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Evaluation of MMSE-Based Iterative Soft Detection Schemes for Coded Massive MIMO System

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ABSTRACT There have been a number of researches on soft iterative detection and decoding scheme for coded multiple-input-multiple-output (MIMO) systems. Minimum mean square error (MMSE)-based detection schemes were often considered for massive MIMO systems, due to their reasonable complexity and performance tradeoff. This paper evaluates a number of MMSE-based soft iterative detection schemes for massive MIMO systems, with new proposals to improve the performances and to reduce the complexity. We consider joint iterative detection and decoding schemes for a coded massive MIMO system, and various kinds of MMSE-based soft detection methods proposed in the literatures are investigated. By utilizing the diagonal approximation of the channel matrix, we propose efficient soft MMSE filtering methods in combination with soft interference cancelation techniques and a compact soft bit estimation method. In addition, new matrix inversion-less soft MMSE methods are proposed for joint iterative detections. The simulation results demonstrate that the proposed methods clearly contribute to the reduction of the complexity of the conventional methods, and performance enhancement.

INDEX TERMS Coded massive MIMO, soft MMSE detection, iterative detection and decoding, symbol mapping.

I. INTRODUCTION

The massive multi-input multi-output (MIMO) technology is an important scheme to achieve high speed transmission in wireless communication systems. The most common use of massive MIMO scheme is to serve multi-users at the base station (BS). In this case, the number of receive antennas at the BS could reach up to 10 times more than the number of serving users. It was reported that simple linear detection algorithms, such as minimum mean squared error (MMSE) and zero forcing (ZF) detection algorithms, could achieve near-optimal performance in the massive MIMO system by virtue of diagonal-like channel matrix resulted from the redundancy of the antennas at the receiver [1].

The above linear detectors are usually combined with forward correction coding (FEC) schemes with sufficiently good decoding performance, which is mainly as a result of soft-input-soft-output (SISO) iterative decoder at the receiver. Therefore, researches were made on the role of linear detector to provide soft bit information (SBI) to the iterative decoder. The SBI is usually estimated in the form of log likelihood ratio (LLR) of bit value 0 and 1. For accurate SBI estimation, in these linear detection algorithms there are complicated complex-matrix operations and exhaustive search processes, resulting in high computational complexity.

Joint iterative detection and decoding (JIDD) schemes that are based on the SISO MMSE method were proposed with various loop types and with a number of performance enhancing mechanisms [2]–[9]. Even though the MMSE based detector is usually considered for JIDD due to its reasonable performance and complexity trade-off, it incurs too much computational complexity in the massive MIMO system due to the complicated complex-matrix inversion processes. For this reason, several computational complexity reduced methods were proposed by approximating the matrix inversion using iterative-based methods [10]–[14].

The idea of the complexity reduction in [10]–[14] utilized the fact that the channel matrix of the massive MIMO system is column full rank and column asymptotically orthogonal [1], and eliminated the matrix inversion processes. Furthermore, in order to combine with a SISO iterative decoder, proper methods to compute SBI values were presented as follows. Jacobi-iteration based soft-output massive MIMO detection algorithms were proposed in [10] and [11], while a Gauss-Seidel (GS)-iteration based soft-output detection algorithm and its improvements were proposed in [12]–[14]. However, to our best knowledge, there have been no attempt to apply these methods to JIDD.

In this paper, we propose a number of efficient MMSE-based detection schemes which can be applied to JIDD for coded massive MIMO systems, with reduced computational complexity and enhanced performance. We focused on the diagonal approximation of the channel matrix, and tailored it to the conventional soft MMSE detection scheme as well as to the matrix inversion-less methods. The techniques used for the conventional soft MMSE detection scheme can be summarized as follow; first, the diagonal approximation of the channel matrix to reduce the complexity; second, symbol mapping technique in the process of estimating SBI from the detected symbol value to further reduce the complexity; and an additional approximate interference cancelation (IC) process to enhance the performance. In addition to this, we propose efficient soft initial solutions for matrix inversion-less methods, which can activate JIDD and eventually enhance the performance.

The remainder of this paper is organized as follows. Section II briefly reviews the massive MIMO system model, and introduces various kinds of conventional soft-output MMSE detection algorithms. Section III presents new proposals; the first one is to improve the computational efficiency and performance of the conventional SISO MMSE detections in combination with the JIDD scheme, and the second one is a new SISO iteration-based matrix inversion-less detection methods applicable for JIDD. Pseudo code for each algorithm is presented, and the complexity comparisons are given. Section IV demonstrates the simulation results. Finally, we draw conclusions in Section V.

II. RELATED WORKS

A. SYSTEM MODEL

We consider a massive coded MIMO system with N receive antennas and M transmit antennas ($N \gg M$). At the transmitter, the bit information vector $\mathbf{u}$ is encoded to produce the codeword $\mathbf{c}$. In this paper, we assume a FEC coding scheme with a SISO iterative decoder, such as turbo codes or low density parity check (LDPC) codes [15], [16]. Then, at each time slot, KM bits of the interleaved codewords from M transmitting antennas, which are denoted as $\mathbf{x} = [x_{1,1}, \cdots, x_{1,K}, x_{2,1}, \cdots, x_{m,k}, \cdots, x_{M,K}]$, are simultaneously modulated to $M \times 1$ complex-valued symbol vector $\mathbf{s} = [s_1, s_2, \cdots, s_m, \cdots, s_M]^T$, where $x_{m,k}$ represents the

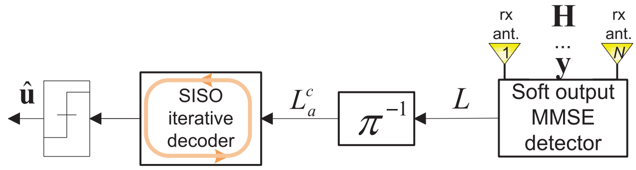

FIGURE 1. Non-iterative MMSE-based MIMO detector serially connected with iterative SISO decoder.

kth bit of the transmitted symbol from the mth transmitting antenna, s_m, which has been independently chosen from a complex constellation O of size 2^K, and the superscript T denotes the transpose of a matrix.

The modulated symbols are transmitted over a massive MIMO channel, and then the received symbol vector can be modeled as:

$$\mathbf{y} = \mathbf{Hs} + \mathbf{n}, \tag{1}$$

where $\mathbf{y} = [y_1, \cdots, y_n, \cdots, y_N]^T$ denotes the complex-valued $N \times 1$ received-symbol vector; the entries of $N \times M$ complex-valued channel matrix $\mathbf{H} = [h_{n,m}]_{N \times M}$ are independent and identically distributed with zero mean and unit variance, where $h_{n,m}$ denotes the channel-fading coefficient between the mth transmit and the nth receive antennas, which are assumed to be perfectly known; and $\mathbf{n}$ is a complex-valued $N \times 1$ vector whose elements are independent zero-mean complex Gaussian random variables with variance N_0 per dimension. The signal-to-noise ratio (SNR) is defined as ME_s/N_0, where E_s is the transmitting power per antenna.

