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Stability of Switched Time-Delay Systems via Mode-Dependent Average Dwell Time Switching

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ABSTRACT The stability problem is explored for a class of switched time-delay systems (STDSs) with unstable subsystems in this paper. The delay-dependent stability criterion of the STDSs with mode-dependent average dwell time (MDADT) switching is developed via the multiple Lyapunov–Krasovskii functional approach. The proposed MDADT switching signal contains both fast switching and slow switching. Finally, the validity of the developed result is certified via a simulation example.

INDEX TERMS Switched system, time delay, mode-dependent average dwell time method, unstable subsystems.

I. INTRODUCTION

Switched systems, which can be used to describe real-world systems, such as chemical procedure control systems, power systems, and navigation systems, have been widely researched in the last few decades [1]–[11]. Many methods and technologies, for example, the common Lyapunov function approach [5], the single Lyapunov function technique [12], and the multiple Lyapunov function scheme [13], have been successively deduced to investigate the stability of switched systems.

Switched systems with unstable subsystems widely exist in many practical system, such as the network control systems, asynchronous switching systems, and so on. The existence of the unstable systems brings some difficulties to analyze the stability of the switched systems. Therefore, it is necessary to discuss in depth the stability problem of the switched systems with unstable subsystems. One of the problems is the design of the switching signal. In [14], for a class of continuous-time switched system which contains stable and unstable subsystems, the stability is investigated. The switching signal belongs to slow switching and is designed via the average dwell time (ADT) method. For the discrete-time switched system with unstable subsystems, some stability conditions are established in [15], where the switched signal is presented by using ADT method. Recently, the mode-dependent average dwell time (MDADT) technique is introduced in [16] and applied to analyze the stability of switched systems. It is presented that MDADT is more flexible than ADT in which each mode has its own ADT in the underlying system. In view of this superiority, the MDADT switching method has been widely applied to investigate many types of problems for the switched systems, see [17]–[20]. Then, for the switched systems with unstable subsystems, in [21], the problem of stability analysis is addressed by designing the switching signal which belongs to slow switching and satisfies MDADT method. In [22], a class of quasi-alternative switching signals is introduced to obtain some improved stability conditions. The slow switching and fast switching are used among stable subsystems and unstable subsystems, respectively.

Note that the studied systems in the above mentioned references do not contain time delay. In many physical processes, the time delay phenomenon unavoidably occurs, and the system performance may be degraded. In order to acquire good performance for the time-delay systems, a number of methods and technologies have been proposed, e.g., the free-weighting-matrix technique [23], [24] and the Wirtinger-based inequality [25]–[27]. Moreover, STDSs have attracted special attention during the past decades. Considerable results have been presented in the literature, see [28]–[31] and the references therein. In the above references, the considered systems assume that all the subsystems are stable. For the

STDSs with unstable subsystems, many scholars have carried out preliminary research, see [32], [33], etc..

As stated previously, MDADT switching method is more applicable. Hence, it is necessary to research the stability issue for STDSs with unstable subsystems via MDADT switching. Under asynchronous switching, the input-to-state stability is investigated for switched delay systems in [34], where the MDADT method is adopted to design the switching signal. The asynchronous L_1 control problem is studied in [35] for a class of switched positive systems with MDADT switching. It is worthwhile to point out that, in the above mentioned references, all the designed ADT (MDADT)-dependent switching signals belong to slow switching, where the total running time of stable subsystems and the total running time of unstable subsystems need to satisfy some conditions. A natural idea is whether this condition can be get rid of or not. If possible, how to remove this restriction? Inspired by the design method of switching signal in [22], we carry out this study.

In this paper, the stability problem is studied for a class of STDSs with unstable subsystems. The MDADT method is adopted to design the switching signal, which is composed of fast MDADT (FMDADT) and slow MDADT (SMDADT). With the help of the constructed multiple Lyapunov-Krasovskii functional, some delay-dependent criteria are presented to obtain the stability of the STDSs. The validity of the developed results is demonstrated via a simulation example finally.

II. PROBLEM FORMULATION

Consider the STDS (Σ)

$$\dot{x}(t) = A_{\sigma(t)}x(t) + B_{\sigma(t)}x(t-d(t)), \tag{1}$$
$$x(t) = \psi(t), t \in [t_0 - h, t_0], \tag{2}$$

where $x(t) \in \mathbb{R}^n$, $\psi(t)$ and t_0 denote the system states, the initial condition and the initial time step, respectively; $\sigma(t)$ is a piecewise constant function and takes its value in a finite set $\mathcal{M} = \{1, 2, \cdots, M\}$, in which M presents the number of the subsystems. The time-varying delay $d(t)$ satisfies

$$0 \le d(t) \le h, \quad \dot{d}(t) \le \tau, \tag{3}$$

where $h > 0$ and $\tau > 0$. Define

$$\chi(t) = \{x(t_0); (i_0, t_0), (i_1, t_1), \cdots, (i_q, t_q), \cdots, \mid i_q \in \mathcal{M}, q \in \mathbb{Z}^+\} \tag{4}$$

as the switching sequence corresponding to $\sigma(t)$, where t_q represents the switching instant. When $t \in [t_q, t_{q+1})$, the i_q-th subsystem works. For any $i_q \in \mathcal{M}$, the i_q-th subsystem (Σ_{i_q}) is described as

$$\dot{x}(t) = A_{i_q}x(t) + B_{i_q}x(t-d(t)), \tag{5}$$
$$x(t) = \psi(t), t \in [t_0 - h, t_0], \tag{6}$$

which maybe stable or unstable. Let $\mathcal{S}$ denote the set of stable subsystems and $\mathcal{U}$ the set of unstable subsystems. A_{i_q} and B_{i_q}, $i_q \in \mathcal{M}$ denote the constant matrices with appropriate dimensions.

In this study, the main concern is to design a MDADT switching signal which is composed of SMDADT switching and FMDADT switching such that the considered system (Σ) is exponentially stable.

III. MAIN RESULTS

For the later development, we first state the following result.

