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New Stability Criteria of Discrete Systems With Time-Varying Delays

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ABSTRACT This paper is concerned with the stability criteria of discrete neural networks with two additive input time-varying delays. By using the time delay division and a new summation inequality, a less conservative criterion is derived. Moreover, to compare the obtained criterion more directly with the existing results, a corollary is proposed accordingly. Finally, some numerical examples are presented to demonstrate that the obtained criteria are less conservative.

INDEX TERMS Discrete neural networks, Lyapunov-Krasovskii functionals, stability, time-varying delays.

I. INTRODUCTION

Neural network is an abstract mathematical model that reflects the structure and function of human brain. It has been widely used in many fields such as pattern recognition, intelligent control, communication and expert system. Therefore, it has attracted the attention of many scholars in recent decades [1]–[4].

Time delay is a common phenomenon in dynamic systems. And it is an important factor leading to the deteriorating performance and even the instability of the system. In the analysis of the stability of time delay system, one of the most important problems is to obtain the maximum allowable upper bound of time delay. As we know, the reason for conservatism is that the conditions derived from the Lyapunov-krasovskii function (LKF) are sufficient and unnecessary. Therefore, the maximum allowable upper bound obtained is a core index to measure the quality of stability conditions or the degree of conservatism. Since many systems are controlled by digital computers without of continuous time delay, the research on discrete model is of more practical significance [5]–[18].

It is well known that reducing conservatism depends on two things: choosing the suitable LKF and making a more accurate estimate of the derivative of the former. In order to select the appropriate LKF, the commonly used methods include constructing an amplified LKF [19] and increasing its integral multiplicity [20] or to divide the time delay interval [21], all of which may obtain more information about the time delay and reduce the conservatism. Jensen inequality [22], [23] is the first powerful tool used to estimate the derivative of LKF in the stability theory of time-delay systems. Subsequently, Seuret and Gouaisbaut [24] proposed the Wirtinger-based integral inequality, which is more accurate than Jensen's inequality in estimating the derivative of LKF. These inequalities are widely used in time-delay systems [6], [8], [16], [25]–[29]. The Free-matrix-based integral inequality was proposed in [30] and [31], which was applied to discussing the global exponential stability of neural networks [1]. The inequality of corresponding discrete version is given in [32].

Based on the above discussions, this paper studies the new stability criteria of discrete systems with two additive time-varying delays. By means of the summation inequality in [32], less conservative criteria than some existing ones are obtained and two examples are presented to demonstrate the effectiveness and superiority of the proposed approach.

Notation: Throughout this paper, $\Re^n$ denotes n-dimensional Euclidean space and $\Re^{n\times m}$ is the set of all $n\times m$ real matrices; I is used to denote an identity matrix with proper dimension; $diag\{\cdots\}$ denotes block diagonal matrix and the Kronecker product is denoted as $\otimes$; $P>0(\geq 0)$ means that P is a real symmetric and positive-definite (semi-positive-definite) matrix; the symmetric term in a matrix is denoted by $*$ and $Sym[A]=A+A^T$; $N^+(N)$ is the set of all positive (nonnegative) integers.

II. PRELIMINARIES

Consider the following discrete-time neural network with additive time-varying delays as

$$\begin{cases} x(k+1)=Ax(k)+Bx(k-d_1(k)-d_2(k)), & k\in N,\\ x(k)=\phi(k), & k=-d_2,-d_2+1,\ldots,0. \end{cases} \tag{1}$$

where $x(k)=(x_1(k),x_2(k),\ldots,x_n(k))^T\in\Re^n$ is state vector and $\phi(k)\in\Re^n$ is the initial condition; $n\in N^+$, A and $B\in\Re^{n\times n}$ are constant matrices; $d_1(k)$ and $d_2(k)$ are the discrete additive time-varying delays that are assumed to satisfy

$$d_{11}\leq d_1(k)\leq d_{12},\quad d_{21}\leq d_2(k)\leq d_{22}, \tag{2}$$

where $d_{ij}(i,j=1,2)$ are nonnegative integers and $d_1=d_{11}+d_{21}$, $d_2=d_{12}+d_{22}$.

Lemma 1 [32]: For any sequence of discrete-time variable $x(i)$ in $[a,a+n]\to\Re^n$, a positive symmetric matrix $R\in\Re^{n\times n}$, and any matrices $N_1,N_2,N_3\in\Re^{4n\times n}$, the following inequality holds:

$$-\sum_{i=a}^{a+n-1}\eta^T(i)R\eta(i)\leq\omega^T\Omega\omega, \tag{3}$$

where

$$\begin{aligned}
\eta(i)&=x(i+1)-x(i),\\
\omega&=\Big(x^T(a+n),\quad x^T(a),\quad \frac{1}{n+1}\sum_{i=a}^{a+n}x^T(i),\\
&\qquad \frac{2}{(n+1)(n+2)}\sum_{i=a}^{a+n}\sum_{j=i}^{a+n}x^T(j)\Big)^T,\\
\Omega&=n(N_1R^{-1}N_1^T+\frac{1}{3}N_2R^{-1}N_2^T+\frac{1}{5}N_3R^{-1}N_3^T)\\
&\quad+sym[N_1\Pi_1+N_2\Pi_2+N_3\Pi_3],\\
\Pi_1&=\varepsilon_1-\varepsilon_2,\ \Pi_2=2\varepsilon_3-\varepsilon_1-\varepsilon_2,\\
\Pi_3&=\varepsilon_1-\varepsilon_2+6\varepsilon_3-6\varepsilon_4,\\
\varepsilon_1&=(I\quad 0\quad 0\quad 0),\qquad \varepsilon_2=(0\quad I\quad 0\quad 0),\\
\varepsilon_3&=(0\quad 0\quad I\quad 0),\qquad \varepsilon_4=(0\quad 0\quad 0\quad I).
\end{aligned}$$

