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Design of H_∞ Output Feedback Controller for Gas Turbine Engine Distributed Control With Random Packet Dropouts

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ABSTRACT The distributed control architecture is expected to dominate the gas turbine engine (GTE) control systems in future, in which the sensors and actuators are connected to the controllers via a network. Hence, the control problem of network-enabled high-performance distributed engine control (DEC) becomes important in modern gas turbine control systems. Due to the properties of the network, the packet dropouts must be considered. This study introduces a distributed control system architecture based on a networked cascade control system (NCCS). Typical turboshaft engine-distributed controllers are designed based on the NCCS framework with a H_∞ output feedback under random packet-dropouts. The sufficient robust mean-stable conditions are derived via the Lyapunov stability theory and linear matrix inequality approach. In addition, the effectiveness of the presented method is illustrated through a series of simulations.

INDEX TERMS Aeroengine, distributed control system, networked control system, packet-dropout, stability.

I. INTRODUCTION

Conventional GTE control systems are designed centralizedly to protect control elements from the extreme environment [1]. However, the requirements of increased performance, more convenient operation, reduction of design and maintenance cost push forward a more effective architecture, used in the control systems. Thus, the distribute engine control (DEC) was presented and it has led to a number of applications [2], [3].

In the distributed architecture, the sensors and controllers are connected by the communication networks, as well between the controllers and the actuators. It may lead to the development of the GTE control system with great extensibility and high capacity for upgrades. The extensive study may refer to the literatures [4]–[9]. An observer-based fuzzy output-feedback control for stochastic nonlinear multiple time-delay systems was provided in [10] and a variable splitting technique was employed to surmount the difficulty occurred in the nonlower-triangular form. Reference [11] provided an adaptive fuzzy hierarchical sliding mode control method for a class of MIMO unknown nonlinear time-delay systems with input saturation. The backstepping and robust adaptive control techniques were provided to design the controller by adopting the structural characteristics of fuzzy systems which were employed to tackle the problem from packaged unknown nonlinearities and the common Lyapunov function approach in [12]. The DEC architecture can be viewed as a Network Cascade Control System (NCCS). For instance, GE T700 turboshaft engine is a two-spool GTE consisting of a gas generator and a free power turbine [13], [14], and the power turbine is connected to the rotor system by a shaft and a gear box.

There are some fundamental factors which affect the GTE DEC system by using the communication networks. They include network-induced time delay, packet dropouts and bandwidth constraints [17], [18]. For control performance and stability, the DEC system should be robust. The network-induced time delay in NCCSs occurs when the sensors, controllers and actuators transfer information/data through the networks. [19]–[26]. Reference [27] provided an algorithm for the desired sampled-data cascade controller for the problem of dissipative fault-tolerant cascade control synthesis of a class of NCCs by constructing the appropriate Lyapunov-Krasovskii function. The reliable sampled-data $L_2 - L_\infty$

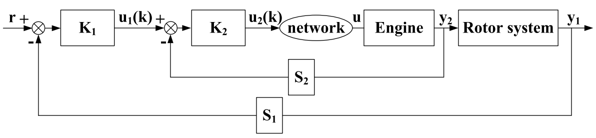

FIGURE 1. Block diagram of the NCCS model.

control problem for a class of NCCs with randomly occurring actuator failures, time-varying delay and disturbances was studied in [28].

However, there are few papers involved with the DEC robust control in GTE. Belapurkar *et al.* [29] analyzed the stability of set-point controller for GTE DEC systems with time delays. Yedavalli *et al.* [18] provided the stability analysis method for DEC systems under packet dropouts. Merrill *et al.* [2] discussed the optimal control issue for the control performance of various types of DEC network configurations.

This paper discusses H_∞ controller design for GTE DEC system design using output feedback control with packet dropouts. In literature, the H_∞ control problem is used to design an GTE DEC appropriate controllers such that the energy-to-energy gain from GTE exogenous disturbance (w) may be restricted less than a desired level. The design problem of DEC H_∞ controllers are designed with the minimum disturbance attenuation level by means of the following energy function described by $\sum_{k=0}^{\infty} E[\|y(k)\|^2] < \gamma^2 \sum_{k=0}^{\infty} E[\|w(k)\|^2]$, and γ is the nonnegative constant. In this paper, the probability-dependent conditions are obtained for H_∞ control design for NCCSs. The main aim of this work is to design the gain-scheduled control with a disturbance attenuation level such that the resulting GTE DEC system is stable. The algorithms for the proposed NCCS strategy are consisted of two loops: an inner loop (Gas generator control loop) with fast dynamic to eliminate the input disturbances and an outer loop (Rotor control loop) to regulate the output performance. The rest of the paper is organized as follows: The output feedback control problem for GTE DEC control system is formed in section II, and the models are described. In III, H_∞ output feedback controllers are obtained based on Lyapunov stability theory and LMI approach. A numerical simulation example is provided in Section IV to illustrate the effectiveness. Conclusion can be found in the last section.

II. PROBLEM FORMULATION

A. DESCRIPTION ON DEC SYSTEM ARCHITECTURE

This study focused on a GE T700 turboshaft engine, and the details described in [30], and Fig. 1 shows the architecture.

