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Finite-Time Non-Fragile Control of a Class of Uncertain Linear Positive Systems

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ABSTRACT The finite-time non-fragile control scheme has received great interest because of its robustness to the controller gain errors. In this paper, we intend to study the finite-time non-fragile control problems for a class of linear positive systems with uncertainties. It devises an appropriate non-fragile control law, such that the closed-loop system is positive and stabilizable and satisfies the given H_∞ performance in a specified time interval. The main issue is to give a sufficient condition for the solution of the designed finite-time non-fragile H_∞ controller associated with the several control techniques applied to the positive system. The design result is described as an optimization problem that can be expressed through a couple of linear matrix inequalities. In the end, we use a practical RL circuit model to evaluate the performance of the proposed controller.

INDEX TERMS Positive system, finite-time, non-fragile control, RL circuit model.

I. INTRODUCTION

When we model some systems in actual engineering applications, we always encounter such kind of controlled objects: when the given initial conditions of the modeled dynamics are non-negative (or positive, strictly), the responses, i.e., the states and outputs are still non-negative (or positive, strictly). Considering the positiveness (or non-negativeness), we always call this dynamic model as positive system. It can be applied in several fields, for example, biological systems, industrial engineering, system control theory, social science and robot industry. The study of positive systems can retrospect to 1970s and many results are available, such as stability analysis [1], [2], controllability [3], observability [4] and the relevant realization problems [5]. Based on these, the research of positive systems has received a lot of concerns in recent years and some publications can be obtained in [6]–[10].

In general, we always assume that the designed controller can be implemented precisely when we study the dynamic systems. However, this assumption is not always the fact because the uncertainties in controller coefficient are often inevitable. These uncertainties will cause some fragile disturbances during the process of controller design. Many factors can cause such fragile disturbances, for example, network environment circumstances [11], round off errors in numerical calculation [12] and the inherent inaccuracies in simulation [13]. Over the years, many scholars have showed great interest in such fragile control problems in theory analysis and practical applications. They are concerned about how to devise a non-fragile control law which can make some errors in feedback control gains be insensitive. Ionescu *et al.* [14] considered a robust non-fragile control strategy with the help of linear matrix inequalities (LMIs) method. For Markovian jumping nonlinear systems, the observer-based passive non-fragile control strategy was studied [15]. Yang and Che [16] investigated the design problems of the non-fragile H_∞ filter for discrete-time systems with FWL conditions. Then, many scholars applied the non-fragile control method to solve some relevant fragile problems of nonlinear systems [17]–[23], jumping systems [13], tracking control [24], neural networks [25]–[27] and etc.

In traditional control fields, many experts and scholars paid more attention to the asymptotic stability analysis, that is, the stability study in an infinite time region. In fact, only the asymptotically stability analysis may not fully

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satisfy the requests of some actual engineering applications. Therefore, the stability and stabilizable problems in a specified finite-time have been received many researchers' consideration and some new results have been achieved, see for instance, [28]–[34] and some relevant published references. Although much research on non-fragile control and finite-time control has devoted to many control systems, little research has been done on positive systems. The finite-time stabilizable (FTS) and the non-fragile control problems of positive dynamic systems have not been intensively noticed by scholars. Based on the previous publications, we intend to fill the research gap of the non-fragile control scheme for positive systems with uncertain parameters in a specified time interval.

The main contributions of this paper as follows. Firstly, an appropriate non-fragile control law is devised to ensure the state trajectories of dynamic systems are FTS and satisfy the given finite-time H_∞ performance index. Secondly, the positiveness of the closed-loop dynamics is guaranteed. Thirdly, an appropriate non-fragile control law is designed to tolerate a certain degree of control gain variations. Applying Lyapunov function methods and LMIs techniques, sufficient conditions are established to achieve the non-fragile control law such that the closed-loop system is positive and stabilizable and satisfies the given H_∞ performance in a specified finite-time. Then, the performance of the proposed controller is illustrated at last through a practical RL circuit model.

II. MAIN DEFINITIONS AND NOTATIONS

Consider the linear dynamic model described by

$$\Theta:\begin{cases}\dot{x}(t)=Ax(t)+Bu(t)\\ z(t)=Cx(t)+Du(t)\\ x(t)=x_0, \quad t=0.\end{cases}$$

The following definitions and lemmas are needed before the study.

Definition 1 [37]: Θ is a positive system, if $x_0 \geq 0$, $u(t) \geq 0$, it concludes $x(t) \geq 0$ and $z(t) \geq 0$, $\forall t > 0$.

Lemma 1 [35]: If matrices A, B, C, D in Θ satisfy that A is a Metzler matrix, $B \succeq 0$, $C \succeq 0$ and $D \succeq 0$, Θ is said to be a positive system.

Definition 2 [37]: If there exists a constant $\varepsilon > 0$ satisfying $A + \varepsilon I \succeq 0$, where A is a real square matrix, A can be called as a Metzler matrix.

Remark 1: From Definition 2, we know that if $A = [a_{ij}] \in A_n$ is a Metzler matrix, it is a real square matrix. The elements in A on the non-diagonal line are non-negative, i.e., $a_{ij} \geq 0, i \neq j$. In other words, if there exists a constant $\varepsilon > 0$ satisfying $A + \varepsilon I \succeq 0$, it means the non-diagonal elements of A are positive.

Lemma 2 [36]: For given proper dimensional matrices M and N, there exists a constant $\varepsilon > 0$ which satisfies $MX(t)N + N^{\mathrm{T}}X^{\mathrm{T}}(t)M^{\mathrm{T}} < \varepsilon^{-1}MM^{\mathrm{T}} + \varepsilon N^{\mathrm{T}}N$.

