Received November 26, 2018, accepted December 6, 2018, date of publication December 24, 2018,
date of current version January 16, 2019.

*Digital Object Identifier 10.1109/ACCESS.2018.2887073*

# Observer-Based Adaptive Neural Control for Non-Triangular Form Systems With Input Saturation and Full State Constraints

**RUI ZHANG**[1,2] **AND JUNMIN LI**[1]

[1]School of Mathematics and Statistics, Xidian University, Xi'an 710126, China
[2]Institute of Mathematics and Information Science, Baoji University of Arts and Sciences, Baoji 721013, China

Corresponding author: Junmin Li (jmli@mail.xidian.edu.cn)

This work was supported in part by the National Science Foundation of China under Grant 61573013.

**ABSTRACT** This paper addresses the problem of adaptive output feedback control for a class of non-triangular time-varying delay system with input constraints and full-state constraints. A variable separation approach is adopted to overcome the design difficulty from the non-triangular structure. A novel Lyapunov function is introduced to compensate the time-delay terms. Unknown functions are approximated by the radial basis function neural networks. Only one parameter needs to be adjusted online, and a dynamic surface control technique is employed to reduce the computation burden. Combining the barrier Lyapunov function with a backstepping technique in the controller design procedure, the proposed controller guarantees that all the signals in the closed-loop system are uniformly ultimately bounded and the full-state constraints are met. The simulation results demonstrate the effectiveness of the proposed approach.

**INDEX TERMS** Adaptive neural control, non-triangular form systems, full state constraints, input saturation.

## I. INTRODUCTION

The adaptive neural or fuzzy control have attracted much attention in past decades. Based on neural networks (NNs) or fuzzy logic systems (FLSs) inherent good approximation ability for uncertain functions of the systems, many research results have been made e.g., see [1]–[30]. For strict-feedback systems, when states were measured, different adaptive methods were given in [2], [4]–[6], [10], and [16]. In [4], tracking control for perturbed strict-feedback nonlinear delay systems has been studied. Reference [20] proposed a distributed coordination control approach for multi-agent systems with dead-zone input. For states were unmeasured, the researchers did some study in [8], [12], [19], [20], and [24]. In [8] an output feedback control method was presented for nonlinear time-varying delay systems with unknown control direction. For pure-feedback systems, [3] investigated the problem of the tracking control design for pure-feedback systems with an ISS-modular approach. Reference [18] addressed the problem of adaptive tracking control design for pure-feedback systems with learning control. Compared to the above two classes of systems, the non-triangular form system which nonlinearities function of it include the whole states [25]–[32] is more general. The strict-feedback and pure-feedback systems can be seen as its special cases. Reference [31] investigated the problem of adaptive tracking control for stochastic nonstrict-feedback switch systems. In [26] an output feedback adaptive control method was proposed for a class of nonstrict-feedback stochastic nonlinear systems. However, the ‘‘explosion of complexity’’ were exist in backstepping design process in [10], [11], [20], [29], and [30]. To overcome this drawback, the DSC technique were employed for strict-feedback systems [13], [17], [34], for pure-feedback system [9] and for nonstrict-feedback systems [32]. In [32], Niu *et al.* gave an output feedback control approach for stochastic interconnected nonlinear nonstrict-feedback systems with dead zone input. The DSC technique which has been applied in these literatures achieved the good tracking performance and really reduces the computation burden.

It is well known, in practice systems, the time-delay is often appears or input saturation is required, if they are handed inappropriately, they can degrade the system performance and even lead to system instability, hence analysis and design of the nonlinear systems with input saturation or time-delay become an important topic, much interesting research results on these issues have been obtained in recent years [33]–[41]. In [33] a control method for uncertain discrete-time nonlinear systems with input saturation was proposed. An dynamic model was employed to describe the saturation nonlinearity with DSC for strict-feedback system was studied in [34]. When the states were unmeasured, [37] proposed control scheme for nonlinear systems with unknown control directions and input saturation In [41], Zhou *et al.* presented adaptive output feedback fuzzy tracking control method for time-delay and input saturation nonlinear systems with DSC. However, the above methods are only feasible in the nonlinear triangular form systems and the constraints for the states were ignored.

Constraints have become an important issues in many control systems, the constraints may appear in the output, input or states, such as physical stoppage, saturation, performance and safety specifications [42]. Recently, partial-state constraints and full state constraints have been explored in this area by using Barrier Lyapuov Functions (BLF) [43]–[50], [53], [54]. In [44] partial state constraints was explored by using BLF for strict-feedback systems. Reference [45] investigated the adaptive output constraints issue for nonlinear strict-feedback systems with states unmeasured. In real systems, the full state constraints may be required, [48] dealt with the tracking control problem for an uncertain n-link robot with full state constraints. For states were unmeasured, [46] and [49] gave the output feedback control methods for strict-feedback systems. For pure-feedback systems, in [47], Liu and Tong proposed a method for pure-feedback systems by employing the mean value theorem transformed it into the strict-feedback form with full state constraints. Reference [50] studied the adaptive tracking control issue for switch pure-feedback systems with full state constraints. From the above observations, we can see that most of the research results on full state constraints are limited in strict-feedback and pure-feedback form without DSC, they may be invalidated on non-triangular form systems. It is natural question how to design the non-triangular form systems especially the nonlinear function include the whole states with DSC, To the best of the authors' knowledge, there is seldom published works for such output feedback system with DSC, especially the time-varying delay and input saturation can be considered simultaneously. This problem motivates the research of this paper. In this paper, we will employ the BLF to investigate the problem of adaptive tracking control design for non-triangular form time-varying delay system with input saturation and full state constraints.

The main contributions of this paper are summarized as follows:

1) By constructing a new Lyapunov function, a DSC-based adaptive output feedback neural control method is presented for uncertain non-triangular form time-varying delay systems with input saturation and full state constraints. Compared with [1]–[8], [10]–[12], [14]–[16], and [18]–[24], the system in this paper is more general and the computation burden is reduced.

2) Only one parameter needs to be adjusted in controller design procedure, this reduce the online computation burden greatly.

3) It can guarantees that all the signals in the closed-loop system are uniformly ultimately bounded and the full state constraints are not violated.

The rest of paper is organized as follows: Section 2 present problem formulation and preliminaries. In Section 3 the state observer, adaptive neural control scheme and stability analysis are given. Two examples are performed to demonstrate the effectiveness of the proposed method in Section 4, and Section 5 provides a summary of the work performed.

## II. PROBLEM FORMULATION AND PRELIMINARIES

### A. SYSTEMS REPRESENTATION

Consider the following non-triangular time-varying delay system:

$$\begin{cases} \dot{x}_i = x_{i+1} + f_i(x) + m_i(x(t-\tau_i(t))) + d_i(x,t), \\ \qquad 1 \le i \le n-1 \\ \dot{x}_n = u(v) + f_n(x) + m_n(x(t-\tau_n(t))) + d_n(x,t), \\ y = x_1. \end{cases} \tag{1}$$

where $x = [x_1, x_2, \cdots, x_n]^T \in R^n$ and $y \in R$ denote the system state and control output, respectively. For $i = 1, \cdots, n$, functions $f_i(\cdot)$ and $m_i(\cdot)$ are unknown smooth functions with $f_i(0) = 0, m_i(0) = 0 \tau_i(t)$ are the time-varying delay of the *ith* subsystems, $d_i(\cdot)$ denote the external disturbance. The full states are limited as $|x_i(t)| < M_i$ with $M_i$ being a positive constant. $u(v(t))$ is the saturation nonlinearly input function, which is defined as:

$$\begin{aligned} u(v(t)) &= sat(v(t)) \\ &= \begin{cases} sign(v(t))\, u_M, & |v(t)| \ge u_M, \\ v(t), & |v(t)| < u_M, \end{cases} \end{aligned} \tag{2}$$

where $v(t) \in R$ is the control input, $u_M$ is an upper bound of $u(t)$, $sign(\cdot)$ and $sat(\cdot)$ are the standard unit sign function and saturation function, respectively. From the definition of the saturation function (2), $u(t)$ has a sharp corner when $|v(t)| = u_M$, thus backstepping technique cannot be directly applied for (1) [51], the saturation nonlinearity with smooth form which can be instead of (2) was described by [35] as follows

$$h(v) = u_M \times \tanh(\frac{v}{u_M}) = u_M \frac{e^{v/u_M} - e^{-v/u_M}}{e^{v/u_M} + e^{-v/u_M}}. \tag{3}$$

To facilitate the control system design, the (2) can be reformulated as

$$u(v) = h(v) + p(v), \tag{4}$$
$$|p(v)| = |u(v) - h(v)| \le u_M(1 - \tanh(1)) \triangleq K_1, \tag{5}$$

where $K_1$ is a positive constant.

*Remark 1:* From (1), we can see that the unknown functions $f_i(\cdot)$ and $m_i(\cdot)$ include all state variables, so it is more general than systems in [1]–[24] and [33]–[50].

 

*Assumption 1 [28]:* For nonlinear functions $f_i(\cdot)$ and $m_i(\cdot)$, there exist positive constants $p_i, q_i$ such that

$$\left|f_i(x) - f_i(\hat{x})\right| \le p_i \left\|x - \hat{x}\right\|,$$
$$\left|m_i(x) - m_i(\hat{x})\right| \le q_i \left\|x - \hat{x}\right\|.$$

for all $x, \hat{x} \in R^n$.

*Assumption 2:* It is assumed that desired signal $y_r(t)$, its 1*th* derivative $y_r^{(1)}(t)$ and 2*th* derivative $y_r^{(2)}(t)$ satisfy $|y_r(t)| \le \kappa_0 < M_1$, $\left|y_r^{(1)}(t)\right| \le \kappa_1$, $\left|y_r^{(2)}(t)\right| \le \kappa_2$ where $\kappa_0, \kappa_1, \kappa_2$ are positive constants.

*Assumption 3:* $d_i(x, t), i = 1, 2, \cdots, n$ are bounded external disturbance, there exists known positive constant $\bar{d}_i$, such that $|d_i(x, t)| \le \bar{d}_i$.

*Assumption 4:* The discrete and distributed time-varying delays $\tau_i(t), i = 1, \cdots, n-1$ satisfy $0 \le \tau_i(t) \le d_1$ and the time derivations of $\tau_i(t)$ satisfies $\dot{\tau}_i(t) \le d_1^* < 1$, respectively, where $d_1, d_1^*$ are constants.

*Assumption 5:* There exists a constant $m > 0$, such that $|\tilde{\rho}| \le m$, $\tilde{\rho}$ is an auxiliary design signal, which dynamic will be described in the later.

*Lemma1 [Young's Inequality] [1]:* For $(x, y) \in R^2$ the following inequality holds:

$$xy \le \frac{\varsigma^p}{p}|x|^p + \frac{1}{q\varsigma^q}|y|^q, \tag{6}$$

where $\varsigma > 0, p > 1, q > 1$ and $(p-1)(q-1) = 1$.

*Lemma 2 [45]:* For any positive constant $k_{b_i}$, if $z_i$ satisfy $|z_i| < k_{b_i}$, the following inequality holds.

$$\log \frac{k_{b_i^2}}{k_{b_i^2} - z_i^2} < \frac{z_i^2}{k_{b_i^2} - z_i^2} \tag{7}$$

The control objective of this paper is to design an adaptive neural output feedback controller such that:

1) the system output $x_1$ tracks the reference trajectory $y_r(t)$ and guarantees that all the signals in the closed-loop system are uniformly ultimately bounded; 2) the full state constraints are not violated.

