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Self-Organizing Approximation Command Filtered Backstepping Control for Higher Order SISO Systems in Internet of Things

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ABSTRACT With the rapid development of the Internet of Things (IOT) and increasing research on tracking control of the affine IOT systems with uncertainty, a nonlinear control method of command filtering backstepping with self-organizing approximation is proposed. A command filter with a saturation structure is used to eliminate the analytic computation of command derivatives, and tracking errors are redefined to compensate the discrepancy between the command filtered signals and the analytic computation of command derivatives. Command filtered backstepping is used to decouple the higher IOT system and simplify the structure of the IOT system. Self-organizing approximation is used to online approximate the uncertain and eliminate the effect of the uncertain in the IOT system. The approximation for the uncertain is finished by a linear product between a set of basis functions and weighted functions on the local region. The weighted sum of compensated tracking errors is used as the prespecified tracking accuracy performance. A new local approximator is added to the control system according to the criterion based on the Lyapunov theory to enhance the capacity of the approximator when the prespecified tracking accuracy performance cannot be achieved. Finally, an application for a simple IOT system is analyzed by simulations. The simulation results show that the control method for the IOT system is effective.

INDEX TERMS Internet of Things system, command filtered backstepping control, nonlinear control systems, self-organizing feature maps.

I. INTRODUCTION

With the rapid development of wireless communication technology and computing penetrate into every field of our life, the IOT and IOT system are born [1]. Testing and defending methods including model-based nonlinear control are given in [2]. The key technologies of IOT are presented by References [3]–[7]. However, the detailed control methods of IOT system are not involved. Due to frequency interference between the Internet of Things and other business systems (Authorized frequency and unlicensed frequency is used in china), parameter perturbation from Internet of Things systems, and unmodeled dynamics for IOT system, it is inevitable that actual nonlinear systems will contain partial uncertainty [8]. Currently, processing these uncertainties is the key to difficulties of model-based nonlinear control. Introducing a universal approximator into the control structure for online approximation uncertainty, and the elimination of its effects, has become an effective way to process uncertainty. The focus of research into this type of control method lies in the selection of controller structure, automatic adjustment of the approximator, and the demonstration of closed loop stability [9]–[13]. These control approaches that contain linear approximator are often based on the following hypothesis: if the primary function in the approximator has an adequate large quantity of nodes, any given $\varepsilon\,(\varepsilon > 0)$ approximation precision can be gained by choosing the appropriate approximator parameters. To gain the expected approximation precision, one ideal approximator requires an adequate number of nodes in the primary function [14], [15]. For the IOT system, this will cause a sharp increase of calculation loads. Moreover, excessive nodes in the primary function might induce the possibility of over parameterization. That is, it will lead to system complications and the system becomes difficult to control.

To overcome the above-mentioned defects, the thought of locally weighted learning was first proposed by Atkeson *et al.* [15] and Schaal and Atkeson [16]. Locally weighted leaning divides the working field into several local approximation fields in which the corresponding local approximator is defined. The local approximator is constructed into a function that is related to the operating points of the system. When the operating points of the system enter into new local approximation fields, the system adds a new local approximator automatically and the approximation precision gained by previous local approximation fields can be accumulated continuously. In References [9], [17], [18], the linearly parameterized model has been applied locally, which is a special case of the receptive field weighted regression proposed in [15] and [16]. These methods are to replace the single approximtor which has abundant nodes in primary function by a local approximator set that is adjusted continuously according to system state, aiming to reduce number of nodes, calculation load and parameter scale in the primary function. However, for IOT system these methods have shortages: 1) They only proved the stability of state variable and approximator parameters, but have not proved the system stability when the number of local approximators change. 2) Adaptive laws of approximator parameters are only determined by system state, but they are unrelated to control performance. Consequently, the IOT system adds new local approximators automatically when the operating points of the system are far away from all local approximation fields, but doesn't take requirements on current control precision into account.

Hence, a control method for self-organizing approximation structure based on current control performance has been proposed in [19]. This method can make online adjustment of approximator structure in view of control precision, and it still maintains the minimum quantity of local approximators. There is no need to increase the gain control and thereby avoids risk of high gains. However, this method may be only applicable to the following scalar SISO IOT system:

$$\dot{x} = f^0(x) + f(x) + u, \quad x \in R$$

In [20], this method has been further expanded to an n-order single-input single-output (SISO) input-state feedback linearization system. The system equations are described below:

$$\begin{cases} \dot{x}_i = x_{i+1} 1 \le i \le n-1 \\ \dot{x}_n = \left[f_n^0(x) + f_n(x)\right] + \left[g_n^0(x) + g_n(x)\right] u \end{cases} \tag{1}$$

The complete closed loop stability has been proved considering instantaneous changes of quantity of local approximators. However, the feedback linearization form in equation (1) is difficult to be met by actual physical systems, thus limiting the applicable range of the method significantly.

But it is generally known that the command-filtered backstepping-based adaptive control method is an effective tool for controller design (see [21]–[27]). However, the traditional command-filtered backstepping design procedure requires the repeated differentiations of virtual controls, which causes a serious instability.

Based on the command filtered backstepping controller [28]–[31], the self-organizing approximation method [22] has been further expanded to a more common type of n-order SISO affine system as shown in equation (1). It realizes the expected tracking precision without use of high gain control and large amplitude switching. The basic principle is that: 1) the expected error allowance in tracking precision is set. 2) Online self-organizing approximation is only triggered when breaking the error allowance. Such approximation is to reduce effects of system model difference and make tracking error meet the requirements. 3) Effects of inherent error of online self-organizing approximation on control precision are eliminated by similar slip form.

The proposed method is mainly consists of command filtered backstepping and online self-organizing approximation. The command filtered backstepping is used to overcome the analytic solution of virtual control command in ordinary backstepping control. As a result, the high-order cascade structure of the control object in IOT system can be processed by relative decoupling. The goal of online self-organizing approximation is to continuously increase the online approximators according to the system error and to reduce negative impacts of the system model difference on system tracking performance.

This paper is organized as follows. The description of control problem of IOT system is briefly introduced in Section II. The locally weigthed learning algorithm applied in IOT system is presented in Section III. Section IV presents the self-organizing online approximation command filtered control. Proof of stability is given in Section V. Simulation case analysis and results discussion are given in Section VI. Finally, Section VII contains the conclusions.

II. DESCIPTION OF PROBLEMS

For the following n-order SISO system in IOT system:

$$\begin{cases} \dot{x}_i = \left[f_i^0(x) + f_i(x)\right] + \left[g_i^0(x) + g_i(x)\right] x_{i+1} \\ \qquad 1 \le i \le n-1 \\ \dot{x}_n = \left[f_n^0(x) + f_n(x)\right] + \left[g_n^0(x) + g_n(x)\right] u \\ i = n \end{cases} \tag{2}$$

where $x = [x_1, \ldots, x_n]^T \in D^n$ and it is a state vector. D^n is the whole working field of the physical system, which might be either known or unknown. x_1 is the output of system, and u is the control input signal. Functions $f_i^0(x)$ and $g_i^0(x)$ are known models of controlled objects during controller design. The unknown functions $f_i(x)$ and $g_i(x)$ are unknown parts of the controlled objects, which refer to errors between the real model of the controlled object and controller design model. Let $f_i^0(x)$, $g_i^0(x)$, $f_i(x)$ and $g_i(x)$ be continuous functions.

In this study, the goal of control is to design the control input signal u to drive x_1 to trace reference input x_{1c}. Meanwhile, all states x_i shall be have boundaries. For some regions, which model errors $f_i(x)$ and $g_i(x)$ might be too large in the system state space, the controller fails to reach the expected tracking precision. In this study, an online approximation controller structure and the compensation model error in the full-state space were used to achieve the expected tracking precision.

To protect controllability of IOT, it is suppose that $g_i^0(x) + g_i(x)$, has a definite sign, boundaries, and is not zero. Without loss of generality, the following hypothesis is proposed: the function $g_i^0(x) + g_i(x)$ has a lower boundary to make $g_i^0(x) + g_i(x) \geq g_l(x) \geq g_l \geq 0 (\forall x \in \Re^n)$ true, where $g_l(x)$ is a known function and g_l is a known constant.

