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An Iterative Reputation Ranking Method via the Beta Probability Distribution

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ABSTRACT Ranking user reputation and object quality has drawn increasing attention for online rating systems. By introducing an iterative reputation-allocation process, in this paper, we present an iterative reputation ranking algorithm in terms of the beta probability distribution (IBeta), where the user reputation is calculated as the probability that the user will give fair ratings to objects and the high reputation users' ratings have larger weights in dominating the corresponding quantity of fair/unfair ratings. User reputation is reallocated based on their ratings and the previous reputations. The user reputation and users' quantities of fair/unfair ratings are iteratively updated until they become stable. The experimental results for the synthetic networks show that both the AUC values and Kendall's tau τ of the IBeta algorithm are larger than those generated by the RBPD method with different fractions of random ratings. Moreover, the results for the empirical networks indicate that the presented algorithm is more accurate and robust than the RBPD method when the rating systems are under spamming attacks. This paper provides a further understanding on the role of the probability for the online user reputation identification.

INDEX TERMS Online rating systems, user reputation, beta probability distribution, iterative ranking algorithm.

I. INTRODUCTION

Identifying user reputation for online social systems is of significance in construction of personal reputation systems [1]–[3], which could influence e-commerce [4], recruitment, recommender systems [5], [6], rumor spreading [7], [8], etc. Nowadays, many rating systems in online websites provide users channels to reflect their preferences of various objects by giving ratings. However, some users give unreasonable ratings as a result of their dishonesty or non-familiarity [9], [10]. Therefore, the measurement of the online user reputation according to rating behaviors is of critical for maintaining the credibility of rating systems and building reputation systems [11]–[14].

In the previous work for reputation measurement, the iterative-oriented mechanisms are popular, for instance Pagerank [15] and HITS algorithms [16]. Moreover, the quality-based ranking methods are widely investigated. Based on the assumption that each object has an intrinsic quality, the user reputation is measured by the difference or the correlation coefficient between his/her rating vector and the corresponding objects' calculated quality vector. In the quality-based ranking methods, user reputation and object quality are interdependent and are updated iteratively until the reputation and quality values become stable. These methods include iterative refinement (namely IR) method [17], improved IR method [18], correlation-based ranking (namely CR) method [19], iterative ranking method with reputation redistribution (IARR for short) [20], iterative ranking method via the user activity (IRUA for short) [21], and some others [22], [23]. In addition, by grouping users in terms of their ratings, the group-based ranking methods [24]–[26] are proposed, in which user reputation is measured based on the sizes of the corresponding groups.

Recently, a reputation ranking algorithm via the beta probability distribution (namely RBPD) [27] was proposed. By introducing Bayesian analysis and probability distribution [28], [29] to the user-object systems, the user reputation is estimated as the probability that he/she will provide fair ratings to objects. Fair rating is determined by the result whether the rating's opinion accounts for more than half of all the opinions given to the corresponding object, regardless of the users' reputations. As a matter of fact, the ratings given by users with higher reputation should take a more important part in the judgment of the fair rating, and the effects of ratings from users with lower reputation should be reduced. Meanwhile, some research on physical dynamics [30]–[34] showed us the way to improve the robustness by adopting a reputation-allocation process to the original method.

Inspired by the above idea, in this paper we present an iterative reputation ranking algorithm in terms of the beta probability distribution (IBeta), by adopting an iterative reputation-allocation process into the RBPD method. In the IBeta algorithm, we use the beta probability distribution to model the user reputation which is estimated as the probability that the user will give fair ratings. Firstly, considering the users' rating personalities, the ratings are extracted to the positive or negative opinions by a normalized method. Secondly, for each rating, the probability that it is the fair rating is calculated by combining the rating opinion and the initial user reputation. Ratings given by high reputation users are assigned with larger weights in dominating the corresponding quantity of fair/unfair ratings. Then, we get the user reputation by calculating the expected value of the probability that the user will give fair ratings. Consequently, the user reputation and the quantity of fair/unfair ratings for user ratings are iteratively updated until they become stable. Finally, object quality is obtained by the received ratings and the corresponding user reputation. Implementing the presented method for the synthetic networks, the results show that the IBeta algorithm could measure the user reputation and object quality more accurately than the RBPD method. Moreover, the results for the empirical networks indicate that the presented algorithm is more accurate and robust than the RBPD method when the rating systems are under spamming attacks.

II. THE IBETA ALGORITHM

The iterative reputation ranking algorithm via the Beta probability distribution (IBeta) is mainly inspired by the original method in terms of the probability theory (RBPD) and the iterative refinement procedures in which user reputation is reallocated based on their ratings and the previous reputations. The IBeta algorithm has several distinguishing characteristics. Figure 1 shows the flowchart of the IBeta algorithm. We will describe the IBeta algorithm in further detail.

