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Incorporating Importance Sampling in EM Learning for Sequence Detection in SPAD Underwater OWC

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ABSTRACT In this paper, an importance sampling-based expectation-maximization (EMIS) algorithm is developed for sequence detection in single-photon avalanche diode underwater optical wireless communication (OWC) systems. To be more specific, the expectation-maximization (EM) algorithm in statistic learning provides a general framework for the sequence detection, and the importance sampling (IS) method is employed for evaluating the minimum mean-square error estimates required in the EM algorithm. Theoretical analysis indicates that the developed EMIS algorithm achieves near-optimal performance with low-complexity symbol-by-symbol detection. The simulation results verify the effectiveness of the proposed EMIS algorithm.

INDEX TERMS Underwater optical wireless communication (OWC), single-photon avalanche diode (SPAD), lognormal channel, sequence detection, importance sampling (IS), expectation-maximization (EM) learning.

I. INTRODUCTION

Optical wireless communication (OWC) has been regarded as a promising technique for the future wireless transmission, due to its appealing advantages such as high data rate and large potential bandwidth [1]–[3]. In recent years, OWC has been applied in underwater environment to realize high-speed underwater transmission [4], [5]. However, optical signal in underwater OWC suffers severely from the effects of absorption, scattering and turbulence over underwater channels [6].

To realize long-distance transmission under the weak optical intensity, single photon avalanche diode (SPAD) is employed for the optical signal detection because of its high sensitivity [7]. Different from traditional photodiodes, the output of SPAD depends on the number of photons, which is typically modeled as Poisson distribution [8], [9]. As a result, conventional detection approaches for Gaussian channels cannot be directly applied in the SPAD based OWC system.

Unlike static indoor OWC channels, the underwater OWC channel is even challenging due to its time-varying and lognormally distributed fading [10], [11]. Because channel estimation may take long time and high overhead in underwater OWC, statistical channel state information (CSI) rather than perfect CSI is generally considered available at the receiver [12]. Meanwhile, the channel can be considered constant in a relatively short observation window under the signalling rates of interest. In consequence, sequence detection techniques using statistical CSI have attracted much interest in underwater OWC. In [13], the maximumlikelihood sequence detection (MLSD) method was adopted to detect symbol sequence in Poisson OWC channels. However, the MLSD method is too complicated to implement in practice, because the MLSD metric cannot be represented in closed form and the search complexity grows exponentially in sequence length. To reduce the detection complexity, the generalized-likelihood ratio test (GLRT) method was

applied in [14]. In spite of its simpler form, the GLRT method fails to fully exploit the statistical CSI and incurs a performance loss, resulting in less satisfactory channel estimation accuracy.

To address these issues in existing detection schemes, we develop an importance sampling based expectationmaximization (EMIS) algorithm for the sequence detection in SPAD underwater OWC with on-off keying (OOK). Typically, the expectation-maximization (EM) algorithm comprises an iterative learning procedure for obtaining the maximum-likelihood (ML) estimates of parameters [15], [16], and the importance sampling (IS) is adopted to effectively compute the expectation of complicate distributions [17]–[19]. The proposed EMIS algorithm can be viewed as an incorporation of the EM algorithm and IS method. Specifically, the EM algorithm provides a general framework for the sequence detection, and the IS method is used for calculating the minimum mean-square error (MMSE) estimates required in the EM algorithm.

It was previously introduced in [20] to design EM algorithm incorporating statistical sampling method in the field of machine learning. This kind of algorithms has the advantage of solving nonlinear problems effectively in linear complexity. In wireless communication, it has been popularly known that optimal sequence detection commonly suffers from exponentially growing complexity and it has attracted many attentions from researchers. To the best of our knowledge, this is the first time that the EM algorithm and the IS method are jointly adopted for sequence detection, especially in SPAD underwater OWC. Through mathematical derivations, we show that the developed EMIS algorithm has a low complexity especially from two aspects: 1) The EM algorithm simplifies sequence detection to a symbol-by-symbol detector; 2) The IS method can be readily implemented by generating samples from an appropriate distribution, which has been optimized in our study for the specific problem. Simulation results show that the performance of the proposed EMIS algorithm approaches that of the ideal receiver knowing perfect CSI with a few iterations and relatively short sequence length. The EMIS algorithm provides a good tradeoff between performance and complexity in practice.

The remainder of this paper is organized as follows. System model is described in Section II. Section III briefly illustrates existing sequence detection methods for Poisson OWC channels. The developed EMIS algorithm is elaborated in Section IV. Simulation results are given in Section V. Finally, conclusions are drawn in Section VI.