Notations: Throughout this paper, we assume that the notations are quite standard and all the matrices have compatible dimensions. We give the meaning of the notations in Table 1.

TABLE 1. Symbols meanings.

Notation	Denotes	Notation	Denotes
$\Re^n$	n-dimensional Euclidean space	$*$	symmetric matrix
$\Re^{n\times m}$	$n\times m$ real matrices	I	unit matrix
A^{T}	matrix transpose	N	number of subsystems
A^{-1}	matrix inverse	0	zero matrix
$\lVert * \rVert$	Euclidean vector norm	$\lambda_{\min}(P)$	minimum eigenvalue of P
$P>(<)0$	positive-(negative) definite matrix	$\lambda_{\max}(P)$	maximum eigenvalue of P
$A\succeq 0$	positive matrix	diag$\{A\ B\}$	block-diagonal matrix of A and B
$\Delta(\square)$	uncertainties	$\Gamma(t)$	time-varying matrix function

III. SYSTEM FORMULATION

Consider the linear positive system with uncertainties described by:

$$\begin{cases}\dot{x}(t)=(A+\Delta A(t))\,x(t)+(B_1+\Delta B_1(t))\,u(t)+B_2\hbar(t)\\ z(t)=(C+\Delta C(t))\,x(t)+(D_1+\Delta D_1(t))\,u(t)+D_2\hbar(t)\\ \dot{\hbar}(t)=(H+\Delta H(t))\,\hbar(t)\\ x(t)=x_0,\quad \hbar(t)=\hbar_0,\ t=0\end{cases} \tag{1}$$

where $x(t) \in \Re^n$ is the state, $z(t) \in \Re^p$ is the controlled output, $\hbar(t) \in \Re^q$ is the unknown disturbance, $u(t) \in \Re^m$ is the controlled input. $(A + \Delta A(t)) \in \Re^{n\times n}$ is a Metzler matrix, $(B_1 + \Delta B_1(t)) \in \Re^{n\times q}$, $B_2 \in \Re^{n\times m}$, $(C + \Delta C(t)) \in \Re^{p\times n}$, $(D_1 + \Delta D_1(t)) \in \Re^{p\times q}$ and $D_2 \in \Re^{p\times m}$ are positive matrices, $(H + \Delta H(t)) \in \Re^{m\times m}$ is a known matrix. The uncertain matrices satisfy:

$$\begin{bmatrix}\Delta A(t) & \Delta B_1(t)\\ \Delta C(t) & \Delta D_1(t)\end{bmatrix}=\begin{bmatrix}M_1\\ M_2\end{bmatrix}\Gamma(t)\,[N_1\quad N_2] \tag{2}$$

$$\Delta H(t)=M_3\Gamma(t)N_3 \tag{3}$$

where $\Gamma(t)$ is a Lebesgue norm measurable function and satisfies $\Gamma^{\mathrm{T}}(t)\Gamma(t) \leq I$; $M_1, M_2, M_3, N_1, N_2, N_3$ are known matrices. For given positive constants T and h, we assume that the unknown disturbance $\hbar(t) \in \Re^q$ satisfies:

$$\int_0^T \hbar^{\mathrm{T}}(t)\hbar(t)dt < h \tag{4}$$

where $T > 0$ and $h > 0$.

Supposing that the states of positive system (1) can be available. Then we devise the state feedback control law as:

$$u(t) = (K + \Delta K(t))\, x(t) \tag{5}$$

where K is the control law gain to be determined and $\Delta K(t)$ is an additive perturbation of the control law gain satisfying:

$$\Delta K(t) = M_4 \Gamma(t) N_4 \tag{6}$$

where M_4, N_4 are known matrices, which incarnate the structural characteristics of the uncertain parameters.

Remark 2: In this paper, the uncertain matrices with the symbol $\Delta(\cdot)$ in Eqs. (2), (3) and (5) can be considered as admissible conditions. In actual applications, we always cannot obtain the exact mathematical model of practical dynamics directly because of some complexity processes, environmental noises and time-varying parameters. Thus, the uncertain dynamics existing in positive system (1) reflect the inexactness in mathematical modeling of such positive system. Moreover, the Lebesgue norm measurable function $\Gamma(t)$ is selected as a full row rank matrix and it also can be considered as state-dependent, i.e., $\Gamma(t) = \Gamma(t, x(t))$ if $\Gamma^{\mathrm{T}}(t, x(t))\Gamma(t, x(t)) \leq I$. For more results of this subject, the readers can refer to literature [15], [17], [13], [22]–[25].

Substituting the state feedback control law (5) into positive system (1), we can get the following closed-loop system:

$$\begin{cases} \dot{x}(t) = Ex(t) + B_2\hbar(t) \\ z(t) = Fx(t) + D_2\hbar(t) \\ \dot{\hbar}(t) = \bar{H}\,\hbar(t) \\ x(t) = x_0, \quad \hbar(t) = \hbar_0, \ t = 0 \end{cases} \tag{7}$$

where

$$\begin{aligned} E &= (A + \Delta A(t)) + (B_1 + \Delta B_1(t))\,(K + \Delta K(t))\,, \\ F &= (C + \Delta C(t)) + (D_1 + \Delta D_1(t))\,(K + \Delta K(t))\,, \\ \bar{H} &= (H + \Delta H(t))\,. \end{aligned}$$

Before giving the results of the paper, the following main definitions are necessary.