III. LOCALLY WEIGTHED LEARNING ALGOTITHM

In this study, for the IOT system, a control method that combines online self-organizing approximation, command filtered backstepping, and is based on locally weighted learning is proposed. In this method, the approximation value of the unknown function $f_i(x)$ at one state point x can be expressed by the normalized weighted mean of the following local approximator $\hat{f}_{ik}(x)$:

$$\hat{f}_i(x) = \frac{\sum_k w_{ik}(x)\hat{f}_{ik}(x)}{\sum_k w_{ik}(x)} \quad (3)$$

where, $\hat{f}_{ik}(x)$ is the k^{th} local approximation function of the unknown function, $f_i(x)$. $w_{ik}(x)$ is only nonzero in the local set S_k (defined in the following text). In the set S_k, $\hat{f}_{ik}(x)$ is used to improve the approximation precision of $f_i(x)$. In the following text, the local weighted learning algorithm is defined by using $f_i(x)$. The definition of the unknown function $g_i(x)$ is similar and is not introduced here.

A. WEIGTHED FUNCTION

For the $\hat{f}_{ik}(x)$, one continuous, nonnegative and locally supported weighting function $w_{ik}(x)$ is defined. Firstly, the supporting set (hereinafter referred to as approximation field) of $w_{ik}(x)$ is defined:

$$S_k = \left\{x \in D^n \,|w_{ik}(x) \neq 0\right\} \quad (4)$$

If the size of S_k is defined as $\rho(S) = (\|x-y\|)$, the locally supporting means that $\rho(S_k)$ is a smaller relative to $\rho(D^n)$. Let $\bar{S}_k$ be the closure of set S_k. It should be noted that $\bar{S}_k$ is a compact set. One case of weight function that meets these conditions is the following biquadratic kernel function:

$$w_{ik}(x) = \begin{cases} \left[1-(\|x-c_k\|/\mu_k)^2\right]^2 & \|x-c_k\| < \mu_k \\ otherwise \end{cases} \quad (5)$$

where c_k is the central vector of k^{th} weighting function, and μ_k is a constant that expresses radius of S_k. The supporting set corresponding to equation (5) is:

$$S_k = \left\{x \in D^n \,|\|x-c_k\| < \mu_k\right\} \quad (6)$$

Since the approximator is online self-organizing, the total quantity $N(t)$ of local approximator $\hat{f}_{ik}(x)$ is not a constant, but is a variable that increases overtime. Conditions for increasing $N(t)$ at discretization will be introduced below. Since $N(t)$ is a variable, the approximation field corresponding to equation (3) changes with time. It is defined as:

$$A^{N(t)} = \bigcup\nolimits_{1\leq k\leq N(t)} S_k \quad (7)$$

When $x(t) \in A^{N(t)}$, there's at least one k that makes $w_{ik}(x) \neq 0$. The normalized weighting function is defined as:

$$\bar{w}_{ik}(x) = \frac{w_{ik}(x)}{\sum_{k=1}^{N(t)} w_{ik}(x)} \quad (8)$$

Therefore, it can be seen that the non-negative function set $\{\bar{w}_{ik}(x)\}_{k=1}^{N(t)}$ can form one unit partition in $A^{N(t)}$:

$$\sum\nolimits_{k=1}^{N(t)} \bar{w}_{ik}(x) = 1 x \in A^{N(t)} \quad (9)$$

The supporting set of $w_{ik}(x)$ is consistent with $\bar{w}_{ik}(x)$. When $x \notin A^{N(t)}$, $w_{ik}(x) = 0$ and $1 \leq k \leq N(t)$. To define all $x \in D^n$ in $\hat{f}_i(x)$ in equation (3), the complete definition of $\hat{f}_i(x)$ is:

$$f_i(x) = \begin{cases} \sum\nolimits_{k=1}^{N(t)} \bar{w}_{ik}(x)\hat{f}_{ik}(x) & x \in A^{N(t)} \\ 0 & x \in D^n - A^{N(t)} \end{cases} \quad (10)$$

The local weighting learning algorithm when $x \in A^{N(t)}$ is defined in the following.

B. LOCAL APPROXIMATOR

When $x \in A^{N(t)}$, the k^{th} local optimal approximator of $f_i(x)$ is defined as:

$$f_{ik}^*(x) = \phi_{f_{ik}}^T \theta_{f_{ik}}^* \quad (11)$$

where $\phi_{f_{ik}}$ is the vector of continuous primary function appointed by the designer. The vector $\theta_{f_{ik}}^*$ refers to the unknown optimal parameter estimation vector (for $x \in \bar{S}_k$):

$$\theta_{f_{ik}}^* = argmin_{\theta_{f_{ik}}} \left\{\int_{\bar{S}_k} w_{ik}(x)\left|f_i(x) - \hat{f}_{ik}(x)\right|^2 dx\right\} \quad (12)$$

$$\hat{f}_{ik}(x) = \phi_{f_{ik}}^T \theta_{f_{ik}} \quad (13)$$

where $\theta_{f_{ik}}$ refers to online updated unknown parameter estimation vector (for $x \in \bar{S}_k$).

It should be noted that $\theta_{f_{ik}}^*$ has definitions to each k. Since $f_i(x)$ and $f_{ik}^*(x)$ have a smoothness function on $\bar{S}_k$. Therefore, $f_{ik}^*(x)$ is the optimal local approximator of $f_i(x)$ on $\bar{S}_k$.

Let local approximation error $\varepsilon_{f_{ik}}(x)$ on D^n be defined as:

$$\varepsilon_{f_{ik}}(x) = \begin{cases} f_i(x) - f_{ik}^*(x) & x \in \bar{S}_k \\ 0 & otherwise \end{cases} \quad (14)$$

Let the constant $\varepsilon_{f_i} > 0$ be the known variable. It is hypothesized that the base vector set ϕ_{f_i} is large enough and μ_k is small enough, such that $\left|\varepsilon_{f_{ik}}(x)\right| \leq \bar{\varepsilon}_{f_i}$ is always true for some unknown positive constant $\bar{\varepsilon}_{f_i} < \varepsilon_{f_i}$ in the range of $x \in \bar{S}_k$. It should be noted that the boundedness of $max_{x\in\bar{S}_k}(\left|\varepsilon_{f_{ik}}(x)\right|)$ is protected by the continuity of $\left|\varepsilon_{f_{ik}}(x)\right|$ in $\bar{S}_k$.

For any $x \in A^{N(t)}$, $f_i(x)$ can be expressed as the following weighted sum of local approximator:

$$f_i(x) = \sum\nolimits_k \bar{w}_{ik}(x) f_{ik}^*(x) + \delta_{f_i}(x) \tag{15}$$

Since ϕ_{f_i} may not be infinitely large, and μ_k may not be infinite small, in actual design, $\delta_{f_i}(x)$ in equation (15) means that the inherent approximation error to $f_i(x)$ on $A^{N(t)}$ when ϕ_{f_i} and μ_k are finite values. Therefore it can be seen that $\left|\delta_{f_i}(x) \leq \bar{\varepsilon}_{f_i}\right|$. Since:

$$\begin{aligned}\left|\delta_{f_i}\right| &= \left|f_i(x) - \sum\nolimits_k \bar{w}_{ik}(x) f_{ik}^*(x)\right| \\ &= \left|\sum\nolimits_k \bar{w}_{ik}(x)\left(f_i(x) - f_{ik}^*(x)\right)\right| \\ &\leq \sum\nolimits_k \bar{w}_{ik}(x)\left|\varepsilon_{f_{ik}}(x)\right|\end{aligned} \tag{16}$$

That is:

$$\left|\delta_{f_i}(x)\right| \leq max_k\left(\left|\varepsilon_{f_{ik}}(x)\right|\right)\sum\nolimits_k \bar{w}_{ik}(x) = \bar{\varepsilon}_{f_i} \tag{17}$$

Each local optimal approximator $f_{ik}^*(x)$ can reach precision $\bar{\varepsilon}_{f_i}$ on $\bar{S}_k$. Therefore, the global precision of $\sum_k \bar{w}_{ik}(x) f_{ik}^*(x)$ on $A^{N(t)}$ is at least $\bar{\varepsilon}_{f_i}$.

