=1}^n \gamma_{bi}\rho_{bi}.$$

Case 3: Some z_i's are belong to $z_i \in \Omega_{z_i}^O \bigcup \Omega_{z_i}^I$, while other z_j's are belong to $z_j \in \Omega_{z_j}$. let

$$I = \{i \mid z_i \in \Omega_{z_i}^O \bigcup \Omega_{z_i}^I\},\ J = \{j \mid z_j \in \Omega_{z_j}\}.$$

a) For those $z_i \in \Omega_{z_i}^o \bigcup \Omega_{z_i}^I$, the corresponding control effort $\alpha_i(t)$ adaptation law for $\hat{W}_i$, $\hat{b}_i$ are invoked, and according to (47)(48), we get

$$V_i(t) \le \rho_i + V_i(0) + d_{i0} + \frac{1}{C_{i1}} l_i^{+2} \sup_{\tau \in (0,t)} z_{i+1}^2(\tau)\ (i \in I/n),$$

where $I/n = I - n$. Letting

$$\upsilon_{i+1} = \begin{cases} c_{z_{i+1}} & \text{if } z_{i+1} \in \Omega_{z_{i+1}}, \\ \sqrt{2\mu_{i+1}} & \text{if } z_{i+1} \in \Omega_{z_{i+1}}^O \bigcup \Omega_{z_{i+1}}^I, \end{cases}$$

then $\sup_{\tau \in (0,t)} z_{i+1}^2(\tau) \le \upsilon_{i+1}^2$. Defining $V_I(t) = \sum_I V_i(t)$ and positive constants

$$C_{BIi} = \begin{cases} \rho_i + V_i(0) + d_{i0} + \frac{1}{C_{i1}} l_i^{+2} \upsilon_{i+1}^2, \\ \qquad \text{if } z_i \in \Omega_{z_i}^O \bigcup \Omega_{z_i}^I,\ (i \in I/n) \\ \mu_{n1}, \quad \text{if } z_n \in \Omega_{z_n}^O \bigcup \Omega_{z_n}^I,\ (i = n) \end{cases}$$

we have that

$$z_i^2 \le 2C_{BIi},\ \ \|\tilde{W}_{I_i}\|^2 \le \frac{2C_{BIi}}{\lambda_{\min}(\Gamma_i^{-1})},\ \ \tilde{b}_i^2 \le 2\gamma_{bi} C_{BIi}. \quad (67)$$

Furthermore, we note that (46)(48).
As $t \to \infty$, we have $V_i(t) \le C_{BIi}^*$,
where

$$C_{BIi}^* = \begin{cases} \rho_i + d_{i0}, \ \text{if } z_i \in \Omega_{z_i}^o \bigcup \Omega_{z_i}^I + \frac{1}{C_{i1}} l_i^{+2} \upsilon_{i+1}^2, \\ \qquad (i \in I/n) \\ \mu_{n1}^*, \qquad \text{if } z_n \in \Omega_{z_n}^o \bigcup \Omega_{z_n}^I,\ (i = n) \end{cases}$$

we have that

$$z_i^2 \le 2C_{BIi}^*, \quad \|\tilde{W}_{I_i}\|^2 \le \frac{2C_{BIi}^*}{\lambda_{\min}(\Gamma_i^{-1})}, \quad \tilde{b}_i^2 \le 2\gamma_{bi} C_{BIi}^*. \quad (68)$$

b) For those $z_j \in \Omega_{z_j}$, i.e. $|z_j| \le c_{z_j}$. The analysis is the same case 2. We get

$$\sum_J^n z_j^2 \le 2\sum_J^n c_{z_j}^2,$$
$$\sum_J^n \|\tilde{W}_j\|^2 \le \sum_J^n \frac{2\mu_{w_j}}{\lambda_{\min}(\Gamma_j^{-1})},$$
$$\sum_J^n \tilde{b}_j^2 \le 2\sum_J^n \gamma_{bj}\mu_{bj}. \quad (69)$$

As $t \to \infty$ we have

$$\sum_J^n z_j^2 \le 2\sum_J^n c_{z_j}^2,$$
$$\sum_J^n \|\tilde{W}_j\|^2 \le \sum_J^n \frac{2\rho_{\omega_j}}{\lambda_{\min}(\Gamma_j^{-1})},$$
$$\sum_J^n \tilde{b}_j^2 \le 2\sum_J^n \gamma_{bj}\rho_{bj}. \quad (70)$$