### B. NEURAL NETWORK APPROXIMATION

The radial basis function neural network (RBFNN) belongs to a class of linearly parameterized network. An unknown and continuous nonlinear function $f_i(Z): \Omega_z \to R$ can be approximated by RBFNN over a compact set $\Omega_Z$ as follows:

$$f_i(Z) = \Phi_i^{*T}\xi_i(Z) + \delta_i(Z), \quad \forall Z \in \Omega_Z \subset R^q \tag{8}$$

where $Z \in \Omega_Z \subset R^q$ is the input vector, $\xi_i(Z) = [\xi_{i1}(Z), \xi_{i2}(Z), \cdots \xi_{il}(Z)] \in R^l$ is basis function vector with the NN node number $l > 1$, $\delta_i(Z)$ is the approximation error satisfying $|\delta_i(Z)| \le \varepsilon_i$, $\varepsilon_i$ are positive constants. $\xi_{ij}(Z)$ are chosen as Gaussian function in the following form

$$\xi_{ij}(Z) = \exp\left[\frac{-(Z - \varpi_{ijk})^T (Z - \varpi_{ijk})}{\eta_{ij}^2}\right] \tag{9}$$

where $\varpi_{ijk} \in \Omega_Z$ are the centers and $\eta_{ij}$ are the width of the receptive field, respectively. $\Phi_i^* = [\phi_{i1}, \phi_{i2}, \cdots, \phi_{il}] \in R^l$ is the optimal weight vector, which are defined as follows:

$$\Phi_i^* = \arg\min_{\Phi_i \in R^l}\left\{\sup_{Z\in\Omega_Z}\left|f_i(Z) - \Phi_i^T\xi_i(Z)\right|\right\}. \tag{10}$$

*Remark 2:* $\xi_i(x)$ are radial basis function vector and satisfy $\xi_i^T(x)\xi_i(x) \le 1$. $\Phi_i^*$ is ideal weights vector, which are unknown, we define $\theta^* = \max\left\{\left\|\Phi_i^*\right\|^2, i = 1, 2, \cdots, n\right\}$, $\hat{\theta}$ is the estimation of the $\theta^*$, and the estimation error $\tilde{\theta} = \hat{\theta} - \theta^*$. Compared with multiple parameters need to be estimated in [25], [29], and [49], in this paper, only unknown parameter $\theta^*$ needs to be estimated, the computational burden can be reduced. However, when different unknown parameters need to be estimated, it should be use the multiple adaptive laws [22].

## III. OUTPUT FEEDBACK ADAPTIVE CONTROLLER DESIGN AND STABILITY ANALYSIS

### A. STATE OBSERVER DESIGN

In this paper, only state $x_1$ is measured and the other states are unavailable, let us consider the linear observer for system (1):

$$\begin{cases}\dot{\hat{x}}_i = \hat{x}_{i+1} + k_i(y - \hat{x}_1), & i = 1, \cdots, n-1, \\ \dot{\hat{x}}_n = u(v) + k_n(y - \hat{x}_1), \end{cases} \tag{11}$$

where $k_i, i = 1, 2, \cdots, n$ are positive design parameters, $\hat{x}_i, i = 1, 2, \cdots, n$ are the observer states. $\hat{x} = [\hat{x}_1, \hat{x}_2, \cdots \hat{x}_n]^T$ and $e = x - \hat{x}$ are the observer state vector and observer error vector, then the observer error dynamic can be described as follows:

$$\dot{e} = Ae + F(x) + M(x(t - \tau(t))) + D(x, t) \tag{12}$$

where

$$A = \begin{bmatrix} -k_1 & & & \\ \vdots & & I_{n-1} & \\ -k_n & 0 & \cdots & 0 \end{bmatrix}, \quad F(x) = \begin{bmatrix} f_1(x) \\ \vdots \\ f_n(x) \end{bmatrix}$$

$$M(x(t - \tau(t))) = \begin{bmatrix} m_1(x(t - \tau_1(t))) \\ \vdots \\ m_n(x(t - \tau_n(t))) \end{bmatrix}$$

$$D(x, t) = \begin{bmatrix} d_1(x, t) \\ \vdots \\ d_n(x, t) \end{bmatrix}$$

Choose positive constants $k_i, i = 1, 2, \cdots, n$ such that $A$ is a strict Hurwitz matrix, hence there exists a matrix $P = P^T > 0$ such that

$$A^T P + PA = -Q. \tag{13}$$

### B. ADAPTIVE NEURAL CONTTOLLER DESIGN AND STABILITY ANALYSIS

In this section, adaptive neural controller will be proposed by using the backstepping technique.

The change of coordinates are given as follow:

$$z_1 = \hat{x}_1 - y_r \tag{14}$$

$$z_i = \hat{x}_i - \alpha_{if}, \quad i = 2, \cdots, n-1, \tag{15}$$

$$z_n = \hat{x}_n - \alpha_{nf} - \tilde{\rho} \tag{16}$$

$$\chi_i = \alpha_{if} - \alpha_{i-1}, \quad i = 2, \cdots, n \tag{17}$$

where $z_i$ is the virtual error, $\alpha_{if}$ is the first-order filter output, $\chi_i$ is the filter error, $\alpha_{i-1}$ is the virtual control input need to be designed later. Let $\alpha_i$ pass through a first-order filter with time constant $\tau_{i+1}$ the $\alpha_{i+1f}$ can be obtained:

$$\begin{cases} \tau_{i+1}\dot{\alpha}_{i+1f} + \alpha_{i+1f} = \alpha_i, \\ \alpha_{i+1f}(0) = \alpha_i(0), \end{cases} \quad i = 1, \cdots n-1 \tag{18}$$

The virtual control input signals $\alpha_i$, actual control input $v$ and parameter adaptive law for $\hat{\theta}$ are developed as:

$$\alpha_i = -c_i z_i - \frac{z_i}{2\left(k_{b_i}^2 - z_i^2\right)} - \frac{\hat{\theta} z_i \xi_i^T(Z_i)\,\xi_i(Z_i)}{2\eta_i^2\left(k_{b_i}^2 - z_i^2\right)}, \quad 1 \le i \le n-1, \tag{19}$$

$$v = -c_n z_n - \frac{z_n}{2\left(k_{b_n}^2 - z_n^2\right)} - \frac{\hat{\theta} z_n \xi_n^T(Z_n)\,\xi_n(Z_n)}{2\eta_n^2\left(k_{b_n}^2 - z_n^2\right)} - \tilde{\rho} \tag{20}$$

$$\dot{\hat{\theta}} = \sum_{i=1}^{n} \frac{p z_i^2 \xi_i^T(Z_i)\,\xi_i(Z_i)}{2\eta_i^2} - \sigma\hat{\theta} \tag{21}$$

where $c_i, \eta_i, c_n, \eta_n, p, \sigma$ are positive design parameters, $\hat{\theta}$ is the estimation of parameter $\theta^*$, $Z_i = \left[\hat{x}_1, \cdots \hat{x}_i, \hat{\theta}, y_r, \dot{y}_r, \ddot{y}_r,\right]^T \in R^{i+4}$.

*Lemma 3:* For change of coordinates $z_1 = \hat{x}_1 - y_r$ $z_i = \hat{x}_i - \chi_i - \alpha_{i-1}, i = 2, \cdots n-1$ and $z_n = \hat{x}_n - \chi_n - \alpha_{n-1} - \tilde{\rho}$, the following inequality holds:

$$\|\hat{x}\| \le \sum_{i=1}^{n} |z_i|\,\beta_i\left(z_i, \hat{\theta}\right) + \sum_{i=2}^{n} \chi_i + \mu \tag{22}$$

where

$$\beta_i\left(z_i, \hat{\theta}\right) = 1 + c_i + \frac{1}{2\left(k_{bi}^2 - z_i^2\right)} + \frac{\hat{\theta}}{2\eta_i^2\left(k_{bi}^2 - z_i^2\right)}$$

$$\mu = \kappa_0 + m$$

*Proof:* $\|\hat{x}\| \le \sum_{i=1}^{n} |\hat{x}_i| = |\hat{x}_1| + \sum_{i=2}^{n} |\hat{x}_i|$

$$\le |z_1 + y_r| + \sum_{i=2}^{n-1}\{|z_i + \alpha_{i-1} + \chi_i|\} + |z_n + \alpha_{n-1} + \chi_n + \tilde{\rho}|$$

$$\le \sum_{i=1}^{n} |z_i| + \sum_{i=1}^{n-1} |\alpha_i| + \sum_{i=2}^{n} \chi_i + |y_r| + |\tilde{\rho}|$$

$$\le \sum_{i=1}^{n} |z_i| + \sum_{i=1}^{n-1}\left|c_i z_i + \frac{z_i}{2\left(k_{bi}^2 - z_i^2\right)} + \frac{\hat{\theta} z_i \xi_i^T \xi_i}{2\eta_i^2\left(k_{bi}^2 - z_i^2\right)}\right|$$

$$+ \sum_{i=2}^{n} \chi_i + \mu \le \sum_{i=1}^{n} |z_i|\,\beta_i\left(z_i, \hat{\theta}\right) + \sum_{i=2}^{n} \chi_i + \mu.$$

*Remark 3:* According to (18), because $\sum_{i=1}^{n} \frac{p z_i^2 \xi_i^T(Z_i)\xi_i(Z_i)}{2\eta_i^2}$ is nonnegative, so we know that if initial condition $\hat{\theta}(t_0) \ge 0$, then the solution $\hat{\theta}(t) \ge 0$ for all $t \ge t_0$.

Combining with (9), (10) and (12), one obtains

$$\begin{cases} \dot{e} = Ae + F(x) + M(x(t - \tau(t))) + D(x, t), \\ \dot{z}_1 = \hat{x}_2 + k_1 e_1 - \dot{y}_r, \\ \dot{\hat{x}}_i = \hat{x}_{i+1} + k_i e_1, \quad i = 2, \cdots, n-1, \\ \dot{\hat{x}}_n = u(v) + k_n e_1. \end{cases} \tag{23}$$

A block diagram of control system is shown in Figure.1.