Primary Plant: The state-space representation of the rotor system is provided by the following equation:

$$\begin{cases} \dot{x}_1(t) = \mathcal{A}_1 x_1(t) + \mathcal{B}_1 y_2(t) \\ y_1(t) = C_1 x_1(t) \end{cases} \tag{1}$$

where, $x_1 = [N_P \; N_{MR} \; Q_{MR}]^T$, and $y_1 = N_P$ are the state vector and the output of the rotor system, respectively. $y_2 = Q_S$ is the gas generator output. The matrices $\mathcal{A}_1$, $\mathcal{B}_1$, and C_1 are

$$\mathcal{A}_1 = \begin{bmatrix} 0 & 0 & -\dfrac{1}{J_T} \\ 0 & -\dfrac{DAM}{J_{MR}} & \dfrac{1}{J_{MR}} \\ KMR & \dfrac{DMR \cdot DAM}{J_{MR}} - KMR & -\dfrac{DMR}{J_T} - \dfrac{DMR}{J_{MR}} \end{bmatrix},$$

$$\mathcal{B}_1 = \begin{bmatrix} \dfrac{2}{J_T} \\ 0 \\ \dfrac{2 \cdot DMR}{J_T} \end{bmatrix} \text{ and } C_1 = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}.$$

Secondary Plant: The continuous-time linear model of the gas generator is shown as follows:

$$\begin{cases} \dot{x}_2(t) = \mathcal{A}_2 x_2(t) + \mathcal{B}_2 u(t) + \mathcal{B}_3 w(t) \\ y_2(t) = C_2 x_2(t) \end{cases} \tag{2}$$

where $x_2 = \begin{bmatrix} N_G \; Q_S \; T_{45} \; P_{S3} \; N_P \end{bmatrix}^T$, $y_2 = Q_S$ are the state and output vectors; $u(t) = W_F$ is the control input; and $w(t)$ is the exogenous disturbance signal belonging to $l_2[0, \infty)$. The matrices $\mathcal{A}_2$, $\mathcal{B}_2$, and C_2 are

$$\mathcal{A}_2 = \begin{bmatrix} \dfrac{1}{J_G} \cdot \dfrac{\delta Q_G}{\delta N_G} 0000 \\ \dfrac{2 \cdot DMR}{J_T} \cdot \dfrac{\delta Q_P}{\delta N_G} 000 \dfrac{2 \cdot DMR}{J_T} \cdot \dfrac{\delta Q_P}{\delta N_P} \\ \dfrac{\delta T_{45}}{\delta N_G} 0000 \\ \dfrac{\delta P_{S3}}{\delta N_G} 0000 \\ \dfrac{2}{J_T} \cdot \dfrac{\delta Q_P}{\delta N_G} - \dfrac{1}{J_T} 00 \dfrac{2}{J_T} \cdot \dfrac{\delta Q_P}{\delta N_P} \end{bmatrix},$$

$$\mathcal{B}_2 = \begin{bmatrix} \dfrac{1}{J_G} \cdot \dfrac{\delta Q_G}{\delta W_F} \\ \dfrac{2 \cdot DMR}{J_T} \cdot \dfrac{\delta Q_P}{\delta W_F} \\ \dfrac{\delta T_{45}}{\delta W_F} \\ \dfrac{\delta P_{S3}}{\delta W_F} \\ \dfrac{2}{J_T} \cdot \dfrac{\delta Q_P}{\delta W_F} \end{bmatrix}, \quad C_2 = [01000]$$

and $\mathcal{B}_3$ is a real constant matrix with an appropriate dimension.

The following assumptions are partially taken from literatures [30]–[32]:

1) The controllers are event driven. The actuator is time driven, and the sensors are also time driven.
2) The data packet transmitted from the controller to the plant may be delayed. The delay is assumed to be constant and less than a sampling period h (i.e., $\tau_k \in [0, h]$).

3) The data packet is assumed to be transmitted between the primary and secondary controllers in a single packet without any loss. However, the data packet transmitted between the secondary controller and the actuator may be delayed or may meet with a possible failure in a random manner.

B. OUTPUT FEEDBACK CONTROL OF DEC SYSTEM

Taking the network-induced delay τ_k into account, the controllers are event-driven and the actuator is time-driven. Under such condition, the piece-wised control input is given by:

$$\begin{cases} \widetilde{u}_2(k-1), & kh \le t < kh + \tau_k \\ \widetilde{u}_2(k), & kh + \tau_k \le t < (k+1)h \end{cases} \tag{3}$$

and

$$\widetilde{u}_2(k) = \begin{cases} \widetilde{u}_2(k-1), & \text{if } u_2(k) \text{ is lost during transmission} \\ u_2(k), & \text{if } u_2(k) \text{ is transmitted successfully} \end{cases} \tag{4}$$

where $u_2(k)$ is the control output of the secondary controller, (3) and (4) the actuator receives the signal $u_2(k)$ if the data is transmitted successfully. Otherwise, the previous value is used in the actuator by Zero-Order-Hold. .

Since the actuator is time driven, the packet loss may happen in a random manner. Then, $\widetilde{u}_2(k)$ can be rewritten by [32], [33]:

$$\widetilde{u}_2(k) = \lambda(k)u_2(k) + (1-\lambda(k))\widetilde{u}_2(k-1) \tag{5}$$

where $\lambda(k)$ is a Bernoulli distributed stochastic variable with the value 0 or 1. $\lambda(k) = 1$ represents the successful state transmission of the delayed packet and $\lambda(k) = 0$ describes the complete packet loss. It is assumed that $\lambda(k)$ satisfies the Bernoulli distribution [34]:

$$\begin{cases} \text{Prob}\{\lambda(k) = 1\} = \alpha \\ \text{Prob}\{\lambda(k) = 0\} = 1 - \alpha \end{cases} \tag{6}$$

where α is defined as packet-dropout probability and it is a positive scalar, and $E[\lambda(k) - \alpha] = 0$, $E[(\lambda(k) - \alpha)^2] = \alpha(1-\alpha)$.

Considering the system reference input $N_{Pr} = 0$, the output feedback controller is utilized in K_1. Actually, the controller uses a discrete-time form in practical applications:

$$u_1(k) = K_1 y_1(k) \tag{7}$$

where $y_1(k)$ is the output vector of a rotor system in discrete-time form, and K_1 is the system output feedback gain. K_2 also uses the feedback form:

$$u_2(k) = u_1(k) + K_2 y_2(k) \tag{8}$$

where $y_2(k)$ is the output vector of the engine in discrete-time form.