A. THE ONLINE USER REPUTATION EVALUATION

The online rating system can be described by a weighted bipartite network $G = \{U, O, E\}$, which consists of the users (set U), objects (set O) and ratings (set E). The Latin and Greek letters are used to indicate the users and objects, respectively. The rating given from user i to object γ is denoted by $r_{i\gamma}$ and all the ratings are recorded as a rating matrix $\mathbf{A}$. The set U_γ denotes the users who rate to object γ, and the set O_i describes the objects rated by user i. In addition, the degrees of user i and object γ are denoted by k_i and ρ_γ, respectively.

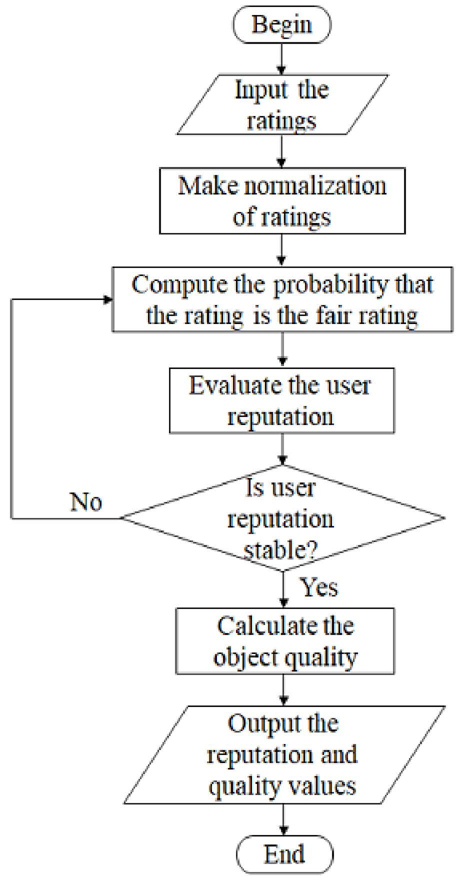

FIGURE 1. The flowchart of the IBeta algorithm.

1) MAKE NORMALIZATION OF RATINGS

The reputation of user i is denoted by R_i. This paper uses the Bayesian analysis in terms of the probability theory to model the user reputation. In the Bayesian analysis [29], the probability that a rating is the fair/unfair rating is adopted to measure each of users' ratings. Opinion of ratings, which is closely related to the identification of fair rating, could be described in the following way.

Two kinds of opinions are set for the ratings to the objects: Positive and negative ones. A normalized method is introduced to distinguish the two kinds of opinions. For a given rating $r_{i\gamma}$, the quantity $r'_{i\gamma}$ represents the extent of fanciness from user i to object γ, from which one can get the opinion (positive or negative). Since different users tend to have different rating criteria, i.e., some users usually give high ratings and others tend to give low ratings, the rating $r_{i\gamma}$ is transformed to the extent of fanciness $r'_{i\gamma}$,

$$r'_{i\gamma} = \begin{cases} 2(r_{i\gamma} - r_i^{\min})/(r_i^{\max} - r_i^{\min}) - 1 & \text{if } r_i^{\max} \neq r_i^{\min} \\ 0 & \text{if } r_i^{\max} = r_i^{\min}, \end{cases} \tag{1}$$

where $r_i^{\max}$ and $r_i^{\min}$ denote the maximum and minimum rating from user i, respectively. By the normalized method, all the ratings would be standardized to $[-1, 1]$, in which the maximum rating one user has given is mapped to 1 and the minimum rating is placed on -1. Meanwhile, the ratings are normalized to 0 if a user always gives the same rating. The normalized results are recorded as a matrix $\mathbf{A}'$, where the element is the rating's extent of fanciness $r'_{i\gamma}$. Non-negative (positive or 0) $r'_{i\gamma}$ represents a positive opinion and the negative $r'_{i\gamma}$ indicates a negative one.

2) COMPUTE THE PROBABILITY THAT THE RATING IS THE FAIR RATING

The probability that a rating is the fair rating, not merely absolutely judge a rating is the fair rating or unfair rating (a binary event), making the measurement of the fair rating more refined. For instance, if the fair rating is defined as the rating's opinion accounts for more than half (50%) of all opinions to the corresponding object, as the definition of fair rating in RBPD method [27], the identification of the case of 49% and 51% may be lack of accuracy. The probability $p_{i\gamma}$ that a rating $r_{i\gamma}$ to an object γ is the fair rating could be calculated by the ratio of the rating's opinion in all the opinions to the object γ, in which the ratings given by high reputation users have larger weights in dominating the opinions to the object γ. The positive opinion PO_γ and negative opinion NE_γ to object γ are calculated by considering both the rating's extent of fanciness $r'_{i\gamma}$ and the user reputation R_i,

