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Wavelets for Electrocardiogram: Overview and Taxonomy

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ABSTRACT Physiological and pathological information within electrocardiogram (ECG) is crucial for the diagnosis of heart diseases. Computer-aided diagnosis for the ECG signals has drawn growing research attention up to date. Automatic ECG analysis mainly includes signal denoising, wave detection, and heartbeat classification. These three issues are relevant that the signal denoising can help attenuate the noises and accentuate the typical waves in ECG signals for wave detection, and wave detection can help locate the typical ECG waves and acquire the diagnostically valuable heartbeats based on these waves for the heartbeat classification. The wavelet-based methods play important roles in the three issues, but these methods are scattered and unorganized in the literature. In order to manifest the value of these methods, this paper contributes an overview and taxonomy on them. This paper does the comprehensive summary and systematic categorization on the methods for signal denoising, wave detection, and heartbeat classification according to the deep analysis of their methodological characteristics. By doing so, this paper not only uncovers the inner mechanism that why wavelet-based methods are suitable for ECG analysis but also reveals the designing principles that these methods potentially follow. Finally, this paper has provided an outlook for the developing prospect of "wavelets for ECG" in the future.

INDEX TERMS Wavelets for electrocardiogram, signal denoising, wave detection, heartbeat classification, overview and taxonomy.

I. OUTLINE AND INTRODUCTION

A. OUTLINE AND CONTRIBUTION

Electrocardiogram (ECG) plays an important role in diagnosing heart diseases, because ECG signals record the cardiac electrical activity, which conveys important pathological information about human heart's condition. By analyzing the characteristics of ECG, doctors are able to judge whether the heart situation is normal or not, and know what troubles the heart confronts with. However, due to the limited ability of naked eyes and the complicated variations of ECG data, it is impractical and even impossible for doctors to cope with large amounts of ECG data in a limited time. Therefore, computer-aided diagnostic systems have drawn growing research interests up to date. By automatically doing ECG analysis, such systems can help doctors to enhance the diagnostic efficiency and reduce the misdiagnosis rate.

ECG analysis mainly includes signal denoising, wave detection, and heartbeat classification. Heartbeat classification can help diagnose the cardiac diseases, such as the arrhythmias, the myocardial infarctions, and so on. Typical waves in ECG signals usually contain abundant information about the cardiac diseases, so wave detection can help acquire the diagnostically valuable heartbeats from the signals. Although ECG signals are easily interfered by a variety of noises, signal denoising can help attenuate these noises and accentuate the typical ECG waves [1], [2]. Wavelet based methods for ECG analysis are classical and famous, but they are scattered and unorganized in the accessible literature resources. This work attempts to summarize these approaches into a complete overview and categorize them into a systemic taxonomy. By summarization and categorization, this overview and taxonomy aims to uncover the philosophy

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behind these methods and the potential designing principles they follow, for the purpose of making these approaches better to be understood, utilized, and improved, so as to further inspire new ideas and innovations in the future.

The framework of this paper is as bellows. As the part of outline and introduction, **Section I** not only clarifies the outline and the contribution of this paper, but also introduces the basic knowledge of ECG and wavelet transform; as the part of main body, **Sections II** provides the in-depth analysis and the systemic summary on the wavelet based methods for signal denoising, wave detection, and heartbeat classification, respectively, and also displays the performances of these methods; as the part of conclusion and outlook, **Section III** concludes the methodologies and provides the outlook for their developing trends in the future.

On the whole, this paper will display a full picture of wavelet methodology for ECG analysis on the basis of the summarized philosophies and principles. Therefore, this paper can also serve as a manual for researchers in the relevant fields to evaluate their works. Such a manual can not only provide readers with an overall understanding of the classical and the popular technologies for ECG analysis from the wavelet perspective, but also help them to fast know the essential ideas and the representative performances of these technologies.

B. INTRODUCTION OF ELECTROCARDIOGRAM

ECG is a time-varying signal reflecting the ionic current flow, which causes the cardiac fibers to contract and subsequently relax. Usually, ECG can be obtained by recording the potential difference between two electrodes placed on the surface of the skin. Generally, there are three main components in ECG: the P wave, the QRS complex, and the T wave. Each of them has a fairly unique pattern.

In early years, researchers inclined to model ECG signals for analyzing them. There exist various mathematical models of ECG in the literature, but almost all of them are limited to perfectly periodic waveforms. These models usually describe ECG by the P wave, the QRS complex, and the T wave separately [3].

One of the most representative ECG models is based on the Gaussian pulse:

$$v(t) = \sum_{n=-\infty}^{n=+\infty} [\phi_P(t-\tau_P-n\theta) + \phi_R(t-\tau_R-n\theta)+\phi_T(t-\tau_T-n\theta)]. \quad (1)$$

In Eq. (1), subscript P indicates the P wave; subscript T indicates the T wave; subscript R indicates the QRS complex for the sake of brevity; θ denotes the period, and thus the periodicity can be verified by $v(t+\theta)$.

Furthermore, the P wave can be modeled as

$$\phi_P(t-\tau_P) = a_P \exp[-\frac{(t-\tau_P)^2}{2b_P^2}]. \quad (2)$$

The QRS complex can be modeled as

$$\phi_R(t-\tau_R) = [a_R^{(0)} + a_R^{(1)}\frac{d}{dt} + a_R^{(2)}\frac{d^2}{dt^2}]\exp[-\frac{(t-\tau_R)^2}{2b_R^2}]. \quad (3)$$

The T wave can be modeled as

$$\phi_T(t-\tau_T) = a_T \exp[-\frac{(t-\tau_T)^2}{2b_T^2}]. \quad (4)$$

In Eq. (2)-(4), a_P, a_R and a_T denote the amplitude coefficients; b_P, b_R and b_T denote the width parameters; τ_P, τ_R and τ_T denote the center positions.

Many other models can also faithfully describe ECG, so long as they can keep the shape similarity between the basis functions and the ECG segments [4]–[6]. For example, some models use the Gaussian pulses to represent each of the P, Q, R, S and T waveforms individually, which thereby breaks the QRS complex into three separate waveforms; some models use sine and cosine pulses to simulate each waveform; other models describe the QRS complex by means of the triangular pulse; still other models adopt the pulses with other shapes to represent the waveforms.

In real applications, ECG signals usually show sophisticated variations for different patients across temporal and physical conditions. Even for the healthy subjects, the signals are not the same from one to the other, either. Although a number of approaches have tried to develop the mathematical models of ECG, they cannot precisely specify the potential variations. Considering the variability of ECG signals, currently, the analytic methods based on mathematical models are rarely used for ECG analysis in comparison with the numerical methods, especially those capable of capturing the local characteristics of ECG in practice. Despite so, these mathematical models still can help us better understand ECG.

C. INTRODUCTION OF WAVELET TRANSFORM

ECG signals can be regarded as the non-stationary random process. Fourier transform is not suitable for ECG analysis, because it characterizes the signals in a global way but ignores the local information. Although short-time Fourier transform divides long time signal into short segments of the equal length and then computes the Fourier transform separately on each segment, the big pitfall of short-time Fourier transform comes from its fixed resolution. Usually, the width of the windowing function relates to how the signal is represented. A wide window gives better a frequency resolution but a poor time resolution. A narrow window gives a good time resolution but a poor frequency resolution. Therefore, short-time Fourier transform is unsuitable to analyze the ECG signals of varying frequency as well.

Wavelet transform expands the signals in terms of the wavelet function that is localized in both time and frequency. Basically, wavelet transform should only allow the changes in time extension instead of the shape. Wavelet transform overcomes the weakness of the fixed resolution of short-time

Fourier transform. Wavelet transform can provide good time resolution for high-frequency events, and good frequency resolution for low-frequency events. Hence, wavelet transform is quite suitable for ECG analysis.

Given a signal $f(t)$, its Continuous Wavelet Transform (CWT) is formulated as:

$$\begin{aligned} F(a,\tau) &= < f(t), \psi_{a,\tau}(t) > \\ &= \frac{1}{|a|^{1/2}} \int_{-\infty}^{+\infty} f(t)\overline{\psi(\frac{t-\tau}{a})}\mathrm{d}t, \end{aligned} \tag{5}$$

In Eq. (5), a is the scaling parameter and τ is the translating parameter, where $a \in \mathbb{R}_+$ and $\tau \in \mathbb{R}$. The scaling parameter can either compress or dilate the signal. When a is relatively small, the signal will be more contracted; when a is large, the signal will be stretched out. The translating parameter is related to the location of the window, as the window is shifted through the signal. Wavelet basis function $\psi_{a,\tau}(t)$ is obtained by translating and scaling the mother wavelet $\psi(t)$ which is continuous in both time and frequency domains. The overline symbol represents the operation of complex conjugate. More concretely, $\psi_{a,\tau}(t)$ is given by:

$$\psi_{a,\tau}(t) = \frac{1}{|a|^{1/2}}\psi(\frac{t-\tau}{a}). \tag{6}$$

Discrete Wavelet Transform (DWT) is any wavelet transform for which the wavelets are discretely sampled. To obtain DWT, in $F(a,\tau)$, the scaling parameter a needs to be discretized by $a = b^j$ where $j \in \mathbb{Z}$ and $b > 0$, and the translating parameter τ should be discretized by $\tau = b^j kT$ where $k \in \mathbb{Z}$ and $T > 0$. Here, j is the frequency scale; k is the time scale; T is the constant depending on mother wavelet. Especially, when $b = 2$ and $j > 0$, DWT is called as "dyadic".

Furthermore, multi-resolution analysis is widely-used for designing DWT in practice. Multi-resolution analysis enables a signal to be decomposed into the coefficients at different scales. Usually, such process can be derived from the filter bank consisting of low-pass and high-pass filters. These filters decompose the input signal into low-frequency and high-frequency components, and the low-frequency component continues to be further decomposed in the similar manner. Such process will be repeated till reaching the expected maximum decomposition level.