Upon receiving $\mathbf{y}$, the job of the receiver is to find the estimation of the information, $\hat{\mathbf{u}}$, at the output of the decoder. For this, a symbol level detection should be made first, and in this paper we assume a MMSE detection scheme to find $\hat{\mathbf{s}}$. Then the detector needs to provide its soft-output, SBI to the decoder so that it can activate iterative SISO decoding process with de-interleaved version of the SBI. The detector estimates SBI with non-iterative or iterative manner as explained in the following sections B or C, respectively.

B. NON-ITERATIVE SOFT-OUTPUT MMSE DETECTION

Figure 1 shows a block diagram of the receiver for an $N \times M$ MIMO system, where a soft-output MMSE detector is directly connected to the iterative SISO decoder with a de-interleaver π^{-1}. We classify the soft-output MMSE detectors reported in the literature into two main streams as follows.

1) CLASSICAL SOFT-OUTPUT MMSE DETECTION

The classical MMSE detector estimates the transmitted symbol vector, $\hat{\mathbf{s}}$ by using the following formula:

$$\hat{\mathbf{s}} = \mathbf{W}^{-1}\mathbf{y}^{MF}, \tag{2}$$

where $\mathbf{y}^{MF} = \mathbf{H}^H\mathbf{y}$ is the matched-filter output and the superscript H denotes the conjugate transpose of a matrix, and $\mathbf{W}^{-1}$ is the MMSE filtering matrix, represented as:

$$\mathbf{W}^{-1} = \left(\mathbf{G} + \sigma^2\mathbf{I}_M\right)^{-1}, \tag{3}$$

where $\mathbf{G} = \mathbf{H}^H\mathbf{H}$ is the Gram matrix.

With the estimated symbol in (2), SBI in the form of LLR for the kth coded bit of the ith symbol can be extracted as:

$$L(x_{i,k}|\mathbf{y},\mathbf{H}) = \gamma_i \left(\min_{a \in O_k^0} \left| \frac{\hat{s}_i}{\mu_i} - a \right|^2 - \min_{a \in O_k^1} \left| \frac{\hat{s}_i}{\mu_i} - a \right|^2 \right), \quad (4)$$

where $\gamma_i = \mu_i^2/v_i^2$ is the post-equalization signal-to-interference-plus-noise ratio (PE-SINR) for the ith transmitted symbol, and μ_i and v_i^2 can be represented by [11]:

$$\mu_i = \mathbf{e}_i^H \mathbf{W}^{-1} \mathbf{G} \mathbf{e}_i, \quad (5)$$

$$v_i^2 = \mathbf{e}_i^H \mathbf{W}^{-1} \mathbf{G} \mathbf{G} \mathbf{W}^{-1} \mathbf{e}_i + \sigma^2 \mathbf{e}_i^H \mathbf{W}^{-1} \mathbf{G} \mathbf{W}^{-1} \mathbf{e}_i - \mu_i^2, \quad (6)$$

where $\mathbf{e}_i$ is the ith column vector of the $M \times M$ identity matrix $\mathbf{I}_M$. The above shows that the PE-SINR is a layer dependent value, and thus they should be individually calculated. In addition, $a \in O_k^0$ and $a \in O_k^1$ denote constellation-symbol sets with the kth bit of 0 and 1, and the search process to find the solution of min$(\cdot)$ needs complexity of $O(2^K)$ for each layer.

2) MATRIX INVERSION-LESS SOFT-OUTPUT MMSE DETECTION

The major computational complexity problem for the MMSE detection lies in matrix multiplication and inversion processes, and it may incur prohibitive computations in massive MIMO systems. Several computational complexity reduction methods were proposed by eliminating the inversion of the MMSE filtering matrix in (2), from the fact that $\mathbf{W}$ is Hermitian positive definite and diagonal dominant. These methods approximate $\mathbf{W}^{-1}$ with iterative approaches, by interpreting (2) as a problem of finding the solution of the following linear system.

$$\mathbf{W}\hat{\mathbf{s}} = \mathbf{y}^{MF}. \quad (7)$$

For example, with the Jacobi based iterative method, the solution of (7) in the αth Jacobi iteration can be represented as [18]:

$$\hat{s}_i^{\alpha} = \frac{1}{w_{i,i}} \left(y_i^{MF} - \sum_{t \neq i} w_{i,t} \hat{s}_t^{\alpha-1} \right), \quad (8)$$

where $\hat{s}_i^{\alpha}$, $\hat{s}_i^{\alpha-1}$ and y_i^{MF} denote the ith element of $\hat{\mathbf{s}}^{\alpha}$, $\hat{\mathbf{s}}^{\alpha-1}$ and $\mathbf{y}^{MF}$, respectively, and $w_{i,t}$ denotes the element of $\mathbf{W}$ in the ith row and tth column. For the above Jacobi-iteration based approach, the initial solution $\hat{\mathbf{s}}^0$ is usually set to zero vector without loss of generality [17]. To accelerate the convergence rate and reduce the complexity, a number of attempts of using proper initial solutions were proposed [10], [12], [14], [19]. With the solution in (8), LLRs are calculated using (4). There were attempts to reduce the computational complexity of estimating the PE-SINR [10], [11].

C. ITERATIVE SOFT OUTPUT MMSE DETECTION

Figure 2 shows the block diagram of the JIDD receiver for an $N \times M$ MIMO system, where a SISO MMSE detector exchanges soft information with the SISO decoder.

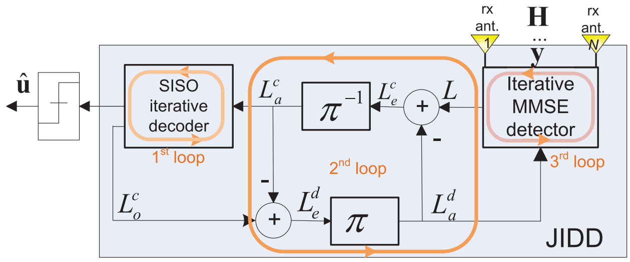

FIGURE 2. Iterative MMSE-based MIMO detector with multiple loops.

Referring to Fig. 2, the SBI from the MMSE detector, L is first calculated and its value is used to subtract the *a priori* information that is produced from the channel decoder, L_a^d, for the estimation of the extrinsic information to the channel decoder, i.e., $L_e^c = L - L_a^d$; then, the extrinsic information, L_e^c is passed through the de-interleaver, and its de-interleaved version L_a^c is used as the *a priori* information by the channel decoder. The channel decoder then estimates the information sequence and generates its soft-output L_o^c. Subsequently, the extrinsic information to the MMSE detector is estimated by $L_e^d = L_o^c - L_a^c$, and its interleaved version, L_a^d, is fed back to the MMSE detector as the *a priori* information.

In addition, the MMSE detector iteratively estimates the SBI inside the detection process as shown in Fig. 2. The self-iteration inside the MMSE detection can be performed by re-feeding L back into the input of the detector, which is usually double iteration purpose [6]. This way, we can activate three loops in JIDD; the first one inside the iterative SISO decoder, the second one between the decoder and the MMSE detector, and the third one inside the MMSE detector.