From the (a) and (b) in case 2, noting (65)(67), we have

$$\sum_{i=1}^n z_i^2 \le 2\left(\sum_I C_{BIi} + \sum_J c_{z_j}^2\right),$$
$$\sum_{i=1}^n \|\tilde{W}_i\|^2 \le \sum_I \frac{2C_{BIi}}{\lambda_{\min}(\Gamma_i^{-1})} + \sum_J \frac{2\mu_{w_j}}{\lambda_{\min}(\Gamma_j^{-1})},$$
$$\sum_{i=1}^n \tilde{b}_i^2 \le 2\left(\sum_I \gamma_{bi} C_{BIi} + \sum_J \gamma_{bi}\mu_{bi}\right).$$

Furthermore, noting (66)(68), when $t \to \infty$ we have

$$\sum_{i=1}^n z_i^2 \le 2\left(\sum_I C_{BIi}^* + \sum_J c_{z_j}^2\right),$$
$$\sum_{i=1}^n \|\tilde{W}_i\|^2 \le \sum_I \frac{2C_{BIi}^*}{\lambda_{\min}(\Gamma_i^{-1})} + \sum_J \frac{2\rho_{\omega_j}}{\lambda_{\min}(\Gamma_j^{-1})},$$
$$\sum_{i=1}^n \tilde{b}_i^2 \le 2\left(\sum_I \gamma_{bi} C_{BIi}^* + \sum_J \gamma_{bi}\rho_{bi}\right).$$

Synthesizing case (1)(2)(3), we have

$$\sum_{i=1}^n z_i^2 \le \max\Bigg\{2\sum_{i=1}^n \mu_{i1}, 2\sum_{j=1}^n c_{z_i}^2, 2\left(\sum_I C_{BIi} + \sum_J c_{z_j}^2\right)\Bigg\} \triangleq A_0, \quad (71)$$

$$\sum_{i=1}^n \|\tilde{W}_i\|^2 \le \max\Bigg\{\sum_{i=1}^n \frac{2\mu_{i1}}{\lambda_{\min}(\Gamma_i^{-1})}, \sum_{i=1}^n \frac{2\mu_{\omega 1}}{\lambda_{\min}(\Gamma_i^{-1})}, \sum_I \frac{2C_{BIi}}{\lambda_{\min}(\Gamma_i^{-1})} + \sum_J \frac{2\mu_{w_j}}{\lambda_{\min}(\Gamma_j^{-1})}\Bigg\} \triangleq A_1, \quad (72)$$

$$\sum_{i=1}^n \tilde{b}_i^2 \le \max\Bigg\{2\sum_{i=1}^n \gamma_{bi}\mu_{i1}, 2\sum_{i=1}^n \gamma_{bi}\mu_{bi}, 2\left(\sum_I \gamma_{bi} C_{BIi} + \sum_J \gamma_{bi}\mu_{bi}\right)\Bigg\} \triangleq A_2. \quad (73)$$

As $t \to \infty$, we have

$$\sum_{i=1}^n z_i^2 \le \max\Bigg\{2\sum_{i=1}^n \mu_{i1}^*, 2\sum_{j=1}^n c_{z_i}^2, 2\left(\sum_I C_{BIi}^* + \sum_J c_{z_j}^2\right)\Bigg\} \triangleq A_0^*, \quad (74)$$

$$\sum_{i=1}^{n}\|\tilde{W}_i\|^2 \leq \max\left\{\sum_{i=1}^{n}\frac{2\mu_{i1}^*}{\lambda_{\min}(\Gamma_i^{-1})}, \sum_{i=1}^{n}\frac{2\rho_{\omega_i}}{\lambda_{\min}(\Gamma_i^{-1})},\right.$$
$$\left.\sum_{I}\frac{2C_{BIi}^*}{\lambda_{\min}(\Gamma_i^{-1})}+\sum_{J}\frac{2\rho_{\omega_j}}{\lambda_{\min}(\Gamma_j^{-1})}\right\} \triangleq A_1^*, \quad (75)$$

$$\sum_{i=1}^{n}\tilde{b}_i^2 \leq \max\left\{2\sum_{i=1}^{n}\gamma_{bi}\mu_{i1}^*, 2\sum_{i=1}^{n}\gamma_{bi}\rho_{bi},\right.$$
$$\left.2\left(\sum_{I}\gamma_{bi}C_{BIi}^*+\sum_{J}\gamma_{bi}\rho_{bi}\right)\right\} \triangleq A_2^*. \quad (76)$$

