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Modeling and Analysis of Aeroelasticity and Sloshing for Liquid Rocket

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ABSTRACT In this paper, based on the advantages of Newtonian mechanics method, mixed coordinate method, and analytical mechanics method, the dynamic model of rigid-aeroelasticity-sloshing coupling is established for longitudinal vibration and liquid fuel sloshing, as well as the coupling between elastic deformation and liquid sloshing and their influence on the liquid rocket motion are analyzed. Considering the coupling effect between aeroelasticity and liquid sloshing, a new flutter analysis model is proposed, and the corresponding flutter analysis is carried out. First, the simplified model of the body and the liquid of the tank are established. By using Newtonian mechanics and analytical mechanics method, the nonlinear structural dynamic model considering the 3-D vibration and liquid sloshing is obtained. Second, influence of rigid-aeroelasticity, rigid-sloshing, aeroelasticity-sloshing, and rigid-aeroelasticity-sloshing coupling is simulated and analyzed. Finally, a new simplified model for the analysis of the flutter of the body is built by taking into account the impact of the liquid fuel sloshing. The results demonstrate that the elastic deformation and liquid sloshing change the motion characteristics of the rocket, and there is a strong coupling between the aeroelasticity and sloshing. Meanwhile, liquid sloshing has different effects on the flutter in all directions.

INDEX TERMS Liquid rocket, elastic body, liquid sloshing, nonlinear dynamic model, rigid-elastic coupling, rigid-sloshing coupling.

I. INTRODUCTION

Due to the increase of weight, thrust and slenderness ratio, the influence of the elastic effect on the vehicle dynamics is more noticeble during the flight [1]–[4]. As the acceleration loads increase, rigid mode, elastic mode and sloshing mode of the liquid rocket coupled together under high axial loads, The interaction among them resulting in the vibrate violently, which threatens the structure of the liquid rocket and the stability of the flight [5]. On the other hand, in order to meet the requirements of large thrust, the large liquid projectile rocket usually carries a number of liquid fuel tanks. The fuel sloshing will produce additional inertial force and torque. Once the inertial excitations exceed the system capability, the control system will be unstable and the structure may be destroyed [6], [7]. Considering these reasons, it is necessary to establish a nonlinear dynamic model considering the rigid-aeroelasticity-sloshing coupling. Note that, the factors of sloshing are highlighted when establishing the flutter analysis model of the liquid rocket. The fundamental problem to be addressed is the dynamic modeling of the complex elastic coupling system. Waszak *et al.* [8] and Schmidt [9] used the Lagrangian equation to establish the flight dynamics model of the elastic vehicle under the mean axes. The mean axes will result elastic translational and rotational equations of motion are formally identical in form to those of the flight dynamics model obtained in the rigid body approximation, except the expression of the aerodynamic load is different. Although the use of an average axis can indeed simplify the equations of motion, the cost of the simplification is huge. Actually, it is difficult to define the limit condition of the mean axis, thus it can not determine the specific direction of the coordinate axis and the size of the inertia. In addition, the aerodynamic loads must also be provided on the same average shafting by using the mean axes, which is also a rather cumbersome task. Du *et al.* [10] derived the vibration equation of the rocket body by using the vibration equation of the beam. This method is simple, clear and easy to understand, but the drawback is that it is necessary to eliminate the binding force and easy to add and leak the inertia force, especially for the complex elastic deformation of the bundled rockets.

The previous studies mainly focus on the elastic vibration, and the impact of fuel sloshing is not accounted. In order

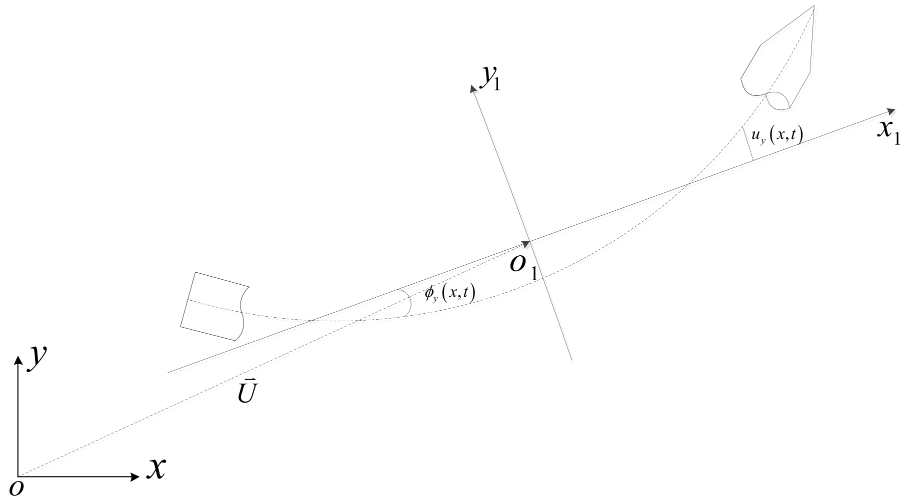

FIGURE 1. The bending deformation of rocket in the pitch plane.

to solve the sloshing problem in the course of spacecraft docking, Bayle [11] used the CFD method and the equivalent mechanical model method to obtain the force of sloshing on the spacecraft, respectively. The results of these two methods are agree well with each other and show the validity of the equivalent mechanical model of liquid sloshing. Krishnaswamy *et al.* [12] and Shekhawat *et al.* [13] analyzed the influence of rocking on the stability of rocket attitude motion based on the kinetic equation of the rocket plane motion by using the sloshing equivalent mechanics model and the rigid body model. However, they did not consider the impact of aeroelasticity. Based on the additional mass method and the mass effect of the longitudinal and lateral propellant in the liquid filled tank, Zhongwen *et al.* [14], [15] established a dynamic model of the longitudinal, lateral and torsion of the liquid rocket. Li *et al.* [16] derived the rigid and elastic coupled equations of motion using the Lagrangian approach. The full process of calculating the trajectory and vibration characteristics of a spinning flexible liquid rocket is explained. Noorian *et al.* [17] developed a numerical model for investigation of coupled dynamics of fuel contained elastic liquid rocket in planar atmospheric flight. A reduced order boundary element model is employed to model the liquid sloshing in tanks. The interaction of sloshing and aeroelasticity is studied using stability analysis of the coupled system. Results show that the slosh–aeroelastic coupling in an elastic liquid rocket occurs for low tank filling ratios and may lead to decreasing the system damping.

In this paper, based on the advantages of Newtonian mechanics method, mixed coordinate method and analytical mechanics method, the dynamic model of rigid-aeroelasticity-sloshing coupling is established for longitudinal, lateral, torsional vibration and liquid fuel sloshing. This ensures that the model has a clear physical meaning and does not miss a particular item. At the same time, numerical simulation is carried out to analyze the influence of the coupling of rigid-aeroelasticity, rigid-sloshing, aeroelasticity-sloshing and rigid-aeroelasticity-sloshing. Finally, in order to study the flutter characteristics of liquid rocket conveniently, a new longitudinal-transverse-torsion flutter analysis model considering both aeroelasticity and liquid sloshing is proposed. Based on this model, the flutter equation and simulation analysis are carried out.

This paper is organized as follows. Section II contains the establishment of rigid-aeroelasticity-sloshing model and some necessary preliminary results. In Sect. III, the simulation analysis results are given. In Sect. IV, a new model for flutter analysis is proposed, and a numerical simulation result is used to demonstrate the performance of the model. The final section contains conclusions.

II. RIGID-AEROELASTICITY-SLOSHING MODEL

A. GENERAL DYNAMIC EQUATION AND ELASTIC EQUATION

As shown in Figure 1, a fixed global frame oxy is firstly defined. A body frame $o_1x_1y_1$ is then built in the global frame to describe the vehicle position and orientation, with the origin at the center of mass. x_1 points to the forward, y_1 is perpendicular to the x_1 in the pitch plane, and z_1 is cross product of x_1 and y_1.