II. WAVELETS FOR ECG ANALYSIS

A. WAVELETS FOR SIGNAL DENOISING

1) PHILOSOPHY

ECG signals are usually polluted by physiological and technological noises, which include baseline wander, power-line interference, electromyographic artifact, motion artifact, and so forth. Wavelet based methods occupy quite an important position for the issue of ECG signal denoising. Basically, these methods include four steps: decomposing the signals by the wavelet transform; deciding the threshold value based on the prescribed rules; using the thresholding function to filter the coefficients; reconstructing the signals based on the filtered coefficients by the inverse wavelet transform. The thresholding function is important for wavelet based denoisers. In general, the hard thresholding function can well preserve the sharpness of the original signal due to the discontinuity characteristics, but such kind of thresholding is vulnerable to the high-frequency noises; on the contrary, the soft thresholding function is smooth, but such kind of thresholding may distort the reconstructed signals. There are also many researchers attempting to design the thresholding function that can integrate the merits of both hard and soft thresholding and at the same time overcome the shortcomings of them, such as the non-negative garrote thresholding function, the polynomial thresholding function, and so on. The threshold calculation rule is crucial as well. If the threshold value is too large, the thresholding function will bear many noisy components of the signals; if the threshold value is too small, the thresholding function will discard the useful components. Therefore, it is appropriate to keep balance between the removal of noises and the conservation of signal waves. Widely-used threshold calculation rules primarily include the sqtwolog rule, the minimax rule, the heursure rule, the rigrsure rule, and so forth [7]–[9].

2) METHODOLOGY

a: MOTHER WAVELET SELECTION IS IMPORTANT FOR SIGNAL DENOISING

Seljuq *et al.* [10] have recommended Daubechies9 (Db9) as the mother wavelet function of the DWT for ECG signal denoising. Lin *et al.* [11] have chosen the Symlets5 (Sym5) based DWT with the soft thresholding and the sqtwolog rule for removing the electromyographic artifact from the ECG signals. Li *et al.* [12] have selected the Sym6 based DWT with the soft thresholding and the sqtwolog rule for reducing the noises in the ECG signals. By using the Genetic Algorithm (GA), El-Dahshan [8] has found that the Db7 or Db8 based DWT is more suitable to cope with the low-noise ECG signals, and that the Sym5 or Sym6 based DWT is more appropriate to deal with the high-noise ones. Sawant and Patii [13] have suggested using the Biorthogonal2.4 (Bior2.4) based DWT with the soft thresholding and the heursure rule for tackling the electromyographic artifact in the ECG signals, and utilizing the Db9 based DWT with the hard thresholding and the sureshrink rule for handling the power-line interference of the signals. Biswas *et al.* [14] have pointed out the particular advantage of the Bior5.5 based DWT in suppressing the power-line interference in the ECG signals.

b: PERFORMING THRESHOLDING ON THE WAVELET COEFFICIENTS IN A SUITABLE MANNER PLAYS A SIGNIFICANT ROLE IN THE PROCESS OF DENOISING

Smith *et al.* [15] have designed the polynomial thresholding function, and then optimized the polynomial coefficients by the least-squares minimization for ECG signal denoising. Tulsani and Gupta [16] have optimized the polynomial coefficients by the artificial bee colony algorithm for denoising the ECG signals. Zhang *et al.* [17] have decomposed the

ECG signals by the DWT based on the Mallat algorithm, and adopted the composite thresholding function that performs hard and soft thresholding separately in different levels of the DWT for tackling the baseline wander, the power-line interference and the electromyographic artifact in the signals. Zhang *et al.* [18] have combined the hard thresholding and the soft thresholding for coping with the noises in the ECG signals. The combining ratio and the regulatory factors are automatically computed according to the maximum and the minimum values of wavelet coefficients. Mithun *et al.* [19] have utilized the Discrete Meyer (Dmey) based DWT with the adjustable thresholding function to suppress the electromyographic artifact and the motion artifact in the ECG signals. This adjustable thresholding function combines the advantages of both hard and soft thresholding functions, and has a parameter to adjust the thresholding from the hard to the soft modes. Mallaparapu *et al.* [20] have crafted a thresholding function by mixing the hard and the non-negative garrote functions to denoise the ECG signals. Wang *et al.* [21] have improved the single-wavelet thresholding based on the non-negative garrote thresholding function by the double-wavelet lifting scheme for handling the baseline wander, the power-line interference and the electromyographic artifact in the ECG signals. Patil and Holambe [22] have set the adaptive threshold value in terms of the level-dependent minimax rule base on the DWT for handling different kinds of noises in the ECG signals. Yi and Song [23] have presented the adjustable thresholding function with the level-dependent sqtwolog rule based on the DWT for tackling different kinds of noises in the ECG signals. This thresholding function can be adjusted from the hard to the soft modes according to the actual signal situations. Poornachandra [24] has devised the Subband level dependent Median (S-median) threshold based on DWT for recovery of the ECG signals contaminated by the noises. Awal *et al.* [25] have put forward the modified S-median threshold with the additional level dependent adaptation factor based on the optimal wavelet function and the optimal decomposition level to cope with the composite noise in the ECG signals.

c: THE STRENGTHS OF WAVELET TRANSFORM AND DIFFERENT TYPES OF TECHNIQUES CAN BE COMBINED TO IMPROVE THE DENOISING PERFORMANCE

Liu *et al.* [26] have carried out ECG signal denoising by the Independent Component Analysis (ICA) in conjunction with the DWT based denoiser. Li *et al.* [27] have combined Coiflets5 (Coif5) based DWT, Fast Fourier Transform (FFT) based bandpass filter, and nonlinear Bayesian filter in one framework to cope with the noises in the ECG signals. The Coif5 based DWT can eliminate the power-line interference and the high-frequency noises; the FFT based bandpass filter can remove the baseline wander; the nonlinear Bayesian filter can reduce the Gaussian noise and the electromyographic artifact. Hao *et al.* [28] have presented the Multivariate Wavelet Denoising (MWD) algorithm based on the techniques of subspace and Principal Component Analysis (PCA) for dealing with the white Gaussian noise in the ECG signals. The key element of this Subspace and PCA based MWD (SP-MWD) algorithm is introducing an orthogonal matrix that divides the observation data into the signal subspace and the noise subspace to facilitate the wavelet based denoising. Rajankar and Talbar [29] have designed the Wavelet Neural Network (WNN), which involves the wavelets into the activation function of the Multi-Layer Perceptron Neural Network (MLP-NN), for eliminating the white Gaussian noise in the ECG signals. Alyasseri *et al.* [30] have suggested combining the DWT with the β-hill climbing technique for suppressing the white Gaussian noise in the ECG signals. β-hill climbing can help find the optimal parameters of the DWT so as to obtain the minimum of the mean square error between the original and the denoised signals. Jenkal *et al.* [31] have employed the Adaptive Dual Threshold Filter (ADTF) in conjunction with the DWT based denoiser for depressing the white Gaussian noise in the ECG signals. ADTF relies on the mean value of the signal in the moving window to calculate the high and the low thresholds. Hesar and Mohebbi [32] have proposed the model based Bayesian denoising framework, which utilizes the DWT based thresholding with the Variational Mode Decomposition (VMD) to lower the noise impact on the ECG signals and then adopts the Marginalized Particle-Extended Kalman Filter (MP-EKF) with the Fuzzy Based Adaptive Particle Weighting (FBAPW) technique to further tackle the noises in the signals. In this framework, the VMD is conducive to reducing the negative influence of the DWT based thresholding on the sharp signal waves, and the FBAPW technique is beneficial for adjusting the MP-EKF to the significant morphological changes and different noise measurements of the signals.

d: DUAL-TREE COMPLEX WAVELET TRANSFORM CAN WORK AS A CAPABLE WAVELET BASED DENOISER

Dual-Tree Complex Wavelet Transform (DTCWT) calculates the complex transform of a signal by means of two separate DWT decompositions. DTCWT can overcome the drawbacks that may be encountered by the DWT, such as the oscillations around singularities, the lack of shift invariance, the poor directional selectivity, and the severe frequency aliasing. Wang and Ji [33] have shown the capability of the DTCWT for ECG signal denoising. Shemi and Shareena [34] have found the capability of the double-density DTCWT to deal with the composite noise in the ECG signals. Zhang *et al.* [35] have leveraged the DTCWT and the median filtering to denoise the ECG signals before doing arrhythmia classification by the recurrent neural network and the density based clustering technique. B'charri *et al.* [36] have tuned the parameters of the DTCWT based denoiser to search for the best threshold function, the optimal threshold value, and the most suitable decomposition level to handle the noises in the ECG signals.

e: THE MODE DECOMPOSITION CAN BE INTEGRATED WITH WAVELET TRANSFORM FOR DENOISING THE SIGNALS

Empirical Mode Decomposition (EMD) decomposes a signal into a reasonably small number of approximately harmonic components, which are referred to as intrinsic mode functions, in the time-frequency plane. Li and Li [37] have presented the ECG signal denoising method based on the cooperation of the EMD and the wavelet adaptive thresholding technique. This method utilizes the EMD to make up the indetermination for choosing the wavelet function, and then adopts the wavelet adaptive thresholding technique to prevent the distortion effect of EMD. Kabir and Shahnaz [38] have resorted to the collaboration of the EMD and the DWT for denoising the ECG signals. This approach carries out windowing in the EMD domain to reduce the noises from initial intrinsic mode functions, and then denoises the yielded signals by the DWT. Nevertheless, as the weakness, EMD lacks the mathematical theory and exhibits too many modes that are difficult to interpret. In contrast to EMD, Empirical Wavelet Transform (EWT) is a fully adaptive and data-driven signal processing technique with well-defined mathematical backgrounds. EWT performs mode decomposition using the adaptive wavelet filter bank based on the boundaries that are computed from the segmentation of the signal Fourier spectrum. Singh and Sunkaria [39] have made use of the EWT with the technique of mode subtraction for dealing with different kinds of noises in the ECG signals. Synchro-Squeezed Wavelet Transform (SSWT) can also realize the adaptive time-frequency decomposition, which is the goal of EMD. But different from EWT, SSWT brings together the wavelet analysis and the reallocation technique. Daubechies *et al.* [40] have proposed the SSWT that reallocates the CWT coefficients to obtain a concentrated time-frequency picture, from which the instantaneous frequency lines can be extracted. SSWT is capable of decomposing the superposition of well-separated modes with the analytic wave shape functions sufficiently close to the exponential function. But as the weakness, this transform is quite easy to mix up the high-frequency modes, which more or less limits its application for the spike-shape signals. Yang [41] has relieved much of such trouble by the synchro-squeezed WPT. Synchro-squeezed WPT combines together the WPT of a certain geometric scaling, the reallocation technique for sharpening the phase space representation, and the clustering algorithm for mode decomposition. This transform not only can provide the effective mode decomposition and the accurate instantaneous property estimation, but also has better resolution than the SSWT to distinguish the harmonic modes with high frequencies. It is worth noting that the SSWT and the synchro-squeezed WPT are also the powerful tools for ECG signal denoising.