Thus, the control state of the secondary controller is re-written by:

$$u_2(k) = K_1 y_1(k) + K_2(\alpha) y_2(k) \tag{9}$$

By using (3), the rotor system and engine with sampling period, $[kh, (k+1)h]$, are discretized to

$$\begin{cases} x_1(k+1) = A_1 x_1(k) + B_1 y_2(k) \\ y_1(k) = C_1 x_1(k) \end{cases} \tag{10}$$

where $A_1 = e^{\mathcal{A}_1 h}$, $B_1 = \int_0^h e^{\mathcal{A}_1 s} ds \mathcal{B}_1$, and the gas generator can be discretized to (11). Where $A_2 = e^{\mathcal{A}_2 h}$, $B_{21} = \int_{h-\tau_k}^h e^{\mathcal{A}_2 s} ds \mathcal{B}_2$, $B_{22} = \int_0^{h-\tau_k} e^{\mathcal{A}_2 s} ds \mathcal{B}_2$, $B_3 = \int_0^h e^{\mathcal{A}_2 s} ds \mathcal{B}_3$. Introduce (5) and (9) into (11), shown at the top of the next page, to obtain (12), shown at the top of the next page. Thus, the discretized system can be further expanded to (13), shown at the top of the next page.

Since the goal of this paper is to design the output controllers to regulate the power turbine speed in presence of disturbances, the output of the closed-loop is determined by $y_1(k)$ and the input is given by exogenous disturbance $w(k)$. Observing (10) and (11), $x_1(k)$, $x_2(k)$, $\widetilde{u}_2(k-1)$ are chosen as the closed-loop state vectors. Therefore, the closed-loop state-space form is given by:

$$\begin{cases} x(k+1) = (A(\alpha) + (\lambda(k) - \alpha)B(\alpha))x(k) + Dw(k) \\ y(k) = Cx(k) \end{cases} \tag{14}$$

where $A(\alpha)$, $B(\alpha)$, C and D can seen in (15), shown at the top of the next page.

III. MAIN RESULTS

A. SYSTEM PERFORMANCE REQUIREMENT

To design the controllers (7) and (8) for the turboshaft engine NCCS, the closed-loop system must satisfy the following performance requirements:

1) The close-loop system (14) is exponentially mean-stable [35],
2) Under the zero-initial condition, the controlled output $y(k)$ satisfies

$$\sum_{k=0}^{\infty} E[\|y(k)\|^2] < \gamma^2 \sum_{k=0}^{\infty} E[\|w(k)\|^2] \tag{16}$$

for all nonzero $w(k)$, where $\gamma > 0$ is a prescribed scalar.

B. CONTROLLER DESIGN

Lemma 1 (Schur Complement): Given constant matrices Ω_1, Ω_2, Ω_3, where $\Omega_1 = \Omega_1^T$ and $\Omega_2 = \Omega_2^T > 0$ we have $\Omega_1 + \Omega_3^T \Omega_2^{-1} \Omega_3 < 0$ if and only if

$$\begin{bmatrix} \Omega_1 & \Omega_3^T \\ \Omega_3 & -\Omega_2 \end{bmatrix} < 0 \quad \text{or} \quad \begin{bmatrix} -\Omega_2 & \Omega_3 \\ \Omega_3^T & \Omega_1 \end{bmatrix} < 0$$

Theorem 2: Given the positive scalar α, the closed-loop system (14) is exponentially mean-square stable with a H_∞ performance index γ. When the definite matrices $P(\alpha)$ and matrices R_1, R_2, R_3, $\mathcal{K}_1$, $\mathcal{K}_{21}$, $\mathcal{K}_{22}$ are symmetric positive,

$$\begin{cases} x_2(k+1) = A_2x_2(k) + B_{21}\widetilde{u}_2(k-1) + B_{22}\widetilde{u}_2(k) + B_3w(k) \\ y_2(k) = C_2x_2(k) \end{cases} \tag{11}$$

$$\begin{cases} x_2(k+1) = A_2x_2(k) + (B_{21} - (1-\lambda(k))B_{22})\widetilde{u}_2(k-1) \\ \qquad\qquad +\lambda(k)B_{22}(K_1y_1(k) + K_2(\alpha)y_2(k)) + B_3w(k) \\ y_2(k) = C_2(k)x_2(k) \end{cases} \tag{12}$$

$$\begin{cases} x_2(k+1) = (A_2 + \alpha B_{22}K_2(\alpha)C_2)x_2(k) + (\lambda(k)-\alpha)B_{22}(K_1C_1x_1(k) \\ \qquad\qquad +K_2(\alpha)C_2x_2(k)) + \alpha B_{22}K_1C_1x_1(k) + (B_{21} + (1-\alpha B_{22}))\widetilde{u}_2(k-1) \\ \qquad\qquad +(\alpha - \lambda(k))B_{22}\widetilde{u}_2(k-1) + B_3w(k) \\ y_2(k) = C_2(k)x_2(k) \end{cases} \tag{13}$$

$$A(\alpha) = \begin{bmatrix} A_1 & B_1C_2 & 0 \\ \alpha B_{22}K_1C_1 & A_2 + \alpha B_{22}K_2(\alpha)C_2 & B_{21} + (1-\alpha)B_{22} \\ \alpha K_1C_1 & \alpha K_2(\alpha)C_2 & (1-\alpha)I \end{bmatrix},$$
$$B(\alpha) = \begin{bmatrix} 0 & 0 & 0 \\ B_{22}K_1C_1 & B_{22}K_2(\alpha)C_2 & -B_{22} \\ K_1C_1 & K_2(\alpha)C_2 & -I \end{bmatrix}, \quad C = \begin{bmatrix} C_1^T \\ 0 \\ 0 \end{bmatrix}^T, \; D = \begin{bmatrix} 0 \\ B_3 \\ 0 \end{bmatrix}. \tag{15}$$