Since $f_i(x)$ is an unknown function and the $\theta_{f_{ik}}^*$ is unknown, the control law uses equation (10) defined on $A^{N(t)}$ and equation (13) defined on $\bar{S}_k$, to make an online approximation. The basic principle of designed the adaptive capability of control laws is that during operation of controller, $\theta_{f_{ik}}$ a real-time automatic adjustment to improve approximation precision of unknown function will be made. To analyze the convergence of the parameter estimation, the parameter error vector is defined as $\tilde{\theta}_{f_{ik}} = \theta_{f_{ik}} - \theta_{f_k}^*$ for the k^{th} local approximator. The self-organizing principle of controller is seen during the system operation, the number of $\hat{f}_{ik}(x)$ will increase automatically according to the system tracking error when the system tracking error exceeds the design requirements, in order to reduce effects of model error on system control precision.

IV. SELF-ORGANIZING ONLINE APPROXIMATION COMMAND FILTERED CONTROL

A. DEFINITION OF ERROR

For $i = 1, 2, \ldots, n$, the tracking errors $\tilde{x}_i$ of different orders in the system (2) are defined as:

$$\tilde{x}_i = x_i - x_{i,c} \tag{18}$$

Here x_{1c} is the actual command trajectory, and $x_{i,c}$ $(2 \leq i \leq n)$ is the dummy control variable or intermediate control variable generated in the process of command filtered backstepping control process.

For $i = 1, 2, \ldots, n$, the compensation tracking error of different orders $(\bar{x}_i)$ in system (2) is defined as:

$$\bar{x}_i = \tilde{x}_i - \xi_i \tag{19}$$

where ξ_i is defined below.

One scalar mapping of compensation tracking error is defined as:

$$e = \sum_{i=1}^{n} \bar{x}_i \tag{20}$$

The goal of controller design is to reach the appointed tracking precision: $|e| \leq \mu_e$.

In the following text, the total time that the system tracking error exceeds the expected precision ($|e| > \mu_e$) is expressed as the total time function that the system tracking error exceeds the expected precision in the time interval $[t_1, t_2]$:

$$\begin{aligned}\bar{\mu}(e, \mu_e, t_1, t_2) &= \int_{t_1}^{t_2} 1\left(|e(t) - \mu_e|\right) \\ 1(\lambda) &= \begin{cases}1 & \lambda > 0 \\ 0 & \lambda \leq 0\end{cases}\end{aligned} \tag{21}$$

B. COMMAND FILTERED BACKSTEPPING

For $i = 1, 2, \ldots, n-1$, it is defined that:

$$x_{i+1,c}^0 = \alpha_i - \xi_{i+1} \tag{22}$$

$$u_c^0 = \alpha_n \tag{23}$$

where α_i is the dummy control variable generated by the command filtered backstepping control.

Signals $x_{i+1,c}$ and $\dot{x}_{i+1,c}$ are gained by following filter:

$$\dot{x}_{i+1,c} = -K_{i+1}(x_{i+1,c} - x_{i+1,c}^0) \tag{24}$$

Signal u_c is gained through the following filter:

$$\dot{u}_c = -K(u_c - u_c^0) \tag{25}$$

where $K, K_{i+1} > 0$ and they are the constant set by controller designers.

The initial condition is $x_{i+1,c}(0) = \alpha_i(0)$. Since equation (24) is a stale linear filter, if the input signal $x_{i+1,c}^0$ is bounded, $x_{i+1,c}$ and $\dot{x}_{i+1,c}$ must be bounded.

For $i = 1, 2, \ldots, n-1$, it defines:

$$\dot{\xi}_i = -k_i\xi_i + (g_i^0 + \hat{g}_i + \beta_{g_i})(x_{i+1,c} - x_{i+1,c}^0) \tag{26}$$

For $i = n$, it defines:

$$\dot{\xi}_n = -k_n\xi_n + (g_n^0 + \hat{g}_n + \beta_{g_n})(u_c - u_c^0) \tag{27}$$

where $k_i > 0$ $(1 \leq i \leq n)$ is the control gain appointed by the designer. The initial condition is $\xi_i(0) = 0$. The final system input is $u = u_c$.

Equations (26) and (27) are a low pass filter and the input is the product of $(g_i^0 + \hat{g}_i + \beta_{g_i})$ and $(x_{i+1,c} - x_{i+1,c}^0)$, where

$(g_i^0+\hat{g}_i+\beta_{g_i})$ is the bounded function and $(x_{i+1,c}-x_{i+1,c}^0)$ has small amplitude. Since $x_{i+1,c}$ and $\dot{x}_{i+1,c}$ are calculated by equation (24), this ensures that $K_{i+1}\gg k_{i+1}$ make $x_{i+1,c}$ and be able to trace $x_{i+1,c}^0$ accurately.

C. CONTROLLER DESIGN BASED ON ONLINE APPROXIMATION

When $x\in D^n$, for $i=1,2,\ldots,n$, the dummy control signal of backstepping process α_i is defined as:

$$\alpha_i=\frac{u_{\alpha_i}}{g_i^0+\hat{g}_i+\beta_{g_i}} \tag{28}$$

For $i=1$,

$$u_{\alpha_1}=-k_1\tilde{x}_1+\dot{x}_{1,c}-f_1^0-\hat{f}_1-\beta_{f_1} \tag{29}$$

For $i\in[2,n-1]$,

$$u_{\alpha_i}=-k_i\tilde{x}_i+\dot{x}_{i,c}-f_i^0-\hat{f}_i-\beta_{f_i}-(g_{i-1}^0+\hat{g}_{i-1}+\beta_{g_{i-1}})\bar{x}_i \tag{30}$$

where β_{f_i} and β_{g_i} $(1\le i\le n-1)$ are used to realize the robustness of the inherent approximation error. They are defined as:

$$\begin{aligned}\beta_{f_i}&=\varepsilon_{f_i}sat\ (e/\mu_e)\\ \beta_{g_i}&=\varepsilon_{g_i}sat(e/\mu_e)sign(x_{i+1})\end{aligned} \tag{31}$$

where $sat(*)$ is saturation function. For $i=n$,

$$u_{an}=-k_n\tilde{x}_n+\dot{x}_{n,c}-f_n^0-\hat{f}_n-\beta_{f_n}-(g_{n-1}^0+\hat{g}_{n-1}+\beta_{g_{n-1}})\bar{x}_n \tag{32}$$

Here β_{f_n} and β_{g_n} are used to realize robustness of inherent approximation error, they are defined as:

$$\begin{aligned}\beta_{f_n}&=\varepsilon_{f_n}sat\ (e/\mu_e)\\ \beta_{g_n}&=\varepsilon_{g_n}sat(e/\mu_e)sign(u)\end{aligned} \tag{33}$$

D. DYNAMICS OF COMPENSATION ERROR

When proving stability of the control system, the state variable was used as the compensation for dynamics of tracking error. Based on equations (18), (19) and (26), the dynamics of the compensation tacking error of the system at all orders can be divided into the following three situations:

For $i=1$,

$$\dot{\bar{x}}_1=-k_1\bar{x}_1+f_1-\hat{f}_1-\beta_{f_1}+\left(g_1^0+\hat{g}_1+\beta_{g_1}\right)\bar{x}_2+(g_1-\hat{g}_1-\beta_{g_i})x_2 \tag{34}$$

For $i\in[2,n-1]$,

$$\dot{\bar{x}}_i=-k_i\bar{x}_i+f_i-\hat{f}_i-\beta_{f_i}-\left(g_{i-1}^0+\hat{g}_{i-1}+\beta_{g_{i-1}}\right)\bar{x}_i+\left(g_i^0-\hat{g}_i-\beta_{g_i}\right)\bar{x}_{i+1}+\left(g_i-\hat{g}_i-\beta_{g_i}\right)x_{i+1} \tag{35}$$

For $i=n$,

$$\dot{\bar{x}}_n=-k_n\bar{x}_n+f_n-\hat{f}_n-\beta_{f_n}-\left(g_{n-1}^0+\hat{g}_{n-1}+\beta_{g_{n-1}}\right)\bar{x}_n+(g_n-\hat{g}_n-\beta_{g_n})u \tag{36}$$

When $x\in D^n$, derivatives of e can be gained from equations (20), (34), (35), and (36):

$$\begin{aligned}\dot{e}&=\sum_{i=1}^{n}\dot{\bar{x}}_i\\&=\sum_{i=1}^{n-1}\left(-k_i\bar{x}_i+f_i-\hat{f}_i-\beta_{f_i}+g_ix_{i+1}-\beta_{g_i}x_{i+1}\right)\\&\quad+(-k_n\bar{x}_n+f_n-\hat{f}_n-\beta_{f_n}+g_nu-\hat{g}_nu-\beta_{g_n}u)\end{aligned} \tag{37}$$