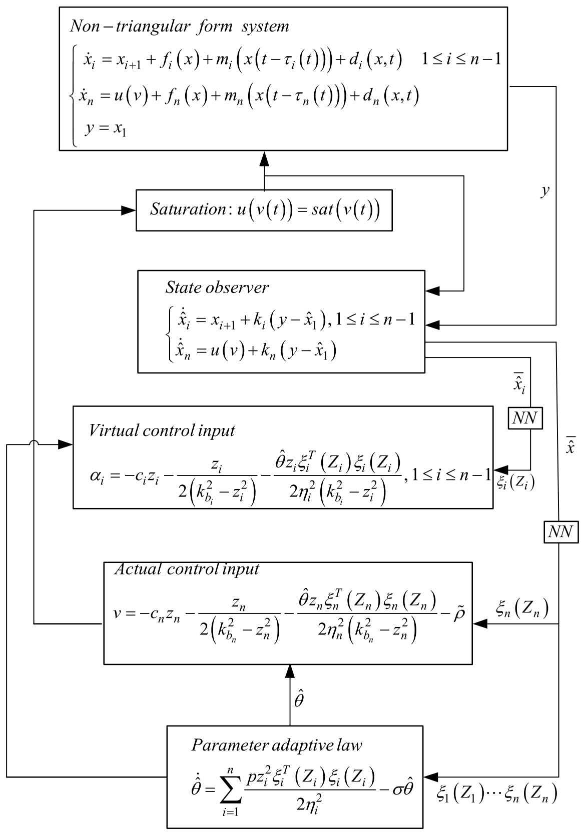


**FIGURE 1.** **Block diagram of controlled system.**

We consider the following Lyapunov function:

$$\tilde{V} = V_e + V_z + V_H \tag{24}$$

with

$$V_e = \frac{1}{2} e^T P e$$

$$V_z = \sum_{i=1}^{n} V_i$$

$$V_i = \frac{1}{2}\log\frac{k_{b_i}^2}{\left(k_{b_i}^2 - z_i^2\right)} + \frac{\chi_{i+1}^2}{2}, \quad i = 1, \cdots, n-1$$

$$V_n = \frac{1}{2}\log\frac{k_{b_n}^2}{\left(k_{b_n}^2 - z_n^2\right)}$$

$$\begin{aligned} V_H &= \frac{e^{-\gamma(t-d_1)}}{1-d_1^*}\sum_{i=1}^{n}\int_{t-\tau_i(t)}^{t}\frac{1}{4}\|P\|^2 e^{\gamma s} q_i^2 e_i^2(s)\,ds \\ &+ \frac{e^{-\gamma(t-d_1)}}{1-d_1^*}\sum_{i=1}^{n}\int_{t-\tau_i(t)}^{t} e^{\gamma s}\bar{c} z_i^2(s)\beta_i^2(s)\,ds \\ &+ \frac{e^{-\gamma(t-d_1)}}{1-d_1^*}\sum_{i=2}^{n}\int_{t-\tau_i(t)}^{t} e^{\gamma s}\bar{c}\chi_i^2(s)\,ds, \end{aligned}$$

The matrix $P$ satisfies (13) and $\gamma$ is a positive constant.

Next, we consider the time derivative of the Lyapunov function $\tilde{V}$.

First, we take the time derivative of $V_e$, from (12) $\dot{V}_e$ is follow:

$$\begin{aligned} \dot{V}_e &= \frac{1}{2}\dot{e}^T P e + \frac{1}{2} e^T P \dot{e} \\ &= \frac{1}{2} e^T \left(A^T P + PA\right) e + e^T PF(x) \\ &\quad + e^T PM(x(t-\tau(t))) + e^T PD(x,t). \qquad (25) \end{aligned}$$

By applying the Young's inequality, Assumptions 1, 3 and Lemma 3, one can obtain the following inequalities:

$$\begin{aligned} e^T P\left(F(x) - F(\hat{x})\right) &\le \|e\|^2 + \frac{1}{4}\|P\|^2\sum_{i=1}^{n} p_i^2\|e\|^2 \\ &= \varsigma_1\|e\|^2, \qquad (26) \end{aligned}$$

where

$$\begin{aligned} \varsigma_1 &= 1 + \frac{1}{4}\|P\|^2\sum_{i=1}^{n} p_i^2 \cdot e^T PF(\hat{x}) \\ &\le \|e\|^2 + \frac{1}{4}\|P\|^2\left\|F(\hat{x})\right\|^2 \\ &\le \|e\|^2 + \frac{1}{4}\|P\|^2\sum_{i=1}^{n} p_i^2\left\|\hat{x}\right\|^2 \\ &\le \|e\|^2 + \frac{1}{4}\|P\|^2\sum_{i=1}^{n} p_i^2\left[\sum_{i=1}^{n}\beta_i(z_i,\theta)|z_i| + \sum_{i=2}^{n}\chi_i + \mu\right]^2 \\ &\le \|e\|^2 + \frac{1}{2}n\|P\|^2\sum_{i=1}^{n} p_i^2\sum_{i=1}^{n}\beta_i^2(z_i,\theta)z_i^2 \\ &\quad + \frac{1}{2}n\|P\|^2\sum_{i=1}^{n} p_i^2\sum_{i=2}^{n}\chi_i^2 + \frac{1}{2}\|P\|^2\sum_{i=1}^{n} p_i^2\mu^2 \\ &\le \|e\|^2 + c\sum_{i=1}^{n} z_i^2\beta_i^2 + c\sum_{i=2}^{n}\chi_i^2 + \varepsilon_0 \qquad (27) \end{aligned}$$

where

$$\begin{aligned} c &= \frac{n}{2}\|P\|^2\sum_{i=1}^{n} p_i^2,\ \varepsilon_0 = \frac{1}{2}\|P\|^2\sum_{i=1}^{n} p_i^2\mu^2 \\ &\quad \cdot e^T P\left(M(x(t-\tau(t))) - M\left(\hat{x}(t-\tau(t))\right)\right) \\ &\le \|e\|^2 + \frac{1}{4}\|P\|^2\sum_{i=1}^{n} q_i^2 e_i^2(t-\tau_i(t)), \qquad (28) \end{aligned}$$

$$\begin{aligned} & e^T PM\left(\hat{x}(t-\tau(t))\right) \\ &\quad \le \|e\|^2 + \bar{c}\sum_{i=1}^{n} z_i^2(t-\tau_i(t))\beta_i^2(t-\tau_i(t)) \\ &\quad\quad + \bar{c}\sum_{i=2}^{n}\chi_i^2(t-\tau_i(t)) + \bar{\varepsilon}_0 \qquad (29) \end{aligned}$$

$$e^T PD(t) \le \frac{1}{2}\|e\|^2 + \frac{1}{2}\|P\|^2\sum_{i=1}^{n}\bar{d}_i^2, \qquad (30)$$

where $\bar{c} = \frac{n}{2}\|P\|^2\sum_{i=1}^{n} q_i^2$, $\bar{\varepsilon}_0 = \frac{1}{2}\|P\|^2\sum_{i=1}^{n} q_i^2\mu^2$.

Substitute (13) and (26)-(30) into (25) produces that

$$\begin{aligned} \dot{V}_e &\le -\lambda_{\min}(Q)\|e\|^2 + \frac{7}{2}\|e\|^2 + \varsigma_1\|e\|^2 + \varepsilon_0 + \bar{\varepsilon}_0 \\ &\quad + \bar{c}\sum_{i=1}^{n} z_i^2(t-\tau_i(t))\beta_i^2(t-\tau_i(t)) + c\sum_{i=2}^{n}\chi_i^2 \\ &\quad + \bar{c}\sum_{i=2}^{n}\chi_i^2(t-\tau_i(t)) + \frac{1}{4}\|P\|^2\sum_{i=1}^{n} q_i^2 e_i^2(t-\tau_i(t)) \\ &\quad + c\sum_{i=1}^{n} z_i^2\beta_i^2 + 2\varepsilon\|P\|^2\sum_{i=1}^{n}\bar{d}_i^2, \qquad (31) \end{aligned}$$

Second, we take the time derivative of $V_z$, from (24) the definition of $V_z$, first of all it need get $\dot{V}_i$, respectively.

*Step 1:* Take the time derivative of $V_1$, $\dot{V}_1$ is given by

$$\begin{aligned} \dot{V}_1 &= \frac{z_1\dot{z}_1}{k_{b_1}^2 - z_1^2} + \chi_2\dot{\chi}_2 \\ &= \frac{z_1}{k_{b_1}^2 - z_1^2}\left(\hat{x}_2 + k_1 e_1 - \dot{y}_r\right) + \chi_2\dot{\chi}_2 \\ &= \frac{z_1}{k_{b_1}^2 - z_1^2}\left(z_2 + \alpha_1 + \chi_2 + k_1 e_1 - \dot{y}_r\right) + \chi_2\dot{\chi}_2, \end{aligned} \qquad (32)$$

$$\frac{z_1}{k_{b_1}^2 - z_1^2}k_1 e_1 \le \frac{\varpi_1}{2}\|e_1\|^2 + \frac{1}{2\varpi_1}\frac{z_1^2}{\left(k_{b_1}^2 - z_1^2\right)^2}k_1^2, \qquad (33)$$

$$\frac{z_1}{k_{b_1}^2 - z_1^2}\chi_2 \le \frac{z_1^2}{2\left(k_{b_1}^2 - z_1^2\right)^2} + \frac{\chi_2^2}{2}. \qquad (34)$$

From (17), we can get

$$\chi_2\dot{\chi}_2 \le -\frac{\chi_2^2}{\tau_2} + \frac{\chi_2^2 K_2^2}{2\delta_2} + \frac{\delta_2}{2}, \qquad (35)$$

where $\varpi_1, \tau_2, \delta_2$ are design positive parameters. $K_2$ is the bound of $\dot{\alpha}_1$

Substituting (33)-(35) into (32) yields

$$\dot{V}_1 \leq \frac{z_1}{k_{b_1}^2 - z_1^2} z_2 + z_1 \left( \frac{1}{k_{b_1}^2 - z_1^2} \alpha_1 + \Theta_1 \right) + \frac{\varpi_1}{2} \|e_1\|^2 + \frac{\chi_2^2}{2} - \frac{\chi_2^2}{\tau_2} + \frac{\chi_2^2 K_2^2}{2\delta_2}, \quad (36)$$

where $\Theta_1 = \frac{z_1}{2\varpi_1\left(k_{b_1}^2 - z_1^2\right)^2} k_1^2 + \frac{z_1}{2\left(k_{b_1}^2 - z_1^2\right)^2} - \frac{1}{k_{b_1}^2 - z_1^2} \dot{y}_r$.

*Step 2:* Take the time derivative of $V_2$, $\dot{V}_2$ is given by

$$\dot{V}_2 = \frac{z_2 \dot{z}_2}{k_{b_2}^2 - z_2^2} + \chi_3 \dot{\chi}_3 = \frac{z_2}{k_{b_2}^2 - z_2^2} \left( z_3 + \chi_3 + \alpha_2 + k_2 e_1 - \dot{\alpha}_{2f} \right) + \chi_3 \dot{\chi}_3, \quad (37)$$

$$\frac{z_2}{k_{b_2}^2 - z_2^2} k_2 e_1 \leq \frac{\varpi_2}{2} \|e_1\|^2 + \frac{1}{2\varpi_2} \frac{z_2^2}{\left(k_{b_2}^2 - z_2^2\right)^2} k_2^2, \quad (38)$$

$$\frac{z_2}{k_{b_2}^2 - z_2^2} \chi_3 \leq \frac{z_2^2}{2\left(k_{b_2}^2 - z_2^2\right)^2} + \frac{\chi_3^2}{2}. \quad (39)$$

From (18) we can get

$$\chi_3 \dot{\chi}_3 \leq -\frac{\chi_3^2}{\tau_3} + \frac{\chi_3^2 K_3^2}{2\delta_3} + \frac{\delta_3}{2}, \quad (40)$$

where $\varpi_2$, $\tau_3$, $\delta_3$ are design positive parameters. $K_3$ is the bound of $\dot{\alpha}_2$.

Substituting (38)-(40) into (37) yields

$$\dot{V}_2 \leq \frac{z_2}{k_{b_2}^2 - z_2^2} z_3 + z_2 \left( \frac{1}{k_{b_2}^2 - z_2^2} \alpha_2 + \Theta_2 \right) + \frac{\varpi_2}{2} \|e_1\|^2 + \frac{\chi_3^2}{2} - \frac{\chi_3^2}{\tau_3} + \frac{\chi_3^2 K_3^2}{2\delta_3}, \quad (41)$$

where $\Theta_2 = \frac{z_2}{2\varpi_2\left(k_{b_2}^2 - z_2^2\right)^2} k_2^2 + \frac{z_2}{2\left(k_{b_2}^2 - z_2^2\right)^2} - \dot{\alpha}_{2f}$.