1) SISO MMSE-PIC DETECTION

During the SISO MMSE detection process, interference cancellation process can be performed in parallel leading to MMSE-parallel interference cancellation (MMSE-PIC) detector [9]. The SISO MMSE-PIC detector performs as follows. First, using the *a priori* information from the decoder, the expected mean $\tilde{s}_i$ and the variance E_i of the transmitted symbol s_i are calculated as follows [2]:

$$\tilde{s}_i = \sum_{a \in O} \frac{a}{2^K} \prod_{k=1}^{K} \left(1 + \tilde{x}_{i,k} \zeta_{i,k}\right), \quad (9)$$

where a is a constellation symbol from O, and $\tilde{x}_{i,k}$ is set to be -1 and 1 according to the kth bits of a that are 0 and 1, respectively. $\zeta_{i,k}$ is the *a priori* information that is either from the decoder or from the third loop at the SISO MMSE-PIC detector, and it is expressed as follows:

$$\zeta_{i,k} = \tanh\left(\frac{L_a^d(x_{i,k}) + L(x_{i,k}|\mathbf{y},\mathbf{H})}{2} \right), \quad (10)$$

where $L_a^d(x_{i,k})$ is the *a priori* information fed back from the decoder. At the initial iteration, $L_a^d(x_{i,k})$ and $L(x_{i,k}|\mathbf{y},\mathbf{H})$ is set to 0 because they are not available. After the first iteration of the third loop, $L(x_{i,k}|\mathbf{y},\mathbf{H})$ is produced, then updated at the next iteration. At the same time, the following equation applies:

$$E_i = \sum_{a\in O} \frac{|a|^2}{2^K} \prod_{k=1}^{K} \left(1+\widetilde{x}_{i,k}\zeta_{i,k}\right) - |\widetilde{s}_i|^2. \tag{11}$$

Then, using the following equation, the PIC process is performed on the received symbol vector $\mathbf{y}$ with the aid of $\widetilde{s}_i$:

$$\hat{\mathbf{y}}_i = \mathbf{y} - \sum_{j\neq i} \mathbf{h}_j\widetilde{s}_j = \mathbf{h}_i s_i + \widetilde{\mathbf{n}}, \tag{12}$$

where $\hat{\mathbf{y}}_i$ is the interference-canceled symbol vector for the ith layer, $\mathbf{h}_i$ is the ith column of the channel matrix $\mathbf{H}$, and $\widetilde{\mathbf{n}}$ denotes the residual noise plus interference (NPI) term that is expressed by $\widetilde{\mathbf{n}} = \sum_{j\neq i}\mathbf{h}_j(s_i - \widetilde{s}_i) + \mathbf{n}$.

While the PIC process is performed, the MMSE filtering matrix for SISO detection is simultaneously calculated as follows [3]:

$$\mathbf{W}^H = \left(\mathbf{H}^H\mathbf{H}\mathbf{\Lambda} + \sigma^2\mathbf{I}_M\right)^{-1}\mathbf{H}^H = \tilde{\mathbf{W}}^{-1}\mathbf{H}^H \tag{13}$$

where $\mathbf{\Lambda}$ is a diagonal matrix with its ith diagonal element $\Lambda_{i,i} = E_i$ that is estimated using (11), and $\tilde{\mathbf{W}} = \mathbf{G}\mathbf{\Lambda} + \sigma^2\mathbf{I}_M$. Using $\mathbf{W}^H$ in (13), the third step is the suppression of the NPI term in (12), and the filtered result for the ith layer will be as follows:

$$\hat{s}_i = \mathbf{w}_i^H\hat{\mathbf{y}}_i, \tag{14}$$

where $\mathbf{w}_i^H$ denotes the ith row of $\mathbf{W}^H$.

The last step is the calculation of the LLR for $x_{i,k}$, of which the channel-compensated value $z_i = \hat{s}_i/\mu_i$, $\mu_i = \mathbf{w}_i^H\mathbf{h}_i$ is used so that the SBI estimation is not subject to the channel gain. The *a priori* information from the decoder L_a^d can be additionally applied, and the SBI for $x_{i,k}$ can then be expressed by [20]:

$$\begin{aligned} L(x_{i,k}|\mathbf{y},\mathbf{H}) &\approx L(x_{i,k}|\hat{s}_i) \\ &\approx \min_{a\in O_k^0}\left\{\gamma_i|z_i-a|^2 + \sum_{k=1}^{K}\ln\left(1+e^{(-x_{i,k}L_a^d(x_{i,k}))}\right)\right\} \\ &\quad - \min_{a\in O_k^1}\left\{\gamma_i|z_i-a|^2 + \sum_{k=1}^{K}\ln\left(1+e^{(-x_{i,k}L_a^d(x_{i,k}))}\right)\right\}, \end{aligned} \tag{15}$$

where

$$\gamma_i = \frac{\mu_i}{1-E_i\mu_i}. \tag{16}$$

The final L value in (15) can be directly sent to the decoder for its iterative decoding process in the first loop, or it can be used to estimate $\zeta_{i,k}$ for the SISO MMSE-PIC detection in the third loop. We summarize the conventional SISO MMSE-PIC algorithm in Fig. 3. In all of the following algorithms shown in this paper, η and ℓ are the indices for the second and third loops in the JIDD scheme in Fig. 2.

```
Initialize: L_a^d = 0, L = 0
G = H^H H
for(1 ≤ η ≤ η_max)
  for(1 ≤ ℓ ≤ ℓ_max)        ◇ Iterative detection
    Calculate ζ_{i,k}, s̃_i, E_i with (10), (9), and (11)
    ŷ_i = y − Σ_{j≠i} h_j s̃_j
    W^H = (GΛ + σ^2 I_M)^{-1} H^H
    ŝ_i = w_i^H ŷ_i
    μ_i = w_i^H h_i, and γ_i = μ_i / (1 − E_i μ_i)
    z_i = ŝ_i / μ_i, L(x_{i,k}|y, H) in (15)
  end for
  SISO iterative decoding, output: L_a^d
  L = 0
end for
```

FIGURE 3. Pseudo–code for conventional MMSE-PIC detection algorithm.