V. PROOF OF STABILITY

A. STABILITY OF CONTROL SYSTEM STATE

When $x\in D^n-A^{N(t)}$, the controller cannot make an online approximation. This means that $\hat{f}_i=\hat{g}_i=0$, but $\beta_{f_i}\neq 0$ and $\beta_{g_i}\neq 0$. When using equations (28) $\sim$ (33) as control laws, the Lyapunov function has been chosen as $V_0(e)=e^2/2$ and its derivative given by [22] is:

$$\begin{aligned}&\dot{V}_0(e)\\&=e\dot{e}\\&=e\sum_{i=1}^{n-1}(-k_i\bar{x}_i+f_i-\beta_{f_i}+g_ix_{i+1}-\beta_{g_i}x_{i+1})\\&\quad+e(-k_n\bar{x}_n+f_n-\beta_{f_n}-g_nu-\beta_{g_n}u)\\&=e\sum_{i=1}^{n}(-k_i\bar{x}_i+f-\beta_{f_i})\\&\quad+e\sum_{i=1}^{n-1}\left(g_i-\beta_{g_i}\right)x_{i+1}+e(g_n-\beta_{g_n})u\\&\le -e\underline{k}_i\sum_{i=1}^{n}\bar{x}_i\\&\quad+e\left[\sum_{i=1}^{n}\left(f_i-\beta_{f_i}\right)+\sum_{i=1}^{n-1}\left(g_i-\beta_{g_i}\right)x_{i+1}+(g_n-\beta_{g_n})u\right]\\&=-\underline{k}_ie^2\\&\quad+e\left[\sum_{i=1}^{n}\left(f_i-\beta_{f_i}\right)+\sum_{i=1}^{n-1}\left(g_i-\beta_{g_i}\right)x_{i+1}+(g_n-\beta_{g_n})u\right]\end{aligned} \tag{38}$$

where $\underline{k}_i$ is the minimum in all control gains k_i $(1\le i\le n)$.

When $|e(t)|>\mu_e$, if $|f_i|\le\varepsilon_{f_i}$ and $|g_i|\le\varepsilon_{g_i}$, the slip forms (31) and (33) can make the following formula true:

$$e\left[\sum_{i=1}^{n}(f_i-\beta_{f_i})+\sum_{i=1}^{n-1}(g_i-\beta_{g_i})x_{i+1}+(g_n-\beta_{g_n})u\right]\le 0 \tag{39}$$

And, equation (38) can be simplified as:

$$\dot{V}_0(e)\le-\underline{k}_ie^2=-2\underline{k}_iV_0(e)<0 \tag{40}$$

Therefore, $|f_i|\le\varepsilon_{f_i}$ and $|g_i|\le\varepsilon_{g_i}$, $V_0(e)$ must decline with time when $|e(t)|>\mu_e$. Otherwise, $V_0(e)$ increases with time when $|e(t)|>\mu_e$. Then, $|f_i|>\varepsilon_{f_i}$ or $|g_i|>\varepsilon_{g_i}$.

This conclusion can be used as conditions for increasing local approximators.

According to the comparison theorem:

$$V_0(t) \leq e^{-2\underline{k}_i(t-T_0)} V(T_0) \tag{41}$$
$$|e(t)| \leq e^{-2\underline{k}_i(t-T_0)} |e(T_0)| \tag{42}$$

Therefore, if $|e(t)| > \mu_e$, there's $|f_i| > \varepsilon_{f_i}$ or $|g_i| > \varepsilon_{g_i}$ for $t \in \left[T_0, T_0 + (1/\underline{k}_i) ln(|e(T_0)| / \mu_e)\right]$.

On this basis, some local working fields which the controlled system known model can provide adequate precision (in other words, effects of the unknown parts are smaller, that is, $f_i \leq \varepsilon_{f_i}$ and $g_i \leq \varepsilon_{g_i}$) can reach expected tracking precision $|e(t)| \leq \mu_e$ without the need of online approximation to unknown system parts (f_i and g_i). This also reflects that the error $e(t)$ can reflect effects of the unknown parts of the system (f_i and g_i) on tracking precision of the controller effectively.

Therefore, the principle to increase $\hat{f}_{ik}$ and $\hat{g}_{ik}$ can be defined when the following conditions are met simultaneously:

① The current working point $x(t)$ has not activated any existing local approximator. Therefore, the following formula is always true when $1 \leq i \leq n$ and $1 \leq k \leq N(t)$:

$$w_{ik}(x) = 0 \tag{43}$$

② One of following two conditions is true:

a) $\dot{V}(t) \geq 0$ and $|e(t)| > \mu_e$.

b) $|e(\tau)| > \mu_e, \tau \in \left[t - \left(1/\underline{k}_i\right) \ln\left(|e(T_0)| / \mu_e\right), t\right]$.

For ease of use in the following, the time to add the jth local approximator is T_j. In other words, $N\left(T_j\right) = j$ and $N\left(T_j - \varepsilon\right) = j - 1$. The center position of approximation field of the newly added local approximator is defined as $c_{N(T_j)} = x(T_j)$. The initial quantity of local approximator is $N(0) = 0$.

According to this definition, $N(T_j)$ is a constant j in the time interval $\in \left[T_j, T_{j+1}\right)$. For a constant j, the approximator has adequate approximation ability when $T_{j+1} = \infty$.

The following is to prove that the total time of $\bar{x}_i$, e, $\tilde{\theta}_{fik}$, θ_{fik}, $\tilde{\theta}_{gik}$ and $\theta_{gik} \in L_\infty$ and $|e(t)| > \mu_e$ in the time period of $t \in \left[T_j, T_{j+1}\right)$ (quantity of local approximators in this period is fixed j) is bounded. To simplify the definition, let $j = N(T_j)$ and $T_{j+1}^- = N(T_{j+1} - \varepsilon)$.

Based on above analysis, the approximation precision of optimal approximators $f_i^*(x) = \sum_k \bar{w}_{ik}(x) f_{ik}^*(x)$ and $g_i^*(x) = \sum_k \bar{w}_{ik}(x) g_{ik}^*(x)$ on A^j can be expressed as:

$$\left|f_i(x) - f_i^*(x)\right| \leq \varepsilon_{f_i}$$
$$\left|g_i(x) - g_i^*(x)\right| \leq \varepsilon_{g_i} \tag{44}$$

In other words, $f_i^*(x)$ and $g_i^*(x)$ on A^j reach precisions at least ε_{f_i} and ε_{g_i}. In the following text, for $x \in D^n$, the use of $\hat{f}_i(x)$ and $\hat{g}_i(x)$ can make system control precision finally reach $|e(t)| \leq \mu_e$.

For $x \in A^j$, the Lyapunov function was chosen as:

$$V_j(t) = \frac{1}{2}e^2 + \frac{1}{2}\sum_{i=1}^{n}\sum_{k=1}^{j}\left(\tilde{\theta}_{fik}^T \Gamma_{fik}^{-1} \tilde{\theta}_{fik} + \tilde{\theta}_{gik}^T \Gamma_{gik}^{-1} \tilde{\theta}_{gik}\right)$$
$$= V_0 + V_\theta^j \tag{45}$$

where $V_\theta^j = \frac{1}{2}\sum_{i=1}^{n}\sum_{k=1}^{j}\left(\tilde{\theta}_{fik}^T \Gamma_{fik}^{-1} \tilde{\theta}_{fik} + \tilde{\theta}_{gik}^T \Gamma_{gik}^{-1} \tilde{\theta}_{gik}\right)$. Γ_{fik}^{-1} and Γ_{gik}^{-1} are diagonal positive definite matrixes and can control rate of parameter approximation.