III. MAIN RESULTS

Theorem 2: For given positive integers $d_{ij}(i,j=1,2)$ satisfying (2), the system (1) is asymptotically stable if there exist matrices $P>0(\in\Re^{5n\times 5n})$, $Q_l>0(l=1,2,3,4)$, $R_m>0(m=1,2,\ldots,6)$ and matrices $N_r=(N_{r1}\quad N_{r2}\quad N_{r3})(N_{rs}\in\Re^{4n\times n}, r=1,2,\ldots,8; s=1,2,3)$, such that the following LMI holds:

$$\Phi(d_1(k),d_2(k))=\begin{pmatrix}\Phi_1(d_1(k),d_2(k)) & \Phi_2(d_1(k),d_2(k))\\ * & \Phi_3\end{pmatrix}<0, \tag{4}$$

where

$$\begin{aligned}
\Phi_1(d_1(k),d_2(k))&=\sum_{i=1}^{8}D_i+G_1(d_1(k),d_2(k)),\\
\Phi_2(d_1(k),d_2(k))&=(\sqrt{d_{11}}\alpha_1N_1,\sqrt{d_1(k)-d_{11}}\alpha_2N_2,\sqrt{d_{12}-d_1(k)}\alpha_3N_3,\\
&\quad\sqrt{d_2-d_{12}}\alpha_4N_4,\sqrt{d_{21}}\alpha_5N_5,\sqrt{d_2(k)-d_{21}}\alpha_6N_6,\\
&\quad\sqrt{d_{22}-d_2(k)}\alpha_7N_7,\sqrt{d_2-d_{22}}\alpha_8N_8),\\
\Phi_3&=diag(C\otimes R_1,C\otimes R_2,C\otimes R_2,C\otimes R_3,C\otimes R_4,\\
&\qquad C\otimes R_5,C\otimes R_5,C\otimes R_6),
\end{aligned}$$

and

$$\begin{aligned}
D_1&=E_2^TPE_2-E_1^TPE_1,\\
D_2&=e_1^T(Q_1+Q_2)e_1-e_2^T(Q_1-Q_3)e_2\\
&\quad-e_5^T(Q_2-Q_4)e_5-e_9^T(Q_3+Q_4)e_9,\\
D_3&=d_{11}(e_0-e_1)^TR_1(e_0-e_1)\\
&\quad+\alpha_1sym[N_{11}\Pi_1+N_{12}\Pi_2+N_{13}\Pi_3]\alpha_1^T,\\
D_4&=(d_{12}-d_{11})(e_0-e_1)^TR_2(e_0-e_1)\\
&\quad+\alpha_2sym[N_{21}\Pi_1+N_{22}\Pi_2+N_{23}\Pi_3]\alpha_2^T\\
&\quad+\alpha_3sym[N_{31}\Pi_1+N_{32}\Pi_2+N_{33}\Pi_3]\alpha_3^T,\\
D_5&=(d_2-d_{12})(e_0-e_1)^TR_3(e_0-e_1)\\
&\quad+\alpha_4sym[N_{41}\Pi_1+N_{42}\Pi_2+N_{43}\Pi_3]\alpha_4^T,\\
D_6&=d_{21}(e_0-e_1)^TR_4(e_0-e_1)\\
&\quad+\alpha_5sym[N_{51}\Pi_1+N_{52}\Pi_2+N_{53}\Pi_3]\alpha_5^T,\\
D_7&=(d_{22}-d_{21})(e_0-e_1)^TR_5(e_0-e_1)\\
&\quad+\alpha_6sym[N_{61}\Pi_1+N_{62}\Pi_2+N_{63}\Pi_3]\alpha_6^T\\
&\quad+\alpha_7sym[N_{71}\Pi_1+N_{72}\Pi_2+N_{73}\Pi_3]\alpha_7^T,\\
D_8&=(d_2-d_{22})(e_0-e_1)^TR_6(e_0-e_1)\\
&\quad+\alpha_8sym[N_{81}\Pi_1+N_{82}\Pi_2+N_{83}\Pi_3]\alpha_8^T,\\
G_1(d_1(k),d_2(k))&=sym[(E_2-E_1)^TPE(d_1(k),d_2(k))],\\
E(d_1(k),d_2(k))&=(0,0,(d_1(k)-d_{11}+1)e_{11}^T+(d_{12}-d_1(k)+1)e_{12}^T,\\
&\qquad 0,(d_2(k)-d_{21}+1)e_{15}^T+(d_{22}-d_2(k)+1)e_{16}^T)^T,\\
E_1&=(e_1^T,(d_{11}+1)e_{10}^T-e_1^T,-e_2^T-e_3^T,\\
&\qquad (d_{21}+1)e_{14}^T-e_1^T,-e_5^T-e_6^T)^T,\\
E_2&=(e_0^T,(d_{11}+1)e_{10}^T-e_2^T,-e_3^T-e_4^T,\\
&\qquad (d_{21}+1)e_{14}^T-e_5^T,-e_6^T-e_7^T)^T,\\
\alpha_1&=(e_1^T,e_2^T,e_{10}^T,e_{18}^T),\quad \alpha_2=(e_2^T,e_3^T,e_{11}^T,e_{19}^T),\\
\alpha_3&=(e_3^T,e_4^T,e_{12}^T,e_{20}^T),\quad \alpha_4=(e_4^T,e_9^T,e_{13}^T,e_{21}^T),
\end{aligned}$$