Proposition 1: For given scalars $h > 0$, $0 < \tau < 1$,

$$\begin{cases} \alpha_{i_\ell} > 0, \ \beta_{i_\ell} = 1, \ \mu_{i_\ell} > 1, & i_\ell \in \mathcal{S}, \\ \alpha_{i_\ell} < 0, \ \beta_{i_\ell} = 0, \ 0 < \mu_{i_\ell} < 1, & i_\ell \in \mathcal{U}, \end{cases} \tag{7}$$

if there exist matrices $P_{i_\ell} > 0$, $Q_{i_\ell} > 0$ and $R_{i_\ell} > 0$, $i_\ell \in \mathcal{M}$, $\ell = 0, 1, 2, \cdots$ such that

$$\Psi_{i_\ell} = \begin{bmatrix} \Psi_{i_\ell}(1,1) & \Psi_{i_\ell}(1,2) & \frac{6}{h}e^{-\alpha_{i_\ell}\beta_{i_\ell}h}R_{i_\ell} & hA_{i_\ell}^T R_{i_\ell} \\ * & \Psi_{i_\ell}(2,2) & \frac{6}{h}e^{-\alpha_{i_\ell}\beta_{i_\ell}h}R_{i_\ell} & hB_{i_\ell}^T R_{i_\ell} \\ * & * & -\frac{12}{h}e^{-\alpha_{i_\ell}\beta_{i_\ell}h}R_{i_\ell} & 0 \\ * & * & * & -hR_{i_\ell} \end{bmatrix} < 0, \tag{8}$$

where

$$\Psi_{i_\ell}(1,1) = A_{i_\ell}^T P_{i_\ell} + P_{i_\ell}A_{i_\ell} + \alpha_{i_\ell}P_{i_\ell} + Q_{i_\ell} - \frac{4}{h}e^{-\alpha_{i_\ell}\beta_{i_\ell}h}R_{i_\ell},$$
$$\Psi_{i_\ell}(1,2) = B_{i_\ell}^T P_{i_\ell} - \frac{2}{h}e^{-\alpha_{i_\ell}\beta_{i_\ell}h}R_{i_\ell},$$
$$\Psi_{i_\ell}(2,2) = -\frac{4}{h}e^{-\alpha_{i_\ell}\beta_{i_\ell}h}R_{i_\ell} - (1-\tau)e^{\alpha_{i_\ell}\beta_{i_\ell}h}Q_{i_\ell},$$

then

$$\mathcal{V}_{i_\ell}(t) \le e^{-\alpha_{i_\ell}(t-t_\ell)}\mathcal{V}_{i_\ell}(t_\ell), \tag{9}$$

where

$$\mathcal{V}_{i_\ell}(t) = x^T(t)P_{i_\ell}x(t) + \int_{t-d(t)}^{t} x^T(\vartheta)e^{\alpha_{i_\ell}(\vartheta-t)}Q_{i_\ell}x(\vartheta)d\vartheta + \int_{-h}^{0}\int_{t+\theta}^{t} \dot{x}^T(\vartheta)e^{\alpha_{i_\ell}(\vartheta-t)}R_{i_\ell}\dot{x}(\vartheta)d\vartheta d\theta. \tag{10}$$

Proof: Calculating the derivative of $\mathcal{V}_{i_\ell}(t)$ along the system (Σ_{i_q}), we have

$$\begin{aligned} \dot{\mathcal{V}}_{i_\ell}(t) = & 2x^T(t)P_{i_\ell}\dot{x}(t) + x^T(t)Q_{i_\ell}x(t) \\ & - (1-\dot{d}(t))x^T(t-d(t))e^{-\alpha_{i_\ell}d(t)}Q_{i_\ell}x(t-d(t)) \\ & - \alpha_{i_\ell}\int_{t-d(t)}^{t} x^T(s)e^{\alpha_{i_\ell}(s-t)}Q_{i_\ell}x(s)ds \\ & + h\dot{x}^T(t)R_{i_\ell}\dot{x}(t) - \int_{t-h}^{t} \dot{x}^T(s)e^{\alpha_{i_\ell}(s-t)}R_{i_\ell}\dot{x}(s)ds \\ & - \alpha_{i_\ell}\int_{-h}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)e^{\alpha_{i_\ell}(s-t)}R_{i_\ell}\dot{x}(s)dsd\theta \end{aligned}$$

$$\begin{aligned}
&\leq 2x^T(t)P_{i_\ell}\dot{x}(t) + x^T(t)Q_{i_\ell}x(t) + h\dot{x}^T(t)R_{i_\ell}\dot{x}(t) \\
&\quad - (1-\tau)x^T(t-d(t))e^{\alpha_{i_\ell}\beta_{i_\ell}h}Q_{i_\ell}x(t-d(t)) \\
&\quad - \alpha_{i_\ell}\int_{t-d(t)}^{t} x^T(s)e^{\alpha_{i_\ell}(s-t)}Q_{i_\ell}x(s)ds \\
&\quad - \int_{t-h}^{t} \dot{x}^T(s)e^{-\alpha_{i_\ell}\beta_{i_\ell}h}R_{i_\ell}\dot{x}(s)ds \\
&\quad - \alpha_{i_\ell}\int_{-h}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)e^{\alpha_{i_\ell}(s-t)}R_{i_\ell}\dot{x}(s)dsd\theta, \\
&\leq 2x^T(t)P_{i_\ell}\dot{x}(t) + x^T(t)Q_{i_\ell}x(t) + h\dot{x}^T(t)R_{i_\ell}\dot{x}(t) \\
&\quad - (1-\tau)x^T(t-d(t))e^{-\alpha_{i_\ell}\beta_{i_\ell}h}Q_{i_\ell}x(t-d(t)) \\
&\quad - \int_{t-d(t)}^{t} \dot{x}^T(s)e^{-\alpha_{i_\ell}\beta_{i_\ell}h}R_{i_\ell}\dot{x}(s)ds \\
&\quad + \alpha_{i_\ell}x^T(t)P_{i_\ell}x(t) - \alpha_{i_\ell}\mathcal{V}_{i_\ell}(t). \qquad (11)
\end{aligned}$$

