IEEE Access
Multidisciplinary | Rapid Review | Open Access Journal

Received October 18, 2018, accepted November 27, 2018, date of publication December 7, 2018, date of current version January 4, 2019.

Digital Object Identifier 10.1109/ACCESS.2018.2885563

A Graph-Theory-Based Method for Topological and Dimensional Representation of Planar Mechanisms as a Computational Tool for Engineering Design

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This work was supported by the Instituto Politécnico Nacional de México, both by COFAA and SIP, with project 20181162. All authors also thank CONACyT of México their grants for research and doctoral studies.

ABSTRACT In this paper, a graph-theory-based approach for representing planar mechanisms is presented, the Santiago Portilla method (SPM). From the corresponding adjacency matrix, SPM generates an extended matrix containing the complete characterization of a planar mechanism, including all the information about both topology and geometry. This matrix representation can be used for the optimal design of mechanisms, allowing simultaneously the topological and dimensional synthesis by means of computational tools such as the metaheuristic algorithms. A case study corresponding to the design of a fixed-linear-trajectory tracker mechanism is included in order to test the efficiency of the proposed approach. It was carried out by addressing the design as an optimization problem and solving it with the differential evolution algorithm, representing the individuals in its population by the matrix form generated by the SPM. The results of the case study show that the SPM and its matrix representation constitute a useful and flexible tool for the solution of the real engineering problems involving the design of planar mechanisms.

INDEX TERMS Graph theory, topological design, dimensional synthesis, matrix representation, optimization.

I. INTRODUCTION

The extensive use of machinery has propitiated the development of new devices for specific tasks, and their design is a motivating challenge because it involves the solution of complex engineering problems. In many cases, traditional design methodologies are surpassed by this complexity and new solution approaches are required, considering computers as a fundamental tool for solving engineering problems in an efficient and effective way. The design of planar mechanisms is a representative case, since they are used in most applications involving machinery.

Generally, the design of planar mechanisms is a sequential process involving three stages: 1) characterization of the functionality requirements, degrees of freedom, workspace, and mechanism type, among other specifications; 2) topological design to establish the number of elements and the union types connecting them; and 3) dimensional synthesis of the resulting mechanism to determine the size of its elements. The experience of the designer is fundamental in the second step, since a wrong topology selection unfulfilling the design and/or function requirements implies to repeat both the topological and dimensional synthesis stages.

Diverse approaches have been developed for the topological synthesis of planar-mechanisms [1]–[4], or for their dimensional synthesis once the topology is selected, from classical techniques such as mathematical programming to new methodologies based on metaheuristics or graph theory [5]–[12]. In [13], a method for solving the topological and dimensional synthesis is developed, but still in a sequential manner. In order to simultaneously carry out both syntheses, an important issue is to find a complete mechanism representation including the information of topology and geometry in such a way that it can be interpreted by computers. Graph theory is a mathematical and computer science branch, also known as theory of graphics, that was introduced in the XVIII century by Swiss mathematician Leonard Euler, to solve the problem of the seven bridges of KÃnisgberg [14], [15]. It is of regular application in areas as diverse as chemistry [16], [17], geology [18], electrical engineering [19], mathematics [20], computing [21], [22] and economy [23], to mention a few, and has been used since the 60s for mechanism representation and synthesis [24], [25]. From then on, diverse works have been developed with this approach, as in [26]–[29].

Tsai [30] proposed two important innovations to this field: the representation of any planar mechanism as a graph, regardless of its components: gears, cams, and bars; and the use of the adjacency and/or incidence matrices for representing the corresponding graph. Yan and Hwang [31] developed an algorithm to generate configurations for different mechanisms, using the adjacency matrix to represent graphs for up to twelve elements (bars) and revolute type joints. In [32], the adjacency and incidence matrices are applied to represent mechanisms, and two algorithms are implemented for isomorphism identification between the corresponding graphs. Spatial mechanisms are represented with graphs and their adjacency matrices in [33], to design hybrid and fractionated structure robots, although only considering revolute joints. In [34], three different algorithms are used in conjunction to the adjacency matrix to generate mechanisms by varying the number and type of the links, obtaining an atlas as a result. An elementary graph is used in [35] to represent the topology of planar and spatial mechanisms, also taking into account mobility criteria to produce unique representations including links with multiple unions. Graph theory is applied in [36] for the structural synthesis of parallel manipulators with subchain coupling, performing simultaneously the processes of fractioning and graph simplification to generate an atlas for the different topologies.