B. WAVELETS FOR WAVE DETECTION

1) PHILOSOPHY

Typical waves of ECG signals mainly refer to the P, Q, R, S and T waves. Because the QRS complexes contain significant information about the cardiac diseases, like most researches in the literature, this paper focuses the issue of ECG wave detection on QRS complex finding. Usually, QRS complex detection can be reduced to the problem of R peak detection, due to that once the R wave, which has the most salient peak in the QRS complex, is located, the Q and the S waves can also be easily found. Hence, in practice, the QRS complex location can be represented by the R peak location as well. Wavelet transform is achieved by the convolution of the wavelet basis function and the target signal, and if there is an event in the signal with a similar morphology to the wavelet, a peak will appear in the convolution result. Thus, the wave-like oscillation property of wavelets makes wavelet transform inherently suitable for QRS complex detection. Moreover, wavelet transform can exploit the multi-scale information of signals. By using wavelet transform, the energy of any QRS complex will tend to concentrate around a specific time point across several successive scales. With such multi-scale information, it can be easy to distinguish the QRS complexes from other high waves as well as different kinds of noises [42]–[44]. Note that, in this work, wavelet based detectors not only include the methods that directly detect the typical waves in ECG signals using the wavelet transform, but also involve those doing detection based on the ECG signals whose waves have been accentuated by the wavelet based denoisers.

2) METHODOLOGY

a: IT IS BEST TO USE THE WAVELET SIMILAR TO THE TARGET WAVEFORM FOR WAVE DETECTION

Haddadi *et al.* [45] have suggested using the Db4 based DWT for QRS complex detection based on the ECG signals due to that this wavelet is similar to the QRS complex. Kaur *et al.* [46] have demonstrated the advantage of the Db6 based DWT for detecting the QRS complexes in the ECG signals. Balachandran *et al.* [47] have leveraged the Db6 based DWT to process the ECG signals for R peak finding. Wang and Eklund [48] have utilized the Db6 based DWT to generate the masks for the potential QRS complexes in the ECG signals, and then narrowed down the searching to the masked regions for accurately finding the QRS complexes. Sabherwal *et al.* [49] have used the Db6 based DWT to deepen the S valleys of the ECG signals firstly, and then detected the S valleys and the R peaks independently; after that, they have estimated the R peak locations using the detected S valleys, and finally fused together the R peak estimation and the R peak detection for QRS complex locating. Das *et al.* [50] have taken advantage of the Morlet based CWT with the zero-crossing technique for R peak detection based on the ECG signals. Li *et al.* [43] have found that the quadratic spline wavelet based DWT is suitable for detecting the QRS complexes in the ECG signals. Chen *et al.* [51] have recommended the DWT with the quadratic Bior spline mother wavelet for locating the QRS complexes in the ECG signals.

b: WAVELET BASED DETECTORS USUALLY CARRY OUT THRESHOLDING OR MAXIMA FINDING BASED ON THE WAVELET COEFFICIENTS AT SUITABLE SCALES

Abdelliche *et al.* [52] have performed thresholding on the fused real parts at scale 3 and scale 4 of CWT for making the QRS complexes of the ECG signals more salient, and then implemented the zero-crossing testing on the imaginary part at scale 4 of CWT for locating the QRS complexes. Aqil *et al.* [53] have suggested detecting the R peaks by means of the maxima finding based on the CWT coefficients at scale 6, which corresponds to the maximum energy of the ECG signals. Yochum *et al.* [54] have located the QRS complexes in the ECG signals based on the threshold that is automatically determined by the histogram representation of the CWT coefficients at scale 38. Pal and Mitra [55] have recommended finding the maxima of the fused detail coefficients from scale 3 to scale 5 of the DWT for QRS complex detection based on the ECG signals. Sahoo *et al.* [56] have carried out thresholding on the fusion of the detail coefficients from scale 3 to scale 5 of the DWT for detecting the QRS complexes in the ECG signals. Rai and Chatterjee [57] have put forward the R peak detection method based on the ECG signals by means of finding the maxima of the fused detail coefficients from scale 3 to scale 5 of the DWT. Banerjee *et al.* [58] have taken advantage of the fusion of the detail coefficients at scale 4 and scale 5 of the DWT to set the threshold for QRS complex detection based on the ECG signals.

c: TEMPORAL THRESHOLDING AND MORPHOLOGICAL THRESHOLDING ARE COMPLEMENTARY TO THE WAVELET BASED THRESHOLDING FOR DETECTING THE WAVES

By taking advantage of the temporal and the morphological characteristics of ECG signals as the thresholds, the temporal thresholding and the morphological thresholding can effectively cooperate with the wavelet based thresholding for wave detection. Here, the temporal characteristics mainly refer to the timing interval properties and the morphological characteristics primarily refer to the amplitude properties of ECG signals. Razavi and Mohammadi [59] have performed thresholding on the maxima of the CWT coefficients in the moving window based on the ECG signals to detect the R peak candidates. If the distance of the two detected R peaks is more than the threshold of 1.5 times the average RR-interval length, this approach returns to the previous R peak location and performs detection using the smaller threshold. Zidelmal *et al.* [60] have fused the detail coefficients of the DWT to set the threshold for detecting the QRS complexes in the ECG signals, and then made use of the smaller threshold to search back for the missed ones if the distance between two QRS complexes is bigger than the threshold of 1.5 times the current RR-interval length. Lin *et al.* [11] have denoised the ECG signals by the DWT based thresholding for making the R peaks more discernable, and then used the amplitude and the interval thresholds in the adaptive searching window for R peak detection. If the maximum value in the searching window is higher than the estimated amplitude threshold, such location should correspond to the R peak; if the distance between two R peaks is longer than the threshold of 1.4 times the average RR-interval length, this method searches back and uses the smaller amplitude threshold to find the R peaks again. Jenkal *et al.* [61] have denoised the ECG signals by the DWT based thresholding for making the QRS complexes more perceptible, and then detected all the possible QRS complexes by the relatively small amplitude threshold, and finally removed the false QRS complexes by a moving threshold window whose size is determined by the maximum QRS complex duration and the maximum heart rate. Qin *et al.* [62] have denoised the ECG signals by the DWT based thresholding for making the R peaks more detectable, and then adopted the signal mirroring technique to convert the large negative R peaks to the positive ones based on the processed signals, and finally truncated the local maxima by the amplitude threshold and the interval threshold to locate the R peaks.

d: THE EXTENSIONS OF WAVELET TRANSFORM ARE ALSO POWERFUL FOR THE DETECTION TASK

As an extension of DWT, Wavelet Packet Transform (WPT) recursively decomposes the approximation and detail coefficients using the same filtering and down-sampling techniques as used in DWT. Attributing to a more detailed decomposition, WPT allows the flexible signal analysis for a desired frequency band in contrast to the prefixed octave frequency bands [63]. Vega-Martinez *et al.* [64] have utilized the WPT to process the ECG signals for making the QRS complexes more distinctive, and then calculated the envelops of the processed signals, and finally carried out thresholding on the enveloped signals to locate the QRS complexes. Chouakri *et al.* [65] have designed the QRS complex detection routine that processes the nodes of both Haar based WPT and Db10 based WPT by the histogram approach based on the ECG signals. Singh and Sunkaria [66] have improved the performance of QRS complex detection based on the ECG signals by using the adaptive wavelet at each scale in the WPT tree and the adaptive threshold in each detection channel before combining the results from different channels. As another extension of DWT, Stationary Wavelet Transform (SWT) maintains a dyadic sampling of the scales, but the time steps are not subsampled at each level and hence are not dyadic. SWT can overcome the lack of translation invariance of DWT [63]. Merah *et al.* [67] have taken advantage of the SWT to detect the R peaks in the ECG signals. Farashi [44] has also utilized the SWT for R peak detection based on the ECG signals. As an extension of CWT, S-Transform (S-T) is derived by the phase correction of the CWT with the window being the Gaussian function. S-T can provide the frequency-dependent resolution whilst maintaining a direct relationship with the Fourier spectrum. Zidelmal *et al.* [68] have computed the Shannon Energy (ShE) from the local spectrum based on the S-T to detect the QRS complexes in the ECG signals. As another extension of CWT, SSWT integrates the synchro-squeezed technique with CWT. SSWT can effectively decompose a class of superpositions of intrinsic mode type components and accurately estimate the instantaneous properties of these components. Herry *et al.* [69] have acquired the SSWT-derived phase information, and incorporated this information into the Optimized Knowledge Based Detector (OKBD) to improve the performance of R peak detection based on the ECG signals.

e: HILBERT TRANSFORM CAN COOPERATE WITH WAVELET TRANSFORM TO IMPROVE THE DETECTION PERFORMANCE

Hilbert Transform (HT) is capable of analyzing the instantaneous attributes of signals. This transform is complementary with wavelet transform for the task of wave detection. Sabherwal *et al.* [70] have used the DWT to denoise the ECG signals, and then adopted the derivatives of the reconstructed signals to enhance the QRS complexes, and finally employed the HT with the peak-finding logic for R peak detection. Rakshit and Das [71] have utilized the DWT and the ShE Transform (ShET) to attenuate the noises and accentuate the QRS complexes in the ECG signals, and then leveraged the peak-finding logic based on the HT to detect the R peaks.