II. SYSTEM MODEL

Consider a SPAD-based underwater OWC system with a light source at the transmitter and a SPAD at the receiver. Assume OOK modulation is employed and let $s(n) \in \{0, 1\}$ denote the OOK symbol in the *n*-th slot. The SPAD detects the number of received photons. The output of SPAD, r(n), in the *n*-th slot follows the Poisson distribution with mean $\lambda(n) = \alpha_r h(n)P_t s(n) + \beta$ [7]–[9], where P_t is the transmit optical power, $h(n) \ge 0$ denotes the channel gain, α_r is the power gain of the SPAD receiver, and β represents the mean number of detected background noise photons. Then, the probability mass function of r(n) follows

$$\Pr(r(n)|s(n), h(n)) = \frac{e^{-\lambda(n)}\lambda(n)^{r(n)}}{r(n)!}, \quad r(n) \ge 0, \quad (1)$$

where $Pr(\cdot)$ is the probability of an event. For notational simplicity, we define in the following $\alpha = \alpha_r P_t$.

Under good water type and signalling rates of interest, the channel gain h(n) can be assumed constant over a relatively short observation window of *N* symbols, i.e., h(n) = hfor $n = 1, \dots, N$ [6]. Typically, the channel gain *h* is well modeled as a lognormal random variable with probability density function (PDF) [10], [11]

$$p_{h}(h) = \frac{1}{h\sqrt{2\pi\sigma_{t}^{2}}} \exp\left\{-\frac{(\ln h - m_{t})^{2}}{2\sigma_{t}^{2}}\right\},$$
 (2)

where m_t and σ_t^2 are mean and variance of $\ln h$, respectively. Without loss of generality, we assume that the mean of the channel gain is normalized, i.e., $\mathbb{E}\{h\} = 1$ with $\mathbb{E}(\cdot)$ denoting the expectation operation, which implies $\sigma_t^2 = -2m_t$.

III. EXISTING DETECTION SCHEMES

Let $\mathbf{s} = [s(1), \dots, s(N)]^T$ be the transmitted signal vector and $\mathbf{r} = [r(1), \dots, r(N)]^T$ be the received vector. With $\lambda(n) = \alpha h s(n) + \beta$ and from (1), the conditional probability of \mathbf{r} given \mathbf{s} and h is

$$\Pr(\mathbf{r}|\mathbf{s},h) = \prod_{n=1}^{N} \frac{e^{-\lambda(n)}\lambda(n)^{r(n)}}{r(n)!}.$$
(3)

The problem in this work is to efficiently detect the symbol sequence, **s**, using the received signal vector, **r**, the statistical CSI, $p_h(h)$, and the conditional probability in (3). Before presenting our proposed EMIS algorithm, we first present a review of three typical sequence detections in this section.

A. SYMBOL DETECTION WITH PERFECT CSI

When perfect CSI is assumed known at the receiver, the problem of MLSD is

$$\hat{\mathbf{s}}_{PC} = \arg \max_{\mathbf{s} \in \{0,1\}^N} \Pr(\mathbf{r}|\mathbf{s}, h)$$

=
$$\arg \max_{\mathbf{s} \in \{0,1\}^N} \prod_{n=1}^N e^{-\lambda(n)} \lambda(n)^{r(n)}, \qquad (4)$$

which can be decoupled and performed in a symbol-bysymbol manner as [21]

$$\hat{s}_{PC}(n) = \arg \max_{s(n) \in \{0, 1\}} e^{-\lambda(n)} \lambda(n)^{r(n)} = \arg \max_{s(n) \in \{0, 1\}} -\lambda(n) + r(n) \ln \lambda(n) = \begin{cases} 0, r(n) \le \tau_{PC} \\ 1, r(n) > \tau_{PC}, \end{cases}$$
(5)

where $\tau_{PC} = \frac{\alpha h}{\ln(1+\alpha h/\beta)}$ is the detection threshold. In underwater OWC systems, however, perfect CSI is generally hard to achieve. Thus, the performance with perfect CSI can be regarded as a performance lower bound and serves as a benchmark in this work.

B. MLSD DETECTION WITH STATISTICAL CSI

When only statistical information (a.k.a. prior information) of the channel, rather than perfect CSI, is available at the receiver, the MLSD metric in (4) is replaced by [13]

$$\Pr(\mathbf{r}|\mathbf{s}) = \mathbb{E}_{h}[\Pr(\mathbf{r}|\mathbf{s}, h)]$$

$$\propto \int p_{h}(h) \prod_{n=1}^{N} e^{-\lambda(n)} \lambda(n)^{r(n)} dh.$$
(6)

The MLSD method finds the symbol sequence maximizing the likelihood function $Pr(\mathbf{r}|\mathbf{s})$ in (6), i.e.,

$$\hat{\mathbf{s}}_{MLSD} = \arg \max_{\mathbf{s} \in \{0,1\}^N} \Pr(\mathbf{r}|\mathbf{s})$$
$$= \arg \max_{\mathbf{s} \in \{0,1\}^N} \int p_h(h) \prod_{n=1}^N e^{-\lambda(n)} \lambda(n)^{r(n)} dh. \quad (7)$$

It is observed from (7) that the computational complexity of the MLSD method in (7) can be very high mainly for two facts: 1) It is difficult to obtain a closed-form expression for evaluating the likelihood function $Pr(\mathbf{r}|\mathbf{s})$ in (6), especially under the lognormal fading in underwater OWC; 2) It requires exhaustive search over 2^N candidate sequences to find $\hat{\mathbf{s}}_{MLSD}$. Consequently, the MLSD method is so computationally complicate that it hardly fits in practical applications, especially for large *N*.