Definition 3 [38]: For given time constant $T > 0$, the closed-loop system (7) is FTS on $(c_1\ c_2\ T\ R\ \delta)$, if there exists constants c_1, c_2 with $c_2 > c_1 > 0$ and positive-define matrix $R > 0$, such that:

$$x^{\mathrm{T}}(t)Rx(t) < c_2, \quad \text{if } x_0^{\mathrm{T}}Rx_0 \leq c_1,\ \hbar_0^{\mathrm{T}}\hbar_0 \leq \delta,\ \forall t \in [0, T]\,. \tag{8}$$

Definition 4 [38]: For given time constant $T > 0$, the state feedback control law (5) can be considered as the finite-time non-fragile H_∞ control law of positive system (1), if under the non-fragile control law $u(t) = (K + \Delta K(t))\,x(t)$, the closed-loop system (7) is FTS on $(c_1\ c_2\ T\ R\ \delta)$ and satisfies the given H_∞ performance index:

$$J = \int_0^T \left[z^{\mathrm{T}}(t)z(t) - \gamma^2\hbar^{\mathrm{T}}(t)\hbar(t)\right]dt < 0. \tag{9}$$

Remark 3: This paper study the non-fragile finite-time H_∞ controller design problem for linear positive system. Although the main results in [4], [15], [18], and [19] consider the nonlinear case, the nonlinear part is bounded and can be linearization representation. Thus, it can be considered as a linearization method to study the nonlinear systems in [4], [15], [18], and [19]. In other words, the method studied in this paper also can be extended to the nonlinear system.

IV. MAIN RESULTS

Theorem 1: For given constants $T > 0$, $\alpha > 0$, $c_1 > 0$, $h > 0$, the closed-loop system (7) is FTS on $(c_1\ c_2\ T\ R\ \delta)$, where $R > 0$, if there exists constants $\lambda > 0$, $c_2 > c_1 > 0$ and positive definite and symmetric matrices $P_1 \in \Re^{n\times n}$, $P_2 \in \Re^{m\times m}$, the following inequalities hold:

$$\begin{bmatrix} E^{\mathrm{T}}P_1 + P_1E - \alpha P_1 & P_1B_2 \\ * & \bar{H}^{\mathrm{T}}P_2 + P_2\bar{H} - \alpha P_2 \end{bmatrix} < 0 \tag{10}$$

$$c_1\lambda_{\max}(\tilde{P}_1) + \delta\lambda_{\max}(P_2) < c_2\lambda_{\min}(\tilde{P}_1)e^{-\alpha T}. \tag{11}$$

Proof: For any positive-definite and symmetric matrices $P_1 \in \Re^{n\times n}$ and $P_2 \in \Re^{m\times m}$, we select the Lyapunov function as:

$$\wp(x(t), \hbar(t)) = x^{\mathrm{T}}(t)P_1x(t) + \hbar^{\mathrm{T}}(t)P_2\hbar(t) \tag{12}$$

Along the track of the closed-loop system (7), the derivative of $\wp(x(t), \hbar(t))$ is:

$$\begin{aligned} &\dot{\wp}(x(t), \hbar(t)) \\ &\quad = x^{\mathrm{T}}(t)(E^{\mathrm{T}}P_1 + P_1E)x(t) + x^{\mathrm{T}}(t)P_1B_2\hbar(t) \\ &\qquad + \hbar^{\mathrm{T}}(t)B_2^{\mathrm{T}}P_1x(t) + \hbar^{\mathrm{T}}(t)(\bar{H}^{\mathrm{T}}P_2 + P_2\bar{H})\hbar(t). \end{aligned} \tag{13}$$

From inequality (10), we have:

$$\dot{\wp}(x(t), \hbar(t)) - \alpha\wp(x(t), \hbar(t)) < 0. \tag{14}$$

Multiplying the inequality (14) by $e^{-\alpha t}$ and integrating inequality (14) from 0 to t, we obtain:

$$\int_0^t \frac{\mathrm{d}}{\mathrm{dt}}\left[e^{-\alpha\tau}\wp(x(t), \hbar(t))\right]\mathrm{d}\tau < 0 \tag{15}$$

which implies $\wp(x(t), \hbar(t)) - e^{\alpha t}\wp(x(0), \hbar(0)) < 0$.

Definite $\tilde{P}_1 = R^{-1/2}P_1R^{-1/2}$ and yield:

$$\begin{aligned} x^{\mathrm{T}}(t)P_1x(t) &\leq \wp(x(t), \quad \hbar(t)) < e^{\alpha t}\wp(x(t), \hbar(t)) \\ &< e^{\alpha t}\wp(x(0), \quad \hbar(0)) < e^{\alpha t}[x_0^{\mathrm{T}}P_1x_0 + \hbar_0^{\mathrm{T}}P_2\hbar_0] \\ &< e^{\alpha T}[\lambda_{\max}(\tilde{P}_1)x_0^{\mathrm{T}}Rx_0 + \lambda_{\max}(P_2)\hbar_0^{\mathrm{T}}\hbar_0]. \end{aligned} \tag{16}$$