In the aforementioned works, graph theory is used to design mechanisms with specific tasks or to generate atlas with revolute-type connections, taking into account just the topology. In this work a novel method for representing planar mechanisms in a matrix form is proposed, the Santiago-Portilla Method, that includes not only the topology of the mechanism but its geometry: the dimension of each element, the link and joint types, and its location referred to a coordinate system. Starting from the corresponding adjacency matrix, an extended matrix is generated to consider these parameters, for using the representation simultaneously in both topological and dimensional synthesis. The paper is organized as follows: Section 2 includes a description of the proposed method for mechanism representation in matrix form, while in Section 3 its practical implementation with a case study corresponding to the design of a fixed-linear-trajectory tracker mechanism is included. Finally, conclusions are drawn in Section 4.

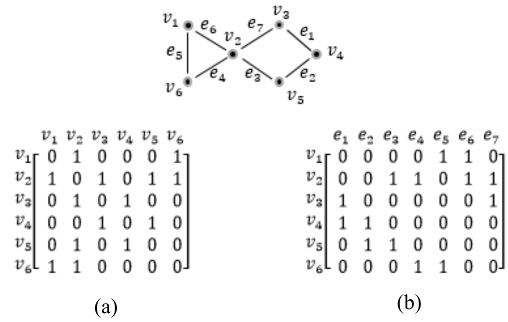

FIGURE 1. Matrix representation of graphs. (a) Adjacency matrix. (b) Incidence matrix.

II. MATRIX REPRESENTATION OF PLANAR MECHANISMS

A graph is a representation of points (vertices) that are related to each other by lines (edges) [14], used to solve logical problems. Graphs are useful for a variety of applications such as trajectory planning, where it is required to reach a final point starting from an initial one with the minimum resource consumption. The incidence and adjacency matrices are commonly used for representing graphs, indicating any binary relationship between the graph elements [15].

An adjacency matrix is a squared array where both rows and columns identify vertices, each position corresponding to an edge e (a connection between vertices v). The value of an edge is 1 or 0 if its respective vertices are related or not, as shown in Fig. 1A. Unlike the adjacency matrix, an incidence matrix is not necessarily squared, since the total of rows and columns is determined by the number of vertices and edges, respectively. The filling process, shown in Fig. 1B, is as follows: if the edge e_i is incident to the vertex v_j, a value 1 is placed in the corresponding place in the matrix, otherwise it is 0.

A. METHODOLOGY OF TSAI

A general methodology to represent planar mechanisms in a matrix form containing the essential information for their description was developed by Tsai [30]. In that method, a mechanism is specified by a graph, taking into account the number of elements, link type, and the ground element. These characteristics are interpreted in a polygonal-type graphic scheme, where each piece of the mechanism corresponds to a vertex labeled with the number of its associated link. A link is a rigid body with at least two nodes, and these nodes are joint points to other links to permit a movement. The joints between elements are called edges, named with an R if their type is revolute, P if it is prismatic, G for a gear, and Cp when it is a cam type. The ground element is identified by an additional circle in the corresponding vertex. Different mechanism types can be represented with this method, as illustrated in Fig. 2.

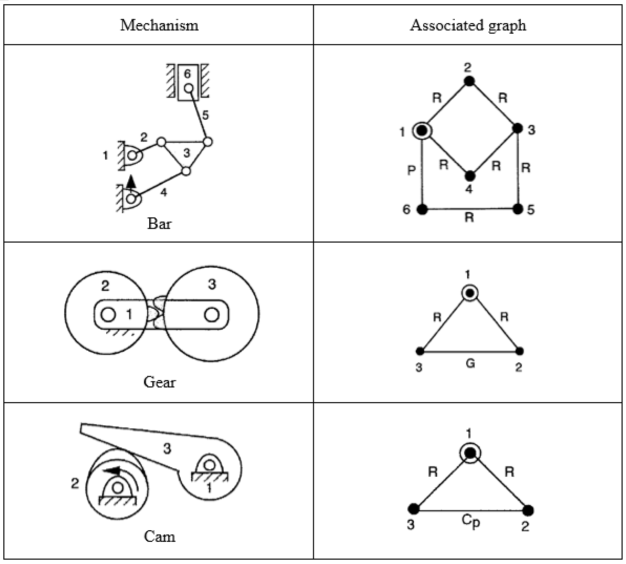

FIGURE 2. Mechanism representation using the methodology of Tsai.