$$\alpha_5 = (e_1^T, e_5^T, e_{14}^T, e_{22}^T), \quad \alpha_6 = (e_5^T, e_6^T, e_{15}^T, e_{23}^T),$$
$$\alpha_7 = (e_6^T, e_7^T, e_{16}^T, e_{24}^T), \quad \alpha_8 = (e_7^T, e_9^T, e_{17}^T, e_{25}^T),$$
$$e_i = (0_{n\times(i-1)n}, I, 0_{n\times(25-i)n}), \quad i = 1, 2, \ldots, 25,$$
$$e_0 = Ae_1 + Be_8, \quad C = diag(-1, -3, -5).$$

Proof: Construct a Lyapunov functional as follows:

$$V(k) = \sum_{i=1}^{8} V_i(k),$$

where

$$V_1(k) = \theta^T(k)P\theta(k),$$
$$V_2(k) = \sum_{i=k-d_{11}}^{k-1} x^T(i)Q_1x(i) + \sum_{i=k-d_{21}}^{k-1} x^T(i)Q_2x(i) + \sum_{i=k-d_2}^{k-d_{11}-1} x^T(i)Q_3x(i) + \sum_{i=k-d_2}^{k-d_{21}-1} x^T(i)Q_4x(i),$$
$$V_3(k) = \sum_{i=-d_{11}}^{-1}\sum_{j=k+i}^{k-1} \eta^T(j)R_1\eta(j),$$
$$V_4(k) = \sum_{i=-d_{12}}^{-d_{11}-1}\sum_{j=k+i}^{k-1} \eta^T(j)R_2\eta(j),$$
$$V_5(k) = \sum_{i=-d_2}^{-d_{12}-1}\sum_{j=k+i}^{k-1} \eta^T(j)R_3\eta(j),$$
$$V_6(k) = \sum_{i=-d_{21}}^{-1}\sum_{j=k+i}^{k-1} \eta^T(j)R_4\eta(j),$$
$$V_7(k) = \sum_{i=-d_{22}}^{-d_{21}-1}\sum_{j=k+i}^{k-1} \eta^T(j)R_5\eta(j),$$
$$V_8(k) = \sum_{i=-d_2}^{-d_{22}-1}\sum_{j=k+i}^{k-1} \eta^T(j)R_6\eta(j),$$

where

$$\theta(k) = \Big(x^T(k), \sum_{i=k-d_{11}}^{k-1} x^T(i), \sum_{i=k-d_{12}}^{k-d_{11}-1} x^T(i), \sum_{i=k-d_{21}}^{k-1} x^T(i), \sum_{i=k-d_{22}}^{k-d_{21}-1} x^T(i)\Big)^T.$$

For simplicity, the following vectors are defined firstly.

$$\xi^T(k) = \Big(x^T(k) \quad x^T(k-d_{11}) \quad x^T(k-d_1(k)) \quad x^T(k-d_{12}) \quad x^T(k-d_{21}) \quad x^T(k-d_2(k)) \quad x^T(k-d_{22}) \quad x^T(k-d(k)) \quad x^T(k-d_2) \quad U_{11}^T \quad U_{12}^T \quad U_{13}^T \quad U_{14}^T \quad U_{21}^T \quad U_{22}^T \quad U_{23}^T \quad U_{24}^T \quad W_{11}^T \quad W_{12}^T \quad W_{13}^T \quad W_{14}^T \quad W_{21}^T \quad W_{22}^T \quad W_{23}^T \quad W_{24}^T\Big),$$

where

$$U_{m1} = \frac{1}{d_{m1}+1}\sum_{i=k-d_{m1}}^{k} x(i),$$
$$U_{m2} = \frac{1}{d_m(k)-d_{m1}+1}\sum_{i=k-d_m(k)}^{k-d_{m1}} x(i),$$
$$U_{m3} = \frac{1}{d_{m2}-d_m(k)+1}\sum_{i=k-d_{m2}}^{k-d_m(k)} x(i),$$
$$U_{m4} = \frac{1}{d_2-d_{m2}+1}\sum_{i=k-d_2}^{k-d_{m2}} x(i),$$
$$W_{m1} = \frac{2}{(d_{m1}+1)(d_{m1}+2)}\sum_{i=k-d_{m1}}^{k}\sum_{j=i}^{k} x(j),$$
$$W_{m2} = \frac{2}{(d_m(k)-d_{m1}+1)(d_m(k)-d_{m1}+2)} \times \sum_{i=k-d_m(k)}^{k-d_{m1}}\sum_{j=i}^{k-d_{m1}} x(j),$$
$$W_{m3} = \frac{2}{(d_{m2}-d_m(k)+1)(d_{m2}-d_m(k)+2)} \times \sum_{i=k-d_{m2}}^{k-d_m(k)}\sum_{j=i}^{k-d_m(k)} x(j),$$
$$W_{m4} = \frac{2}{(d_2-d_{m2}+1)(d_2-d_{m2}+2)} \times \sum_{i=k-d_2}^{k-d_{m2}}\sum_{j=i}^{k-d_{m2}} x(j),$$

and $m = 1, 2$, for the formulas above.