Using Wirtinger-based inequality yields

$$\begin{aligned}
&-\int_{t-d(t)}^{t} e^{-\alpha_{i_\ell}\beta_{i_\ell}h}\dot{x}^T(s)R_{i_\ell}\dot{x}(s)ds \\
&\leq -\frac{1}{h}\eta(t)\begin{bmatrix} e^{-\alpha_{i_\ell}\beta_{i_\ell}h}R_{i_\ell} & 0 \\ 0 & 3e^{-\alpha_{i_\ell}\beta_{i_\ell}h}R_{i_\ell}\end{bmatrix}\eta^T(t) \\
&= \xi^T(t) \\
&\quad \times \frac{e^{-\alpha_{i_\ell}\beta_{i_\ell}h}}{h}\begin{bmatrix} -4R_{i_\ell} & -2R_{i_\ell} & 6R_{i_\ell} \\ * & -4R_{i_\ell} & 6R_{i_\ell} \\ * & * & -12R_{i_\ell}\end{bmatrix}\xi(t), \qquad (12)
\end{aligned}$$

where

$$\begin{aligned}
\eta(t) &= \mathrm{col}\Big\{x(t)-x(t-d(t)), \\
&\qquad x(t)+x(t-d(t)) - \frac{2}{d(t)}\int_{t-d(t)}^{t} x(s)ds\Big\}, \\
\xi(t) &= \mathrm{col}\Big\{x(t),\ x(t-d(t)), \frac{1}{d(t)}\int_{t-d(t)}^{t} x(s)ds\Big\}.
\end{aligned}$$

From (11) and (12), one gets

$$\begin{aligned}
&\dot{\mathcal{V}}_{i_\ell}(t) \\
&\leq 2x^T(t)P_{i_\ell}(A_{i_\ell}x(t) + B_{i_\ell}x(t-d(t))) + x^T(t)Q_{i_\ell}x(t) \\
&\quad - (1-\tau)x^T(t-d(t))e^{\alpha_{i_\ell}\beta_{i_\ell}h}Q_{i_\ell}x(t-d(t)) \\
&\quad + h(A_{i_\ell}x(t) + B_{i_\ell}x(t-d(t)))^T R_{i_\ell} \\
&\quad \times (A_{i_\ell}x(t) + B_{i_\ell}x(t-d(t))) \\
&\quad + \xi^T(t)\frac{e^{-\alpha_{i_\ell}\beta_{i_\ell}h}}{h}\begin{bmatrix} -4R_{i_\ell} & -2R_{i_\ell} & 6R_{i_\ell} \\ * & -4R_{i_\ell} & 6R_{i_\ell} \\ * & * & -12R_{i_\ell}\end{bmatrix} \\
&\quad \times \xi(t) + \alpha_{i_\ell}x^T(t)P_{i_\ell}x(t) - \alpha_{i_\ell}\mathcal{V}_{i_\ell}(t) \\
&= \xi^T(t)\begin{bmatrix} \Psi_{i_\ell}(1,1) & \Psi_{i_\ell}(1,2) & \frac{6}{h}e^{-\alpha_{i_\ell}\beta_{i_\ell}h}R_{i_\ell} \\ * & \Psi_{i_\ell}(2,2) & \frac{6}{h}e^{-\alpha_{i_\ell}\beta_{i_\ell}h}R_{i_\ell} \\ * & * & -\frac{12}{h}e^{-\alpha_{i_\ell}\beta_{i_\ell}h}R_{i_\ell}\end{bmatrix}\xi(t) \\
&\quad + h\xi^T(t)\begin{bmatrix} A_{i_\ell}^T \\ B_{i_\ell}^T \\ 0\end{bmatrix} R_{i_\ell}\begin{bmatrix} A_{i_\ell} & B_{i_\ell} & 0\end{bmatrix}\xi(t) - \alpha_{i_\ell}\mathcal{V}_{i_\ell}(t). \\
&\qquad (13)
\end{aligned}$$

Combining the Schur complement lemma with (8) gives rise to

$$\begin{aligned}
&\begin{bmatrix} \Psi_{i_\ell}(1,1) & \Psi_{i_\ell}(1,2) & \frac{6}{h}e^{-\alpha_{i_\ell}\beta_{i_\ell}h}R_{i_\ell} \\ * & \Psi_{i_\ell}(2,2) & \frac{6}{h}e^{-\alpha_{i_\ell}\beta_{i_\ell}h}R_{i_\ell} \\ * & * & -\frac{12}{h}e^{-\alpha_{i_\ell}\beta_{i_\ell}h}R_{i_\ell}\end{bmatrix} \\
&\quad + h\begin{bmatrix} A_{i_\ell}^T \\ B_{i_\ell}^T \\ 0\end{bmatrix} R_{i_\ell}\begin{bmatrix} A_{i_\ell} & B_{i_\ell} & 0\end{bmatrix} < 0, \qquad (14)
\end{aligned}$$

which together with (13) leads to (9). □

Remark 1: In this study, the Lyapunov-Krasovskii functional (10) is given, in which the parameter α_{i_ℓ} is provided to distinguish the differences between stable and unstable subsystems (5). In addition, in order to obtain a unitary expression of (8) and the derivative of $\mathcal{V}_{i_\ell}(t)$, the parameter β_{i_ℓ} in (7) is introduced.

Remark 2: The inequality (9) reveals the relationship between $\mathcal{V}_{i_\ell}(t)$ and $\mathcal{V}_{i_\ell}(t_\ell)$ for subsystem i_ℓ, from which we obtain that when $i_\ell \in \mathcal{S}$, the energy of subsystem i_ℓ is descendant; while when $i_\ell \in \mathcal{U}$, the energy of subsystem i_ℓ is incremental. In the following, the system (Σ) composed of stable subsystems and unstable subsystems shall be considered, and the result of the exponential stability of the system (Σ) is given via the MDADT switching signal.

Theorem 1: For given scalars $h > 0$, $0 < \tau < 1$, and α_{i_ℓ}, β_{i_ℓ}, μ_{i_ℓ} satisfying (7), if there exist matrices $P_{i_\ell} > 0$, $Q_{i_\ell} > 0$ and $R_{i_\ell} > 0$, $i_\ell, i_p \in \mathcal{M}$, $\ell, p = 0, 1, 2, \cdots$ such that (8) and the following inequalities

$$P_{i_\ell} \leq \mu_{i_\ell}P_{i_p},\ \ Q_{i_\ell} \leq \mu_{i_\ell}Q_{i_p}, R_{i_\ell} \leq \mu_{i_\ell}R_{i_p}, i_\ell \in \mathcal{S}, i_p \in \mathcal{M}, \qquad (15)$$

$$P_{i_\ell} \leq \mu_{i_\ell}P_{i_p},\ \ Q_{i_\ell} \leq \mu_{i_\ell}Q_{i_p}, R_{i_\ell} \leq \mu_{i_\ell}R_{i_p}, i_\ell \in \mathcal{U}, i_p \in \mathcal{S} \qquad (16)$$

hold, then the system (Σ) is exponentially stable under the MDADT switching signal satisfying

$$\begin{cases} \tau_{i_\ell}^a \geq \tau_{i_\ell}^{a*} = \dfrac{\ln \mu_{i_\ell}}{\alpha_{i_\ell}}, & i_\ell \in \mathcal{S}, \\ \tau_{i_\ell}^a \leq \tau_{i_\ell}^{a*} = \dfrac{\ln \mu_{i_\ell}}{\alpha_{i_\ell}}, & i_\ell \in \mathcal{U}. \end{cases} \qquad (17)$$