From (71-73), $V_i(t), z_i, \hat{W}_i$, and $\hat{b}$ is bounded. And x_1 is bounded because y_d is bounded and $z_1 = x_1 - y_d$. For $z_2 = x_2 + \alpha_1$, α_1 and x_2 are both bounded. Same as before, it can be come true that all α_{i-1} and x_i, $i = 3, \ldots, n$ are bounded. Thence, system's states x_i, $i = 1, 2, \ldots, n$ are bounded.

Considering (57)(71-73), we have Ω_Z defined in (56) over which NN approximation is done under conditions that guarantee its feasibility.

From (74-76), as $t \to \infty$, we can conclude

$$\sum_{i=1}^{n}\|z_i\|^2 \leq A_0^*, \quad \sum_{i=1}^{n}\|\tilde{W}_i\|^2 \leq A_1^*, \quad \sum_{i=1}^{n}\tilde{b}_i^2 \leq A_2^*.$$

i.e., 2) is hold. ■

IV. SIMULATION STUDIES

Consider a second-order system rule.

$$\begin{cases}\dot{x}_1 = \varphi_1(x_1)x_2 + f_1(x_1) + \xi_1(x_1(t-\tau_1)) + \Lambda_1(x,t),\\ \dot{x}_2 = \varphi_2(x)u + f_2(x) + \xi_2(x(t-\tau_2)) + \Lambda_2(x,t),\\ y = x_1 + d(t),\end{cases}$$

where

$$\begin{aligned}\varphi_1(x_1) &= 0.6 + 0.1\sin x_1, \quad f_1(x_1) = 0.1e^{x_1},\\ \varphi_2(x) &= 4.5 + 0.4\sin(x_1x_2), \quad f_2(x) = 0.4x_1^2 + x_1x_2,\\ \xi_1(x_1) &= 0.2x_1^2\cos x_1, \quad \beta_1(x_1) = 0.2x_1^2,\\ \xi_2(x) &= 0.1x_2^2\sin x_1\cos x_2, \quad \beta_2(x) = 0.1x_2^2,\\ \Lambda_1(x,t) &= \frac{0.4\sin x_2}{x_1^2+x_1+7}, \quad \phi_1(t) = \phi_2(t) = 0,\\ \Lambda_2(x,t) &= \frac{0.3(1-e^{-x_2^2})}{1+e^{x_1^2x_2}}, \quad \tau_1 = \tau_2 = 3\,sec,\\ y_d &= 0.5(\cos(t)+\cos(0.3t)), \quad d(t) = sin(t).\end{aligned}$$

Λ_1 and Λ_2 satisfy the following inequalities

$$|\Lambda_1(x,t)| \leq p_1^*\Psi_1(x_1),$$
$$|\Lambda_2(x,t)| \leq p_2^*\Psi_2(x_2),$$

where

$$p_1^* = 0.4, \quad p_2^* = 0.3,$$
$$\Psi_1(x_1) = \frac{1}{x_1^2+x_1+7},$$
$$\Psi_2(x) = \frac{1-e^{-x_2^2}}{1+e^{x_1^2x_2}}.$$

The initial conditional laws of the previous design was chosen as:

