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Single Fuzzy Parameter Seepage Model of Oil and Gas Reservoir

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ABSTRACT In the process of modeling, the reservoir permeability is often regarded as a certain value in the traditional flow model of oil and gas reservoir, but most reservoir areas are uneven and complex and so this idealization will lead to a great decrease in the accuracy of the model. Given this, this paper puts forward fuzzy permeability, which makes the model more in line with the seepage situation, and then improves the accuracy of the model. At the same time, the fuzzy function theory is studied deeply and systematically. On this basis, according to the seepage law and physical property of the oil and gas reservoir, the fuzzy seepage differential equation and the fuzzy boundary value condition of the reservoir are established. Then, the solution of the fuzzy seepage problem of the oil and gas reservoir is achieved. Compared with the traditional seepage model, the fuzzy seepage model can describe the flow in the reservoir more accurately. The definition of fuzzy permeability and the construction of fuzzy seepage equation provide a new method and a novel idea for studying the seepage of the reservoir.

INDEX TERMS Fuzzy permeability, fuzzy differential equation, fuzzy seepage, oil and gas reservoir.

I. INTRODUCTION

A large number of scholars at home and abroad have studied the flow pattern of fractured horizontal wells [1]–[7]. Giger first discussed the problem of horizontal well fracturing in 1985. He pointed out that horizontal well is a useful method for developing heterogeneous reservoir [8]. Therefore, as long as cementing technology is applied industrially, the assumption of horizontal well fracturing will be implemented, which indicates the enormous development potential of horizontal well fracturing technology. In 1993, Larsen L. and Guo G et al. obtained the analytical solution of the bottom hole pressure of fractured horizontal well based on the point source function of the real space, utilizing the superposition principle and the Newmann product principle. In 1973, Gringarten and Raghavan [9] and Gringarten and Remay [10], [11] introduced the point source solution [12] of the heat conduction equation into the petroleum industry for the first time. It has been successfully implemented to partially penetrating vertical wells, fractured vertical wells, and horizontal wells. It is demonstrated that the method is convenient and easy to understand, and greatly promotes the development of the theory of unsteady well test.

Because of the significant advantages of fuzzy theory in solving uncertainty problems, many scholars both at home and abroad have carried out in-depth research on fuzzy theory [16]–[19]. In 2007, G Papaschinopoulos, G Stefanidou and P Efraimidis studied the existence and the uniqueness of the solutions of the fuzzy differential equation with piecewise constant argument of the form $x'(t) = px(t) + qx([t]), t \in [0, \infty)$, where p, q are constant real numbers and the initial value x_0 is a fuzzy number [20]. They prove that every nontrivial solution of this form is unbounded. In 2011, Allahviranloo and Salahshour [18] studied the numerical method for solving hybrid fuzzy differential using Euler method under generalized Hukuhara differentiability. They identify the Euler method for both cases of H-differentiability. Also, the convergence of the proposed plan is studied, and the characteristic theorem is provided for both cases.

At present, the traditional seepage model of oil and gas reservoir can only describe the entire state roughly by classical mathematical and empirical methods. It results in the inability to accurately describe some of the local conditions in the reservoir area. Moreover, it is found the exact solution of the seepage problem expressed by the mathematical physics equation does not share the seepage condition of the reservoir. The traditional mathematical and physical methods have made an idealized assumption of the flow state in the reservoir. For example, extremely irregular reservoir space is regarded as homogeneous porous space, and even in ample space, the permeability of the medium is considered to be constant. This not only makes the model have a great error with reality but also lacks the adaptability to complex and changeable reservoir area, which makes it difficult to generalize the conclusion of the study.

In this paper, fuzzy permeability is advanced, the fuzzy seepage equation is established, and the approximate solution of fuzzy seepage equation is generated. The mathematical expression of the fuzzy seepage problem is a fuzzy differential equation. There have been many types of research on the theory of fuzzy differential equations [13]–[15]. However, from the essential meaning of fuzzy differential equation in practical engineering, the theoretical research is not thick enough, and complete application system has not meant to form.

The remainder of this paper is organized as follows:

In the second part, the definition and related properties of fuzzy triangular permeability are introduced. In Secton3, we first construct a single fracture fuzzy seepage model, then solve the fuzzy differential equation and prove related properties of the fuzzy solution. In the fourth part, the point source solutions under different boundary conditions are also addressed. In Secton5, we give two examples to illustrate the broad applicability of this method. In the last section, we present our conclusion and a further research topic.

A. TRIANGULAR FUZZY PERMEABILITY

We begin this section with defining the notation we will use in the paper. First of all, some basic definitions and properties of fuzzy numbers are reviewed [22]. We place a "$\sim$" sign over a letter to denote a fuzzy subset of the real numbers. We remember $\widetilde{A}(x)$,a number in [0, 1], for the membership function of $\widetilde{A}$ at x. An α-cut of $\widetilde{A}$, written by $\widetilde{A_\alpha}$, for $0 < \alpha \leq 1$ and α-cut of fuzzy numbers are always closed and bounded. The related definitions and properties of fuzzy numbers and fuzzy differential are found in Appendix A, Appendix B, and Appendix C.

From the preceding discussion, we can see that the difference between fuzzy seepage problem and traditional seepage problem is that fuzzy permeability is introduced and fuzzy permeability is a differentiable fuzzy function on Ω of the reservoir area. Fuzzy permeability means the permeability of the medium is a fuzzy number at any point in the reservoir area. The forms of fuzzy numbers are various, such as regular fuzzy numbers, fuzzy triangular numbers and so on, among which the kind of fuzzy triangular numbers is the simplest. In this paper, fuzzy triangular numbers are used. If the value of a fuzzy function at any point is a triangular fuzzy number, it is called a triangular-valued fuzzy function. We take the permeability of the reservoir as triangular-valued fuzzy function and call it triangular-fuzzy permeability.

Suppose that $A_{K,m} = A(K^m, K, K^{1/m}) = A(K^m, K) \cup A(K, K^{1/m})$ has $K^* = \alpha K + (1-\alpha)(K^m \vee K^{1/m})$ ($K^m \vee K^{1/m}$ means K^m or $K^{1/m}$) for $\forall K^* \in A_{K,m}$. If $K^* \in A(K^m, K)$, has $K^* = \alpha K + (1-\alpha)(K^m)$, then [22]

$$\begin{aligned} \frac{1}{K^*} &= \frac{1}{\alpha K + (1-\alpha)(K^m)} \\ &= \frac{\alpha K^m}{K \cdot K^m + \alpha(1-\alpha)(K-K^m)^2} \\ &\quad + \frac{(1-\alpha)K}{K \cdot K^m + \alpha(1-\alpha)(K-K^m)^2} \\ &= \frac{\alpha}{K + \frac{\alpha(1-\alpha)(K-K^m)^2}{K^m}} \\ &\quad + \frac{1-\alpha}{K^m + \frac{\alpha(1-\alpha)(K-K^m)^2}{K}} \end{aligned} \tag{1}$$

if $p_K = K + \frac{\alpha(1-\alpha)(K-K^m)^2}{K^m}$ and $p_{K^m} = K^m + \frac{\alpha(1-\alpha)(K-K^m)^2}{K}$, has

$$\begin{aligned} \frac{1}{K^*} &= \frac{1}{\alpha K + (1-\alpha)(K^m)} \\ &= \frac{\alpha K^m}{K \cdot K^m + \alpha(1-\alpha)(K-K^m)^2} \\ &\quad + \frac{(1-\alpha)K}{K \cdot K^m + \alpha(1-\alpha)(K-K^m)^2} \\ &= \frac{\alpha}{p_K} + \frac{1-\alpha}{p_{K^m}} \end{aligned} \tag{2}$$