Proof: From switching sequence (4), when $t \in [t_q, t_{q+1})$, $\sigma(t) = i_q$. For the i_q subsystem, choose the Lyapunov-Krasovskii functional as

$$\begin{aligned}
\mathcal{V}_{i_q}(t) &= x^T(t)P_{i_q}x(t) \\
&\quad + \int_{t-d(t)}^{t} x^T(\vartheta)e^{\alpha_{i_q}(\vartheta-t)}Q_{i_q}x(\vartheta)d\vartheta \\
&\quad + \int_{-h}^{0}\int_{t+\theta}^{t} \dot{x}^T(\vartheta)e^{\alpha_{i_q}(\vartheta-t)}R_{i_q}\dot{x}(\vartheta)d\vartheta d\theta. \qquad (18)
\end{aligned}$$

Combining (9) with (18), we have

$$\begin{aligned}\mathcal{V}_{i_q}(t) &\leq e^{-\alpha_{i_q}(t-t_q)}V_{i_q}(t_q)\\&= e^{-\alpha_{i_q}(t-t_q)}\Big(x^T(t_q)P_{i_q}x(t_q)\\&\quad+\int_{t_q-d(t_q)}^{t_q}x^T(\vartheta)e^{\alpha_{i_q}(\vartheta-t_q)}Q_{i_q}x(\vartheta)d\vartheta\\&\quad+\int_{-h}^{0}\int_{t_q+\theta}^{t_q}\dot{x}^T(\vartheta)e^{\alpha_{i_q}(\vartheta-t_q)}R_{i_q}\dot{x}(\vartheta)d\vartheta d\theta\Big)\\&= e^{-\alpha_{i_q}(t-t_q)}\Big(x^T(t_q)P_{i_q}x(t_q)\\&\quad+\int_{t_q-d(t_q)}^{t_q}x^T(\vartheta)e^{\alpha_{i_{q-1}}(\vartheta-t_q)}\\&\quad\times e^{(\alpha_{i_q}-\alpha_{i_{q-1}})(\vartheta-t_q)}Q_{i_q}x(\vartheta)d\vartheta\\&\quad+\int_{-h}^{0}\int_{t_q+\theta}^{t_q}\dot{x}^T(\vartheta)e^{\alpha_{i_{q-1}}(\vartheta-t_q)}\\&\quad\times e^{(\alpha_{i_q}-\alpha_{i_{q-1}})(\vartheta-t_q)}R_{i_q}\dot{x}(\vartheta)d\vartheta d\theta\Big).\end{aligned}\tag{19}$$

At the switching instant t_q, the following cases are discussed.

Case I: $i_{q-1}\in\mathcal{U}$ and $i_q\in\mathcal{S}$

In this case, $\alpha_{i_q}-\alpha_{i_{q-1}}>0$. By (7), we can obtain $e^{(\alpha_{i_q}-\alpha_{i_{q-1}})(s-t_q)}<1$, where $s-t_q\in[-h,0]$. Combining (15) with (19) implies

$$\mathcal{V}_{i_q}(t)\leq e^{-\alpha_{i_q}(t-t_q)}\mu_{i_q}\mathcal{V}_{i_{q-1}}(t_q).\tag{20}$$

Case II: $i_{q-1}\in\mathcal{S}$ and $i_q\in\mathcal{U}$

In this case, $\alpha_{i_q}-\alpha_{i_{q-1}}<0$. From (19), we have

$$\begin{aligned}&\mathcal{V}_{i_q}(t)\\&\leq e^{-\alpha_{i_q}(t-t_q)}\Big(x^T(t_q)P_{i_q}x(t_q)\\&\quad+\int_{t_q-d(t_q)}^{t_q}x^T(\vartheta)e^{\alpha_{i_{q-1}}(\vartheta-t_q)}e^{(\alpha_{i_{q-1}}-\alpha_{i_q})h}Q_{i_q}x(\vartheta)d\vartheta\\&\quad+\int_{-h}^{0}\int_{t_q+\theta}^{t_q}\dot{x}^T(\vartheta)e^{\alpha_{i_{q-1}}(\vartheta-t_q)}\widetilde{e}R_{i_q}\dot{x}(\vartheta)d\vartheta d\theta\Big)\\&= e^{-\alpha_{i_q}(t-t_q)}e^{(\alpha_{i_{q-1}}-\alpha_{i_q})h}\Big(x^T(t_q)e^{(\alpha_{i_q}-\alpha_{i_{q-1}})h}P_{i_q}x(t_q)\\&\quad+\int_{t_q-d(t_q)}^{t_q}x^T(\vartheta)e^{\alpha_{i_{q-1}}(\vartheta-t_q)}Q_{i_q}x(\vartheta)d\vartheta\\&\quad+\int_{-h}^{0}\int_{t_q+\theta}^{t_q}\dot{x}^T(\vartheta)e^{\alpha_{i_{q-1}}(\vartheta-t_q)}R_{i_q}\dot{x}(\vartheta)d\vartheta d\theta\Big)\\&\leq e^{-\alpha_{i_q}(t-t_q)}e^{(\alpha_{i_{q-1}}-\alpha_{i_q})h}\Big(x^T(t_q)P_{i_q}x(t_q)\\&\quad+\int_{t_q-d(t_q)}^{t_q}x^T(\vartheta)e^{\alpha_{i_{q-1}}(\vartheta-t_q)}Q_{i_q}x(\vartheta)d\vartheta\\&\quad+\int_{-h}^{0}\int_{t_q+\theta}^{t_q}\dot{x}^T(\vartheta)e^{\alpha_{i_{q-1}}(\vartheta-t_q)}R_{i_q}\dot{x}(\vartheta)d\vartheta d\theta\Big)\\&\leq e^{-\alpha_{i_q}(t-t_q)}e^{(\alpha_{i_{q-1}}-\alpha_{i_q})h}\mu_{i_q}\mathcal{V}_{i_{q-1}}(t_q),\end{aligned}\tag{21}$$

where $\widetilde{e}=e^{(\alpha_{i_{q-1}}-\alpha_{i_q})h}$.

