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SER Analysis of Adaptive Threshold-Based Relay Selection With Limited Feedback for Type II Relay

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ABSTRACT In this paper, via the exact closed-form expressions, we statistically investigate the symbol-error-rate performance of the limited feedback-based adaptive threshold-based relay selection (ATRS) scheme under multiple type II relay environments. Our analytical results are thoroughly validated by cross-checking them against the results obtained via the Monte Carlo simulations. Some selected results show that the performance degradation caused by minimizing the feedback can be compensated by refining the feedback information, specifically by applying a 1-bit increase in terms of the feedback data rate and an additional single-comparison process in terms of complexity. Note that our derived results are valid for both symmetric and asymmetric link conditions. Additionally, the analytical results can be easily evaluated numerically with the standard mathematical packages. Furthermore, the analytical framework and the derived results can be applied for the expansion of the n-bit feedback-based ATRS scheme. Finally, for practical consideration, we provide useful discussion on the plausibility that the extended scheme can be implemented with only a 1-bit feedback.

INDEX TERMS SER, closed-form expressions, limited feedback, relay selection, type II relay.

I. INTRODUCTION

Cooperative relaying has been introduced in wireless networks to provide significant transmission benefits [1]–[3]. To improve this benefit further, the diversity from multiple relays can be exploited through various relay selection schemes [4], which operate based on the channel status information (CSI) of the related links. However, since cooperative relaying inevitably introduces redundant hops or links, the necessary CSI triggers the bottleneck state in practical implementations. Hence, reduction of the overhead caused by the required CSI exchanges is crucial. Many approaches to reduce both the overhead in the CSI exchanges and the effect of the outdated CSI have been considered and analyzed by comparing the trade-offs between the performance and the CSI overhead (or the quality of CSI) [5]–[8].

In the 3rd generation partnership project (3GPP), mainly two types of relaying strategies, namely the type I (or infrastructure) relay and the type II (or user equipment (UE)) relay, have been investigated [9]–[11]. While the type I relay presents two-hop half-duplex relaying (or non-transparent relay) scheme, the type II relay describes multi-cast cooperative relaying (or transparent relay) scheme. The type I relay forms an independent cell with a small coverage for the coverage extension. The type II relay, meanwhile, forwards the overheard messages to increase the data rate of the end user having weaker signal quality. The current LTE-Advanced specification does not define any detailed functionality of the type II relay. This is because it was decided to focus on the type I relay during the standardization of the current Release and table the type II relay as a study item for the future Releases in 3GPP. Hence, in this study, we are focusing on the type II relay which must be transparent to the end user ($\mathcal{D}$) and the retransmitted signal from the selected relay ($\mathcal{R}$) is seen at $\mathcal{D}$ as it is from the source ($\mathcal{S}$) [10]–[19].

Note that the design of the relay selection schemes should also consider the industry standard compliance to be applicable in the real wireless systems. One of the possible solutions for the type II relay to keep the backward compatibility is to perform the relay selection process at the transmitter (i.e., $\mathcal{S}$).[1] In this transmitter-oriented scheme, unlike the

[1]In [20], the relay selection process is performed at the end user, which increases the complexity and is therefore undesirable for a mobile UE with limited complexity.

conventional receiver-oriented method, $\mathcal{R}$s can send the limited feedback information regarding the channel status to $\mathcal{S}$, so that $\mathcal{S}$ can utilize it when selecting the best $\mathcal{R}$.

Based on these observations, in [21], the adaptive threshold-based relay selection (ATRS) scheme with the minimum feedback information (i.e., the 1-bit feedback information) was proposed in compliance with the specifications for the type II relay. The proposed scheme in [21] does not need to feedback the full channel information to $\mathcal{S}$. Instead, each $\mathcal{R}$ simply reports its $\mathcal{R}-\mathcal{D}$ link status (e.g., unacceptable or acceptable) back to $\mathcal{S}$. Further, $\mathcal{S}$ selects the relay with the highest $\mathcal{S}-\mathcal{R}$ link gain among the relays that report the "acceptable" $\mathcal{R}-\mathcal{D}$ link status. Therefore, $\mathcal{R}$s and $\mathcal{D}$ do not need to exchange the control (scheduling) messages. Thus, the backward compatibility with the LTE-Advanced standard [22] is maintained. Further, this scheme provide the relatively low-complexity compared to the recently proposed type II relay based relay selection schemes [17], [19].

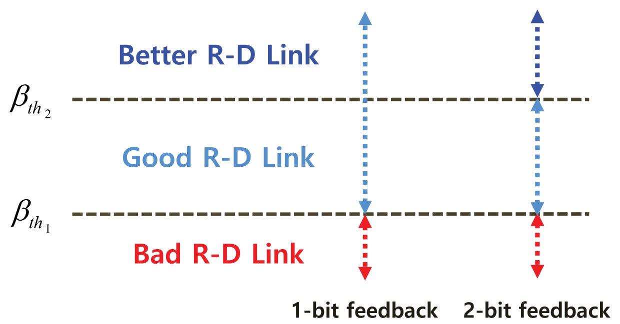

FIGURE 1. ATRS with the 2-bit feedback.

Although the ATRS scheme in [21] can provide an acceptable performance with high bandwidth efficiency and low complexity, $\mathcal{S}$ only has the average channel gain information about the $\mathcal{R}-\mathcal{D}$ link. Thus, due to this insufficient information, the end-to-end performance may be significantly degraded by mistakenly discarding some links with better $\mathcal{R}-\mathcal{D}$ channel quality. For example, as shown in Fig. 1, when selecting candidates using only two levels of the $\mathcal{R}-\mathcal{D}$ link status information, there may exist an $\mathcal{R}$ that does not provide a better $\mathcal{S}-\mathcal{R}$ link but has a better $\mathcal{R}-\mathcal{D}$ link compared to the scheduled $\mathcal{R}$, while providing a better performance overall in terms of the modified harmonic mean applied in [21]. Note that with the 1-bit feedback based scheme, these two $\mathcal{R}$s may be regarded as relays providing the same performance.

To overcome this drawback, in [23] a more refined feedback information is applied to compensate for the potential performance degradation. Instead of identifying the acceptable or unacceptable $\mathcal{R}$s based on a single threshold, the status of each $\mathcal{R}$ can be classified into one of the three levels (e.g., unacceptable, good, or better) according to its $\mathcal{R}-\mathcal{D}$ link gain by adopting one more threshold as shown in Fig. 1. After selecting the best two $\mathcal{R}$s from the "good" candidate group and the "better" candidate group, respectively, $\mathcal{S}$ chooses the better $\mathcal{R}$ between the two. Since each $\mathcal{R}$ needs to classify its $\mathcal{R}-\mathcal{D}$ link status into one of the three status levels, a 2-bit feedback is required. Therefore, with this limited feedback-based ATRS scheme, the potential performance degradation can be compensated with a 1-bit increase in terms of the feedback data rate and an additional single-comparison process in terms of complexity. By adopting one additional threshold to the minimum feedback-based ATRS scheme in [21], the system can provide the symbol error rate (SER) performance very close to that of the perfect feedback case by eliminating the possibility of missing better channel links. However, the SER performance results in [23] only rely on the computer simulations without any analytical derivation and validation of these results. Since the simulation results show some selected results only for the tested parameter range, they are not suitable for the versatile performance analysis that can provide an important insight on the general range of the parameter values [24]–[26]. The timely adoption of these technologies in the real-world systems relies heavily on the analytical predictions.

In this context, we statistically analyze the SER performance of a limited feedback-based ATRS scheme in a closed-form expression and these results are validated with the results obtained via the Monte-Carlo simulations. The derived closed-form results can be used to predict the SER performance under both symmetric and asymmetric link conditions. Additionally, the analytical results can be easily evaluated numerically with the standard mathematical packages, such as Mathematica, MATLAB, and Maple. Further, the analytical framework and the derived results can be applied for the expansion of the n-bit feedback-based ATRS scheme. Finally, for practical consideration, we provide useful discussion on the plausibility that the extended scheme can be implemented with a 1-bit feedback.

The remainder of this study is structured as follows. In section II, we present the system and the channel models including the mode of operation. Further, in section III, we provide the exact closed-form expressions of SER for the M-ary phase-shift keying (PSK) signaling based on the statistical analysis. Finally, some selected results are provided in section IV, followed by the concluding remarks in section V.

II. SYSTEM AND CHANNEL MODELS

Similar to [21], we assume that all the channel links are quasi-static (or block flat fading) and mutually independent, where the fading coefficients remain constant for at least one transmission duration, but vary independently over different transmission durations. Additionally, the fading conditions follow the Rayleigh fading model. We also assume that all $\mathcal{R}$s are statistically identical. We consider the network with multiple type II UE $\mathcal{R}$s and the decode-and-forward (DF) protocols that are distinguished and remarkable due to their simple and intuitive design [1]–[3]. With the DF protocols, $\mathcal{S}$ communicates with $\mathcal{D}$ with the help of multiple UE $\mathcal{R}$s ($\mathcal{R}_i$) where N possible UE $\mathcal{R}$s exist. The channel gains of the $\mathcal{S}-\mathcal{D}$, $\mathcal{S}-\mathcal{R}_\rangle$, and $\mathcal{R}_\rangle-\mathcal{D}$ links are denoted by $h_{s,d}$, h_{s,r_i}, and $h_{r_i,d}$, respectively, which are assumed to be the zero-mean

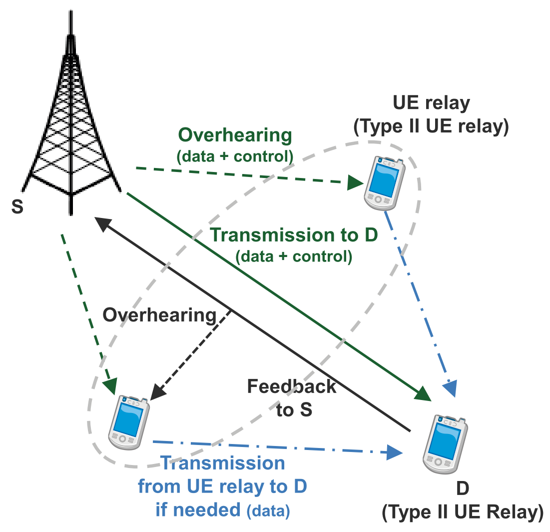

FIGURE 2. System model of Type II relay.

complex Gaussian random variables with variances $\delta^2_{s,d}$, δ^2_{s,r_i}, and $\delta^2_{r_i,d}$, respectively.