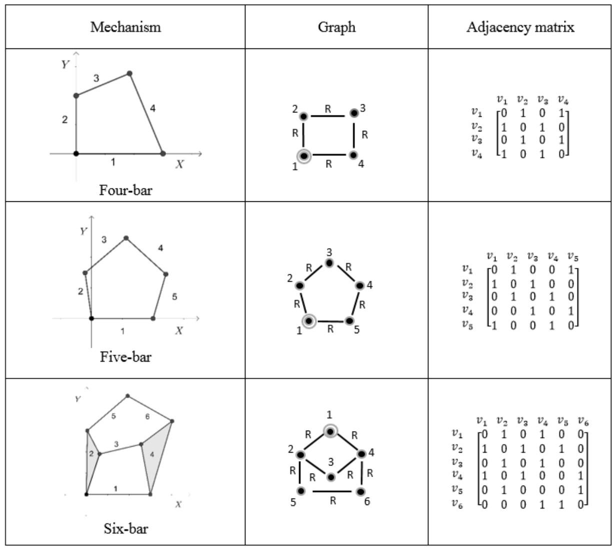

FIGURE 4. Mechanism representation by the adjacency matrix.

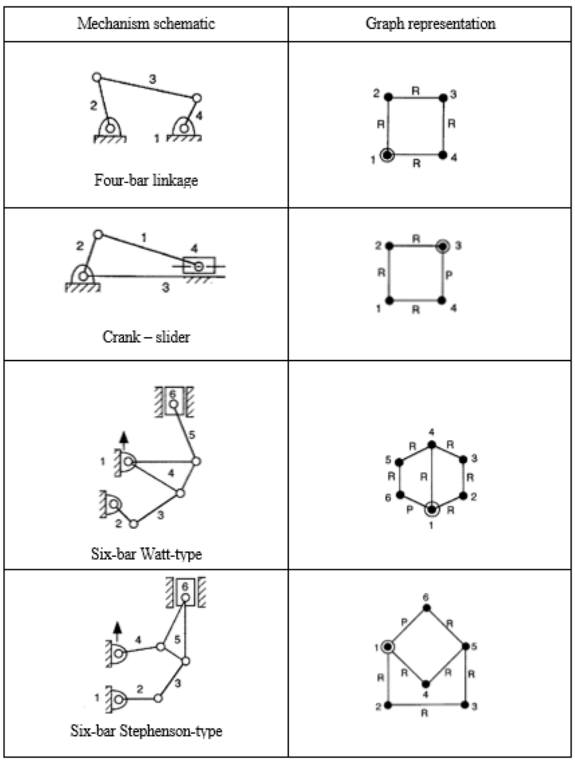

FIGURE 3. Graph representation of planar mechanisms using the methodology of Tsai.

Fig. 3 corresponds to the graph representation of diverse planar mechanisms, using the methodology of Tsai. The resulting graphs look similar, but have differences in the order and/or labeling of the edges. As an example, the figure includes two four-bar mechanisms with the same number of vertices and edges, but differing in the location of the ground bar and that the second mechanism has a prismatic connection. In the same manner, there are two ternary links in the six-bar mechanisms, where the vertex is the ground link for both graphs but the ternary links are in different locations, producing dissimilar associated graphs. With this methodology, a graph can be interpreted by means of its adjacency and incidence matrices. Nevertheless, it is not specified how to perform a complete matrix representation that includes all the graph information. That is, the matrices contain information about the number of links and joints (mechanism topology) but not about the joint types, link functions, or dimension of elements, among other data.

Eberhard *et al.* [13] presented modifications to the methodology of Tsai, to include the information about the joint types and the link functions. They proposed an extended adjacency matrix applied to topology optimization, where unique numbers are used for labeling the types of joints and bars: 2 and 3 indicate revolute and prismatic joints, while 9, 10 and 11 correspond to ground (fixed), input (driver) and output bars, respectively.

B. SANTIAGO-PORTILLA METHOD (SPM) FOR MATRIX REPRESENTATION OF PLANAR MECHANISMS

As established before, mechanisms with diverse topologies and different number of elements can be represented by a graph and its corresponding adjacency matrix, as shown in Fig. 4. This approach has been used for the development of applications such as finding the spatial structure for a specific task [33], or the generation of atlas for different planar mechanisms with a single union type [31], [32], [34]. In this work a novel technique for representing planar mechanisms by extended matrices is developed, the Santiago-Portilla Method (SPM), facilitating the design of mechanisms since it includes joint types, identification and size of bars and coupler points, spatial position of the overall system, and the identification of link types. The stages of the SPM are described in the following sections. In the first two steps, the identification

TABLE 1. Joint types.