In the modeling process, the liquid rocket will be simplified to a one-dimensional beam model for the study of elastic vibration. We have

$$m\frac{d\vec{V}}{dt} = \vec{F} \tag{1}$$

$$\frac{d\vec{H}}{dt} = \vec{M} \tag{2}$$

where $\vec{V}$ represents the velocity vector of the origin of body axis system relative to global frame. $\vec{F}$ denotes the force vector acting on the liquid rocket. $\vec{H}$ and $\vec{M}$ are angular momentum and moment, respectively.

Here, we choose the natural mode of the beam as the generalized coordinates. Taking the elastic displacement in Y direction as an example, the linear displacement and the angular displacement at arbitrary point can be represented by the following series:

$$u_y(x,t) = \sum_{i=1}^{\infty} q_{iy}(t)\,\Phi_{iy}(x) \tag{3}$$

$$\phi_y(y,t) = u'_y(y,t) = \sum_{i=1}^{\infty} q_{iy}(t)\,\Phi'_{iy}(x) \tag{4}$$

In the formula, $\Phi_{iy}(x)$ is the i-th natural vibration mode (elastic DOF); q_{iy} is the i-th order generalized coordinates; $u'_y(y,t)$ and $\Phi'_{iy}(x)$ represent the derivative of x. The generalized coordinate form of the elastic vibration equation is established by using Eq. (3) and Lagrange mechanics method,

$$\ddot{q}_{iy} + 2\xi_{iy}\omega_{iy}\dot{q}_{iy} + \omega_{iy}^2 q_{iy} = \frac{Q_{iy}}{\bar{m}_{iy}} \tag{5}$$

where Q_{iy} and $\bar{m}_{iy}$ are the generalized force and mass. ω_{iy} and ξ_{iy}, in sequence, denotes the natural frequency and damping coefficient of the i-th order mode. Generalized force is obtained via the virtual work principle. Q_{iy} and $\bar{m}_{iy}$ are given as

$$Q_{iy} = \int_0^l f_y(x,t)\Phi_{iy}(x)\,dx \tag{6}$$

$$\bar{m}_{iy} = \int_0^l m(x,t)\Phi_{iy}^2(x)\,dx \tag{7}$$

where $f_y(x,t)$ and $m(x,t)$ are the force in the Y direction and the mass per unit length, respectively. l is the length of the rocket. Note that, the elastic equations in X and Z directions can be obtained in the same method.

B. SLOSHING EQUATION

In Figure 2, $o_p x_p y_p$ is the sloshing coordinate system fixed on the tank. The origin is the initial position of the sloshing mass. x_p is parallel to the $x_1 \cdot y_p$ is perpendicular to the x_p in the pitch plane. z_p is cross product of x_p and y_p.

The dynamic characteristics of the sloshing liquid can be closely approximated by the equivalent dynamic model in the form of spring-mass-damper system. The parameters are the functions of the tank shape, liquid level, etc. m_p, Ω_p, ζ_p are sloshing mass, sloshing frequency and damping ratio of the p-th tank, respectively. By the Rayleigh formula, the equivalent stiffness coefficient k_p and the sloshing inertia tensor I_p of sloshing are converted. The kinetic energy of liquid sloshing is

$$T = \sum_{p=1}^{p} \frac{1}{2} m_p \left(\dot{\vec{U}} + \dot{\vec{u}}_p + \dot{\vec{r}}_p\right)^2 + \sum_{p=1}^{p} \frac{1}{2} m_p \left[\vec{\omega} \times \left(\vec{u}_p + \vec{r}_p\right)\right]^2 \tag{8}$$

Here, $\vec{\omega}$ is angular velocity vector; $\vec{u}_p$ is the position vector of sloshing mass block in $o_p\_x_p y_p z_p$ coordinate system; $\vec{r}_p$ is the position of the $o_p x_p y_p$ frame resolved in the body frame. Potential energy is

$$U = \frac{1}{2} \sum_{p=1}^{p} k_p (y_p^2 + z_p^2) \tag{9}$$

where y_p and z_p are the components of $\vec{u}_p$ in the $o_p\_x_p y_p z_p$ coordinate system, respectively.

The dissipation energy of the system is

$$D = \frac{1}{2} \sum_{p=1}^{p} m_p \zeta_p k_p (\dot{y}_p^2 + \dot{z}_p^2) \tag{10}$$

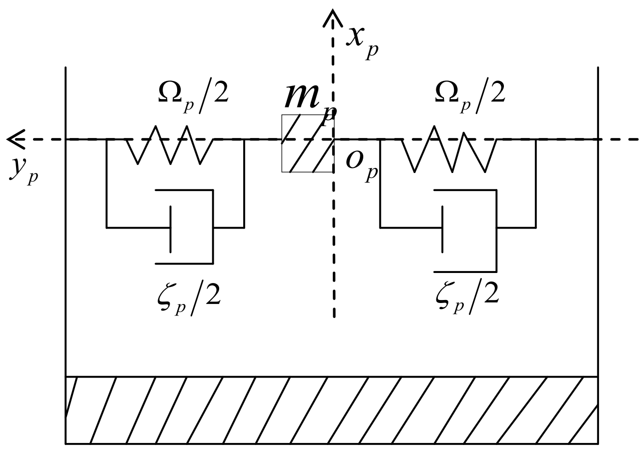

FIGURE 2. Equivalent mechanical model of liquid sloshing.

$$\begin{aligned} m_p\left[\ddot{y}_p + \dot{V}_y + X_p\dot{\omega}_z - Z_p\dot{\omega}_x + \sum_{i=1}^{\infty}\Phi_{iy}\left(x_p\right)\ddot{q}_{iy}\right] - m_{p1}Z_{p1}\sum_{i=1}^{\infty}\Phi_{ix}\left(x_{p1}\right)\ddot{q}_{ix} + 2m_p\zeta_p k_p\dot{y}_p \\ + k_p y_p - m_p V_x\omega_z + m_p V_z\omega_x - m_p V_x\sum_{i=1}^{\infty}\Phi'_{iy}(x,t)\,\dot{q}_{iy} - m_{p1}V_z\sum_{i=1}^{\infty}\Phi_{ix}\left(x_{p1}\right)\ddot{q}_{ix} = Q_{yp} \end{aligned} \tag{11a}$$

$$\begin{aligned} m_p\left[\ddot{z}_p + \dot{V}_z + Y_p\dot{\omega}_x - X_p\dot{\omega}_y + \sum_{i=1}^{\infty}\Phi_{iz}\left(x_p\right)\ddot{q}_{iz}\right] - m_{p1}Y_{p1}\sum_{i=1}^{\infty}\Phi_{ix}\left(x_{p1}\right)\ddot{q}_{ix} + 2m_p\zeta_p k_p\dot{z}_p \\ + k_p z_p - m_p V_x\omega_y + m_p V_y\omega_x - m_p V_x\sum_{i=1}^{\infty}\Phi'_{iz}(x,t)\,\dot{q}_{iz} - m_{p1}V_y\sum_{i=1}^{\infty}\Phi_{ix}\left(x_{p1}\right)\ddot{q}_{ix} = Q_{zp} \end{aligned} \tag{11b}$$

Thus, the sloshing equation can be obtained by the Lagrange equation (11), shown at the bottom of the previous page.

C. FORCE, MOMENT AND GENERALIZED FORCE

This part will focus on the derivation of the force and moment acting on the liquid rocket, e.g., gravity, thrust, engine swing inertia force, aerodynamic force, aerodynamic damping force, and sloshing force. In the fixed global frame, the expression of gravity is

$$\vec{G} = \begin{bmatrix} 0 \\ -m\vec{g} \\ 0 \end{bmatrix} \tag{12}$$

In order to facilitate the derivation of the thrust of the engine, the local elastic coordinate system $o_e\_x_e y_e z_e$ is introduced. Local elastic coordinate system is shown in Figure 3 and 4.