We also assume that each node has a single antenna and operates on the half-duplex mode which means they can either receive or transmit messages but cannot do both simultaneously. To ensure the half-duplex operation, we consider the two-phase protocol for the cooperative transmission. Furthermore, it is assumed that each $\mathcal{R}$ knows the CSI of its $\mathcal{S}-\mathcal{R}_{\rangle}$ and $\mathcal{R}_{\rangle}-\mathcal{D}$ links based on ACK/NACK from the end user ($\mathcal{D}$) [14], [21]. According to the system model of the type II relay shown in Fig. 2, each UE $\mathcal{R}$ can overhear the reference signals including the ACK/NACK signals that are periodically sent from $\mathcal{D}$ to $\mathcal{S}$. Such overheard signals can be compared with the predetermined thresholds to estimate the $\mathcal{R}-\mathcal{D}$ link quality. Thus, it is possible to partially feedback the $\mathcal{R}-\mathcal{D}$ link channel conditions or status to $\mathcal{S}$ [21]. Note that given some forms of the network-block synchronization, the carrier and the symbol synchronization for the network can be built equally between the individual transmitters and receivers. Questions like how this synchronization is achieved and how the small synchronization errors affect the performance are beyond the scope of this paper.

In the relay-selection scheme, we consider the time division duplex (TDD) mode [21] where $h_{s,d}$ and h_{s,r_i} are known at $\mathcal{S}$ and $h_{r_i,d}$ is known at $\mathcal{R}$, and $h_{r_i,d}$ must be fed back to $\mathcal{S}$ by $\mathcal{R}$, thereby causing system overhead. As noted in section I, the ATRS scheme based on the minimum (i.e., 1-bit) feedback information in [21] is sophisticated to the scheme with the limited (i.e., 2-bit) feedback information in [23] by adopting two thresholds to improve the SER performance while slightly increasing the system complexity. Note that both [21] and [23] are basically adopting the protocol proposed in [27] to compensate for the performance degradation caused by the limited feedback.

In [27], a relay-selection protocol using the modified harmonic mean is proposed, which is an appropriate metric to represent the $\mathcal{R}$s' ability to offer help. According to [27], the optimal $\mathcal{R}$ is the one that has the maximum value of a relay's metric defined as a modified version of the harmonic mean function of its $\mathcal{S}-\mathcal{R}$ and $\mathcal{R}-\mathcal{D}$ instantaneous channel gains. Therefore, if multiple $\mathcal{R}$s are available, then $\mathcal{R}$ with the maximum value in terms of the modified harmonic mean is selected. More specifically, i) $\mathcal{S}$ transmits data to $\mathcal{D}$; ii) $\mathcal{D}$ transmits an ACK/NACK message to $\mathcal{S}$, which is overheard by multiple $\mathcal{R}$s to acquire information about the $\mathcal{R}-\mathcal{D}$ links; iii) the eligible $\mathcal{R}$s must communicate their eligibility to $\mathcal{S}$; and iv) $\mathcal{S}$ selects the best $\mathcal{R}$ in terms of the modified harmonic mean. The quantized $\mathcal{R}-\mathcal{D}$ channel gain values can be written as $\beta_{r_i,d} \in \{0, \beta_{th_1}, \beta_{th_2}\}$ (for $i = 1, 2, \ldots, k$ and $k \leq N$), where $\beta_{th_1}, \beta_{th_2}$ are the thresholds as shown in Fig. 1. In the first stage of the scheduling process, each $\mathcal{R}$ notifies $\mathcal{S}$ of being "better" or "good" as candidates for the best relay if

$$\beta_{r_i,d} \geq \beta_{th_2} \longrightarrow \text{available with a better link condition}$$
or
$$\beta_{th_2} > \beta_{r_i,d} \geq \beta_{th_1} \longrightarrow \text{available with a good link condition}$$

where $\beta_{x,y} = |h_{x,y}|^2$ and $\beta_{th_1} < \beta_{th_2}$. Then, if there are k_2 candidate relays in the "better" candidate group and k_1 candidate relays in the "good" candidate group among the total k candidate relays, where $k_1 + k_2 = k$, $\mathcal{S}$ selects one candidate from the "better" candidate group, $\beta_b = \max\{\beta_{s,r_1}, \beta_{s,r_2}, \cdots, \beta_{s,r_{k_2}}\}$, and another candidate from the "good" candidate group, $\beta_a = \max\{\beta_{s,r_1}, \beta_{s,r_2}, \cdots, \beta_{s,r_{k_1}}\}$. Finally, to maximize the modified harmonic mean, from these two candidates, $\mathcal{S}$ chooses $\mathcal{R}$ with the best $\mathcal{S}-\mathcal{R}$ link. Consequently, by applying (6) in [27], the optimum $\mathcal{R}$ will have a metric which is equal to the max $\left\{\beta_{k_1^*}, \beta_{k_2^*}\right\}$ where

$$\beta_{k_1^*} = \frac{2q_1q_2\beta_{th_1}\beta_a}{q_1\beta_{th_1} + q_2\beta_a}, \quad \beta_{k_2^*} = \frac{2q_1q_2\beta_{th_2}\beta_b}{q_1\beta_{th_2} + q_2\beta_b}, \tag{1}$$

and

$$q_1 = \left(\frac{M-1}{M} + \frac{\sin\left(2\frac{\pi}{M}\right)}{2\pi}\right)^2,$$
$$q_2 = \left(\frac{3(M-1)}{2M} + \frac{\sin\left(2\frac{\pi}{M}\right)}{\pi} - \frac{\sin\left(4\frac{\pi}{M}\right)}{8\pi}\right), \tag{2}$$

where M is the modulation order of PSK. Note that if we assume that the system has the total power constraint of $P = P_1 + P_2$, where P is the total maximum transmit power available, and P_1 and P_2 are the transmit powers at $\mathcal{S}$ and the selected $\mathcal{R}$, respectively; then (2) is available especially for $P_1 = P_2$. In case there exists no candidate ($k = 0$), $\mathcal{S}$ randomly chooses one $\mathcal{R}$ among N $\mathcal{R}$s [21].

For the cooperative transmission (upon reception of NACK) in the second stage, the selected $\mathcal{R}$ forwards data to $\mathcal{D}$ if decoding is performed correctly. Otherwise, $\mathcal{R}$ remains idle. We also assume maximal-ratio combining (MRC) for the signals from $\mathcal{S}$ and the selected $\mathcal{R}$ to $\mathcal{D}$, in which $\mathcal{D}$ estimates the CSI coherently. The instantaneous signal-to-noise ratio (SNR) of the MRC output can be evaluated [21] as

$$\gamma_{s,r_i,d} = \frac{P_1\beta_{s,d} + P_2\beta_{r_i,d}}{\sigma_n^2}. \quad (3)$$

III. PERFORMANCE ANALYSIS

In this section, the SER performance of the ATRS scheme with a limited feedback is analyzed for the M-PSK signaling. Extending the performance analysis framework applied in [21], we can formulate the average SER conditioned on the number of candidates, k_1 and k_2, as

$$\overline{\text{SER}}_{\text{total}} = \sum_{k_1=0}^{N}\sum_{k_2=0}^{N-k_1} \overline{\text{SER}}\,(k_1,k_2)\,P\,(K_1=k_1,K_2=k_2), \quad (4)$$

where $\overline{\text{SER}}\,(k_1,k_2)$ is SER at $\mathcal{D}$ when there are k_1 candidate $\mathcal{R}$s with the "good" $\mathcal{R}-\mathcal{D}$ links and k_2 candidate $\mathcal{R}$s with the "better" $\mathcal{R}-\mathcal{D}$ links, and $P\,(K_1=k_1,K_2=k_2)$ is the probability of having the candidate relay subsets of size k_1 and k_2. In deriving $P\,(K_1=k_1,K_2=k_2)$, the problem can be simplified to "How many candidate $\mathcal{R}$s in each group ("good" and "better") exist?" because we assume that all $\mathcal{R}$s are statistically identical. As a result, the probability of having k_1 and k_2 candidates follows the multinomial distribution [28] as

$$\begin{aligned} &P\,(K_1=k_1,K_2=k_2) \\ &= \frac{N!}{k_1!k_2!\,(N-k_1-k_2)!} \times \left(e^{-\frac{\beta_{th_1}}{\delta_{r,d}^2}} - e^{-\frac{\beta_{th_2}}{\delta_{r,d}^2}}\right)^{k_1} \\ &\quad \times \left(e^{-\frac{\beta_{th_2}}{\delta_{r,d}^2}}\right)^{k_2}\left(1-e^{-\frac{\beta_{th_1}}{\delta_{r,d}^2}}\right)^{N-k_1-k_2}. \end{aligned} \quad (5)$$

In deriving $\overline{\text{SER}}\,(k_1,k_2)$, based on the mode of operation, if there is no $\mathcal{R}$ (i.e., $k_1=k_2=0$) for the cooperation mode, then the direct transmission mode is performed. Otherwise (i.e., $k_1 \neq 0$ or $k_2 \neq 0$), the relay cooperation mode is performed. Considering these two modes, as a result, we can formulate $\overline{\text{SER}}\,(k_1,k_2)$ as

$$\begin{aligned} \overline{\text{SER}}\,(k_1,k_2) &= P_e\,(s,\ r_i|\,k_1,k_2)\,P_e\,(s,d) \\ &\quad + [1-P_e\,(s,\ r_i|\,k_1,k_2)]\,P_e\,(s,r_i,d|k_1,k_2), \end{aligned} \quad (6)$$

where $P_e\,(s,r_i|k_1,k_2)$, $P_e\,(s,d)$, and $P_e\,(s,r_i,d|k_1,k_2)$ represent the conditional decoding error at the best $\mathcal{R}$, the SER for the direct transmission, and the conditional SER for the cooperative transmission, respectively. To evaluate these three terms, we need to consider four separate cases based on the number of candidate $\mathcal{R}$s with the "good" $\mathcal{R}-\mathcal{D}$ links, k_1, and the number of candidate $\mathcal{R}$s with the "better" $\mathcal{R}-\mathcal{D}$ links, k_2, as follows:

- $k_1 \neq 0$ and $k_2 \neq 0$: at least one candidate $\mathcal{R}$ in both "better" and "good" candidate groups

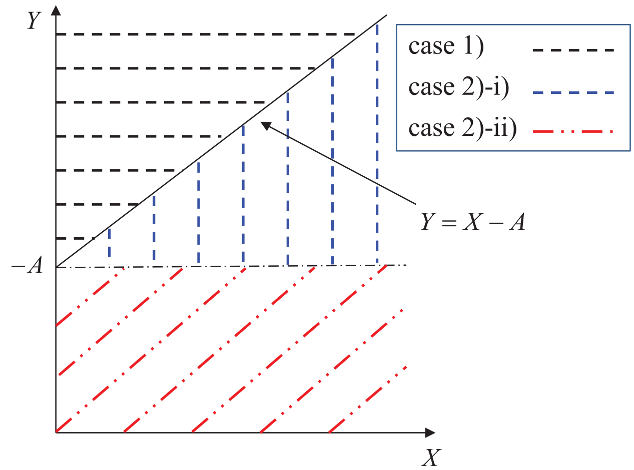

FIGURE 3. Integral regions to determine the overall SER.