*Step i* $(3 \leq i \leq n-1)$*:* Take the time derivative of $V_i$, $\dot{V}_i$ is given by

$$\dot{V}_i = \frac{z_i \dot{z}_i}{k_{b_i}^2 - z_i^2} + \chi_{i+1} \dot{\chi}_{i+1} = \frac{z_i}{k_{b_i}^2 - z_i^2} \left( z_{i+1} + \chi_{i+1} + \alpha_i + k_i e_1 - \dot{\alpha}_{if} \right) + \chi_{i+1} \dot{\chi}_{i+1} \quad (42)$$

$$\frac{z_i}{k_{b_i}^2 - z_i^2} k_i e_1 \leq \frac{\varpi_i}{2} \|e_1\|^2 + \frac{1}{2\varpi_i} \frac{z_i^2}{\left(k_{b_i}^2 - z_i^2\right)^2} k_i^2, \quad (43)$$

$$\frac{z_i}{k_{b_i}^2 - z_i^2} \chi_{i+1} \leq \frac{z_i^2}{2\left(k_{b_i}^2 - z_i^2\right)^2} + \frac{\chi_{i+1}^2}{2}, \quad (44)$$

$$\chi_{i+1} \dot{\chi}_{i+1} \leq -\frac{\chi_{i+1}^2}{\tau_{i+1}} + \frac{\chi_{i+1}^2 K_{i+1}^2}{2\delta_{i+1}} + \frac{\delta_{i+1}}{2}, \quad (45)$$

where $\varpi_i$, $\tau_{i+1}$, $\delta_{i+1}$ are design positive parameters. $K_{i+1}$ is the bound of $\dot{\alpha}_i$.

Substituting (43)-(45) into (42) yields

$$\dot{V}_i \leq \frac{z_i}{k_{b_i}^2 - z_i^2} z_{i+1} + z_i \left( \frac{1}{k_{b_i}^2 - z_i^2} \alpha_i + \Theta_i \right) + \frac{\varpi_i}{2} \|e_1\|^2 + \frac{\chi_{i+1}^2}{2} - \frac{\chi_{i+1}^2}{\tau_{i+1}} + \frac{\chi_{i+1}^2 K_{i+1}^2}{2\delta_{i+1}}, \quad (46)$$

where $\Theta_i = \frac{z_i}{2\varpi_i\left(k_{b_i}^2 - z_i^2\right)^2} k_i^2 + \frac{z_i}{2\left(k_{b_i}^2 - z_i^2\right)^2} - \dot{\alpha}_{if}$.

*Step n:* Take the time derivative of $V_n$ is

$$\dot{V}_n = \frac{z_n \dot{z}_n}{k_{b_n}^2 - z_n^2} = \frac{z_n}{k_{b_n}^2 - z_n^2} \left( u(v) + k_n e_1 - \dot{\tilde{\rho}} - \dot{\alpha}_{nf} \right), \quad (47)$$

$\tilde{\rho}$ can be obtained from the following dynamic systems:

$$\dot{\tilde{\rho}} = -\tilde{\rho} + (h(v) - v), \quad (48)$$

$$\frac{z_n}{k_{b_n}^2 - z_n^2} k_n e_1 \leq \frac{\varpi_n}{2} \|e_1\|^2 + \frac{1}{2\varpi_n} \frac{z_n^2}{\left(k_{b_n}^2 - z_n^2\right)^2} k_n^2, \quad (49)$$

$$\frac{z_n}{k_{b_n}^2 - z_n^2} p(v) \leq \frac{z_n^2}{2\left(k_{b_n}^2 - z_n^2\right)^2} + \frac{K_1^2}{2}. \quad (50)$$

Substituting (48)-(50) into (47) yields

$$\dot{V}_n \leq \frac{z_n}{k_{b_n}^2 - z_n^2} (v + \tilde{\rho}) + z_n \Theta_n + \frac{\varpi_n}{2} \|e_1\|^2 + \frac{K_1^2}{2}, \quad (51)$$

where $\Theta_n = \frac{z_n}{2\varpi_n\left(k_{b_n}^2 - z_n^2\right)^2} k_n^2 + \frac{z_n}{2\left(k_{b_n}^2 - z_n^2\right)^2} - \dot{\alpha}_{nf}$ $\varpi_n > 0$ is a design parameter.

*Remark 4:* In this paper, we employ the DSC technique to avoid the "explosion of complexity" arisen in the controller design procedure, compared with [25]–[31], it reduces the computation burden. There is few published works on this issue in the non-triangular form systems.

Finally, we take the time derivative of $V_H$, it can be obtained

$$\begin{aligned} \dot{V}_H \leq\ & \frac{e^{\gamma d_1}}{1 - d_1^*} \frac{1}{4} \|P\|^2 q_{\max}^2 \|e\|^2 \\ & + \sum_{i=1}^{n} \frac{e^{\gamma(d_1 - \tau_i(t))} \left(1 - \dot{\tau}_i(t)\right)}{1 - d_1^*} \frac{1}{4} \|P\|^2 q_i^2 e_i^2 (t - \tau_i(t)) \\ & + \frac{e^{\gamma d_1}}{1 - d_1^*} \sum_{i=1}^{n} \bar{c} z_i^2 \beta_i^2 \\ & - \sum_{i=1}^{n} \frac{e^{\gamma(d_1 - \tau_i(t))} \left(1 - \dot{\tau}_i(t)\right)}{1 - d_1^*} \bar{c} z_i^2 (t - \tau_i(t)) \\ & \times \beta_i^2 (t - \tau_i(t)) + \frac{\bar{c} e^{\gamma d_1}}{1 - d_1^*} \sum_{i=1}^{n} \chi_i^2 \\ & - \sum_{i=2}^{n} \frac{e^{\gamma(d_1 - \tau_i(t))} \left(1 - \dot{\tau}_i(t)\right)}{1 - d_1^*} \bar{c} \chi_i^2 (t - \tau_i(t)) \\ & - \gamma V_H, . \end{aligned} \quad (52)$$

From Assumption 4 we can obtain the following inequality

$$e^{\gamma(d_1-\tau_i(t))} > 1, \quad -\frac{1-\dot{\tau}_i(t)}{1-d_1^*} \leq -1. \tag{53}$$

According to (31), (36), (41), (46), (51), (52) and (53), the time derivation of the Lyapunov function $\tilde{V}$ is

$$\begin{aligned}\dot{\tilde{V}} \leq &-\Bigg(\lambda_{\min}(Q) - \frac{7}{2} - \varsigma_1 \\ &- \frac{e^{\gamma d_1}}{4\left(1-d_1^*\right)}\|P\|^2 q_{\max}^2 - \sum_{i=1}^{n}\frac{\varpi_i}{2}\Bigg)\|e\|^2 \\ &+ c\sum_{i=1}^{n} z_i^2\beta_i^2 + \frac{1}{k_{b_1}^2 - z_1^2}z_1 z_2 + z_1\left[\frac{1}{k_{b_1}^2 - z_1^2}\alpha_1 + \Theta_1\right] \\ &+ \frac{1}{k_{b_2}^2 - z_2^2}z_2 z_3 + z_2\left[\frac{1}{k_{b_2}^2 - z_2^2}\alpha_2 + \Theta_2\right] \\ &+ \sum_{i=3}^{n-1}\left\{\frac{1}{k_{b_i}^2 - z_i^2}z_i z_{i+1} + z_i\left(\frac{1}{k_{b_i}^2 - z_i^2}\alpha_i + \Theta_i\right)\right\} \\ &+ z_n\left(\frac{1}{k_{b_n}^2 - z_n^2}(v+\tilde{\rho}) + \Theta_n\right) + \frac{e^{\gamma d_1}}{1-d_1^*}\sum_{i=1}^{n}\bar{c}z_i^2\beta_i^2 \\ &- \gamma V_H + \frac{1}{2}\|P\|^2\sum_{i=1}^{n}\bar{d}_i^2 + \frac{K_1^2}{2} + \varepsilon_0 + \bar{\varepsilon}_0 \\ &- \sum_{i=2}^{n}\left(\frac{1}{\tau_i} - \frac{1}{2} - c - \frac{\bar{c}e^{\gamma d_1}}{1-d_1^*} - \frac{K_i^2}{2\delta_i}\right)\chi_i^2.. \end{aligned} \tag{54}$$

To facilitate the adaptive controller design, denote

$$\bar{f}_1 = \Theta_1 + cz_1\beta_1^2 + \frac{e^{\gamma d_1}}{1-d_1^*}\bar{c}z_1\beta_1^2, \tag{55}$$

$$\bar{f}_2 = \Theta_2 + cz_2\beta_2^2 + \frac{e^{\gamma d_1}}{1-d_1^*}\bar{c}z_2\beta_2^2 + \frac{z_1}{k_{b_1}^2 - z_1^2}, \tag{56}$$

$$\bar{f}_i = \Theta_i + cz_i\beta_i^2 + \frac{e^{\gamma d_1}}{1-d_1^*}\bar{c}z_i\beta_i^2 + \frac{z_{i-1}}{k_{b_{i-1}}^2 - z_{i-1}^2}, \quad 3 \leq i \leq n-1 \tag{57}$$

$$\bar{f}_n = \Theta_n + cz_n\beta_n^2 + \frac{e^{\gamma d_1}}{1-d_1^*}\bar{c}z_n\beta_n^2 + \frac{z_{n-1}}{k_{b_{n-1}}^2 - z_{n-1}^2}, \tag{58}$$

The (54) can be rewritten as

$$\begin{aligned}\dot{\tilde{V}} \leq &-k\|e\|^2 + \sum_{i=1}^{n-1} z_i\left(\frac{1}{\left(k_{b_i}^2 - z_i^2\right)}\alpha_i + \bar{f}_i(Z_i)\right) \\ &+ z_n\left(\frac{1}{\left(k_{b_n}^2 - z_n^2\right)}(v+\tilde{\rho}) + \bar{f}_n(Z_n)\right) \\ &- \gamma V_H + 2\varepsilon\|P\|^2\sum_{i=1}^{n}\bar{d}_i^2 \\ &+ \frac{K_1^2}{2} + \varepsilon_0 + \bar{\varepsilon}_0 \\ &- \sum_{i=2}^{n}\left(\frac{1}{\tau_i} - c - \frac{\bar{c}e^{\gamma d_1}}{1-d_1^*} - \frac{1}{2} - \frac{K_i^2}{2\delta_i}\right)\chi_i^2, \end{aligned} \tag{59}$$

where $k = \lambda_{\min}(Q) - \frac{7}{2} - \varsigma_1 - \frac{e^{\gamma d_1}}{4(1-d_1^*)}\|P\|^2 q_{\max}^2 - \sum_{i=1}^{n}\frac{\varpi_i}{2}$ choosing the proper design parameters $k_i$, $d_1$, $d_1^*$, $\gamma$, $\omega_i$, such that $k > 0$.

The unknown smooth function $\bar{f}_i(Z_i)$ is approximated by the NNs $\Phi_i^{*T}\xi_i(Z_i)$, so there exists an NNs on a compact set $\Omega_{Z_i}$, such that

$$\bar{f}_i(Z_i) = \Phi_i^{*T}\xi_i(Z_i) + \delta_i(Z_i) \tag{60}$$

where $\Phi_i^*$ is the ideal weight vector, $\delta_i(Z_i)$ is a neural approximation error, and $|\delta_i(Z_i)| \leq \varepsilon_i$

$$z_i\bar{f}_i(Z_i) \leq \frac{\theta^* z_i^2\xi_i^T(Z_i)\xi_i(Z_i)}{2\eta_i^2} + \frac{\eta_i^2}{2} + \frac{z_i^2}{2} + \frac{\varepsilon_i^2}{2} \tag{61}$$

where $\eta_i > 0$, $\varepsilon_i > 0$ are design constants.