C. GLRT METHOD

To reduce the detection complexity, the GLRT method was adopted in [14]. It jointly estimates the channel and detect the sequence. Mathematically, the GLRT method is expressed as

$$(\hat{\mathbf{s}}_{GLRT}, \hat{h}_{GLRT}) = \arg \max_{\mathbf{s} \in \{0, 1\}^N, h} \Pr(\mathbf{r} | \mathbf{s}, h)$$
$$= \arg \max_{\mathbf{s} \in \{0, 1\}^N, h} \sum_{n=1}^N -\lambda(n) + r(n) \ln \lambda(n).$$

By substituting the derived relationship $\alpha \hat{h}_{GLRT} = \frac{\mathbf{r}^T \hat{s}_{GLRT}}{\mathbf{1}^T \hat{s}_{GLRT}} - \beta$ into (8), the decision rule of the GLRT method is obtained as

$$\hat{\mathbf{s}}_{GLRT} = \arg \max_{\mathbf{s} \in \{0,1\}^N} \sum_{n=1}^N -s(n) \left(\frac{\mathbf{r}^T \mathbf{s}}{\mathbf{1}^T \mathbf{s}} - \beta \right) + r(n) \ln \left[s(n) \left(\frac{\mathbf{r}^T \mathbf{s}}{\mathbf{1}^T \mathbf{s}} - \beta \right) + \beta \right] = \arg \max_{\mathbf{s} \in \{0,1\}^N} \sum_{\{n:s(n)=1\}} - \left(\frac{\mathbf{r}^T \mathbf{s}}{\mathbf{1}^T \mathbf{s}} - \beta \right) + r(n) \ln \frac{\mathbf{r}^T \mathbf{s}}{\mathbf{1}^T \mathbf{s}} + \sum_{\{n:s(n)=0\}} r(n) \ln \beta.$$
(9)

Despite its simpler form in (9) compared to (7), the GLRT method in (9) has the following drawbacks:

- The prior knowledge of statistics of *h* is not exploited in (8), resulting in a detection performance degradation;
- Regarding the estimation of the continuous random variable *h* conditioned on s, the MMSE estimator, rather than the ML estimator in (8), is proven optimal in terms of estimation accuracy.

IV. EMIS ALGORITHM FOR SEQUENCE DETECTION

To overcome the aforementioned problems of existing sequence detection schemes, in this section, we develop an EMIS algorithm to detect the symbol sequence, **s**, for the SPAD underwater OWC system. Specifically, in the EMIS algorithm, the EM learning algorithm determines a general framework for sequence detection, and the IS method is adopted to help evaluate the MMSE estimates of parameters. It is shown that the proposed EMIS algorithm provides a good performance-complexity tradeoff with substantially low complexity.

A. EM ALGORITHM FOR SEQUENCE DETECTION

The EM algorithm is an iterative method for obtaining the ML estimates of parameters [15]. The EM algorithm can be applied to scenarios where it is difficult to obtain a theoretical expression for the ML estimator. For elaborating the EM algorithm, it is convenient to define some parameters as follows. Let $\Gamma = (\mathbf{r}, \mathbf{u})$ denote a complete data set comprising the observed data (a.k.a. the incomplete data) \mathbf{r} and missing data \mathbf{u} . The data Γ follows a probability distribution, denoted by $p(\Gamma|\mathbf{s})$, where \mathbf{s} is the set of unknown parameters to be estimated from \mathbf{r} . The EM algorithm is used for acquiring an estimate of \mathbf{s} approaching its ML estimator. Briefly, each iteration of the EM algorithm includes two steps, termed the expectation step (E-step) and the maximization step (M-step). In the E-step, the expectation of the complete log-likelihood function is calculated as

$$Q(\mathbf{s}|\hat{\mathbf{s}}^{(i)}) = \int p(\Gamma|\mathbf{r}, \hat{\mathbf{s}}^{(i)}) \ln p(\Gamma|\mathbf{s}) d\Gamma$$
$$\triangleq \mathbb{E}[\ln p(\Gamma|\mathbf{s})|\mathbf{r}, \hat{\mathbf{s}}^{(i)}], \qquad (10)$$

where $\hat{\mathbf{s}}^{(i)}$ denotes the estimate of \mathbf{s} in the previous *i*-th iteration. In the M-step, $\hat{\mathbf{s}}^{(i+1)}$ is obtained by maximizing the expectation $Q(\mathbf{s}|\hat{\mathbf{s}}^{(i)})$, i.e., $\hat{\mathbf{s}}^{(i+1)} = \arg \max_{\mathbf{s}} Q(\mathbf{s}|\hat{\mathbf{s}}^{(i)})$. For more details on the EM procedure, the readers are referred to [15] and [16].

Particularly, in the considered sequence detection problem, we have $\Gamma = (\mathbf{r}, h)$ as the complete data and sequence \mathbf{s} as the parameter vector to be estimated. Then, the EM algorithm for the sequence detection in the SPAD underwater OWC is derived as follows.