- $k_1 = 0$ and $k_2 \neq 0$: at least one candidate $\mathcal{R}$ only in the "better" candidate group
- $k_1 \neq 0$ and $k_2 = 0$: at least one candidate $\mathcal{R}$ only in the "good" candidate group
- $k_1 = 0$ and $k_2 = 0$: no candidate $\mathcal{R}$ in both "better" and "good" candidate groups

Note that the common functions used to derive $P_e(s,r_i|k_1,k_2)$, $P_e\,(s,d)$, and $P_e\,(s,r_i,d|k_1,k_2)$ are obtained in Appendix I.

A. $k_1 \neq 0$ AND $k_2 \neq 0$

1) $P_e\,(s,r_i|\,k_1,k_2)$

Note that if $\beta_{k_2^*} > \beta_{k_1^*}$, $\mathcal{R}$ for cooperation is selected from the k_2 candidate $\mathcal{R}$s. Otherwise, $\mathcal{R}$ is selected from the k_1 candidate $\mathcal{R}$s as

$$\begin{aligned} &P_e\,(s,\ r_i|\,k_1,k_2) \\ &\quad = \int_0^\infty P_e(\beta)p_{s,r_{1,i}}\,(\beta\,|K_1=k_1,K_2=k_2)\,d\beta \\ &\qquad + \int_0^\infty P_e(\beta)p_{s,r_{2,i}}\,(\beta\,|K_1=k_1,K_2=k_2)\,d\beta, \end{aligned} \quad (7)$$

where $P_e(\cdot)$ is the SER formula for the M-PSK signaling [21], [26], and is represented as

$$P_e(\gamma) = \frac{1}{\pi}\int_0^{\frac{(M-1)\pi}{M}} e^{-\frac{b\gamma}{\sin^2\theta}}\,d\theta, \quad (8)$$

where $b = \sin^2(\pi/M)$. Two integral terms in (7) can be calculated by integrating β_a and β_b over the three different shaded regions shown in Fig. 3. We can rewrite $\beta_{k_2^*} > \beta_{k_1^*}$ as a function of β_a and β_b as

$$\begin{aligned} \beta_{k_1^*} > \beta_{k_2^*} &\Leftrightarrow \frac{1}{q_2*\beta_b} + \frac{1}{q_1*\beta_{th2}} > \frac{1}{q_2*\beta_a} + \frac{1}{q_1*\beta_{th1}} \\ &\Leftrightarrow \frac{1}{\beta_a} - \frac{1}{\beta_b} < \frac{q_2}{q_1}\left(\frac{1}{\beta_{th2}} - \frac{1}{\beta_{th1}}\right). \end{aligned} \quad (9)$$

If we let $X = \frac{1}{\beta_a}$ and $Y = \frac{1}{\beta_b}$, the valid integral regions of β_a and β_b become $0 < \beta_a < \infty$ and $0 < \beta_b < \frac{\beta_a}{1-A\beta_a}$, respectively, as shown in Fig. 3, where $A = \frac{q_2}{q_1}\left(\frac{1}{\beta_{th_2}} - \frac{1}{\beta_{th_1}}\right)$ ($A < 0$). Similarly, we can rewrite $\beta_{k_2^*} < \beta_{k_1^*}$ as a function of β_a and β_b as

$$\beta_{k_1^*} < \beta_{k_2^*} \Leftrightarrow \frac{1}{q_2 * \beta_b} + \frac{1}{q_1 * \beta_{th2}} < \frac{1}{q_2 * \beta_a} + \frac{1}{q_1 * \beta_{th1}}$$
$$\Leftrightarrow \frac{1}{\beta_a} - \frac{1}{\beta_b} > \frac{q_2}{q_1}\left(\frac{1}{\beta_{th2}} - \frac{1}{\beta_{th1}}\right). \quad (10)$$

For the mathematical convenience, we consider two cases separately as shown in Fig 3. For case 2)-i) (i.e., $Y > -A$), the valid integral regions of β_a and β_b become $0 < \beta_a < \frac{\beta_b}{1+A\beta_b}$ and $0 < \beta_b < \frac{1}{-A}$, respectively. Otherwise (i.e., for case 2)-ii)), the valid integral regions of β_a and β_b become $0 < \beta_a < \infty$ and $\frac{1}{-A} < \beta_b < \infty$, respectively. Therefore, the first integral term in (7) can be calculated by integrating β_a, and β_b, over the shaded region accordingly as

$$\int_0^\infty P_e(\beta) p_{s,r_{1,i}}(\beta \,|K_1 = k_1, K_2 = k_2)\, d\beta$$
$$= \int_0^\infty \frac{1}{\pi} \int_0^{\frac{(M-1)\pi}{M}} e^{\left(\frac{-b\beta_a}{\sin^2\theta}\right)} \times \int_0^{\frac{\beta_a}{1-A\beta_a}} \frac{k_2}{\delta_{s,r_i}^2} e^{-\frac{\beta_b}{\delta_{s,r_i}^2}} \left(1 - e^{-\frac{\beta_b}{\delta_{s,r_i}^2}}\right)^{k_2-1}$$
$$\times \frac{k_1}{\delta_{s,r_i}^2} e^{-\frac{\beta_a}{\delta_{s,r_i}^2}} \left(1 - e^{-\frac{\beta_a}{\delta_{s,r_i}^2}}\right)^{k_1-1} d\beta_b d\theta d\beta_a, \quad (11)$$

and similarly the second integral term in (7) can be rewritten as

$$\int_0^\infty P_e(\beta) p_{s,r_{2,i}}(\beta \,|K_1 = k_1, K_2 = k_2)\, d\beta$$
$$= \int_0^{-\frac{1}{A}} \frac{1}{\pi} \int_0^{\frac{(M-1)\pi}{M}} e^{\left(\frac{-b\beta_b}{\sin^2\theta}\right)} \times \int_0^{\frac{\beta_b}{1+A\beta_b}} \frac{k_1}{\delta_{s,r_i}^2} e^{-\frac{\beta_a}{\delta_{s,r_i}^2}} \left(1 - e^{-\frac{\beta_a}{\delta_{s,r_i}^2}}\right)^{k_1-1}$$
$$\times \frac{k_2}{\delta_{s,r_i}^2} e^{-\frac{\beta_b}{\delta_{s,r_i}^2}} \left(1 - e^{-\frac{\beta_b}{\delta_{s,r_i}^2}}\right)^{k_2-1} d\beta_a d\theta d\beta_b$$
$$+ \int_{-\frac{1}{A}}^\infty \frac{1}{\pi} \int_0^{\frac{(M-1)\pi}{M}} e^{\left(\frac{-b\beta_b}{\sin^2\theta}\right)} \times \frac{k_2}{\delta_{s,r_i}^2} e^{-\frac{\beta_b}{\delta_{s,r_i}^2}} \left(1 - e^{-\frac{\beta_b}{\delta_{s,r_i}^2}}\right)^{k_2-1} d\theta d\beta_b. \quad (12)$$

As derived in Appendix II, the closed-form expression of (11) can be obtained as

$$\sum_{j=0}^{k_2-1}\sum_{l=0}^{k_1-1} \binom{k_2-1}{j}\binom{k_1-1}{l} k_2 k_1 \frac{(-1)^{j+l+1}}{1+j}$$
$$\times \left[\sum_{n=0}^{\infty} F_1\left(\frac{\left(1+l+\frac{b\delta_{s,r_i}^2}{\sin^2\theta}\right)\left(\frac{A\delta_{s,r_i}^2}{1+j}\right)^n}{U\left(n, 0, -\frac{1}{A}\left(\frac{b}{\sin^2\theta} + \frac{1+l}{\delta_{s,r_i}^2}\right)\right)}\right) - F_1\left(1 + l + \frac{b\delta_{s,r_i}^2}{\sin^2\theta}\right)\right] \quad (13)$$

where

$$F_1(x(\theta)) = \frac{1}{\pi}\int_0^{\frac{(M-1)\pi}{M}} \frac{1}{x(\theta)}\, d\theta, \quad (14)$$

which also can be evaluated in the standard mathematical packages. Note that although (13) involves an infinite summation, the summand decays exponentially (or slightly faster) with the increase of n because Stirling's approximation specifies that $n!$ grows as $\exp(n \ln n)$ [29]. Moreover, due to the factorial term in the Tricomi's confluent hypergeometric function as a function of n, the truncated summation with a finite number of terms can reliably achieve the required accuracy.