C. WAVELETS FOR HEARTBEAT CLASSIFICATION

1) PHILOSOPHY

The main challenges for the issue of ECG heartbeat classification are the large intra-class variation and small inter-class difference of ECG data, which are caused by the facts that the heartbeats in the same class may present different characteristics across subjects and conditions but that those from different classes may exhibit similar temporal and morphological properties. Feature designing and classifier learning are the essential stages in the methodological pipeline for heartbeat classification in general. Feature designing is important and fundamental in the pipeline, because this stage establishes a platform for the subsequent procedures. Note that, in this work, wavelet based features not only include the features that are directly extracted from the coefficients of wavelet transform for ECG data, but also involve those acquired based on the ECG characteristic points that have been located by the wavelet based detectors. Wavelet based features been widely used for heartbeat classification, because they can capture the local characteristics of ECG data. Good features can project ECG heartbeats into a discriminatory space where the samples in the same class stay close together and those from different classes reside far apart. However, in real applications, the heuristics of designing process and the complicacy of ECG data unavoidably limit the feature power. Therefore, learning the classifiers that can further improve the intra-class compactness and inter-class separation of heartbeat samples in the feature space is also important and indispensable for classification [1], [72], [73].

2) METHODOLOGY

a: BY USING WAVELET BASED FEATURES, NEURAL NETWORKS ARE QUITE DISCRIMINATIVE FOR HEARTBEAT CLASSIFICATION

Sarkaleh and Shahbahrami [74] have designed the Wavelet Coefficient Feature (WCF) for the ECG heartbeats using the maximum, the minimum and the variance of the DWT coefficients, and then leveraged the MLP-NN based on this feature for arrhythmia classification. Sarma *et al.* [75] have acquired the features of the normalized subband energy, the average subband energy, and the relative average subband energy based on the DWT of the ECG heartbeats, and then learned the MLP-NN based on these features for arrhythmia recognition. Özbay *et al.* [76] have carried out the T2FCM clustering to group the ECG heartbeats into the clusters with different memberships, and then used the DWT to extract the WCF which is made up of the decomposition coefficients from these heartbeats with the training patterns obtained from fuzzy clustering, and finally performed the MLP-NN based on this feature for the arrhythmia discrimination. Das and Ari [77] have taken advantage of the mixture of the WCF, the Temporal Feature (TF), and the S-T Feature (STF) as the representation of each ECG heartbeat, and then used the MLP-NN based on this representation for classifying the arrhythmias. Here, the WCF contains the mean, the maximum, the minimum and the standard deviation of the DWT coefficients; the TF consists of the pre-RR, the post-RR, the local-RR and the average-RR intervals; the STF is comprised of the statistical attributes of the time-frequency contour and the time maximum amplitude plot from the S-T. Rai *et al.* [78] have concatenated the WCF, the TF and the Morphological Feature (MF) together as the representation of each ECG heartbeat, and then applied the MLP-NN based on this representation for recognizing the arrhythmias. Here, the WCF is comprised of the mean, the variance and the standard deviation of the DWT coefficients; the MF is composed of the amplitudes of P, Q, R, S and T waves and the number of R peaks, where these signal characteristic points are detected by means of the DWT; the TF is formed by the standard deviations of the RR, the PR, the PT, the ST, the TT and the QT intervals. Thomas *et al.* [79] have performed the FFT on the absolute values of the DTCWT coefficients of the ECG heartbeats, and then calculated the logarithm of the Fourier spectrum to obtain the WCF; after that, they have combined the WCF with the TF and the Higher Order Statistics (HOS) feature together to construct the feature space, and ultimately employed the MLP-NN in the feature space for discriminating the arrhythmias. Here, the TF is the ratio between the pre-RR and the post-RR intervals; the HOS feature consists of the variance, the skewness and the kurtosis of the QRS complexes. Rai and Chatterjee [57] have crafted the MF for the ECG heartbeats based on the uniformly sampled points around the R peaks which are detected by means of the DWT, and then made use of the Probabilistic Neural Network (PNN) in the feature space for identifying the arrhythmias. Here, the MF contains the amplitudes of 21 points sampled around each R peak with the equal step. Patil and Singh [80] have combined the TF and the MF together to construct the feature space of the ECG heartbeats, and then adopted the wavelet based classifier, WNN, in the feature space for discriminating the arrhythmias. Here, the TF and the MF are extracted from the QRS complexes detected by means of the DWT; the TF is comprised of the pre-RR and the post-RR intervals, the QRS complex duration, and the ratio between the post-RR and the pre-RR intervals; the MF is the R peak amplitude. Sumathi *et al.* [81] have obtained the MF of the ECG heartbeats from the QRS complexes detected by means of the DWT, and then utilized the Adaptive Neuro-Fuzzy Inference System (ANFIS) in the feature space for arrhythmia identification. Here, the MF contains the amplitudes of Q, R and S waves and the two slope values of each R wave; the ANFIS is a kind of neural network that is based on the fuzzy inference system.

b: DEEP LEARNING FURTHER EXPLOITS THE POTENTIALITIES OF NEURAL NETWORKS FOR WAVELET BASED CLASSIFICATION

Luo *et al.* [82] have extracted the Time-Frequency Representation (TFR) from each ECG heartbeat by the Modified Frequency Slice Wavelet Transform (MFSWT), and then adopted the Stacked Denoising Autoencoder (SDA) based on this representation to do patient-specific arrhythmia classification. Rahhal *et al.* [83] have resorted to the Convolutional Neural Network (CNN), which is made up of the alternating convolutional and pooling layers as well as the additional fully-connected layers, for classifying the arrhythmias. This method pre-trains the CNN model by a large amount of auxiliary image data at first, and then learns the pre-trained model based on the over-complete TFR which is extracted from each ECG heartbeat using the CWT with different mother wavelets. Yildirim [84] has proposed the Deep Bidirectional Long-Short Term Memory (DBLSTM) network with the input layer of Wavelet Sequence (WS) to classify the arrhythmias. This approach utilizes the WS layer to produce the sequences that contain the main ECG segments and their DWT subbands, and then delivers these sequences into the DBLSTM network for the classification. Li and Li [85] have constructed the Local Deep Field (LDF) for multiple-beat arrhythmia classification. This method learns the deep MLP-NN model within each local chart of the ECG data manifold that has been denoised by means of the DWT. Local regionalization can help tackle local variations of the data, and deep models can explore the hidden class information within local distributions.

c: BY USING WAVELET BASED FEATURES, SUPPORT VECTOR MACHINE IS VERY DISCRIMINATORY FOR CLASSIFYING THE HEARTBEATS

Li *et al.* [86] have designed the ECG heartbeat features which include the WCF optimized by the Linear Discriminant Analysis (LDA) and the Kernel ICA (KICA) feature extracted from the PCA subspace of the raw data, and then learned the Support Vector Machine (SVM) based on these features for arrhythmia classification. Here, the WCF consists of the mean, the maximum, the minimum and the standard deviation of the DWT coefficients. Ye *et al.* [87] have concatenated the WCF, the TF and the ICA feature together to construct the ECG heartbeat feature space, and then performed the SVM in the feature space for arrhythmia recognition. Here, the WCF is formed by the DWT coefficients; the TF is comprised of the pre-RR, the post-RR, the local-RR and the average-RR intervals; the ICA feature is calculated based on the raw data. Elhaj *et al.* [88] have combined the PCA of the WCF and the ICA of the HOS feature together to construct the feature space of the ECG heartbeats, and then used the SVM with the kernel of Radial Basis Function (RBF) in the feature space for arrhythmia discrimination. Here, the WCF is made up of the DWT coefficients; the HOS feature is comprised of the skewness and the kurtosis of the QRS complexes. Daamouche *et al.* [89] have formulated the polyphase representation of the wavelet filter bank and the SVM within a Particle Swarm Optimization (PSO) framework based on the ECG heartbeats for arrhythmia identification. This framework applies the generated DWT to each heartbeat with the normalized periodic length to obtain the WCF, and then adds the TF to the WCF before doing classification by the SVM. Here, the WCF consists of the DWT coefficients; the TF is composed of the pre-RR and the local-RR intervals and the QRS complex duration. Garcia *et al.* [90] have extracted the Temporal Vectorcardiogram (TVCG) feature by the complex network, and then concatenated this feature with the DWT based autocorrelation feature, the TF and the MF together as the representation of each ECG heartbeat, and finally employed the PSO to optimize this representation and the SVM for classifying the arrhythmias. Here, the DWT based autocorrelation feature is obtained by applying the autocorrelation function to the DWT subbands; the TF is composed of the pre-RR, the post-RR and the local-RR intervals, the QRS complex and the T wave durations, and the presence or absence of the P-wave; the MF is made up of the amplitudes of 37 points sampled from each heartbeat in a certain way. Cheng and Dong [91] have computed the feature using the correlation coefficients between the normal ECG heartbeat template and each testing heartbeat sample, and then implemented the SVM based on the combination of this feature and other kinds of features for arrhythmia classification. Alickovic and Subasi [92] have performed the multi-scale PCA based on the DWT coefficients for ECG signal denoising, and then employed the autoregressive model for feature extraction based on the denoised signals, and finally utilized the SVM with the sequential minimal optimization in the feature space for recognizing the arrhythmias. Venkatesan *et al.* [2] have acquired the frequency-domain feature of the Heart Rate Variability (HRV) signals, which are extracted from the ECG data, based on the R peaks detected by the DWT, and then utilized the SVM in the feature space for discriminating the arrhythmias. Herry *et al.* [69] have integrated the SSWT-derived phase feature, the TF and the MF together as the representation of the ECG heartbeats, and then carried out the SVM based on this representation for identifying the arrhythmias. Here, the SSWT-derived phase feature, the TF and the MF are extracted from the QRS complexes detected by the collaboration of the OKBD and the SSWT; the TF contains the pre-RR, the post-RR and the average-RR intervals and the QRS complex duration; the MF is the R peak amplitude.

d: BESIDES NEURAL NETWORKS AND SUPPORT VECTOR MACHINE, MANY OTHER KINDS OF CLASSIFIERS ALSO SHOW STRONG CAPABILITY FOR THE CLASSIFICATION BASED ON WAVELET TRANSFORM