the following LMI holds:

$$\begin{bmatrix} O_1 & 0 & O_2 & O_3 & O_4 \\ * & -\gamma^2 I & O_5 & 0 & 0 \\ * & * & O_6 & 0 & 0 \\ * & * & * & O_6 & 0 \\ * & * & * & * & -I \end{bmatrix} < 0 \tag{17}$$

where

$$O_1 = P - diag\{R_1 + R_1^T, R_2 + R_2^T, R_3 + R_3^T\},$$
$$O_2 = \begin{bmatrix} R_1A_1^T & \alpha\mathcal{K}_1^TB_{22}^T & \alpha\mathcal{K}_1^T \\ R_2^TC_2^TB_1^T & R_2^TA_2^T + \alpha\mathcal{K}_2^T(\alpha)B_{22}^T & \mathcal{K}_2^T(\alpha) \\ 0 & R_3^TB_{21}^T + (1-\alpha)R_3^TB_{22}^T & (1-\alpha)R_3^T \end{bmatrix},$$
$$O_3 = \begin{bmatrix} 0 & \mathcal{K}_1^TB_{22}^T & \mathcal{K}_1^T \\ 0 & \mathcal{K}_2^T(\alpha)B_{22}^T & \mathcal{K}_2^T(\alpha) \\ 0 & -R_3^TB_{22}^T & -R_3^T \end{bmatrix}, \; O_4 = \begin{bmatrix} R_1^TC_1^T \\ 0 \\ 0 \end{bmatrix},$$
$$O_5 = \begin{bmatrix} 0 \\ B_3 \\ 0 \end{bmatrix}^T, \quad \mathcal{K}_2(\alpha) = \mathcal{K}_{21} + \alpha\mathcal{K}_{22}, \quad O_6 = -P,$$

and the state feedback gain matrices can be gained by:

$$K_1 = \mathcal{K}_1(C_1R_1)^{-1}, \quad K_{21} = \mathcal{K}_{21}(C_2R_2)^{-1},$$
$$K_{22} = \mathcal{K}_{22}(C_2R_3)^{-1}. \tag{18}$$

Proof 1: In order to conclude the controller design conditions, the following function can be defined

$$V(k) = x^T(k)P^{-1}x(k) \tag{19}$$

For any nonzero $w(k)$, if

$$\begin{aligned} &E[V(k+1)] - E[V(k)] + E[y(k)^Ty(k)] - \gamma^2E[w(k)^Tw(k)] \\ &= E[(A(\alpha) + (\lambda(k)-\alpha)B(\alpha)x(k+1) \\ &\quad + Dw(k))^TP^{-1}(A(\alpha) + (\lambda(k)-\alpha)B(\alpha)x(k+1) \\ &\quad + Dw(k))] - x(k)^TP^{-1}x(k) + y(k)^Ty(k) - \gamma^2w(k)^Tw(k) \\ &= (A(\alpha x(k))^TP^{-1}(A(\alpha x(k)) \\ &\quad + \alpha(1-\alpha)x^T(k)B^T(\alpha)P^{-1}B(\alpha)x(k) \\ &\quad - x(k)^TP^{-1}x(k) + y(k)^Ty(k) - \gamma^2w(k)^Tw(k) \\ &= \begin{bmatrix} x(k) \\ w(k) \end{bmatrix}^T \cdot O(\alpha) \cdot \begin{bmatrix} x(k) \\ w(k) \end{bmatrix} \end{aligned} \tag{20}$$

where $O(\alpha)$ can be seen in (21), shown at the bottom of the next page, and $O(\alpha)$ can be rewritten in (22), shown at the bottom of the next page.

Thus, by applying the Lemma 1, we get (23).

$$O(\alpha) = \begin{bmatrix} -P^{-1} & 0 & A(\alpha)^T & \alpha(1-\alpha)B(\alpha)^T & C^T \\ * & -\gamma^2 I & D^T & 0 & 0 \\ * & * & -P & 0 & 0 \\ * & * & * & -P & 0 \\ * & * & * & * & -I \end{bmatrix} \tag{23}$$

Now, the goal is to prove $O(\alpha) < 0$. By means of the partition matrices $R = diag\{R_1, R_2, R_3\}$, P, the values of $A(\alpha)$, $B(\alpha)$, C and $\mathcal{K}_1$, $\mathcal{K}_{21}$, $\mathcal{K}_{22}$ in LMI (17), the following inequality (24), shown at the bottom of the next page, can be gotten.

If (24) holds, $P - 2R \geq -R^TP^{-1}R$, the following inequality (25), shown at the bottom of the next page, can be gotten.