$$\begin{aligned}\bar{\Psi}_1 &= 1 + \Psi_1,\\ \bar{\Psi}_2 &= 1 + \Psi_2 + [\frac{1}{4}(\frac{\partial\alpha_1}{\partial x_1})^2 + 1]\Psi_1,\\ \dot{\hat{b}}_i &= \gamma_{b_i}(z_i\bar{\Psi}_i(\bar{x}_i)\tanh[\frac{z_i\bar{\Psi}_i(\bar{x}_i)}{\epsilon_i}] - \sigma_{b_i}\hat{b}_i),\\ \dot{\hat{W}}_i &= \Gamma_i(S(Z_i)z_i - \sigma_{wi}\hat{W}_i);\\ \dot{\zeta}_i &= k_i(t)z_i^2 + \hat{W}_i^TS_i(Z_i)z_i\\ &\quad + \hat{b}_iz_i\bar{\Psi}_i(\bar{x}_i)\tanh[\frac{z_i\bar{\Psi}_i(\bar{x}_i)}{\epsilon_i}],\\ \alpha_1 &= q_1(z_1)N(\zeta_1)(k_1(t)z_1 + \hat{W}_1^TS(Z_1)\\ &\quad + \hat{b}_1\bar{\Psi}_1(x_1)\tanh[\frac{z_1\bar{\Psi}_1(x_1)}{\epsilon_1}]),\\ u &= q_2(z_2)N(\zeta_2)(k_2(t)z_2 + \hat{W}_2^TS(Z_2)\\ &\quad + \hat{b}_2\bar{\Psi}_2(\bar{x}_2)\tanh[\frac{z_2\bar{\Psi}_2(\bar{x}_2)}{\epsilon_2}]),\end{aligned}$$

where

$$N(\zeta_i) = e^{\zeta_i^2}\cos((\pi/2)\zeta_i), \ (i = 1, 2)$$

are Nassbaum functions,

$$Z_1 = [x_1, y_d, \dot{y}_d]^T,$$
$$Z_2 = [x_1, x_2, \alpha_1, \partial\alpha_1/\partial x_1, w_1]^T$$

and

$$k_i(t) = \frac{3}{4} + k_{i0} + k_{i1}(t)$$

with constant $k_{i0} > 0$ and $k_{i1}(t)$ being chosen as

$$k_{i1}(t) = \frac{\varepsilon_{i0}\cosh(z_i)}{2(1+z_i^2)}\int_{t-\tau_{\max}}^{t}\sum_{j=1}^{i}U_j(\bar{x}_j(\tau))d\tau, \ (i = 1, 2)$$

where

$$\begin{aligned}x_1(0) &= 0.3, \quad x_2(0) = 0,\\ b_1(0) &= b_2(0) = 0, \quad \hat{W}_1(0) = \hat{W}_2(0) = 0,\\ \Gamma_1 &= diag[1.5], \quad \Gamma_2 = diag[0.2],\\ \sigma_{\omega_1} &= 1.5, \quad \sigma_{w_2} = 0.1,\\ \sigma_{b_1} &= \sigma_{b_2} = 0.1, \quad \epsilon_1 = 0.1,\\ \epsilon_2 &= 1.2, \quad k_{10} = 1.2,\\ k_{20} &= 2.5, \quad \varepsilon_{10} = 0.1,\\ \varepsilon_{20} &= 0.5, \quad \gamma_{b_1} = \gamma_{b_2} = 0.5.\end{aligned}$$

The performance of a controller is greatly affected by the center and width of the RBF. It has been indicated [24], [28] that Gaussian RBFNNs can evenly approximate a sufficiently smooth function over a closed bounded subset. Therefore we can select the centers and widths in the following simulation studies. Specifically, $\hat{W}_1^TS(Z_1)$ contains 27 nodes (i.e. $l_1 = 27$) with centers $\eta_l(l = 1, \ldots, l_1)$ evenly spaced in $[-2.5, 2.5]\times[-3.5, 3.5]\times[-4.5, 4.5]$, and widths

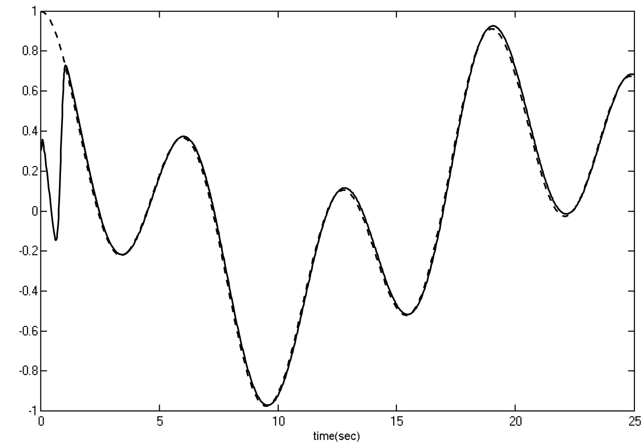

FIGURE 1. Output $y(t)$("–") and reference y_d("- -").