2) MMSE-EI DETECTION

A concise representation is proposed for iterative soft-output MMSE detection algorithm by using extrinsic information (EI) [7], [21], and we refer to this method as the MMSE-EI detection in this paper. The *a posteriori* mean and variance of the transmitted symbols can be represented as follows:

$$\mathbf{\Lambda}^p = \left(\mathbf{\Lambda}^{-1} + \frac{1}{\sigma^2}\mathbf{H}^H\mathbf{H}\right)^{-1}, \tag{17}$$

$$\mathbf{s}^p = \widetilde{\mathbf{s}} + \frac{1}{\sigma^2}\mathbf{\Lambda}^p\left(\mathbf{H}^H\mathbf{y} - \mathbf{H}^H\mathbf{H}\widetilde{\mathbf{s}}\right). \tag{18}$$

Then, the i-th extrinsic mean s_i^e and variance $\Lambda_{i,i}^e$ of the transmitted symbol vector can be calculated by, respectively:

$$s_i^e = \Lambda_{i,i}^e\left(\frac{s_i^p}{\Lambda_{i,i}^p} - \frac{\widetilde{s}_i}{\Lambda_{i,i}}\right), \tag{19}$$

$$\Lambda_{i,i}^e = \left(\frac{1}{\Lambda_{i,i}^p} - \frac{1}{\Lambda_{i,i}}\right)^{-1}. \tag{20}$$

With these extrinsic mean and variance values, LLR can be directly calculated by:

$$L(x_{i,k}|\mathbf{y},\mathbf{H}) \approx \ln\frac{\sum_{a\in O_k^1}\exp\left(-\frac{|s_i^e-a|^2}{\Lambda_{i,i}^e}\right)\prod_{k=1}^{K}\left(1+e^{(-x_{i,k}L_a^d(x_{i,k}))}\right)}{\sum_{a\in O_k^0}\exp\left(-\frac{|s_i^e-a|^2}{\Lambda_{i,i}^e}\right)\prod_{k=1}^{K}\left(1+e^{(-x_{i,k}L_a^d(x_{i,k}))}\right)}. \tag{21}$$

We summarize the MMSE-EI detection algorithm in Fig. 4.

III. PROPOSED METHODS FOR JIDD

A. COMPLEXITY REDUCTION AND PERFORMANCE ENHANCEMENT OF MMSE-PIC DETECTIONS FOR JIDD

In this subsection, we first propose to simplify the conventional MMSE-PIC with two complexity reduced

```
Initialize: L_a^d = 0, L = 0
G = H^H H, y^MF = H^H y
for(1 ≤ η ≤ η_max)
    Calculate ζ_{i,k}, s̃_i, E_i with (10), (9), and (11)
    Λ^p = (Λ^{-1} + (1/σ^2) G)^{-1}
    s^p = s̃ + (1/σ^2) Λ^p (y^MF − G s̃)
    Λ^e_{i,i} = (1/Λ^p_{i,i} − 1/Λ_{i,i})^{-1}
    s^e_i = Λ^e_{i,i} (s^p_i/Λ^p_{i,i} − s̃_i/Λ_{i,i})
    L(x_{i,k}|y, H) with (21)
    SISO iterative decoding, output: L_a^d
    L = 0
end for
```

FIGURE 4. **Pseudo–code for the MMSE-EI detection algorithm.**

```
Initialize: L_a^d = 0, L = 0
G = H^H H, y^MF = H^H y
for(1 ≤ η ≤ η_max)
  for(1 ≤ ℓ ≤ ℓ_max)        ◇ Iterative detection
    Calculate ζ_{i,k}, s̃_i, E_i with (10), (9), and (11)
    w̃_{i,i} = g_{i,i} Λ_{i,i} + σ^2
    ŝ_i = (y_i^MF − Σ_{j≠i} g_{i,j} s̃_j) / w̃_{i,i}
    μ_i = g_{i,i}/w̃_{i,i} , and γ_i = μ_i/(1−E_i μ_i)
    z_i = ŝ_i/μ_i, L(x_{i,k}|y, H) in (25)
  end for
  SISO iterative decoding, output: L_a^d
  L = 0
end for
```

FIGURE 5. **Pseudo–code for the modified MMSE-PIC algorithm.**

techniques and then to enhance the performance of the MMSE-EI with the utilization of an additional approximated IC, which can be applied to JIDD for coded massive MIMO systems, with reduced computational complexity and enhanced performance.

1) MODIFIED MMSE-PIC

For the efficient application of the SISO MMSE-PIC, we apply an efficient modification to reduce the computational complexity. Utilizing the property of $\mathbf{G}$ that is almost diagonal-like matrix for a massive MIMO system, $\mathbf{W}^H$ in (13) can be approximated as follows:

$$\mathbf{W}^H = \left(\mathbf{H}^H\mathbf{H}\mathbf{\Lambda} + \sigma^2\mathbf{I}_M\right)^{-1}\mathbf{H}^H \approx \begin{bmatrix} \tilde{w}_{1,1} & 0 & \cdots & 0 \\ 0 & \tilde{w}_{2,2} & \cdots & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & \tilde{w}_{M,M} \end{bmatrix}^{-1} \mathbf{H}^H, \quad (22)$$

where $\tilde{w}_{i,i}$ denotes the ith diagonal element of $\tilde{\mathbf{W}}$ used in (13). Then, $\hat{s}_i$ in (14) can be approximated as follows:

$$\hat{s}_i = \mathbf{w}_i^H\hat{\mathbf{y}}_i \approx \frac{1}{\tilde{w}_{i,i}}\mathbf{h}_i^H(\mathbf{y} - \sum_{j\neq i}\mathbf{h}_j\tilde{s}_j) \approx \frac{1}{\tilde{w}_{i,i}}\left(y_i^{MF} - \sum_{j\neq i} g_{i,j}\tilde{s}_j\right), \quad (23)$$

where $\mathbf{h}_i^H$ is the ith row of $\mathbf{H}^H$. Thus, the matrix inversion and multiplications in (13) do not need anymore, and instead we only need scalar multiplications as in (23) and a simple estimation of $\tilde{w}_{i,i} = g_{i,i}\Lambda_{i,i} + \sigma^2$.

Afterwards, SBI estimation can be made by using just one distance calculation per bit, if we use symbol-mapping-based technique [22]. Referring to (4), estimation of LLR was done so as to find a symbol, a with the minimum distance from the channel-compensated value $z_i = \hat{s}_i/\mu_i$ for the kth bit. In addition, we derive a complexity reduced estimation of μ_i by approximating $\tilde{\mathbf{W}}$ with its diagonal elements as follows:

$$\mu_i = \mathbf{w}_i^H\mathbf{h}_i \approx \left[0, 0, \cdots, \frac{1}{\tilde{w}_{i,i}}, \cdots, 0\right]\mathbf{H}^H\mathbf{h}_i \approx \frac{g_{i,i}}{\tilde{w}_{i,i}}. \quad (24)$$

Instead of finding the minima, we map z_i to a target unit range where there is only one constellation symbol to estimate the distance from z_i. If we apply this concept, (4) can be written as:

$$L(x_{i,k}|\mathbf{y}, \mathbf{H}) = \eta'_{i,k}\gamma_i\left(\left|F(z_i, \varepsilon_k^0) - q_k^0\right|^2 - \left|F(z_i, \varepsilon_k^1) - q_k^1\right|^2\right), \quad (25)$$

where $\eta'_{i,k}$ is the sign change due to the symbol mapping process, for the kth bit of the ith symbol; ε_k^b, $b \in \{0, 1\}$ is the phase of the mapped symbol used for the kth bit of b, i.e., $F(z_i, \varepsilon_k^b) = |z_i|e^{j\varepsilon_k^b}$; and q_k^b are the unique symbols nearest to the mapped version of z_i, $F(z_i, \varepsilon_k^b)$. Carrying out (25) to estimate SBI only requires the mapping process and almost one distance estimation per bit. We summarize the modified MMSE-PIC algorithm as shown in Fig. 5.