Considering that:

$$x^{\mathrm{T}}(t)P_1x(t) \geq \lambda_{\min}(\tilde{P}_1)x^{\mathrm{T}}(t)Rx(t) \tag{17}$$

$$\begin{bmatrix} E^{\mathrm{T}}P_1 + P_1E + F^{\mathrm{T}}F - \alpha P_1 & P_1B_2 + F^{\mathrm{T}}D_2 \\ * & \bar{H}^{\mathrm{T}}P_2 + P_2\bar{H} + D_2^{\mathrm{T}}D_2 - \alpha P_2 - \gamma^2 I \end{bmatrix} < 0 \tag{21}$$

and

$$e^{\alpha T}[\lambda_{\max}(\tilde{P}_1)x_0^{\mathrm{T}}Rx_0 + \lambda_{\max}(P_2)\hbar_0^{\mathrm{T}}\hbar_0] > \lambda_{\min}(\tilde{P}_1)x^{\mathrm{T}}(t)Rx(t) \tag{18}$$

we have:

$$x^{\mathrm{T}}(t)Rx(t) < \frac{e^{\alpha T}[\lambda_{\max}(\tilde{P}_1)x_0^{\mathrm{T}}Rx_0 + \lambda_{\max}(P_2)\hbar_0^{\mathrm{T}}\hbar_0]}{\lambda_{\min}(\tilde{P}_1)}. \tag{19}$$

From inequality (8), we can get:

$$e^{\alpha T}[c_1\lambda_{\max}(\tilde{P}_1) + \delta\lambda_{\max}(P_2)] < c_2\lambda_{\min}(P_1). \tag{20}$$

It is obviously that inequality (20) can be guaranteed by inequality (11) for $\forall\, t \in [0, T]$. This completes the proof.

Recalling to Definition 4, we give the following Theorem 2.

Theorem 2: For given constants $T > 0$, $\alpha > 0$, $c_1 > 0$, $h > 0$, the closed-loop system (7) is FTS on $(c_1\ c_2\ T\ R\ \delta)$ and satisfies the given H_∞ performance index (9), if there exists constants $\gamma > 0$, $\lambda > 0$, $c_2 > c_1 > 0$ and positive-definite and symmetric matrices $P_1 \in \Re^{n\times n}$, $P_2 \in \Re^{m\times m}$, such that inequality (11) and the following relation hold, (21), as shown at the top of this page.

Proof: We select the similar Lyapunov function in (12) and introduce the following inequality:

$$\dot{\wp}(x(t), \hbar(t)) < \alpha\wp(x(t), \hbar(t)) + \gamma^2\hbar^{\mathrm{T}}(t)\hbar(t) - z^{\mathrm{T}}(t)z(t). \tag{22}$$

Multiplying inequality (22) by $e^{-\alpha t}$ and integrating inequality (22) from 0 to t, it yields:

$$\gamma^2\int_0^t e^{-\alpha\tau}[\gamma^2\hbar^{\mathrm{T}}(\tau)\hbar(\tau) - z^{\mathrm{T}}(\tau)z(\tau)]\mathrm{d}\tau > \int_0^t \frac{\mathrm{d}}{\mathrm{d}t}[e^{-\alpha\tau}\wp(x(t), \hbar(t))]\mathrm{d}\tau. \tag{23}$$

According to Newton-Leibniz formula, we have:

$$\gamma^2\int_0^t e^{-\alpha\tau}[\gamma^2\hbar^{\mathrm{T}}(\tau)\hbar(\tau) - z^{\mathrm{T}}(\tau)z(\tau)]\mathrm{d}\tau > e^{-\alpha\tau}\wp(x(t), \hbar(t)) - \wp(x_0, \hbar_0) \tag{24}$$

which means,

$$\gamma^2\int_0^t e^{-\alpha\tau}\hbar^{\mathrm{T}}(\tau)\hbar(\tau)\mathrm{d}\tau > \int_0^t e^{-\alpha\tau}z^{\mathrm{T}}(\tau)z(\tau)\mathrm{d}\tau. \tag{25}$$

For $\forall t \in [0, T]$, we have:

$$\gamma^2 e^{\alpha T}\int_0^T \hbar^{\mathrm{T}}(\tau)\hbar(\tau)\mathrm{d}\tau > \int_0^T z^{\mathrm{T}}(\tau)z(\tau)\mathrm{d}\tau. \tag{26}$$

It is evidently that the finite-time H_∞ performance index (9) can be guaranteed by $\bar{\gamma} = \sqrt{e^{\alpha T}}\gamma$ for $\forall t \in [0, T]$. This completes the proof.

Theorem 3: For given constants $T > 0$, $\alpha > 0$, $c_1 > 0$, there exists a finite-time non-fragile control law with $K = YX^{-1}$ and the closed-loop system (7) is positive and FTS and satisfies the given H_∞ performance index (9) on $(c_1\ c_2\ T\ R\ \delta)$, where $R > 0$, if there exists constants $c_2 > c_1 > 0$, $\delta > 0$, $\varepsilon > 0$, $\nu > 0$, $\mu > 0$, $\lambda_1 > 0$, $\lambda_2 > 0$, $\lambda_3 > 0$, $\sigma > 0$, $\varphi > 0$, $\theta > 0$, positive-definite and symmetric matrix $X \in \Re^{n\times n}$ and matrix $Y \in \Re^{m\times n}$, such that:

$$\begin{bmatrix} \Theta_{11} & \Theta_{12} \\ * & \Theta_{22} \end{bmatrix} < 0 \tag{27}$$

$$\sigma R^{-1} < X < R^{-1} \tag{28}$$

$$\begin{bmatrix} \delta\lambda_2 - c_2\lambda_3 e^{-\alpha T} & \sqrt{c_1} \\ \sqrt{c_1} & -\lambda_1 \end{bmatrix} < 0 \tag{29}$$

$$AX + B_1Y + \mu I \succeq 0 \tag{30}$$

$$CX + D_1Y \succeq 0 \tag{31}$$

$$N_1X + N_2Y \succeq 0 \tag{32}$$

$$N_4X \succeq 0 \tag{33}$$

where

$$\Theta_{11} = \begin{bmatrix} \psi_{11} & B_2 & \psi_{13} \\ * & \psi_{22} & D_2^{\mathrm{T}} \\ * & * & \psi_{33} \end{bmatrix}$$