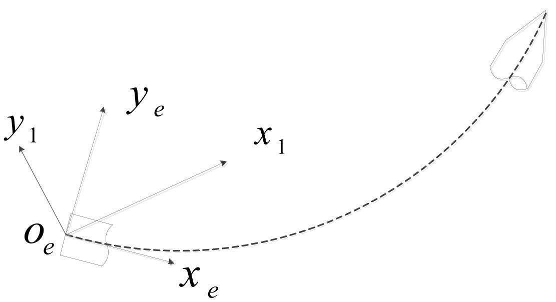

FIGURE 3. Local elastic coordinate system.

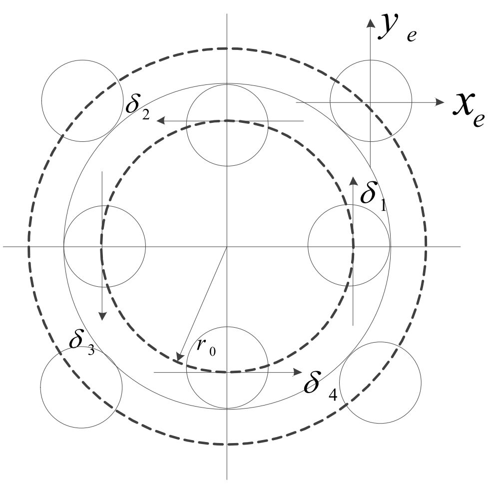

FIGURE 4. Local elastic coordinate system in roll plane.

The local elastic coordinate system is fixed on the engine nozzle. In the body frame, the three translational components of the elastic deformation belonging to engine position are $u_x(x_F)$, $u_y(x_F)$, $u_z(x_F)$, and the three rotational components are $R_x(x_F)$, $R_y(x_F)$, $R_z(x_F)$. Then we can get the transformation matrix C_e^B,

$$C_e^B = \begin{bmatrix} 1 & R_z(x) & -R_y(x) \\ -R_z(x) & 1 & R_x(x) \\ R_y(x) & -R_x(x) & 1 \end{bmatrix} \tag{13}$$

Here, the transformation matrix of boost and core engine will be the same one. The component of the thrust in the local elastic coordinate system is

$$\vec{P}_e = \begin{bmatrix} \vec{P} \\ 0 \\ 0 \end{bmatrix} \tag{14}$$

We can obtain the expression of thrust in the body coordinate system by the transformation matrix,

$$\vec{P}_B = C_e^B M_\delta \vec{P}_e \tag{15}$$

where M_δ is swing angle transformation matrix of rocket engine. The displacement in the body coordinate of engine is

$$\vec{r}_B = C_e^B \vec{r}_e \vec{u}(x_F) \tag{16}$$

where $\vec{u}(x_F)$ is elastic displacement. Then, the thrust torque can be obtained as

$$\vec{M}_p = \vec{P}_B \vec{r}_B \tag{17}$$

Next, we will derive the inertia force of the engine swing. The inertia force of the engine includes two parts. The first one is the swing of the engine itself, which plays the role of controlling the attitude motion of the rocket. The second one is caused by the rotation of the rocket. By D'Alembert's principle, we can obtain

$$\vec{I}_{core} = -m_{core}\ddot{\vec{\delta}} \times \vec{r}_{core-e} - m_{core}\dot{\vec{\omega}} \times \vec{r}_{core-B} \tag{18a}$$

$$\vec{I}_{booster} = -m_{booster}\dot{\vec{\omega}} \times \vec{r}_{booster-B} \tag{18b}$$

where $\ddot{\vec{\delta}}$ and $\dot{\vec{\omega}}$ are the acceleration of the engine swing and the rotation of the rocket. $\vec{r}_{core-e}$, $\vec{r}_{core-B}$ are the displacement in local elastic coordinate system and body coordinate system. The total engine swing inertia force is

$$\vec{I} = \vec{I}_{core} + \vec{I}_{booster}$$

Similar to the engine swing inertia force, sloshing force is also composed of two parts. One part is the relative inertia force, the other is the convected inertial force. We can get the total sloshing inertia force as

$$\vec{S} = -\sum_{p=1}^{p} \left(m_p \ddot{\vec{r}}_{p-S} + m_p \dot{\vec{\omega}} \times \vec{r}_{p-B} \right) \tag{19}$$

where $\vec{r}_{p-S}$, $\vec{r}_{p-B}$ are the displacement vector of tank in sloshing coordinate system and body coordinate system. Then, the sloshing moment will be obtained

$$\vec{M}_s = \vec{S}\left(\vec{r}_{p-S} + \vec{r}_{p-B}\right) \tag{20}$$

Finally, we will derive the aerodynamic and aerodynamic damping forces of the liquid rocket in the velocity coordinate system. The velocity coordinate system origin O_1 is located on the instantaneous center of mass of the rocket. O_1X_V axis coincides with the velocity vector $\vec{V}$ of the centroid of the body, and O_1Y_V axis is located in the OXY plane of the body coordinate system, which is perpendicular to O_1X_V axis. O_1Z_V axis is determined by the right hand rule. Aerodynamic and aerodynamic damping force calculation is mainly based on the strip principle. Elastic deformation can produce additional flight attitude which will affect the angle of attack and sideslip angle. From Figure 1, we can get the additional angle of attack caused by elastic deformationčž

$$\alpha_1(x) = \sum_{i=1}^{n} \Phi'_{iy}(x)\, q_{iy}(t) \tag{21}$$

The aerodynamic damping angle caused by the rotation of the body and the elastic deformation velocity is

$$\alpha_2(x) = -\omega_z \frac{x}{V} - \frac{\dot{u}(x, y, z, t)}{V} \tag{22}$$

So we obtain total angle of attack

$$\alpha_T(x) = \alpha + \sum_{i=1}^{n} \Phi'_{iy}(x)\, q_{iy}(t) - \omega_z \frac{x}{V} - \frac{\dot{u}(x, y, z, t)}{V} \tag{23}$$

According to the hypothesis of strip theory, the lift coefficient $C_A(x)$ of the vehicle is

$$C_A(x) = \frac{dC_A}{d\alpha}\alpha(x) = C'_A\alpha(x) \tag{24}$$

Thus, the lift of the wing strip whose width is x is

$$d_A(x) = \frac{\rho}{2} V_\infty^2 l(x)\, C'_A \alpha(x)\, dx \tag{25}$$

Taking Eq. (24) into Eq. (25)then we can obtain the total lift A after the integration

$$A = \int_0^l \frac{\rho}{2} V_\infty^2 l(x)\, C'_A \left(\alpha + \sum_{i=1}^{n} \Phi'_{iy}(x)\, q_{iy} - (t) \right.$$
$$\left. \omega_z \frac{x}{V} - \frac{\dot{u}(x, y, z, t)}{V} \right) dx \tag{26}$$

Lateral aerodynamic force and aerodynamic damping force can be obtained by the similar method. Put the expression of forces in Eq. (6), we can obtain generalized forces. Then elastic vibration equation will be obtained by substitute the generalized forces in Eq. (5). Finally, a nonlinear dynamic model of the rocket considering elastic vibration and sloshing is established.

III. SIMULATION, RESULTS AND ANALYSIS

To illustrate the effects of elastic vibration, liquid fuel sloshing and bouncing coupling on the rocket body, the analyses based on the above cases are given below. The simulations can be used to design the subsequent rocket control system with elastic vibration, liquid shaking and their coupling, so as to suppress the system coupling and improve the stability and control performance of the rocket (Control system is not discussed in this article.). It also provides a basis for the simplified flutter model and analysis of liquid sloshing.