(17) can then be obtained by pre- and post-multiplying (25) by $diag(R^{-1}, I, I, I, I)$. Therefore, for zero to ∞ with respect to k yields:

$$\sum_{k=0}^{\infty} E[\|y(k)\|^2] < \gamma^2 \sum_{k=0}^{\infty} E[\|w(k)\|^2] + E[V(0)] - E[V(\infty)] \tag{26}$$

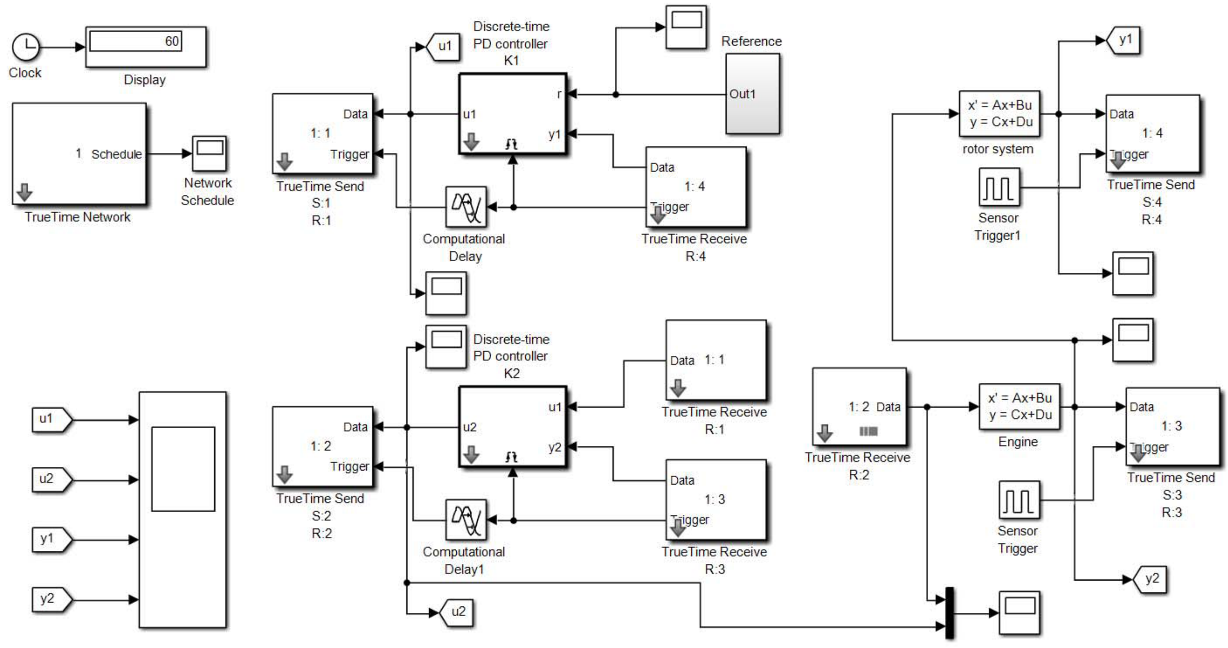

FIGURE 2. **Diagram of MATLAB/Simulink model.**

Since $\begin{bmatrix} x(0) \\ w(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, the closed-loop system (14) is exponentially mean-square stable, and it satisfies (16).

Algorithm for the controllers design is:

1) The continuous closed-loop system parameters are derived based on [30].
2) The continuous system parameters are discretized.
3) The convex optimization problem (17) is solved to obtain the feasible solutions in terms of positive definite matrix P, nonsingular slack matrices R_i, ($i = 1, 2, 3$) and matrices $\mathcal{K}_1$, $\mathcal{K}_{21}$, $\mathcal{K}_{22}$ and γ.
4) The controller parameters K_1, K_{21}, and K_{22} are derived from Theorem 2.
5) Stop.

IV. SIMULATION EXAMPLE

This section presents the evaluation on effectiveness of the proposed method via simulations to the GE T700 turboshaft GTE DEC control systems. The DEC system is simulated

$$O(\alpha) = \begin{bmatrix} A(\alpha)^T P^{-1} A(\alpha) + \alpha(1-\alpha) B(\alpha)^T P^{-1} B(\alpha) + C^T C - P^{-1} & A(\alpha)^T P^{-1} D \\ D^T P^{-1} A(\alpha) & D^T P^{-1} D - \gamma^2 I \end{bmatrix} \tag{21}$$

$$\begin{aligned} O(\alpha) = \Bigg(\begin{bmatrix} -P^{-1} & 0 \\ * & -\gamma^2 I \end{bmatrix} + \begin{bmatrix} A(\alpha)^T \\ D^T \end{bmatrix} \cdot P^{-1} \cdot \begin{bmatrix} A(\alpha) & D \end{bmatrix} \\ + \alpha(1-\alpha) \begin{bmatrix} B(\alpha)^T \\ 0 \end{bmatrix} \cdot P^{-1} \cdot \begin{bmatrix} B(\alpha) & 0 \end{bmatrix} + \begin{bmatrix} C^T \\ 0 \end{bmatrix} \cdot \begin{bmatrix} C & 0 \end{bmatrix} \Bigg) \end{aligned} \tag{22}$$

$$\begin{bmatrix} P - R + R^T & 0 & R^T A(\alpha)^T & \alpha(1-\alpha) R^T B(\alpha)^T & R^T C^T \\ * & -\gamma^2 I & D^T & 0 & 0 \\ * & * & -P & 0 & 0 \\ * & * & * & -P & 0 \\ * & * & * & * & -I \end{bmatrix} < 0 \tag{24}$$

$$\begin{bmatrix} -R^T P^{-1} R & 0 & R^T A(\alpha)^T & \alpha(1-\alpha) R^T B(\alpha)^T & R^T C^T \\ * & -\gamma^2 I & D^T & 0 & 0 \\ * & * & -P & 0 & 0 \\ * & * & * & -P & 0 \\ * & * & * & * & -I \end{bmatrix} < 0 \tag{25}$$

$$\begin{cases} \begin{bmatrix} \dot{N}_P \\ \dot{N}_{MR} \\ \dot{Q}_{MR} \end{bmatrix} = \begin{bmatrix} 0 & 0 & -285.7143 \\ 0 & -0.4533 & 9.0662 \\ 5.2650 & -5.2131 & -42.5958 \end{bmatrix} \begin{bmatrix} N_P \\ N_{MR} \\ Q_{MR} \end{bmatrix} + \begin{bmatrix} 571.4286 \\ 0 \\ 82.5714 \end{bmatrix} Q_S \\ N_P = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} N_P \\ N_{MR} \\ Q_{MR} \end{bmatrix} \end{cases} \tag{27}$$