The first integral term in (12) can be rewritten by changing the integration regions based on Fig. 3 as

$$\int_0^\infty \frac{1}{\pi}\int_0^{\frac{(M-1)\pi}{M}} e^{\left(\frac{-b\beta_b}{\sin^2\theta}\right)} \int_{\frac{\beta_b}{1-A\beta_b}}^{-\frac{1}{A}} \frac{k_2}{\delta_{s,r_i}^2} e^{-\frac{\beta_b}{\delta_{s,r_i}^2}} \left(1 - e^{-\frac{\beta_b}{\delta_{s,r_i}^2}}\right)^{k_2-1}$$
$$\times \frac{k_1}{\delta_{s,r_i}^2} e^{-\frac{\beta_a}{\delta_{s,r_i}^2}} \left(1 - e^{-\frac{\beta_a}{\delta_{s,r_i}^2}}\right)^{k_1-1} d\beta_b d\theta d\beta_a. \quad (15)$$

After applying the similar approaches used in (57) and then some manipulations, we can rewrite (15) as

$$\sum_{j=0}^{k_2-1}\sum_{l=0}^{k_1-1} \binom{k_2-1}{j}\binom{k_1-l}{j} \frac{(-1)^{j+l}}{\pi}$$
$$\times \int_0^{\frac{(M-1)\pi}{M}} \frac{k_2 \sin^2\theta}{(1+j)\sin^2\theta + b\delta_{s,r_i}^2} e^{\frac{\frac{b}{\sin^2\theta} + \frac{1+j}{\delta_{s,r_i}^2}}{A}}$$
$$\times \left(\int_0^\infty e^{\frac{(1+j)\sin^2\theta + b\delta_{s,r_i}^2}{A\sin^2\theta(-1+A\beta_a)\delta_{s,r_i}^2}} \frac{k_1}{\delta_{s,r_i}^2} e^{-\frac{\beta_a(l+1)}{\delta_{s,r_i}^2}} d\beta_a - \frac{k_1}{1+l}\right) d\theta. \quad (16)$$

Expanding the exponential function in the inner integral term as a Taylor series and then integrating over β_a with the help of [30, eq. (3.381.3)], we can express (16) as the single integral expression

$$\sum_{j=0}^{k_2-1}\sum_{l=0}^{k_1-1}\binom{k_2-1}{j}\binom{k_1-1}{l}\frac{(-1)^{j+l}}{\pi}$$
$$\times\int_0^{\frac{(M-1)\pi}{M}}\frac{e^{\frac{\frac{b}{\sin^2\theta}+\frac{1+j}{\delta^2_{s,r_i}}}{A}}k_1k_2\sin^2\theta}{(1+l)(1+j)\sin^2\theta+b\delta^2_{s,r_i}}$$
$$\times\left(\sum_{n=0}^{\infty}\frac{e^{-\frac{1+l}{A\delta^2_{s,r_i}}}}{(-A)^n n!}\Gamma\left(1-n,-\frac{1+l}{A\delta^2_{s,r_i}}\right)\right.$$
$$\left.\times\left(-\frac{(1+j)}{A\delta^2_{s,r_i}}-\frac{b}{A\sin^2\theta}\right)^n\left(\frac{+l}{\delta^2_{s,r_i}}\right)^n-1\right)d\theta. \quad (17)$$

Similar to the previous cases, the closed-form expression of the first integral term in (12) as a function of $F_1(\cdot)$ in (14) can be obtained as

$$\sum_{j=0}^{k_2-1}\sum_{l=0}^{k_1-1}\binom{k_1-1}{l}\binom{k_2-1}{j}\frac{(-1)^{j+l}k_1k_2}{1+l}$$
$$\times\left[\sum_{n=0}^{\infty}\frac{1}{n!}\Gamma\left(1-n,-\frac{1+l}{A\delta^2_{s,r_i}}\right)\left(\frac{A^2\left(\delta^2_{s,r_i}\right)^2}{1+l}\right)^{-n}e^{\frac{j-l}{A\delta^2_{s,r_i}}}\right.$$
$$\times F_1\left(e^{-\frac{b}{A\sin^2\theta}}\left(1+j+\frac{b\delta^2_{s,r_i}}{\sin^2\theta}\right)^{1-n}\right)$$
$$\left.-e^{\frac{1+j}{A\delta^2_{s,r_i}}}F_1\left(e^{-\frac{b}{A\sin^2\theta}}\left(1+j+\frac{b\delta^2_{s,r_i}}{\sin^2\theta}\right)\right)\right]. \quad (18)$$

With the similar approach, the second integral term in (12) can also be rewritten as the following single integral form

$$\sum_{j=0}^{k_2-1}\binom{k_2-1}{j}(-1)^j\frac{k_2}{\delta^2_{s,r_i}}\frac{1}{\pi}$$
$$\times\int_0^{\frac{(M-1)\pi}{M}}\frac{e^{\frac{\frac{b}{\sin^2\theta}+\frac{1+j}{\delta^2_{s,r_i}}}{A}}\sin^2\theta\,\delta^2_{s,r_i}}{(1+j)\sin^2\theta+b\delta^2_{s,r_i}}d\theta. \quad (19)$$

With the help of (14), the closed-form expression of (19) can be obtained as

$$\sum_{j=0}^{k_2-1}\binom{k_2-1}{j}k_2(-1)^j e^{\frac{1+j}{A\delta^2_{s,r_i}}}$$
$$\times F_1\left(e^{-\frac{b}{A\sin^2\theta}}\left(1+j+\frac{b\delta^2_{s,r_i}}{\sin^2\theta}\right)\right). \quad (20)$$

2) $P_e(s,d)$

In this case, the probability density function (PDF) of the $\mathcal{S}-\mathcal{D}$ link is independent to the number of candidates and its channel gain follows the exponential distribution. Therefore, the SER of the direct transmission from $\mathcal{S}$ to $\mathcal{D}$ can be evaluated as

$$\begin{aligned}&P_e(s,d)\\&=\int_0^{\infty}P_e(\beta)p_{s,d}(\beta)d\beta\\&=\int_0^{\infty}\frac{1}{\pi}\int_0^{\frac{(M-1)\pi}{M}}\exp\left(\frac{-b\beta}{\sin^2\theta}\right)\frac{1}{\delta^2_{s,d}}\exp\left(-\frac{\beta}{\delta^2_{s,d}}\right)d\theta d\beta\\&=\frac{1}{\pi}\int_0^{\frac{(M-1)\pi}{M}}\frac{\sin^2\theta}{\sin^2\theta+b\delta^2_{s,d}}d\theta\\&=F_1\left(1+\frac{b\delta^2_{s,d}}{\sin^2\theta}\right).\end{aligned} \quad (21)$$

3) $P_e(s,r_i,d|k_1,k_2)$

In this case, the conditional SER for the cooperative transmission at $\mathcal{D}$ with k_1 and k_2 candidates can be formulated by considering two cases, "good" or "better", as

$$\begin{aligned}&P_e(s,r_i,d|k_1,k_2)\\&=P\left[\beta_{k_1^*}>\beta_{k_2^*}|K_1=k_1,K_2=k_2\right]\int_0^{\infty}P_e(\beta)p_{s,r_{1,i},d}(\beta)d\beta\\&\quad+P\left[\beta_{k_1^*}<\beta_{k_2^*}|K_1=k_1,K_2=k_2\right]\\&\quad\times\int_0^{\infty}P_e(\beta)p_{s,r_{2,i},d}(\beta)d\beta.\end{aligned} \quad (22)$$

Inserting (51) into (22) and then simply performing the integration of the exponential function over β will give

$$\begin{aligned}&\int_0^{\infty}P_e(\beta)p_{s,r_{1,i},d}(\beta)d\beta\\&=\frac{1}{\pi}\int_0^{\frac{(M-1)\pi}{M}}\\&\times\left[\frac{\left(e^{-\frac{b\beta_{th_2}}{\sin^2\theta}+\frac{\beta_{th_1}}{\delta^2_{r,d}}}-e^{-\frac{b\beta_{th_1}}{\sin^2\theta}+\frac{\beta_{th_2}}{\delta^2_{r,d}}}\right)\sin^4\theta}{\left(e^{\frac{\beta_{th_1}}{\delta^2_{r,d}}}-e^{\frac{\beta_{th_2}}{\delta^2_{r,d}}}\right)\left(\sin^2\theta+b\delta^2_{r,d}\right)\left(\sin^2\theta+b\delta^2_{s,d}\right)}\right]d\theta.\end{aligned} \quad (23)$$

With the help of (14), we can obtain the closed-form expression of (23) as

$$\begin{aligned}&\int_0^{\infty}P_e(\beta)p_{s,r_{1,i},d}(\beta)d\beta\\&=F_1\left(\left(\frac{e^{\frac{\beta_{th_1}}{\delta^2_{r,d}}}-e^{\frac{\beta_{th_2}}{\delta^2_{r,d}}}}{e^{-\frac{b\beta_{th_2}}{\sin^2\theta}+\frac{\beta_{th_1}}{\delta^2_{r,d}}}-e^{-\frac{b\beta_{th_1}}{\sin^2\theta}+\frac{\beta_{th_2}}{\delta^2_{r,d}}}}\right.\right.\\&\left.\left.\quad\times\left(1+\frac{b\delta^2_{r,d}}{\sin^2\theta}\right)\left(1+\frac{b\delta^2_{s,d}}{\sin^2\theta}\right)\right)\right).\end{aligned} \quad (24)$$

Similarly, with (55), the second integral term in (22) can be also rewritten as

$$\begin{aligned}\int_0^\infty & P_e(\beta) p_{s,r_{2,i},d}(\beta) d\beta \\ &= \frac{1}{\pi}\int_0^{\frac{(M-1)\pi}{M}} \frac{1}{\delta_{r,d}^2-\delta_{s,d}^2}\int_{\beta_{th_2}}^\infty e^{\left(\frac{-b\beta}{\sin^2\theta}\right)} \\ &\quad\times\left(e^{\frac{-\beta+\beta_{th_2}}{\delta_{r,d}^2}} - e^{\frac{-\beta+\beta_{th_2}}{\delta_{s,d}^2}}\right) d\beta d\theta \\ &= \frac{1}{\pi}\int_0^{\frac{(M-1)\pi}{M}} \frac{e^{-\frac{b\beta_{th_2}}{\sin^2\theta}}\sin^4\theta}{\left(\sin^2\theta+b\delta_{r,d}^2\right)\left(\sin^2\theta+b\delta_{s,d}^2\right)} d\theta \quad (25)\end{aligned}$$

whose closed-form expression can be obtained as

$$\begin{aligned}\int_0^\infty & P_e(\beta) p_{s,r_{2,i},d}(\beta) d\beta \\ &= F_1\left(e^{\frac{b\beta_{th_2}}{\sin^2\theta}}\left(1+\frac{b\delta_{r,d}^2}{\sin^2\theta}\right)\left(1+\frac{b\delta_{s,d}^2}{\sin^2\theta}\right)\right). \quad (26)\end{aligned}$$