A. COMPARISON OF SIMULATION RESULTS BETWEEN RIGID AND RIGID-AEROELASTICITY-SLOSHING MODEL

Figure 5 shows the change of the angle of attack (a) and side slip (b). It can be seen from the figure that in the early stage of simulation, under the combined action of elastic deformation and fuel sloshing, the angle of attack after elastic deformation fluctuates relatively to the rigid body, and the side slip angle has a small deviation. At the end of the simulation, the angle of attack is far away from the rigid body model curve, and the side slip angle also has a big change. This indicates that in the absence of control input, the elastic deformation gradually increases and eventually diverges.

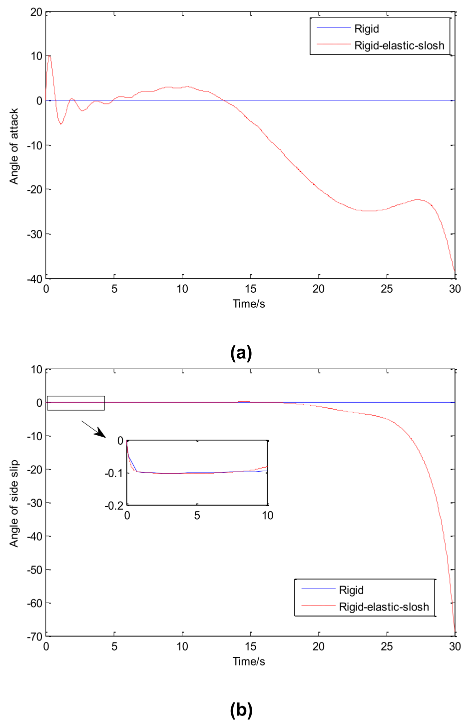

FIGURE 5. Simulation results of angle of attack and side slip.

Figure 6 is a comparison of the component of the force and moment act on the liquid rocket under the rigid body model and the rigid-aeroelasticity-sloshing model. In the start stage of simulation, due to the influence of elastic deformation and fuel sloshing, a larger additional force appears in the Y direction. However, in X or Z direction, the force curve does not change much. It is shown that the elastic deformation is larger in Y direction. This is also consistent with the simulation results of attitude angle. The additional load on Y axis has a great influence on the moment of X and Z axes, which results in the fluctuation of torque. It can be consulted on Figure 7 that the angular velocity of X axis fluctuates with respect to the angular velocity of the rigid model. This fluctuation is affected by the change of X axis torque. Z axis also produces a larger change, while Y axis of the amplitude is relatively small.

Figure 8 is the aerodynamic and aerodynamic damping force simulation results. Aerodynamic force is the main force

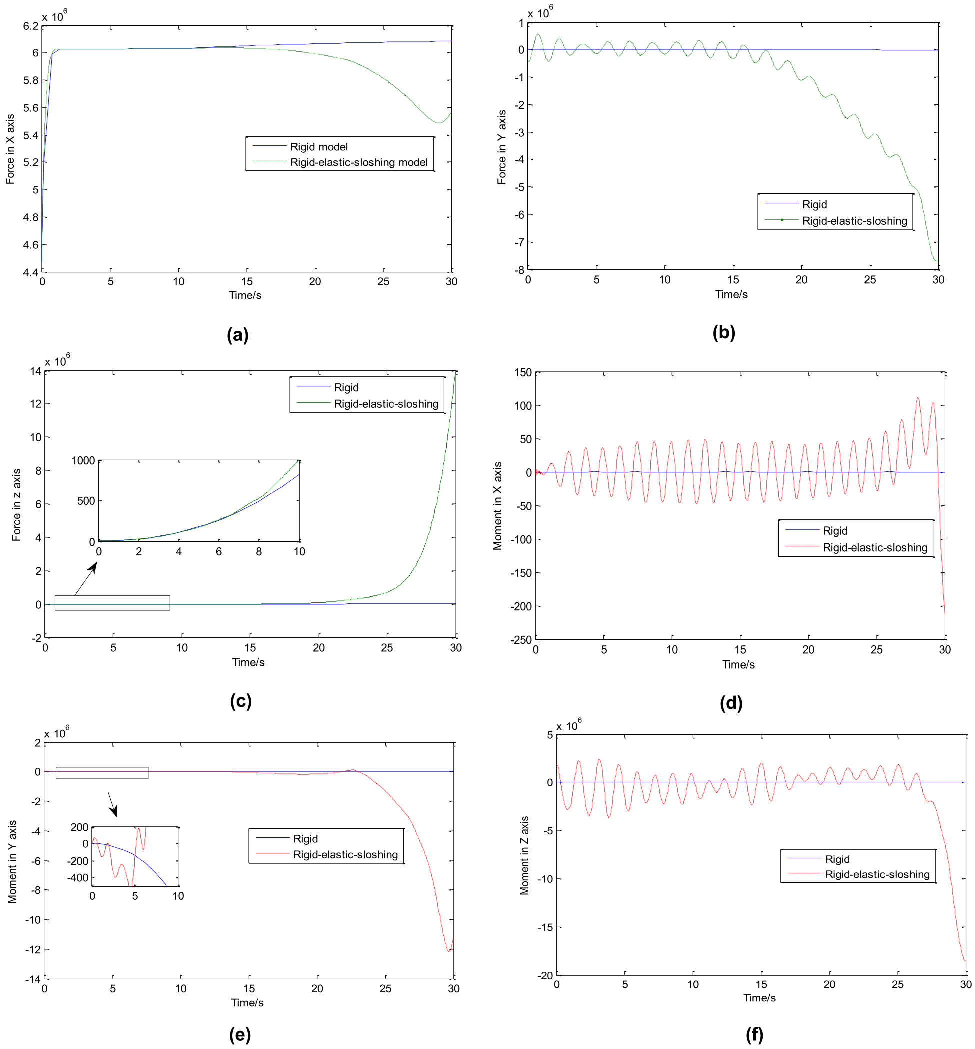

FIGURE 6. **The total force and moment in rigid and rigid-elastic-sloshing model.**

in the direction of Y and Z. From the simulation results, it is found that the elastic deformation and liquid sloshing have a significant influence on the aerodynamic force in the Y direction, while the influence of the Z direction is smaller. This demonstrates that in Y, Z directions, the influence of the aeroelasticity-sloshing is not symmetrical.

B. ANALYSIS OF THE INFLUENCE OF AEROELASTICITY

Next, we will analyze the simulation results of the elastic vibration of the liquid rocket. Figure 9 shows the first 4 mode shapes of the liquid rocket.

Figure 10 shows the simulation results of the first 4 order generalized coordinates in Y and Z axes, the first order is dominant in the first four order generalized coordinates, which makes the greatest contribution to the elastic deformation. However, in the X axis direction, the fourth order generalized coordinates occupies the dominant position.

Figures 11 and 12 are additional aerodynamic and damping forces caused by elastic deformation. The coupling of elastic vibration and rigid body is carried out by means of generates additional aerodynamic forces and moments. Elastic deformation of the body will produce additional aerodynamic, and this part of aerodynamic force will cause the body to produce elastic deformation at the same time. This positive feedback effect will eventually lead to the flutter divergence.

C. ANALYSIS OF LIQUID SLOSHING

In this section, we will analyze the effect of sloshing on the liquid rocket. In order to facilitate the study, only two tanks were selected for simulation analysis.