$$\Theta_{12} = \begin{bmatrix} 0 & XN_1^{\mathrm{T}} + Y^{\mathrm{T}}N_2^{\mathrm{T}} & XN_4^{\mathrm{T}} \\ P_2M_3 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$\Theta_{22} = \begin{bmatrix} -\nu I & 0 & 0 \\ * & -\varepsilon I & 0 \\ * & * & \Psi_{33} \end{bmatrix},$$

$$\psi_{11} = AX + XA^{\mathrm{T}} + B_1Y + Y^{\mathrm{T}}B_1^{\mathrm{T}} + \varepsilon^{-1}M_1M_1^{\mathrm{T}} + \varphi^{-1}B_1M_4M_4^{\mathrm{T}}B_1^{\mathrm{T}} + \theta M_1M_1^{\mathrm{T}} - \alpha X,$$

$$\psi_{13} = XC^{\mathrm{T}} + Y^{\mathrm{T}}D_1^{\mathrm{T}} + \varepsilon^{-1}M_1M_2^{\mathrm{T}} + \varphi^{-1}B_1M_4M_4^{\mathrm{T}}D_1^{\mathrm{T}} + \theta M_1M_2^{\mathrm{T}},$$

$$\psi_{22} = H^{\mathrm{T}}P_2 + P_2H - \alpha P_2 - \gamma^2 I - \nu^{-1}N_3^{\mathrm{T}}N_3,$$

$$\psi_{33} = -I + \varepsilon^{-1}M_2M_2^{\mathrm{T}} + \varphi^{-1}D_1M_4M_4^{\mathrm{T}}D_1^{\mathrm{T}} + \theta M_2M_2^{\mathrm{T}},$$

$$\Psi_{33} = -\varphi I + \theta^{-1}M_4^{\mathrm{T}}N_2^{\mathrm{T}}N_2M_4.$$

Proof: Substituting

$$E = (A + \Delta A(t)) + (B_1 + \Delta B_1(t))\,(K + \Delta K(t)),$$
$$F = (C + \Delta C(t)) + (D_1 + \Delta D_1(t))\,(K + \Delta K(t)),$$
$$\bar{H} = (H + \Delta H(t))$$

into inequality (21), we can get:

$$\begin{bmatrix} \Xi_{11} & \Xi_{12} \\ * & \Xi_{22} \end{bmatrix} < 0 \tag{34}$$

where

$$\begin{aligned}\Xi_{11} &= [(A+\Delta A(t))+(B+\Delta B(t))(K+\Delta K(t))]^{\mathrm{T}} P_1 \\ &\quad +[(C+\Delta C(t))+(D+\Delta D(t))(K+\Delta K(t))]^{\mathrm{T}} \\ &\quad \cdot[(C+\Delta C(t))+(D+\Delta D(t))(K+\Delta K(t))] \\ &\quad +P_1[(B+\Delta B(t))(K+\Delta K(t))]+P_1[(A+\Delta A(t))] \\ &\quad -\alpha P_1 \\ \Xi_{12} &= P_1B_2+[(C+\Delta C(t))+(D+\Delta D(t))(K+\Delta K(t))]^{\mathrm{T}}D_2, \\ \Xi_{22} &= -\gamma^2 I+D_2^{\mathrm{T}}D_2+[H+\Delta H(t)]^{\mathrm{T}}P_2+P_2[H+\Delta H(t)] \\ &\quad -\alpha P_2.\end{aligned}$$

Using diag $\{P_1^{-1}, I\}$ to pre- and post-multiply matrix (34), and considering $X = P_1^{-1}$, $Y = KX$, we have:

$$\Omega_1 + \Delta\Omega_1 < 0 \tag{35}$$

where

$$\begin{aligned}\Omega_1 &= \begin{bmatrix}\Phi_{11} & B_2 \\ * & \Phi_{22}\end{bmatrix}, \\ \Phi_{11} &= [(A+\Delta A(t))+(B_1+\Delta B_1(t))(K+\Delta K(t))]X \\ &\quad +X[(A+\Delta A(t))+(B_1+\Delta B_1(t))(K+\Delta K(t))]^{\mathrm{T}} \\ &\quad -\alpha X, \\ \Phi_{22} &= (H+\Delta H(t))^{\mathrm{T}}P_2+P_2(H+\Delta H(t))-\alpha P_2-\gamma^2 I, \\ \Delta\Omega_1 &= \begin{bmatrix}\Delta\Phi_{11} & \Delta\Phi_{12} \\ * & D_2^TD_2\end{bmatrix}, \\ \Delta\Phi_{11} &= X[(C+\Delta C(t))+(D_1+\Delta D_1(t))(K+\Delta K(t))]^{\mathrm{T}} \\ &\quad \cdot[(C+\Delta C(t))+(D_1+\Delta D_1(t))(K+\Delta K(t))]X, \\ \Delta\Phi_{12} &= X[(C+\Delta C(t))+(D_1+\Delta D_1(t))(K+\Delta K(t))]^{\mathrm{T}}D_2.\end{aligned}$$