In (22), the probability that the $\mathcal{R}-\mathcal{D}$ link of the selected $\mathcal{R}$ is "good" can be evaluated as

$$\begin{aligned}P&\left[\beta_{k_1^*} > \beta_{k_2^*} | K_1 = k_1, K_2 = k_2\right] \\ &= \int_0^\infty \frac{k_1}{\delta_{s,r_i}^2 x^2} e^{-\frac{1}{\delta_{s,r_i}^2 x}}\left(1-e^{-\frac{1}{\delta_{s,r_i}^2 x}}\right)^{k_1-1} \\ &\quad\times\left(\int_{x-A}^\infty \frac{k_2}{\delta_{s,r_i}^2 y^2} e^{-\frac{1}{\delta_{s,r_i}^2 y}}\left(1-e^{-\frac{1}{\delta_{s,r_i}^2 y}}\right)^{k_2-1} dy\right) dx, \quad (27)\end{aligned}$$

where $A = \frac{q_2}{q_1}\left(\frac{1}{\beta_{th_2}} - \frac{1}{\beta_{th_1}}\right)$. With the binomial theorem, the inner integral term in (27) can be rewritten as

$$\sum_{l=0}^{k_2-1}\binom{k_2-1}{l}\int_{x-A}^\infty \frac{k_2}{\delta_{s,r_i}^2 y^2} e^{-\frac{1}{\delta_{s,r_i}^2 y}}\left(-e^{-\frac{1}{\delta_{s,r_i}^2 y}}\right)^l dy. \quad (28)$$

After performing the inner integral and mathematical simplifications, we can rewrite (27) as

$$\begin{aligned}&\sum_{l=0}^{k_2-1}\binom{k_2-1}{l}\frac{k_2}{1+l} \\ &\times\left[\int_0^\infty (-1)^l \frac{k_1}{\delta_{s,r_i}^2 x^2} e^{-\frac{1}{\delta_{s,r_i}^2 x}}\left(1-e^{-\frac{1}{\delta_{s,r_i}^2 x}}\right)^{k_1-1} dx\right. \\ &\left.+\int_0^\infty \frac{k_1}{\delta_{s,r_i}^2 x^2} e^{-\frac{1}{\delta_{s,r_i}^2 x}}\left(1-e^{-\frac{1}{\delta_{s,r_i}^2 x}}\right)^{k_1-1}\left(-e^{\frac{1}{(A-x)\delta_{s,r_i}^2}}\right)^{1+l} dx\right] \\ &\quad (29)\end{aligned}$$

whose simplified form can be obtained via the binomial theorem and some manipulations as

$$\begin{aligned}&\sum_{l=0}^{k_2-1}\binom{k_2-1}{l}(-1)^l\frac{k_2}{1+l}\left[1+\sum_{j=0}^{k_1-1}\binom{k_1-1}{j}\right. \\ &\left.\times(-1)^{j+1}\frac{k_1}{\delta_{s,r_i}^2}\int_0^\infty \frac{1}{x^2} e^{-\frac{1+j}{\delta_{s,r_i}^2 x}} e^{-\frac{1+l}{(x-A)\delta_{s,r_i}^2}} dx\right]. \quad (30)\end{aligned}$$

To evaluate the integral term in (30), with the help of the Taylor series expansions of exponential functions [29], we rewrite (30) as

$$\begin{aligned}&\sum_{l=0}^{k_2-1}\binom{k_2-1}{l}(-1)^l\frac{k_2}{1+l}\left[1+\sum_{j=0}^{k_1-1}\sum_{n=0}^{\infty}\binom{k_1-1}{j}\right. \\ &\left.\times\frac{(-1)^{j+1}k_1}{\delta_{s,r_i}^2}\int_0^\infty \frac{1}{x^2} e^{-\frac{1+j}{\delta_{s,r_i}^2 x}}\frac{1}{n!}\left(\frac{-1-l}{(x-A)\delta_{s,r_i}^2}\right)^n dx\right]. \\ &\quad (31)\end{aligned}$$

Note that the expression in (31) involves an infinite summation for the term of n. However, similar to the derived result in (13), a truncated summation with the finite number of terms can reliably achieve the required accuracy. Thus, with the help of the integral identity [31, eq. (07.33.07.0001.01)], we can finally obtain the closed-form expression of (27) as

$$\begin{aligned}P&\left[\beta_{k_1^*} > \beta_{k_2^*} | K_1 = k_1, K_2 = k_2\right] \\ &= \sum_{l=0}^{k_2-1}\binom{k_2-1}{l}\frac{(-1)^l k_2}{1+l}\left[1+\sum_{j=0}^{k_1-1}\sum_{n=0}^{\infty}\binom{k_1-1}{j}\right. \\ &\left.\times\frac{(-1)^{j+1}k_1}{1+j}A^{-n}U\left(n, 0, \frac{-1-j}{A\delta_{s,r_i}^2}\right)\left(\frac{1+l}{\delta_{s,r_i}^2}\right)^n\right]. \quad (32)\end{aligned}$$

From (32), the probability that the $\mathcal{R}-\mathcal{D}$ link of the selected relay is "better" in (22) can be evaluated as

$$\begin{aligned}P&\left[\beta_{k_1^*} < \beta_{k_2^*} | K_1 = k_1, K_2 = k_2\right] \\ &= 1 - P\left[\beta_{k_1^*} > \beta_{k_2^*} | K_1 = k_1, K_2 = k_2\right]. \quad (33)\end{aligned}$$

B. $k_1 = 0$ AND $k_2 \neq 0$

In this case, there exist candidate $\mathcal{R}$s only with "better" $\mathcal{R}-\mathcal{D}$ links (i.e., $k_1 = 0$). Therefore, $P_e\left(s, r_i | k_1 = 0, k_2\right)$ can be represented as a form of the simple exponential integration

$$\begin{aligned}P_e&\left(s, r_i | 0, k_2\right) \\ &= \int_0^\infty P_e(\beta) p_{s,r_i}\left(\beta | K = k_2\right) d\beta \\ &= \int_0^\infty \frac{1}{\pi}\int_0^{\frac{(M-1)\pi}{M}} e^{-\frac{b\beta}{\sin^2\theta}}\frac{k_2}{\delta_{s,r_i}^2} e^{-\frac{\beta}{\delta_{s,r_i}^2}}\left(1-e^{-\frac{\beta}{\delta_{s,r_i}^2}}\right)^{k_2-1} \\ &\quad\times d\theta d\beta. \quad (34)\end{aligned}$$

Performing the simple exponential integral with respect to β, we can obtain

$$P_e\left(s, r_i | 0, k_2\right) = \frac{1}{\pi}\int_0^{\frac{(M-1)\pi}{M}} \frac{k_2!\left(\frac{b\delta_{s,r_i}^2}{\sin^2\theta}\right)!}{\left(k_2+\frac{b\delta_{s,r_i}^2}{\sin^2\theta}\right)!} d\theta. \quad (35)$$

By applying (14) as in the previous case, we can easily obtain the closed-form expression of (34) as

$$P_e\left(s,\, r_i|\, 0, k_2\right) = F_1\left(\frac{\left(k_2+\frac{b\delta_{s,r_i}^2}{\sin^2\theta}\right)!}{k_2!\left(\frac{b\delta_{s,r_i}^2}{\sin^2\theta}\right)!}\right). \tag{36}$$

For $P_e(s, d)$, the PDF of the $\mathcal{S}-\mathcal{D}$ link is also independent of the number of candidates. Therefore, $P_e(s, d)$ in this case has the same result as the previous result given in (21). Note that since $k_1 = 0$ and $k_2 \neq 0$, from (22), we can express $P_e\left(s, r_i, d|k_1 = 0, k_2\right)$ as

$$P_e\left(s, r_i, d|0, k_2\right) = \int_0^\infty P_e(\beta) p_{s,r_{2,i},d}(\beta)d\beta \tag{37}$$

whose closed-from expression is obtained in (26).

C. $k_1 \neq 0$ AND $k_2 = 0$

This is the case that there exist candidate $\mathcal{R}$s only with "good" $\mathcal{R}-\mathcal{D}$ links (i.e., $k_2 = 0$). Hence, for $P_e\left(s, r_i|\, k_1, 0\right)$, the closed-form result can be obtained by simply replacing k_2 in (36) with k_1, and for $P_e(s, d)$, the closed-form result is the same as the previous result given in (21). For $P_e\left(s, r_i, d|k_1, k_2 = 0\right)$, from (22), we can easily write

$$P_e\left(s, r_i, d|k_1, 0\right) = \int_0^\infty P_e(\beta) p_{s,r_{1,i},d}(\beta)d\beta \tag{38}$$

whose closed-from expression is obtained in (24).

D. $k_2 = 0, k_1 = 0$

In this case, if there is no $\mathcal{R}$ whose received SNR exceeds the threshold, β_{th_1}, $\mathcal{R}$ is selected randomly among N $\mathcal{R}$s for the transmission. Therefore, we can directly apply the derived result in [21] especially for $k = 0$ (i.e., $k = k_1 + k_2$ becomes 0). As a result, $p_{s,r_i}\left(\beta|K = 0\right)$ and $p_{r_i,d}\left(\beta\right)$ can be written, respectively, as

$$p_{s,r_i}(\beta|K=0) = \frac{1}{\delta_{s,r_i}^2} e^{-\frac{\beta}{\delta_{s,r_i}^2}}, \tag{39}$$

and

$$p_{r_i,d}(\beta) = \frac{1}{\delta_{r,d}^2}\cdot\frac{e^{-\frac{\beta}{\delta_{r,d}^2}}}{1-e^{-\frac{\beta_{th_1}}{\delta_{r,d}^2}}}. \tag{40}$$