Figure 13 is the sloshing displacement of the liquid in two tanks. Figure 14 is the sloshing inertia force and moment. It can be observed in Figure 13 that the sloshing in the Y direction is significantly stronger than that in the Z direction. The effect of sloshing in Z-axis is negligible relative to the overall motion of the liquid rocket. From the force and torque diagram, it can also be seen that in the Y direction of the inertial force is stronger than that of the Z direction, while

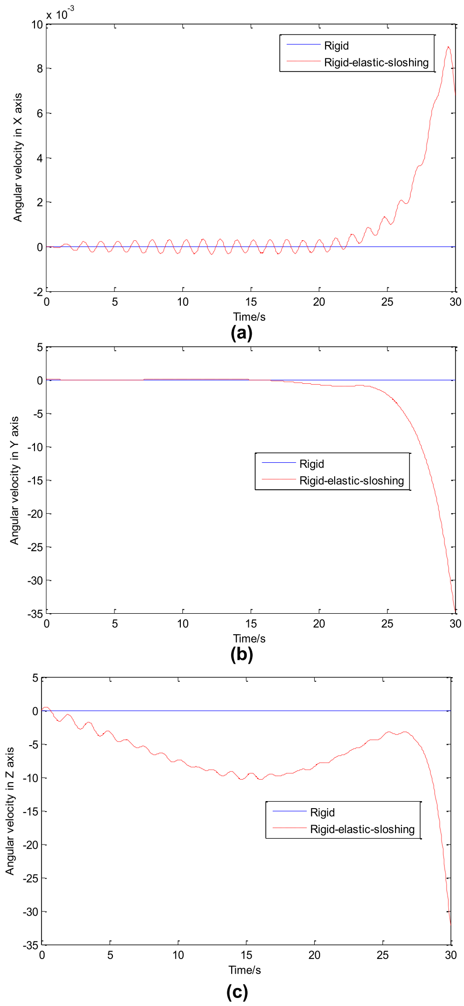

FIGURE 7. Angular velocity.

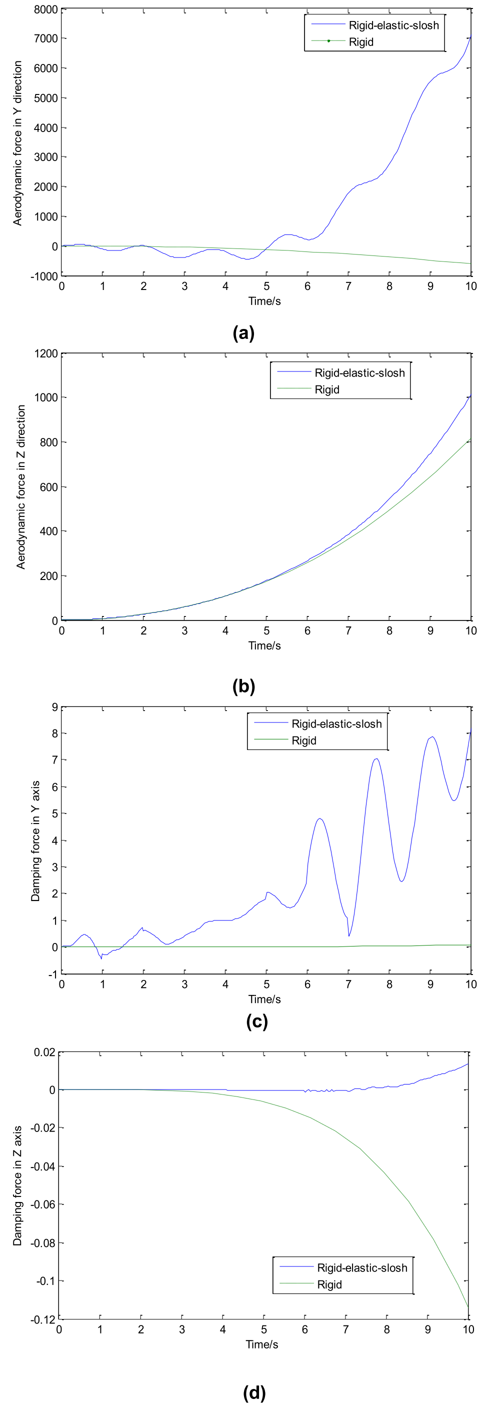

FIGURE 8. Aerodynamic and damping force in two models.

the Z-axis torque is much stronger than those of the X, Y directions. This shows that the pitch plane is more strongly affected by the sloshing. This also explains the change of angle of attack in Figure 5 is stronger than the side slip angle. At the same time, it is shown that the effect of liquid sloshing cannot be ignored.

Figure 15 shows the sloshing inertia force and aerodynamic force in Y and Z directions. In the Y direction, it can be seen that the sloshing inertia force plays a dominant role at the beginning of the simulation. However, with the movement of the liquid rocket, the aerodynamic force increases gradually, and the effect of the inertial force is gradually replaced.

D. COUPLED ANALYSIS OF AEROELASTICITY AND LIQUID SLOSHING

There are strong couplings between aeroelasticity-sloshing and rigid body and between aeroelasticity and liquid sloshing. Elastic deformation of the body is not only affected by aeroelasticity, but also affected by the liquid fuel sloshing. On the other hand, the sloshing of liquid fuel is also affected by the elastic deformation.

Figure 16 shows the comparison of the first order generalized coordinates in the rigid-aeroelasticity and rigid-aeroelasticity-sloshing model. It is found that the liquid sloshing enhances the elastic vibration. The coupling between sloshing and aeroelasticity is realized by the additional load caused by sloshing inertia force and elastic vibration.

Figure 17 is the simulation of the generalized force of sloshing. Relative to Z axis, the liquid sloshing on Y axis

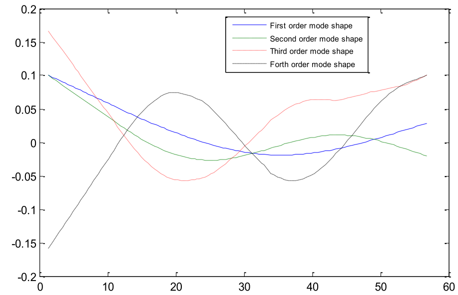

FIGURE 9. The first 4 order modes of rocket.

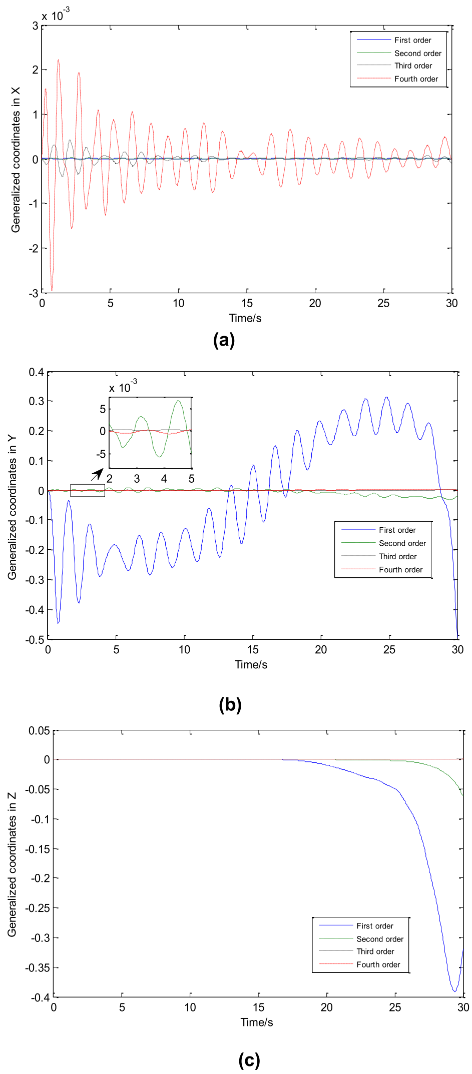

FIGURE 10. Generalized coordinates in X, Y and Z axis.