$$PO_\gamma = \Sigma_{r'_{i\gamma}\geq 0, i\in U_\gamma} R_i, \tag{2}$$

$$NE_\gamma = \Sigma_{r'_{i\gamma}< 0, i\in U_\gamma} R_i, \tag{3}$$

where each user i is assigned with the equal reputation $R_i = 1$ in the initial configuration. The probability $p_{i\gamma}$ that a rating $r_{i\gamma}$ is the fair rating can be expressed as

$$p_{i\gamma} = \begin{cases} PO_\gamma/(PO_\gamma + NE_\gamma) & \text{if } r'_{i\gamma} \geq 0 \\ NE_\gamma/(PO_\gamma + NE_\gamma) & \text{if } r'_{i\gamma} < 0, \end{cases} \tag{4}$$

then $1 - p_{i\gamma}$ reports the probability to be an unfair rating. For all the ratings, after determining the probability that they are the fair ratings, the results are represented by a matrix $\mathbf{B}$, in which the element lies in [0, 1].

3) EVALUATE THE USER REPUTATION VIA THE BETA PROBABILITY

The reputation R_i of user i is calculated as the probability θ_i that user i will give fair ratings to objects, which is limited to [0, 1]. Considering that there is incomplete users' information, we know nothing about users except their ratings, the probability p_i could be estimated by its expected value,

$$R_i = E(\theta_i). \tag{5}$$

The expected value $E(\theta_i)$ of the probability θ_i is calculated based on the probability density function used to model θ_i. In Bayesian analysis, the beta family of probability density functions [28], [29] is commonly used as a prior distribution for continuous random variables with values in interval [0, 1]. The vector $D_i = \{X_i(1), X_i(2), \ldots, X_i(k_i)\}$ for user i can be regarded as the prerequisites in the Bayesian analysis, where the element $X_i(j)$ represents the probability that the jth rating given from user i is the fair rating. The beta probability distribution can be used as the prior probability distribution $f(\theta_i)$ of θ_i,

$$f(\theta_i) = \text{Beta}(\theta_i|\alpha_s, \alpha_f) = \frac{\Gamma(\alpha_s + \alpha_f)}{\Gamma(\alpha_s)\Gamma(\alpha_f)}\theta_i^{\alpha_s - 1}(1-\theta_i)^{\alpha_f - 1}, \tag{6}$$

where $0 \leq \theta_i \leq 1$, α_s and α_f are two parameters of the probability density function $f(\theta_i)$ and $\alpha_s, \alpha_f > 0$. $\Gamma(\cdot)$ is defined as the Gamma function, in which $\Gamma(x+1) = x\Gamma(x)$ and $\Gamma(1) = 1$.

By the Bayesian analysis, we can get the posteriori distribution of the probability θ_i,

$$f(\theta_i|D_i) = \text{Beta}(\theta_i; \alpha_s + s_i, \alpha_f + f_i), \tag{7}$$

where the quantities s_i and f_i denote the sum of the probability that the rating from user i is fair and unfair, respectively, and $s_i + f_i = k_i$. In addition, the results that the sum of the probability that the rating is fair/unfair, the quantities of fair/unfair ratings from each of users, are denoted by a matrix $\mathbf{D}$. Accordingly, the reputation R_i of user i can be expressed as

$$R_i = E(\theta_i|D_i) = \frac{\alpha_s + s_i}{\alpha_s + s_i + \alpha_f + f_i}. \tag{8}$$

Considering that the beta probability distribution can be regarded as uniformly distributed when $\alpha_s = \alpha_f = 1$ in Eq. (6), which is acceptable when no ratings are given. Thus, we can get the reputation R_i,

$$R_i = E(\theta_i|D_i) = \frac{1 + s_i}{1 + s_i + 1 + f_i} = \frac{s_i + 1}{k_i + 2}. \tag{9}$$

One can find that the larger the quantity of fair ratings given by user i, the higher reputation he/she will have.

At each step, the matrix $\mathbf{B}$, matrix $\mathbf{D}$ and user reputation will be updated via Eqs. (2)-(9). The reputation matrices of the current step and the previous step are denoted by $\mathbf{R}$ and $\mathbf{R}'$, respectively. The iteration process will stop when the difference between the reputation vectors,

$$diff(R, R') = \frac{1}{|U|}\Sigma_{i\in U}(R_i - R'_i)^2 \tag{10}$$

is smaller than the threshold $\delta = 10^{-5}$.

B. THE ONLINE OBJECT QUALITY EVALUATION

The quality of object γ is denoted as Q_γ. Regarding to the IARR2 method [20] and RBPD method, the object quality is not only relied on the weighted average rating of the object, but also determined by the maximum reputation of the users who rate it,

$$Q_\gamma = \max_{i\in U_\gamma}\{R_i\}\frac{\Sigma_{i\in U_\gamma} R_i r_{i\gamma}}{\Sigma_{i\in U_\gamma} R_i}, \tag{11}$$

where $\max_{i\in U_\gamma}\{R_i\}$ is designed based on the hypothesis: The object could not be assigned with high quality if it is always rated by low reputation users, though the ratings might be high. A visual representation of the IBeta algorithm is shown in Fig. 2.