Li *et al.* [93] have designed the WCF for the ECG heartbeats using the coefficients of the DWT with the mother wavelet selected from Reverse Bior6.8 (RBior6.8), Fejer-Korovkin22 (FK22), and so forth, and then adopted the Metric Learning to Rank (MLR) to improve the discriminative ability of the feature space, and finally measured the Minority Based Dissimilarity (MBD) between the feature sets for multiple-beat arrhythmia classification. Abdullah *et al.* [94] have concatenated the WCF with the TF which is extracted based on the R peaks detected by the DWT together as the representation of each ECG heartbeat, and then implemented the quadratic discriminant analysis on the basis of this representation for arrhythmia recognition. Here, the WCF contains the mean, the standard deviation, the skewness and the kurtosis of the DWT coefficients; the TF consists of the pre-RR and the post-RR intervals and the ratio of the pre-RR interval over the total period of each heartbeat. Kumar and Inbarani [95] have devised the MF based on the ECG characteristic points which are detected by means of the DWT, and then taken advantage of the Neighborhood Rough Set (NRS) to classify this feature for arrhythmia discrimination. Here, the MF is composed of the amplitudes of P, Q, R, S and T waves. Jung and Lee [96] have acquired the ECG heartbeat features by performing both PCA and LDA on the WCF that is comprised of the DWT coefficients firstly, and then classified these features by the K-Nearest Neighbor (KNN) algorithm for arrhythmia identification; after that, they have tested the classified samples using the fitness rule, and ultimately reclassified the misclassified ones by the weighted KNN technique. Elhaj *et al.* [97] have combined the PCA of the WCF and the ICA of the HOS feature together to construct the ECG heartbeat feature space, and then utilized the Bayesian and Extreme Learning Machine (B-ELM) technique in the feature space for classifying the arrhythmias. Here, the WCF is made up of the DWT coefficients; the HOS feature is formed by the skewness and the kurtosis of the QRS complexes. Pan *et al.* [98] have used different kinds of features together to describe the ECG heartbeats, and then carried out the Random Forest (RF) for recognizing the arrhythmias. Here, these features include the variance of the raw data, the variance of the DWT coefficients, the PCA of the raw data, the PCA of the DWT coefficients, the min-max ratio of the DWT coefficients, the autocorrelation feature of the DWT coefficients, and the TF which is the adjacent heartbeat intervals. Banerjee and Mitra [99] have obtained the Wavelet Cross Spectrum Feature (WCSF) and the Wavelet Coherence Feature (WCoF) from the Cross Wavelet Transform (XWT) between each pair of the ECG heartbeats, and then classified these features by the thresholding technique for identifying the inferior myocardial infarction.

e: SUBSPACE METHODS CAN REMOVE THE REDUNDANT DIMENSIONS OF WAVELET BASED FEATURES BEFORE CLASSIFIER LEARNING

Martis *et al.* [100] have designed the WCF using the DWT coefficients of the ECG heartbeats, and then reduced the feature dimension by the technique selected from the PCA, the ICA, and the LDA, and finally performed arrhythmia classification by the classifiers of the MLP-NN, the PNN, and the SVM, respectively, based on the processed feature. By comparison, they have found that, based on the WCF, the combination of the ICA and the PNN produces the best performance. Martis *et al.* [101] have acquired the feature subspaces of the ECG heartbeats by applying the PCA to the WCF and the discrete cosine transform coefficient feature, respectively, and also by applying the ICA to each of these two features; after that, they have employed the KNN, the Decision Tree (DT), and the MLP-NN independently in the each feature subspace for arrhythmia recognition. By comparison, they have found that, based on the discrete cosine transform coefficient feature, the cooperation of the ICA and the KNN yields the best performance. Martis *et al.* [102] have devised the WCF for the ECG heartbeats using the coefficients of the DWT with the carefully selected basis function, and then used the PCA to reduce the dimension of the WCF before doing arrhythmia discrimination by the SVM, the MLP-NN, and the Gaussian Mixture Model (GMM), respectively. By comparison, they have found that, in combination with the PCA and the SVM, the Sym2 based WCF generate the best performance. Martis *et al.* [103] have applied the PCA to the ECG heartbeats, the error signals of linear prediction model, and the WCF composed of the DWT coefficients independently to obtain three feature subspaces for arrhythmia identification. By comparison, they have found that, in cooperation with the SVM, the raw data based PCA achieves the best performance. Rajagopal and Ranganathan [104] have evaluated different linear and nonlinear subspace approaches in collaboration with the PNN for classifying the arrhythmias based on the WCF that consists of the DWT coefficients of the ECG heartbeats. By comparison, they have found that, based on the WCF, the collaboration of the tangential contrast function based fast ICA and the PNN leads to the best performance. Nazarahari *et al.* [105] have generated the suitable wavelet functions for the ECG heartbeats by means of the polyphase representation of the wavelet filter bank within a hybrid GA and PSO (GA-PSO) framework, and then taken advantage of the PCA to reduce the dimension of the WCF that is comprised of the generated DWT coefficients, and finally adopted the MLP-NN in this feature subspace for recognizing the arrhythmias.

f: WAVELET PACKET TRANSFORM PROVIDES AN EFFECTIVE TOOL FOR FEATURE EXTRACTION

Li and Zhou [106] have calculated the Shannon Entropy (ShEnt) of each terminal node of the WPT for the ECG heartbeats at first, and then composed the entropies as the feature; after that, they have combined this feature with two adjacent RR intervals together to construct the feature space, and at last utilized the RF in the feature space for the classification. Li *et al.* [107] have used the WPT to decompose the ECG heartbeats into different frequency bands, and then calculated the Approximate Entropy (AppEnt) of the decomposition coefficients as the feature, and finally classified this feature by the SVM with the parameters estimated by the PSO for arrhythmia recognition. Kutlu and Kuntalp *et al.* [108] have designed the HOS feature that contains the variance, the skewness and the kurtosis of the WPT coefficients of the ECG heartbeats, and then carried out the KNN based on this feature for arrhythmia discrimination. Shahnaz *et al.* [109] have acquired the intrinsic mode functions by the EMD firstly, and then implemented the WPT merely on the selected dominant intrinsic mode functions; after that, they have extracted the HOS feature that is formed by the variance, the skewness and the kurtosis of the WPT coefficients, and ultimately classified this feature by the KNN algorithm. Li *et al.* [110] have crafted the WCF for the ECG heartbeats using the maximum, the standard deviation and the singular value of the WPT coefficients, and then implemented the MLP-NN in the feature space with the use of the GA to decrease the feature dimension and optimize the classifier parameters for classifying the arrhythmias.

g: FLEXIBLE ANALYTIC WAVELET TRANSFORM CAN BE UTILIZED TO OBTAIN THE USEFUL ENTROPY FEATURES

Flexible Analytic Wavelet Transform (FAWT) is a rational-dilation wavelet transform, which is allowed to easily adjust the parameters of the quality-factor, the redundancy, and the dilation factor for analyzing the signals. FAWT can be implemented by the iterative filter bank. At each level of the iterative filter bank, this wavelet transform has one low-pass and two high-pass channels. The two high-pass channels separate the negative and positive frequencies and provide the analytic bases for the signals. For diagnosing the atrial fibrillation, Kumar *et al.* [111] have used the FAWT to obtain the subbands from the ECG heartbeats, and then extracted the Log Energy Entropy (LEEnt) and the Permutation Entropy (PeEnt) features from the subbands, and finally adopted the RF to classify these features. For diagnosing the myocardial infarction, Kumar *et al.* [112] have utilized the FAWT to decompose the ECG heartbeats into the subbands, and then computed the Sample Entropy (SaEnt) feature from these subbands, and finally adopted the SVM with the linear, the polynomial, the RBF, and the Morlet wavelet kernels, respectively, to classify this feature. It is worth mentioning that the SVM with the wavelet kernel can also be seen as a kind of wavelet based classifier. From another perspective, HRV signals usually carry useful information related to the nature of heart diseases. For diagnosing the congestive heart failure, Kumar *et al.* [113] have employed the FAWT to decompose the HRV signals into the subbands at first, and then computed the Accumulated Fuzzy Entropy (AFEnt) and the Accumulated Permutation Entropy (APEnt) features from the cumulative sums of these subbands; after that, they have ranked the feature samples according to their discrimination ability, and at last classified the ones of comparatively high discrimination ability by the SVM with the linear, the RBF, and the Morlet wavelet kernels, respectively. For diagnosing the coronary artery disease, Kumar *et al.* [114] have decomposed the HRV signals using the FAWT firstly, and then extracted two nonlinear features, the K-Nearest Neighbor Entropy Estimator (KNNEE) and the Fuzzy Entropy (FuEnt), from the decomposed subbands; after that, they have implemented the ranking technique to select the discriminative feature samples, and ultimately classified the selected ones by the SVM with the RBF and the Morlet wavelet kernels, respectively. Besides, Kumar *et al.* [115] have applied the FAWT to decomposing the ECG heartbeats, and then calculated the Cross Information Potential (CIP) feature from the real values of the decomposition coefficients, and finally made use of the SVM with the RBF and the Morlet wavelet kernels, respectively, to classify this feature. In addition, Tunable-Q Wavelet Transform (TQWT) can also be treated as a kind of FAWT. TQWT facilitates the signal analysis by means of the easily adjustable parameters which are the quality-factor, the redundancy, and the number of decomposition levels. Also for diagnosing the coronary artery disease, Patidar *et al.* [116] have decomposed the heart rate signals that are acquired from the ECG data into various subbands using the TQWT at first, and then extracted the nonlinear feature, centered correntropy, from the decomposed detail subbands; after that, they have carried out the PCA to transform the feature space, and at last taken advantage of the SVM with the RBF, the Morlet wavelet, the Mexican hat wavelet kernels, respectively, to classify this feature.

D. METHOD PERFORMANCE

Commonly-used databases for research on ECG analysis mainly include the MIT-BIH Arrhythmia Database, the MIT-BIH Noise Stress Test Database, the MIT-BIH Normal Sinus Rhythm Database, the MIT-BIH Atrial Fibrillation Database, the MIT-BIH Malignant Ventricular Arrhythmia Database, the PTB Diagnostic ECG Database, the Fantasia Database, the QT Database, the Apnea-ECG Database, the BIDMC Congestive Heart Failure Database, the St.-Petersburg Institute of Cardiological Technics 12-lead Arrhythmia Database, and so forth. Their introductions and resources are available on the website http://www.physionet.org/physiobank/database.