According to (15), (16), (51) and (53), we have

$$\begin{aligned}\dot{\tilde{V}} \leq &-k\|e\|^2 - \sum_{i=1}^{n} c_i\frac{z_i^2}{\left(k_{b_i}^2 - z_i^2\right)} - \sum_{i=1}^{n}\frac{\tilde{\theta}z_i^2\xi_i^T(Z_i)\xi(Z_i)}{2\eta_i^2} \\ &- \gamma V_H + 2\varepsilon\|P\|^2\sum_{i=1}^{n}\bar{d}_i^2 + \varepsilon_0 + \bar{\varepsilon}_0 + \frac{K_1^2}{2} \\ &- \sum_{i=1}^{n-1}\pi_{i+1}\chi_{i+1}^2, \end{aligned} \tag{62}$$

where

$$\pi_{i+1} = \frac{1}{\tau_{i+1}} - c - \frac{\bar{c}e^{\gamma d_1}}{1-d_1^*} - \frac{1}{2} - \frac{K_{i+1}^2}{2\delta_{i+1}}, \quad i = 1, \cdots, n-1,$$

select proper parameter such that $\pi_{i+1} > 0$.

Select the Lyapunov function $V$ as follows:

$$V = \tilde{V} + \frac{\tilde{\theta}^2}{2p}, \tag{63}$$

Combining with (62), one can get the $\dot{V}$ as follows:

$$\begin{aligned}\dot{V} \leq &-k\|e\|^2 - \sum_{i=1}^{n} c_i\frac{z_i^2}{\left(k_{b_i}^2 - z_i^2\right)} - \gamma V_H + 2\varepsilon\|P\|^2\sum_{i=1}^{n}\bar{d}_i^2 \\ &+ \varepsilon_0 + \bar{\varepsilon}_0 + \frac{K_1^2}{2} - \sum_{i=1}^{n-1}\pi_{i+1}\chi_{i+1}^2 \\ &- \sum_{i=1}^{n}\frac{\tilde{\theta}z_i^2\xi_i^T(Z_i)\xi(Z_i)}{2\eta_i^2} + \frac{\tilde{\theta}\dot{\hat{\theta}}}{p} \end{aligned} \tag{64}$$

Substitute (21) into (64) yields

$$\begin{aligned}\dot{V} \leq &-k\|e\|^2 - \sum_{i=1}^{n} c_i\frac{z_i^2}{b_i^2 - z_i^2} - \sum_{i=1}^{n-1}\pi_{i+1}\chi_{i+1}^2 \\ &- \gamma V_H - \frac{\sigma\tilde{\theta}\hat{\theta}}{p} + 2\varepsilon\|P\|^2\sum_{i=1}^{n}\bar{d}_i^2 + \varepsilon_0 + \bar{\varepsilon}_0 + \frac{K_1^2}{2} \end{aligned} \tag{65}$$

*Theorem 1:* Assume that the Assumptions 1-4 held, the closed-loop system consisting of the system (1) and observer (11) with input saturation (4), the virtual controller (19), actual controller (20) and the parameter adaptive law (21), if the initial conditions are bounded, then we can obtain that all the signals in the closed-looped system are uniformly ultimately bounded and the full state constraints are not violated.

*Proof:* Using the following inequality

$$-\frac{\sigma\tilde{\theta}\hat{\theta}}{p} \leq -\frac{\sigma\tilde{\theta}^2}{2p} + \frac{\sigma\theta^{*2}}{2p}, \tag{66}$$

(65) can be rewritten as

$$\dot{V} \leq -k\left\|e\right\|^2 - \sum_{i=1}^{n} c_i \frac{z_i^2}{\left(k_{b_i}^2 - z_i^2\right)} - \gamma V_H - \frac{\sigma\tilde{\theta}^2}{2p} - \sum_{i=1}^{n-1} \pi_{i+1}\chi_{i+1}^2 + 2\varepsilon\left\|P\right\|^2 \sum_{i=1}^{n} \bar{d}_i^2 + C \tag{67}$$

According to Lemma 2, we have

$$\frac{-c_i z_i^2}{k_{b_i}^2 - z_i^2} < \log \frac{-c_i k_{b_i}^2}{k_{b_i}^2 - z_i^2} \tag{68}$$

then (68) can be expressed as

$$\dot{V} \leq -\rho_1 V + C, \tag{69}$$

where

$$\rho_1 = \min\left(\frac{2k}{\lambda_{\max}(P)}, 2c_i\,(i = 1, \cdots n), 2\pi_{i+1}\,(i=2, \cdots n-1), \gamma, \sigma\right)$$

$$C = \frac{K_1^2}{2} + 2\varepsilon\left\|P\right\|^2 \sum_{i=1}^{n} \bar{d}_i^2 + \frac{\sigma\theta^{*2}}{2p} + \varepsilon_0 + \bar{\varepsilon}_0$$

Resorting to (69), it can be conclude that

$$0 \leq V(t) \leq \left[V(0) - \frac{C}{\rho_1}\right] e^{-\rho_1 t} + \frac{C}{\rho_1} \leq V(0) + \frac{C}{\rho_1}, \tag{70}$$

From the definition of $V(t)$, we get $|\chi_i| \leq \|(\chi_2, \cdots \chi_n)\| \leq \sqrt{2\left(V(0) + \frac{C}{\rho_1}\right)} = \rho_i, i = 2, \cdots, n$. According to definition of $V$ and (68), we know that $e, \tilde{\theta}, \log \frac{k_{b_i}^2}{k_{b_i}^2 - z_i^2}$ are bounded and

$$\frac{1}{2}e^T P e \leq V(0) + \frac{C}{\rho_1},$$

so

$$|e_i| \leq \|e\| \leq \sqrt{\frac{2\left(V(0) + \frac{C}{\rho_1}\right)}{\lambda_{\min}(P)}} = \Delta_q.$$

Since $\hat{x}_1 = z_1 + y_r(t)$ and $|y_r(t)| \leq \kappa_0$, then we have $|\hat{x}_1| \leq |z_1| + |y_r(t)| < k_{b_1} + \kappa_0$,from the definition of $e_1$, we get $|x_1| \leq |\hat{x}_1| + |e_1| < k_{b_1} + \kappa_0 + \Delta_q$. Let $M_1 = \kappa_0 + \Delta_q + k_{b_1}$ then $|x_1| < M_1$. $\tilde{\theta}$ is bounded and $\theta^*$ is constant, so $\hat{\theta}$ is bounded. Since $\hat{x}_1, \theta, y_r, \dot{y}_r$ are bounded, so $\alpha_1$ is bounded, suppose $\bar{\alpha}_1$ is the bound of $\alpha_1$, so $|\alpha_1| \leq \bar{\alpha}_1$, from $z_2 = \hat{x}_2 - \chi_2 - \alpha_1$ and $|z_2| < k_{b_2}$ we can get $|\hat{x}_2| < k_{b_2} + \bar{\alpha}_1 + \rho_2$ from the definition of $e_2$, we have $|x_2| \leq |\hat{x}_2| + |e_2| < k_{b_2} + \bar{\alpha}_1 + \rho_2 + \Delta_q$. Let $M_2 = \bar{\alpha}_1 + \rho_2 + \Delta_q + k_{b_2}$ then $|x_2| < M_2$. Similarly, it can be in turn to prove that $|x_{i+1}| < M_{i+1}, i = 2, 3, \cdots, n-2$ after verifying $|\alpha_i| \leq \bar{\alpha}_i$. Since $|\hat{x}_i| < k_{b_i} + \bar{\alpha}_i + \rho_i$, $|z_i| < k_{b_i}$, we know $|\alpha_i| \leq \bar{\alpha}_i$ from (15) and (16), change of coordinates $z_{i+1} = \hat{x}_{i+1} - \alpha_i - \chi_{i+1}$, then $|\hat{x}_{i+1}| < k_{b_{i+1}} + \bar{\alpha}_i + \rho_{i+1}$ and $|x_{i+1}| < k_{b_{i+1}} + \bar{\alpha}_i + \rho_{i+1} + \Delta_q$. Which implies that $|x_{i+1}| < M_{i+1}$ if $M_{i+1} = \bar{\alpha}_i + \rho_{i+1} + \Delta_q + k_{b_{i+1}}$. Since $\hat{x}_{n-1}$ and $\hat{\theta}$ are bounded, so $|\alpha_{n-1}| \leq \bar{\alpha}_{n-1}$, from $z_n = \hat{x}_n - \tilde{\rho} - \chi_n - \alpha_{n-1}$, we have $|\hat{x}_n| < k_{b_n} + m + \bar{\alpha}_{n-1} + \rho_n$, so $|x_n| < k_{b_n} + m + \bar{\alpha}_{n-1} + \rho_n + \Delta_q$ as long as $M_n = \bar{\alpha}_{n-1} + \rho_n + \Delta_q + m + k_{b_n}$ from (20) we can see that $v$ is bounded. Therefore we can conclude that all the signal $u$, $x$ and $\hat{\theta}$ are bounded and the system states are not violated.

From (24) and (70), one can obtain $\log k_{b_1}^2 / \left(k_{b_1}^2 - z_1^2\right) \leq 2\left[V(0) - \frac{C}{\rho_1}\right] e^{-\rho_1 t} + 2\frac{C}{\rho_1}$, then it produces $k_{b_1}^2 / \left(k_{b_1}^2 - z_1^2\right) \leq e^{2\left[V(0) - \frac{C}{\rho_1}\right] e^{-\rho_1 t} + 2\frac{C}{\rho_1}}$, further we have $|z_1| \leq k_{b_1}^2 \sqrt{1 - e^{-2\left[V(0) - \frac{C}{\rho_1}\right] e^{-\rho_1 t} - 2\frac{C}{\rho_1}}} = \Delta_1$ if $V(0) = \frac{C}{\rho_1}$, then $|z_1| \leq k_{b_1}^2 \sqrt{1 - e^{-2\frac{C}{\rho_1}}} = \Delta_{11}$ holds. If $V(0) \neq \frac{C}{\rho_1}$, from $k_{b_1}^2 \sqrt{1 - e^{-2\left[V(0) - \frac{C}{\rho_1}\right] e^{-\rho_1 t} - 2\frac{C}{\rho_1}}}$ we can get it is a decreasing function about $t$, so given any $\Delta_{12} > k_{b_1}^2 \sqrt{1 - e^{-2\frac{C}{\rho_1}}}$, there exist $T_1 = -\frac{1}{\rho_1} \times In\left[\left(2C_1 + In\left(1 - \left(\frac{\Delta_{12}}{k_{b_1}^2}\right)^2\right)\right) / \left(-2\left(V(0) - \frac{C}{\rho_1}\right)\right)\right]$ such that $t > T_1$, $|z_1| \leq \Delta_{12}$ holds, which implies that $\lim_{t\to\infty} \sup |z_1| \leq \Delta_{11}$, then $t > T_1$, it holds $|z_1| \leq \Delta_{11}$. This implies that $z_1$ can be arbitrarily small by selecting proper design parameter.

## IV. SIMULATION EXAMPLES

In this section, two simulation examples are presented to verify the effectiveness of our proposed scheme for non-triangular form time-varying delay systems.