For the direct transmission, as in the previous cases, $P_e(s, d)$ can be obtained as in (21). Therefore, by directly adopting the derived results in [21], the SER terms for the direct transmission mode and for the relay cooperation mode in (4) can be represented as

$$\begin{aligned} &P_e\left(s, r_i|k_1 = 0, k_2 = 0\right) P_e(s, d) P\left(K_1 = 0, K_2 = 0\right) \\ &= \left(1-e^{-\frac{\beta_{th_1}}{\delta_{r,d}^2}}\right)^N F_1\left(\frac{b\delta_{s,r}^2}{\sin^2\theta}+1\right) F_1\left(\frac{b\delta_{s,d}^2}{\sin^2\theta}+1\right), \end{aligned} \tag{41}$$

$$\begin{aligned} &P_e\left(s, r_i, d\left|k_1 = 0, k_2 = 0\right.\right) P\left(K_1 = 0, K_2 = 0\right) \\ &= \left(1-e^{-\frac{\beta_{th_1}}{\delta_{r,d}^2}}\right)^{N-1} F_1\left[\left(\frac{b\delta_{r,d}^2}{\sin^2\theta}+1\right)\left(\frac{b\delta_{s,d}^2}{\sin^2\theta}+1\right)\right. \\ &\quad \left.\times\left(1-e^{\left(-\frac{b\beta_{th_1}}{\sin^2\theta}-\frac{\beta_{th_1}}{\delta_{r,d}^2}\right)}\right)^{-1}\right], \end{aligned} \tag{42}$$

and

$$\begin{aligned} &P_e\left(s, r_i|k_1 = 0, k_2 = 0\right) P_e\left(s, r_i, d\left|k_1 = 0, k_2 = 0\right.\right) \\ &\quad\times P\left(K_1 = 0, K_2 = 0\right) \\ &= \left(1-e^{-\frac{\beta_{th_1}}{\delta_{r,d}^2}}\right)^{N-1} F_1\left(\frac{b\delta_{s,d}^2}{\sin^2\theta}+1\right) \\ &\quad\times F_1\left[\left(\frac{b\delta_{r,d}^2}{\sin^2\theta}+1\right)\left(\frac{b\delta_{s,d}^2}{\sin^2\theta}+1\right)\right. \\ &\quad\quad \left.\times\left(1-e^{\left(-\frac{b\beta_{th_1}}{\sin^2\theta}-\frac{\beta_{th_1}}{\delta_{r,d}^2}\right)}\right)^{-1}\right]. \end{aligned} \tag{43}$$

IV. NUMERICAL RESULTS

As a validation of our derivations, we compare the analytical results with the results obtained via the Monte-Carlo simulation over independent and identically distributed (i.i.d.) Rayleigh fading channels. For a fair comparison between the SER performance of the minimum and limited feedback-based schemes, we consider an equal power allocation and a fixed threshold, to show the effect of more refined feedback information on the SER performance. In the following figures, the lines and the markers represent the simulation and the analytical results, respectively. Note that the simulation results match the derived analytical results well. Further, N represents the number of relays, $\overline{\gamma}_{SR}$, $\overline{\gamma}_{RD}$, and $\overline{\gamma}_{SD}$ represent the average SNRs of $\mathcal{S}-\mathcal{R}$, $\mathcal{R}-\mathcal{D}$, and $\mathcal{S}-\mathcal{D}$ links, respectively, and $\{\beta_{th_1}, \beta_{th_2}\}$ represent the quantized $\mathcal{R}-\mathcal{D}$ channel gain thresholds for "good" and "better" candidate $\mathcal{R}$s.

In Fig. 4, we assume that there are 6 $\mathcal{R}$s and the average SNRs of $\mathcal{S}-\mathcal{R}$, $\mathcal{R}-\mathcal{D}$, and $\mathcal{S}-\mathcal{D}$ links are identical. This figure shows that the proposed ATRS scheme with a limited feedback information achieves better performance than the minimum feedback-based scheme. Through the more-refined feedback information with only one additional comparison operation in terms of complexity, the limited feedback-based scheme has a higher ability to compensate for the potential performance degradation. We can also observe that if we consider the $\mathcal{S}-\mathcal{R}$ link conditions of each candidate $\mathcal{R}$s selected from both the "better" and "good" candidate groups are similar, the performance improvements are increasing as

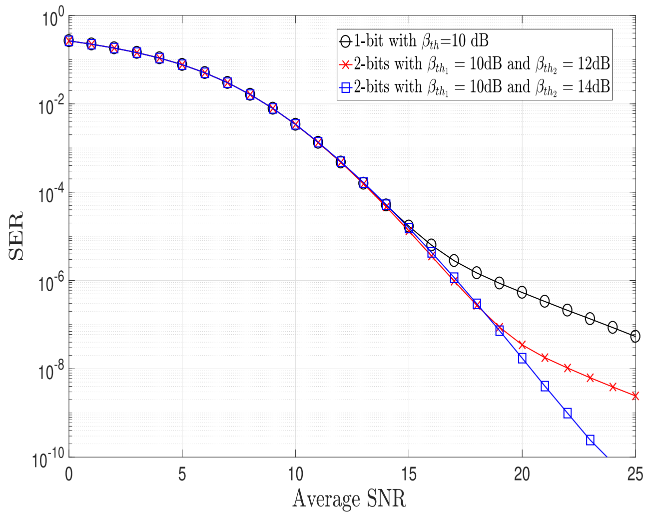

FIGURE 4. **SER performance for the various values of threshold when $\overline{\gamma}_{SR} = \overline{\gamma}_{RD} = \overline{\gamma}_{SD} = \overline{\gamma}$ and $N = 6$.**

the threshold, β_{th_2}, increases since the possibility that the selected candidate has a better performance increases.

Note that for the low average SNR region, both minimum and limited feedback-based schemes provide similar performance while, in the middle average SNR region, the limited feedback-based scheme with a relatively lower value of β_{th_2} may provide slightly better performance compared to that with a relatively higher value of β_{th_2}. This is because, in the latter case, the possibility that most of the candidate $\mathcal{R}$s are in the "good" candidate group is high. However, as the average value of the SNR increases, the possibility that the selected $\mathcal{R}$ is chosen from the "better" candidate group also gradually increases, and eventually leads to the performance improvement. As a result, we can say that the limited feedback-based scheme with relatively higher value of β_{th_2} may provide better performance, especially in the high average SNR region.

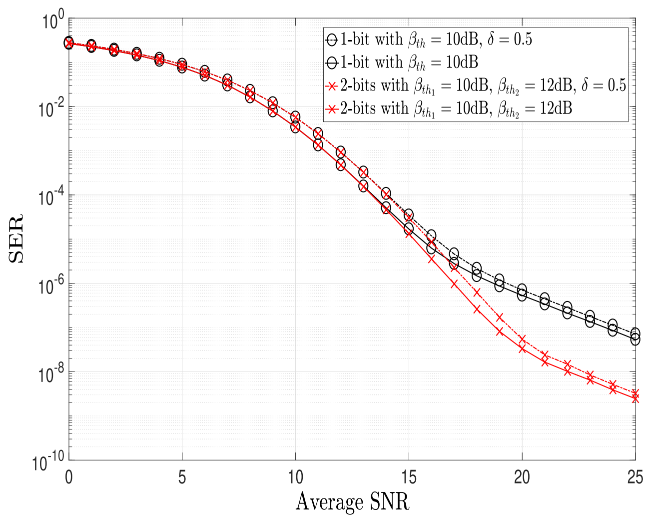

FIGURE 5. **SER performance for the non-identical fading scenario when $N = 6$.**

In Fig. 5, we present some results under asymmetric link assumptions, especially for non-identical fading assumptions in the $\mathcal{R} - \mathcal{D}$ links. We assume that there are 6 $\mathcal{R}$s and the $\mathcal{R} - \mathcal{D}$ channels of three $\mathcal{R}$s among total six $\mathcal{R}$s have an exponentially decaying multipath intensity profile (MIP) with $\overline{\gamma}_l = \overline{\gamma} \cdot \exp\left(-\delta\left(l-1\right)\right)$, $(l = 1, \cdots, 3)$,where $\overline{\gamma}$ is the strongest average SNR and δ is the average fading power decay factor. As expected, Fig. 5 shows that in the low average SNR region, the performance under the independent but non-identically distributed (i.n.d.) fading scenario is slightly degraded compared to the performance under the i.i.d. fading scenario due to the effect of the decaying factor. However, as the average SNR increases, this performance gap between them is decreased. Additionally, the performance of the limited feedback-based scheme under the i.n.d. fading scenario is slightly worse than the minimum feedback-based scheme under the identical fading scenario due to the decreased SNR caused by the effect of the decaying factor in the limited feedback-based scheme. However, as the average SNR increases, we can observe that the limited feedback-based scheme over non-identical fading conditions provides better performance than the minimum feedback-based scheme over both identical and non-identical fading conditions since the influence of the decay factors is reduced due to the effect of the increased SNR.

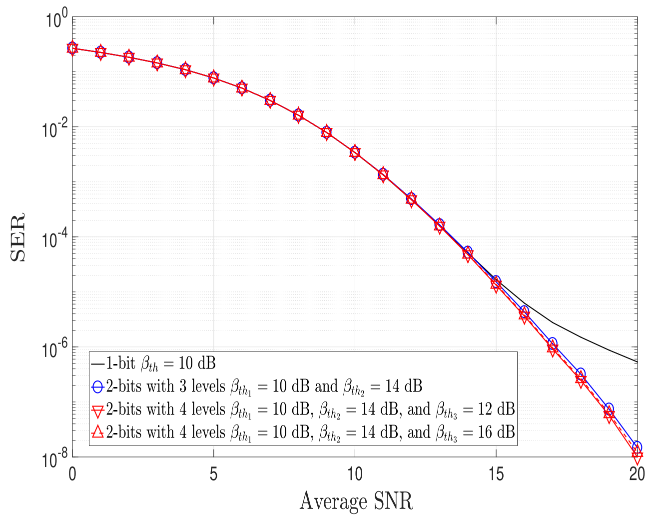

FIGURE 6. **SER performance comparison between 3-level and 4-level when $\overline{\gamma}_{SR} = \overline{\gamma}_{RD} = \overline{\gamma}_{SD} = \overline{\gamma}$ and $N = 6$.**

One may expect that, with the 2-bit feedback, it is possible to have four different levels with three thresholds. However, Fig. 6 shows that under same channel and system assumptions, compared with the 3-level based scheme (i.e., one level added to the 1-bit based scheme), 4-level based scheme does not provide a better improvement at the cost of complexity. This tendency will be maintained even if more links are to be selected because the performance is mainly affected by the threshold values not by the number of links. Further, for practical consideration, the extended ATRS scheme with a limited feedback information can also be implemented with the 1-bit feedback. More specifically, only $\mathcal{R}$ with the "acceptable"

$\mathcal{R}-\mathcal{D}$ link status reports to $\mathcal{S}$ about "good" or "better". Further, after selecting the best $\mathcal{R}$ from both the "good" candidate group and from the "better" candidate group, $\mathcal{S}$ can choose the best $\mathcal{R}$ from between these two using the 1-bit-based feedback information.