It is called p_K and p_{K^m} are based permeability. They are nonlinear expressions of K and K^m with α as a parameter, can also be written as $p_K(\alpha)$, $p_{K^m}(\alpha)$. In the process of solving the seepage model, the calculation process is extremely complicated due to the parameter α in the based permeability. In this article, we select $0 < \alpha < 1, 0 < K < 1$ and K is usually a tiny positive number. So $\alpha(1-\alpha)(K-K^m)^2$ is very small for any point in oil and gas reservoirs. Then the approximate formula true is as follows:

$$p_K \approx K \quad p_{K^m} \approx K^m \tag{3}$$

So we can get:

$$\begin{aligned} \frac{1}{K^*} &= \frac{1}{\alpha K + (1-\alpha)(K^m)} \\ &= \frac{\alpha}{p_K} + \frac{1-\alpha}{p_{K^m}} \approx \frac{\alpha}{K} + \frac{1-\alpha}{K^m} \end{aligned} \tag{4}$$

Similar to this process, we can get if $K^* \in A(K, K^{\frac{1}{m}})$, has $K^* = \alpha K + (1-\alpha)(K^{\frac{1}{m}})$, then

$$\begin{aligned} \frac{1}{K^*} &= \frac{1}{\alpha K + (1-\alpha)(K^{\frac{1}{m}})} \\ &= \frac{\alpha}{p_K} + \frac{1-\alpha}{p_{K^{\frac{1}{m}}}} \approx \frac{\alpha}{K} + \frac{1-\alpha}{K^{\frac{1}{m}}} \end{aligned} \tag{5}$$

II. A SINGLE FRACTURE SYSTEM

In the reservoir, under the condition of strong plasticity, horizontal well fracturing to form wings type after hydraulic fracturing. Take the artificial fracture and horizontal well, and vertical fractures are rectangular, in horizontal well around the symmetrical distribution. At the same time, the artificial fracture is regular, so the analytical method can be used to solve the problem. First, the Laplace space point source solution is obtained according to the Ozkan idea. The mirror image is utilized to eliminate the influence of the surrounding boundary, and then the integral solution of the single fracture is obtained. Then using the superposition principle considering the interference between fractures, multiple fractures of bottom hole pressure solution is achieved when the production yield.

A. MODEL OF SINGLE FRACTURE SYSTEM

First, do the following assumptions [23]:

- Formation fluid meet Darcy flow;
- The rock and fluid are slightly compressible, and the compression coefficient is constant C_t;
- For any point in the reservoir, the permeability of the porous medium at this point is regarded as a triangular fuzzy number $A_{Km} = A < K^m, K, K^{\frac{1}{m}} >$. This permeability in the reservoir area coefficient is a fuzzy function.

The mathematical model of seepage at any point in the stratum (continuous point source) is as follows:

$$\begin{cases} \frac{1}{r_D^2}\frac{\partial}{\partial r_D}(r_D^2 \cdot \frac{\partial \triangle P}{\partial r_D}) = \frac{\partial \triangle P}{\partial t_D}, \\ \triangle P(r_D, 0) = 0, \\ \lim\limits_{\varepsilon \to 0^+} \frac{4\pi \widetilde{K} L}{\mu}(r_D^2 \cdot \frac{\partial \triangle P}{\partial r_D})|_{r_D=\varepsilon} = -\widetilde{q}, \\ \triangle P(r_D \to \infty, t_D) = 0. \end{cases} \tag{6}$$

Research [23] pointed out that choosing fuzzy permeability (function) by the $A_{Km} = A < K^m, K, K^{\frac{1}{m}} >$ generation, and $|K - K^m|$ and $|K - K^{\frac{1}{m}}|$ are very small, in solving the gas flow in goaf of fuzzy differential equations, first calculate the medium permeability were K^m, K and $K^{\frac{1}{m}}$ corresponding to the three solution $H(r, K^m)$, $H(r, K)$ and $H(r, K^{\frac{1}{m}})$. If $K^* = \alpha K + (1 - \alpha)(K^m \vee K^{\frac{1}{m}})$, when the permeability coefficient is K^*, the corresponding solution of the seepage equation can be approximately expressed as:

$$H(r, K^*) = \alpha H(r, K) + (1 - \alpha)[H(r, K^m) \vee H(r, K^{\frac{1}{m}})] \tag{7}$$

We suspect that the conclusion [23] in the model of oil and gas reservoir seepage is still valid. First, the three corresponding solutions $\overline{\triangle P}(r_D, K)$, $\overline{\triangle P}(r_D, K^m)$ and $\overline{\triangle P}(r_D, K^{\frac{1}{m}})$ are calculated when the permeability is K^m, K and $K^{\frac{1}{m}}$ respectively. The seepage model of permeability K is first solved.

$$\begin{cases} \frac{1}{r_D^2}\frac{\partial}{\partial r_D}(r_D^2 \cdot \frac{\partial \triangle P}{\partial r_D}) = \frac{\partial \triangle P}{\partial t_D}, \\ \triangle P(r_D, 0) = 0, \\ \lim\limits_{\varepsilon \to 0^+} \frac{4\pi KL}{\mu}(r_D^2 \cdot \frac{\partial \triangle P}{\partial r_D})|_{r_D=\varepsilon} = -\widetilde{q}, \\ \triangle P(r_D \to \infty, t_D) = 0. \end{cases} \tag{8}$$

Among them, dimensionless parameters are defined as $\triangle P = P_0 - P$; $t_D = \frac{Kt}{\phi \mu C_t L^2}$; $r_D = \frac{r}{L}$. In order to facilitate the derivation, no special instructions, this paper adopt international standard unit. P is the formation pressure value, Pa; P_0 is the original formation pressure, Pa; $\widetilde{q}$ is the flow value of a continuous point source, m^3/s; μ is the viscosity of fluids, $Pa \cdot s$; K is the permeability value of oil reservoir, m^2; h is the thickness of oil reservoir, m; t is the time, s; ϕ is the reservoir porosity; C_t is a comprehensive compression coefficient, Pa^{-1}; r is the radius of seepage, m; L is the reference length, m.