Case III: $i_{q-1}\in\mathcal{S}$ and $i_q\in\mathcal{S}$

Under this circumstance, $\alpha_{i_q}-\alpha_{i_{q-1}}\geq 0$ and $\alpha_{i_q}-\alpha_{i_{q-1}}<0$ are considered, respectively.

When $\alpha_{i_q}-\alpha_{i_{q-1}}\geq 0$, $e^{(\alpha_{i_q}-\alpha_{i_{q-1}})(s-t_q)}\leq 1$. The above inequality implies that

$$\begin{aligned}\mathcal{V}_{i_q}(t)&\leq e^{-\alpha_{i_q}(t-t_q)}\Big(x^T(t_q)P_{i_q}x(t_q)\\&\quad+\int_{t_q-d(t_q)}^{t_q}x^T(\vartheta)e^{\alpha_{i_{q-1}}(\vartheta-t_q)}Q_{i_q}x(\vartheta)d\vartheta\\&\quad+\int_{-h}^{0}\int_{t_q+\theta}^{t_q}\dot{x}^T(\vartheta)e^{\alpha_{i_{q-1}}(\vartheta-t_q)}R_{i_q}\dot{x}(\vartheta)d\vartheta d\theta\Big)\\&\leq e^{-\alpha_{i_q}(t-t_q)}\mu_{i_q}\mathcal{V}_{i_{q-1}}(t_q).\end{aligned}\tag{22}$$

When $\alpha_{i_q}-\alpha_{i_{q-1}}<0$, the following inequality is true

$$\mathcal{V}_{i_q}(t)\leq e^{-\alpha_{i_q}(t-t_q)}\times e^{(\alpha_{i_{q-1}}-\alpha_{i_q})h}\mu_{i_q}\mathcal{V}_{i_{q-1}}(t_q).\tag{23}$$

Define

$$\nu_{i_q}=\begin{cases}\mu_{i_q}, & \alpha_{i_q}-\alpha_{i_{q-1}}\geq 0,\\ \bar{\mu}_{i_q}, & \alpha_{i_q}-\alpha_{i_{q-1}}<0,\end{cases}\tag{24}$$

where $\bar{\mu}_{i_q}=e^{(\alpha_{i_{q-1}}-\alpha_{i_q})h}\mu_{i_q}$. From (19)-(23), one gets

$$\mathcal{V}_{i_q}(t)\leq e^{-\alpha_{i_q}(t-t_q)}\nu_{i_q}\mathcal{V}_{i_{q-1}}(t_q),\tag{25}$$

which further implies that

$$\begin{aligned}\mathcal{V}_{i_q}(t)&\leq e^{-\alpha_{i_q}(t-t_q)}\nu_{i_q}\mathcal{V}_{i_{q-1}}(t_q)\\&\leq e^{-\alpha_{i_q}(t-t_q)}\nu_{i_q}e^{-\alpha_{i_{q-1}}(t_q-t_{q-1})}\times\mathcal{V}_{i_{q-1}}(t_{q-1})\\&\leq e^{-\alpha_{i_q}(t-t_q)}\nu_{i_q}e^{-\alpha_{i_{q-1}}(t_q-t_{q-1})}\times\nu_{i_{q-1}}\mathcal{V}_{i_{q-2}}(t_{q-1})\\&\leq\cdots\\&\leq e^{-\alpha_{i_q}(t-t_q)}\nu_{i_q}e^{-\alpha_{i_{q-1}}(t_q-t_{q-1})}\\&\quad\times\nu_{i_{q-1}}\cdots\nu_{i_1}e^{-\alpha_{i_0}(t_1-t_0)}\mathcal{V}_{i_0}(t_0)\\&= e^{-\alpha_{i_q}(t-t_q)}\prod_{s=0}^{q-1}\nu_{i_{s+1}}e^{-\alpha_{i_s}(t_{s+1}-t_s)}\times\mathcal{V}_{i_0}(t_0).\end{aligned}\tag{26}$$

Let $H_1\triangleq\alpha_{i_{s+1}}-\alpha_{i_s}\geq 0$, $H_2\triangleq\alpha_{i_{s+1}}-\alpha_{i_s}<0$, and $\widetilde{N}=N^{\sigma}_{i_{s+1}}(t_0,t)$. Note that

$$\begin{aligned}&\prod_{s=0}^{q-1}\nu_{i_{s+1}}\\&=\exp\Big(\sum_{s=0}^{q-1}\ln\nu_{i_{s+1}}\Big)\\&=\exp\Big(\sum_{H_1}\ln\nu_j+\sum_{H_2}\ln\nu_j\Big)\\&=\exp\Big(\sum_{H_1}\Big(\sum_{\substack{i_{s+1}\in\mathcal{S}\\ i_s\in\mathcal{U}}}\ln\nu_{i_{s+1}}+\sum_{\substack{i_s,\,i_{s+1}\in\mathcal{S}\\ i_{s+1}\neq i_s}}\ln\nu_{i_{s+1}}\Big)\end{aligned}$$

$$+\sum_{H_2}\Big(\sum_{\substack{i_{s+1}\in\mathcal{U}\\ i_s\in\mathcal{S}}}\ln\nu_{i_{s+1}}+\sum_{\substack{i_s,i_{s+1}\in\mathcal{S}\\ i_s\neq i_{s+1}}}\ln\nu_{i_{s+1}}\Big)\Big)$$

$$=\exp\Big(\sum_{H_1}\Big(\sum_{\substack{i_{s+1}\in\mathcal{S}\\ i_s\in\mathcal{U}}}\ln\mu_{i_{s+1}}+\sum_{\substack{i_s,i_{s+1}\in\mathcal{S}\\ i_s\neq i_{s+1}}}\ln\mu_{i_{s+1}}\Big)$$

$$+\sum_{H_2}\Big(\sum_{i_{s+1}\in\mathcal{U},i_s\in\mathcal{S}}\ln(e^{(\alpha_{i_s}-\alpha_{i_{s+1}})h}\mu_{i_{s+1}})$$