Representative wavelet based methods for signal denoising are summarized in Tables 1-3. The evaluation metrics

TABLE 1. Representative wavelet based methods for signal denoising (Part 1).

Researches	Denoisers	Noises	Results	Databases
Alyasseri et al. [30]	DWT(Sym3) with Soft Thresholding (Heursure Rule)	WGN	SNR_{in}=0 dB SNR_{out}=8.9649 dB	MIT-BIH Arrhythmia
	DWT(RBior4.4) with Soft Thresholding (Rigrsure Rule)		SNR_{in}=5 dB SNR_{out}=12.6986 dB	
	DWT(Coif3) with Soft Thresholding (Rigrsure Rule)		SNR_{in}=10 dB SNR_{out}=16.6867 dB	
	DWT(Bior6.8) with Soft Thresholding (Heursure Rule)		SNR_{in}=15 dB SNR_{out}=20.404 dB	
	DWT(Sym8) with Soft Thresholding (Rigrsure Rule)		SNR_{in}=20 dB SNR_{out}=24.1728 dB	
	DWT(Db36) with Soft Thresholding (Heursure Rule)		SNR_{in}=25 dB SNR_{out}=27.7325 dB	
	DWT(Db43) with Soft Thresholding (Heursure Rule)		SNR_{in}=30 dB SNR_{out}=31.3455 dB	
Hao *et al.* [28]	SP-MWD(Sym4) with Soft Thresholding	WGN	SNR_{in}=2 dB SNR_{out}=13.5 $\sim$ 14 dB	PTB Diagnostic ECG
Jenkal *et al.* [31]	DWT(Db6)+ADTF	WGN	SNR_{in}=-15 dB SNR_{out}=22.27 dB MSE_{out}=0.0019 dB	Record 115 of MIT-BIH Arrhythmia
Rajankar and Talbar [29]	DWT(Db6) based WNN	WGN	SNR_{in}=-100 dB SNR_{out}=27.19 dB	Record 100 of MIT-BIH Arrhythmia
Patil and Holambe [22]	DWT(Bior4.4) with Hard Thresholding (Level-Dependent Minimax Rule)	WGN	SNR_{in}=9.6564 dB SNR_{out}=10.265 dB $RMSE_{out}$=0.043998 dB	Record 103 of MIT-BIH Arrhythmia
		MA	SNR_{in}=4.0372 dB SNR_{out}=11.250 dB $RMSE_{out}$=0.045176 dB	
Hesar and Mohebbi [32]	DWT(Db4) with Soft Thresholding (Heursure Rule) based on VMD+ MP-EKF(FBAPW)	WGN	SNR_{in}=-1 dB SNR_{out}=1.395 $\pm$ 0.539 dB	MIT-BIH Arrhythmia
			SNR_{in}=-3 dB SNR_{out}=1.393 $\pm$ 0.542 dB	
			SNR_{in}=-5 dB SNR_{out}=1.3905 $\pm$ 0.539 dB	
		WGN	SNR_{in}=-1 dB SNR_{out}=0.901 $\pm$ 0.176 dB	MIT-BIH Normal Sinus Rhythm
			SNR_{in}=-3 dB SNR_{out}=0.993 $\pm$ 0.187 dB	
			SNR_{in}=-5 dB SNR_{out}=1.039 $\pm$ 0.192 dB	
		EMG	SNR_{in}=-1 dB SNR_{out}=1.113 $\pm$ 0.162 dB	
			SNR_{in}=-3 dB SNR_{out}=1.237 $\pm$ 0.170 dB	
			SNR_{in}=-5 dB SNR_{out}=1.292 $\pm$ 0.176 dB	

TABLE 2. Representative wavelet based methods for signal denoising (Part 2).

Researches	Denoisers	Noises	Results	Databases
Lin *et al.* [11]	DWT(Sym5) with Soft Thresholding (Sqtwolog Rule)	EMG	$\mathrm{SNR_{in}}=-10$ dB $\mathrm{SNR_{imp}}=12.9$ dB	MIT-BIH Arrhythmia
			$\mathrm{SNR_{in}}=-5$ dB $\mathrm{SNR_{imp}}=11.0$ dB	
			$\mathrm{SNR_{in}}=0$ dB $\mathrm{SNR_{imp}}=9.8$ dB	
Biswas *et al.* [14]	DWT(Bior5.5) with Thresholding (Heursure Rule)	PLI	$\mathrm{SNR_{imp}}=80.9410$ dB	Record 201 of MIT-BIH Arrhythmia
Singh and Sunkaria [39]	EWT with Mode Subtraction	PLI	$\mathrm{SNR_{in}}=-10$ dB $\mathrm{SNR_{out}}=0.9963$ dB	Record 100 of MIT-BIH Arrhythmia, Generated Sine Waves of 50Hz
			$\mathrm{SNR_{in}}=-5$ dB $\mathrm{SNR_{out}}=0.9948$ dB	
			$\mathrm{SNR_{in}}=0$ dB $\mathrm{SNR_{out}}=0.9943$ dB	
			$\mathrm{SNR_{in}}=5$ dB $\mathrm{SNR_{out}}=0.9932$ dB	
			$\mathrm{SNR_{in}}=10$ dB $\mathrm{SNR_{out}}=0.9898$ dB	
			$\mathrm{SNR_{in}}=-10$ dB $\mathrm{SNR_{out}}=0.9940$ dB	Record 103 of MIT-BIH Arrhythmia, Generated Sine Waves of 50Hz
			$\mathrm{SNR_{in}}=-5$ dB $\mathrm{SNR_{out}}=0.9974$ dB	
			$\mathrm{SNR_{in}}=0$ dB $\mathrm{SNR_{out}}=0.9985$ dB	
			$\mathrm{SNR_{in}}=5$ dB $\mathrm{SNR_{out}}=0.9989$ dB	
			$\mathrm{SNR_{in}}=10$ dB $\mathrm{SNR_{out}}=0.9991$ dB	
		BW	$\mathrm{SNR_{in}}=-15$ dB $\mathrm{SNR_{out}}=7.0259$ dB	Record 100 of MIT-BIH Arrhythmia, MIT-BIH Noise Stress Test
			$\mathrm{SNR_{in}}=-10$ dB $\mathrm{SNR_{out}}=10.8476$ dB	
			$\mathrm{SNR_{in}}=-5$ dB $\mathrm{SNR_{out}}=14.2190$ dB	
			$\mathrm{SNR_{in}}=0$ dB $\mathrm{SNR_{out}}=16.4016$ dB	
			$\mathrm{SNR_{in}}=5$ dB $\mathrm{SNR_{out}}=17.4193$ dB	
			$\mathrm{SNR_{in}}=-15$ dB $\mathrm{SNR_{out}}=6.5331$ dB	Record 103 of MIT-BIH Arrhythmia, MIT-BIH Noise Stress Test
			$\mathrm{SNR_{in}}=-10$ dB $\mathrm{SNR_{out}}=9.6127$ dB	
			$\mathrm{SNR_{in}}=-5$ dB $\mathrm{SNR_{out}}=11.8098$ dB	
			$\mathrm{SNR_{in}}=0$ dB $\mathrm{SNR_{out}}=12.8956$ dB	
			$\mathrm{SNR_{in}}=5$ dB $\mathrm{SNR_{out}}=13.3047$ dB	
Zhang *et al.* [17]	DWT(Bior3.7) with Composite Thresholding	EMG+ PLI+BW	$\mathrm{SNR_{in}}=121.7965$ dB $\mathrm{SNR_{out}}=150.0776$ dB $\mathrm{MSE_{in}}=0.0222$ dB $\mathrm{MSE_{out}}=0.0049$ dB	Record 124 of MIT-BIH Arrhythmia

TABLE 3. Representative wavelet based methods for signal denoising (Part 3).

Researches	Denoisers	Noises	Results	Databases
Wang *et al.* [21]	{DWT(Sym4)+ DWT(Db8)}with Non-Negative Garrote Thresholding	EMG+ PLI+BW	SNR_{in}=10 dB SNR_{out}=19.33 dB	Record 105 of MIT-BIH Arrhythmia
			SNR_{in}=15 dB SNR_{out}=22.84 dB	
			SNR_{in}=20 dB SNR_{out}=26.57 dB	
Mithun *et al.* [19]	DWT(Dmey) with Adjustable Thresholding	EMG+MA	SNR_{in}=−10 dB SNR_{imp}=11.4 dB	MIT-BIH Arrhythmia, MIT-BIH Noise Stress Test
			SNR_{in}=−5 dB SNR_{imp}=8.3 dB	
			SNR_{in}=0 dB SNR_{imp}=4.9 dB	
Yi and Song [23]	DWT(Db2) with Adjustable Thresholding (Level-Dependent Sqtwolog Rule)	EMG+ MA+BW	SNR_{in}=75.8468 dB SNR_{out}=129.4051 dB MSE_{in}=0.0554 dB MSE_{out}=0.00418 dB	Record 100 of MIT-BIH Arrhythmia
		WGN	SNR_{in}=82.0675 dB SNR_{out}=103.0249 dB MSE_{in}=0.0383 dB MSE_{out}=0.0216 dB	Record 100 of MIT-BIH Arrhythmia
Awal *et al.* [25]	DWT(Sym6) with Soft Thresholding (Modified S-median)	CN	SNR_{in}=5 dB SNR_{imp}=8.443084 dB	Record 101 of MIT-BIH Arrhythmia, MIT-BIH Noise Stress Test
	DWT(Sym6) with Hard Thresholding (Modified S-median)		SNR_{in}=5 dB SNR_{imp}=6.738051 dB	
Kabir and Shahnaz [38]	DWT(Sym7) with Soft Thresholding (Level-Dependent Sqtwolog Rule) based on EMD	CN	SNR_{in}=20 dB SNR_{imp}=5.5 $\sim$ 6 dB	MIT-BIH Arrhythmia
			SNR_{in}=15 dB SNR_{imp}=6 $\sim$ 6.5 dB	
			SNR_{in}=10 dB SNR_{imp}=7 $\sim$ 7.5 dB	
Li *et al.* [27]	DWT(Coif5) with Soft Thresholding (Heursure Rule)+ Bandpass Filter (FFT)+ Nonlinear Bayesian Filter	CN	SNR_{in}=5.4578 dB SNR_{out}=13.1222 dB	Record 118 of MIT-BIH Arrhythmia, MIT-BIH Noise Stress Test
	DWT(Coif5) with Soft Thresholding (Minimax Rule)+ Bandpass Filter (FFT)+ Nonlinear Bayesian Filter		SNR_{in}=5.4578 dB SNR_{out}=13.1395 dB	
B'charri *et al.* [36]	DTCWT with Hyperbolic Thresholding	CN	SNR_{in}=−5 dB SNR_{imp}=10.85304 dB MSE_{out}=0.00120 dB	Records 108, 109, 111, 119, 203 of MIT-BIH Arrhythmia, MIT-BIH Noise Stress Test
Shemi and Shareena [34]	Double-Density DTCWT(Db4) with Soft Thresholding	CN	SNR_{in}=57.9091 dB SNR_{out}=60.9477 dB $RMSE_{in}$=0.3244 dB $RMSE_{out}$=0.2295 dB	Records 100, 200, 213 of MIT-BIH Arrhythmia