$$\begin{cases} \begin{bmatrix} \dot{N}_G \\ \dot{Q}_S \\ \dot{T}_{45} \\ \dot{P}_{S3} \\ \dot{N}_P \end{bmatrix} = \begin{bmatrix} -126.8 & 27.04 & 12.36 & 22.17 & 16.72 \\ 54.67 & 57.21 & -77.02 & -76.21 & 50.81 \\ -336.6 & 223.3 & -130.7 & -83.32 & 172.1 \\ 161.2 & 2.459 & -21.8 & -63.09 & 1.799 \\ 62.42 & -73.55 & -104.2 & -91.44 & -102.3 \end{bmatrix} \begin{bmatrix} N_G \\ Q_S \\ T_{45} \\ P_{S3} \\ N_P \end{bmatrix} + \begin{bmatrix} -11.7 \\ 44.24 \\ 53.56 \\ 17.45 \\ 59.35 \end{bmatrix} W_F + \begin{bmatrix} 0.02 \\ 0.02 \\ 0.02 \\ 0.02 \\ 0.02 \end{bmatrix} w \\ Q_S = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} N_G \\ Q_S \\ T_{45} \\ P_{S3} \\ N_P \end{bmatrix} \end{cases} \tag{28}$$

$$P = 10^{-4} \times \begin{bmatrix} -0.3072 & -0.0064 & 0.0087 & -0.0000 & 0.0026 & -0.0005 & 0.0012 & -0.0014 & -0.0002 \\ -0.0064 & -0.3819 & -0.0248 & 0.0204 & -0.0146 & -0.0058 & 0.0030 & -0.0122 & 0.0002 \\ 0.0087 & -0.0248 & -0.2889 & -0.0026 & 0.0056 & -0.0002 & 0.0020 & -0.0013 & -0.0005 \\ -0.0000 & 0.0204 & -0.0026 & -0.3719 & -0.0263 & 0.0323 & 0.0020 & 0.0250 & -0.0258 \\ 0.0026 & -0.0146 & 0.0056 & -0.0263 & -0.5233 & -0.0094 & -0.0608 & 0.1694 & -0.0102 \\ -0.0005 & -0.0058 & -0.0002 & 0.0323 & -0.0094 & -0.2819 & -0.0002 & 0.0126 & -0.0005 \\ 0.0012 & 0.0030 & 0.0020 & 0.0020 & -0.0608 & -0.0002 & -0.2965 & 0.0640 & 0.0013 \\ -0.0014 & -0.0122 & -0.0013 & 0.0250 & 0.1694 & 0.0126 & 0.0640 & -0.4445 & 0.0104 \\ -0.0002 & 0.0002 & -0.0005 & -0.0258 & -0.0102 & -0.0005 & 0.0013 & 0.0104 & -0.3702 \end{bmatrix},$$
$$R_1 = \begin{bmatrix} 0.0020 & 0.0001 & 0.0001 \\ 0.0001 & 0.0001 & -0.0002 \\ 0.0001 & -0.0002 & 0.0018 \end{bmatrix},$$
$$R_2 = \begin{bmatrix} 0.0002 & -0.0000 & 0.0006 & 0.0002 & 0.0000 \\ -0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0006 & 0.0000 & 0.0019 & -0.0001 & -0.0000 \\ 0.0002 & 0.0000 & -0.0001 & 0.0020 & -0.0000 \\ 0.0000 & 0.0000 & -0.0000 & -0.0000 & 0.0020 \end{bmatrix},$$
$$R_3 = 3.6081 \times 10^{-5}, \quad K_1 = -1.5188 \times 10^{-6},$$
$$K_{21} = 0.0253, \quad K_{22} = -3.6452 \times 10^{-4}, \quad \gamma_{opt} = 0.0928. \tag{29}$$

herein by using the TrueTime network simulation software under MATLAB/Simulink [36]. The simulation model is shown in Fig. 2.

The rotor system in continuous time form is provided in (27), shown at the top of this page, and the gas generator model is given in (28), shown at the top of this page. The coefficients after the discretization are provided as follows ($h = 0.1s$):

$$A_1 = \begin{bmatrix} -0.0962 & 1.0556 & 0.1728 \\ 0.0338 & 0.9231 & -0.0084 \\ -0.0032 & 0.0048 & -0.1039 \end{bmatrix}, \; B_1 = \begin{bmatrix} 1.3929 \\ 1.7354 \\ 2.1425 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix},$$
$$A_2 = \begin{bmatrix} 0.1753 & 0.1167 & -0.0512 & -0.0109 & 0.0005 \\ 0.9670 & 0.7028 & -0.3030 & -0.1120 & -0.0056 \\ -0.2834 & -0.1361 & 0.0643 & -0.0289 & -0.0085 \\ 0.5923 & 0.3504 & -0.1576 & 0.0021 & 0.0082 \\ -0.8309 & -0.6204 & 0.2661 & 0.1109 & 0.0072 \end{bmatrix},$$
$$B_{21} = \begin{bmatrix} 0.0099 \\ 0.3108 \\ 0.5111 \\ 0.1203 \\ -0.2493 \end{bmatrix}, \quad B_{22} = \begin{bmatrix} 0.0107 \\ 0.0652 \\ -0.0148 \\ 0.0328 \\ -0.0600 \end{bmatrix},$$

TABLE 1. Optimized γ_{opt} and control parameters for various values of α.