The Laplace transformation of equation Eq.(8) on t_D can be obtained:

$$\begin{cases} \frac{1}{r_D^2}\frac{\partial}{\partial r_D}(r_D^2 \cdot \frac{\partial \overline{\triangle P}}{\partial r_D}) = s\overline{\triangle P}, \\ \overline{\triangle P}(r_D, 0) = 0, \\ \lim\limits_{\varepsilon \to 0^+} \frac{4\pi KL}{\mu}(r_D^2 \cdot \frac{\partial \overline{\triangle P}}{\partial r_D})|_{r_D=\varepsilon} = -\overline{\widetilde{q}}, \\ \overline{\triangle P}(r_D \to \infty, s) = 0. \end{cases} \tag{9}$$

Similar to this process, can get the corresponding seepage model for permeability are K^m and $K^{\frac{1}{m}}$ respectively. When the permeability is K^m, the fuzzy seepage equation is as follows:

$$\begin{cases} \frac{1}{r_D^2}\frac{\partial}{\partial r_D}(r_D^2 \cdot \frac{\partial \overline{\triangle P}}{\partial r_D}) = s\overline{\triangle P}, \\ \overline{\triangle P}(r_D, 0) = 0, \\ \lim\limits_{\varepsilon \to 0^+} \frac{4\pi K^m}{\mu}(r_D^2 \frac{\partial \overline{\triangle P}}{\partial r_D})|_{r_D=\varepsilon} = -\overline{\widetilde{q}}, \\ \overline{\triangle P}(r_D \to \infty, s) = 0. \end{cases} \tag{10}$$

When the permeability is $K^{\frac{1}{m}}$, the fuzzy seepage equation is as follows:

$$\begin{cases} \frac{1}{r_D^2}\frac{\partial}{\partial r_D}(r_D^2 \cdot \frac{\partial \overline{\triangle P}}{\partial r_D}) = s\overline{\triangle P}, \\ \overline{\triangle P}(r_D, 0) = 0, \\ \lim\limits_{\varepsilon \to 0^+} \frac{4\pi K^{\frac{1}{m}}}{\mu}(r_D^2 \frac{\partial \overline{\triangle P}}{\partial r_D})|_{r_D=\varepsilon} = -\overline{\widetilde{q}}, \\ \overline{\triangle P}(r_D \to \infty, s) = 0. \end{cases} \tag{11}$$

Furthermore, when the permeability is K^*, the fuzzy seepage equation is as follows:

$$\begin{cases} \frac{1}{r_D^2}\frac{\partial}{\partial r_D}(r_D^2 \cdot \frac{\partial \overline{\triangle P}}{\partial r_D}) = s\overline{\triangle P}, \\ \overline{\triangle P}(r_D, 0) = 0, \\ \lim\limits_{\varepsilon \to 0^+} \frac{4\pi K^*}{\mu}(r_D^2 \frac{\partial \overline{\triangle P}}{\partial r_D})|_{r_D=\varepsilon} = -\overline{\widetilde{q}}, \\ \overline{\triangle P}(r_D \to \infty, s) = 0. \end{cases} \tag{12}$$

B. SOLVING THE SINGLE FRACTURE SYSTEM MODEL

By solving Eq.(9),Eq.(10),Eq.(11)and Eq.(12), the solutions of Laplace point source can be obtained as follows:

$$\begin{cases} \overline{\triangle P}(r_D, K) = \dfrac{\mu\tilde{\overline{q}}}{4\pi KL}\dfrac{\exp(-\sqrt{s}r_D)}{r_D} \\ \quad \text{the permeability is } K \\ \overline{\triangle P}(r_D, K^m) = \dfrac{\mu\tilde{\overline{q}}}{4\pi K^m L}\dfrac{\exp(-\sqrt{s}r_D)}{r_D} \\ \quad \text{the permeability is } K^m \\ \overline{\triangle P}(r_D, K^{\frac{1}{m}}) = \dfrac{\mu\tilde{\overline{q}}}{4\pi K^{\frac{1}{m}} L}\dfrac{\exp(-\sqrt{s}r_D)}{r_D} \\ \quad \text{the permeability is } K^{\frac{1}{m}} \\ \overline{\triangle P}(r_D, K^*) = \dfrac{\mu\tilde{\overline{q}}}{4\pi K^* L}\dfrac{\exp(-\sqrt{s}r_D)}{r_D} \\ \quad (\text{the permeability is } K^*) \end{cases} \tag{13}$$

Next, we first prove the representation of the fuzzy solution. Let $K_1(x, y)$ and $K_2(x, y)$ be an arbitrary nonzero bounded function defined on the reservoir area Ω. The solutions of the corresponding equations are as follows:

$$\begin{cases} \overline{\triangle P}(r_D, K_1) = \dfrac{\mu\tilde{\overline{q}}}{4\pi K_1 L}\dfrac{\exp(-\sqrt{s}r_D)}{r_D} \\ \quad \text{the permeability is } K_1 \\ \overline{\triangle P}(r_D, K_2) = \dfrac{\mu\tilde{\overline{q}}}{4\pi K_2 L}\dfrac{\exp(-\sqrt{s}r_D)}{r_D} \\ \quad \text{the permeability is } K_2 \end{cases} \tag{14}$$

Numerical inversion of the above results leads to the following results:

$$\begin{cases} \triangle P(r_D, K_1) = \dfrac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty} \dfrac{N_1}{K_1} e^{st} ds \\ \triangle P(r_D, K_2) = \dfrac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty} \dfrac{N_1}{K_2} e^{st} ds \end{cases} \tag{15}$$

where

$$N_1 = \frac{\mu\tilde{\overline{q}}}{4\pi L}\frac{\exp(-\sqrt{s}r_D)}{r_D} \tag{16}$$

According to the property of definite integral, if $K_1(x, y) \leq K_2(x, y)$, there is

$$\frac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty}(\frac{N_1}{K_1})e^{st}ds \geq \frac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty}(\frac{N_1}{K_2})e^{st}ds \tag{17}$$

Thus, according to the Theorem 6 we can see that the resolution of the fuzzy seepage problem can be shown. It is also proved by the theorem that the exact solution of the fuzzy seepage problem is a fuzzy function on the reservoir space Ω. Since $\forall K^* \in A_{K,m}$, has $K^* = \alpha K + (1-\alpha)(K^m \vee K^{1/m})$. If $K^* \in A(K^m, K)$, has $K^* = \alpha K + (1-\alpha)(K^m)$, then

$$\begin{aligned} \frac{1}{K^*} &= \frac{1}{\alpha K + (1-\alpha)(K^m)} \\ &= \frac{\alpha}{p_K} + \frac{1-\alpha}{p_{K^m}} \approx \frac{\alpha}{K} + \frac{1-\alpha}{K^m} \end{aligned} \tag{18}$$

this formula can be used in the point source solution:

$$\begin{aligned} \overline{\triangle P}(r_D, K^*) &= \frac{N_1}{K^*} = N_1(\frac{\alpha}{p_K} + \frac{1-\alpha}{p_{K^m}}) \\ &= (1-\alpha)\frac{N_1}{p_{K^m}} + \alpha\frac{N_1}{p_K} \\ &\approx (1-\alpha)\frac{N_1}{K^m} + \alpha\frac{N_1}{K} \\ &= \alpha\overline{\triangle P}(r_D, K) + (1-\alpha)\overline{\triangle P}(r_D, K^m) \end{aligned}$$