Joint	Label
Revolute	4
Prismatic	5

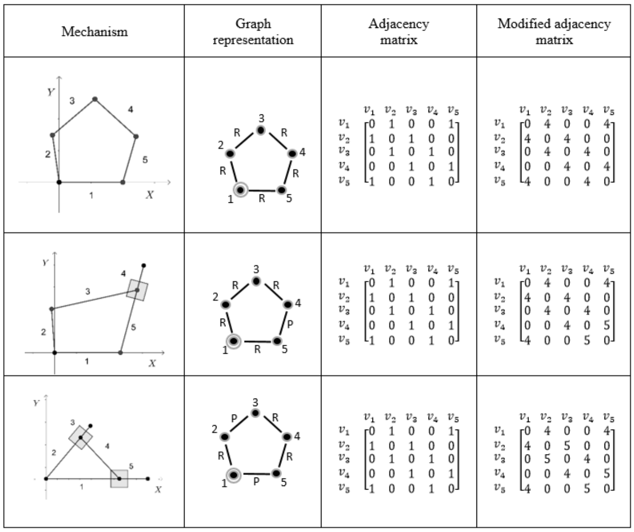

FIGURE 5. Identification of joint types in the adjacency matrix.

of joint types and bar functions is similar to the approach presented in [13], but applying different labeling conventions.

1) IDENTIFICATION OF JOINT TYPES

A graph representing a mechanism contains information about the joint types between its links, but this data is missing in the corresponding adjacency matrix. In the first stage of SPM, it is included in the matrix by a unique number. There are two types of joints for planar mechanisms: revolute and prismatic [37], and they are marked as indicated in Table 1. The convention permits to properly identify mechanisms with the same number of elements but different joints. Fig. 5 shows five-bar mechanisms with their corresponding graphs and adjacency matrices, both original and modified. As can be seen, the different configurations look the same in the original matrix presentation.

2) IDENTIFICATION OF INPUT, OUTPUT, AND GROUND BARS

The elements that introduce motion into a planar mechanism are called inputs, while the ground bar refers to the fixed element. Additionally, the output bar is responsible for transmitting force or motion, and in some cases it requires special geometries as is the case of a coupling point. In the original adjacency matrix these elements are unidentified, so in the next stage the links are inserted in the matrix accordingly to the conventions in Table 2. The labels for identifying the bar function are placed on the main diagonal of the matrix in $A_{r,r}$, where r is the bar number or the label of the vertex. These positions usually contain a value zero because a different

TABLE 2. Link functions.

Link	Label
Ground	1
Input	2
Output	3

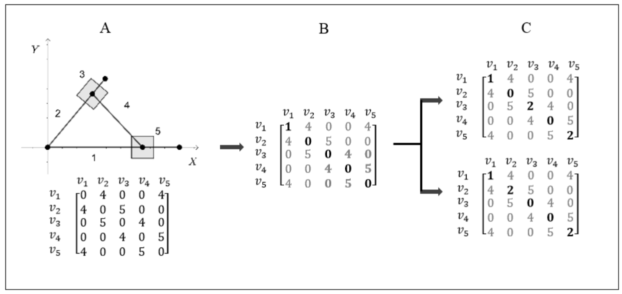

FIGURE 6. Identification of the ground and input bars.

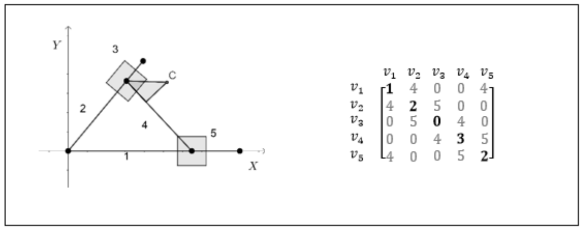

FIGURE 7. Identification of the output bar.

number would indicate the connection of a component with itself, which is not physically possible. An example for a five-bar mechanism is presented in Fig. 6, developed as follows:

1) Fig. 6A shows the mechanism and its matrix representation, including the identification of joint types.
2) A label 1 is assigned to the ground bar r_1, corresponding to the matrix position $A_{1,1}$, as seen in Fig. 6B.
3) This configuration of the five-bar mechanism has two inputs in three different distributions: two sliders (r_3 & r_5), or a rotational bar and a slider (r_2 & r_3, r_2 & r_5). For this example, the combinations r_3 & r_5 and r_2 & r_5 were considered, labeling the corresponding elements as 2 within the matrices in Fig. 6C.
4) Some planar mechanisms need an additional element as output, requiring the identification of its supporting bar; for this example, a coupler is added to the r_2 & r_5 combination (rotational bar and slider). The additional piece is placed on r_4, and $A_{4.4}$ is labeled with 3, as shown in Fig. 7.