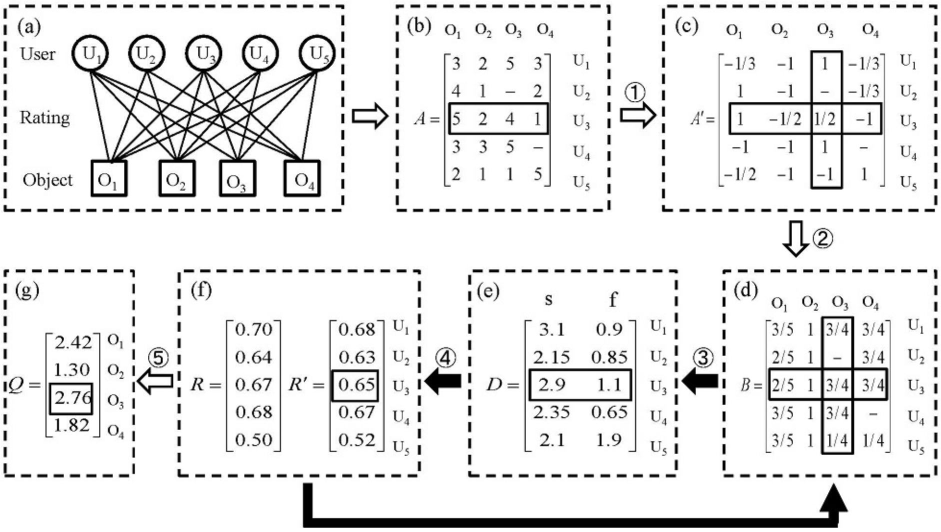

FIGURE 2. A schematic illustration of the IBeta algorithm. The black arrow shows the steps of the calculation procedure. (a) The rating system described by the weighted bipartite network. (b) The corresponding rating matrix, A. The row and column correspond to users and objects, respectively. The symbol "-" indicates that there is no rating behaviors, which is neglected. (c) The normalized rating matrix, A′. Take u_3 as an example, $r_{32} = 2, r'_{32} = 2 * (2-1)/(5-1) - 1 = -1/2$. (d) The probability that the rating is a fair rating for each of ratings, represented by matrix B. Take o_3 as an example, $r'_{13} > 0, r'_{33} > 0, r'_{43} > 0, r'_{53} < 0$, the probability that the ratings given by u_1, u_3 and u_4 to o_3 are the fair ratings is 3/4 and the probability is 1/4 for the rating given from u_5. (e) The results that the sum of the probability that the rating is fair and unfair given by each of users, say s and f, could be denoted as matrix D. Take u_3 as an example, the quantity of fair ratings is $2/5 + 1 + 3/4 + 3/4 = 2.9$. The quantity of unfair ratings is $k_3 - s_3 = 1.1$. (f) The reputation matrix of the current step, R′. $R'_3 = (1 + s_3)/(2 + k_3) = 0.65$. The reputation matrix R can be obtained by iterating (d-f) until the user reputation becomes stable. (g) The quality matrix, Q. Take o_3 as an example, $Q_3 = 0.7 * (5 * 0.7 + 4 * 0.67 + 5 * 0.68 + 1 * 0.5)/(0.7 + 0.67 + 0.68 + 0.5) = 2.76$.

III. RESULTS FOR SYNTHETIC NETWORKS

We investigate the ranking performance of the IBeta algorithm compared with the RBPD method for the synthetic networks, in which we firstly add the weighted links by employing the preferential attachment mechanism and then involve different number of distorted ratings.

When generating the synthetic networks, we set $|U| = 6000$, $|O| = 4000$. The weighted links (ratings) will be added one by one until the network sparsity η reaches 0.02, 0.04, 0.06, respectively. Thus, the total number of the ratings is $|E| = \eta \times |U||O| = 4.8 \times 10^5, 9.6 \times 10^5, 1.44 \times 10^6$, respectively. In the synthetic networks, the users give ratings to objects according to the object degree preferentially. The probability of selecting user i and object γ at each time step t can be expressed as

$$p_i(t) = \frac{k_i(t) + 1}{\Sigma_{j \in U} k_i(t) + 1}, \tag{12}$$

$$p_\gamma(t) = \frac{k_\gamma(t) + 1}{\Sigma_{\beta \in O} k_\beta(t) + 1}, \tag{13}$$

where $k_i(t)$ and $k_\gamma(t)$ are the degree of user i and object γ, respectively, at the time step t.