According to the inversion formula of Laplace transform $L^{-1}[\overline{f(t)}] = f(t) = \frac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty}\overline{f(t)}e^{st}ds$, there are:

$$\begin{aligned} \triangle P(r_D, K^*) &= \frac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty}\overline{\triangle P}(r_D, K^*)e^{st}ds \\ &\approx \frac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty}(\alpha\overline{\triangle P}(r_D, K) \\ &\quad + (1-\alpha)\overline{\triangle P}(r_D, K^m))e^{st}ds \\ &= \alpha\frac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty}\overline{\triangle P}(r_D, K)e^{st}ds \\ &\quad + (1-\alpha)\frac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty}\overline{\triangle P}(r_D, K^m)e^{st}ds \\ &= \alpha\triangle P(r_D, K) + (1-\alpha)\triangle P(r_D, K^m) \end{aligned}$$

similar to this, if $K^* \in A(K, K^{\frac{1}{m}})$, has $K^* = \alpha K + (1-\alpha)(K^{\frac{1}{m}})$, then

$$\begin{aligned} \triangle P(r_D, K^*) &= \frac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty}\overline{\triangle P}(r_D, K^*)e^{st}ds \\ &\approx \frac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty}(\alpha\overline{\triangle P}(r_D, K) \\ &\quad + (1-\alpha)\overline{\triangle P}(r_D, K^{\frac{1}{m}}))e^{st}ds \\ &= \alpha\frac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty}\overline{\triangle P}(r_D, K)e^{st}ds \\ &\quad + (1-\alpha)\frac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty}\overline{\triangle P}(r_D, K^{\frac{1}{m}})e^{st}ds \\ &= \alpha\triangle P(r_D, K) + (1-\alpha)\triangle P(r_D, K^{\frac{1}{m}}) \end{aligned}$$

so if $K^* \in A_{K,m}$, and $K^* = \alpha K + (1-\alpha)[K^m \vee K^{\frac{1}{m}}]$, has:

$$\begin{aligned} &\triangle P(r, K^*) \\ &\quad = \alpha\triangle P(r, K) + (1-\alpha)[\triangle P(r, K^m) \vee \triangle P(r, K^{\frac{1}{m}})] \end{aligned}$$

The above form solves the assumptions we have put forward at the beginning of this section. The solution set of the fuzzy seepage definite solution problem can be generated by the corresponding solutions $\triangle P(r_D, K)$, $\triangle P(r_D, K^m)$ and $\triangle P(r_D, K^{\frac{1}{m}})$ with K, K^m and $K^{\frac{1}{m}}$ as the seepage permeability. Because the fuzzy permeability $\widetilde{K}$ in the fuzzy seepage equation (11) is a fuzzy function generated by $A < K^m, K, K^{\frac{1}{m}} >$ on Ω. If $K^* = \alpha K + (1-\alpha)(K^m \vee K^{1/m})$, has

TABLE 1. Parameters for numerical simulation.

Parameter	Unit	value
porosity, φ	fraction	0.07
permeability, K	md	0.0005
formation temperature, T	K	366.15
formation pressure, P_e	MPa	24.13
pressure relief radil, r_e	m	400
rock density, ρ_c	g/cm^3	2.9
compression factor, Z	number	0.89
gas viscosity, μ	MPa s	0.027
radius of the wellbore, r_ω	m	0.1
flowing bottomhole pressure, P_ω	MPa	6
formation thickness, h	m	30.5
diffusion coefficient, D_k	m^2/s	3×10^{-7}

K^* is a boundary of the α-cuts of $\widetilde{K}$. The solution $\triangle P(r_D, K^*)$ belongs to the definite solution $\widetilde{\triangle P}$ of fuzzy seepage equation, and the membership degree of $\triangle P(r_D, K^*)$ is α, that is:

$$\begin{aligned}\mu_{\widetilde{\triangle P}}(\triangle P(r_D, K^*)) &= \mu_{\widetilde{\triangle P}}[\alpha \triangle P(r_D, K) + (1-\alpha) \\ &\quad \times (\triangle P(r_D, K^m) \vee \triangle P(r_D, K^{\frac{1}{m}}))] \\ &= \alpha \qquad (19)\end{aligned}$$

Different ΔP can be obtained under different membership degrees, as showed in figure 1.

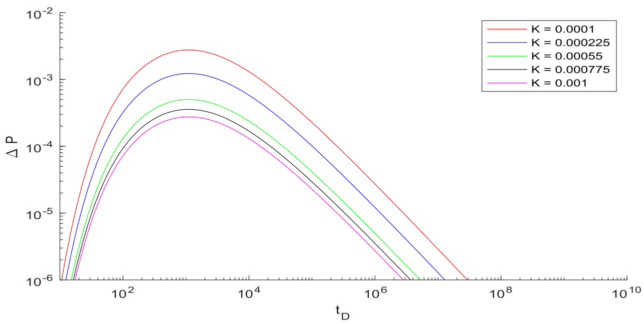

FIGURE 1. Relation curve between time t_D and Δp under different permeability K.

As can be seen from Figure1, the ΔP - value corresponding to the permeability under different membership degrees at different times can be obtained by the fuzzy analytic evaluation. Thus, the range of ΔP can be collected and then the range of formation pressure P can be derived. These data will provide a useful reference for practical engineering and related well test analysis. At the same time, from Figure 2, we can see the membership degree corresponding to different ΔP.

It can be seen from Figure 3 that when t_D and r_D are increased to a certain extent, P tends to be stable.

From the representation of fuzzy differential equations and the decomposition theorem of fuzzy functions, we can obtain

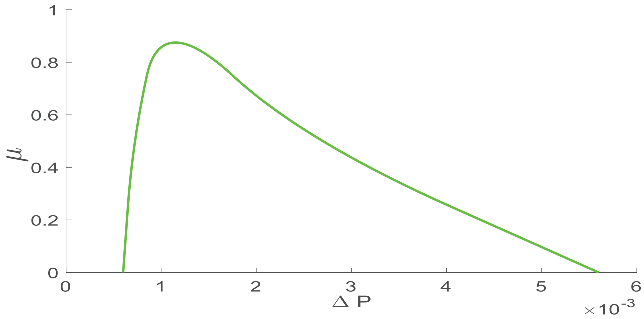

FIGURE 2. Relation curve between Δp and membership μ.

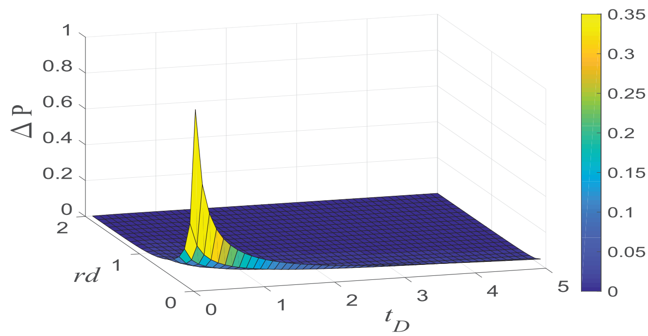

FIGURE 3. 3D relation plot of t_D, r_D and Δp.

the fuzzy definite solution of the fuzzy seepage equation:

$$\widetilde{\triangle P} = \bigcup_{K^* \in A_{K,m}} \mu_{\widetilde{\triangle P}}(\triangle P(r_D, K^*)) * \triangle P(r_D, K^*) \qquad (20)$$

The solution of fuzzy seepage definite solution problem with fuzzy triangular permeability can be approximately expressed as a trigonometric valued fuzzy function. This conclusion is of great practical significance. The following three theorems prove that the solution of the fuzzy seepage definite solution problem obtained by the Eq.(20) is the triangular valued fuzzy function on the reservoir area.