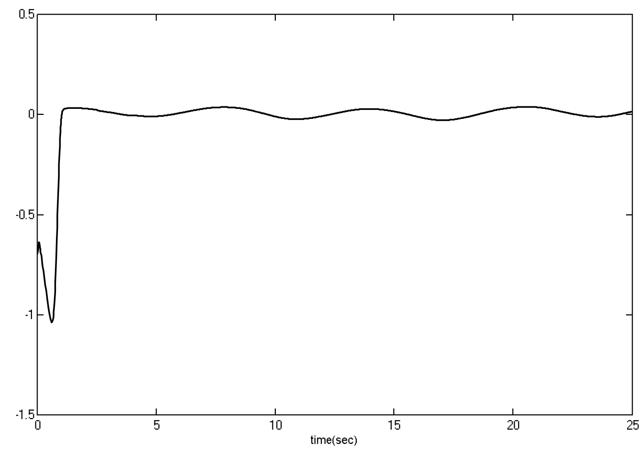

FIGURE 2. Tracking error.

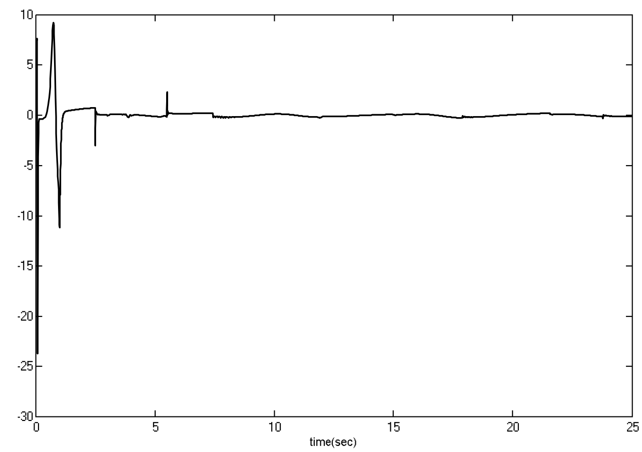

FIGURE 3. Control input u.

$\varpi_l = 0.5(l = 1, \ldots, l_1)$. $\hat{W}_2^T S(Z_2)$ contains 243 nodes (i.e. $l_2 = 243$) with centers $\eta_l(l = 1, \ldots, l_2)$ evenly spaced in $[-4, 4] \times [-4, 4] \times [-4, 4] \times [-4, 4] \times [-4, 4]$, and widths $\varpi_l = 3(l = 1, \ldots, l_1)$.

The effectiveness of design is illustrated by the Fig.1–Fig.6. Good tracking performance is shown in Fig.1 and Fig.2, it is clear that the system output signal can quickly

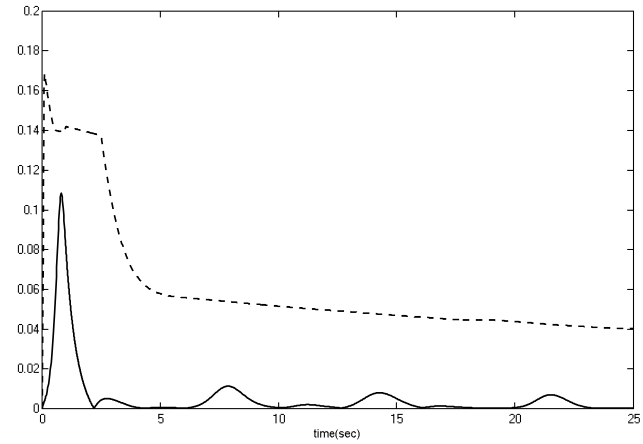

FIGURE 4. Boundedness of weights $\|\hat{W}_1\|$:"solid line." $\|\hat{W}_2\|$:"dash line".

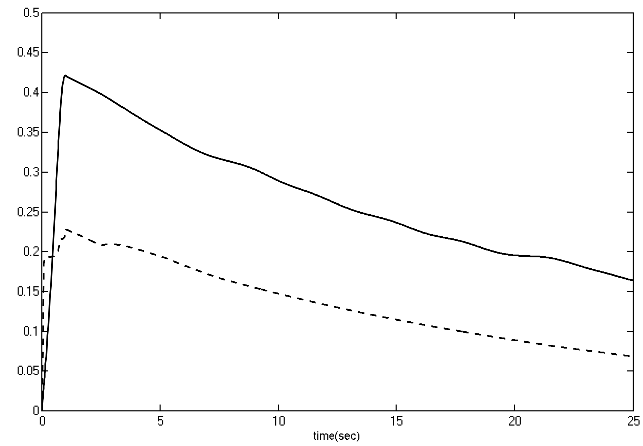

FIGURE 5. Boundedness of parameters $\|\hat{b}_1\|$:"solid line." $\|\hat{b}_2\|$:"dash line".