1) In the E-step, we calculate the expectation of the complete log-likelihood function

$$Q(\mathbf{s}|\hat{\mathbf{s}}^{(i)}) = \mathbb{E}[\ln p(\mathbf{r}, h|\mathbf{s})|\mathbf{r}, \hat{\mathbf{s}}^{(i)}].$$
(11)

As *h* is independent of **s**, it follows that $p(\mathbf{r}, h|\mathbf{s}) = \Pr(\mathbf{r}|h, \mathbf{s})p(h|\mathbf{s}) = \Pr(\mathbf{r}|h, \mathbf{s})p_h(h)$. Then, we have

$$\ln p(\mathbf{r}, h|\mathbf{s}) = \ln \Pr(\mathbf{r}|h, \mathbf{s}) + \ln p_h(h)$$
$$= \sum_{n=1}^{N} -\alpha h s(n) + r(n) \ln \lambda(n) + C,$$
(12)

where *C* is a constant term irrelevant to \mathbf{s} , and the last equality comes from the substitution of (2) and (3). The term *C* remains constant in the M-step and can be safely dropped to yield an equivalent expression for evaluation:

$$\tilde{Q}(\mathbf{s}|\hat{\mathbf{s}}^{(i)}) = \mathbb{E}\left\{\sum_{n=1}^{N} -\alpha h s(n) + r(n) \ln \lambda(n) |\mathbf{r}, \hat{\mathbf{s}}^{(i)}\right\}$$
$$= \sum_{n=1}^{N} -\alpha s(n) \mathbb{E}[h|\mathbf{r}, \hat{\mathbf{s}}^{(i)}]$$
$$+ r(n) \mathbb{E}[\ln(\alpha h s(n) + \beta)|\mathbf{r}, \hat{\mathbf{s}}^{(i)}], \quad (13)$$

where $\mathbb{E}[h|\mathbf{r}, \hat{\mathbf{s}}^{(i)}]$ and $\mathbb{E}[\ln(\alpha h s(n) + \beta)|\mathbf{r}, \hat{\mathbf{s}}^{(i)}]$ are respectively the MMSE estimates of *h* and $\ln(\alpha h s(n) + \beta)$ for given \mathbf{r} and $\hat{\mathbf{s}}^{(i)}$.

2) In the M-step, the maximization of $\tilde{Q}(\mathbf{s}|\hat{\mathbf{s}}^{(i)})$ over $\mathbf{s} \in \{0, 1\}^N$ is equivalent to the maximization of $-\alpha s(n)\mathbb{E}[h|\mathbf{r}, \hat{\mathbf{s}}^{(i)}] + r(n)\mathbb{E}[\ln(\alpha h s(n) + \beta)|\mathbf{r}, \hat{\mathbf{s}}^{(i)}]$ over each $s(n) \in \{0, 1\}$. As a result, we obtain the following recursion

$$\hat{s}^{(i+1)}(n) = \arg \max_{s(n) \in \{0,1\}} -\alpha s(n) \mathbb{E}[h|\mathbf{r}, \hat{\mathbf{s}}^{(i)}] + r(n) \mathbb{E}[\ln(\alpha h s(n) + \beta)|\mathbf{r}, \hat{\mathbf{s}}^{(i)}] = \begin{cases} 0, r(n) \le \tau_{EM} \\ 1, r(n) > \tau_{EM}, \end{cases}$$
(14)

where the detection threshold τ_{EM} is directly obtained as

$$\tau_{EM} = \frac{\alpha \mathbb{E}[h|\mathbf{r}, \hat{\mathbf{s}}^{(i)}]}{\mathbb{E}[\ln(1 + \alpha h/\beta)|\mathbf{r}, \hat{\mathbf{s}}^{(i)}]}.$$
 (15)

Observing the binary detection in (14) for each symbol s(n), the implementation complexity of the EM algorithm can be fairly low due to the symbol-by-symbol decisions. Moreover, since the prior information of h is exploited, the performance of the EM algorithm is as expected superior to that of the GLRT method.

Let g(h) be an arbitrary function of h (in (14), g(h) is h or $\ln(\alpha h s(n) + \beta)$). According to the Bayes' rule, $\mathbb{E}[g(h)|\mathbf{r}, \hat{\mathbf{s}}^{(i)}]$ can be calculated as

$$\mathbb{E}[g(h)|\mathbf{r}, \hat{\mathbf{s}}^{(i)}] = \int g(h)p(h|\mathbf{r}, \hat{\mathbf{s}}^{(i)})dh$$
$$= \frac{\int g(h)\Pr(\mathbf{r}|h, \hat{\mathbf{s}}^{(i)})p_h(h)dh}{\int\Pr(\mathbf{r}|h, \hat{\mathbf{s}}^{(i)})p_h(h)dh}.$$
(16)

In the EM algorithm, the remaining task is to evaluate $\mathbb{E}[g(h)|\mathbf{r}, \hat{\mathbf{s}}^{(i)}]$ at each step, e.g., in (14). However, closed-form

expression for $\mathbb{E}[g(h)|\mathbf{r}, \hat{\mathbf{s}}^{(i)}]$ in (16) is generally not attainable. To address this difficulty, we resort to the IS method to acquire an accurate numerical evaluation of $\mathbb{E}[g(h)|\mathbf{r}, \hat{\mathbf{s}}^{(i)}]$.