Theorem 1: The support set $\{\triangle P | \mu_{\widetilde{\triangle P}}(\triangle P) > 0\}$ of fuzzy solution sets $\widetilde{\triangle P}$ of fuzzy equations (11) is totally ordered set. That is to say, if $\forall K_1, K_2 \in A_{K,m}$ meets $K_1 < K_2$, there is $\triangle P(r_D, K_1) > \triangle P(r_D, K_2)$.

The conclusion can be seen from the above relation, and the specific proof process can be found in [22].

Theorem 2: The fuzzy solution set $\widetilde{\triangle P}$ defined by Eq.(20) is a normal convex fuzzy set on $C_1(R^2)$.

Proof: First of all, it is proved that $\widetilde{\triangle P}$ is normal. If $A_{K,m} = A(K^m, K, K^{\frac{1}{m}})$, $\triangle P_0 = \triangle P(r_D, K)$, then $\mu_{\widetilde{\triangle P}}(\triangle P_0) = 1$ can be obtained by definition.

Next, it is proved that $\widetilde{\triangle P}$ is fuzzy convex. If $\triangle P_i = TK_i$, $\triangle P_j = TK_j$, $K_i, K_j \in A_{K,m}$ and $\triangle P_i > \triangle P_j > \triangle P_0$ or $\triangle P_i < \triangle P_j < \triangle P_0$, has $\| \triangle P_i - \triangle P_0 \| > \| \triangle P_j - \triangle P_0 \|$. That is to say $\triangle P_j$ is closer to $\triangle P_0$ than $\triangle P_i$. At this time there is $\mu_{\widetilde{\triangle P}}(\triangle P_j) > \mu_{\widetilde{\triangle P}}(\triangle P_i)$.

If $K_1, K_2, K_3 \in A_{K,m}$ and $K_1 < K_2 < K_3$, $\triangle P_i = TK_i$, $i = 1, 2, 3$, $\triangle P_3 < \triangle P_2 < \triangle p_1$ can be obtained. In other words, no matter when $\triangle P_0$ is, there is always one of $\|\triangle P_1 - \triangle P_0\| > \|\triangle P_2 - \triangle P_0\|$ or $\|\triangle P_3 - \triangle P_0\| > \|\triangle P_2 - \triangle P_0\|$. So $\mu_{\widetilde{\triangle P}}(\triangle P_2) \geq min\{\mu_{\widetilde{\triangle P}}(\triangle P_1), \mu_{\widetilde{\triangle P}}(\triangle P_3)\}$.

From the definition of convex fuzzy sets, $\widetilde{\triangle P}$ is convex. □

Theorem 3: The fuzzy set $\widetilde{\triangle P}|_{x,y}$ restricted by $\widetilde{\triangle P}$ at $(x, y) \in \Omega$ is a triangular fuzzy number.

Proof: In fact, $\widetilde{\triangle P}|(x, y)$ is a fuzzy number. By definition, the support set of $\widetilde{\triangle P}|_{x,y}$ for a given $(x, y) \in \Omega$ is:

$$\{y|\mu_{\widetilde{\triangle P}|(x,y)}(y)\} = (\bigtriangleup P^m(x, y), \bigtriangleup P^{\frac{1}{m}}(x, y)) \tag{21}$$

There are also $\bigtriangleup P^m(x, y) < \bigtriangleup P(x, y) < \bigtriangleup P^{\frac{1}{m}}(x, y)$, $\mu_{\widetilde{\triangle P}|(x,y)}(\triangle P(x, y)) = 1$. That is $\widetilde{\triangle P}$ is normal. Written $a = \triangle P^m(x, y), b = \triangle P^{\frac{1}{m}}(x, y), M = \triangle P(x, y)$, if $y = f(x, y) > b = \triangle P^{\frac{1}{m}}(x, y)$ or $y = f(x, y) < a = \triangle P^m(x, y)$, has $\mu_{\widetilde{\triangle P}}(f) = 0$, that is $\mu_{\widetilde{\triangle P}|(x,y)}(y) = 0$. If $y \in [M, b]$, then $\exists f \in \tau, y = f(x, y), f = \alpha K + (1-\alpha)K^{\frac{1}{m}}$, has $y = \alpha M + (1-\alpha)Kb$. That is:

$$\begin{aligned}\mu_{\widetilde{\triangle P}|(x,y)}(y) = \alpha &= \frac{\alpha(b-M)}{\mu_{\widetilde{\triangle P}|(x,y)}(y)} = \frac{b - \alpha M - b + \alpha b}{b - M} \\ &= \frac{b - [\alpha M + (1-\alpha)b]}{b-M} = \frac{b-y}{b-M}\end{aligned} \tag{22}$$

Analogously, if $y \in [a, M]$, has $\mu_{\widetilde{\triangle P}|(x,y)}(y) = \frac{y-a}{M-a}$. So the membership function of $\widetilde{\triangle P}|_{(x,y)}$ is:

$$\mu_{\widetilde{\triangle P}|(x,y)}(y) = \begin{cases} \dfrac{y-a}{M-a} & a < y \leq M \\ \dfrac{b-y}{b-M} & M < y \leq b \\ 0 & \text{other} \end{cases}$$

It is shown that $\widetilde{\triangle P}|(x, y)$ is triangular fuzzy set on [0, 1]. □

The solution $\triangle P(r_D, K)$,$\triangle P(r_D, K^m)$ and $\triangle P(r_D, K^{\frac{1}{m}})$ of the three special cases of the fuzzy seepage definite solution problem are given. For any given $\lambda \in (0, 1]$, we can easily obtain the α-cuts $\widetilde{\triangle P}(x, y)_\lambda = [\triangle P_1(x, y)_\lambda, \triangle P_2(x, y)_\lambda]$ of fuzzy solution of fuzzy seepage equation. According to the formula (7) there is:

$$\begin{aligned}\triangle P_1(x, y)_\lambda &= \lambda \triangle P(r, K) + (1-\lambda) \triangle P(r, K^m) \\ \triangle P_2(x, y)_\lambda &= \lambda \triangle P(r, K) + (1-\lambda) \triangle P(r, K^{\frac{1}{m}})\end{aligned}$$

among them, $r = (x, y)$. Thus, the fuzzy solution of the fuzzy seepage problem is represented by the expression theorem as follows:

$$\widetilde{\triangle P} = \bigcup_{\lambda \in [0,1]} \lambda * [\triangle P_1(x, y)_\lambda, \triangle P_2(x, y)_\lambda] \tag{23}$$

Therefore, to solve the fuzzy seepage problem (6), the main task is to find the definite solution $\triangle P(r, K)$, $\triangle P(r, K^m)$, $\triangle P(r, K^{\frac{1}{m}})$of three individual cases