TABLE 4. Representative wavelet based methods for wave detection (Part 1).

Researches	Detectors	Parameters	Results	Databases
Razavi and Mohammadi [59]	CWT(Db2) with Thresholding	selected from $S_1 \sim S_{32}$	SEN=99.74% PPV=99.72% ACC=99.72%	MIT-BIH Arrhythmia
Yochum *et al.* [54]	CWT(Db3) with Thresholding	S_{38}	SEN=99.85% PPV=99.48% ACC=99.33%	MIT-BIH Arrhythmia
Das *et al.* [50]	CWT(Morlet) with Thresholding	S_4	SEN=99.96% PPV=98.61% ACC=96.28%	MIT-BIH Arrhythmia
			SEN=100% PPV=100%	PTB Diagnostic ECG
			SEN=100% PPV=100%	Fantasia
Aqil *et al.* [53]	CWT(Morlet) with Maxima Finding	S_6	ACC=100%	Record a18 of Apnea-ECG
Abdelliche *et al.* [52]	CWT(Morlet) with Thresholding	S_3, S_4	SEN=99.51% PPV=99.77% ACC=99.28%	MIT-BIH Arrhythmia
	CWT(Fractional) with Thresholding		SEN=99.57% PPV=99.85% ACC=99.42%	
Haddadi *et al.* [45]	DWT(Db4) with Thresholding	D_4	SEN=98.1%	MIT-BIH Arrhythmia
Kaur *et al.* [46]	DWT(Db6) with Thresholding	$D_1 \sim D_5, A_5$	SEN=99.85% PPV=99.92% ACC=99.779%	MIT-BIH Arrhythmia
Wang and Eklund [48]	DWT(Db6) with Thresholding	D_3, D_4	SEN>99%	MIT-BIH Arrhythmia
Balachandran *et al.* [47]	DWT(Db6) with Maxima Finding	$D_3 \sim D_5$	SEN=98.64% PPV=100% ACC=99.87%	Record 100 of MIT-BIH Arrhythmia
Pal and Mitra [55]	DWT(Db6) with Maximal Finding	$D_3 \sim D_5$	ACC=99.38%	PTB Diagnostic ECG
Sahoo *et al.* [56]	DWT(Db6) with Thresholding	$D_3 \sim D_5$	SEN=99.873% PPV=99.696% ACC=99.5732%	MIT-BIH Arrhythmia
Rai and Chatterjee [57]	DWT(Db6) with Maxima Finding	$D_3 \sim D_5$	SEN=99.83% PPV=99.81% ACC=99.6197%	MIT-BIH Arrhythmia
Banerjee *et al.* [58]	DWT(Db6) with Thresholding	D_4, D_5	SEN=99.6% PPV=99.5%	MIT-BIH Arrhythmia
			SEN=99.84% PPV=99.92%	PTB Diagnostic ECG
Sabherwal *et al.* [70]	{DWT(Db6)+Derivatives +HT} with Thresholding	D_4, D_5	SEN=99.9% PPV=99.9% ACC=99.8%	MIT-BIH Arrhythmia, QT, MIT-BIH Noise Stress Test
Rakshit and Das [71]	{DWT(Db10)+ShET+HT} with Maxima Finding	D_4, D_5	SEN=99.93% PPV=99.91% ACC=99.83%	MIT-BIH Arrhythmia
Lin *et al.* [11]	DWT(Sym5) with Thresholding	$D_2 \sim D_4$	SEN=99.94% PPV=99.65%	QT

TABLE 5. Representative wavelet based methods for wave detection (Part 2).

Researches	Detectors	Parameters	Results	Databases
Jenkal *et al.* [61]	DWT(Sym7) with Thresholding	$D_3 \sim D_5$	SEN=99.67% PPV=99.84%	MIT-BIH Arrhythmia
Zidelmal *et al.* [60]	DWT(Haar) with Thresholding	D_4, D_5	SEN=99.64% PPV=99.82% ACC=99.46%	MIT-BIH Arrhythmia
Qin *et al.* [62]	DWT(Bior6.8) with Thresholding	$D_1 \sim D_8$	SEN=99.83% PPV=99.90% ACC=99.73%	QT
		$D_2 \sim D_8$	SEN= 99.39% PPV=99.49% ACC=98.89%	MIT-BIH Arrhythmia
Chen *et al.* [51]	DWT(Quadratic Bior Spline) with Thresholding	S_1, S_3	ACC=99.91%	MIT-BIH Arrhythmia
Li *et al.* [43]	DWT(Quadratic Spline) with Thresholding	$S_1 \sim S_4$	ACC=99.85%	MIT-BIH Arrhythmia
Singh and Sunkaria [66]	WPT(Db2) with Thresholding	S_4	SEN=99.99% PPV=99.92%	Fantasia
	WPT(Db3) with Thresholding		SEN=99.96% PPV=99.92%	
Vega-Martinez *et al.* [64]	WPT(Db10) with Thresholding	S_5	SEN=99.87% PPV=99.85%	MIT-BIH Arrhythmia
Chouakri *et al.* [65]	{WPT(Db10)+ WPT(Haar)} with Thresholding	S_4	SEN=98.68% PPV=97.24% ACC=95.88%	MIT-BIH Arrhythmia
Farashi *et al.* [44]	SWT with Thresholding	$D_1 \sim D_3$	ACC=98.45%	MIT-BIH Normal Sinus Rhythm
Merah *et al.* [67]	SWT(Db2) with Thresholding	D_4	SEN=99.84% PPV=99.88% ACC=99.72%	MIT-BIH Arrhythmia
			SEN=99.94% PPV=99.89% ACC=99.82%	QT
			SEN=95.30% PPV=93.98% ACC=89.19%	MIT-BIH Noise Stress Test
Zidelmal *et al.* [68]	S-T with Thresholding	ShE from 5∼22.5 Hz Signal Frequency	SEN=99.84% PPV=99.91% ACC=99.75%	MIT-BIH Arrhythmia
Herry *et al.* [69]	{OKBD+SSWT} with Thresholding	8∼20 Hz Signal Frequency	SEN=99.82% PPV=99.88%	MIT-BIH Arrhythmia

for comparing the denoised and the original ECG signals include Signal-to-Noise Ratio (SNR), Mean Square Error (MSE), and Root MSE (RMSE). For these metrics, subscript "in" means input, subscript "out" means output, and subscript "imp" means improvement. For them, larger SNR and smaller MSE/RMSE indicate better performances. Besides, the negative SNR is used to measure the added noises rather than the original signals. In Tables 1-3, abbreviations for different kinds of noises are as follows: White Gaussian Noise – WGN; Baseline Wander – BW; Power-Line Interference – PLI; Electromyographic artifact – EMG; Motion Artifact – MA; Composite Noise – CN.

Representative wavelet based methods for wave detection are summarized in Tables 4-5. The evaluation metrics for QRS complex detection include Sensitivity (SEN), Positive Predictive Value (PPV), and Accuracy (ACC). Due to

TABLE 6. Representative wavelet based methods for heartbeat classification (Part 1).

Researches	Features	Classifiers	Classes	Results	Databases
Rai *et al.* [78]	WCF from DWT(Db6)+ {TF+MF} detected by DWT(Db6)	MLP-NN	2	SEN=100% PPV=100% SPE=100% ACC=100%	MIT-BIH Arrhythmia
Sarkaleh and Shahbahrami [74]	WCF from DWT(Db6)	MLP-NN	3	ACC=96.5%	MIT-BIH Arrhythmia
Das and Ari [77]	WCF from DWT(Db2)+ TF+STF	MLP-NN	5	ACC=97.5%	MIT-BIH Arrhythmia
Thomas *et al.* [79]	WCF from DTCWT(Db2) +TF+HOS	MLP-NN	5	SEN=88.60% PPV=91.99% SPE=96.18% ACC=97.86%	MIT-BIH Arrhythmia
Nazarahari *et al.* [105]	PCA based on {WCF from Generated DWT (optimized by GA-PSO)}	MLP-NN	6	SEN=99.44% PPV=99.61% SPE=99.13% ACC=98.77%	MIT-BIH Arrhythmia
Özbay *et al.* [76]	WCF from DWT(Db2)	T2FCM Clustering +MLP-NN	10	ACC=99%	MIT-BIH Arrhythmia
Li *et al.* [110]	{WCF from WPT(Db6)} optimized by GA	MLP-NN optimized by GA	6	SEN=97.86% PPV=97.81% SPE=99.54% ACC=97.78%	MIT-BIH Arrhythmia
Martis *et al.* [100]	ICA based on {WCF from DWT(Dmey)}	PNN	5	SEN=97.97% PPV=99.21% SPE=99.83% ACC=99.28%	MIT-BIH Arrhythmia
Rai and Chatterjee [57]	MF detected by DWT(Db6)	PNN	5	SEN=99.49% PPV=99.52% SPE=99.88% ACC=99.53%	MIT-BIH Arrhythmia
Sumathi *et al.* [81]	MF detected by DWT(Sym)	ANFIS	5	ACC=98.24%	MIT-BIH Arrhythmia, MIT-BIH Atrial Fibrillation, MIT-BIH Malignant Ventricular Arrhythmia
Patil and Singh [80]	{TF+MF} detected by DWT(Db4)	WNN	5	ACC=99.42%	MIT-BIH Arrhythmia
Kumar *et al.* [112]	SaEnt from FAWT	SVM(Morlet Wavelet)	2	SEN=99.64% SPE=97.92% ACC=99.30%	PTB Diagnostic ECG
		SVM(RBF)		SEN=99.62% SPE=98.12% ACC=99.31%	
		SVM (Polynomial)		SEN=96.44% SPE=97.92% ACC=96.74%	
		SVM(Linear)		SEN=81.83% SPE=89.02% ACC=83.32%	

TABLE 7. Representative wavelet based methods for heartbeat classification (Part 2).