Let $t \in [t_1, t_2] \subset \left[T_j, T_{j+1}\right)$ be the time interval when the system tracking error exceeds the expected precision (that is $|e(t)| > \mu_e$). In this period, the state $x(t)$ might be either in A^j or outside A^j. Without loss of generality, it can be seen in two conditions:

① For any sub-interval $t \in [\tau_1, \tau_2] \subset [t_1, t_2]$ that meets $x \notin A^j$, parameter adaptation will stop automatically since $\bar{w}_{ik}(x) = 0$. Therefore, V_θ^j is always a constant. Based on the above analysis, V_0 declines with time in this time interval. Therefore, $V_j(t)$ also declines with time when $t \in [\tau_1, \tau_2] \subset [t_1, t_2]$ (that is, $V_j(\tau_2) \leq V_j(\tau_1)$). Therefore

$$\dot{V}_j(t) = \dot{V}_0 \leq -\underline{k}_i e^2 < 0 \tag{46}$$

② For any sub-interval $t \in [\tau_2, \tau_3] \subset [t_1, t_2]$ that meets $x \in A^j$, the derivative of V_j along equation (37) is

$$\begin{aligned}
&\dot{V}_j(t) \\
&= \dot{V}_0 + \dot{V}_\theta^j \\
&\leq -\underline{k}_i e^2 + e\sum_{i=1}^{n}\left(f_i - f_i^*\right) + e\sum_{i=1}^{n-1}(g_i - g_i^*)x_{i+1} \\
&\quad - e\sum_{i=1}^{n-1}\beta_{g_i}x_{i+1} + e\left(g_n - g_n^*\right)u + e\sum_{i=1}^{n}\left(f_i^* - \hat{f}_i\right) \\
&\quad + e\sum_{i=1}^{n-1}\left(g_i^* - \hat{g}_i\right)x_{i+1} + e\left(g_n^* - \hat{g}_n\right)u - e\sum_{i=1}^{n}\beta_{f_i} \\
&\quad + \sum_{i=1}^{n}\sum_{k=1}^{j}\left(\tilde{\theta}_{fik}^T \Gamma_{fik}^{-1}\dot{\theta}_{fik} + \tilde{\theta}_{gik}^T \Gamma_{gik}^{-1}\dot{\theta}_{gik}\right) - e\beta_{g_n}u \\
&= -\underline{k}_i e^2 + e\sum_{i=1}^{n}\left(f_i - f_i^*\right) + e\left(g_n - g_n^*\right)u - e\beta_{g_n}u \\
&\quad - e\sum_{i=1}^{n-1}\beta_{g_i}x_{i+1} + e\sum_{i=1}^{n-1}(g_i - g_i^*)x_{i+1} \\
&\quad - e\left(\sum_{k=1}^{j}\overline{w}_{ik}\phi_{gik}^T\tilde{\theta}_{gik}\right)u \\
&\quad - e\sum_{i=1}^{n}\left(\sum_{k=1}^{j}\overline{w}_{ik}\phi_{fik}^T\tilde{\theta}_{fik}\right) - e\sum_{i=1}^{n-1}\left(\sum_{k=1}^{j}\overline{w}_{ik}\phi_{gik}^T\tilde{\theta}_{gik}\right)x_{i+1} \\
&\quad - e\sum_{i=1}^{n}\beta_{f_i} + \sum_{i=1}^{n}\sum_{k=1}^{j}\left(\tilde{\theta}_{fik}^T \Gamma_{fik}^{-1}\dot{\theta}_{fik} + \tilde{\theta}_{gik}^T \Gamma_{gik}^{-1}\dot{\theta}_{gik}\right)
\end{aligned}$$

$$
\begin{aligned}
\le & -\underline{k}_i e^2 - \sum_{i=1}^{n-1}\left[e\varepsilon_{g_i} sat\left(e/\mu_e\right) sign(x_{i+1})x_{i+1}\right] \\
& - \sum_{i=1}^{n}\left[e\varepsilon_{f_i} sat\left(e/\mu_e\right)\right] - e\varepsilon_{g_i} sat\left(e/\mu_e\right) sign(u)u \\
& + \sum_{i=1}^{n-1}\left|ex_{i+1}\right|\delta_{g_i} + \sum_{i=1}^{n}\sum_{k=1}^{j}\left[\tilde{\theta}_{fik}^T\Gamma_{gik}^{-1}(\dot{\theta}_{fik} - \Gamma_{fik}\bar{w}_{ik}e\phi_{fik})\right] \\
& + \left|eu\right|\delta_{g_n} + \sum_{i=1}^{n-1}\sum_{k=1}^{j}\left[\tilde{\theta}_{gik}^T\Gamma_{gik}^{-1}(\dot{\theta}_{gik} - \Gamma_{gik}\bar{w}_{ik}ex_{i+1}\phi_{gik})\right] \\
& + \sum_{i=1}^{n}\left|e\right|\delta_{f_i} + \sum_{k=1}^{j}\left[\tilde{\theta}_{gnk}^T\Gamma_{gnk}^{-1}(\dot{\theta}_{gnk} - \Gamma_{gnk}\bar{w}_{nk}eu\phi_{gnk})\right]
\end{aligned} \tag{47}
$$

Therefore, updating the law of approximator parameter can be chosen as:

$$
\dot{\theta}_{fik} = \begin{cases} \Gamma_{fik}\overline{w}_{ik}e\phi_{fik} & |e(t)| > \mu_e \\ 0 & otherwise \end{cases} \tag{48}
$$

$$
\begin{aligned}
\dot{\theta}_{gik} &= Proj\left\{T_{gik}\right\} \\
T_{gik} &= \begin{cases} \Gamma_{gik}\overline{w}_{ik}ex_{i+1}\phi_{gik} & |e(t)| > \mu_e \\ 0 & otherwise \end{cases}
\end{aligned} \tag{49}
$$

$$
\begin{aligned}
\dot{\theta}_{gnk} &= Proj\left\{T_{gnk}\right\} \\
T_{gnk} &= \begin{cases} \Gamma_{gnk}\overline{w}_{nk}eu\phi_{gnk} & |e(t)| > \mu_e \\ 0 & otherwise \end{cases}
\end{aligned} \tag{50}
$$

where the mapping $Proj\{.\}$ is to ensure that $g_i^0 + \hat{g}_i + \beta_{g_i}$ is a bounded function and is not zero.

It can get by bringing equations (48)~(50) into the equation (47):

$$
\begin{aligned}
\dot{V}_j(t) \le & -\underline{k}_i e^2 + \sum_{i=1}^{n}\left[|e|\,\delta_{f_i} - e\varepsilon_{f_i} sat\left(e/\mu_e\right)\right] \\
& + \sum_{i=1}^{n-1}\left[\left|ex_{i+1}\right|\delta_{g_i} - e\varepsilon_{g_i} sat\left(e/\mu_e\right) sign\left(x_{i+1}\right)x_{i+1}\right] \\
& + |eu|\,\delta_{g_n} - e\varepsilon_{g_i} sat\left(e/\mu_e\right) sign\left(u\right)u
\end{aligned} \tag{51}
$$

The mapping $Proj\{\cdot\}$ expressed in equations (49) and (50) ensures that $g_n^0 + \hat{g}_n + \beta_{g_n} > g_l > 0$. Zhao and Farrell [19] reported that for any $t\epsilon\,[\tau_2, \tau_3]$, equation (51) can deduce that:

$$
\dot{V}_j(t) \le -\underline{k}_i e^2 \tag{52}
$$

Therefore, for $|e(t)| > \mu_e$ and $\forall t\epsilon\left[t_1, t_2\right]$, there is:

$$
\dot{V}_j(t) \le -\underline{k}_i e^2 < -\underline{k}_i\mu_e^2 \tag{53}
$$

It can be seen from equation (53) that

$$
\begin{aligned}
V_j(t_2) - V_j(t_1) &\le -\underline{k}_i\mu_e^2\left(t_2 - t_1\right) \\
&= -\underline{k}_i\mu_e^2\bar{\mu}\left(e, \mu_e, t_1, t_2\right)
\end{aligned} \tag{54}
$$

That is,

$$
\bar{\mu}\left(e, \mu_e, t_1, t_2\right) \le \frac{V_j(t_1) - V_j(t_2)}{\underline{k}_i\mu_e^2} \tag{55}
$$

It can be seen from equation (55) that in $[t_1, t_2]$, the total time of $|e(t)| > \mu_e$ is bounded.

Next, suppose e begins to meet requirements of control precision from t_2 to t_3 (in the interval of $|e(t)| \le \mu_e$). This reflects that e enters into the interval $|e(t)| \le \mu_e$ at t_2, but leaves the interval at t_3. Therefore, $t\epsilon\,[t_2, t_3] \subset \left[T_j, T_{j+1}\right)$ refers to $|e(t)| \le \mu_e$ and the time period when $N(t)$ is a constant. The total time that $|e(t)| > \mu_e$ in the time interval is:

$$
\bar{\mu}\left(e, \mu_e, t_2, t_3\right) = 0 \tag{56}
$$

In addition, the following conditions are true: ① in the time interval $[t_2, t_3]$, the approximator parameter is a constant. In other words, the functional approximation stops. ②$|e(t_2)| = |e(t_3)| = \mu_e$. ③$|e(t)| \le |e(t_3)|$, $\forall t \in [t_2, t_3]$.