III. POINT SOURCE SOLUTION UNDER DIFFERENT RESERVOIR BOUNDARIES

According to the principle of superposition, the solution of Laplace point source at any point of the stratum is eliminated after the upper and lower boundary is eliminated:

$$\overline{\triangle P} = N_2 + \frac{\mu \tilde{\bar{q}}}{4\pi KL}$$

where

$$N_2 = \sum_{n=-\infty}^{+\infty} \frac{\exp(-\sqrt{s}\sqrt{r_D^2 + z_{D1}^2})}{\sqrt{r_D^2 + z_{D1}^2}} + \frac{\exp(-\sqrt{s}\sqrt{r_D^2 + z_{D2}^2})}{\sqrt{r_D^2 + z_{D2}^2}}$$

and $r_D = \sqrt{(x_D - x_{\omega D})^2 + (y_D - y_{\omega D})^2}$, $h_D = \frac{h}{L}$, $z_{D1} = z_D - z_{\omega D} - 2nh_D$, $z_{D2} = z_D + z_{\omega D} - 2nh_D$. Let $K_1(x, y)$ and $K_2(x, y)$ be an arbitrary nonzero bounded function defined on the reservoir area Ω. The solutions of the corresponding equations are as follows:

$$\begin{cases} \overline{\bigtriangleup P}(r_D, K_1) = N_2 + \dfrac{\mu \tilde{\bar{q}}}{4\pi K_1 L} \\ \overline{\bigtriangleup P}(r_D, K_2) = N_2 + + \dfrac{\mu \tilde{\bar{q}}}{4\pi K_2 L} \end{cases} \tag{24}$$

Numerical inversion of the above results leads to the following results:

$$\begin{cases} \bigtriangleup P(r_D, K_1) = \dfrac{1}{2\pi i} \int_{\gamma - i\infty}^{\gamma + i\infty} (\dfrac{\mu \tilde{\bar{q}}}{4\pi K_1 L} N_2) e^{st} ds \\ \bigtriangleup P(r_D, K_2) = \dfrac{1}{2\pi i} \int_{\gamma - i\infty}^{\gamma + i\infty} (\dfrac{\mu \tilde{\bar{q}}}{4\pi K_2 L} N_2) e^{st} ds \end{cases}$$

According to the property of definite integral, if $K_1(x, y) \leq K_2(x, y)$, there is

$$\frac{1}{2\pi i} \int_{\gamma - i\infty}^{\gamma + i\infty} \frac{\mu \tilde{\bar{q}} N_2 e^{st}}{4\pi L K_1} ds \geq \frac{1}{2\pi i} \int_{\gamma - i\infty}^{\gamma + i\infty} \frac{\mu \tilde{\bar{q}} N_2 e^{st}}{4\pi L K_2} ds \tag{25}$$

Thus, according to Theorem 6, we can see that the resolution of the fuzzy seepage problem can be expressed. It is also proved that the correct solution of the fuzzy seepage problem is a fuzzy function on the reservoir space Ω. If $K^* \in A(K^m, K)$, has $K^* = \alpha K + (1-\alpha)K^m$, then according to the basic permeability P_K, P_{K^m}, $P_{K^{\frac{1}{m}}}$ defined in the front, we can get:

$$\begin{aligned}\overline{\triangle P}(r_D, K^*) &= \frac{\mu \tilde{\bar{q}} N}{4\pi K^* L} \\ &= \frac{\mu \tilde{\bar{q}} N}{4\pi(\alpha K + (1-\alpha)(K^m))L} \\ &\approx \alpha \frac{\mu \tilde{\bar{q}} N}{4\pi KL} + (1-\alpha)\frac{\mu \tilde{\bar{q}} N}{4\pi K^m L} \\ &= \alpha \overline{\triangle P}(r_D, K) + (1-\alpha)\overline{\triangle P}(r_D, k^m)\end{aligned}$$

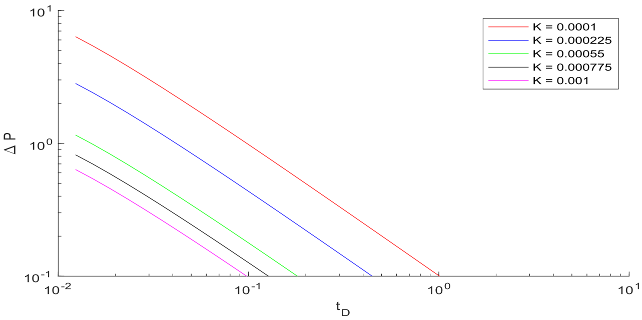

FIGURE 4. Relation curve between time t_D and Δp under different permeability K.

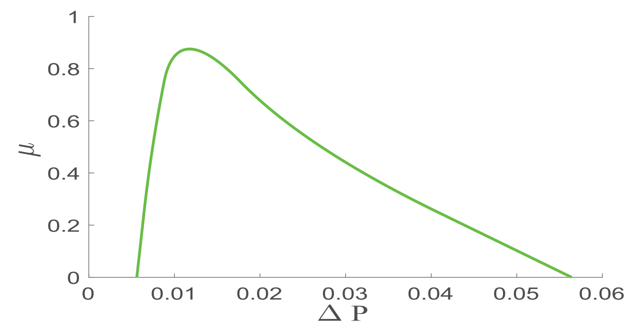

FIGURE 5. Relation curve between Δp and membership μ.

According to the inversion formula of Laplace transform $L^{-1}[\overline{f(t)}] = f(t) = \frac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty}\overline{f(t)}e^{st}ds$, there are:

$$\begin{aligned}\triangle P(r_D, K^*) &= \frac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty}\overline{\triangle P(r_D, K^*)}e^{st}ds \\ &\approx \frac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty}\{\alpha\overline{\triangle P(r_D, K)} \\ &\quad +(1-\alpha)\overline{\triangle P(r_D, K^m)}\}e^{st}ds \\ &= \alpha\triangle P(r_D, K) + (1-\alpha)\triangle P(r_D, K^m)\end{aligned}$$

similar to this, if $K^* \in A(K, K^{\frac{1}{m}})$, has $K^* = \alpha K + (1-\alpha)(K^{\frac{1}{m}})$, then

$$\triangle P(r_D, K^*) = \alpha\triangle P(r_D, K) + (1-\alpha)\triangle P(r_D, K^{\frac{1}{m}}) \tag{26}$$

so if $K^* \in A_{K,m}$, and $K^* = \alpha K + (1-\alpha)[K^m \vee K^{\frac{1}{m}}]$, has:

$$\begin{aligned}&\triangle P(r, K^*) \\ &\quad = \alpha\triangle P(r, K) + (1-\alpha)[\triangle P(r, K^m) \vee \triangle P(r, K^{\frac{1}{m}})]\end{aligned}$$

As can be seen from Figure4, the ΔP - value corresponding to the permeability under different membership degrees at different times can be obtained by the fuzzy analytic evaluation. Thus, the range of ΔP can be obtained and then the range of formation pressure P can be derived. These data will provide a useful reference for practical engineering and related well test analysis. At the same time, from Figure 5, we can see the membership degree corresponding to different ΔP.

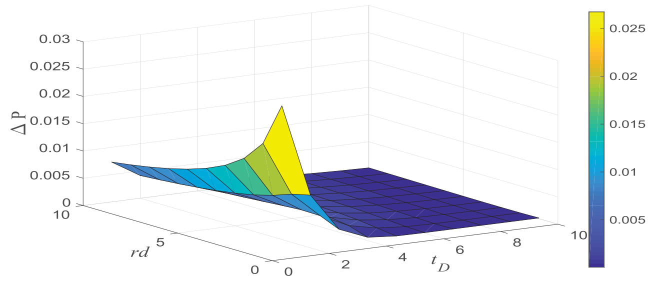

FIGURE 6. 3D relation plot of t_D,r_D and Δp.