$$+\sum_{\substack{i_s,i_{s+1}\in\mathcal{S}\\ i_s\neq i_{s+1}}}\ln(e^{(\alpha_{i_s}-\alpha_{i_{s+1}})h}\mu_{i_{s+1}})\Big)\Big)$$

$$\leq\exp\Big(\sum_{\substack{i_{s+1}\in\mathcal{S}\\ i_s\in\mathcal{U}\\ H_1}}\widetilde{N}\ln\mu_{i_{s+1}}+\sum_{\substack{i_s,i_{s+1}\in\mathcal{S}\\ i_s\neq i_{s+1}\\ H_1}}\widetilde{N}\ln\mu_{i_{s+1}}$$

$$+\sum_{\substack{i_{s+1}\in\mathcal{U}\\ i_s\in\mathcal{S}\\ H_2}}\widetilde{N}\ln(e^{(\alpha_{i_s}-\alpha_{i_{s+1}})h}\mu_{i_{s+1}})$$

$$+\sum_{\substack{i_s,i_{s+1}\in\mathcal{S}\\ i_s\neq i_{s+1}\\ H_2}}\widetilde{N}\ln(e^{(\alpha_{i_s}-\alpha_{i_{s+1}})h}\mu_{i_{s+1}})\Big)$$

$$=\exp\Big(\sum_{\substack{i_{s+1}\in\mathcal{S}\\ i_s\in\mathcal{U}\\ H_1}}\widetilde{N}\ln\mu_{i_{s+1}}+\sum_{\substack{i_s,i_{s+1}\in\mathcal{S}\\ i_s\neq i_{s+1}\\ H_1}}\widetilde{N}\ln\mu_{i_{s+1}}$$

$$+\sum_{\substack{i_{s+1}\in\mathcal{U}\\ i_s\in\mathcal{S}\\ H_2}}\widetilde{N}(\alpha_{i_s}-\alpha_{i_{s+1}})h+\sum_{\substack{i_{s+1}\in\mathcal{U}\\ i_s\in\mathcal{S}\\ H_2}}\widetilde{N}\ln\mu_{i_{s+1}}$$

$$+\sum_{\substack{i_s\in\mathcal{S}\\ i_{s+1}\in\mathcal{S}\\ i_s\neq i_{s+1}\\ H_2}}\widetilde{N}(\alpha_{i_s}-\alpha_{i_{s+1}})h+\sum_{\substack{i_s\in\mathcal{S}\\ i_{s+1}\in\mathcal{S}\\ i_s\neq i_{s+1}\\ H_2}}\widetilde{N}\ln\mu_{i_{s+1}}\Big)$$

$$=\exp\Big(\sum_{\substack{i_{s+1}\in\mathcal{U}\\ i_s\in\mathcal{S}\\ H_2}}\widetilde{N}(\alpha_{i_s}-\alpha_{i_{s+1}})h$$

$$+\sum_{\substack{i_s,i_{s+1}\in\mathcal{S}\\ i_s\neq i_{s+1}\\ H_2}}\widetilde{N}(\alpha_{i_s}-\alpha_{i_{s+1}})h\Big)$$

$$\times\exp\Big(\sum_{\substack{i_{s+1}\in\mathcal{S}\\ i_s\in\mathcal{U}\\ H_1}}\widetilde{N}\ln\mu_{i_{s+1}}+\sum_{\substack{i_s,i_{s+1}\in\mathcal{S}\\ i_s\neq i_{s+1}\\ H_1}}\widetilde{N}\ln\mu_{i_{s+1}}$$

$$+\sum_{\substack{i_{s+1}\in\mathcal{U}\\ i_s\in\mathcal{S}\\ H_2}}\widetilde{N}\ln\mu_{i_{s+1}}+\sum_{\substack{i_s,i_{s+1}\in\mathcal{S}\\ i_s\neq i_{s+1}\\ H_2}}\widetilde{N}\ln\mu_{i_{s+1}}\Big)$$

$$=\exp\Big(\sum_{\substack{i_{s+1}\in\mathcal{U}\\ i_s\in\mathcal{S}}}\widetilde{N}(\alpha_{i_s}-\alpha_{i_{s+1}})h$$

$$+\sum_{\substack{i_s,i_{s+1}\in\mathcal{S}\\ i_s\neq i_{s+1}\\ H_2}}\widetilde{N}(\alpha_{i_s}-\alpha_{i_{s+1}})h\Big)$$

$$\times\exp\Big(\sum_{\substack{i_{s+1}\in\mathcal{S}\\ i_s\in\mathcal{U}}}\widetilde{N}\ln\mu_{i_{s+1}}+\sum_{\substack{i_s,i_{s+1}\in\mathcal{S}\\ i_s\neq i_{s+1}}}\widetilde{N}\ln\mu_{i_{s+1}}$$

$$+\sum_{\substack{i_{s+1}\in\mathcal{U}\\ i_s\in\mathcal{S}}}\widetilde{N}\ln\mu_{i_{s+1}}\Big)$$

$$=\exp\Big(\sum_{\substack{i_{s+1}\in\mathcal{U}\\ i_s\in\mathcal{S}}}\widetilde{N}(\alpha_{i_s}-\alpha_{i_{s+1}})h$$

$$+\sum_{\substack{i_s,i_{s+1}\in\mathcal{S}\\ i_s\neq i_{s+1}\\ H_2}}\widetilde{N}(\alpha_{i_s}-\alpha_{i_{s+1}})h\Big)$$

$$\times\exp\Big(\sum_{i_{s+1}\in\mathcal{S}}\widetilde{N}\ln\mu_{i_{s+1}}+\sum_{i_{s+1}\in\mathcal{U}}\widetilde{N}\ln\mu_{i_{s+1}}\Big)$$

$$\leq\exp\Big(\sum_{i_{s+1}\in\mathcal{U},i_s\in\mathcal{S}}\widetilde{N}(\alpha_{i_s}-\alpha_{i_{s+1}})h$$

$$+\sum_{\substack{i_s,i_{s+1}\in\mathcal{S}\\ i_s\neq i_{s+1}\\ H_2}}\widetilde{N}(\alpha_{i_s}-\alpha_{i_{s+1}})h\Big)$$

$$\times\exp\Big(\sum_{i_{s+1}\in\mathcal{S}}(N^0_{i_{s+1}}+\frac{T_{i_{s+1}}(t_0,t)}{\tau^a_{i_{s+1}}})\ln\mu_{i_{s+1}}$$