Obviously, $V_j(t_2) = V_j(t_3)$ and $V_j(t) \le V_j(t_3)$, $\forall t\epsilon\,[t_2, t_3]$. These conclusions are unrelated where x enters into A^j in the interval of $[t_2, t_3]$.

In the following text, stability at any time $t\epsilon\left[T_j, T_{j+1}\right]$ is considered. According to the adding principle, the jth local approximator shall be added when $t = T_j$ and $t = T_{j+1}^-$. It is supposed $e(t)$ leaves the interval $|e(t)| \le \mu_e$ at $t_1 = T_j$ and enters into $|e(t)| \le \mu_e$ at t_m. $e(t)$ leaves the interval $|e(t)| \le \mu_e$ at t_{m+1}. $e(t)$ falls outside the interval of $|e(t)| \le \mu_e$ from t_{m+1} to T_{i+1}^-. Let $\bar{t} \in \left[T_j, T_{j+1}\right)$ be the final time for $|e(t)| \le \mu_e$ in this time interval. Therefore, in the interval of $t\epsilon\left[T_j, T_{j+1}\right)$, the total time that $e(t)$ falls outside the interval of $|e(t)| \le \mu_e$ is:

$$
\begin{aligned}
&\bar{\mu}\left(e, \mu_e, T_j, T_{j+1}^-\right) \\
&= \sum_{j\ge 1}\left[\left(T_{j+1}^- - t_{m+1}\right) + \left(t_{m+1} - \bar{t}\right) + \left(t_m - T_j\right)\right] \\
&\le \frac{1}{\underline{k}_i\mu_e^2}\left[\sum_{j\ge 1}\left[\left(V_j\left(T_{j+1}^-\right) - V_j\left(t_{m+1}\right)\right) + \left(V_j\left(t_{m+1}\right) - V_j\left(\bar{t}\right)\right) + \left(V_j\left(t_m\right) - V_j\left(T_j\right)\right)\right]\right] \\
&\le \frac{1}{\underline{k}_i\mu_e^2}\left[\sum_{j\ge 1}\left[\left(V_j\left(T_{j+1}^-\right) - V_j\left(t_{m+1}\right)\right) + \left(V_j\left(t_{m+1}\right) - V_j\left(t_m\right)\right) + \left(V_j\left(t_m\right) - V_j\left(T_j\right)\right)\right]\right] \\
&= \frac{1}{\underline{k}_i\mu_e^2}\left[\sum_{j\ge 1}\left[\left(V_j\left(T_{j+1}^-\right) - V_j\left(t_{m+1}\right)\right) + \left(V_j\left(t_{m+1}\right) - V_j\left(t_m\right)\right) + \left(V_j\left(t_m\right) - V_j\left(T_j\right)\right)\right]\right] \\
&= \frac{1}{\underline{k}_i\mu_e^2}\left[V_j\left(T_{j+1}^-\right) - V_j\left(T_j\right)\right]
\end{aligned} \tag{57}
$$

This proves that the total time is limited when $e(t)$ is outside the interval of $|e(t)| \le \mu_e$ in the time interval $[T_j, T_{j+1})$. Therefore, either T_{j+1} is infinite to make $|e(t)| \le \mu_e$ or T_{j+1} is finite and $N(t)$ is increased by 1 at $t = T_{j+1}$.

Another important conclusion is that $\forall t \in [T_j, T_{j+1})$ makes the following equation true:

$$V_j(t) \le V_j(T_j) \tag{58}$$

Equation (58) is derived from the above analysis directly no matter whether $|e(t)| > \mu_e$ or $|e(t)| \le \mu_e$ Therefore, these features will be maintained continually for any N, $\bar{x}_i$, e, $\tilde{\theta}_{fik}$, θ_{fik}, $\tilde{\theta}_{gik}$ and $\theta_{gik} \in L_\infty$ even though x enters or leaves A_j for finite times or $e(t)$ enters or levels the interval for $|e(t)| \le \mu_e$ for finite times.

Since D^n is a compact set and each increment of $N(t)$ includes one division of D^n with a radius of μ, only a limited increment of $N(t)$ occurs. Thus, $|e(t)| \le \mu_e$, and $\bar{x}_i$, e, $\tilde{\theta}_{fik}$, θ_{fik}, $\tilde{\theta}_{gik}$, as well as $\theta_{gik} \in L_\infty$.

B. STABILITY OF SELF-ORGANIZING APPROXIMATION

Theorem 1: the system in equation (2) uses the control laws (28) ∼ (33), self-organizing functional approximation and parameter updating laws (48) ∼ (50). It has following characteristics:

① $\bar{x}_i, e, \tilde{\theta}_{fik}, \theta_{fik}, \tilde{\theta}_{gik}, \theta_{gik}$ and $N(t) \epsilon L_\infty$.

② $e = \sum_{i=1}^{n} \bar{x}_i$ finally converges at $|e| \le \mu_e$.

Proof: let the working time of system is $[T_0, T_f]$, where T_f can be infinite. Number of initial approximator is $N(T_0) = 0$. As shown above, $N(t)$ increases by 1 every $T_j.N(t) = 0$ and $\hat{f}_i(x) = \hat{g}_i(x) = 0$ when $t \in [T_0, T_1)$. As stated above, either the total time of $|e(t)| > \mu_e$ is smaller than $(1/\underline{k}_i)\ln(|e(T_0)|/\mu_e)(T_1 = \infty)$, or T_1 is a finite evaluate. Under these two conditions,

$$V_0(T_1^-) \le max\left(V_0(T_0), \frac{1}{2}\mu_e^2\right) \tag{59}$$

For $j \ge 1$, the jth local approximator field is added at $t = T_j$. In the above text, ① and ② in Theorem 1 have been proved under the condition of $t \in [T_j, T_{j+1})$. The only part that has not been proved is where $V_j(T_j)$ in the transition from $V_{j-1}(T_j^-)$ to $V_j(T_j)$ is bounded.

In the following text, $V_j(T_j)$ is a finite value. Let the Lyapunov function at $t = T_j$ be:

$$V_j(T_j) = \frac{1}{2}e^2(T_j) + \frac{1}{2}\sum_{i=1}^{n}\sum_{k=1}^{j}\left[\tilde{\theta}_{fik}^T(T_j)\Gamma_{fik}^{-1}\tilde{\theta}_{fik}(T_j) + \tilde{\theta}_{gik}^T(T_j)\Gamma_{gik}^{-1}\tilde{\theta}_{gik}(T_j)\right] \tag{60}$$

It is seen that $e(T_j) = e(T_j^-)$. Since $e(t)$ is continuous from T_j^- to T_j and $x(T_j)$ has not activated the previous $j-1$ local approximator when the jth local approximator is added into the system when $t = T_j$. Parameters θ_{fik} and θ_{gik} $(k = 1, \cdots, j-1)$ remain the same from T_j^- to T_j. Therefore,

$$\begin{aligned} V_j(T_j) &= \frac{1}{2}e^2(T_j^-) \\ &\quad + \frac{1}{2}\sum_{i=1}^{n}\sum_{k=1}^{j-1}\left[\tilde{\theta}_{fik}^T(T_j^-)\Gamma_{fik}^{-1}\tilde{\theta}_{fik}(T_j^-) + \tilde{\theta}_{gik}^T(T_j^-)\Gamma_{gik}^{-1}\tilde{\theta}_{gik}(T_j^-)\right] \\ &\quad + \frac{1}{2}\left[\tilde{\theta}_{fik}^T(T_j)\Gamma_{fik}^{-}\tilde{\theta}_{fik}(T_j) + \tilde{\theta}_{gik}^T(T_j)\Gamma_{gik}^{-1}\tilde{\theta}_{gik}(T_j)\right] \\ &= V_{j-1}(T_j^-) + \frac{1}{2}\left[\tilde{\theta}_{fik}^T(T_j)\Gamma_{fik}^{-1}\tilde{\theta}_{fik}(T_j) + \tilde{\theta}_{gik}^T(T_j)\Gamma_{gik}^{-1}\tilde{\theta}_{gik}(T_j)\right] \end{aligned} \tag{61}$$