*Example 1:* Consider the following nonlinear system:

$$\begin{cases} \dot{x}_1 = x_2 + 0.1x_1x_2^2 + x_1^2(t - \tau_1(t)) \\ \qquad \sin(x_2(t - \tau_1(t))) + d_1(x, t), \\ \dot{x}_2 = u(v) + x_1^2(t - \tau_2(t)) \sin(x_2(t - \tau_2(t))) + d_2(x, t), \\ y = x_1. \end{cases} \tag{71}$$

where the state constraints are $|x_1| < 0.2, |x_2| < 0.5$, uncertainties terms are $d_1(t) = 0.05\sin(2t)\sin(x_1x_2)$, $d_2(t) = 0.1\sin(0.5t)\sin\left(x_1x_2^2\right)$, time-varying delay term are $\tau_1(t) = \tau_2(t) = 0.3\sin t$, the reference trajectory $y_r = 0.1\sin(t) + 0.1\sin(0.5t)$. Control objective is to guarantee that all the signals in the closed-loop systems are bounded and the full state constraints are not violated. The input $u(v(t))$is described by

$$u(v(t)) = sat(v(t)) = \begin{cases} sign(v(t))\, u_M, & |v(t)| \geq u_M \\ v(t), & |v(t)| < u_M, \end{cases} \quad (72)$$

with $\mu_M = 0.2$.

Choose observer gains $k_1 = k_2 = 3$, from (11) the state observer is designed as:

$$\begin{cases} \dot{\hat{x}}_1 = \hat{x}_2 + 3\left(y - \hat{x}_1\right), \\ \dot{\hat{x}}_2 = u(v) + 3\left(y - \hat{x}_1\right). \end{cases} \quad (73)$$

The virtual control input signal $\alpha_1$, actual control input $v$ and the adaptive law $\dot{\hat{\theta}}$ are given as follows:

$$\alpha_1 = -c_1 z_1 - \frac{z_1}{2\left(k_{b_1}^2 - z_1^2\right)} - \frac{\hat{\theta} z_1 \xi_1^T(Z_1)\,\xi_1(Z_1)}{2\eta_1^2\left(k_{b_1}^2 - z_1^2\right)}, \quad (74)$$

$$v = -c_2 z_2 - \frac{z_2}{2\left(k_{b_2}^2 - z_2^2\right)} - \frac{\hat{\theta} z_2 \xi_2^T(Z_2)\,\xi_2(Z_2)}{2\eta_2^2\left(k_{b_2}^2 - z_2^2\right)} - \tilde{\rho} \quad (75)$$

$$\dot{\hat{\theta}} = \sum_{i=1}^{2} \frac{p}{2\eta_i^2} z_i^2 \xi_i^T(Z_i)\,\xi_i(Z_i) - \sigma\hat{\theta}, \quad (76)$$

where $z_1 = \hat{x}_1 - y_r$, $z_2 = \hat{x}_2 - \alpha_{2f} - \tilde{\rho}$, $Z_1 = \left[\hat{x}_1, y_r, \dot{y}_r, \theta\right]^T$, $Z_2 = \left[\hat{x}_1, \hat{x}_2, y_r, \dot{y}_r, \ddot{y}_r, \theta\right]^T$, two NNs are employed in this simulation. Neural networks $\hat{\Phi}_1^T \xi_1(Z_1)$ contains 324 nodes (i.e $l_1 = 324$) with centers $\varpi_l$ $(l = 1, \cdots, l_1)$ evenly spaced in $[-0.2, 0.2] \times [-0.5, 0.5] \times [-0.2, 0.2] \times [0, 1]$ and widths $\eta_l = 0.2$. The other neural networks $\hat{\Phi}_2^T \xi_2(Z_2)$contain 1944 nodes (i.e. $l_2 = 1944$)with centers $\varpi_l$ $(l = 1, \cdots l_2)$ evenly spaced in $[-0.2, 0.2] \times [-0.5, 0.5] \times [-0.2, 0.2] \times [-0.2, 0.2] \times \times [-0.15, 0.15] \times [0, 1]$ and widths $\eta_l = 0.2$ The design parameters applied in the simulation are set as $c_1 = 6$, $c_2 = 6$, $\sigma = 2$, $k_1 = 3$, $k_2 = 3$, $p = 2$, $\eta_1 = 1$, $\eta_2 = 1$, $\pi_2 = 0.02$. The initial conditions are chosen as $x(0) = [0.1, 0.1]^T$, $\hat{x}(0) = [0, 0.1]^T$ and $\hat{\theta}(0) = 0.5$.

The simulation results are shown in the Figs.2-9. Fig.2 shows the good tracking performance of the system output and the tracking signal. Fig.3 and Fig.4 show the control input signal $v$ and $u$. Fig.5 and Fig.6 show the system state and observer state, it can be seen that the observer have good achieved performance. Fig.7 show the system states. The trajectories of $z_1$, $z_2$, and $\hat{\theta}$ are demonstrated in Fig.8 and Fig.9. From the simulation results, it is clearly that the proposed controller guarantees all the signals in the closed-loop systems are bounded and achieve good control performance.

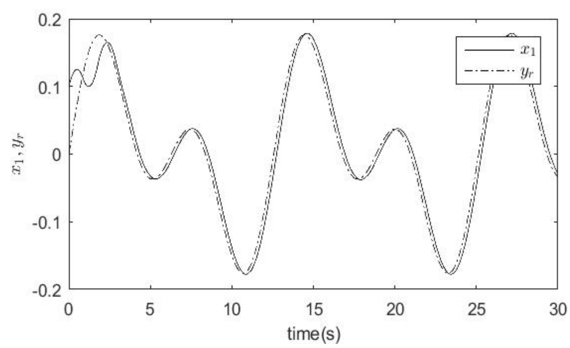


**FIGURE 2.** Trajectories of $x_1$ and $y_r$ in Example 1.

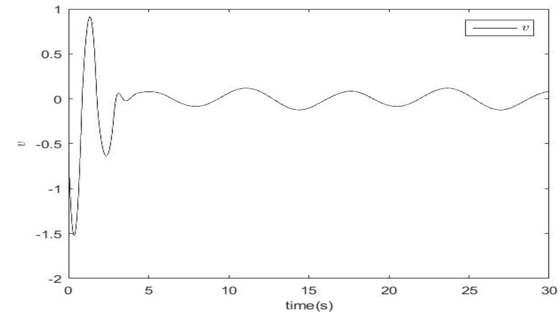


**FIGURE 3.** Trajectory of control $v$ in Example 1.

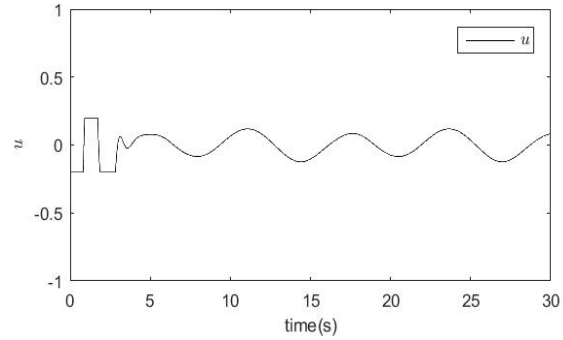


**FIGURE 4.** Trajectory of control $u$ in Example 1.

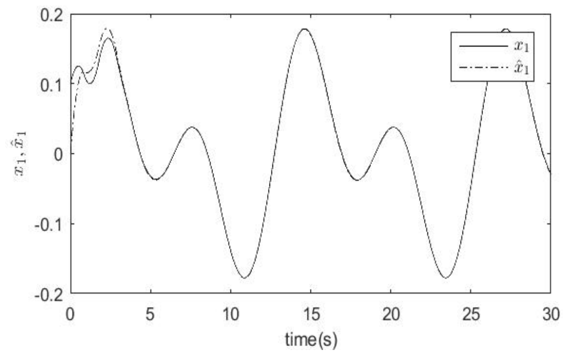


**FIGURE 5.** Trajectories of $x_1$ and $\hat{x}_1$ in Example 1.

*Example 2:* Consider the following Brusselator model in [52]: which can be described by

$$\begin{cases} \dot{x}_1 = A - (B+1)x_1 + x_1^2 x_2 + x_2 + \frac{2}{3}x_1 \\ \qquad + q_1\left(x\left(t - \tau_1(t)\right)\right) + d_1(x, t) \\ \dot{x}_2 = Bx_1 - x_1^2 x_2 + u + q_2\left(x\left(t - \tau_2(t)\right)\right) + d_2(x, t) \\ y = x_1 \end{cases} \quad (77)$$