The rating $r_{i\gamma}$ given from user i to object γ, the link between user i and object γ, is composed of two parts: the object intrinsic quality Q'_γ and the rating error $\Delta\delta_{i\gamma}$, where Q'_γ is drawn from a uniform distribution $U(0.5, 5.5)$ and $\Delta\delta_{i\gamma}$ obeys the normal distribution $N(0, \Delta\delta_i)$. The parameter $\Delta\delta_i$ is the rating error of user i and it obeys a uniform distribution $U(\Delta\delta_{min}, \Delta\delta_{max})$, in which $\Delta\delta_{min} = 0.5$, $\Delta\delta_{max} = 5.5$. The rating $r_{i\gamma}$ is expressed as,

$$r_{i\gamma} = [Q'_\gamma + \Delta\delta_{i\gamma}], \tag{14}$$

where the function [ ] indicates rounding. The rating $r_{i\gamma}$ lies in the set {1, 2, 3, 4, 5} and the ratings lying out the set {1, 2, 3, 4, 5} will be truncated.

In the synthetic networks, we suppose there are some random ratings (distorted ratings), which are given from users (e.g. naughty users or test engineers) who rate objects totally random. After adding the weighted links (ratings) by employing the preferential attachment mechanism, we replace c fraction of the original links (ratings) with the random values in the set {1, 2, 3, 4, 5}. Consequently, the larger the parameter c, the more noisy information in the synthetic networks. $c = 0$ indicates that there is all true information. While $c = 1$ represents there is totally noisy information. In the experiments for the synthetic networks, the parameter c is set to 0.05, 0.1, 0.15, ..., 0.95, respectively.

Two metrics are used to measure the ranking performance of the IBeta algorithm and RBPD method: AUC curve [36] (the area under a receiver operating characteristic curve) and Kendall's tau [37]. When calculating the AUC value, we should select a part of objects as benchmark objects and the others as non-benchmark objects, in which the benchmark objects are considered to be generally with high qualities in the networks. To calculate the AUC value, one should conduct n times independent comparisons of one pair of

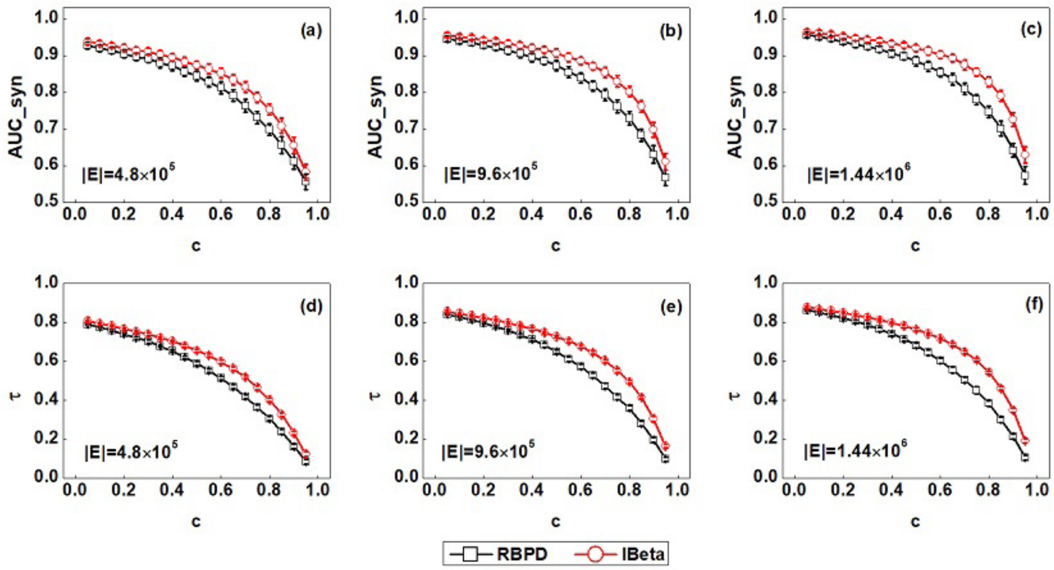

FIGURE 3. (Color online) The (a-c) AUC values *AUC_syn* and (d-f) Kendall's tau τ of the IBeta algorithm and RBPD method for the synthetic networks in the random rating attack case, in which the parameter *c* denotes the ratio of random ratings. One can find that both the AUC values *AUC_syn* and Kendall's tau τ of the IBeta algorithm are larger than those of RBPD method with different parameter *c* and different parameter $|E|$ (size of networks). One can also find that the difference of the AUC values/Kendall's tau between the IBeta algorithm and RBPD method is greater in the case of larger network size. The results are averaged over 50 independent realizations. The error bars are the corresponding standard deviations.

benchmark and non-benchmark objects, which are randomly selected from the benchmark and non-benchmark object sets, respectively. After the n times comparisons, the number of times when the benchmark object has higher quality than the non-benchmark object is denoted by n_1, and the number of times that the benchmark and non-benchmark objects have the same quality is denoted as n_2. The final AUC value is determined by

$$AUC\_syn = \frac{n_1 + n_2 \times 0.5}{n}, \tag{15}$$

here the parameter n is set to $n = 1 \times 10^9$ in the experiments. $AUC\_syn = 1$ represents that each benchmark object selected is ranked higher than the non-benchmark object selected in the n times comparisons. While $AUC\_syn = 0.5$ means that all the objects are ranked randomly. We select top 5% high-quality objects as benchmark objects according to their intrinsic qualities Q'. A higher AUC indicates a higher ranking accuracy.