B. IS METHOD FOR CALCULATING MMSE ESTIMATES

IS is one of common sampling techniques in Monte Carlo methods. Assume *X* as a random variable with PDF $p_X(x)$ and one intends to calculate the expectation of function g(X). With *K* independent and identically distributed (i.i.d.) samples $\{x_1, \dots, x_K\} \sim p_X(x)$, $\mathbb{E}[g(X)]$ can be numerically approximated by $\mathbb{E}[g(X)] \approx \frac{1}{K} \sum_{k=1}^{K} g(x_k)$ and the approximation accuracy increases with the sample size *K*. In many cases, sampling directly from $p_X(x)$ is difficult, while sampling from another PDF $q_Y(y)$ is much more tractable. Then, we resort to using *K* i.i.d. samples $\{y_1, \dots, y_K\} \sim q_Y(y)$ to approximate $\mathbb{E}[g(X)]$ as [17] and [18]

$$\mathbb{E}[g(X)] \approx \frac{\sum_{k=1}^{K} w(y_k)g(y_k)}{\sum_{k=1}^{K} w(y_k)},$$
(17)

where $w(y_k) = \frac{cp_X(y_k)}{q_Y(y_k)} \propto \frac{p_X(y_k)}{q_Y(y_k)}$ denotes the importance weight, and *c* can be any nonzero constant. Note that the importance weight measures the quality of the corresponding sample and a relatively small weight indicates an ineffective sample. To measure the sampling efficiency of the distribution $q_Y(y)$, one commonly used metric is the effective sample size, which is defined as

$$K_{\rm eff} = \frac{K}{(1+v^2)},$$
 (18)

where $v^2 = \frac{1}{K} \sum_{k=1}^{K} \left(\frac{w(y_k)}{\bar{w}} - 1 \right)^2$ is the normalized variation of $\{w(y_k)\}$ with $\bar{w} = \frac{1}{K} \sum_{k=1}^{K} w(y_k)$.

In our problem, closed-form expression for $\mathbb{E}[g(h)|\mathbf{r}, \hat{\mathbf{s}}^{(i)}]$ in (14) is generally not available, and sampling directly from $p(h|\mathbf{r}, \hat{\mathbf{s}}^{(i)})$ is difficult. Now we employ the IS method to acquire an approximation of $\mathbb{E}[g(h)|\mathbf{r}, \hat{\mathbf{s}}^{(i)}]$. The major challenge of implementing IS for evaluating $\mathbb{E}[g(h)|\mathbf{r}, \hat{\mathbf{s}}^{(i)}]$, instead of sampling directly from $p(h|\mathbf{r}, \hat{\mathbf{s}}^{(i)})$, is to find an alternative PDF in the following which admits efficient sampling. Here, efficient sampling means the shape of the sampling distribution is close to that of the distribution $p(h|\mathbf{r}, \hat{\mathbf{s}}^{(i)})$, i.e., the importance weights are balanced [22]. To choose an appropriate sampling distribution, we need a specific characterization of the distribution $p(h|\mathbf{r}, \hat{\mathbf{s}}^{(i)})$.

Let us start with the PDF in (3). For any received \mathbf{r} , it follows

$$\Pr(\mathbf{r}|h, \hat{\mathbf{s}}^{(i)}) = \prod_{n=1}^{N} \frac{e^{-(\alpha h \hat{s}^{(i)}(n) + \beta)} (\alpha h \hat{s}^{(i)}(n) + \beta)^{r(n)}}{r(n)!}$$
$$= \prod_{n=1}^{N} \frac{1}{r(n)!} \times \prod_{\substack{\{n: \hat{s}^{(i)}(n) = 0\}\\ \forall n \in \mathcal{I}, n \in \mathcal$$

$\propto \prod_{\{n:\hat{s}^{(i)}(n)=1\}} e^{-(\alpha h+\beta)} (\alpha h+\beta)^{r(n)}$ = $e^{-(\alpha h+\beta)\hat{N}_{1}^{(i)}} (\alpha h+\beta)^{[n:\hat{s}^{(i)}(n)=1]},$ (19)

where $\hat{N}^{(i)} = \sum_{n=1}^{N} \hat{s}^{(i)}(n)$ represents the number of '*n*'s with $\hat{s}^{(i)}(n) = 1$. Let $\{r(n)\}_{n=1}^{N}$ be sorted in descending order such that $\tilde{r}(N) \ge \tilde{r}(N-1) \ge \cdots \ge \tilde{r}(1)$. According to (14), we have $\sum_{\{n:\hat{s}^{(i)}(n)=1\}} r(n) = \sum_{n=1}^{\hat{N}^{(i)}} \tilde{r}(n)$. Then, (19) can be rewritten as

$$\Pr(\mathbf{r}|h, \hat{\mathbf{s}}^{(i)}) \propto e^{-(\alpha h + \beta)\hat{N}_1^{(i)}} (\alpha h + \beta)^{\sum_{n=1}^{\hat{N}^{(i)}} \tilde{r}(n)}.$$
 (20)