It can be seen from Figure 6 that when t_D and r_D are increased to a certain extent, P tends to be stable.

IV. CONCLUSION

The ideas of this paper can be summarized as follows. Firstly, fuzzy triangular permeability is defined because of the extreme irregularity of porous medium space in the oil and gas reservoir. Secondly, based on fuzzy triangular permeability, the definite fuzzy solution of a single fracture system under different outer boundary conditions is established. This fuzzy solution problem is based on fuzzy analysis and is closer to the percolation condition of oil and gas reservoir. Finally, a mathematical method for calculating reservoir fuzzy percolation problem is obtained.

The solution of fuzzy seepage definite solution problem with fuzzy triangular permeability can be approximately expressed as a fuzzy function. This conclusion is of major practical significance. No doubt, compared with the traditional seepage model, the permeability expressed by a triangular fuzzy number is more in line with the reservoir percolation situation. At the same time, as long as the solution of $\triangle P(r, K)$,$\triangle P(r, K^m)$ and $\triangle P(r, K^{\frac{1}{m}})$ of three individual cases of fuzzy seepage definite solution problem are given, for any given K^*, we can get the fuzzy solution of fuzzy seepage equation conveniently. This method offers a new idea for the study of the seepage of oil and gas reservoirs.

A perfect oil and gas reservoir seepage model should also consider the process of gas dispersion, temperature propagation and other models of calculation. These problems still exist vague, which need to be solved in future research. On the other hand, not only the permeability of fracture is fuzzy, and other main control parameters of matrix permeability are also fuzzy, so the formation of multiple fuzzy parameter differential equations also needs our subsequent research.

APPEDIX A
FUZZY NUMBERS

Definition 1: Let E^1 denote the set of all functions $\mu : R \to [0, 1]$ such that μ satisfies(1)~(4):

(1)u is normal (there exists an $x_0 \in R$ such that $u(x_0) = 1$),

(2)u is fuzzy convex (for $x, y \in R$ and $\lambda \in [0, 1]$ $u(\lambda x + (1-\lambda)y) \geq min\{u(x), u(y)\}$),

(3)u is upper semicontinuous,

(4)$[u]^0$ the closure of $\{x \in R : u(x) > 0\}$ is compact.

In the application of fuzzy mathematics, the most frequently used fuzzy number has two kinds, one is called normal fuzzy number, and the other is triangular fuzzy number. The membership function of normal fuzzy number is:

$$\mu_A(x) = exp\{-(\frac{x-a}{b})^2\}(b > 0, x \in R)$$

The membership function of triangular fuzzy number $A \in N(R)$ is:

$$\mu_A(x) = \begin{cases} \frac{x-a}{m-a}, & a \le x \le m, \\ \frac{x-b}{m-b}, & m < x \le b, \\ 0, & x < a, x > b. \end{cases} \tag{27}$$

In the fuzzy number defined above, m is the kernel of the triangular fuzzy number, a and b are the infimum and supremum of the supper set. Since a triangular fuzzy number can be uniquely represented by these three numbers, a triangular fuzzy number is represented by symbols $A < a, m, b >$.

A triangular fuzzy number is one of the most individual and straightforward fuzzy numbers. Both its expression and the two element operation given by the expansion principle are very concise. At the same time, it is also a fuzzy number which is often used in some practical problems. Given the limitations of the traditional oil and gas reservoir seepage model, this paper uses fuzzy triangular permeability $\tilde{K}$ to represent the permeability of oil and gas reservoir fractures.

The study [22] points out that if the selection of fuzzy permeability by $A_{Km} = A < K^m, K, K^{\frac{1}{m}} >$ generation, and $|K - K^m|$ and $|K - K^{\frac{1}{m}}|$ are very small in solving the fuzzy differential equation of gas flow in goaf. Then, in addressing the fuzzy differential equation of oil and gas reservoir seepage, we can first calculate the three similar solutions when the permeability of medium is K^m, K and $K^{\frac{1}{m}}$ respectively. When the average permeability coefficient is F, the corresponding solution of the seepage equation can be approximately expressed by the three specific solutions. This method converts the problem of solving fuzzy differential equations into ordinary differential equations. Therefore, we will naturally guess whether the technique is still suitable for the question of reservoir seepage. The following will prove that this method is still ideal for seepage problems in oil and gas reservoirs. This will significantly promote the development of percolation theory in oil and gas reservoirs. Given this, to solve the fuzzy differential equation, the three definite solutions must be first obtained.

APPEDIX B
GENERAL FORM OF FUZZY DIFFERENTIAL EQUATIONS

The permeability of the spatial medium is a critical parameter in the seepage equation describing the internal migration law of oil and gas reservoirs, and the permeability is related to the spatial characteristics of the medium. Because the reservoir is unobservable, any accurate estimation of permeability is unreliable, and the reliable method is to use fuzzy numbers as permeability. Therefore, the solution of the equation can only be a fuzzy-valued function, and its derivative is also a fuzzy-valued function. There is no doubt that the expression of this solution provides a reliable guarantee for further analysis of the internal migration law of reservoir.

Fuzzy differential equations are the conditional equality between unknown fuzzy functions and its fuzzy derivative functions and the known fuzzy functions. The unknown functions in fuzzy differential equations are fuzzy functions, mainly due to the existence of known fuzzy functions or fuzzy numbers in the parameter set of equations. In fact, for a differential equation written in the following form:

$$\tilde{f}'(x) + p(x)\tilde{f}(x) = g(x)$$

$p(x)$ and $g(x)$ are known real-valued functions, and the general solution of the equation is

$$\tilde{f}(x) = e^{-\int p(x)dx}(\int g(x)e^{\int p(x)dx}dx + c)$$

Eq.(28) is not a fuzzy-valued function. If the $p(x)$ or $g(x)$ of the expression Eq.(28) is at least one fuzzy function, the integral of the fuzzy function is a fuzzy function. At this point, the general solution of the equation must be the fuzzy-valued function. So we say that the differential equations with fuzzy functions or fuzzy coefficients in the parameter set (a set of known functions and constants in Fuzzy Differential Equations) are fuzzy differential equations.

APPEDIX C
THE EXPRESSION FORMS OF SOLUTIONS

A known function or constant in the ordinary differential equation is replaced by a fuzzy function or a fuzzy number (collectively referred to the as fuzzy parameter), then the equation becomes a fuzzy differential equation.

Definition 2: If the equation contains only one fuzzy parameter, it is called a single parameter fuzzy differential equation. If the fuzzy parameter in fuzzy differential equation is replaced by the definite parameter, the ordinary differential equation is called the description equation of this fuzzy differential equation. The specific parameter of substitution is called characterization parameter.

If we use $F(T; x; D^m; \tilde{H}(t)) = 0$ to denote a fuzzy differential equation with fuzzy parameter terms $\tilde{H}(t)$, its descriptive equation is denoted as $F(T; x; D^m) = 0$.