Then for (35), we have the following inequality by considering Lemma 2 and using Schur complement lemma:

$$\Omega_2 + \Delta\Omega_2 + \Delta\Omega_3 + \Delta\Omega_4 < 0 \tag{36}$$

where

$$\begin{aligned}\Omega_2 &= \begin{bmatrix}\Psi_{11} & B_2 & \Psi_{13} \\ * & \Psi_{22} & D_2^{\mathrm{T}} \\ * & * & -I\end{bmatrix}, \\ \Delta\Omega_2 &= \begin{bmatrix}0 & 0 & 0 \\ * & \Delta H^{\mathrm{T}}P_2+P_2\Delta H & 0 \\ * & * & 0\end{bmatrix}, \\ \Delta\Omega_3 &= \begin{bmatrix}\Xi_{11} & 0 & X\Delta C^{\mathrm{T}}+Y^{\mathrm{T}}\Delta D_1^{\mathrm{T}} \\ * & 0 & 0 \\ * & * & 0\end{bmatrix}, \\ \Delta\Omega_4 &= \begin{bmatrix}\prod_{11} & 0 & X\Delta K^{\mathrm{T}}D_1^{\mathrm{T}}+X\Delta K^{\mathrm{T}}\Delta D_1^{\mathrm{T}} \\ * & 0 & 0 \\ * & * & 0\end{bmatrix}, \\ \Psi_{11} &= XA^{\mathrm{T}}+AX-\alpha X+Y^{\mathrm{T}}B_1^{\mathrm{T}}+B_1Y, \\ \Psi_{13} &= XC^{\mathrm{T}}+Y^{\mathrm{T}}D_1^{\mathrm{T}}, \\ \Psi_{22} &= P_2H+H^{\mathrm{T}}P_2-\gamma^2 I-\alpha P_2, \\ \Xi_{11} &= X\Delta A^{\mathrm{T}}(t)+\Delta A(t)X+Y^{\mathrm{T}}\Delta B_1^{\mathrm{T}}(t)+\Delta B_1(t)Y, \\ \prod\nolimits_{11} &= B_1\Delta K(t)X+\Delta B_1(t)\Delta K(t)X \\ &\quad +X\Delta K^{\mathrm{T}}(t)B_1+X\Delta K^{\mathrm{T}}(t)\Delta B_1(t).\end{aligned}$$

According to relations (2), (3), (6) and Lemma 2, $\Delta\Omega_2$, $\Delta\Omega_3$ and $\Delta\Omega_4$ can be rewritten as:

$$\begin{cases}\Delta\Omega_2 = Z_1\Gamma(t)Z_2+Z_2^{\mathrm{T}}\Gamma^{\mathrm{T}}(t)Z_1^{\mathrm{T}} < \nu^{-1}Z_1Z_1^{\mathrm{T}}+\nu Z_2^{\mathrm{T}}Z_2 \\ \Delta\Omega_3 = Z_3\Gamma(t)Z_4+Z_4^{\mathrm{T}}\Gamma^{\mathrm{T}}(t)Z_3^{\mathrm{T}} < \varepsilon^{-1}Z_3Z_3^{\mathrm{T}}+\varepsilon Z_4^{\mathrm{T}}Z_4 \\ \Delta\Omega_4 = Z_5\Gamma(t)Z_6+Z_6^{\mathrm{T}}\Gamma^{\mathrm{T}}(t)Z_6^{\mathrm{T}} < \varphi^{-1}Z_5Z_5^{\mathrm{T}}+\varphi Z_6^{\mathrm{T}}Z_6,\end{cases}$$

where

$$\begin{aligned}&Z_1 = [0 \quad P_2M_3 \quad 0]^{\mathrm{T}}, \quad Z_2 = [0 \quad N_3 \quad 0], \\ &Z_3 = [M_1 \quad 0 \quad M_2]^{\mathrm{T}}, \quad Z_4 = [M_1^{\mathrm{T}} \quad 0 \quad M_2^{\mathrm{T}}], \\ &Z_5 = [B_1M_4+\Delta B_1M_4 \quad 0 \quad D_1M_4+\Delta D_1M_4]^{\mathrm{T}}, \\ &Z_6 = [N_4X \quad 0 \quad 0].\end{aligned}$$

Then for (36), we can get the following inequality:

$$\Omega_3 + \Delta\Omega_5 < 0 \tag{37}$$

where

$$\begin{aligned}\Omega_3 &= \begin{bmatrix}\coprod_{11} & \coprod_{12} \\ * & \coprod_{22}\end{bmatrix}, \quad \coprod_{11} = \begin{bmatrix}\nabla_{11} & B_2 & \nabla_{13} \\ * & \nabla_{22} & D_2^{\mathrm{T}} \\ * & * & \nabla_{33}\end{bmatrix}, \\ \coprod_{12} &= \begin{bmatrix}0 & XN_1^{\mathrm{T}}+Y^{\mathrm{T}}N_2^{\mathrm{T}} & XN_4^{\mathrm{T}} \\ P_2M_3 & 0 & 0 \\ 0 & 0 & 0\end{bmatrix}, \\ \coprod_{22} &= \begin{bmatrix}-\nu I & 0 & 0 \\ * & -\varepsilon I & 0 \\ * & * & \kappa_{33}\end{bmatrix}, \\ \nabla_{11} &= AX+XA^{\mathrm{T}}+B_1Y+Y^{\mathrm{T}}B_1^{\mathrm{T}}+\varepsilon^{-1}M_1M_1^{\mathrm{T}} \\ &\quad +\varphi^{-1}B_1M_4M_4^{\mathrm{T}}B_1^{\mathrm{T}}-\alpha X, \\ \nabla_{13} &= XC^{\mathrm{T}}+Y^{\mathrm{T}}D_1^{\mathrm{T}}+\varepsilon^{-1}M_1M_2^{\mathrm{T}}+\varphi^{-1}B_1M_4M_4^{\mathrm{T}}D_1^{\mathrm{T}}, \\ \nabla_{22} &= H^{\mathrm{T}}P_2+P_2H-\alpha P_2-\gamma^2 I-\nu^{-1}N_3^{\mathrm{T}}N_3, \\ \nabla_{33} &= -I+\varepsilon^{-1}M_2M_2^{\mathrm{T}}+\varphi^{-1}D_1M_4M_4^{\mathrm{T}}D_1^{\mathrm{T}}, \quad \kappa_{33} = -\varphi I, \\ \Delta\Omega_5 &= \begin{bmatrix}0 & \vartheta_{12} \\ * & 0\end{bmatrix}, \quad \vartheta_{12} = \begin{bmatrix}0 & 0 & \Delta B_1M_4 \\ 0 & 0 & 0 \\ 0 & 0 & \Delta D_1M_4\end{bmatrix}.\end{aligned}$$