Considering the relationship $p(h|\mathbf{r}, \hat{\mathbf{s}}^{(i)}) \propto \Pr(\mathbf{r}|h, \hat{\mathbf{s}}^{(i)})p_h(h)$ and using (20), it yields

$$p(h|\mathbf{r}, \hat{\mathbf{s}}^{(i)}) \propto e^{-(\alpha h+\beta)\hat{N}_{1}^{(i)}} (\alpha h+\beta)_{n=1}^{\tilde{N}^{(i)}} p_{h}(h).$$
(21)

It is observed that $e^{-(\alpha h+\beta)N_1^{(*)}}(\alpha h+\beta)^{n=1}$ in (21) is relevant to the sequence length N and the term $p_h(h)$ is independent of N. Then, one intuitionistic idea on the IS $\sum \tilde{r}(n)$ method is to adopt $q_h(h) \propto e^{-(\alpha h + \beta)\hat{N}_1^{(i)}} (\alpha h + \beta)_{n=1}^{\sum}$

as the sampling distribution and $p_h(h)$ as the importance weight in the IS method. In this manner, a stable sampling can be realized as the importance weight is independent of N.

Remark 1: There are some other choices of sampling distributions in the IS method. For instance, we can also adopt $q_h^1(h) = p_h(h)$ as the sampling distribution and $w^1(h) \propto$ $e^{-(\alpha h+\beta)\hat{N}_{1}^{(i)}}(\alpha h+\beta)^{\hat{N}_{1}^{(i)}}$ as the corresponding importance weight. However, one main drawback of the sampling distribution $q_h^1(h)$ (as well as many other sampling distributions) is that the importance weight $w^{1}(h)$ depends on the sequence length N. As N increases, the discrepancy between $q_{h}^{1}(h)$ and $p(h|\mathbf{r}, \hat{\mathbf{s}}^{(i)})$ also increases and the importance weights $\{w^1(h)\}$ are more likely to be imbalanced, implying an inefficient sampling. In contrast, the importance weight $w(h) = p_h(h)$ is independent of the sequence length N which indicates a stable sampling, if the sampling distribution is chosen as

$$q_h(h) \propto e^{-(\alpha h+\beta)\hat{N}_1^{(i)}} (\alpha h+\beta)^{\sum_{n=1}^{N^{(i)}} \tilde{r}(n)}.$$

With (21) and the above analysis, we seek to efficiently generate samples of the distribution $q_h(h) \propto$ $e^{-(\alpha h+\beta)\hat{N}_{1}^{(i)}}(\alpha h+\beta)^{\sum_{n=1}^{\tilde{r}(n)}}$. Define $\eta \triangleq \alpha h+\beta$. It is inter-

esting to find that

$$e^{-(\alpha h+\beta)\hat{N}_{1}^{(i)}}(\alpha h+\beta)_{n=1}^{\hat{N}_{1}^{(i)}} = e^{-\eta \hat{N}_{1}^{(i)}} \eta_{n=1}^{\hat{N}_{1}^{(i)}} \alpha Gam(\eta; \sum_{n=1}^{\hat{N}^{(i)}} \tilde{r}(n) + 1, \hat{N}^{(i)}),$$
(22)

where $Gam(x; \gamma, \mu) = \frac{\mu^{\gamma} x^{\gamma-1} e^{-\mu x}}{\Gamma(\gamma)}$ denotes the PDF of the Gamma distribution with shape parameter γ and rate parameter μ , and $\Gamma(\gamma)$ is the Gamma function. Note that when the shape parameter γ is an integer, the Gamma distribution reduces to an Erlang distribution. It implies that we can obtain a sample h^q of the distribution $q_h(h) \propto e^{-(\alpha h + \beta)\hat{N}_1^{(i)}}(\alpha h + \beta)$

 $\beta^{\hat{N}^{(i)}}_{p=1} \tilde{r}^{(n)}$

by first drawing a sample η^q from the Gamma $\hat{N}^{(i)}$ distribution $Gam(\eta; \sum_{n=1}^{\hat{N}^{(i)}} \tilde{r}(n) + 1, \hat{N}^{(i)})$, or equivalently from an Erlang distribution $Erl(\eta; \sum_{n=1}^{\hat{N}^{(i)}} \tilde{r}(n) + 1, \hat{N}^{(i)})$ as $\sum_{n=1}^{\hat{N}^{(i)}} \tilde{r}(n) + 1$ 1 is an integer, and then performing the transformation $h^q = \frac{\eta^q - \beta}{q}$. Either Gamma-distributed samples or Erlangdistributed samples can be easily generated by existing methods, e.g., in [23] and [24]. For instance, an Erlang-distributed sample can be generated by summing independent samples of an exponential distribution.