2) MODIFIED MMSE-EI WITH PIC

In this section, we propose a modified scheme for the MMSE-EI scheme described in Section II.C.2). First, to reduce the computational complexity and memory requirement for (18) and (17), we approximate them using the property of the Gram matrix for the massive MIMO system, as follows:

$$\Lambda_{i,i}^p = \mathbf{e}_i^H\mathbf{\Lambda}^p\mathbf{e}_i = \mathbf{e}_i^H\left(\mathbf{\Lambda}^{-1} + \frac{1}{\sigma^2}\mathbf{H}^H\mathbf{H}\right)^{-1}\mathbf{e}_i \approx \left(\frac{1}{\Lambda_{i,i}} + \frac{g_{i,i}}{\sigma^2}\right)^{-1} \approx \frac{\Lambda_{i,i}\sigma^2}{\sigma^2 + g_{i,i}\Lambda_{i,i}}, \quad (26)$$

$$
\begin{aligned}
s_i^p &= \mathbf{e}_i^H \mathbf{s}_i^p = \mathbf{e}_i^H \left(\widetilde{\mathbf{s}} + \frac{1}{\sigma^2} \mathbf{\Lambda}^p \left(\mathbf{H}^H \mathbf{y} - \mathbf{H}^H \mathbf{H} \widetilde{\mathbf{s}} \right) \right) \\
&\approx \widetilde{s}_i + \frac{1}{\sigma^2} \begin{bmatrix} 0 \\ 0 \\ \vdots \\ \Lambda_{i,i}^p \\ \vdots \\ 0 \end{bmatrix}^T \begin{bmatrix} y_1^{MF} - \sum_j g_{1,j}\widetilde{s}_j \\ y_2^{MF} - \sum_j g_{2,j}\widetilde{s}_j \\ \vdots \\ y_i^{MF} - \sum_j g_{i,j}\widetilde{s}_j \\ \vdots \\ y_M^{MF} - \sum_j g_{M,j}\widetilde{s}_j \end{bmatrix} \\
&\approx \widetilde{s}_i + \frac{\Lambda_{i,i}^p}{\sigma^2} \left(y_i^{MF} - \sum_j g_{i,j}\widetilde{s}_j \right).
\end{aligned} \tag{27}
$$

Subsequently, the extrinsic variance $\Lambda_{i,i}^e$ in (20) can be approximated by:

$$
\Lambda_{i,i}^e \approx \frac{\Lambda_{i,i}\sigma^2}{\sigma^2 + g_{i,i}\Lambda_{i,i} - \sigma^2} \approx \frac{\sigma^2}{g_{i,i}}. \tag{28}
$$

Substituting (26) and (27) into (19), a simple expression for s_i^e can be derived to further reduce the computational complexity as follows:

$$
\begin{aligned}
s_i^e &= \Lambda_{i,i}^e \left(\frac{s_i^p}{\Lambda_{i,i}^p} - \frac{\widetilde{s}_i}{\Lambda_{i,i}} \right) = \Lambda_{i,i}^e \left(\frac{s_i^p \Lambda_{i,i} - \widetilde{s}_i \Lambda_{i,i}^p}{\Lambda_{i,i}^p \Lambda_{i,i}} \right) \\
&\approx \Lambda_{i,i}^e \left(\frac{\widetilde{s}_i}{\Lambda_{i,i}^e} + \frac{1}{\sigma^2} \left(y_i^{MF} - \sum_j g_{i,j}\widetilde{s}_j \right) \right) \\
&\approx \widetilde{s}_i + \frac{\Lambda_{i,i}^e}{\sigma^2} \left(y_i^{MF} - \sum_j g_{i,j}\widetilde{s}_j \right).
\end{aligned} \tag{29}
$$

With these approximations, we can avoid division with $\Lambda_{i,i}$, $\Lambda_{i,i}^p$, and $\Lambda_{i,i}^e$ which can be near-zero values.

Second, to improve the BER performance, we additionally apply PIC process on the received symbol vector $\mathbf{y}$ with the aid of s_i^e as follows,

$$
\hat{\mathbf{y}}_i = \mathbf{y} - \sum_{j \neq i} \mathbf{h}_j s_j^e. \tag{30}
$$

Then, the suppression of the NPI term and LLR estimation with (25) are followed consecutively. With the proof that γ_i in (16) is approximated to $1/\Lambda_{i,i}^e$ in (28) as in Appendix A, unnecessary calculations can be eliminated, resulting in a negligible computational complexity increase. We summarize the modified MMSE-EI with PIC algorithm as shown in Fig. 6.

B. MATRIX INVERSION-LESS MMSE DETECTION FOR JIDD

In this subsection, we present efficient initial solutions for the Jacobi and GS-iteration based schemes, which enables the application of matrix inversion-less (iteration based) scheme to JIDD.

Initialize: $\mathbf{L}_a^d = \mathbf{0}$, $\mathbf{L} = \mathbf{0}$
$\mathbf{G} = \mathbf{H}^H\mathbf{H}$, $\mathbf{y}^{MF} = \mathbf{H}^H\mathbf{y}$, $\gamma_i = \frac{1}{\Lambda_{i,i}^e} \approx \frac{g_{i,i}}{\sigma^2}$
for$(1 \le \eta \le \eta_{max})$
  for$(1 \le \ell \le \ell_{max})$ $\diamondsuit$ **Iterative detection**
    Calculate $\zeta_{i,k}$, $\widetilde{s}_i$, E_i with (10), (9), and (11)
    $\widetilde{w}_{i,i} = g_{i,i}\Lambda_{i,i} + \sigma^2$, $\mu_i = g_{i,i}/\widetilde{w}_{i,i}$
    $s_i^e \approx \widetilde{s}_i + \frac{\Lambda_{i,i}^e}{\sigma^2}\left(y_i^{MF} - \sum_j g_{i,j}\widetilde{s}_j\right)$
    $\hat{s}_i = \left(y_i^{MF} - \sum_{j \neq i} g_{i,j}s_j^e\right)/\widetilde{w}_{i,i}$
    $z_i = \hat{s}_i/\mu_i$, $L(x_{i,k}|\mathbf{y}, \mathbf{H})$ with (25)
  end for
  SISO iterative decoding, output: $\mathbf{L}_a^d$
  $\mathbf{L} = \mathbf{0}$
end for

FIGURE 6. **Pseudo–code for the modified MMSE-EI with PIC algorithm.**

1) JACOBI-ITERATION BASED DETECTION

In the proposed scheme, the expected mean $\widetilde{\mathbf{s}}$ or the extrinsic mean $\mathbf{s}^e$ is used for the initial solution $\hat{\mathbf{s}}^0$ of the Jacobi-iteration based detection. Our investigation showed that the latter outperforms the former, and this will be demonstrated by the simulation results in Section IV. The computation of (8) in each Jacobi iteration is almost equivalent to the PIC process with the suppression of the NPI term. This means that the Jacobi-iteration based detection with α of 1 will behave almost the same way as the MMSE-EI PIC detection, if we set $\hat{\mathbf{s}}^0 = \mathbf{s}^e$. Using this concept, the solution of the Jacobi-iteration based detection for JIDD, with $\alpha = 1$ and $\hat{\mathbf{s}}^0 = \mathbf{s}^e$, can be represented as

$$
\hat{s}_i = \frac{\Lambda_{i,i}^e}{\sigma^2} \left(y_i^{MF} - \sum_{j \neq i} g_{i,j} s_j^e \right). \tag{31}
$$

Afterwards, the LLR is estimated by (25) using the symbol mapping technique with $z_i = \hat{s}_i$. We summarize the proposed Jacobi-iteration based algorithm for JIDD with the initial solution as $\mathbf{s}^e$, as shown in Fig. 7.