α	K_1	$K_{21}\&K_{22}$	γ_{opt}
0.0	-5.9621×10^{-5}	$-0.0476\&0$	0.0930
0.5	-1.5188×10^{-6}	$0.0253\& -3.6452 \times 10^{-4}$	0.0928
0.9	2.6447×10^{-5}	$0.0553\&3.4712 \times 10^{-4}$	0.0937

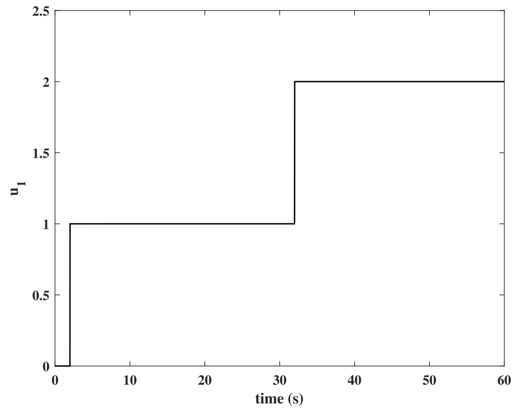

FIGURE 3. Rotor system controller output u_1.

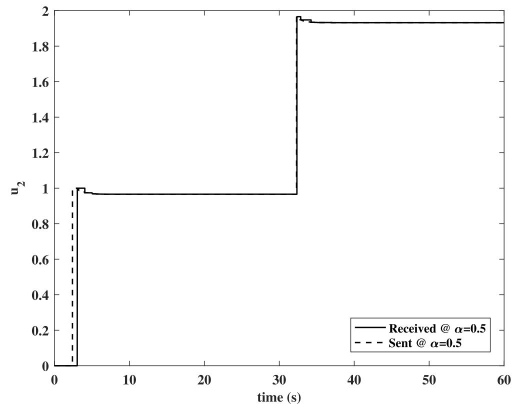

FIGURE 4. Gas generator controller output u_2.

$$B_3 = 10^{-3} \times \begin{bmatrix} 0.1181 \\ 0.8112 \\ 0.0181 \\ 0.2517 \\ -0.6940 \end{bmatrix}, \quad C_2 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \end{bmatrix}.$$

Scenario 1: Let the packet-dropout probability value $\alpha = 0.5$, and given the initial conditions as $x_1(0) = [1\ 0.2\ 0.2]^T$, $x_2(0) = [0.9000\ 0.4189\ 0.7843\ 0.6498\ 1.0000]^T$, the simulation time is $T = 20s$. The goal of this study is to design the controller gains such that the closed-loop system is robustly exponentially mean-stable with a disturbance attenuation level $\gamma > 0$. And system input is superimposed two unit step inputs.The feasible solution of (17) can be calculated based on (29), shown at the top of the previous page, by using the LMI toolbox of MATLAB.

Fig. 3 and Fig. 4 illustrate that the gas generator control loop (inner loop) runs much faster than the rotor system control loop (outer loop). And Fig. 5 shows the partial view of u_2. As it can be seen, the received signal (solid line) delays the sent signal (dash line) at current sampling time, and due to the packet-dropout, the received signal keeps the value of the last sampling time.

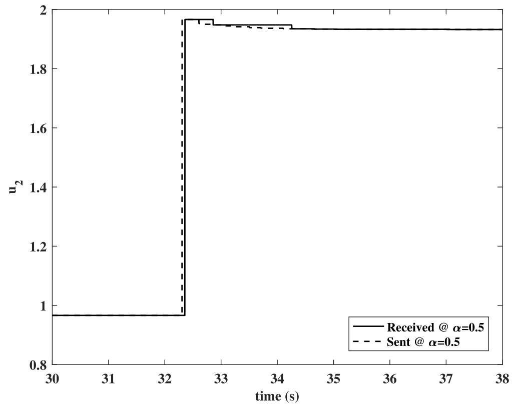

FIGURE 5. Gas generator controller output u_2 (partial view).

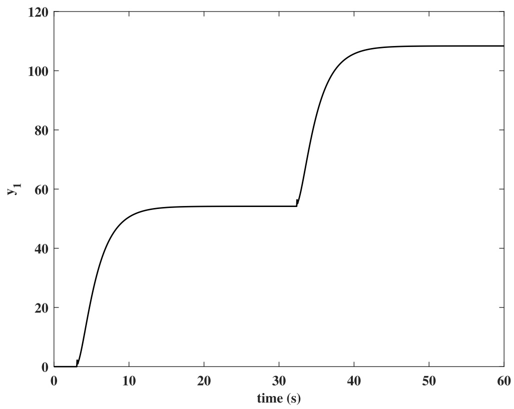

FIGURE 6. Output response y_1.

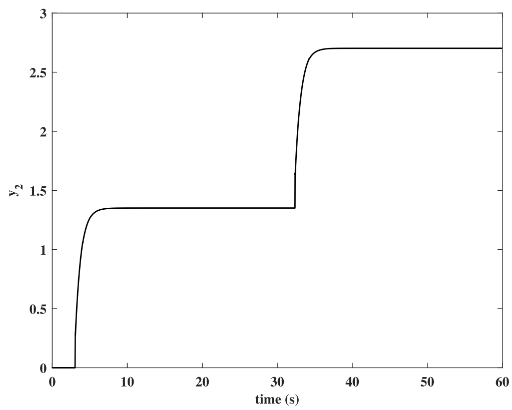

FIGURE 7. Output response y_2.

Fig. 6 and Fig. 7 show the closed-loop system outputs. By Theorem 2, the closed-loop system (14) is robustly mean-stable with a H_∞ disturbance-rejection-attenuation level γ. It should be noted that the obtained controllers make sure the fast response of inner loop to eliminate disturbances.