The forward difference of $V_i(k)(i = 1, 2, \ldots, 8)$ is given by

$$\begin{aligned}\triangle V_1(k) &= \theta^T(k+1)P\theta(k+1) - \theta^T(k)P\theta(k)\\ &= \xi^T(k)\{E_2^TPE_2 - E_1^TPE_1\\ &\quad + sym[(E_2-E_1)^TPE(d_1(k), d_2(k))]\}\xi(k)\\ &= \xi^T(k)[D_1 + G_1(d_1(k), d_2(k))]\xi(k), \end{aligned} \tag{5}$$
$$\begin{aligned}\triangle V_2(k) &= \xi^T(k)[e_1^T(Q_1+Q_2)e_1 - e_2^T(Q_1-Q_3)e_2\\ &\quad - e_5^T(Q_2-Q_4)e_5 - e_9^T(Q_3+Q_4)e_9]\xi(k)\\ &= \xi^T(k)D_2\xi(k).\end{aligned} \tag{6}$$

Using Lemma 2.1, one can obtain

$$\begin{aligned}&\triangle V_3(k)\\ &= \sum_{i=-d_{11}}^{-1}\sum_{j=k+i+1}^{k} \eta^T(j)R_1\eta(j) - \sum_{i=-d_{11}}^{-1}\sum_{j=k+i}^{k-1} \eta^T(j)R_1\eta(j)\\ &= d_{11}\eta^T(k)R_1\eta(k) - \sum_{i=k-d_{11}}^{k-1} \eta^T(i)R_1\eta(i)\\ &\le \xi^T(k)[d_{11}(e_0-e_1)^TR_1(e_0-e_1)]\xi(k)\\ &\quad + \xi^T(k)(e_1^T, e_2^T, e_{10}^T, e_{18}^T)\{d_{11}(N_{11}R_1^{-1}N_{11}^T\end{aligned}$$

$$+\frac{1}{3}N_{12}R_1^{-1}N_{12}^T+\frac{1}{5}N_{13}R_1^{-1}N_{13}^T)+sym[N_{11}\Pi_1+N_{12}\Pi_2+N_{13}\Pi_3]\}(e_1^T,e_2^T,e_{10}^T,e_{18}^T)^T\xi(k)$$
$$=\xi^T(k)[D_3+H_1]\xi(k), \quad (7)$$

$$\begin{aligned}\triangle V_4(k)&=\sum_{i=-d_{12}}^{-d_{11}-1}\sum_{j=k+i+1}^{k}\eta^T(j)R_2\eta(j)-\sum_{i=-d_{12}}^{-d_{11}-1}\sum_{j=k+i}^{k-1}\eta^T(j)R_2\eta(j)\\
&=(d_{12}-d_{11})\eta^T(k)R_2\eta(k)-\sum_{i=k-d_{12}}^{k-d_1(k)-1}\eta^T(i)R_2\eta(i)-\sum_{i=k-d_1(k)}^{k-d_{11}-1}\eta^T(i)R_2\eta(i)\\
&\le\xi^T(k)[(d_{12}-d_{11})(e_0-e_1)^TR_2(e_0-e_1)]\xi(k)\\
&\quad+\xi^T(k)(e_2^T,e_3^T,e_{11}^T,e_{19}^T)\{(d_1(k)-d_{11})(N_{21}R_2^{-1}N_{21}^T+\frac{1}{3}N_{22}R_2^{-1}N_{22}^T+\frac{1}{5}N_{23}R_2^{-1}N_{23}^T)+sym[N_{21}\Pi_1+N_{22}\Pi_2+N_{23}\Pi_3]\}(e_2^T,e_3^T,e_{11}^T,e_{19}^T)^T\xi(k)\\
&\quad+\xi^T(k)(e_3^T,e_4^T,e_{12}^T,e_{20}^T)\{(d_{12}-d_1(k))(N_{31}R_2^{-1}N_{31}^T+\frac{1}{3}N_{32}R_2^{-1}N_{32}^T+\frac{1}{5}N_{33}R_2^{-1}N_{33}^T)+sym[N_{31}\Pi_1+N_{32}\Pi_2+N_{33}\Pi_3]\}(e_3^T,e_4^T,e_{12}^T,e_{20}^T)^T\xi(k)\\
&=\xi^T(k)[D_4+G_2(d_1(k),d_2(k))+G_3(d_1(k),d_2(k))]\xi(k),\end{aligned} \quad (8)$$

$$\begin{aligned}\triangle V_5(k)&\le\xi^T(k)(e_4^T,e_9^T,e_{13}^T,e_{21}^T)\{(d_2-d_{12})(N_{41}R_3^{-1}N_{41}^T+\frac{1}{3}N_{42}R_3^{-1}N_{42}^T+\frac{1}{5}N_{43}R_3^{-1}N_{43}^T)+sym[N_{41}\Pi_1+N_{42}\Pi_2+N_{43}\Pi_3]\}(e_4^T,e_9^T,e_{13}^T,e_{21}^T)^T\xi(k)\\
&\quad+\xi^T(k)[(d_2-d_{12})(e_0-e_1)^TR_3(e_0-e_1)]\xi(k)\\
&=\xi^T(k)[D_5+H_2]\xi(k),\end{aligned} \quad (9)$$