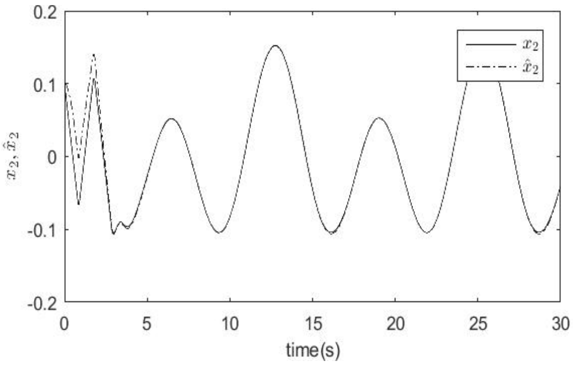


**FIGURE 6.** Trajectories of $x_2$ and $\hat{x}_2$ in Example 1.

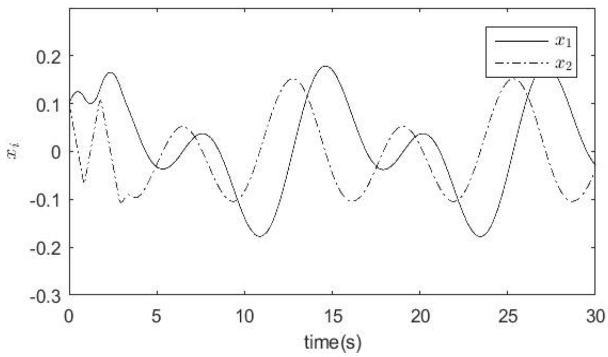


**FIGURE 7.** States trajectories $x_1$ and $x_2$ in Example 1.

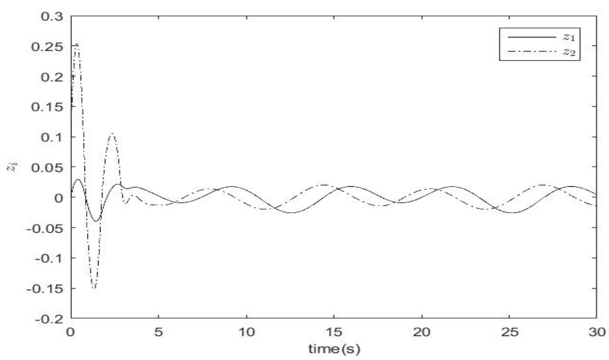


**FIGURE 8.** Trajectories $z_1$ and $z_2$ in Example 1.

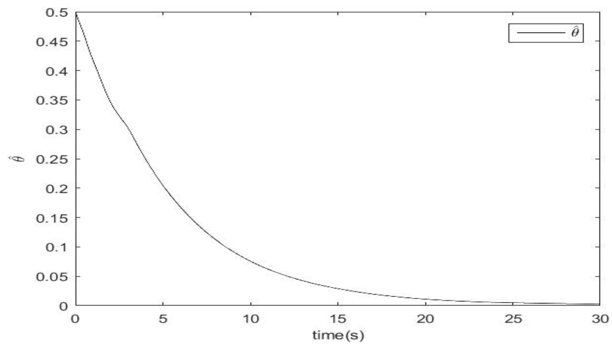


**FIGURE 9.** Trajectory of $\hat{\theta}$ in Example 1.

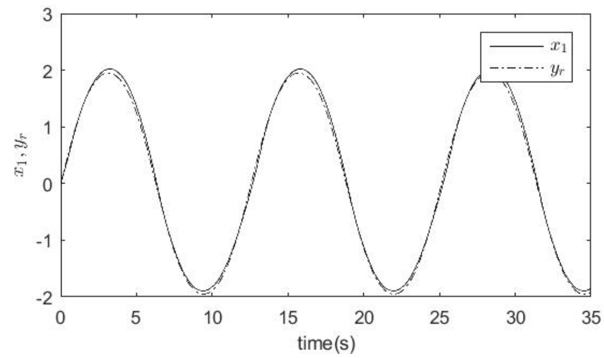


**FIGURE 10.** Trajectories of $x_1$ and $y_r$ in Example 2.

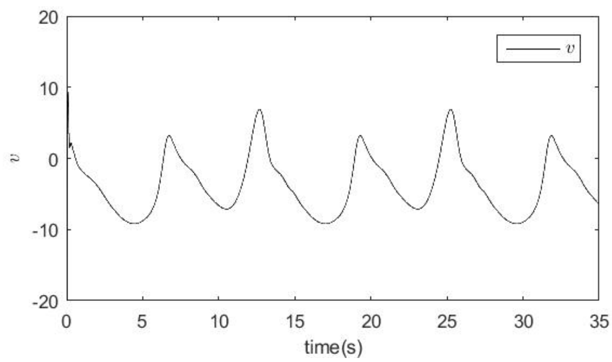


**FIGURE 11.** Trajectory of control $v$ in Example 2.

where

$$\begin{aligned} f_1(x) &= A-(B+1)x_1+x_1^2x_2+\frac{2}{3}x_1 \\ f_2(x) &= Bx_1-x_1^2x_2 \\ q_1(x(t-\tau_1(t))) &= 2x_1(t-\tau_1(t)) \\ \tau_1(t) &= 1+0.2\sin(t) \\ q_2(x(t-\tau_2(t))) &= 0.2x_2(t-\tau_2(t)) \\ \tau_2(t) &= 0.5+\sin(t) \\ d_1(x,t) &= 0.1x_1x_2^2\cos(1.5t) \\ d_2(x,t) &= 0.1\left(x_1^2+x_2^2\right)\sin\left(t^2\right) \end{aligned}$$

We assume the states are constrained in $|x_1| < k_{c_1} = 3\,|x_2| < k_{c_2} = 6$, the reference signal $y_r = 2\sin(0.5t)+0.05\sin(1.5t)$. Two NNs are employed in this simulation. Neural networks $\hat{\Phi}_1^T\xi_1(Z_1)$ contains 144 nodes (i.e $l_1 = 144$) with centers $\varpi_l\ (l=1,\cdots,l_1)$ evenly spaced in $[-3,3]\times[-3,3]\times[-2,2]\times[0,4]$ and widths $\eta_l = 2$. The other neural networks $\hat{\Phi}_2^T\xi_2(Z_2)$contain 360nodes (i.e. $l_2 = 360$) with centers $\varpi_l\ (l=1,\cdots l_2)$ evenly spaced in $[-3,3]\times[-6,6]\times[-3,3]\times[-2,2]\times\times[-2,2]\times[0,4]$ and widths $\eta_l = 3$. The design The initial condition and the design parameters are chosen as $A=1, B=3, c_1=6, c_2=8, \sigma=2, k_1=3, k_2=3, p=4, \eta_1=2, \eta_2=2, \pi_2=0.02$. The initial conditions are chosen as $x(0)=[0.02,0]^T, \hat{x}(0)=[0,0.1]^T, \hat{\theta}(0)=0.3u_M=6$. The simulation results are shown in the Figs.10-14.

Fig.10 shows the good tracking performance of the system output and the tracking signal. Fig.11 and Fig.12 show the control input signal $v$ and $u$. Fig.13 show the system states. The trajectories of $z_1, z_2$, and $\hat{\theta}$ are demonstrated in Fig.14 and Fig.15.

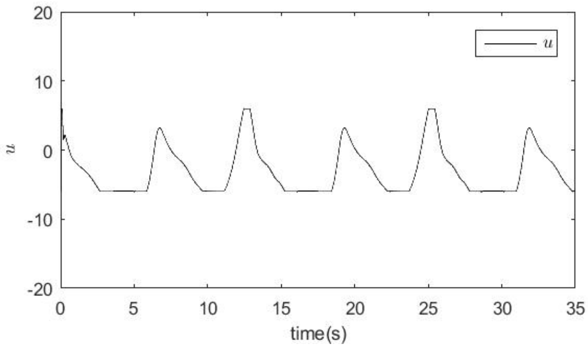


**FIGURE 12.** Trajectory of control $u$ in Example 2.

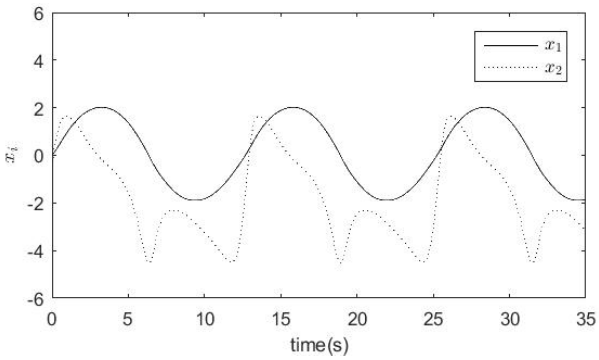


**FIGURE 13.** States trajectories $x_1$ and $x_2$ in Example 2.

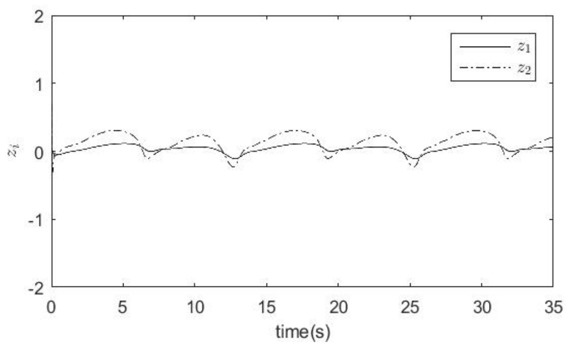


**FIGURE 14.** Trajectories $z_1$ and $z_2$ in Example 2.

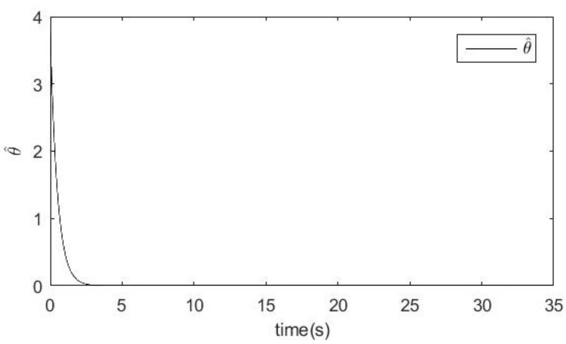


**FIGURE 15.** Trajectory of $\hat{\theta}$ in Example 2.

## V. CONCLUSIONS

In this paper an output feedback adaptive neural control approach for a class of non-triangular time-delay systems with input saturation and full state constraints has been proposed. A variable separation approach is introduced to overcome the non-triangular structure. The state observer and BLF have been used to deal with the immeasurable states and full state constraints. The DSC technique was employed to overcome the ''explosion of complexity''. The proposed controller guarantees that all the signals in the closed-loop system are uniformly ultimate bounded and the full state constraints are not violated. Two simulation results show that the presented control method can perform successful control and achieve desired performance.

## REFERENCES

[1] M. M. Polycarpou, ''Stable adaptive neural control scheme for nonlinear systems,'' *IEEE Trans. Autom. Control*, vol. 41, no. 3, pp. 447–451, Mar. 1996.

[2] T. Zhang, S. S. Ge, and C. C. Hang, ''Adaptive neural network control for strict-feedback nonlinear systems using backstepping design,'' *Automatica*, vol. 36, no. 12, pp. 1835–1846, Dec. 1996.

[3] C. Wang, D. J. Hill, S. S. Ge, and G. Chen, ''An ISS-modular approach for adaptive neural control of pure-feedback systems,'' *Automatica*, vol. 42, no. 5, pp. 723–731, 2006.

[4] M. Wang, B. Chen, X. Liu, and P. Shi, ''Adaptive fuzzy tracking control for a class of perturbed strict-feedback nonlinear time-delay systems,'' *Fuzzy Sets Syst.*, vol. 159, no. 8, pp. 949–967, Apr. 2008.

[5] B. Chen, X. Liu, K. Liu, and C. Lin, ''Direct adaptive fuzzy control of nonlinear strict-feedback systems,'' *Automatica*, vol. 45, no. 6, pp. 1530–1535, 2009.

[6] Y.-J. Liu, G.-X. Wen, and S.-C. Tong, ''Direct adaptive NN control for a class of discrete-time nonlinear strict-feedback systems,'' *Neurocomputing*, vol. 73, nos. 13–15, pp. 2498–2505, Aug. 2010.

[7] C. Wang, M. Wang, T. Liu, and D. J. Hill, ''Learning from ISS-modular adaptive NN control of nonlinear strict-feedback systems,'' *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 23, no. 10, pp. 1539–1550, Oct. 2012.

[8] H. Yue and J. Li, ''Output-feedback adaptive fuzzy control for a class of non-linear time-varying delay systems with unknown control directions,'' *IET Control Theory Appl.*, vol. 6, no. 9, pp. 1266–1280, Jun. 2012.

[9] G. Sun, D. Wang, X. Li, and Z. Peng, ''A DSC approach to adaptive neural network tracking control for pure-feedback nonlinear systems,'' *Appl. Math. Comput.*, vol. 219, no. 11, pp. 6224–6235, Feb. 2013.

[10] G. Sun, D. Wang, T. Li, Z. Peng, and H. Wang, ''Single neural network approximation based adaptive control for a class of uncertain strict-feedback nonlinear systems,'' *Nonlinear Dyn.*, vol. 72, nos. 1–2, pp. 175–184, Apr. 2013.

[11] J. Wu, W. Chen, D. Zhao, and J. Li, ''Globally stable direct adaptive backstepping NN control for uncertain nonlinear strict-feedback systems,'' *Neurocomputing*, vol. 122, pp. 134–147, Dec. 2013.

[12] H. Yue and J. Li, ''Output-feedback adaptive fuzzy control for a class of nonlinear systems with input delay and unknown control directions,'' *J. Franklin Inst.*, vol. 350, no. 1, pp. 129–154, Feb. 2013.

[13] D. Wang and J. Huang, ''Neural network-based adaptive dynamic surface control for a class of uncertain nonlinear systems in strict-feedback form,'' *IEEE Trans. Neural Netw.*, vol. 16, no. 1, pp. 195–202, Jan. 2005.

[14] S.-L. Dai, C. Wang, and M. Wang, ''Dynamic learning from adaptive neural network control of a class of nonaffine nonlinear systems,'' *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 25, no. 1, pp. 111–123, Jan. 2014.