Recalling to Lemma 2, we can re-write $\Delta\Omega_5$ as:

$$\Delta\Omega_4 = Z_7\Gamma(t)Z_8+Z_8^{\mathrm{T}}\Gamma^{\mathrm{T}}(t)Z_7^{\mathrm{T}} < \theta Z_7Z_7^{\mathrm{T}}+\theta^{-1}Z_8^{\mathrm{T}}Z_8,$$

where $Z_7 = [M_1\ 0\ M_2\ 0\ 0\ 0]^{\mathrm{T}}$, $Z_8 = [0\ 0\ 0\ 0\ 0\ N_2M_4]$.

Then for (37), using Schur complement lemma, we can obtain (27).

Definite $\bar{P}_1 = R^{1/2}XR^{1/2}$ and consider $\max\sigma(x) = \frac{1}{\min\sigma(P)}$, it concludes to get condition (11) by inequality (29).

Then, we prove the positiveness. From (30), we know that $(AX+B_1Y)$ is a Metzler matrix. Since $Y = KX$, $(A+B_1K)X$ is also a Metzler matrix. From (31) and (32), we know that N_1X+N_2Y and $CX+D_1Y$ are positive matrices, i.e., $(N_1+N_2K)X$ and $(C+D_1K)X$ are positive, which means N_1+N_2K and $C+D_1K$ are positive. Therefore, E is a Metzler matrix and F is a positive matrix. Recalling to $B_2 \succeq 0$ and $D_2 \succeq 0$, we get that the closed-loop system (7) is positive. This completes the proof.

Remark 4: Theorem 1 and 2 give sufficient conditions to ensure that the closed-loop system (7) is FTS and satisfies the H_∞ performance index from the unknown disturbance to the controlled output. In Theorem 3, we devise the sufficient conditions to obtain the non-fragile H_∞ control law by a couple of matrix inequalities in a specified time interval.

Remark 5: In order to obtain sufficient conditions to the finite-time non-fragile H_∞ controller, we make the appropriate scaling for the selected Lyapunov function, which will bring somewhat conservative for the controller design. The conservative problem can be reduced by selecting different Lyapunov functions [20]–[22].

Corollary 1: A sufficient condition to solve the non-fragile H_∞ control law of positive system (1) in a specified time interval is given in Theorem 3. Considering that the coupling inequalities (27)-(34) are related to X, Y, c_1, c_2, δ, T, α, ν, μ, λ_1, λ_2, λ_3, θ, ε, σ, φ and γ^2, we have the optimization algorithm by setting γ^2 as an optimization variable value:

$$\min_{X,Y,c_1,c_2,\delta,T,\alpha,\nu,\mu,\lambda_1,\lambda_2,\lambda_3,\theta,\varepsilon,\sigma,\phi,\gamma^2} \gamma^2 \quad \text{s.t. LMIs}(27)-(33). \tag{38}$$

Remark 6: Similar to the simple optimal control, we describe the design result as an optimization problem in Corollary 1. However, simple optimal control has no good robustness to controller gain error. Different with linear model predictive control, which focuses on open-loop optimal control problem. The finite-time non-fragile control studied in this paper not only concerned with the optimal solution problem of the closed-loop control system in a specified time interval but also ensured the result has good robustness to the controller gain error.

V. EXAMPLE

Considering a circuit model described as Fig.1, where R_1, R_2, R_3 stand for the resistances, L_1, L_2 stand for the inductances, $u_1(t)$, $u_2(t)$ stand for the controlled sources. Assume that the inductance values and the resistance values are linear time-invariant. $i_{1L}(t)$, $i_{2L}(t)$ are the currents through L_1 and L_2, respectively.

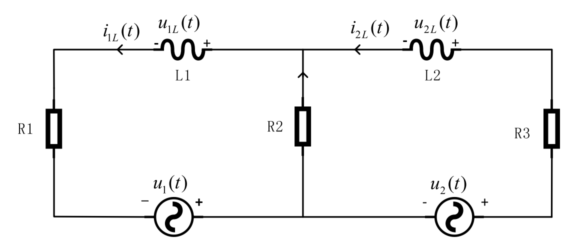

FIGURE 1. The RL circuit model.