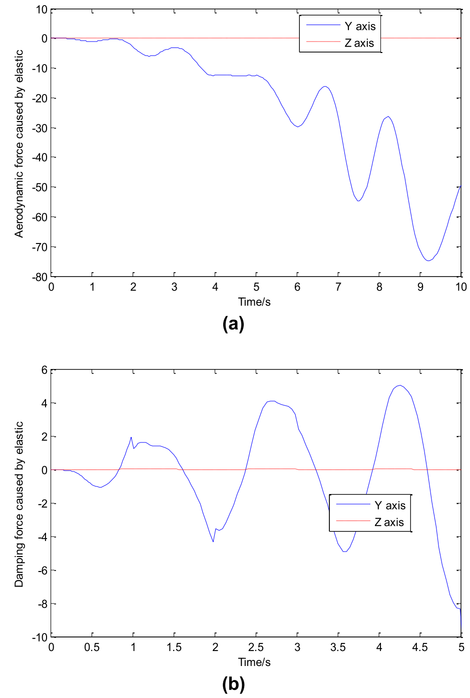

FIGURE 11. Aerodynamic and damping force caused by elastic.

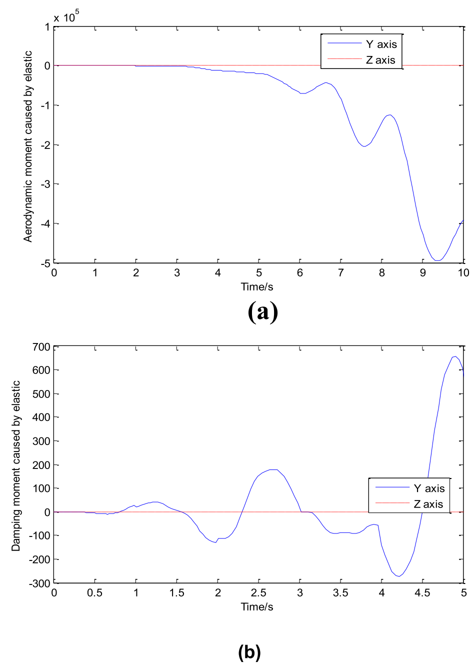

FIGURE 12. Aerodynamic and damping moment caused by elastic.

produces a stronger generalized force. Because of the moment of inertia force in the direction of X axis, the generalized force is produced in the direction of X axis, which affects the torsional vibration of the rocket.

The simulation result of Figure 18 is an additional term caused by elastic deformation in the sloshing equation. The elastic vibration acceleration and the elastic rotation angle will induce the additional load on the sloshing direction, which will affect the dynamic characteristics of the sloshing.

As we seen above, the coupling of sloshing and aeroelasticity has great influence on the stability of the rocket, the interaction between sloshing and aeroelasticity cannot be ignored. To facilitate the study of the interaction between

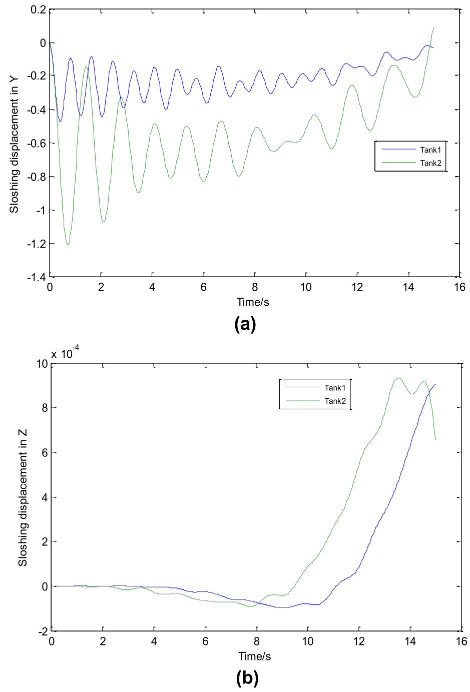

FIGURE 13. Sloshing displacement.

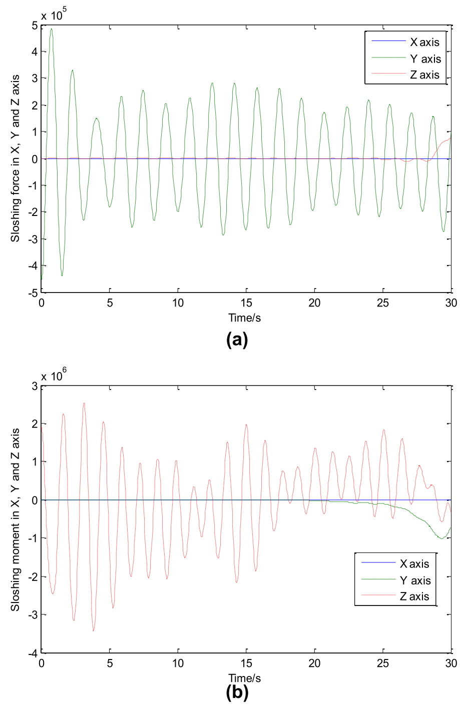

FIGURE 14. Sloshing force and moment.

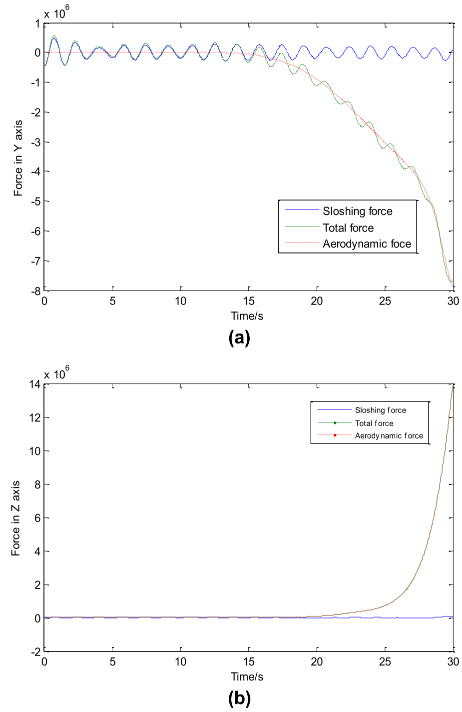

FIGURE 15. Force in Y and Z direction.

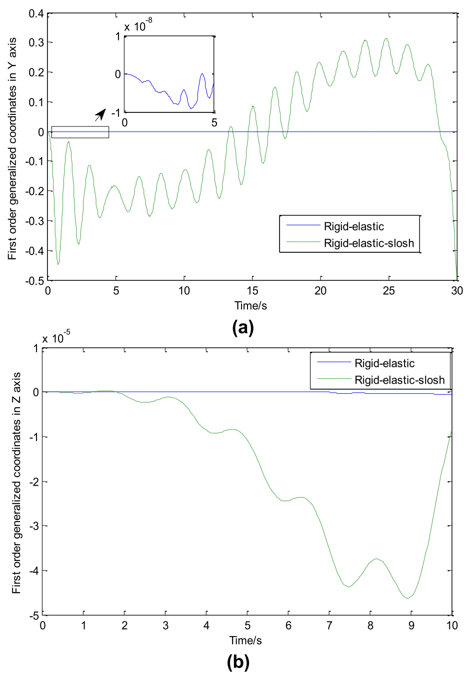

FIGURE 16. First order generalized coordinates in Y and Z axis.

sloshing and aeroelasticity, a simplified analytical model is proposed.

IV. FLUTTER ANALYSIS CONSIDERING FUEL SLOSHING

In the usual sense, the flutter analysis of liquid rocket is mainly to study the elastic vibration of the body. However, it can be seen from these discussions that the inertia force of the fuel sloshing has an important influence on the elastic vibration of the liquid rocket. Therefore, in this part, we will analyze the rocket's flutter problem by taking into account the factors of fuel sloshing based on the rigid-aeroelasticity-sloshing model.