$$\begin{aligned}\triangle V_6(k)&\le\xi^T(k)[d_{21}(e_0-e_1)^TR_4(e_0-e_1)]\xi(k)\\
&\quad+\xi^T(k)(e_1^T,e_5^T,e_{14}^T,e_{22}^T)\{d_{21}(N_{51}R_4^{-1}N_{51}^T+\frac{1}{3}N_{52}R_4^{-1}N_{52}^T+\frac{1}{5}N_{53}R_4^{-1}N_{53}^T)+sym[N_{51}\Pi_1+N_{52}\Pi_2+N_{53}\Pi_3]\}(e_1^T,e_5^T,e_{14}^T,e_{22}^T)^T\xi(k)\\
&=\xi^T(k)[D_6+H_3]\xi(k),\end{aligned} \quad (10)$$

$$\begin{aligned}\triangle V_7(k)&\le\xi^T(k)[(d_{22}-d_{21})(e_0-e_1)^TR_5(e_0-e_1)]\xi(k)\\
&\quad+\xi^T(k)(e_5^T,e_6^T,e_{15}^T,e_{23}^T)\{(d_2(k)-d_{21})(N_{61}R_5^{-1}N_{61}^T+\frac{1}{3}N_{62}R_5^{-1}N_{62}^T+\frac{1}{5}N_{63}R_5^{-1}N_{63}^T)+sym[N_{61}\Pi_1+N_{62}\Pi_2+N_{63}\Pi_3]\}(e_5^T,e_6^T,e_{15}^T,e_{23}^T)^T\xi(k)\\
&\quad+\xi^T(k)(e_6^T,e_7^T,e_{16}^T,e_{24}^T)\{(d_{22}-d_2(k))(N_{71}R_5^{-1}N_{71}^T+\frac{1}{3}N_{72}R_5^{-1}N_{72}^T+\frac{1}{5}N_{73}R_5^{-1}N_{73}^T)+sym[N_{71}\Pi_1+N_{72}\Pi_2+N_{73}\Pi_3]\}(e_6^T,e_7^T,e_{16}^T,e_{24}^T)^T\xi(k)\\
&=\xi^T(k)[D_7+G_4(d_1(k),d_2(k))+G_5(d_1(k),d_2(k))]\xi(k),\end{aligned} \quad (11)$$

$$\begin{aligned}\triangle V_8(k)&\le\xi^T(k)(e_7^T,e_9^T,e_{17}^T,e_{25}^T)\{(d_2-d_{22})(N_{81}R_6^{-1}N_{81}^T+\frac{1}{3}N_{82}R_6^{-1}N_{82}^T+\frac{1}{5}N_{83}R_6^{-1}N_{83}^T)+sym[N_{81}\Pi_1+N_{82}\Pi_2+N_{83}\Pi_3]\}(e_7^T,e_9^T,e_{17}^T,e_{25}^T)^T\xi(k)\\
&\quad+\xi^T(k)[(d_2-d_{22})(e_0-e_1)^TR_6(e_0-e_1)]\xi(k)\\
&=\xi^T(k)[D_8+H_4]\xi(k),\end{aligned} \quad (12)$$

where

$$\begin{aligned}H_1&=\alpha_1[d_{11}(N_{11}R_1^{-1}N_{11}^T+\frac{1}{3}N_{12}R_1^{-1}N_{12}^T+\frac{1}{5}N_{13}R_1^{-1}N_{13}^T)]\alpha_1^T,\\
H_2&=\alpha_4[(d_2-d_{12})(N_{41}R_3^{-1}N_{41}^T+\frac{1}{3}N_{42}R_3^{-1}N_{42}^T+\frac{1}{5}N_{43}R_3^{-1}N_{43}^T)]\alpha_4^T,\\
H_3&=\alpha_5[d_{21}(N_{51}R_4^{-1}N_{51}^T+\frac{1}{3}N_{52}R_4^{-1}N_{52}^T+\frac{1}{5}N_{53}R_4^{-1}N_{53}^T)]\alpha_5^T,\\
H_4&=\alpha_8[(d_2-d_{22})(N_{81}R_6^{-1}N_{81}^T+\frac{1}{3}N_{82}R_6^{-1}N_{82}^T+\frac{1}{5}N_{83}R_6^{-1}N_{83}^T)]\alpha_8^T,\\
G_2(d_1(k),d_2(k))&=\alpha_2[(d_1(k)-d_{11})(N_{21}R_2^{-1}N_{21}^T+\frac{1}{3}N_{22}R_2^{-1}N_{22}^T+\frac{1}{5}N_{23}R_2^{-1}N_{23}^T)]\alpha_2^T,\\
G_3(d_1(k),d_2(k))&=\alpha_3[(d_{12}-d_1(k))(N_{31}R_2^{-1}N_{31}^T+\frac{1}{3}N_{32}R_2^{-1}N_{32}^T+\frac{1}{5}N_{33}R_2^{-1}N_{33}^T)]\alpha_3^T,\\
G_4(d_1(k),d_2(k))&=\alpha_6[(d_2(k)-d_{21})(N_{61}R_5^{-1}N_{61}^T+\frac{1}{3}N_{62}R_5^{-1}N_{62}^T+\frac{1}{5}N_{63}R_5^{-1}N_{63}^T)]\alpha_6^T,\\
G_5(d_1(k),d_2(k))&=\alpha_7[(d_{22}-d_2(k))(N_{71}R_5^{-1}N_{71}^T+\frac{1}{3}N_{72}R_5^{-1}N_{72}^T+\frac{1}{5}N_{73}R_5^{-1}N_{73}^T)]\alpha_7^T.\end{aligned}$$