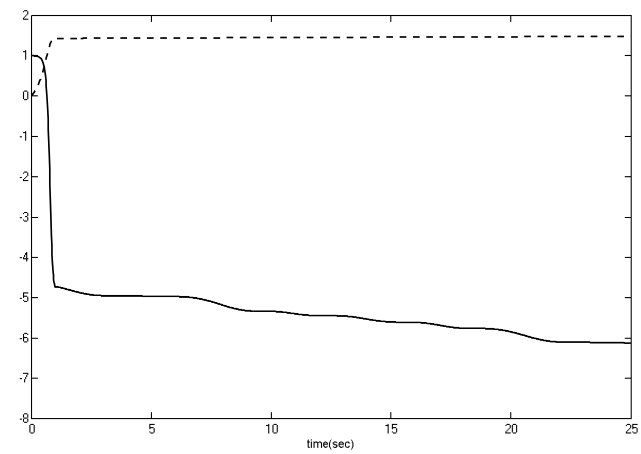

FIGURE 6. Adapting parameters ζ_1:"solid line." $N(\zeta_1)$:"dash line".

track the reference signal. These imply that a great performance of tracking can be obtained based on the designed NNs feedback control scheme. The boundedness of input is represented in Fig.3. Fig.3 shows that during the initial

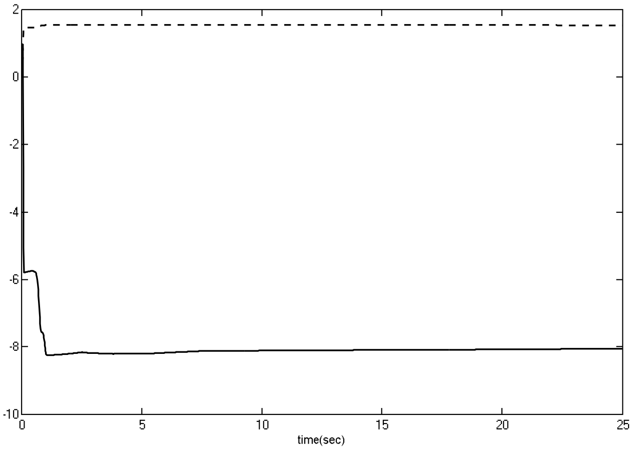

FIGURE 7. Adapting parameters ζ_2:"solid line." $N(\zeta_2)$:"dash line".

tracking of the control system, the system control generates a small amount of jitter, mainly because the system is still in the adjustment phase. When the slope of the reference signal changes greatly, especially when the slope is in the positive and negative alternation region, there will be having a large effect on the control signal of the system. However, as can be seen from Fig.3, the system can obtain excellent tracking effect just after 6 seconds, which indicates that the control method of this paper can achieve a good control effect. In Figs.4 the boundness of weights $\hat{W}_1$, $\hat{W}_2$ are shown. And $\hat{b}_1$ and $\hat{b}_2$ are illustrated in Figs.5. Fig.6 and Figs.7 show the variations of Nussbaum gain $N(\zeta_1)$, $N(\zeta_2)$ and parameters ζ_1, ζ_2 respectively, which are also bounded.

V. CONCLUSION

For the nonlinear system with strict feedback of unknown time delay and unknown output disturbances, an control method is designed to solve. In this design method, a priori knowledge of the symbols is not required to be mastered. By using Lyapunov-Krasovskii functionals, we can make up for the unknown time delays. Nussbaum function is used to handle unknown virtual control directions. Practical robust control is utilized to solve controller singularity problems. The backstepping design method can ensure SGUUB of all the signals. Furthermore, the output can converge to the attachment of the origin. The feasibility of the method is demonstrated by simulation results.

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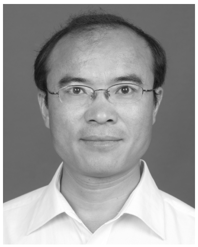

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