The Kendall's tau τ calculates the rank correlation between the intrinsic quality Q' and the estimated object quality Q,

$$\tau = \frac{2}{|O|(|O|-1)} \sum_{\mu<\nu} sgn[(Q'_\mu - Q'_\nu)(Q_\mu - Q_\nu)], \tag{16}$$

where $sgn(x)$ is the sign function: $sgn(x) = 1$ if $x > 0$; $sgn(x) = -1$ for $x < 0$; and $sgn(x) = 0$ when $x = 0$. $(Q'_\alpha - Q'_\beta)(Q_\alpha - Q_\beta) > 0$ means concordant and negative means discordant. $\tau \in [-1, 1]$ and a higher τ shows a more accurate measurement of object quality.

Figure 3 shows the AUC values *AUC_syn* and Kendall's tau τ of the IBeta algorithm and RBPD method with different fractions of random ratings, from which one can find that both the AUC values and Kendall's tau τ of the IBeta algorithm are larger than the results obtained by the RBPD method for different parameter c (fraction of random ratings), with different size of networks($|E| = 4.8 \times 10^5, 9.6 \times 10^5, 1.44 \times 10^6$, respectively). For instance, in the case of $c = 0.5$ and $|E| = 9.6 \times 10^5$, the AUC values of the IBeta algorithm and RBPD method could reach 0.906 and 0.873, respectively, and the Kendall's tau τ could reach 0.727 and 0.650, respectively. Another example, in the case of $c = 0.5$ and $|E| = 1.44 \times 10^6$, the AUC values of the IBeta algorithm and RBPD method could reach 0.920 and 0.885, respectively, and the Kendall's tau τ could reach 0.764 and 0.683, respectively. The results indicate that the IBeta algorithm could measure the user reputation and object quality more accurately than the RBPD method. Meanwhile, one can also find that the IBeta algorithm and RBPD method could reach larger AUC values *AUC_syn* and Kendall's tau τ in larger size of networks with the same sizes of users and objects for the synthetic networks, and the difference of the AUC values/Kendall's tau between the IBeta algorithm and RBPD method is greater in the case of larger network size. For instance, in the case of $c = 0.5$, the AUC values of the IBeta algorithm and RBPD method could reach 0.873 and 0.844, respectively when $|E| = 4.8 \times 10^5$, and the AUC values are 0.920 and 0.885 when $|E| = 1.44 \times 10^6$, $0.873 - 0.844 < 0.920 - 0.885$. Another example, in the case of $c = 0.5$, the Kendall's tau of the IBeta algorithm and RBPD method are 0.655 and 0.589, respectively when $|E| = 4.8 \times 10^5$, and the Kendall's tau are 0.764 and 0.683 when $|E| = 1.44 \times 10^6$, $0.655 - 0.589 < 0.764 - 0.683$. The results indicate that the advantage of the IBeta algorithm over the RBPD method is more obvious in reputation measurement for larger scale rating systems.