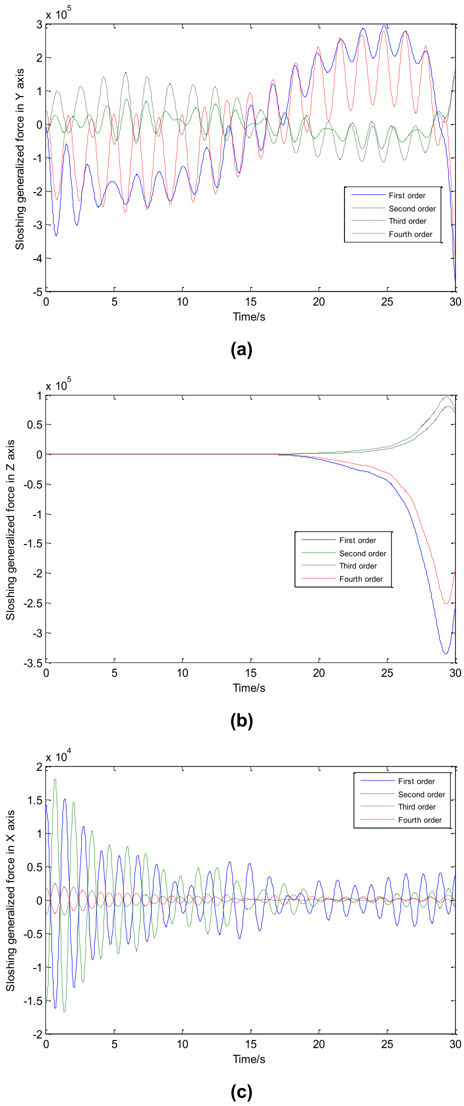

FIGURE 17. **Sloshing generalized force.**

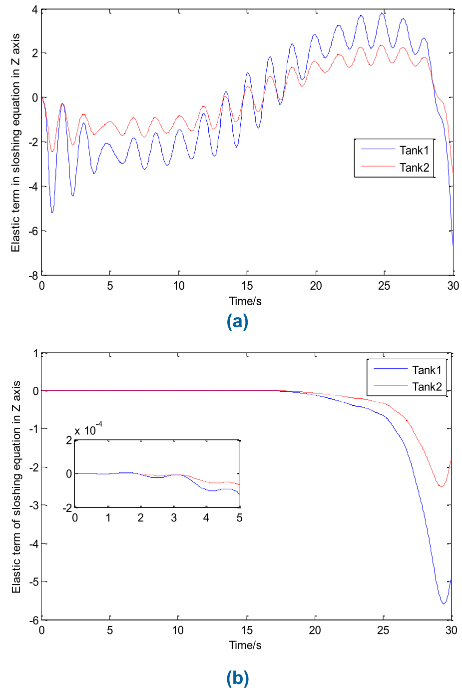

FIGURE 18. **Elastic term in sloshing equation.**

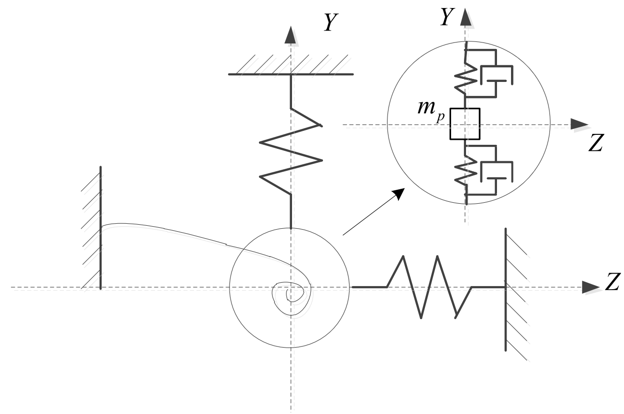

FIGURE 19. **Simplified structure model of rocket flutter analysis.**

Based on the previous study, the impact of sloshing in the Z direction is small. Therefore, we ignore the sloshing in the Z direction for simplification.

We divided the liquid rocket along the X-axis, and took an element of them as the research object. The two-dimensional flutter analysis model is established in Figure 19. There are four degrees of freedom in the model, i.e., three degrees of freedom of elastic and a degree of freedom of sloshing. We use h_1, h_2, γ, h_3 to express these four degrees of freedom. When the liquid sloshing is not considered, the kinetic energy of the mass element dm_c with a distance ξ_c from the origin is

$$dW_c = \frac{1}{2}\left(\dot{h}_1 + \dot{h}_2 + \xi_c \dot{\gamma}\right) dm_c \tag{27}$$

The total kinetic energy W_c is obtained by integral along the cross-section of the liquid rocket as

$$W_c = \frac{1}{2}m_c\dot{h}_1^2 + \frac{1}{2}m_c\dot{h}_2^2 + \frac{1}{2}I\dot{\gamma}^2 + S_c\left(\dot{h}_1\dot{h}_2 + \dot{h}_1\dot{\gamma} + \dot{h}_2\dot{\gamma}\right) \tag{28}$$

Here, m_c is the mass of the element. I is the inertia of the liquid rocket about the axial direction. S_c is the sectional moment.

Considering the kinetic energy generated by the equivalent mass of the sloshing m_p, there is

$$W_{m_p} = \frac{1}{2}m_p\left[\left(\dot{h}_1 + \dot{h}_2\right)^2 + (\dot{\gamma}h_3)^2 + \dot{h}_2^2\right] \tag{29}$$

Therefore, the total kinetic energy is

$$W = W_c + W_{m_p} \tag{30}$$

Using $K_{h_1}, K_{h_2}, K_\gamma, K_{h_3}$ to represent the spring constant, the system potential energy is

$$U_c = \frac{1}{2}K_{h_1}h_1^2 + \frac{1}{2}K_{h_2}h_2^2 + \frac{1}{2}K_\gamma\gamma^2 + \frac{1}{2}K_{h_3}h_3^2 \tag{31}$$

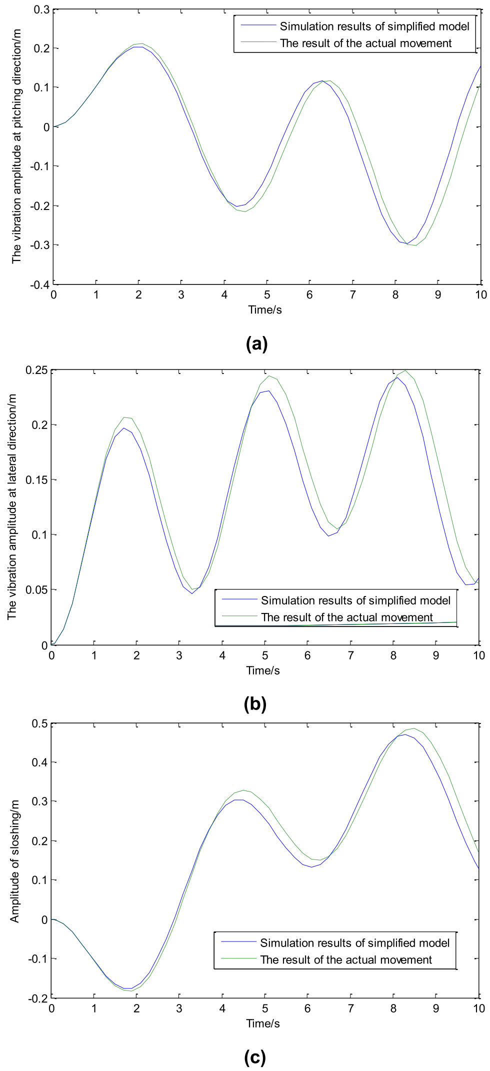

FIGURE 20. Comparison of simulation results between simplified flutter model and rigid-elastic-sloshing model.