[15] W. X. Shi, ''Adaptive fuzzy control for MIMO nonlinear systems with nonsymmetric control gain matrix and unknown control direction,'' *IEEE Trans. Fuzzy Syst.*, vol. 22, no. 5, pp. 1288–1300, Oct. 2014.

[16] J. Li and H. Yue, ''Adaptive fuzzy tracking control for stochastic nonlinear systems with unknown time-varying delays,'' *Appl. Math. Comput.*, vol. 256, pp. 514–528, Apr. 2015.

[17] B. Xu, Z. Shi, C. Yang, and F. Sun, ''Composite neural dynamic surface control of a class of uncertain nonlinear systems in strict-feedback form,'' *IEEE Trans. Cybern.*, vol. 44, no. 12, pp. 2626–2634, Dec. 2014.

[18] M. Wang and C. Wang, ''Neural learning control of pure-feedback nonlinear systems,'' *Nonlinear Dyn.*, vol. 79, no. 4, pp. 2589–2608, 2015, doi: 10.1007/s11071-014-1834-y.

[19] T. Li, Z. Li, D. Wang, and C. L. P. Chen, ''Output-feedback adaptive neural control for stochastic nonlinear time-varying delay systems with unknown control directions,'' *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 26, no. 6, pp. 1188–1201, Jun. 2015.

[20] F. Wang, Z. Liu, and G. Lai, ''Fuzzy adaptive control of nonlinear uncertain plants with unknown dead zone output,'' *Fuzzy sets Syst.*, vol. 263, pp. 27–48, Mar. 2015.

[21] Y. Li, S. Sui, and S. Tong, "Adaptive fuzzy control design for stochastic nonlinear switched systems with arbitrary switchings and unmodeled dynamics," *IEEE Trans. Cybern.*, vol. 47, no. 2, pp. 403–414, Feb. 2017.

[22] B. Niu, D. Wang, H. Li, X. Xie, N. D. Alotaibi, and F. E. Alsaadi, "A novel neural-network-based adaptive control scheme for output-constrained stochastic switched nonlinear systems," *IEEE Trans. Syst., Man, Cybern., Syst.*, to be published. [Online]. Available: http://www.ieee.org/publications_standards/publications/rights/index.html

[23] F. Wang, Z. Liu, Y. Zhang, and C. L. P. Chen, "Adaptive quantized controller design via backstepping and stochastic small-gain approach," *IEEE Trans. Fuzzy Syst.*, vol. 24, no. 2, pp. 330–343, Apr. 2016.

[24] S. Tong, L. Zhang, and Y. Li, "Observed-based adaptive fuzzy decentralized tracking control for switched uncertain nonlinear large-scale systems with dead zones," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 46, no. 1, pp. 37–47, Jan. 2016.

[25] B. Chen, X. Liu, K. Liu, and C. Lin, "Fuzzy approximation-based adaptive control of nonlinear delayed systems with unknown dead zone," *IEEE Trans. Fuzzy Syst.*, vol. 22, no. 2, pp. 237–248, Apr. 2014.

[26] H. Wang, K. Liu, X. Liu, B. Chen, and C. Lin, "Neural-based adaptive output-feedback control for a class of nonstrict-feedback stochastic nonlinear systems," *IEEE Trans. Cybern.*, vol. 45, no. 9, pp. 1977–1987, Sep. 2015.

[27] H. Yang, P. Shi, X. Zhao, and Y. Shi, "Adaptive output-feedback neural tracking control for a class of nonstrict-feedback nonlinear systems," *Inf. Sci.*, vols. 334–335, pp. 205–218, Mar. 2016.

[28] B. Chen, C. Lin, X. Liu, and K. Liu, "Observer-based adaptive fuzzy control for a class of nonlinear delayed systems," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 46, no. 1, pp. 27–36, Jan. 2016.

[29] Y. Li and S. Tong, "Adaptive fuzzy output-feedback stabilization control for a class of switched nonstrict-feedback nonlinear systems," *IEEE Trans. Cybern.*, vol. 47, no. 4, pp. 1007–1016, Apr. 2017.

[30] Z. Yu, S. Li, Z. Yu, and F. Li, "Adaptive neural output feedback control for nonstrict-feedback stochastic nonlinear systems with unknown backlash-Like hysteresis and unknown control directions," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 29, no. 4, pp. 1147–1160, Apr. 2018. [Online]. Available: http://www.ieee.org/publications_standards /publications/rights/index.html

[31] B. Niu, D. Wang, N. D. Alotaibi, and F. E. Alsaadi, "Adaptive neural state-feedback tracking control of stochastic nonlinear switched systems: An average dwell-time method," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 26, no. 10, pp. 2311–2322, Oct. 2018. [Online]. Available: http://www.ieee.org/publications_standards/publications/rights/index.html

[32] B. Niu, H. Li, Z. Q. Zhang, J. Q. Li, T. Hayat, and F. E. Alsaadi, "Adaptive neural-network-based dynamic surface control for stochastic interconnected nonlinear nonstrict-feedback systems with dead zone," *IEEE Trans. Syst., Man, Cybern. Syst.*, vol. 46, no. 1, pp. 27–36, Jan. 2018. [Online]. Available: http://www.ieee.org/publications_standards/publications/rights/index.html

[33] X. Wang, Z. Liu, and Y. Cai, "Adaptive single neural network control for a class of uncertain discrete-time nonlinear strict-feedback systems with input saturation," *Nonlinear Dyn.*, vol. 82, no. 4, pp. 2021–2030, Aug. 2015.

[34] J. Ma, Z. Zheng, and P. Li, "Adaptive dynamic surface control of a class of nonlinear systems with unknown direction control gains and input saturation," *IEEE Trans. Cybern.*, vol. 45, no. 4, pp. 728–741, Apr. 2015.

[35] S. Sui, Y. Li, and S. Tong, "Adaptive fuzzy control design and applications of uncertain stochastic nonlinear systems with input saturation," *Neurocomputing*, vol. 156, pp. 42–51, May 2015.

[36] Y.-F. Gao, X.-M. Sun, C. Wen, and W. Wang, "Adaptive tracking control for a class of stochastic uncertain nonlinear systems with input saturation," *IEEE Trans. Autom. Control*, vol. 62, no. 5, pp. 2498–2504, May 2017.

[37] M. R. Askari, M. Shahrokhi, and M. K. Talkhoncheh, "Observer-based adaptive fuzzy controller for nonlinear systems with unknown control directions and input saturation," *Fuzzy Sets Syst.*, vol. 314, pp. 24–45, May 2017.

[38] C.-S. Ting, "A robust fuzzy control approach to stabilization of nonlinear time-delay systems with saturating inputs," *Int. J. fuzzy Syst.*, vol. 10, no. 1, pp. 50–60, Mar. 2008.

[39] Y. Li, S. Tong, and T. Li, "Hybrid fuzzy adaptive output feedback control design for uncertain MIMO nonlinear systems with time-varying delays and input saturation," *IEEE Trans. Fuzzy Syst.*, vol. 24, no. 4, pp. 841–853, Aug. 2016.

[40] T. Li, R. Li, and J. Li, "Decentralized adaptive neural control of nonlinear interconnected large-scale systems with unknown time delays and input saturation," *Neurocomputing*, vol. 74, nos. 14–15, pp. 2277–2283, 2011.

[41] Q. Zhou, C. Wu, and P. Shi, "Observer-based adaptive fuzzy tracking control of nonlinear systems with time delay and input saturation," *Fuzzy Sets Syst.*, vol. 316, pp. 49–68, Jun. 2017.

[42] S. I. Han and J. M. Lee, "Fuzzy echo state neural networks and funnel dynamic surface control for prescribed performance of a nonlinear dynamic system," *IEEE Trans. Ind. Electron.*, vol. 61, no. 2, pp. 1099–1112, Feb. 2014.

[43] K. B. Ngo, R. Mahony, and Z.-P. Jiang, "Integrator backstepping using barrier functions for systems with multiple state constraints," in *Proc. 44th IEEE Conf. Decis. Control*, Seville, Spain, Dec. 2005, pp. 8306–8312.

[44] K. P. Tee and S. S. Ge, "Control of nonlinear systems with partial state constraints using a barrier Lyapunov function," *Int. J. Control*, vol. 84, no. 12, pp. 2008–2023, Dec. 2011.

[45] B. Ren, S. S. Ge, K. P. Tee, and T. H. Lee, "Adaptive neural control for output feedback nonlinear systems using a barrier Lyapunov function," *IEEE Trans. Neural Netw.*, vol. 21, no. 8, pp. 1339–1345, Aug. 2010.

[46] Y.-J. Liu, D.-J. Li, and S. Tong, "Adaptive output feedback control for a class of nonlinear systems with full-state constraints," *Int. J. Control*, vol. 87, no. 2, pp. 281–290, Jan. 2014.

[47] Y.-J. Liu and S. Tong, "Barrier Lyapunov functions-based adaptive control for a class of nonlinear pure-feedback systems with full state constraints," *Automatica*, vol. 64, pp. 70–75, Feb. 2016.

[48] W. He, Y. Chen, and Z. Yin, "Adaptive neural network control of an uncertain robot with full-state constraints," *IEEE Trans. Cybern.*, vol. 46, no. 3, pp. 620–629, 2016.

[49] Y.-J. Liu, M. Z. Gong, S. Tong, C. L. P. Chen, and D.-J. Li, "Adaptive fuzzy output feedback control for a class of nonlinear systems with full state constraints," *IEEE Trans. Fuzzy Syst.*, vol. 26, no. 5, pp. 2607–2617, Oct. 2018, doi: 10.1109/TFUZZ.2018.2798577..

[50] Y. Bian, Y. Chen, and L. Long, "Full state constraints-based adaptive control for switched nonlinear pure-feedback systems," *Int. J. Control*, vol. 49, no. 15, pp. 3094–3107, 2018, doi: 10.1080/00207721.2018.1533050.

[51] J. Zhou and C. Wen, *Adaptive Backstepping Control of Uncertain Systems*. Berlin, Germany: Springer-Verlag, 2008.

[52] S. S. Ge and C. Wang, "Uncertain chaotic system control via adaptive neural design," *Int. J. Bifurcation Chaos*, vol. 12, no. 5, pp. 1097–1109, 2002.

[53] C. Hua, S. Liu, Y. Li, and X. Guan, "Output feedback control based on event triggered for nonlinear systems with full state constraints," in *Proc. 36th Chin. Control Conf.*, Dalian, China, Jul. 2017, pp. 1091–1096.

[54] Z. Zhang, Y. Song, and K. Zhao, "Neuroadaptive cooperative control without velocity measurement for multiple humanoid robots under full-state constraints," *IEEE Trans. Ind. Electron.*, vol. 66, no. 4, pp. 2956–2964, Apr. 2019, doi: 10.1109/TIE.2018.2844791.

**RUI ZHANG** was born in Xi'an, Shaanxi, China, in 1979. She received the B.S. degree in basic mathematics from Northwest University, China, in 2002, and the M.S. degree in applied mathematics from the North China University of Water Resources and Hydropower, China, in 2008. She is currently pursuing the Ph.D. degree with the College of Mathematics and Statistics, Xidian University, Xi'an. Her research interests include adaptive control and constraints control.

**JUNMIN LI** graduated from Xidian University, China, in 1987. He received the M.Sc. degree from Xidian University, in 1990, and the Ph.D. degree from Xi'an Jiaotong University, China, in 1997. He is currently a Professor with the Department of Applied Mathematics, Xidian University. His research interests include adaptive control, learning control of multi-agent systems, hybrid system, control theory, and networked control systems.