It has therefore been proved that for any $t \in [T_{j-1}, T_j^-]$, $V_{j-1}(t) \le V_{j-1}(T_{j-1})$. Then, it can be deduced that:

$$\begin{aligned} V_j(T_j) &\le V_{j-1}(T_{j-1}) + \frac{1}{2}\left[\tilde{\theta}_{fik}^T(T_j)\Gamma_{fik}^{-1}\tilde{\theta}_{fik}(T_j) + \tilde{\theta}_{gik}^T(T_j)\Gamma_{gik}^{-1}\tilde{\theta}_{gik}(T_j)\right] \\ &\le \frac{1}{2}e^2(T_1) + \frac{1}{2}\sum_{i=1}^{n}\sum_{k=1}^{j}\left[\tilde{\theta}_{fik}^T(T_k)\Gamma_{fik}^{-1}\tilde{\theta}_{fik}(T_k) + \tilde{\theta}_{gik}^T(T_k)\Gamma_{gik}^{-1}\tilde{\theta}_{gik}(T_k)\right] \end{aligned} \tag{62}$$

For $k = 1, \cdots, j$, every $\tilde{\theta}_{fik}(T_k) = \theta_{fik}(T_k) - \theta_{fik}^*$ is a finite value as long as the initial parameter estimation $\theta_{fik}(T_k)$ is finite at $t = T_k$. Under similar conditions, $\tilde{\theta}_{gik}(T_k) = \theta_{gik}(T_k) - \theta_{gik}^*$ is finite. Since N can only grow finitely, that is, $N(T_j) = j < \infty$, the sum of right items in equation (62) is limited. Since $e(T_1)$ is a finite value, it can deduce $V_j(T_j) < \infty$ directly. This means $\bar{x}_i$, e, $\tilde{\theta}_{fik}$, θ_{fik}, $\tilde{\theta}_{gik}$, and $\theta_{gik} \in L_\infty$.

It has to be pointed out that the above proof process only proves the boundedness of system compensation tracking error $\bar{x}_i$ in equation (19), rather than the boundedness of the actual system tracking error $\tilde{x}_i$ in equation (18). It can be seen from equations (18), (19), (22), (24) and (26) that $\bar{x}_i$ approaches to $\tilde{x}_i$ infinitely by choosing filter gain K_i and control law gain k_i reasonably. Therefore, the boundedness of $\tilde{x}_i$ also can be assured.

VI. SIMULATION CASE ANALYSIS

For the following 2-order SISO system [28]:

$$\begin{cases} \dot{x}_1 = \sin(x_1 + x_2) + (2 + g_1(x))\,x_2 \\ \dot{x}_2 = \sin(x_2) + (2 + g_2(x))u \end{cases} \tag{63}$$

where

$$g_1(x) = g_2(x) = \frac{1}{20}\left(x_1^2 + |x_1|\right)\cos(0.01\pi x_1) \tag{64}$$

The system state vector is $x = [x_1, x_2]^T$ and the rating working field is $D^2 = [-3, 3] \times [-3, 3]$. It is supposed that the known design model of the system is:

$$\begin{cases} \dot{x}_1 = 2x_2 \\ \dot{x}_2 = 2u \end{cases} \tag{65}$$

where the known parts are $f_1^0 = f_2^0 = 0$ and $g_1^0 = g_2^0 = 2$. The unknown parts are $f_1 = \sin(x_1 + x_2)$, $f_2 = \sin(x_2)$ and $g_1 = g_2 = \frac{1}{20}\left(x_1^2 + |x_1|\right)\cos(0.01\pi x_1)$. Each unknown function is applied to the above mentioned online self-organizing approximation method for online approximation. $\hat{f}_{1k}, \hat{f}_{2k}, \hat{g}_{1k}$ and $\hat{g}_{2k}$ all use the same primary function vector of normalized biquadratic kernel $w_k(x)$:

$$w_k(x) \begin{cases} \left(1 - R^2\right)^2 & R < 1 \\ 0 & R \geq 1 \end{cases} \tag{66}$$

$$R = \left\| \frac{|x_1 - c_{k,1}|}{\mu_{k,1}}, \frac{|x_2 - c_{k,2}|}{\mu_{k,2}} \right\|_\infty \tag{67}$$

where $c_k = \left[c_{k,1}, c_{k,2}\right]^T$ is the center of kth primary function, $\mu_{k,1}$ and $\mu_{k,2}$ are radii of the kth local approximation field on the x_1 and x_2 direction, $\mu_{k,1} = \mu_{k,2} = 0.3$. The continuous primary function vector is appointed as:

$$\phi_{f_{ik}} = \phi_{g_{ik}} = \left[1 x_1 - c_{k,1} x_2 - c_{k,2}\right]^T \tag{68}$$

Other simulation parameters are set: $k_1 = 2$, $k_2 = 4$, $\mu_e = 0.1$, $\varepsilon_{f_i} = \varepsilon_{g_i} = 0.3$, and $\Gamma_{f_{1k}} = \Gamma_{f_{2k}} = 10\, I_{3\times3} \Gamma_{g_{1k}} = \Gamma_{g_{2k}} = 10 I_{3\times3}$. Here, the first-order filter in equation (24) is used and the gain is $K_2 = K = 40$. The initial value of online estimation parameter is:

$$\theta_{f_{1k}}(0) = \theta_{f_{2k}}(0) = \theta_{g_{1k}}(0) = \theta_{g_{2k}}(0) = [0, 0, 0,]^T.$$

The reference trajectory $x_c(t)$ and its derivative $\dot{x}_c(t)$ can be generated by the following 2-order low pass filter [23]:

$$\dot{z}_1 = z_1 \tag{69}$$

$$\dot{z}_2 = a_1\left[sat\left(a_1\left(sat(r) - z_1\right)\right) - z_2\right]$$

$$\begin{bmatrix} x_c \\ \dot{x}_c \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} \tag{70}$$

$$r = 3sin(0.2\pi t) \tag{71}$$

where the function $sat(\cdot)$ is the amplitude and the rate of restricted input signal to ensure that $(x_c, \dot{x}_c) \in D^2$, $a_1 = 2\xi\omega$, $a_2 = \omega^2/(2\xi\omega)$, and $\xi = 0.9$, $\omega = 5$.

When equations (69)~(71) are used as input, the first four periods of $x_1 - x_2$ plane path orbit of the control system are shown in Fig.1.

In Fig.1, all unknown functions in the grey parallelogram are relatively small. This shows that $|f_i| \leq \varepsilon_{f_i}$ and $|g_i| \leq \varepsilon_{g_i}$. The symbol $\times$ is the center of local approximation field and the square area is the approximation field of local approximator $\hat{f}_{ik}$ and $\hat{g}_{ik}$ (the approximation field $\bar{S}_k$).

It can be seen that equations (69) ~ (71) are periodic input signals. Thus, the phase path of the closed loop system in Fig.1 also shows periodic changes. The control system adds 13 local approximators automatically according to error e. The added local approximators $\hat{f}_{ik}$ and $\hat{g}_{ik}$ are used for online

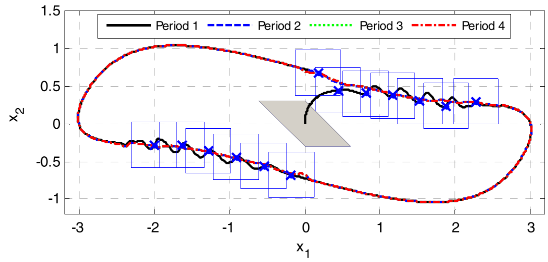

FIGURE 1. First four periods of $x_1 - x_2$ plane path orbit of the control system.

approximation of the unknown functions f_i and g_i. The horizontal distance between the centers of two adjacent local approximation fields is approximately $\mu_{k,1} = \mu_{k,2} = 0.3$. After the system has started, its state approaches quickly from the original position of the origin to the final stable phase path. When the unknown function has large impacts on the system error e, and the sliding-mode control items β_{f_i} and β_{g_i} cannot offset the local region of this reach independently, the control system will add local approximators automatically for the online approximation of unknown function in order to offset adverse impacts of the unknown functions on the system error e. Therefore, the system state returns to the expected phase path quickly. It can be seen from Fig.1 that the system has only state fluctuation in some regions during the first period. Subsequently, the system state can be run continuously in the final stable phase path.