With K i.i.d. samples
$$\{h_1^q, \cdots, h_K^q\} \sim q_h(h) \propto \sum_{\hat{N}^{(i)}} \tilde{\Sigma}^{(i)}(h)$$

 $e^{-(\alpha h+\beta)\hat{N}_{1}^{(i)}}(\alpha h+\beta)^{\sum_{n=1}^{\tilde{r}(n)}}$, we can use (17) and (21) to approximately calculate $\mathbb{E}[g(h)|\mathbf{r}, \hat{\mathbf{s}}^{(i)}]$ as

$$\mathbb{E}[g(h)|\mathbf{r}, \hat{\mathbf{s}}^{(i)}] \approx \frac{\sum_{k=1}^{K} w(h_k^q) g(h_k^q)}{\sum_{k=1}^{K} w(h_k^q)}$$
$$= \frac{\sum_{k=1}^{K} p_h(h_k^q) g(h_k^q)}{\sum_{k=1}^{K} p_h(h_k^q)}, \qquad (23)$$

where $w(h_k^q) = p_h(h_k^q) \propto \frac{p(h_k^q | \mathbf{r}, \hat{\mathbf{s}}^{(i)})}{e^{-(\alpha h_q + \beta)\hat{N}_1^{(i)}} (\alpha h_q + \beta)^{n=1}}$. Using

(23), τ_{FM} in (14) can be evaluate

$$\tau_{EM} = \frac{\alpha \mathbb{E}[h|\mathbf{r}, \hat{\mathbf{s}}^{(i)}]}{\mathbb{E}[\ln(1 + \alpha h/\beta)|\mathbf{r}, \hat{\mathbf{s}}^{(i)}]} \approx \frac{\alpha \sum_{k=1}^{K} p_h(h_k^q) h_k^q}{\sum_{k=1}^{K} p_h(h_k^q) \ln(1 + \alpha h_k^q/\beta)}.$$
(24)

Note that (24) exhibits a simple form, thanks to the choice of the sampling distribution $q_h(h) \propto e^{-(\alpha h + \beta)\hat{N}_1^{(i)}} (\alpha h + \beta)_{n=1}^{\hat{N}^{(i)}} \tilde{r}^{(n)}$ based on (21) Consequently (1)

based on (21). Consequently, the complexity of the IS method can be very low.

Remark 2: To further improve the sampling efficiency in the IS method, it is generally useful to adopt a resampling procedure. Here, we use the systematic resampling procedure in [22], which is very computationally efficient.

The overall procedure of the EMIS algorithm for detecting s is summarized in Algorithm 1, where lines 2-4 correspond to

Algorithm 1 The Proposed EMIS Algorithm for Detecting s

1: Initialize the sequence length *N*, the received signal vector **r**, the sample size *K*, the estimate $\hat{\mathbf{s}}^{(0)}$, the iteration number i = 0, the maximum number of iterations *I*. Sort $\{r(n)\}_{n=1}^{N}$ in descending order such that $\tilde{r}(N) \geq \cdots \geq \tilde{r}(1)$.

2: Calculate
$$\hat{N}^{(i)} = \sum_{n=1}^{N} \hat{s}^{(i)}(n)$$
 and $\sum_{n=1}^{\hat{N}(i)} \tilde{r}(n)$.
3: Generate K i.i.d. samples $\{\eta_{1}^{q}, \dots, \eta_{K}^{q}\} \sim Erl(\eta; \sum_{n=1}^{\hat{N}^{(i)}} \tilde{r}(n) + 1, \hat{N}^{(i)})$. Let $h_{k}^{q} = \frac{\eta_{k}^{q} - \beta}{\alpha}$ for $k = 1, \dots, K$.
4: Calculate τ_{EM} from (24).
5: for $n = 1 : N$ do
6: Detect $\hat{s}^{(i+1)}(n)$ according to (14).
7: end for
8: if $i > I$ or $\hat{s}^{(i+1)} = \hat{s}^{(i)}$ then
9: Output $\hat{s} = \hat{s}^{(i+1)}$, and stop.
10: else
11: Set $i \leftarrow i + 1$, and go to Step 2.
12: end if

the implementation of the IS method, and line 6 is the symbol detection according to the EM.

C. COMPLEXITY ANALYSIS

In this subsection, we analyze the computational complexity of the proposed EMIS algorithm. In each iteration of the EMIS algorithm, the complexity of the IS method is $\mathcal{O}(K)$ for generating *K* i.i.d. samples $\{h_1^q, \dots, h_K^q\}$ and calculating $\mathbb{E}[g(h)|\mathbf{r}, \hat{\mathbf{s}}^{(i)}]$. Meanwhile, the detection of \mathbf{s} in the EM algorithm involves a complexity of $\mathcal{O}(N)$ per iteration. Let *I* represent the maximum number of iterations in the EMIS algorithm. The overall complexity of the EMIS algorithm is $\mathcal{O}(I \cdot K + I \cdot N)$.

V. SIMULATION RESULTS

In this section, the effectiveness of the EMIS algorithm is verified via simulation results. In the simulations, the power gain of the SPAD receiver is set as $\alpha_r = 4.52 \times 10^{14}$ s/J, the background radiation intensity is $\beta = 7.27$ [25], and the variance of $\ln h$ is set to be $\sigma_t^2 = 0.09$. All the simulation results are obtained by averaging over 50000 independent channel realizations.