3) DIMENSION OF THE BARS

The dimension of each element can also be included in the representation. Since the adjacency matrix is symmetric, the upper and lower triangular matrices have the same information. A stage is implemented to use the lower triangular matrix for placing the dimensions, by replacing the label of each joint with its size. Fig. 8 shows the representation of a four-bar mechanism with specific dimensions, using a

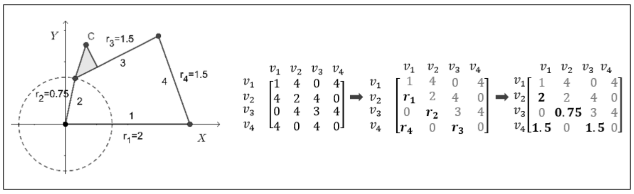

FIGURE 8. **Dimension assignment of the elements.**

TABLE 3. **Label assignation for auxiliary table.**

Label	Mean
-1	Assigned data
0	Data not assigned without priority
1	Data not assigned with priority

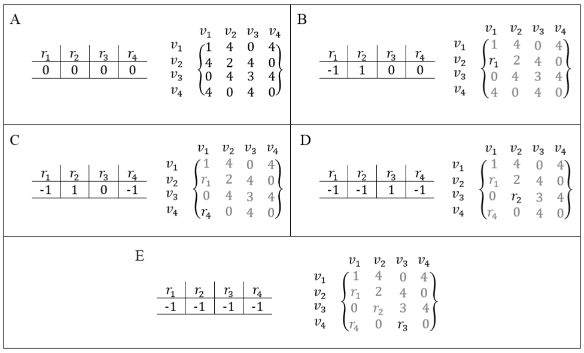

FIGURE 9. **Dimension assignment process.**

4x4 matrix that contains the following information: revolute joint-type, labeled as 4; ground bar r_1 is 1; input bar r_2 is 2; and the location of the coupler point (output) r_3 is 3. The leftmost matrix is modified to include the dimensions, with the center and rightmost arrays indicating the corresponding positions and values in the lower triangular matrix.

An auxiliary table is required in the dimension-assignment process to indicate the present state of each datum, using the labels of Table 3. The size of the auxiliary table depends on the number of links in the mechanism; the following example has four links presented as r_1 to r_4 with an initial value zero, shown in the table of Fig. 9A.

The positions different to zero in the upper triangular matrix are identified, in this case $A_{1,2}$, $A_{1,4}$, $A_{2,3}$ and $A_{3,4}$. The dimensions of the bars are placed in the lower part of the matrix, considering that if $A_{i,j} \neq 0$ is found in the upper zone its value is saved in $A_{j,i}$. The value is determined by the following rules:

1) If $r_i = 0$ and $r_j = 0$ in the auxiliary table, the dimension of bar i is assigned to $A_{j,i}$, otherwise dimension of bar j is assigned.
2) If $r_i = 1$ or $r_j = -1$, the dimension of bar i is assigned to $A_{j,i}$.
3) The values of r_i and/or r_j are updated after any modification to the matrix using -1 for an assigned value, and 1 if it was not.

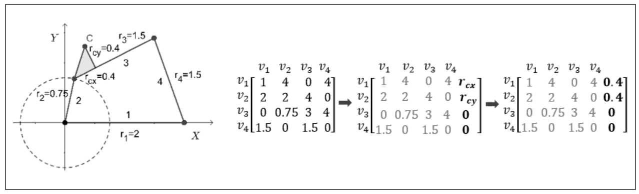

FIGURE 10. **Dimension assignment of a coupler point.**

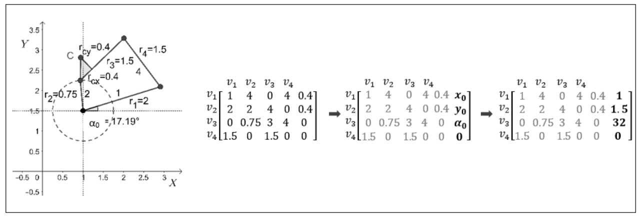

FIGURE 11. **Spatial location data.**

Applying the rules to the example, in Fig. 9B the dimension of bar r_1 is assigned to $A_{2,1}$ and the auxiliary table is changed to $r_1 = -1$, $r_2 = 1$. In the second case, the dimension of bar r_4 is assigned to $A_{4,1}$ since the value of r_1 was already assigned, and only $r_4 = -1$ is updated (Fig. 9C). For $A_{2,3}$, $A_{3,2}$ is assigned the value of bar r_2 since it has priority ($r_2 = 1$), as can be seen in Fig. 9D. Finally, position $A_{4,3}$ is assigned the dimension of bar r_3, because $r_4 = -1$ was already assigned and r_3 has priority; then $r_3 = -1$, as shown in Fig. 9E.