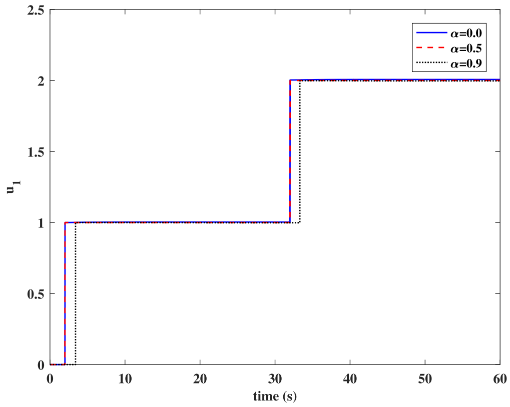

FIGURE 8. Rotor system controller output u_1 under different packet-dropout probabilities.

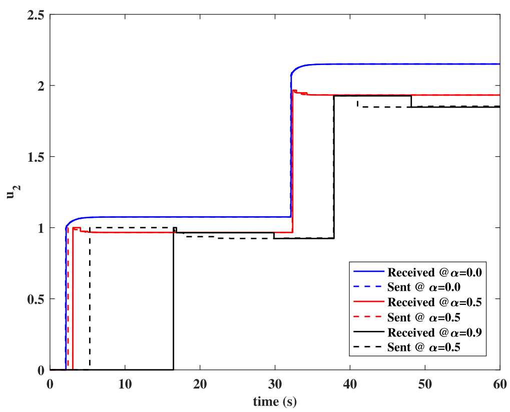

FIGURE 9. Gas generator controller output u_2 under different packet-dropout probabilities.

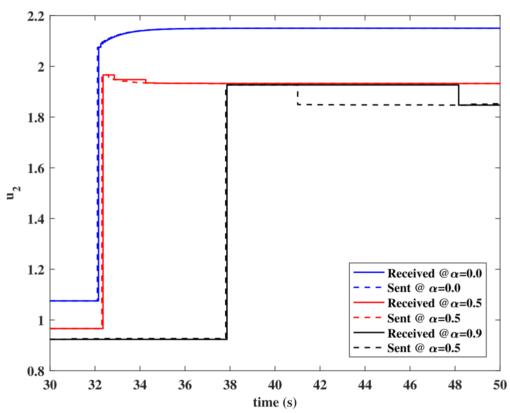

FIGURE 10. Gas generator controller output u_2 under different packet-dropout probabilities (partial view).

It is easy to see that the controlled outputs of both loops are converged quickly by means of the obtained controllers. It can be concluded that the designed controllers are suitable for the GTE NCCS model, which is performed well over the network-induced imperfections like time delay, packet loss and also attenuated the imposed external disturbance on both the plants.

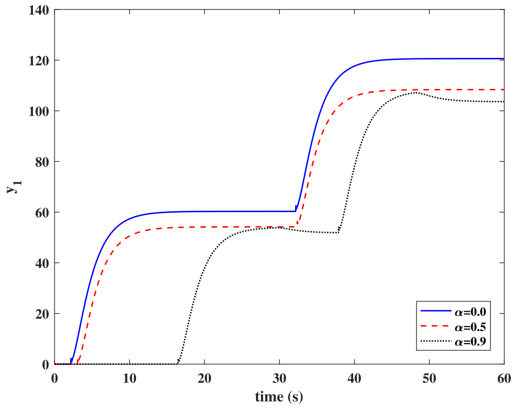

FIGURE 11. Output response y_1 under different packet-dropout probabilities.

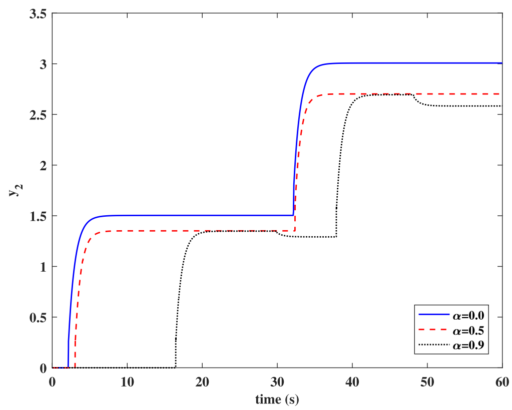

FIGURE 12. Output response y_2 under different packet-dropout probabilities.

Scenario 2: In order to show the evaluation on effectiveness of the proposed method under different values of packet-dropout probability, α, the obtained output feedback controller parameters and disturbance attenuation level γ are presented in Table 1. It should be noted that, K_1, K_{21} and K_{22} can be easily obtained the values of probability for the values of α. Fig. 8 and Fig. 9 show the control signals under different packet-dropout probabilities, and Fig. 10 shows the partial view of u_2. The system input is the same with the Scenario 1.

As it can be seen in Fig. 11 and Fig. 12, the control dynamic becomes poor with increasing packet-dropout probability. However, the closed-loop system stability can be guaranteed with or without packet dropouts by using the design method provided in the article.

V. CONCLUSIONS AND FUTURE WORK

This study considered the novel robust H_∞ DEC problem to guarantee the engine performance with random packet dropouts and disturbance. A distributed control system architecture of a typical turboshaft engine was described accordingly to address the importance of robust H_∞. This distributed architecture can be transformed into a networked cascade control system. The state feedback controllers were designed to robustly mean-stabilize the closed-loop system under packet dropouts and disturbance. The sufficient conditions for mean-stable stability were derived based on the Lyapunov stability and the LMI approach. The controller design problem under consideration is solvable if the LMIs were feasible.Simulation example was provided to show the effectiveness of the approach. One of our future research topics would be the DEC system fault-tolerant control with simultaneous packet dropout and network-induced delays, where the latest delay-dependent techniques can be employed.

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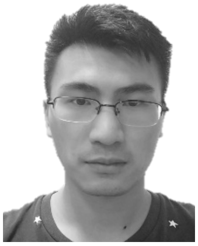

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