Define the state variables as $x_1(t) = i_{1L}(t)$, $x_2(t) = i_{2L}(t)$ and the output variable as $z(t) = \begin{bmatrix} R_1 i_{1L}(t) \\ R_2 i_{2L}(t) \end{bmatrix}$. Using the Kirchhoff voltage law and considering $u_{1L}(t) = L_1 \frac{di_{1L}(t)}{dt}$, $u_{2L}(t) = L_2 \frac{di_{2L}(t)}{dt}$, we can get:

$$\begin{cases} \dot{x}_1(t) == -\dfrac{R_1+R_3}{L_1}x_1(t) + \dfrac{R_3}{L_1}x_2(t) + \dfrac{R_3}{L_1}u_1(t) \\ \dot{x}_2(t) == \dfrac{R_3}{L_2}x_1(t) - \dfrac{R_2+R_3}{L_2}x_2(t) + \dfrac{1}{L_2}u_2(t) \\ z(t) = \begin{bmatrix} R_1 x_1(t) \\ R_2 x_2(t) \end{bmatrix}. \end{cases} \tag{39}$$

Select the RL circuit model parameters as $R_1 = 1, R_2 = 2$, $R_3 = 3, L_1 = L_2 = \frac{1}{3}$. Thus, the circuit model can be derived as:

$$\begin{cases} x(t) = Ax(t) + B_1 u(t) \\ z(t) = Cx(t) + D_1 u(t) \end{cases} \tag{40}$$

where

$$A = \begin{bmatrix} -12 & 9 \\ 9 & -15 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 9 \\ 3 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}, \quad D_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

The other parameters are given as:

$$B_2 = \begin{bmatrix} 0.4 \\ 0.2 \end{bmatrix}, \quad D_2 = \begin{bmatrix} 0.2 \\ 0.1 \end{bmatrix}, \quad H = [1], \quad R = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$
$$M_1 = \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix}, \quad M_2 = \begin{bmatrix} 0.3 \\ 0.2 \end{bmatrix}, \quad M_3 = [2],$$
$$M_4 = \begin{bmatrix} 0.2 & 0.4 \end{bmatrix}, \quad N_1 = \begin{bmatrix} 0.5 & 0.6 \end{bmatrix}, \quad N_2 = [0.02],$$
$$N_3 = [2], \quad N_4 = [0.02], \quad c_1 = 4, \quad \alpha = 9.3, \quad T = 2.$$

Applying the optimization algorithm in Corollary 1 and solving LMIs (27)-(33), we can get the non-fragile control gain as $K = [0.4595 \ \ 0.7102]$ with $c_2 = 4.5615$ and H_∞ performance parameter $\gamma = 7.2162$.

With the initial conditions $x_0 = [1.3 \ \ 1.2]^{\mathrm{T}}$, we can obtain the simulation plots in Fig 2 and Fig 3.

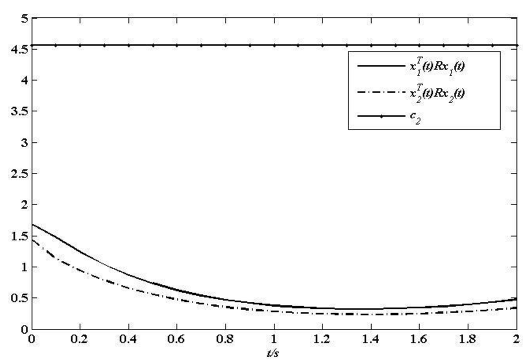

FIGURE 2. The state trajectories of $x^{\mathrm{T}}(t)Rx(t)$.

From Fig.2, we can see that the RL circuit model system is stabilizable within 1 second and the dynamic trajectory $x^{\mathrm{T}}(t)Rx(t)$ of the system is positive and bounded in a specified time interval [0 2] and satisfies $x^T(t)Rx(t) < c_2$ with

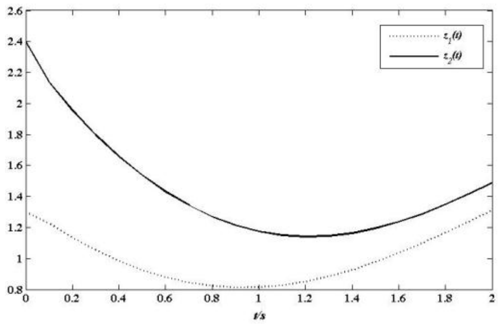

FIGURE 3. The output trajectories of z(*t*).

$c_2 = 4.5615$; It is obvious from Fig.3 to see that the output signal is satisfies condition of positive system in Definition 1 and the given H_∞ performance (9) with $\gamma = 7.2162$ in a specified time interval [0 2]. The stabilizable of the output needs to be further improved in the future work.

Remark 7: A practical RL circuit model is given to testify the effectiveness of the proposed control scheme in simulation results. In the simulation results, we consider the circuit model (39) and (40) containing uncertainties and external disturbances due to the aging and inaccurate measurement of the system devices. Moreover, the designed finite-time non-fragile controller not only applies to the RL circuit but also to any other linear control systems described by (40).

Remark 8: The uncertain parameters and unknown disturbances are given values in the RL circuit model, which will bring somewhat conservative for the simulation results. From Fig.3, we know that the finite-time stabilization of RL circuit model can be studied further improvement. Thus, we will focus on these problems in the future work to improve the proposed methods.

VI. CONCLUSION

In this paper, the research gap of the non-fragile H_∞ controller design problem of uncertain positive systems in a specified time interval is filled. The message of the research is to devise a suitable non-fragile H_∞ controller such that the close-loop system be FTS and satisfy the given finite-time H_∞ performance index. Moreover, the positiveness of the close-loop system is also proved. In order to obtain the sufficient conditions of the designed finite-time non-fragile H_∞ controller, the LMIs technique is used and the control law design problem is formulated as an optimized problem. A practical RL circuit model example is illustrated to evaluate the performance of the proposed controller. In the future research, we can apply the proposed control methods on other linear positive system, such as linear positive Markov jump system and linear positive switching system.

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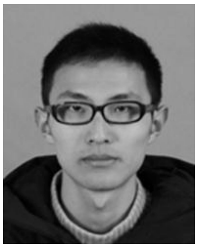

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