4) DIMENSIONS OF THE COUPLER POINT

As mentioned before, some mechanisms require an additional component called coupling point, for transferring force or motion. The dimensions of its mounting on the X and Y axis (r_{cx}, r_{cy}) are included in an additional column at the rightmost extreme of the representation matrix; if there is no coupler the values in this column are zero. Fig. 10 shows this modified matrix for the mechanism described as example in the previous section.

5) LOCATION OF THE MECHANISM

The location of mechanisms referenced to a coordinate system in real world applications, usually implies a translation/rotation from the coordinate system. This information is included in the representation matrix by inserting a new column that contains the values of translation (x_0, y_0) and rotation (α_1). So, the mechanism can be placed in any part of the plane with any orientation; Fig. 11 presents an example where the mechanism is displaced and rotated with respect to the origin coordinate (0,0). The figure also includes the matrix representation and the adjustment done to include the location data.

6) IDENTIFICATION OF THE LINK TYPE

Up to this point, the proposed matrix representation describes mechanisms with only binary links. However, links can

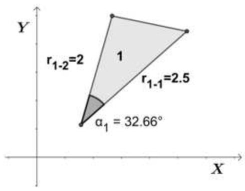

FIGURE 12. Ternary link with its dimensions.

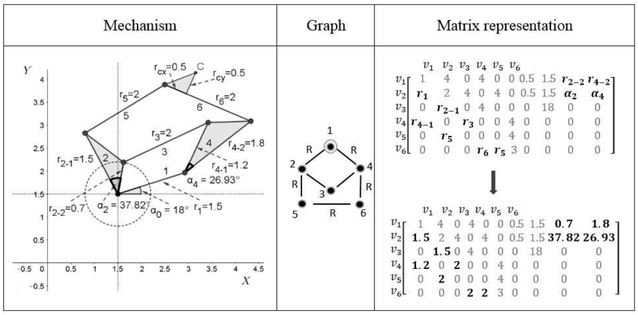

FIGURE 13. Matrix representation of a six-bar Watt-type mechanism.

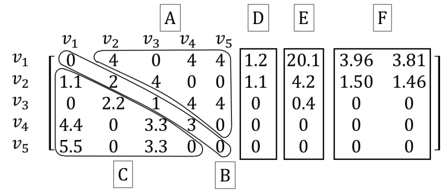

FIGURE 14. General matrix for planar-mechanism representation in the SPM.

include more than one pair of nodes as is the case of the six-bar Watt-type mechanism that contains two ternary links. The last stage in SPM is the description of links with more than one pair of nodes, by incorporating additional columns to the representation matrix.

The number of data required to describe a link depends on the number of nodes k in that link. Since a link can be represented as a geometric figure, it is necessary to know the dimension of the k sides and $k - 1$ angles. For $k > 3$, $2k - 1$ data are needed; e.g., a quaternary link involves four dimensions and three angles. As a special case, a ternary link only requires two sides and an angle, as shown in Fig. 12.

Any mechanism can be represented in matrix form with the SPM if the condition in (1) is accomplished, where ne is the number of elements and k is the number of nodes in the largest link. As a complete example, Fig. 13 presents a six-bar Watt-type mechanism with two ternary links and its representation matrix, where four columns were added corresponding to the coupler point and the links.

$$ne - (2k - 1) \geq 0 \tag{1}$$

Equation (1) is related to both the number of elements and the information required to describe the largest of them. The number of elements determines the number of rows in the matrix and how many columns correspond to the mechanism topology; for this reason, if the information of the largest element is longer than the total of elements then new rows would be required, modifying the matrix and generating a misinterpretation of it.

7) DESCRIPTION OF THE REPRESENTATION MATRIX

Any planar mechanism can be represented with a matrix generated by the SPM, specifying its topology, dimensions, and translation/rotation in the plane. Fig. 14 highlights the sections of the proposed matrix, where: A)

1) Type of the joints: revolute (4), prismatic (5).
2) Function of the bars: ground (1), input (2), output (3).
3) Dimension of the bars.
4) Dimension of the coupler.
5) Location of the mechanism (rotation and translation.)
6) Dimension of the n-ary links.