Definition 3: Consider the ordinary differential equation (ordinary differential equation or partial differential equation) with parameter term $h(t)$, t is the vector on n-dimensional real space T (if the equation is ordinary differential equation, then $n = 1$). Let the solution of differential equation be $f(t)$ under the given initial condition D_0. In order to study the effect of the parameter term represented by $h(t)$ on the solution of equation, we remember $f(t) = F_s(h(t); D_0)$, and call $f(t)$ the associated solution of the function $h(t)$ and the conditional D_0.

The interval valued function in a single parameter interval differential equation is $\bar{g}(t) = [g_1(t), g_2(t)]$, and

$f_1(t) = F_s(g_1(t); D_0), f_2(t) = F_s(g_2(t); D_0)$ are given. If $\forall t \in T, f_1(t) \le f_2(t)$ or $f_2(t) \le f_1(t)$ is constant, we can construct an interval valued function $\overline{f}(t) = [f^{(1)}(t), f^{(2)}(t)]$ in form, where $f^{(1)}(t) = min\{f_1(t), f_2(t)\}, f^{(2)}(t) = max\{f_1(t), f_2(t)\}$. But $\overline{f}(t)$ can not be called the solution of the interval equation. If the interval value function $\overline{f}(t)$ is the solution of the equation, then the condition of the following definition must be satisfied.

Definition 4: Interval valued function $\overline{f(t)} = [f^1(t), f^2(t)]$ is the definite solution of interval differential equation with $\overline{g(t)} = [g_1(t), g_2(t)]$ as the parameter of a single interval(the solutions of $f^{(1)}(t)$ and $f^{(2)}(t)$ are the corresponding equations of the parameter terms $g_1(t)$ and $g_2(t)$ or $g_2(t)$ and $g_1(t)$, respectively, under the same fixed solution condition D_0, if and only if the function $g(t)$ on the given T, if $g_1(t) \le g(t) \le g_2(t)$, has $f^{(1)}(t) \le f(t) \le f^{(2)}(t)$, and $f(t) = F_s(g(t); D_0)$ in here.

Theorem 4: In the single parameter interval differential equation with $\overline{g(t)} = [g_1(t), g_2(t)]$ as parameter, let $f_1(t) = F_s(g_1(t); D_0)$, $f_2(t) = F_s(g_2(t); D_0)$. If the solution of this equation is monotone about the parameter, then the interval valued function $\overline{f(t)} = [f^1(t), f^2(t)]$ is the solution of the interval differential equation, and $f^1(t) = min\{f_1(t), f_2(t)\} f^2(t) = max\{f_1(t), f_2(t)\}$ in here.

Proof: Let $f_1(t) = F_s(g_1(t); D_0), f_2(t) = F_s(g_2(t); D_0)$, and $\forall t \in T$, $g_1(t) \le g_2(t)$, $g(t)$ be a function of $T, f(t) = F_s(g(t); D_0)$. If the solution and the parameter of the equation are in the same order, then $\forall t \in T$, there is $f_1(t) \le f_2(t)$. So, when $g_1(t) \le g(t) \le g_2(t)$, there will be $f_1(t) \le f(t) \le f_2(t)$ set up. If the solution and the parameter of the equation are in inverse order, then $\forall t \in T$, there is $f_1(t) \ge f_2(t)$. So, when $g_1(t) \le g(t) \le g_2(t)$, there will be $f_1(t) \ge f(t) \ge f_2(t)$ set up. It is shown that if the parameter $g(t) \in [g_1(t), g_2(t)]$, there must be a corresponding solution $f(t) \in [f^1(t), f^2(t)]$. □

Definition 5: Fuzzy differential equations with single fuzzy parameters(t is an n-dimensional real vector and $n = 1$ when it is an ordinary differential equation). For a given $\lambda \in (0, 1]$, $\overline{f_\lambda}(t)$ is the solution of the interval differential equation obtained by replacing the equation parameter $\widetilde{g}(t)$ with the λ cut set $\overline{g_\lambda}(t)$ of $\widetilde{g}(t)$. And for $\forall 0 < \lambda_1 \le \lambda_2 \le 1$, there is $\overline{f}_{\lambda_1} \supseteq \overline{f}_{\lambda_2}$. The fuzzy solutions of fuzzy differential equations are represented by the representation theorem as follows:

$$\widetilde{f}(t) = \bigcup_{\lambda \in [0,1]} \lambda * \overline{f_\lambda}(t) \tag{28}$$

Moreover, the solution of fuzzy differential equation can be expressed.

Theorem 5: If the solution of the fuzzy differential equation with fuzzy parameter $\widetilde{g}(t)$ can be expressed, The solution $\widetilde{f}(t)$ of the equation of form like eq(5) is a fuzzy function on T.

Proof: By the definition of $\lambda * \overline{f_\lambda}(t)$, for the given $\lambda \in (0, 1]$, the α-cuts of the solution function $\widetilde{f}(t)$ of the equation is $[\widetilde{f}(t)]_\lambda = \overline{f_\lambda}(t)$, suppose $\overline{f_\lambda}(t) = [f_1(t), f_2(t)]$. The assumption $\overline{f_\lambda}(t)$ is an interval solution of the interval equation corresponding to the parameter term $\overline{g_\lambda}(t)$. By definition, let $f(t) = F_s(g(t); D_0)$, if $f_1(t) \le f(t) \le f_2(t)$, then there must be $g(t) \in \overline{g_\lambda}(t)$. Thus, the membership degree of $g(t)$ on $\widetilde{g}(t)$ is greater than or equal to λ. Furthermore, the membership degree of $f(t)$ about $\widetilde{f}(t)$ is greater than or equal to λ. This shows that for any $t_0 \in T, \widetilde{f}(t_0)$ is convex. Moreover, since $\widetilde{g}(t_0)$ is a fuzzy number, the interval solution $\widetilde{f_\lambda}(t_0)$ of $\lambda = 1$ exists, and then $\widetilde{f}(t_0)$ is normal, so $\widetilde{f}(t_0)$ is fuzzy number. So $\widetilde{f}(t)$ is a fuzzy function on T. □

Theorem 6: For a fuzzy differential equation with single fuzzy parameter, if the parameter of the corresponding description equation is monotone with the equation solution, the solution of the fuzzy differential equation can be expressed.

Proof: The fuzzy parameter of the equation may be set as $\widetilde{h}(t)$, and for $\forall\lambda \in (0, 1]$, the λ-cuts of the fuzzy function $\widetilde{h}(t)$ is $\overline{h_\lambda}(t) = [h_\lambda^{(1)}(t), h_\lambda^{(2)}(t)]$, $f_\lambda^{(1)}(t) = F_s(h_\lambda^{(1)}(t); D_0)$, $f_\lambda^{(2)}(t) = F_s(h_\lambda^{(2)}(t); D_0)$. Because the description parameter and the equation solution of the descriptive equation are monotone, it is known by Theorem 4.4 that the interval valued function $\overline{f_\lambda}(t)$ with $f_\lambda^{(1)}(t)$ and $f_\lambda^{(2)}(t)$ as the end point must be the solution of the interval equation corresponding to the fuzzy differential equation with $\overline{h_\lambda}(t)$ as the parameter term. And for $0 < \lambda_1 \le \lambda_2 \le 1$, there's $\overline{f_{\lambda_1}} \supseteq \overline{f_{\lambda_2}}$. Therefore, the solution of the fuzzy differential equation can be expressed. □

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