From (5)-(12), we have

$$\triangle V(k)\le\xi^T(k)[\sum_{i=1}^{8}D_i+\sum_{i=1}^{5}G_i(d_1(k),d_2(k))+\sum_{i=1}^{4}H_i]\xi(k).$$

According to (4) and Schur complement, it can be obtained $\triangle V(k)<0$ for any $\xi(k)\neq 0$, which guarantees the system (1) is asymptotically stable. This completes the proof.

Remark 1: When system (1) has no additive input delays, we can consider the following system:

$$\begin{cases} x(k+1) = Ax(k) + Bx(k-d(k)), & k \in N, \\ x(k) = \phi(k), & k = -d_2, -d_2+1, \dots, 0. \end{cases} \quad (13)$$

where the discrete delay $d(k)$ satisfies $d_1 \le d(k) \le d_2$. From Theorem 3.1, the stability criterion of system (13) can be obtained similarly.

Corollary 3: For given positive integers d_1 and d_2 satisfying $d_1 \le d(k) \le d_2$, the system (13) is asymptotically stable if there exist matrices $\tilde{P} > 0 (\in \Re^{3n\times 3n})$, $\tilde{Q}_i > 0$, $\tilde{R}_i > 0 (i = 1, 2)$ and matrices $\tilde{N}_r = (\tilde{N}_{r1} \quad \tilde{N}_{r2} \quad \tilde{N}_{r3})(\tilde{N}_{rs} \in \Re^{4n\times n}, r, s = 1, 2, 3)$, such that the following LMI holds:

$$\tilde{\Phi}(d(k)) = \begin{pmatrix} \tilde{\Phi}_1(d(k)) & \tilde{\Phi}_2(d(k)) \\ * & \tilde{\Phi}_3 \end{pmatrix} < 0, \quad (14)$$

where

$$\begin{aligned}
\tilde{\Phi}_1(d(k)) &= \sum_{i=1}^{4} \tilde{D}_i + \tilde{G}_1(d(k)), \\
\tilde{\Phi}_2(d(k)) &= (\sqrt{d_1}\tilde{\alpha}_1\tilde{N}_1, \quad \sqrt{d(k)-d_1}\tilde{\alpha}_2\tilde{N}_2, \\
&\qquad \sqrt{d_2-d(k)}\tilde{\alpha}_3\tilde{N}_3), \\
\tilde{\Phi}_3 &= diag(\tilde{C}\otimes\tilde{R}_1, \quad \tilde{C}\otimes\tilde{R}_2, \quad \tilde{C}\otimes\tilde{R}_2),
\end{aligned}$$