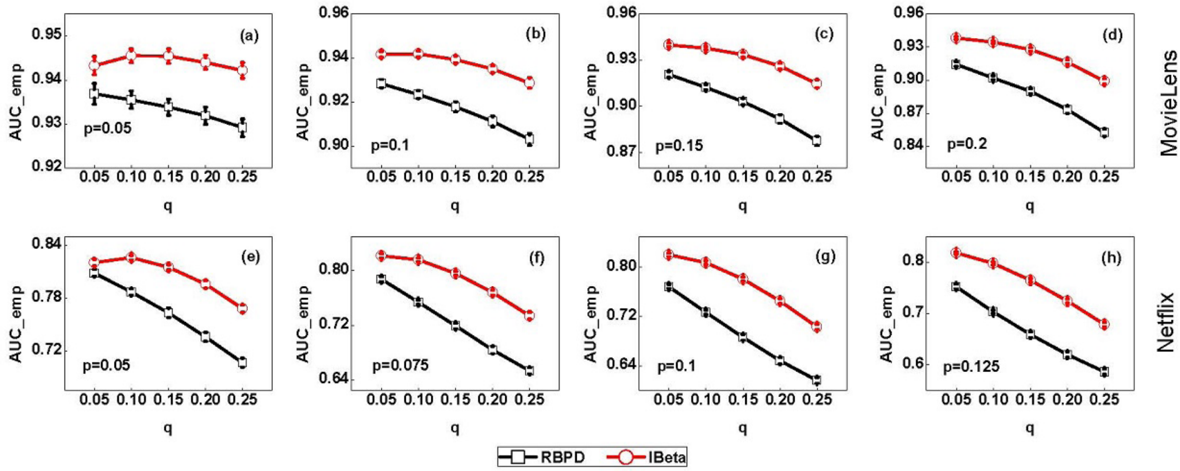

FIGURE 4. (Color online) The AUC values *AUC_emp* of the IBeta algorithm and RBPD method with different parameters *p* and *q* in the random rating attack case for (a-d) MovieLens and (e-h) Netflix data sets, respectively. The parameter *q* is the ratio of spammers and the parameter *p* is the ratio of objects rated by spammers. One can find that the AUC value of the IBeta algorithm is larger than the one obtained by the RBPD method for each parameter pair (p, q). The results are averaged over 50 independent realizations. The error bars are the corresponding standard deviations.

IV. RESULTS FOR EMPIRICAL NETWORKS

Besides the synthetic networks, we investigate the ranking performance of the IBeta algorithm compared with the RBPD method for two empirical data sets which contain ratings for movies: MovieLens and Netflix. The MovieLens data is provided from the GroupLens.[1] We extract a smaller data set in which there are 1 million ratings and each user has at least 20 movies. The Netflix data is downloaded from the Netflix Prize.[2] We sample a subset by choosing 10000 users who have rated at least 20 movies. The ratings in MovieLens and Netflix data sets are given by the integer ratings scaling from 1 to 5. Some basic statistical properties of two data sets are shown in Table 1.

TABLE 1. Basic statistical properties of the empirical data sets (MovieLens and Netflix) used in this paper. $|U|$, $|O|$ and $|E|$ denote the number of users, objects and ratings, respectively. $\langle k_U \rangle$ and $\langle \rho_O \rangle$ are the average degree of users and objects, respectively. η denotes the network sparsity.

Data Sets	$\|U\|$	$\|O\|$	$\|E\|$	$\langle k_U \rangle$	$\langle \rho_O \rangle$	η
MovieLens	6040	3706	1000209	166	270	0.0447
Netflix	10000	6000	824802	82	137	0.0137

In the empirical networks, we suppose that there exist some random ratings, the same as in the synthetic networks. We generate the artificial spammers in the empirical networks: Randomly selecting some users and assign them some distorted ratings (random values in the set $\{1, 2, 3, 4, 5\}$), in which the ratio of spammers is q and the activity of spammers is p. Thus, the number of spammers in the empirical networks is $q|U|$ and the degree of each spammer is $p|O|$. If the quantity $p|O|$ is no more than the original degree (k_i) of a selected spammer i, we randomly select his/her $p|O|$ ratings and replace them with random ratings and the unselected $k_i - p|O|$ ratings are truncated. Otherwise, we replace all the spammers' original ratings, then randomly select $p|O| - k_i$ of his/her non-rated objects and assign them random ratings.

For the empirical networks, two metrics are used to measure the ranking accuracy of the IBeta algorithm and RBPD method: AUC curve [36] and recall [38]. When calculating the AUC values, the benchmark is the spammers, which is different from the experiments for the synthetic networks. We conduct n' times independent comparisons of one pair of users: One spammer and one non-spammer, which are randomly selected from the spammers and the non-spammers, respectively. After n' times comparisons, the number of times when the spammer has lower reputation than the non-spammer is denoted by n'_1, and the number of times that the spammer and the non-spammer have the same reputation is denoted as n'_2. The AUC value is determined by

$$AUC\_emp = \frac{n'_1 + n'_2 \times 0.5}{n'}, \tag{17}$$

here we also set $n' = 1 \times 10^9$ and a higher AUC_emp indicates a higher ranking accuracy.

The recall describes to what degree that the spammers can be identified in the top-L ranking list (top-L low reputation users),

$$R_c(L) = \frac{d'(L)}{q|U|}, \tag{18}$$

where $d'(L)$ is the number of identified spammers in the top-L ranking list. Here the parameter L is set as $L = q|U|$. $d'(L) \leq q|U|$ and $R_c(L)$ is limited in [0, 1]. Higher $R_c(L)$ means a more accurate reputation ranking list.