2) GS-ITERATION BASED DETECTION

Assuming the sequential symbol detection process from the first layer, for detection of s_i, all the previously detected values of $\hat{s}_t^{\alpha}$, $t < i$ can be additionally used as the *a posteriori* information. This is a similar concept of using the second iteration of the third loop in JIDD scheme [8]. The GS-iteration based scheme can be understood as the one which additionally applies extrinsic information to the Jacobi-iteration based detection. We also note that the PE-SINR of the conventional MMSE detector can be approximated to that of the modified MMSE-PIC detector as proved in Appendix B. Using the above concept, the solution of the GS based detection for JIDD can be found by setting $\hat{\mathbf{s}}^0 = \widetilde{\mathbf{s}}$, and represented as:

$$
\hat{s}_i = \frac{1}{w_{i,i}} \left(y_i^{MF} - \sum_{t<i} w_{i,t}\hat{s}_t - \sum_{t>i} w_{i,t}\widetilde{s}_t \right). \tag{32}
$$

Initialize: $\mathbf{L}_a^d = \mathbf{0}$, $\mathbf{L} = \mathbf{0}$
$\mathbf{G} = \mathbf{H}^H\mathbf{H}$, $\mathbf{y}^{MF} = \mathbf{H}^H\mathbf{y}$, $\gamma_i = \frac{1}{\Lambda_{i,i}^e} \approx \frac{g_{i,i}}{\sigma^2}$
for$(1 \leq \eta \leq \eta_{max})$
  for$(1 \leq \ell \leq \ell_{max})$ ◇ **Iterative detection**
    Calculate $\zeta_{i,k}$, $\widetilde{s}_i$, E_i with (10), (9), and (11)
    $\hat{s}_i^0 = \widetilde{s}_i + \frac{\Lambda_{i,i}^e}{\sigma^2}\left(y_i^{MF} - \sum_j g_{i,j}\widetilde{s}_j\right)$
    $\hat{s}_i = \frac{\Lambda_{i,i}^e}{\sigma^2}\left(y_i^{MF} - \sum_{j\neq i} g_{i,j}s_j^e\right)$
    end for
    $L(x_{i,k}|\mathbf{y},\mathbf{H})$ with (25), where $z_i = \hat{s}_i$
  end for
  SISO iterative decoding, output: $\mathbf{L}_a^d$
  $\mathbf{L} = \mathbf{0}$
end for

FIGURE 7. Pseudo–code for the proposed Jacobi-iteration based algorithm for JIDD.

Initialize: $\mathbf{L}_a^d = \mathbf{0}$, $\mathbf{L} = \mathbf{0}$
$\mathbf{G} = \mathbf{H}^H\mathbf{H}$, $\mathbf{y}^{MF} = \mathbf{H}^H\mathbf{y}$, $\gamma_i = \frac{g_{i,i}}{\sigma^2}$
for$(1 \leq \eta \leq \eta_{max})$
  for$(1 \leq \ell \leq \ell_{max})$ ◇ **Iterative detection**
    Calculate $\zeta_{i,k}$, $\widetilde{s}_i$, E_i with (10), (9), and (11)
    $w_{i,i} = g_{i,i} + \sigma^2$, $\mu_i = g_{i,i}/w_{i,i}$
    $\hat{s}_i = \frac{1}{w_{i,i}}\left(y_i^{MF} - \sum_{t<i} w_{i,t}\hat{s}_t - \sum_{t>i} w_{i,t}\widetilde{s}_t\right)$
    $L(x_{i,k}|\mathbf{y},\mathbf{H})$ with (25), where $z_i = \hat{s}_i/\mu_i$
  end for
  SISO iterative decoding, output: $\mathbf{L}_a^d$
  $\mathbf{L} = \mathbf{0}$
end for

FIGURE 8. Pseudo–code for the proposed GS-iteration based algorithm for JIDD.

Afterwards, the LLR is estimated by (25) with $z_i = \hat{s}_i/\mu_i$. We summarize the proposed GS-iteration based algorithm for JIDD as shown in Fig. 8.

C. COMPLEXITY COMPARISON

In this paper, we focus on the number of complex-value multiplications in the detection process with known Gram matrix $\mathbf{G}$ and $\mathbf{y}^{MF}$, of which computational complexities are $O(NM^2)$ and $O(NM)$, respectively. However, we do not consider the computations required for the calculation of $\zeta_{i,k}$, $\widetilde{s}_i$, and E_i with (9) to (11), because they are common to all the schemes considered in the simulations. For a matrix with a size of $M \times M$ and a vector with a size of M, it is generally known that the complexity of the matrix inversion based on Cholesky factorization is $O(M^3)$; that of the diagonal matrix inversion is $O(M)$; that of the square matrix multiplication is $O(M^3)$; that of the matrix multiplication with a diagonal matrix is $O(M^2)$; that of the matrix multiplication with a vector is $O(M^2)$; that of the vector multiplication with a vector/scalar is $O(M)$; that of the scalar multiplication and division are $O(1)$.

TABLE 1. Complexity comparisons of various detection algorithms.

Section	Algorithm	Number of multiplications
II.B.1)	MMSE	$M^3 + 2M^2 + 2M$
II.B.2)	Jacobi	$(\alpha + 1)(M^2 + M) + M$
II.C.1)	SISO MMSE-PIC	$\eta\ell(2M^2N + M^3 + 2MN + M^2 + 3M)$
II.C.2)	MMSE-EI	$\eta(M^3 + 4M^2 + 7M)$
III.A.1)	Modified MMSE-PIC	$\eta\ell(M^2 + 6M)$
III.A.2)	MMSE-EI PIC	$\eta\ell(2M^2 + 5M) + 2M$
III.B.1)	Jacobi JIDD	$\eta\ell(2M^2 + 2M) + 2M$
III.B.2)	GS JIDD	$\eta\ell(M^2 + 3M) + M$

Using these facts, Table 1 compares the complexity of various detection schemes presented in this paper. Taking the complexity of the MMSE detection method in Section II.B.1) for example, $M^3 + M^2 + 2M$, is obtained by the summation of $O(M^3)$ for the matrix inversion of (3) based on the Cholesky factorization, $O(M^2)$ for the matrix multiplication with a vector as in (2), $O(M^2)$ for M times of a vector multiplication with a vector as in (5), and $O(2M)$ for the PE-SINR estimation of $\gamma_i = \mu_i^2/v_i^2 = \mu_i/(1-\mu_i)$. As shown in the Table the complexity of the proposed schemes are reduced from $O(M^3)$ to $O(M^2)$. In the next section, we demonstrate more specific values by using a MIMO system used in the performance simulations.