$$+\sum_{i_{s+1}\in\mathcal{U}}(N^0_{i_{s+1}}+\frac{T_{i_{s+1}}(t_0,t)}{\tau^a_{i_{s+1}}})\ln\mu_{i_{s+1}}\Big)$$

$$=\exp\Big(\sum_{\substack{i_{s+1}\in\mathcal{U}\\ i_s\in\mathcal{S}}}\widetilde{N}(\alpha_{i_s}-\alpha_{i_{s+1}})h$$

$$+\sum_{\substack{i_s,i_{s+1}\in\mathcal{S}\\ i_s\neq i_{s+1}\\ H_2}}\widetilde{N}(\alpha_{i_s}-\alpha_{i_{s+1}})h\Big)$$

$$\times\exp\Big(\sum_{i_{s+1}\in\mathcal{S}}N^0_{i_{s+1}}\ln\mu_{i_{s+1}}+\sum_{i_{s+1}\in\mathcal{U}}N^0_{i_{s+1}}\ln\mu_{i_{s+1}}\Big)$$

$$\times \exp\Big(\sum_{i_{s+1}\in\mathcal{S}} \frac{T_{i_{s+1}}(t_0,t)}{\tau^a_{i_{s+1}}} \ln \mu_{i_{s+1}} + \sum_{i_{s+1}\in\mathcal{U}} \frac{T_{i_{s+1}}(t_0,t)}{\tau^a_{i_{s+1}}} \ln \mu_{i_{s+1}}\Big), \tag{27}$$

and

$$e^{-\alpha_{i_q}(t-t_q)} \prod_{s=0}^{q-1} e^{-\alpha_{i_s}(t_{s+1}-t_s)} = \exp\Big(\sum_{i_s\in\mathcal{S}} -\alpha_{i_s} T_{i_s}(t_0,t) + \sum_{i_s\in\mathcal{U}} -\alpha_{i_s} T_{i_s}(t_0,t)\Big). \tag{28}$$

Consequently, because of the inequalities (26)-(28) and $\mathcal{V}_{\sigma(t)}(t) = \mathcal{V}_{i_q}(t)$ when $t \in [t_q, t_{q+1})$, we obtain

$$\begin{aligned}
\mathcal{V}_{\sigma(t)}(t) \le\ & \exp\Big(\sum_{i_\ell\in\mathcal{U}, i_p\in\mathcal{S}} N^\sigma_{i_\ell}(t_0,t)(\alpha_{i_p}-\alpha_{i_\ell})h + \sum_{\substack{i_p, i_\ell \in \mathcal{S}\\ i_p \neq i_\ell\\ \alpha_{i_\ell}-\alpha_{i_p}<0}} N^\sigma_{i_\ell}(t_0,t)(\alpha_{i_p}-\alpha_{i_\ell})h\Big)\\
&\times \exp\Big(\sum_{i_\ell\in\mathcal{S}} N^0_{i_\ell}\ln\mu_{i_\ell} + \sum_{i_\ell\in\mathcal{U}} N^0_{i_\ell}\ln\mu_{i_\ell}\Big)\\
&\times \exp\Big(\sum_{i_\ell\in\mathcal{S}} \frac{T_{i_\ell}(t_0,t)}{\tau^a_{i_\ell}}\ln\mu_{i_\ell} + \sum_{i_\ell\in\mathcal{U}} \frac{T_{i_\ell}(t_0,t)}{\tau^a_{i_\ell}}\ln\mu_{i_\ell}\Big)\\
&\times \exp\Big(\sum_{i_\ell\in\mathcal{S}} -\alpha_{i_\ell}T_{i_\ell}(t_0,t) + \sum_{i_\ell\in\mathcal{U}} -\alpha_{i_\ell}T_{i_\ell}(t_0,t)\Big)\\
&\times V_{\sigma(t_0)}(t_0)\\
=\ & \exp\Big(\sum_{i_\ell\in\mathcal{U}, i\in\mathcal{S}} N^\sigma_{i_\ell}(t_0,t)(\alpha_i-\alpha_{i_\ell})h + \sum_{\substack{i_p, i_\ell \in \mathcal{S}\\ i_p \neq i_\ell\\ \alpha_{i_\ell}-\alpha_{i_p}<0}} N^\sigma_{i_\ell}(t_0,t)(\alpha_i-\alpha_{i_\ell})h\Big)\\
&\times \exp\Big(\sum_{i_\ell\in\mathcal{S}} N^0_{i_\ell}\ln\mu_{i_\ell} + \sum_{i_\ell\in\mathcal{U}} N^0_{i_\ell}\ln\mu_{i_\ell}\Big)\\
&\times \exp\Big(\sum_{i_\ell\in\mathcal{S}} \Big(\frac{\ln\mu_{i_\ell}}{\tau^a_{i_\ell}} - \alpha_{i_\ell}\Big)T_{i_\ell}(t_0,t) + \sum_{i_\ell\in\mathcal{U}} \Big(\frac{\ln\mu_{i_\ell}}{\tau^a_{i_\ell}} - \alpha_{i_\ell}\Big)T_{i_\ell}(t_0,t)\Big)\mathcal{V}_{\sigma(t_0)}(t_0).
\end{aligned} \tag{29}$$

According to (17), we have $V_{\sigma(t)}(t) \to 0$ as $t \to \infty$. By the definition in [36], we conclude that the system (Σ) is exponentially stable. □

Remark 3: From Theorem 1, where the MDADT switching signal (17) is designed, we observe that the SMDADT switching is applied to a stable subsystem whereas the FMDADT switching is applied to an unstable subsystem. However, this does not mean that the FMDADT is certainly smaller than the SMDADT. This argument will be explained via a numerical example.

The following result is obtained when all the subsystems are stable.

Corollary 1: For given scalars $h > 0$, $0 < \tau < 1$, and $\alpha_{i_\ell} > 0$, $\mu_{i_\ell} > 1$, $i_\ell \in \mathcal{M}$, if there exist matrices $P_{i_\ell} > 0$, $Q_{i_\ell} > 0$ and $R_{i_\ell} > 0$, $i_\ell \in \mathcal{M}$, $\ell, p = 0, 1, 2, \cdots$ such that the inequality (8) and the following inequalities

$$P_{i_\ell} \le \mu_{i_\ell} P_{i_p}, \quad Q_{i_\ell} \le \mu_{i_\ell} Q_{i_p}, \quad R_{i_\ell} \le \mu_{i_\ell} R_{i_p}, \tag{30}$$

hold, then the switched delay system (Σ) is exponentially stable under the MDADT switching signal satisfying

$$\tau^a_{i_\ell} \ge \tau^{a*}_{i_\ell} = \frac{\ln\mu_{i_\ell}}{\alpha_{i_\ell}}. \tag{31}$$

IV. A NUMERICAL EXAMPLE

In order to present the effectiveness of the developed result, a simulation example is shown in this section.