and

$$\begin{aligned}
\tilde{D}_1 &= \tilde{E}_2^T\tilde{P}\tilde{E}_2 - \tilde{E}_1^T\tilde{P}\tilde{E}_1, \\
\tilde{D}_2 &= \tilde{e}_1^T\tilde{Q}_1\tilde{e}_1 - \tilde{e}_2^T(\tilde{Q}_1 - \tilde{Q}_2)\tilde{e}_2 - \tilde{e}_4^T\tilde{Q}_2\tilde{e}_4, \\
\tilde{D}_3 &= d_1(\tilde{e}_0 - \tilde{e}_1)^T\tilde{R}_1(\tilde{e}_0 - \tilde{e}_1) \\
&\quad + \tilde{\alpha}_1 sym[\tilde{N}_{11}\Pi_1 + \tilde{N}_{12}\Pi_2 + \tilde{N}_{13}\Pi_3]\tilde{\alpha}_1^T, \\
\tilde{D}_4 &= (d_2 - d_1)(\tilde{e}_0 - \tilde{e}_1)^T\tilde{R}_2(\tilde{e}_0 - \tilde{e}_1) \\
&\quad + \tilde{\alpha}_2 sym[\tilde{N}_{21}\Pi_1 + \tilde{N}_{22}\Pi_2 + \tilde{N}_{23}\Pi_3]\tilde{\alpha}_2^T \\
&\quad + \tilde{\alpha}_3 sym[\tilde{N}_{31}\Pi_1 + \tilde{N}_{32}\Pi_2 + \tilde{N}_{33}\Pi_3]\tilde{\alpha}_3^T, \\
\tilde{G}_1(d(k)) &= sym[(\tilde{E}_2 - \tilde{E}_1)^T\tilde{P}\tilde{E}(d(k))], \\
\tilde{E}(d(k)) &= (0, 0, (d(k) - d_1 + 1)\tilde{e}_6^T \\
&\quad + (d_2 - d(k) + 1)\tilde{e}_7^T)^T, \\
\tilde{E}_1 &= (\tilde{e}_1^T, (d_1+1)\tilde{e}_5^T - \tilde{e}_1^T, -\tilde{e}_2^T - \tilde{e}_3^T)^T, \\
\tilde{E}_2 &= (\tilde{e}_0^T, (d_1+1)\tilde{e}_5^T - \tilde{e}_2^T, -\tilde{e}_3^T - \tilde{e}_4^T)^T, \\
\tilde{H} &= \tilde{\alpha}_1[d_1(\tilde{N}_{11}\tilde{R}_1^{-1}\tilde{N}_{11}^T + \frac{1}{3}\tilde{N}_{12}\tilde{R}_1^{-1}\tilde{N}_{12}^T \\
&\quad + \frac{1}{5}\tilde{N}_{13}\tilde{R}_1^{-1}\tilde{N}_{13}^T)]\tilde{\alpha}_1^T, \\
\tilde{G}_2(d(k)) &= \tilde{\alpha}_2[(d(k) - d_1)(\tilde{N}_{21}\tilde{R}_2^{-1}\tilde{N}_{21}^T \\
&\quad + \frac{1}{3}\tilde{N}_{22}\tilde{R}_2^{-1}\tilde{N}_{22}^T + \frac{1}{5}\tilde{N}_{23}\tilde{R}_2^{-1}\tilde{N}_{23}^T)]\tilde{\alpha}_2^T, \\
\tilde{G}_3(d(k)) &= \tilde{\alpha}_3[(d_2 - d(k))(\tilde{N}_{31}\tilde{R}_2^{-1}\tilde{N}_{31}^T \\
&\quad + \frac{1}{3}\tilde{N}_{32}\tilde{R}_2^{-1}\tilde{N}_{32}^T + \frac{1}{5}\tilde{N}_{33}\tilde{R}_2^{-1}\tilde{N}_{33}^T)]\tilde{\alpha}_3^T, \\
\tilde{\alpha}_1 &= (\tilde{e}_1^T, \tilde{e}_2^T, \tilde{e}_5^T, \tilde{e}_8^T), \quad \tilde{\alpha}_2 = (\tilde{e}_2^T, \tilde{e}_3^T, \tilde{e}_6^T, \tilde{e}_9^T), \\
\tilde{\alpha}_3 &= (\tilde{e}_3^T, \tilde{e}_4^T, \tilde{e}_7^T, \tilde{e}_{10}^T), \quad \tilde{e}_0 = A\tilde{e}_1 + B\tilde{e}_3, \\
\tilde{e}_i &= (0_{n\times(i-1)n}, I, 0_{n\times(10-i)n}), \quad i = 1, 2, \dots, 10, \\
\tilde{C} &= C = diag(-1, -3, -5).
\end{aligned}$$

Proof: Construct a Lyapunov functional as follows:

$$V(k) = \sum_{i=1}^{4} V_i(k),$$

where

$$\begin{aligned}
V_1(k) &= \theta^T(k)\tilde{P}\theta(k), \\
V_2(k) &= \sum_{i=k-d_1}^{k-1} x^T(i)\tilde{Q}_1x(i) + \sum_{i=k-d_2}^{k-d_1-1} x^T(i)\tilde{Q}_2x(i), \\
V_3(k) &= \sum_{i=-d_1}^{-1}\sum_{j=k+i}^{k-1} \eta^T(j)\tilde{R}_1\eta(j), \\
V_4(k) &= \sum_{i=-d_2}^{-d_1-1}\sum_{j=k+i}^{k-1} \eta^T(j)\tilde{R}_2\eta(j),
\end{aligned}$$

and

$$\theta(k) = \Big(x^T(k), \sum_{i=k-d_1}^{k-1} x^T(i), \sum_{i=k-d_2}^{k-d_1-1} x^T(i)\Big)^T.$$

Let

$$\begin{aligned}
\xi^T(k) = \Big(&x^T(k) \quad x^T(k-d_1) \quad x^T(k-d(k)) \\
&x^T(k-d_2) \quad U_1^T \quad U_2^T \quad U_3^T \quad W_1^T \quad W_2^T \quad W_3^T\Big),
\end{aligned}$$

where

$$\begin{aligned}
U_1 &= \frac{1}{d_1+1}\sum_{i=k-d_1}^{k} x(i), \\
U_2 &= \frac{1}{d(k)-d_1+1}\sum_{i=k-d(k)}^{k-d_1} x(i), \\
U_3 &= \frac{1}{d_2-d(k)+1}\sum_{i=k-d_2}^{k-d(k)} x(i), \\
W_1 &= \frac{2}{(d_1+1)(d_1+2)}\sum_{i=k-d_1}^{k}\sum_{j=i}^{k} x(j), \\
W_2 &= \frac{2}{(d(k)-d_1+1)(d(k)-d_1+2)}\sum_{i=k-d(k)}^{k-d_1}\sum_{j=i}^{k-d_1} x(j), \\
W_3 &= \frac{2}{(d_2-d(k)+1)(d_2-d(k)+2)}\sum_{i=k-d_2}^{k-d(k)}\sum_{j=i}^{k-d(k)} x(j).
\end{aligned}$$

The remainder of the proof is similar to that of Theorem 3.1, which is omitted for brevity. This completes the proof.

IV. NUMERICAL ILLUSTRATIONS

In this section, a numerical example that often appears in the literatures is presented to demonstrate the effectiveness and sophistication of the proposed approach.