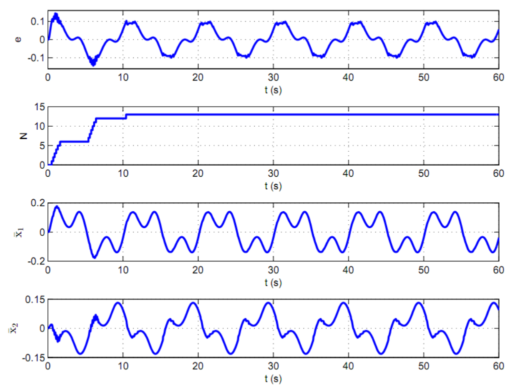

FIGURE 2. e, N and $\bar{x}_i$.

The scalar mapping e of the system compensation tracking error, the number of local approximator N and compensation tracking error $\bar{x}_i$ are shown in Fig.2. State x_i, control command $x_{i,c}$, and the actual tracking error $\tilde{x}_i$ of the system are shown in Fig.3.

It can be seen from Fig.2 that the error e exceeds the required range of $\mu_e = 0.1$ in the time interval of 0.5~1.63s, 5.4~6.41s and 10.41~10.47s, and the total time exceeding the error range is absolutely limited. The three time intervals correspond to the local approximation field expressed by square area as shown in Fig.2. Under the circumstances, β_{f_i} and β_{g_i} cannot offset the impacts of the unknown function on error e independently.

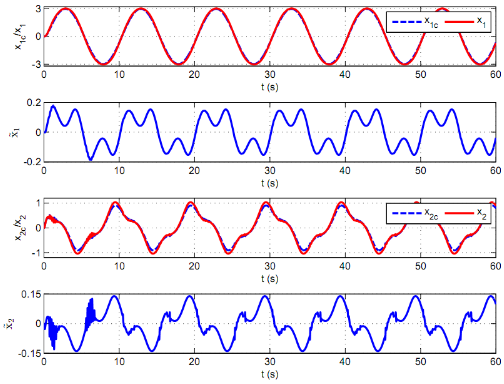

FIGURE 3. x_i, $x_{i,c}$ and $\bar{x}_i$.

The control system adds 6, 6, and 1 local approximators automatically to offset adverse impacts of the unknown function on error e, respectively. After 10.47s, e shows periodic changes and always meet $|e| \leq \mu_e = 0.1$ due to the addition of the approximator. The compensation tracking error $\bar{x}_i$ of the system has a similar variation law with e and is always remained bounded after stable system running (after 10.47s). It can be seen from the comparison between Fig.2 and Fig.3 that since the command filters (24) and (25) use the large gains K_2 and K, the compensation tracking error $\bar{x}_i$ and the actual tracking error $\tilde{x}_i$ are very close.

The unknown functions (f_i and g_i), online approximation functions of the unknown function ($\hat{f}_i$ and $\hat{g}_i$), and β_{f_i} and β_{g_i} are shown in Fig.6 and Fig.7.

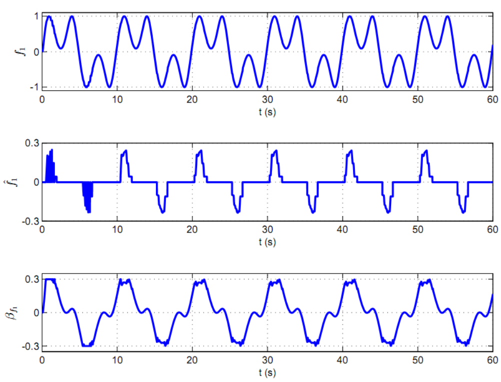

FIGURE 4. f_1, $\hat{f}_1$ and β_{f_1}.

It can be seen from Fig.4 to 7 that the online approximator $\hat{f}_i$ and $\hat{g}_i$ make an online approximation of f_i and g_i with respect to positions (position of cycles in the interval of 0.5~1.63s, 5.4~6.41s and subsequent interval) in the local approximation field in Fig.1. In the time interval of 0.5~1.63s and 5.4~6.41s, $\hat{f}_i$ and $\hat{g}_i$ are in starting periods and have not been developed completely. Under this circumstance, although β_{f_i} and β_{g_i} have reached the saturation state, they are still unable to offset the effects of the excessive unknown function on e. After stable running of the system, $\hat{f}_i$ and $\hat{g}_i$ begin to develop completely. At this moment, β_{f_i} and β_{g_i} always maintain

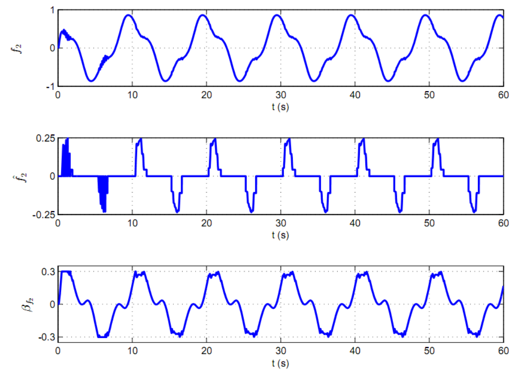

FIGURE 5. f_2, $\hat{f}_2$ and β_{f_2}.

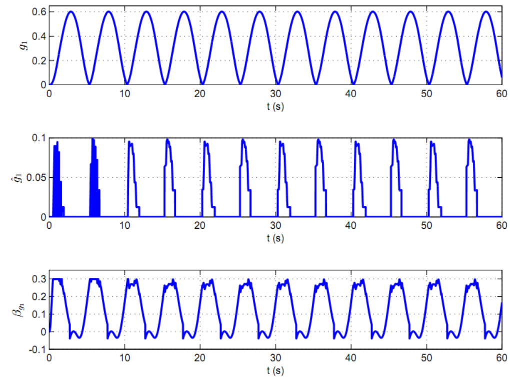

FIGURE 6. g_1, $\hat{g}_1$ and β_{g_1}.

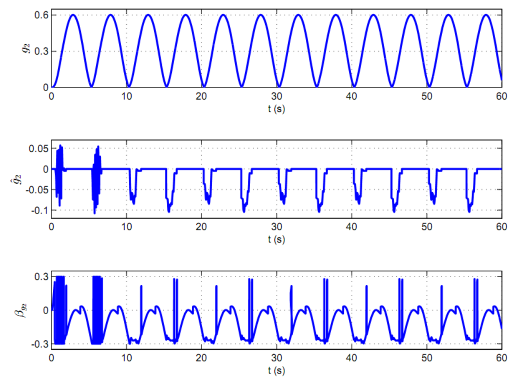

FIGURE 7. g_2, $\hat{g}_2$ and β_{g_2}.

the unsaturated states and the error e can always be maintained in the expected precision. All system states are stably bounded.

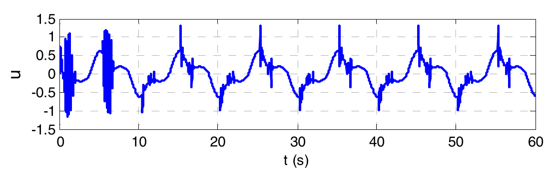

FIGURE 8. System input.

The system input u is shown in Fig.8. It has slight high-frequency shaking in the working time period of approximator, which might be caused by incompletely unreasonable setting of controller parameters.

It should be pointed out that the error e in Equation (20) is the algebraic sum of compensation tracking errors ($\bar{x}_i$) of all levels and cannot reflect the actual tracking error $\tilde{x}_1$ of the system directly. Therefore, control parameters have to be determined according to requirements of the actual tracking error of the system and simulation effect. In addition, the quality of control methods is determined by the selection of control parameters during control system design. In this simulation case, only one reference parameter value is given, but it is not the optimal value. Further optimization can be made to gain the optimal control effect.

VII. CONCLUSION

Based on the command filtered backstepping controller structure, the self-organizing approximation in IOT system is expanded to one type of more common n-order SISO affine system. Its application range of this method is expanded significantly compared to the method in [29], [31], [33], and [34].

The proposed method includes command filtered backstepping and online self-organizing approximation. The command filtered backstepping is to overcome the analytic solution to virtual control command in ordinary backstepping control. As a result, the high-order cascade structure of controlled objects can be processed by relative decoupling. The online self-organizing approximation is to increase local approximators continuously according to system error in order to reduce adverse impacts of IOT system model difference on tracking performance of the IOT system.

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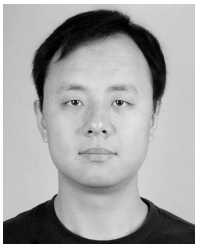

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