First, we investigate the sampling efficiency of the distri-

bution $q_h(h) \propto e^{-(\alpha h + \beta)\hat{N}_1^{(i)}}(\alpha h + \beta)_{n=1}^{\hat{N}_1^{(i)}}$ compared with the distribution $q_h^1(h) = p_h(h)$. Fig. 1 shows the symbol error rate (SER) of the EMIS algorithm with sampling distributions $q_h(h)$ and $q_h^1(h)$ under different sequence length N. The transmit optical power is $P_t = -100$ dBm, the sample size in the IS method is K = 20, and the maximum iteration number of the EMIS algorithm is set as I = 3. The performance lower bound given by detection with perfect CSI is also



FIGURE 1. SER of the EMIS algorithm with sampling distributions $q_h(h)$ and $q_h^1(h)$ ($P_t = -100$ dBm).



FIGURE 2. Effective sample sizes of distributions $q_h(h)$ and $q_h^1(h)$ without resampling ($P_t = -100$ dBm).

plotted as a benchmark. It can be observed that the SER of the EMIS algorithm decreases with the sequence length N. When the sequence length is $N \ge 12$, the SER of the EMIS algorithm with the sampling distribution $q_h(h)$ is close to the performance lower bound, indicating the sampling efficiency of $q_h(h)$ even with relatively short sequence length. However, when using $q_h^1(h)$ as the sampling distribution, we can always observe a non-ignorable performance gap between the EMIS algorithm and performance lower bound, which implies the inefficiency of $q_h^1(h)$. Fig. 2 compares the effective sample sizes defined in (18) of the distributions $q_h(h)$ and $q_h^1(h)$. To reflect the inherent sampling efficiencies of the distributions, the effective sample sizes are considered in the case without resampling. It is seen that the effective sample size of the distribution $q_h(h)$ always takes a high value, while that of the distribution $q_h^1(h)$ is much smaller. The results again demonstrate the sampling efficiency of the distribution $q_h(h)$.



FIGURE 3. SER of the EMIS algorithm with different numbers of iterations (N = 6).



 $(P_t = -100 \text{ dBm}).$

Then, we investigate the convergence behavior of the EMIS algorithm. Fig. 3 shows the SER of the EMIS algorithm with different numbers of iterations. The length of the observation window is set to be N = 6, and the sample size is K = 20. It can be seen from Fig. 3 that the performance of the EMIS algorithm can be improved by increasing the number of iterations *I*. Meanwhile, the SER of the EMIS algorithm with I = 3 is almost the same as that with I = 4, and very little power gain can be achieved by further increasing *I*. Thus, setting I = 3 is enough to harvest considerable performance gain in the EMIS algorithm, while maintaining relatively low complexity and delay.

Next, we study the performance of the EMIS algorithm with different sample sizes in the IS-method. Fig. 4 shows the SER of the EMIS algorithm versus the sample size K when the transmit optical power is $P_t = -100$ dBm. It is seen that the SER of the EMIS algorithm decreases with the sample size K, which mainly results from the increased approximation accuracy of the IS method. In addition, when $K \ge 20$, the SER of the EMIS algorithm almost remains constant as K increases. Therefore, we can just set the sample size to be K = 20 in the IS method.



FIGURE 5. SER of different sequence detection schemes.

Finally, we compare the EMIS algorithm with the GLRT method as well as the performance lower bound. The MLSD method is not considered as it is too computationally complicated to implement. Fig. 5 presents the SER of different sequence detection schemes for N = 6, 8, 12. It is seen from Fig. 5 that the EMIS algorithm outperforms the GLRT method in any case. Specifically, for N = 8, the EMIS algorithm yields a power gain about 3 dB over the GLRT method at the SER of 10^{-3} . The significant power gain mainly results from the increased estimation accuracy by using the MMSE estimate $E[g(h)|\mathbf{r}, \hat{\mathbf{s}}^{(i)}]$ in the EMIS algorithm, as analyzed in Section IV. In addition, when the sequence length is N = 12, the SER of the EMIS algorithm is close to the performance lower bound, indicating the good performance achieved by the EMIS algorithm with relatively short sequence length.



FIGURE 6. MSE of the channel estimation using the EMIS algorithm and the GLRT method versus the sequence length *N*.

To further investigate the superiority of the EMIS algorithm over the GLRT method, the mean-square error (MSE) of the channel estimation with the two methods versus the sequence length is plotted in Fig. 6. It is seen that the channel estimation accuracy increases with the sequence length N. Besides, the EMIS algorithm can estimate h more accurately than the GLRT method, which results in the performance superiority of the EMIS algorithm.

VI. CONCLUSION

We have developed an EMIS algorithm for the sequence detection in SPAD underwater OWC systems. Mathematical derivations indicate that the developed EMIS algorithm has a very low complexity. Furthermore, simulation results have verified the good performance achieved by the EMIS algorithm. Therefore, the developed EMIS algorithm can provide a good performance-complexity tradeoff and is appealing for practical applications.

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