IV. SIMULATION RESULTS

The bit error rate (BER) performances of the proposed methods are compared to those of the conventional schemes using simulation results for a MIMO system with 16 transmit antennas and 128 receive antennas. A 16-quadrature amplitude modulation (QAM) scheme was used and the modulated signals were transmitted over a Rayleigh faded MIMO channel, where an independent fading coefficient is applied to each modulated symbol. As an FEC scheme the LDPC code with a length of 16200 bits and a code rate of 1/2 was used. At the SISO iterative decoder, the min-sum product decoding algorithm with a correction factor was used [16], and the maximum number of iteration was limited to 10.

The performance comparisons between the classic non-iterative MMSE, the conventional SISO MMSE-PIC, and the conventional MMSE-EI are shown in Fig. 9. In the legend, (η, ℓ) denotes (the number of joint iterations, the number of detector iterations), i.e., η and ℓ are the numbers of the iterations in the second and third loops in Fig. 2, respectively. It is shown that the effect of the third loop is not important for the conventional SISO MMSE-PIC scheme. The third loop played important role for the MIMO system with lower number of antennas as in [8], but it may not be necessary for the massive MIMO system. On the other hand, the third loop plays an important role for the conventional MMSE-EI scheme, especially in early iterations in the second loop. Comparing with the conventional SISO MMSE-PIC scheme, the MMSE-EI scheme shows slightly worse performance.

Figure 10 shows performance comparison between the conventional SISO MMSE-PIC and the proposed MMSE-PIC with modification for complexity reduction. The proposed scheme shows appreciable performance

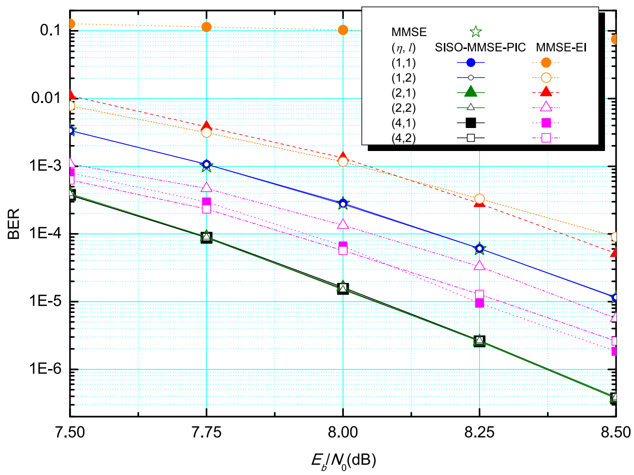

FIGURE 9. Performance comparison of the conventional MMSE detectors.

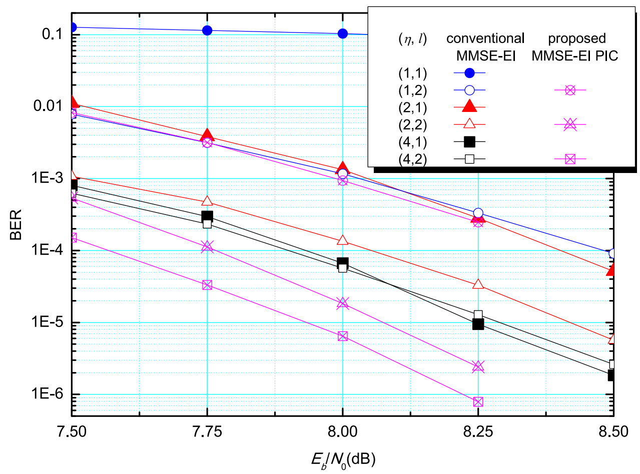

FIGURE 11. Performance comparison of the conventional MMSE-EI and modified MMSE-EI with PIC for JIDD scheme.

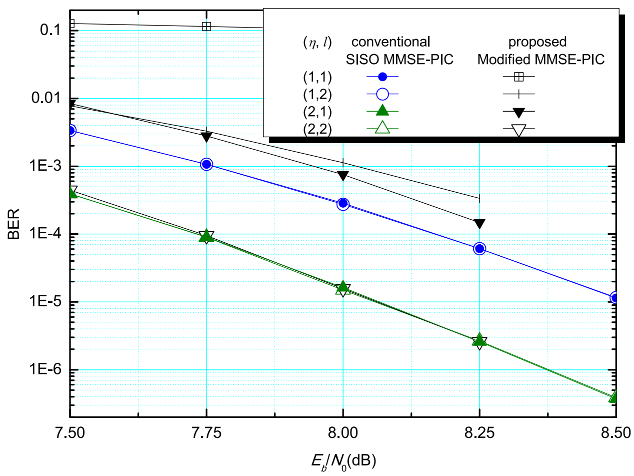

FIGURE 10. Performance comparison of the conventional SISO MMSE-PIC and modified MMSE-PIC detectors for JIDD scheme.

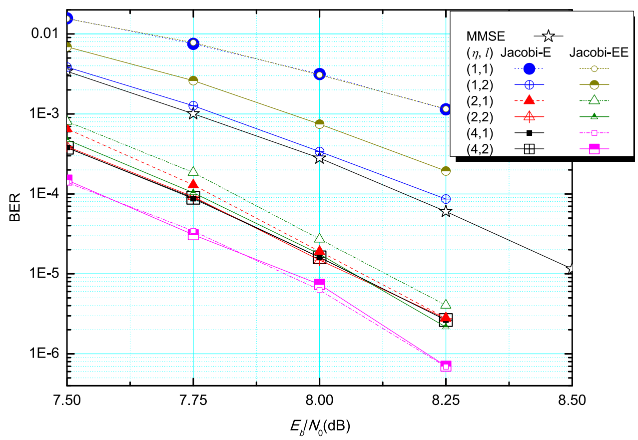

FIGURE 12. Performance comparison of the proposed Jacobi-iteration based detection for JIDD.

degradation compared to the conventional scheme with $\eta = 1$ and $\ell = 1$ due to the approximation. Nevertheless, the performance of the proposed scheme is almost the same as the conventional scheme, with $\eta = 2$ and $\ell = 2$ as the posteriori information from both of the channel decoder and the detector become available. Figure 11 shows performance comparison between the conventional MMSE-EI and the proposed MMSE-EI PIC with modification for complexity reduction and performance enhancement. The proposed scheme shows appreciable performance improvement with less number of iterations, by virtue of the additional approximate IC whose complexity is minor. That is, the complexity is reduced due to the diagonal approximation, while the performance is improved due to the added PIC process.