III. PRACTICAL IMPLEMENTATION OF THE SANTIAGO-PORTILLA METHOD

A. SUMMARY OF THE DESIGN PROCESS

The process for generating the representation matrix of a planar mechanism with the SPM can be summarized as follows:
A)

1) Identification of the mechanism.
2) Graph representation of the mechanism, with bars indicated as vertices and edges for the joints.
3) Conversion of the graph representation into the adjacency matrix, labeling as 1 every connection between bars.
4) Identification of the joint types by labeling them in the matrix, assigning 4 and 5 for revolute and prismatic types, respectively.
5) Identification of the bar functions with labels 1, 2, and 3 for ground, input, and output, respectively, placing them on the main diagonal.
6) Assignment of the bar dimensions, replacing the labels in the lower triangular matrix with the corresponding lengths. This stage is detailed in Algorithm 1; before this step, only the topology was considered in the representation matrix, so this algorithm is in charge of processing the geometry. When assigning dimensions it is necessary to use the auxiliary table for specifying which bars have already been assigned a value.
7) The matrix is added one column to include the parameters of the coupler point.
8) The matrix is extended one column to include the displacement and rotation of the mechanism.
9) The matrix is extended m columns to include the information of the links with more than one pair of nodes, where m is the number of these links.

Fig. 15 shows a six-bar planar mechanism and its matrix representation, obtained with the SPM.

Algorithm 1 Assignment of Element Dimensions in SPM

```
set to zero every position in the auxiliary table;
for (i = 1 to ne − 1) do
    for (j = i + 1 to ne) do
        A_{j,i} ← 0;
        if (A_{i,j} ≠ 0) then
            aux1 ← j, aux2 ← i;
            if ((r_i = 0 and r_j = 0 and i < j) or (r_i = 1
            or r_j = −1)) then
                aux1 ← i, aux2 ← j; // in
                auxiliary table
            end
            A_{j,i} ← sizebar_{aux1} // assign
            dimension of the bar;
            r_{aux1} ← −1; // in auxiliary
            table;
            if (r_{aux2} = 0) then
                r_{aux2} ← 1; // in auxiliary
                table
            end
        end
    end
end
```

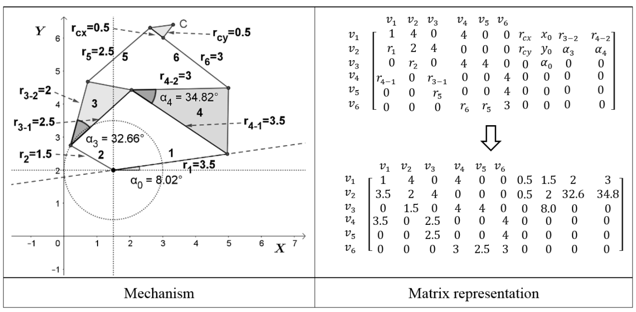

FIGURE 15. Matrix representation of a six-bar Watt-type mechanism.

B. CASE STUDY

The proposed Santiago-Portilla Method for matrix representation of planar mechanisms was applied to a case study corresponding to the design of a trajectory tracker. The path is a straight line described by a set of five precision points, ω={(20,20),(20,25),(20,30),(20,35),(20,40),(20,45)}. The design objective is to find the planar mechanism that follows the trajectory with the minimum error, including its dimensional synthesis.

An constrained optimization problem is proposed to determine the shape and geometry of the mechanism, by minimizing the quadratic error between each precision point and the real position of the coupler; a complete description of such problem is in [38]. Due to this solution approach, the tracking problem may include a higher number of points or different locations for them. It was solved using a metaheuristic algorithm, Differential Evolution (DE), developed by Storn and Price [39]. DE is based in the evolution process of a set of individuals, each one representing a proposed solution. The usual representation of the individuals is as a design vector, but for the study case the algorithm was modified to use the matrix representation obtained with SPM, in such a way that every individual in the population corresponds to a possible mechanism.

DE was also modified to address the problem constraints by applying the rules of Deb [40]. Additionally, a data base was integrated to the algorithm containing the mathematical model of every planar mechanism considered as a valid solution, identifying them by the number of elements, type and number of joints, and corresponding kinematic. Three different configurations of the four-bar mechanism were considered for this case study: crank-slider, crank-conrod-slider, and crank-conrod-rocker.

The general operation of the algorithm is as follows, and its complete pseudocode is in Algorithm 2:

A)

1) A set of individuals is randomly generated, where each individual is a mechanism in a matrix representation corresponding to a proposed a solution.
2) Every individual is analyzed to determine if the matrix represents a mechanism, a structure or an open kinematic chain. In the case of mechanisms, their quality is evaluated by the objective function, while the constraint violation sum (CVS) validates their feasibility; otherwise, the individuals are highly penalized.
3) An offspring is obtained from the initial population, based on the variation operators in the algorithm. DE includes crossover and mutation as its variation operators (lines 8-17 in Algorithm 2).
4) A repair operation is applied on the data in the upper triangular matrix; since those values represent the union types, they are rounded to the closer integer (line 18).
5) These new individuals are evaluated (line 19) and a selection process is carried out, applying the rules of Deb.
6) The stages of offspring production and selection are repeated for an specific number of cycles (generations).