The system parameters are listed as follows:

$$A_1 = \begin{bmatrix} -2.5 & -0.5 \\ 0.2 & -1.5 \end{bmatrix}, \quad B_1 = \begin{bmatrix} -0.1 & 0.1 \\ -0.1 & 0.2 \end{bmatrix},$$
$$A_2 = \begin{bmatrix} -1.8 & 0.1 \\ 0.1 & -1.5 \end{bmatrix}, \quad B_2 = \begin{bmatrix} -0.2 & -0.1 \\ -0.4 & -0.1 \end{bmatrix},$$
$$A_3 = \begin{bmatrix} 0.5 & 0.1 \\ -0.3 & 0.5 \end{bmatrix}, \quad B_3 = \begin{bmatrix} 0.1 & -0.3 \\ -0.2 & 0.1 \end{bmatrix}.$$

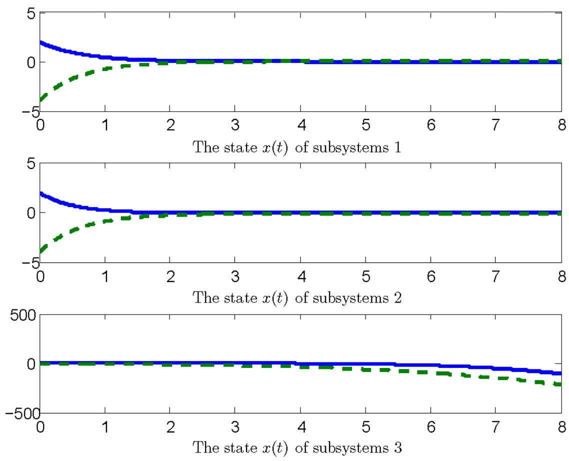

FIGURE 1. The state trajectories of subsystems.

The state trajectories of the subsystems are presented in FIGURE 1. From FIGURE 1, the subsystems 1 and 2 are stable, and the subsystem 3 is unstable. The state trajectories of subsystems. Subsequently, by Theorem 1, the parameters $\mu_{i_\ell}, \alpha_{i_\ell}, \beta_{i_\ell}, i_\ell = 1, 2, 3, \ell = 0, 1, 2, \cdots$ and the corresponding MDADTs are shown in TABLE 1.

TABLE 1 shows that the dwell time of the 2nd subsystem is shorter than the one of the 3rd subsystem, which signifies that

TABLE 1. Parameters μ_{i_ℓ}, α_{i_ℓ}, β_{i_ℓ} and MDADTs $\tau_{i_\ell}^{a*}$.

i_ℓ	μ_{i_ℓ}	α_{i_ℓ}	β_{i_ℓ}	$\tau_{i_\ell}^{a*}$
1	3.8	2	1	0.6675
2	1.7	2.6	1	0.2041
3	0.6	-2	0	0.2554

TABLE 2. Parameters τ_2^{a*} and h.

τ_2^{a*}	0.1895	0.1965	0.2123	0.2211
h	14.2	14.5	14.9	16.2

TABLE 3. Parameters τ_3^{a*} and h.

τ_3^{a*}	0.2433	0.2322	0.2221	0.2128
h	16	16.5	16.7	17.1

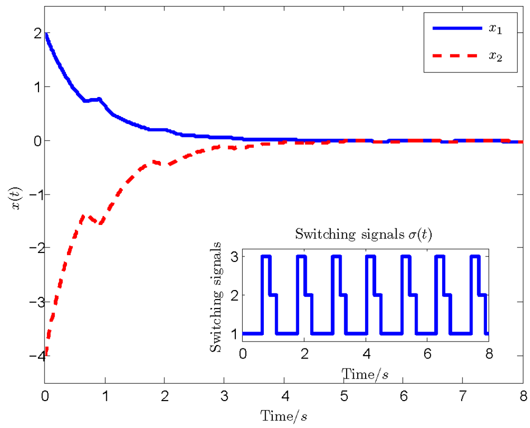

FIGURE 2. Response curves of $x(t)$ and $\sigma(t)$.

the resident time of the unstable subsystem is not necessarily smaller than that of the stable subsystem.

In the sequel, the relations between the dwell time $\tau_{i_\ell}^{a*}$ and the maximum upper boundedness h of the time delay $d(t)$ are analyzed by virtue of TABLE 2 and TABLE 3. TABLE 2 presents relations between the parameter h and the dwell time τ_2^{a*} of the stable subsystem 2, which shows that the longer the dwell time of the stable subsystems, the bigger the maximum upper boundedness h. In the same way, from TABLE 3, we can obtain that a shorter dwell time of the unstable subsystem results in a bigger upper boundedness of time delay.

For $h = 2$, $d(t) = 0.2\sin(t)$, and $\tau_1^a = 0.67 > \tau_1^{a*}$, $\tau_2^a = 0.21 > \tau_2^{a*}$, $\tau_3^a = 0.25 < \tau_3^{a*}$, on the basis of Theorem 1, the state response curves of system (Σ) under the switching sequence $1 \to 3 \to 2 \to 1 \to 3 \to 2 \cdots$ and the initial value $x(0) = \begin{bmatrix} 2 & -4 \end{bmatrix}^T$ are depicted in FIGURE 2. Furthermore, the response of the switching signal is illustrated in FIGURE 2 , from which we can see that the state of the system (Σ) converges to zero with the developed switching signal. Response curves of $x(t)$ and $\sigma(t)$.

V. CONCLUSION

The stability problem has been studied for a class of STDSs with unstable subsystems. The switching signal has been designed via MDADT method, which contains fast switching with unstable subsystems and slow switching with stable subsystems. Some delay-dependent stability results have been developed. A simulation example has been presented to demonstrate the validity of the obtained results. Considering the influence of the constructed Lyapunov-Krasovskii functional and the methods adopted to deal with time-delay on the stable conditions, in the future research, we will provide some new methods and techniques to deal with the delay to obtain better results.

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