Figures 12 and 13 show the performance of the proposed Jacobi and GS-iteration based detections for JIDD, respectively in comparison with the classic non-iterative MMSE detector. In the legend of Fig. 12, the Jacobi-E denotes the proposed Jacobi-iteration based method with $\hat{\mathbf{s}}^0 = \tilde{\mathbf{s}}$, while the Jacobi-EE denotes the one with $\hat{\mathbf{s}}^0 = \mathbf{s}^e$. At the initial joint iteration, i.e., $\eta = 1$, the Jacobi-E achieves almost the same and better performance than that of the Jacobi-EE, when $\ell = 1$ and $\ell = 2$, respectively. As η iteration continues, the performance of the Jacobi-E is saturated, whereas the proposed Jacobi-EE keeps improving the performance. The performance improvement of the proposed Jacobi-EE compared to the Jacobi-E is resulted from the utilization of $\mathbf{s}^e$ which is an approximate IC. The proposed GS-iteration based detection for JIDD improves the performance as η iteration goes by as in Fig. 13, at the cost of the time required by the sequential detection.

The performance comparisons between our proposed methods are given in Fig. 14. Across all the methods investigated in this paper, as η increases we can hardly find performance improvement by ℓ. For this reason, we set $\ell = 1$ in Fig. 14. The performance of the Jacobi-EE with $\eta = 4$ is almost same as that of the MMSE-EI PIC detection, and it is better than that of the modified MMSE-PIC detector and slightly worse than that of the GS-iteration based

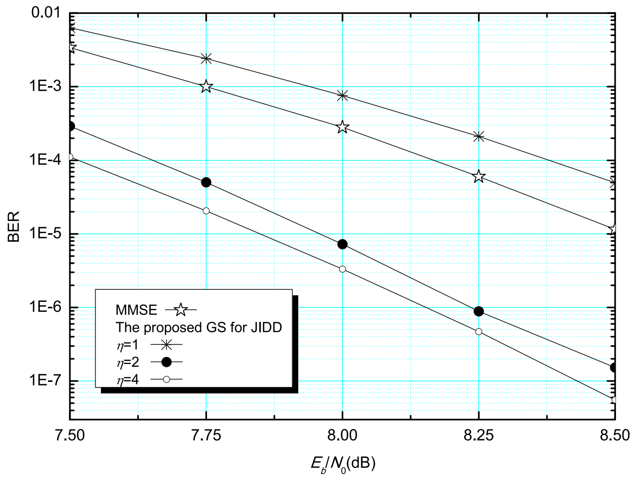

FIGURE 13. Performance comparison of the proposed GS-iteration based detection for JIDD.

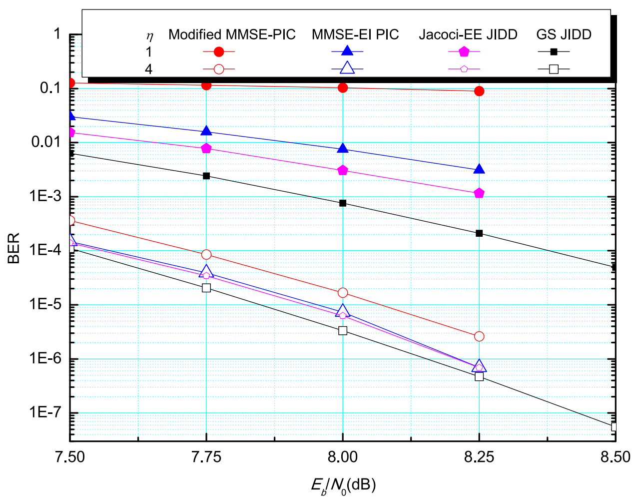

FIGURE 14. Performance comparisons of the proposed detectors for JIDD scheme 128 × 16 massive MIMO system.

detector for JIDD. The proposed GS-iteration based detector achieves the best performance among all the proposed schemes, because the posteriori information is provided not only from the channel decoder but also from the previously detected layers at the cost of the detection time.

For the schemes used in the BER performance simulations, complexity is compared using the complexity estimation in Table 1. Table 2 shows its result, with $\eta = 4$, $\ell = 1$, for the investigated MIMO system with 16 transmit and 128 receive antennas, except the complexity required for estimation of $\mathbf{G}$ and $\mathbf{y}^{MF}$ which are common to all the JIDD methods. We note that the complexity value was estimated as 34816 for the investigated MIMO system, by using total complexity of $O(NM^2)$ for $\mathbf{G}$ and $O(NM)$ for $\mathbf{y}^{MF}$. For the complexity comparison, the GS-iteration based method shows the least complexity. However, symbol detection should be performed sequentially in the proposed GS-based method, while parallel processing can be made in all the other proposed methods.

TABLE 2. Complexity comparisons of various detection algorithms, $\eta = 4$, $\ell = 1$, $\alpha = 1$.

Section	Algorithm	Number of multiplications
II.B.1)	MMSE	4640
II.B.2)	Jacobi	560
II.C.1)	SISO MMSE-PIC	296128
II.C.2)	MMSE-EI	20928
III.A.1)	Modified MMSE-PIC	1408
III.A.2)	Modified MMSE-EI PIC	2400
III.B.1)	Jacobi JIDD	2208
III.B.2)	GS JIDD	1232

V. CONCLUSION

The objective of this study was to investigate conventional soft output MMSE detection schemes for a coded massive MIMO system, and to propose a number of efficient methods to reduce the computational complexity as well as to improve the performance of the conventional one. We presented formulas to approximate MMSE filtering and IC using the characteristics of the channel matrix, with which we can achieve almost the same performance as the conventional scheme with orders of less computational complexity. As an effective means to enhance the performance, we proposed application of an additional IC to the MMSE based scheme or a proper initial solution to the matrix inversion-less method with minor complexity. All the proposed methods can be utilized with JIDD for efficient performance improvement. Simulation results demonstrated that the proposed detectors can achieve approximating or improved performance compared to the conventional schemes, with reduced complexity.

APPENDIX A

The PE-SINR of the proposed schemes can be approximated to a simple expression by substituting (24) into (16), and it gives

$$\begin{aligned}\gamma_i &= \frac{\mu_i}{1 - E_i\mu_i} \approx \frac{g_{i,i}/w_{i,i}}{1 - E_i g_{i,i}/w_{i,i}} \\ &= \frac{g_{i,i}}{w_{i,i} - \Lambda_{i,i} g_{i,i}} = \frac{g_{i,i}}{\sigma^2} = \frac{1}{\Lambda^e_{i,i}}. \qquad (33)\end{aligned}$$

APPENDIX B

The PE-SINR $\gamma_i = \frac{\mu_i^2}{v_i^2}$ of the conventional MMSE detector with (5) - (6) can be approximated to be the same as that of the proposed scheme, as follows:

$$\begin{aligned}\gamma_i &= \frac{\mu_i^2}{v_i^2} = \frac{\mu_i}{1-\mu_i} \approx \frac{1 - \sigma^2 \mathbf{e}_i^H \mathbf{D}^{-1} \mathbf{e}_i}{1 - (1 - \sigma^2 \mathbf{e}_i^H \mathbf{D}^{-1} \mathbf{e}_i)} \\ &= \frac{1 - \sigma^2/(g_{i,i} + \sigma^2)}{\sigma^2/(g_{i,i} + \sigma^2)} = \frac{g_{i,i}}{\sigma^2}. \qquad (34)\end{aligned}$$

This is true when the filtering matrix is approximated to a diagonal matrix in a massive MIMO system.

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