Figure 4 shows the AUC values AUC_emp of the IBeta algorithm and RBPD method with different parameters p and q in the random rating attack case for the MovieLens and Netflix data sets, respectively. The selection of the

[1]http://www.grouplens.org
[2]http://www.netflixprize.com

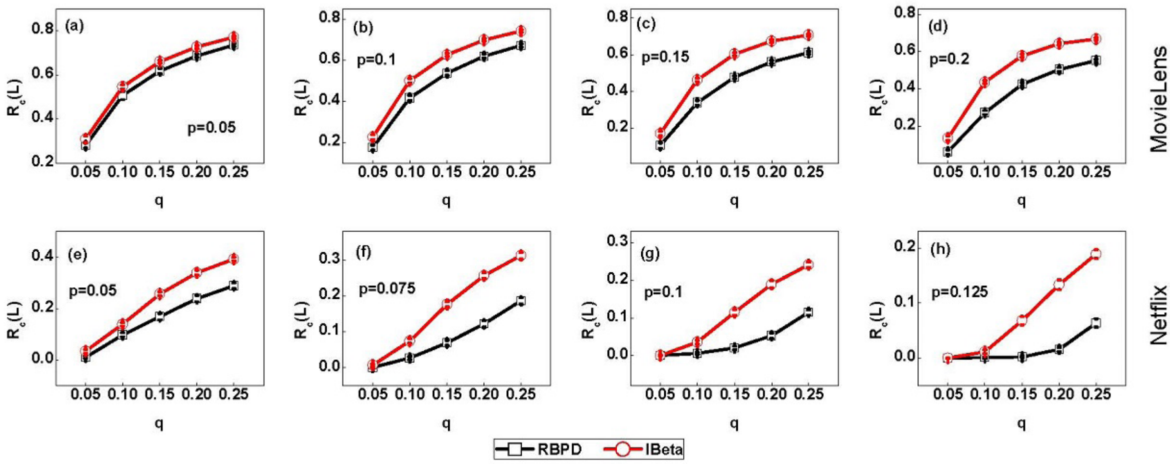

FIGURE 5. **(Color online) The recall $R_c(L)$ of the IBeta algorithm and RBPD method with different parameters p and q in the random rating attack case for (a-d) MovieLens and (e-h) Netflix data sets, respectively. The parameter q is the ratio of spammers, the parameter p is the ratio of objects rated by spammers and the parameter $L = q|U|$. One can find that the recall $R_c(L)$ of the IBeta algorithm is larger than that of the RBPD method for each parameter pair (p, q). The results are averaged over 50 independent realizations. The error bars are the corresponding standard deviations.**

parameter p is different for two empirical data sets since the Netflix data set is more sparser than the MovieLens data set. From Fig. 4 one can find that the AUC values of the IBeta algorithm are larger than those obtained by the RBPD method with different parameters p and q. For example, the AUC values of the IBeta algorithm and RBPD method could reach 0.926 and 0.892, respectively, when $(p, q) = (0.15, 0.2)$ for the MovieLens data set. When $(p, q) = (0.05, 0.2)$ for the Netflix data set, the AUC values of the IBeta algorithm and RBPD method could reach 0.796 and 0.736, respectively. The results of the recall $R_c(L)$ for two empirical data sets are shown in Fig. 5, from which one can find that the recall $R_c(L)$ of the IBeta algorithm is larger than that of the RBPD method for each parameter pair (p, q). For instance, the recall $R_c(L)$ of the IBeta algorithm is larger than the one obtained by the RBPD algorithm by 20.0% and 42.6% with $(p, q) = (0.15, 0.2)$ for the MovieLens data set and $(p, q) = (0.05, 0.2)$ for the Netflix data set, respectively. The results indicate that the IBeta algorithm performs better in detecting random spammers than the RBPD method.

V. CONCLUSION AND DISCUSSIONS

By adopting an iterative reputation-allocation process, in this paper we present an iterative reputation ranking algorithm in terms of the beta probability distribution (IBeta) based on the Bayesian analysis. The user reputation is calculated as the probability that he/she will give fair ratings to objects. In the IBeta algorithm, ratings given by higher reputation users are assigned with larger weights in dominating the corresponding quantities of fair/unfair ratings (the sum of the probability that the rating is fair/unfair according to users' historical ratings). User reputation is reallocated based on their ratings and the previous reputations. The user reputation and the quantity of fair/unfair ratings for users are iteratively updated until they become stable. The experimental results for the synthetic networks show that the IBeta algorithm could produce more accurate ranking lists for user reputation and object quality than the RBPD method. Moreover, the results for the empirical networks indicate that the presented algorithm is more accurate and robust in detecting random spammers than the RBPD method when the rating systems are under spammer attacks. This work improves the role of Bayesian analysis and probability theory in designing more accurate reputation ranking methods.

The following points should be paid attention in the future research. Firstly, the empirical networks investigated in this work are ratings on movies, ratings on other kinds of objects (books, stories, music, etc.) could be used as the empirical data sets. Secondly, the empirical and synthetic experiments in this paper suggest that the IBeta algorithm could converge, while the theoretical convergence still need to be proved. Besides, the effect of continuous and discrete ratings on evaluating the user reputation is also worthy of further investigation. Accordingly, our future work will concern of designing more accurate reputation ranking algorithms which can objectively reflect the real reputation of online users based on Bayesian analysis and probability theory.

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