If the damping coefficient of the equivalent model is c, the dissipated energy of the system will be

$$\zeta = \frac{1}{2}c\dot{h}_3^2 \tag{32}$$

Substituting potential energy, kinetic energy and dissipated energy expressions into Lagrange equation, we can obtain the flutter equation considering liquid fuel sloshing:

$$m_c\ddot{h}_1 + S_c\ddot{h}_2 + S_c\ddot{\gamma} + m_p\ddot{h}_1 + m_p\ddot{h}_3 + K_{h_1}h_1 = Q_{h_1} \tag{33a}$$

$$m_c\ddot{h}_2 + S_c\ddot{h}_1 + S_c\ddot{\gamma} + m_p\ddot{h}_2 + K_{h_2}h_2 = Q_{h_2} \tag{33b}$$

$$I\ddot{\gamma} + S_c\ddot{h}_1 + S_c\ddot{h}_2 + m_p\ddot{\gamma}h_3^2 + 2m_p\dot{\gamma}\dot{h}_3h_3 + K_\gamma\gamma = Q_\gamma \tag{33c}$$

$$m_p\ddot{h}_1 + m_p\ddot{h}_3 - m_p\dot{\gamma}^2h_3 + c\dot{h}_3 + K_{h_3}h_3 = Q_{h_3} \tag{33d}$$

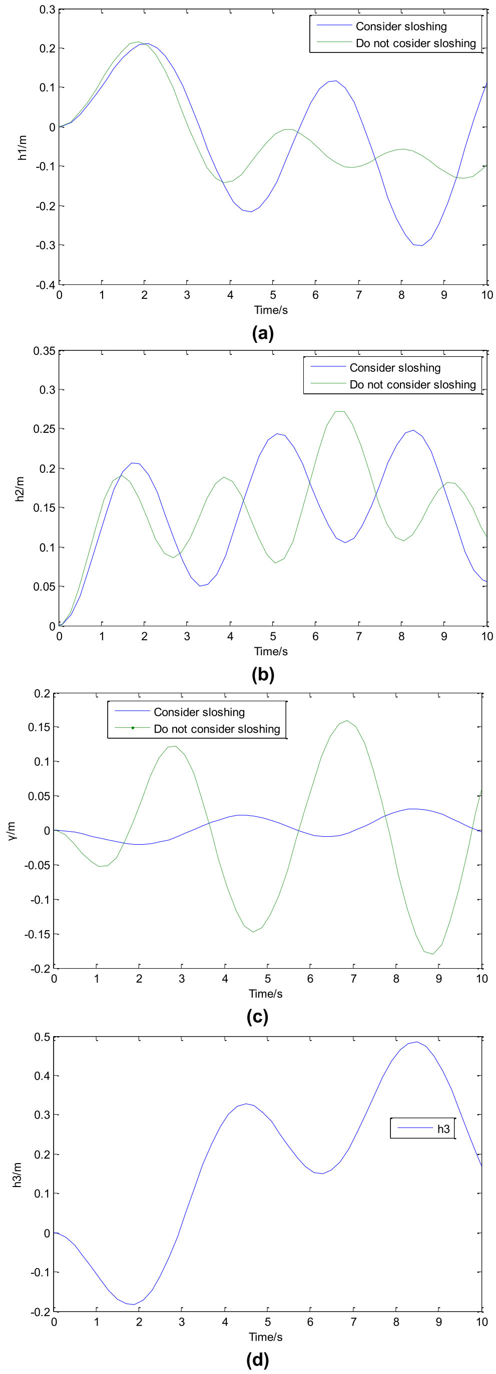

FIGURE 21. Flutter simulation results.

In these formulas, Q_{h_1}, Q_{h_2}, Q_γ, Q_{h_3} represent the generalized force of the corresponding generalized coordinates. We assume that the essence of these generalized forces is aerodynamic force, sloshing inertia force and flutter inertia force. The lift force, the lateral aerodynamic force, the inertia force in the Y direction, the aerodynamic moment of the X axis, the inertia moment of sloshing and the flutter inertia force in the direction of Y are described by A, B, S_Y, M_A, M_S, and C_Y, respectively. The aerodynamic formula can be obtained by strip theory. By the principle of virtual work, we can get

$$\delta E = (A + S_Y)\,\delta h_1 + B\delta h_2 + (M_A + M_S)\,\delta\gamma + C_Y \delta h_3 \tag{34}$$

Then

$$Q_{h_1} = \frac{\delta E}{\delta h_1} = A + S_Y \tag{35a}$$

$$Q_{h_2} = \frac{\delta E}{\delta h_2} = B \tag{35b}$$

$$Q_{\gamma} = \frac{\delta E}{\delta \gamma} = M_A + M_S \tag{35c}$$

$$Q_{h_3} = \frac{\delta E}{\delta h_3} = C_Y \tag{35d}$$

From the above deduction, we can see that the equations (27-35d) describe the simplified model of flutter analysis considering sloshing factors.

In order to verify the rationality and validity of the model, the simulation results of the model are compared with the results of the rigid-aeroelasticity-sloshing model.

Figure 20 shows that the simplified model can reflect the flutter movement of the body and reduce the complexity of the dynamic equations. Based on this model, under the condition of with and without sloshing inertia force, the simulations are carried out respectively.

From Figure 21, we can see that the sloshing inertia force enhances the flutter in the Y direction. The sloshing of liquid fuel results in the increase of the flutter amplitude. However, the sloshing has little effect on the flutter amplitude in the Z direction. Note that the flutter frequency is decreased. It can be seen from Figure (21c), the sloshing has an inhibitory effect on the torsional flutter. The simulation results show that the sloshing inertia force has different influence on each degree of freedom. Due to the enhancement of the sloshing on the Y direction, the flutter critical velocity will be reduced, which will have a devastating impact on the liquid rocket.

V. CONCLUSIONS

In this paper, the equations of elastic vibration and liquid sloshing are established by using the method of mixed coordinates and analytical mechanics. At the same time, the motion equation of the rigid body is obtained by using Newton's mechanics method. The rigid-aeroelasticity-sloshing model of the liquid rocket is developed. Then, the coupling of rigid-aeroelasticity-sloshing and its influence on the motion of rocket are analyzed. Furthermore, the coupling between aeroelastic and liquid sloshing is also studied. The results show that there is an interaction between sloshing and aeroelasticity. The sloshing of liquid fuel can reduce the critical condition of elastic divergence, while the elastic vibration can enhance the fuel sloshing.

A novel model for the analysis of longitudinal-transverse-torsional flutter is proposed. Meanwhile, the equation of the flutter is derived, and the simulation analysis is carried out. The simulation results demonstrate that the simplified model can reflect the flutter movement of the liquid rocket. At the same time, the liquid sloshing has an enhanced effect on the flutter in the Y and Z directions, but it has an inhibitory effect in the torsional direction.

NOMENCLATURE

SYMBOLS

x_R	= distance from the rear of the launch vehicle to the theoretical point, m
x_T	= Distance from the center of mass of the launch vehicle to the theoretical cusp, m
$\Phi_i\,(\chi, y, z)$	= the i-th natural vibration mode, m
$q_{i(\chi,y,z)}$	= the i-th order generalized coordinates, 1
$Q_{i(\chi,y,z)}$	= generalized force, N·m
$\bar{m}_{i(\chi,y,z)}$	= generalized mass, Kg·m^2
$\omega_{i(\chi,y,z)}$	= natural frequency, Hz
$\xi_{i(\chi,y,z)}$	= viscous damping coefficient of the i-th mode, 1
m_p	= sloshing mass, Kg
Ω_p	= sloshing frequency, Hz
ζ_p	= damping ratio, 1
φ	= pitch angle, deg
ψ	= yaw angle, deg
γ	= roll angle, deg
$c_A\,(x)$	= lift coefficient, 1
h_1, h_2, h_3, γ	= four degrees of freedom of flutter model, m
m_c	= the mass of the cross section, Kg
I	= inertia of the launch vehicle about the axial direction, Kg·m^2
S_c	= sectional moment, m^3
$K_{h_1}, K_{h_2}, K_{\gamma}, K_{h_3}$	= spring constant, N/m

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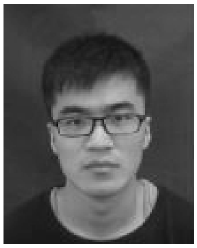

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