The algorithm is executed thirty times, and the best solution of each one is considered for the final analysis. The algorithm was tuned as follows: population NP=100 individuals, G_{max}=5,000 generations, crossover CR=[0.8,1] for run, and mutation factor F=[0.4,0.8] for generation. The matrix representation shown in Fig. 16A corresponds to the optimum solution obtained for the case study, with an objective function OF=0, representing no deviation between the calculated and the ideal precision points. The resulting four-bar mechanism appears in the part B of the same figure. The matrix includes the complete information of the mechanism, represented with SPM. As can be seen, the topology includes four bars reflected in four vertices (v_1 to v_4), in a crank-conrod-slider configuration with three revolutes and a prismatic joint, labeled with 4 and 5 in the upper triangular matrix, respectively. The dimensions of the

Algorithm 2 Modified DE Algorithm

set algorithm parameters ($Gmax, F, Cr, Np$);
generate randomly $X_i^0, i = 1, \ldots, Np$;
  `// initial population using matrix representation by SPM;`
evaluate initial fitness population for $f\left(X_i^0\right), i = 1, \ldots, Np$;
for ($g = 0; g < Gmax; g{+}{+}$) **do**
  for ($i = 1; i \leq Np; i{+}{+}$) **do**
    select randomly $r_0, r_1, r_2 \in [1, Np]$, with $r_0 \neq r_1 \neq r_2 \neq i$
    select randomly $j_{rand} \in [1, D]$, $k_{rand} \in [1, E]$, `// D and E are matrix dimensions`
    for ($j = 1; j \leq D; j{+}{+}$) **do**
      for ($k = 1; k \leq E; k{+}{+}$) **do**
        generate randomly $rand_j \in [0, 1]$
        if $\left(rand_j \leq Cr\right)$ *or* $\left(j_{rand} = j \text{ and } k_{rand} = k\right)$ **then**
          $U_{i,j,k}^g = X_{r_0,j,k}^g + F\left(X_{r_1,j,k}^g - X_{r_2,j,k}^g\right)$
        else
          $U_{i,j,k}^g = X_{i,j,k}^g$
        end
      end
    end
    repair U_i^g
    evaluate solution $f\left(U_i^g\right)$
    if $f\left(U_i^g\right) < f\left(X_i^g\right)$ `// by rules of Deb` **then**
      $X_i^{g+1} = U_i^g$
    else
      $X_i^{g+1} = X_i^g$
    end
  end
end

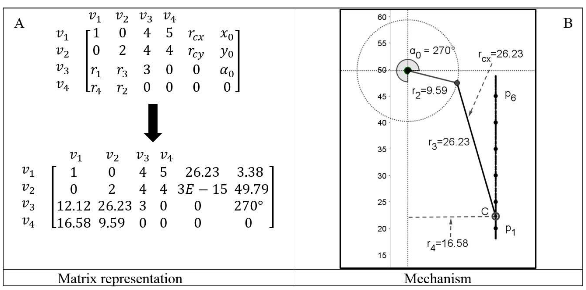

FIGURE 16. Matrix representation of the optimum solution for the case study.

coupler point are the same than the r_3 bar, so they both appear mounted, and the mechanism has a rotation of 270° and a translation of (3.38,49.79) with respect to the origin. Finally, the dimensions of the bars are in the lower triangular matrix.

IV. CONCLUSIONS

In this work, a novel technique for representing planar mechanisms is presented, the Santiago-Portilla Method (SPM). The method uses a matrix for representing any type of planar mechanisms, independently of the number of elements or the link type. The representation matrix contains all the information regarding the topology (types, number and function of bars and joints) and geometry (dimension) of the mechanism, even including the spatial position and translation with respect to the reference coordinate system.

The SPM generates mechanism representations that can be interpreted and processed by computer devices; in this way, it is a useful and flexible tool for a wide variety of engineering applications, that requires a complete characterization of the specific problem . As an example, a case study was solved using the SPM representation in order to carry out simultaneously the topological and dimensional synthesis of a planar mechanism, in a real problem corresponding to a trajectory tracker. The results show a simplification of the design process, since it is effected in a single stage in contrast to the classical approach requiring each synthesis to be developed individually and in a sequential manner.

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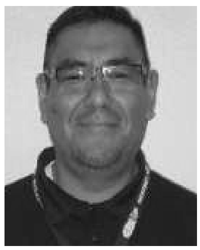

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