V. CONCLUSION

In this paper, we analyzed the SER performance of the extended ATRS scheme based on the "limited" (i.e., 2-bit) feedback information in the closed-form expressions. The derived analytical results were verified by cross-checking them via Monte-Carlo simulations. Based on some selected results, we confirmed that the potential performance degradation caused by applying the minimum feedback can be compensated with a more-refined feedback information, especially with a 1-bit increase in terms of the feedback data rate and an additional single-comparison process in terms of complexity. Note that the derived closed-form results of SER can be easily evaluated numerically in the standard mathematical packages. Additionally, the derived closed-form results can be used to predict the SER performance under both symmetric and asymmetric link conditions. It can also be noted that similar to the 1-bit feedback-based scheme in [21], the extended ATRS scheme with the limited feedback information can also be implemented with the 1-bit feedback. This means that only $\mathcal{R}$s with the "acceptable" $\mathcal{R}-\mathcal{D}$ link status can report to $\mathcal{S}$ about "good" or "better". Further, after selecting the best $\mathcal{R}$ from both the "good" candidate group and the "better" candidate group, $\mathcal{S}$ can choose the best $\mathcal{R}$ from between these two using the 1-bit-based feedback information.

APPENDIX I
COMMON STATISTICAL FUNCTIONS

We statistically address the derivations of the common functions used in section III. These common statistical functions are evaluated to derive the closed-form expressions of the conditional decoding error at the best $\mathcal{R}$, $P_e(s, r_i|k_1, k_2)$; the SER for the direct transmission, $P_e(s, d)$; and the conditional SER for the cooperative transmission, $P_e(s, r_i, d|k_1, k_2)$. We assume that the fading conditions follow the i.i.d. Rayleigh fading model. Therefore, we can first obtain the following basic statistical functions for each link:

i) $p_{s,d}(\beta)$:

$$p_{s,d}(\beta) = \frac{1}{\delta_{s,d}^2} e^{-\frac{\beta}{\delta_{s,d}^2}}. \tag{44}$$

ii) $p_{s,r_i}(\beta|K=k)$:

$$p_{s,r_i}(\beta|K=k) = \frac{k}{\delta_{s,r_i}^2} e^{-\frac{\beta}{\delta_{s,r_i}^2}} \left(1-e^{-\frac{\beta}{\delta_{s,r_i}^2}}\right)^{k-1}. \tag{45}$$

iii) $p_{r_{1,i},d}(\beta)$:

$$p_{r_{1,i},d}(\beta) = \frac{1}{\delta_{r,d}^2} \cdot \frac{e^{-\frac{\beta}{\delta_{r,d}^2}}}{e^{-\frac{\beta_{th_1}}{\delta_{r,d}^2}} - e^{-\frac{\beta_{th_2}}{\delta_{r,d}^2}}}. \tag{46}$$

iv) $p_{r_{2,i},d}(\beta)$:

$$p_{r_{2,i},d}(\beta) = \frac{1}{\delta_{r,d}^2} \cdot \frac{e^{-\frac{\beta}{\delta_{r,d}^2}}}{e^{-\frac{\beta_{th_2}}{\delta_{r,d}^2}}}. \tag{47}$$

Additionally, we can also evaluate the following commonly used statistical functions:

i) $p_{s,r_{1,i}}(\beta|K_1=k_1, K_2=k_2)$: With the help of the derivation approach used in [21], we can formulate the equation as

$$\begin{aligned} p_{s,r_{1,i}}&(\beta|K_1=k_1, K_2=k_2) \\ &= \left(\int_0^{\frac{\beta}{1-A\beta}} P_{s,r_i}(\beta'|K=k_2)\,d\beta'\right) P_{s,r_i}(\beta|K=k_1) \\ &= \int_0^{\frac{\beta}{1-A\beta}} \frac{k_2}{\delta_{s,r_i}^2} e^{-\frac{\beta'}{\delta_{s,r_i}^2}} \left(1-e^{-\frac{\beta'}{\delta_{s,r_i}^2}}\right)^{k_2-1} \\ &\quad \times \frac{k_1}{\delta_{s,r_i}^2} e^{-\frac{\beta}{\delta_{s,r_i}^2}} \left(1-e^{-\frac{\beta}{\delta_{s,r_i}^2}}\right)^{k_1-1} d\beta'. \end{aligned} \tag{48}$$

By integrating over β', the closed-form expression of (48) can be obtained as

$$\begin{aligned} p_{s,r_{1,i}}&(\beta|K_1=k_1, K_2=k_2) \\ &= \frac{k_1}{\delta_{s,r_i}^2} e^{-\frac{\beta}{\delta_{s,r_i}^2}} \left(1-e^{-\frac{\beta}{(1-A\beta)\delta_{s,r_i}^2}}\right)^{k_2} \left(1-e^{-\frac{\beta}{\delta_{s,r_i}^2}}\right)^{k_1-1}. \end{aligned} \tag{49}$$

ii) $p_{s,r_{2,i}}(\beta|K_1=k_1, K_2=k_2)$: Similarly, we can also represent the formula as

$$\begin{aligned} p_{s,r_{2,i}}&(\beta|K_1=k_1, K_2=k_2) \\ &= \left(\int_0^{\frac{\beta}{1+A\beta}} P_{s,r_i}(\beta'|K=k_1)\,d\beta'\right) P_{s,r_i}(\beta|K=k_2) \\ &= \int_0^{\frac{\beta}{1+A\beta}} \frac{k_1}{\delta_{s,r_i}^2} e^{-\frac{\beta'}{\delta_{s,r_i}^2}} \left(1-e^{-\frac{\beta'}{\delta_{s,r_i}^2}}\right)^{k_1-1} \\ &\quad \times \frac{k_2}{\delta_{s,r_i}^2} e^{-\frac{\beta}{\delta_{s,r_i}^2}} \left(1-e^{-\frac{\beta}{\delta_{s,r_i}^2}}\right)^{k_2-1} d\beta' \\ &= \frac{k_2}{\delta_{s,r_i}^2} e^{-\frac{\beta}{\delta_{s,r_i}^2}} \left(1-e^{-\frac{\beta}{(1+A\beta)\delta_{s,r_i}^2}}\right)^{k_1} \left(1-e^{-\frac{\beta}{\delta_{s,r_i}^2}}\right)^{k_2-1}. \end{aligned} \tag{50}$$

iii) $p_{s,r_{1,i},d}(\beta)$: In this case, the instantaneous SNR of the MRC output at $\mathcal{D}$ is the sum of the SNRs of the $\mathcal{R}-\mathcal{D}$ and $\mathcal{S}-\mathcal{D}$ links. Based on our assumptions, these two links are independent. Therefore, it is possible to find an expression for the distribution of the combined SNRs, $p_{s,r_{1,i},d}(\beta)$, that results by convoluting these two distributions, $p_{s,d}(\beta)$ and $p_{r_{1,i},d}(\beta)$, as

$$p_{s,r_{1,i},d}(\beta) = \int_{-\infty}^{\infty} p_{s,d}(\beta - z)p_{r_{1,i},d}(z)dz. \quad (51)$$

If β is larger than β_{th_2}, then the valid integration region becomes the whole domain of $p_{r_{1,i},d}(z)$ (i.e., $\beta_{th_1} < z < \beta_{th_2}$). Otherwise, the valid integration region becomes $\beta_{th_1} < z < \beta$. As a result, (51) can be rewritten as

$$p_{s,r_{1,i},d}(\beta) = \begin{cases} \int_{\beta_{th_1}}^{\beta} p_{s,d}(\beta - z)p_{r_{1,i},d}(z)dz, \\ \quad \text{for } \beta_{th_1} < \beta < \beta_{th_2} \\ \int_{\beta_{th_1}}^{\beta_{th_2}} p_{s,d}(\beta - z)p_{r_{1,i},d}(z)dz, \\ \quad \text{for } \beta > \beta_{th_2}. \end{cases} \quad (52)$$

Substituting (44) and (46) in (52), we can simply obtain the closed-form expression of (51) as

$$p_{s,r_{1,i},d}(\beta) = \begin{cases} \dfrac{e^{-\frac{\beta}{\delta_{s,d}^2}}\left(e^{\beta\left(-\frac{1}{\delta_{r,d}^2}+\frac{1}{\delta_{s,d}^2}\right)} - e^{\beta_{th_1}\left(-\frac{1}{\delta_{r,d}^2}+\frac{1}{\delta_{s,d}^2}\right)}\right)}{\left(e^{-\frac{\beta_{th_1}}{\delta_{r,d}^2}} - e^{-\frac{\beta_{th_2}}{\delta_{r,d}^2}}\right)\left(\delta_{r,d}^2 - \delta_{s,d}^2\right)}, \\ \quad \text{for } \beta_{th_1} < \beta < \beta_{th_2} \\ \dfrac{e^{-\frac{\beta}{\delta_{s,d}^2}}\left(e^{\beta_{th_2}\left(-\frac{1}{\delta_{r,d}^2}+\frac{1}{\delta_{s,d}^2}\right)} - e^{\beta_{th_1}\left(-\frac{1}{\delta_{r,d}^2}+\frac{1}{\delta_{s,d}^2}\right)}\right)}{\left(e^{-\frac{\beta_{th_1}}{\delta_{r,d}^2}} - e^{-\frac{\beta_{th_2}}{\delta_{r,d}^2}}\right)\left(\delta_{r,d}^2 - \delta_{s,d}^2\right)}, \\ \quad \text{for } \beta > \beta_{th_2}. \end{cases} \quad (53)$$