Example 1: Consider the system (13) with the following parameters:

$$A = \begin{pmatrix} 0.8 & 0 \\ 0.05 & 0.9 \end{pmatrix}, \quad B = \begin{pmatrix} -0.1 & 0 \\ -0.2 & -0.1 \end{pmatrix}.$$

Example 2: Consider the system (13), where

$$A = \begin{pmatrix} 0.7 & 0.1 \\ 0.05 & 0.7 \end{pmatrix}, \quad B = \begin{pmatrix} -0.1 & 0.1 \\ -0.1 & -0.2 \end{pmatrix}.$$

The allowable delay upper bound d_2 can be found for given d_1 or vice versa. The simulation results are listed in Table 1 and Table 2.

TABLE 1. **Allowable delay upper bound d_2 for various d_1 for Example 1.**

Method	d_1	4	6	10	20	25	30
[16]	d_2	17	18	20	27	31	35
[13]	d_2	18	19	20	26	30	35
[9]	d_2	19	20	21	27	31	35
[18]	d_2	21	21	22	27	29	34
[6]	d_2	21	21	22	27	31	35
[8]	d_2	22	22	23	28	32	36
[12] (Remark 4)	d_2	21	21	23	29	32	36
[12](Theorem 1)	d_2	22	22	23	29	32	36
Corollary 3.1	d_2	22	22	23	29	32	36

TABLE 2. **Allowable delay upper bound d_2 for various d_1 for Example 2.**

Method	d_1	5	6	7	10	20
[13]	d_2	12	13	14	17	27
[9](Theorem 1)	d_2	14	15	16	19	29
[18]	d_2	14	15	16	19	29
[6](Theorem 4, l=4)	d_2	14	15	16	19	29
Corollary 3.1	d_2	16	17	17	20	30

Remark 2: It is easy to see from Table 1 that the result of Corollary 3.1 (without delay-decomposition) in this paper and the result of Theorem 1 (with delay-decomposition) in [12] are the same, which are better than the other results in the table. Furthermore, how to derive a less conservative result remains a challenging and fascinating task.

Remark 3: The method of this paper can be applied to analyzing the networked-Markov jump systems easily.

Consider the following system

$$\begin{cases} x(k+1) = A(r(k))x(k) + B(r(k)) \\ \quad \times x(k - d_1(k) - d_2(k)), \quad k \in N, \\ x(k) = \phi(k), \quad k = -d_2, -d_2+1, \ldots, 0, \end{cases}$$

where $\{r(k)\}$ is a discrete-time homogeneous Markov chain taking values in a finite set $\mathbb{S} = \{1, 2, ..., n\}$ $(n \in N^+)$ with transition probability matrix $\Pi = (\pi_{ij})(i, j \in \mathbb{S})$ given by

$$P\{r(k+1) = j | r(k) = i\} = \pi_{ij},$$

where $0 \le \pi_{ij} \le 1$ is the transition probability from mode i to mode j and $\sum_{j=1}^{n} \pi_{ij} = 1$.

We use the following Lyapunov functional:

$$V(k) = \sum_{i=1}^{8} V_i(k),$$

where

$$V_1(k) = \theta^T(k)P(r(k))\theta(k),$$

$$V_2(k) = \sum_{i=k-d_{11}}^{k-1} x^T(i)Q_1x(i) + \sum_{i=k-d_{21}}^{k-1} x^T(i)Q_2x(i) + \sum_{i=k-d_2}^{k-d_{11}-1} x^T(i)Q_3x(i) + \sum_{i=k-d_2}^{k-d_{21}-1} x^T(i)Q_4x(i),$$

$$V_3(k) = \sum_{i=-d_{11}}^{-1} \sum_{j=k+i}^{k-1} \eta^T(j)R_1(r(k))\eta(j),$$

$$V_4(k) = \sum_{i=-d_{12}}^{-d_{11}-1} \sum_{j=k+i}^{k-1} \eta^T(j)R_2(r(k))\eta(j),$$

$$V_5(k) = \sum_{i=-d_2}^{-d_{12}-1} \sum_{j=k+i}^{k-1} \eta^T(j)R_3(r(k))\eta(j),$$

$$V_6(k) = \sum_{i=-d_{21}}^{-1} \sum_{j=k+i}^{k-1} \eta^T(j)R_4(r(k))\eta(j),$$

$$V_7(k) = \sum_{i=-d_{22}}^{-d_{21}-1} \sum_{j=k+i}^{k-1} \eta^T(j)R_5(r(k))\eta(j),$$

$$V_8(k) = \sum_{i=-d_2}^{-d_{22}-1} \sum_{j=k+i}^{k-1} \eta^T(j)R_6(r(k))\eta(j).$$

Then, we can obtain some similar results for the networked-Markov jump systems, which are omitted for brevity.

V. CONCLUSION

In this paper, the problem of stability criteria of discrete systems with two additive time-varying delay components has been investigated. By means of the discrete form of Free-matrix-based integral inequality, a criterion less conservative than some existing ones is derived from a tighter estimation of the new inequality. Then, a corollary is proposed to compare the obtained criterion more directly with the existing results. Finally, two illustrative examples are presented to demonstrate the effectiveness of the obtained method.

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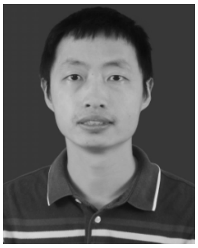

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