Note that, for $\delta_{s,d}^2 \approx \delta_{r,d}^2$, the approximate expression can be obtained as

$$P_{s,r_{1,i},d}(\beta) \approx \begin{cases} \dfrac{\beta - \beta_{th_1}}{\delta_{s,d}^4} \cdot \dfrac{e^{-\frac{\beta}{\delta_{s,d}^2}}}{e^{-\frac{\beta_{th_1}}{\delta_{s,d}^2}} - e^{-\frac{\beta_{th_2}}{\delta_{s,d}^2}}} \\ \quad \text{for } \beta_{th_1} < \beta < \beta_{th_2} \\ \dfrac{\beta_{th_2} - \beta_{th_1}}{\delta_{s,d}^4} \cdot \dfrac{e^{-\frac{\beta}{\delta_{s,d}^2}}}{e^{-\frac{\beta_{th_1}}{\delta_{s,d}^2}} - e^{-\frac{\beta_{th_2}}{\delta_{s,d}^2}}} \\ \quad \text{for } \beta > \beta_{th_2}. \end{cases} \quad (54)$$

iv) $p_{s,r_{2,i},d}(\beta)$: Similar to iii) ($p_{s,r_{1,i},d}(\beta)$), the distribution of the combined SNRs, $p_{s,r_{2,i},d}(\beta)$, can be obtained by performing the convolution of the two distributions, $p_{s,d}(\beta)$ and $p_{r_{2,i},d}(\beta)$. In this case, based on the mode of operation, the valid integration region becomes $\beta_{th_2} < z < \beta$. Therefore, the distribution of the combined SNRs can be obtained as

$$\begin{aligned} p_{s,r_{2,i},d}(\beta) &= \int_{\beta_{th_2}}^{\beta} p_{s,d}(\beta - z)p_{r_{2,i},d}(z)dz \\ &= \frac{1}{\delta_{r,d}^2 - \delta_{s,d}^2}\left(e^{\frac{-\beta+\beta_{th_2}}{\delta_{r,d}^2}} - e^{\frac{-\beta+\beta_{th_2}}{\delta_{s,d}^2}}\right). \end{aligned} \quad (55)$$

When $\delta_{s,d}^2 \approx \delta_{r,d}^2$, the approximate expression can also be obtained as

$$p_{s,r_{2,i},d}(\beta) \approx \frac{\beta - \beta_{th_2}}{\delta_{s,d}^4} e^{\frac{-\beta+\beta_{th_2}}{\delta_{s,d}^2}}. \quad (56)$$

APPENDIX II
DERIVATION OF (13)

Applying (49) in (11) and then with the help of the binomial theorem [29], we can rewrite (11) as the following double integral form

$$\sum_{j=0}^{k_2-1}\sum_{l=0}^{k_1-1}\binom{k_2-1}{j}\binom{k_1-1}{l}\frac{k_2k_1}{\delta_{s,r_i}^2}\cdot\frac{(-1)^{j+l+1}}{1+j} \times \frac{1}{\pi}\int_0^{\frac{(M-1)\pi}{M}}\left(\int_0^{\infty} e^{-\beta_a\left(\frac{b}{\sin^2\theta}+\frac{(l+1)}{\delta_{s,r_i}^2}\right)} \times\left(e^{\frac{(1+j)\beta_a}{(-1+A\beta_a)\delta_{s,r_i}^2}} - 1\right)d\beta_a\right)d\theta. \quad (57)$$

Further, with the help of the integral identity [31, eq. (07.33.07.0001.01)] and by applying the Taylor series expansions of exponential functions [29], the double integral term in (57) can be rewritten as the following single integral form

$$\left[\sum_{n=0}^{\infty}\frac{1}{\pi}\int_0^{\frac{(M-1)\pi}{M}}\frac{\sin^2\theta\, U\left(n, 0, -\frac{\frac{b}{\sin^2\theta}+\frac{1+l}{\delta_{s,r_i}^2}}{A}\right)}{(1+l)\sin^2\theta + b\delta_{s,r_i}^2}\left(\frac{A\delta_{s,r_i}^2}{1+j}\right)^{-n}d\theta - \frac{1}{\pi}\int_0^{\frac{(M-1)\pi}{M}}\frac{\sin^2\theta}{(1+l)\sin^2\theta + b\delta_{s,r_i}^2}d\theta\right], \quad (58)$$

where $U(a, b, z)$ is Tricomi's confluent hypergeometric function [31, eq. (07.33.02.0001.01)], which is available in any standard mathematical package, such as Mathematica, MATLAB, and Maple. Finally, applying [21, eq. (22)] in (58) gives the closed-form expression of (11) as shown in (13).

REFERENCES

[1] A. Sendonaris, E. Erkip, and B. Aazhang, "User cooperation diversity—Part I: System description," *IEEE Trans. Commun.*, vol. 51, no. 11, pp. 1927–1938, Nov. 2003.

[2] A. Sendonaris, E. Erkip, and B. Aazhang, "User cooperation diversity—Part II: Implementation aspects and performance analysis," *IEEE Trans. Commun.*, vol. 51, no. 11, pp. 1939–1948, Nov. 2003.

[3] J. N. Laneman, G. W. Wornell, and D. N. C. Tse, "An efficient protocol for realizing cooperative diversity in wireless networks," in *Proc. IEEE Int. Symp. Inf. Theory (ISIT)*, Washington, DC, USA, Jun. 2001, p. 294.

[4] B. Bai, W. Chane, K. B. Letaief, and Z. Cao, "Joint relay selection and subchannel allocation for amplify-and-forward OFDMA cooperative networks," in *Proc. IEEE Int. Conf. Commun. (ICC)*, Ottawa, ON, Canada, Jun. 2012, pp. 4192–4196.

[5] T. Islam, D. S. Michalopoulos, R. Schober, and V. K. Bhargava, "Delay constrained buffer-aided relaying with outdated CSI," in *Proc. IEEE Wireless Commun. Netw. Conf. (WCNC)*, Istanbul, Turkey, Apr. 2014, pp. 875–880.

[6] J. M. Moualeu, W. Hamouda, and F. Takawira, "Selection relaying in coded cooperation with delayed CSI over Rayleigh fading channels," in *Proc. IEEE Wireless Commun. Netw. Conf. (WCNC)*, Istanbul, Turkey, Apr. 2014, pp. 881–886.

[7] B. K. Charlise, L. Vandendorpe, Y. D. Zhang, and M. G. Amin, "Local CSI based selection beamforming for amplify-and-forward MIMO relay networks," *IEEE Trans. Signal Process.*, vol. 60, no. 5, pp. 2433–2446, May 2012.

[8] Z. Zhang, T. Lv, and S. Yang, "Round-robin relaying with diversity in amplify-and-forward multisource cooperative communications," *IEEE Trans. Veh. Technol.*, vol. 62, no. 3, pp. 1251–1266, Mar. 2013.

[9] K. Loa *et al.*, "IMT-advanced relay standards [WiMAX/LTE update]," *IEEE Commun. Mag.*, vol. 48, no. 8, pp. 40–48, Aug. 2010.

[10] *Further Advancements for E-UTRA Physical Layer Aspects*, document 36.814. V2.0.1, 3GPP, Mar. 2010.

[11] *PUSCH Forwarding in Type II Relay*, document R1-100237, 3GPP, Jan. 2010.

[12] K. Azarian, H. El Gamal, and P. Schniter, "On the achievable diversity-multiplexing tradeoff in half-duplex cooperative channels," *IEEE Trans. Inf. Theory*, vol. 51, no. 12, pp. 4152–4172, Dec. 2005.

[13] T. Bui and J. Yuan, "A decode and forward cooperation scheme with soft relaying in wireless communication," in *Proc. IEEE Signal Process. Adv. Wireless Commun. (SPAWC)*, Helsinki, Finland, Jun. 2007, pp. 1–5.

[14] E. Hossain, D. I. Kim, and V. K. Bhargava, *Cooperative Cellular Wireless Networks*, 1st ed. New York, NY, USA: Cambridge Univ. Press, 2011.

[15] S. S. Nam, D. I. Kim, and H.-C. Yang, "Modified dynamic DF for type-2 UE relays," in *Proc. IEEE Wireless Commun. Netw. Conf. (WCNC)*, Paris, France, Apr. 2012, pp. 1392–1397.

[16] G. Y. Dai and W. H. Mow, "Soft forwarding for cooperative wireless communication," U.S. Patent US8 787 428 B2, Jul. 22, 2014.

[17] S. S. Nam, M.-S. Alouini, and S. Choi, "Modified dynamic decode-and-forward relaying protocol for Type II relay in LTE-advanced and beyond," *PLoS ONE*, vol. 11, no. 11, pp. 1–21, Nov. 2016.

[18] S. Choi, M.-S. Alouini, and S. S. Nam, "Impact of co-channel interference on the outage performance under multiple type II relay environments," *IEEE Access*, vol. 5, pp. 26201–26210, Nov. 2017.

[19] S. S. Nam, M.-S. Alouini, and S. Choi, "Iterative relay scheduling with hybrid ARQ under multiple user equipment (type II) relay environments," *IEEE Access*, vol. 6, pp. 6455–6463, Jan. 2018.

[20] F. A. Onat, T. Fan, H. Yanikomeroglu, and H. V. Poor, "Threshold based relay selection in cooperative wireless networks," in *Proc. IEEE Global Telecommun. Conf. (GLOBECOM)*, New Orleans, LO, USA, Nov./Dec. 2008, pp. 1–5.

[21] S. C. Park, D. I. Kim, and S. S. Nam, "Adaptive threshold based relay selection for minimum feedback and channel usage," *IEEE Trans. Wireless Commun.*, vol. 10, no. 11, pp. 3620–3625, Nov. 2011.

[22] Y. Yuan, *LTE-Advanced Relay Technology and Standardization*, 1st ed. Berlin, Germany: Springer, 2013.

[23] K. Oh and D. I. Kim, "Relay selection with limited feedback for multiple UE relays," in *Proc. Int. Conf. Comput. Sci. Appl. (ICCSA)*, Santander, Spain, Jun. 2011, pp. 157–166.

[24] G. L. Stüber, *Principles of Mobile Communication*, 2nd ed. Norwell, MA, USA: Kluwer, 2001.

[25] T. S. Rapappport, *Wireless Communications: Principles and Practice*, 2nd ed. Upper Saddle River, NJ, USA: Prentice-Hall, 2002.

[26] M. K. Simon and M.-S. Alouini, *Digital Communication over Fading Channels*, 2nd ed. New York, NY, USA: Wiley, 2005.

[27] A. S. Ibrahim, A. K. Sadek, W. Su, and K. J. R. Liu, "Cooperative communications with relay-selection: when to cooperate and whom to cooperate with?" *IEEE Trans. Wireless Commun.*, vol. 7, no. 7, pp. 2814–2827, Jul. 2008.

[28] A. Papoulis, *Probability, Random Variables, and Stochastic Processes*, 3rd ed. New York, NY, USA: McGraw-Hill, 1991.

[29] M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions*, 1st ed. New York, NY, USA: Dover, 1972.

[30] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products*, 7th ed. San Diego, CA, USA: Academic, 2007.

[31] Wolfram-Research, *Mathematica Edition: Version 8.0*. Champaign, IL, USA: Wolfram-Research, 2010.

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