Researches	Features	Classifiers	Classes	Results	Databases
Kumar *et al.* [113]	{AFEnt+APEnt} from FAWT based on HRV (500-Point)	SVM(Morlet Wavelet)	2	SEN=98.07% SPE=98.33% ACC=98.21%	MIT-BIH Normal Sinus Rhythm, BIDMC Congestive Heart Failure, Fantasia
	{AFEnt+APEnt} from FAWT based on HRV (1000-Point)	SVM(Morlet Wavelet)		SEN=97.95% SPE=98.07% ACC=98.01%	
	{AFEnt+APEnt} from FAWT based on HRV (2000-Point)	SVM(RBF)		SEN=97.76% SPE=97.67% ACC=97.71%	
Kumar *et al.* [114]	{KNNEE+FuEnt} from FAWT based on HRV	SVM(Morlet Wavelet)	2	SEN=100% SPE=100% ACC=100%	10 Healthy Subjects and 10 Coronary Artery Disease Patients
		SVM(RBF)		SEN=100% SPE=100% ACC=100%	
Kumar *et al.* [115]	CIP from FAWT	SVM(Morlet Wavelet)	2	SEN=99.57% SPE=99.61% ACC=99.60%	St.-Petersburg Institute of Cardiological Technics 12-lead Arrhythmia, Fantasia
		SVM(RBF)		SEN=99.50% SPE=99.58% ACC=99.56%	
Patidar *et al.* [116]	PCA based on {Centered Correntropy from TQWT}	SVM(Mexican Hat Wavelet)	2	SEN=96.83% SPE=97.35% ACC= 97.09%	10 Healthy Subjects and 10 Coronary Artery Disease Patients
		SVM(Morlet Wavelet)		SEN=99.63% SPE=99.81% ACC=99.72%	
		SVM(RBF)		SEN= 99.49% SPE=99.88% ACC=99.69%	
Herry *et al.* [69]	{SSWT-Derived Phase+TF+MF} detected by {OKBD+SSWT}	SVM(RBF)	2	SEN=98.50% PPV=97.31% ACC=96.24%	MIT-BIH Arrhythmia
			4	ACC=83.20%	
Li *et al.* [86]	LDA based on {WCF from DWT(Db2)}+ KICA based on PCA	SVM(RBF)	5	SEN=98.50% PPV=98.91% SPE=99.69% ACC=98.80%	MIT-BIH Arrhythmia
Li *et al.* [107]	AppEnt from WPT(Db6)	SVM(RBF)	5	ACC=97.78%	MIT-BIH Arrhythmia
Garcia *et al.* [90]	{TVCG+TF+MF +Autocorrelation from DWT} optimized by PSO	SVM(RBF) optimized by PSO	5	ACC=92.4%	MIT-BIH Arrhythmia
Daamouche *et al.* [89]	{WCF from Generated DWT +TF} optimized by PSO	SVM(RBF) optimized by PSO	6	SEN=91.75% PPV=74.26% SPE=96.14% ACC=88.84%	MIT-BIH Arrhythmia

TABLE 8. Representative wavelet based methods for heartbeat classification (Part 3).

Researches	Features	Classifiers	Classes	Results	Databases
Ye *et al.* [87]	WCF from DWT(Db8) +TF+ICA	SVM(RBF)	16	ACC=99.32%	MIT-BIH Arrhythmia
Elhaj *et al.* [88]	PCA base on {WCF from DWT(Db6)}+ ICA based on HOS	SVM(RBF)	5	SEN=98.91% SPE=97.85% ACC=98.91%	MIT-BIH Arrhythmia
		MLP-NN		SEN=98.90% SPE=98.90% ACC=98.90%	
Martis *et al.* [102]	LDA based on {WCF from DWT(Sym2)}	SVM(Linear)	2	ACC=97.23%	MIT-BIH Arrhythmia, MIT-BIH Normal Sinus Rhythm
		MLP-NN		ACC=95.17%	
		GMM		ACC=89.78%	
Elhaj *et al.* [97]	PCA based on {WCF from DWT(Db6)}+ ICA based on HOS	B-ELM	5	ACC=98.09%	MIT-BIH Arrhythmia
Kumar *et al.* [111]	LEEnt from FAWT	RF	4	SEN=95.8% SPE=97.6% ACC=96.84%	MIT-BIH Atrial Fibrillation
	PeEnt from FAWT			SEN=84.5% SPE=86.8% ACC=85.84%	
Li and Zhou [106]	ShEnt from WPT(Db4)	RF	5	ACC=94.61%	MIT-BIH Arrhythmia
Pan *et al.* [98]	{Variance+PCA+TF} based on Raw Data +{Variance+PCA +Autocorrelation +Min-Max Ratio} based on DWT(Sym10)	RF	9	SEN=95.56% SPE=99.83% ACC=99.77%	MIT-BIH Arrhythmia
Rahhal *et al.* [83]	TFR from {CWT(Db4)+ CWT(Coif3)+ CWT(Bior3.5)}	CNN	2	SEN=98.7% PPV=99.3% SPE=99.9% ACC=99.7%	MIT-BIH Arrhythmia
Luo *et al.* [82]	TFR from MFSWT	SDA	4	SEN=85.9% PPV=84.4% ACC=97.5%	MIT-BIH Arrhythmia
Yildirim [84]	WS based on DWT(Db6)	DBLSTM	5	ACC=99.39%	MIT-BIH Arrhythmia
Li and Li [85]	Raw Data denoised by DWT(Bior6.8)	LDF	5	ACC=99.01%	MIT-BIH Arrhythmia
Li *et al.* [93]	WCF from DWT(Bior6.8)	MLR+MBD	16	ACC=96.93%	MIT-BIH Arrhythmia
	WCF from DWT(Db14)			ACC=97.83%	
	WCF from DWT(Sym8)			ACC=97.68%	
	WCF from DWT(Coif5)			ACC=97.64%	
	WCF from DWT(FK22)			ACC=97.83%	
	WCF from DWT(RBior6.8)			ACC=97.55%	

TABLE 9. Representative wavelet based methods for heartbeat classification (Part 4).

Researches	Features	Classifiers	Classes	Results	Databases
Kumar and Inbarani [95]	MF detected by DWT(Db4)	NRS	5	SEN=99.23% PPV=99.14% SPE=99.47% ACC=99.32%	MIT-BIH Arrhythmia
Jung and Lee [96]	{PCA+LDA} based on {WCF from DWT(Db2)}	Weighted KNN +Fitness Rule	4	SEN=97.57% PPV=94.41% SPE=99.43% ACC=98.12%	MIT-BIH Arrhythmia
Shahnaz *et al.* [109]	HOS from {WPT(Haar) based on EMD}	KNN	5	SEN=99.2% PPV=99.2% SPE=99.8% ACC=99.2%	MIT-BIH Arrhythmia
Kutlu and Kuntalp [108]	HOS from WPT(Db6)	KNN	5	SEN=90% PPV=92% SPE=98%	MIT-BIH Arrhythmia
Martis *et al.* [101]	PCA based on {WCF from DWT(Dmey)}	KNN	3	SEN=98.18% PPV=100% SPE=100% ACC=96.43%	MIT-BIH Arrhythmia, MIT-BIH Atrial Fibrillation
		DT		SEN=93.82% PPV=100% SPE=100% ACC=92.07%	
		MLP-NN		SEN=98.34% PPV=99.19% SPE=97.78% ACC=96.01%	
	ICA based on {WCF from DWT(Dmey)}	KNN		SEN=98.08% PPV=100% SPE=100% ACC=96.43%	
		DT		SEN=94.45% PPV=100% SPE=100% ACC=92.82%	
		MLP-NN		SEN=98.68% PPV=99.42% SPE=98.42% ACC=96.43%	
Banerjee and Mitra [99]	{WCSF+WCoF} from XWT(Morlet)	Thresholding	2	SEN=97.3% SPE=98.8% ACC=97.6%	PTB Diagnostic ECG

the close relationship between the QRS complex and the R peak, in some works, the performance of QRS complex detection is evaluated by R peak detection instead. In Tables 4-5, abbreviations of the detector parameters used for thresholding or maxima finding are as follows: Scale – S; Detail – D; Approximation – A. Subscript numbers for S, D, and A indicate the level in wavelet transform.

Representative wavelet based methods for heartbeat classification are summarized in Tables 6-9. The classification performance is measured by SEN, PPV, Specificity (SPE), and ACC. As the special cases, if the denotation is accompanied with the phrase "detected by", such denotation indicates the wavelet based detector, and in this case, the corresponding features are extracted from the characteristic points located by this detector; if the denotation is accompanied with the phrase "denoised by", such denotation indicates the wavelet based denoiser, and in this case, the corresponding features are extracted from the signals processed by this denoiser.

The results in Tables 1-9 are on the basis of widely-used benchmark ECG databases. Although there may be multiple results from more than one group of experiments for each method in the literature, we only report the most significant and representative results for conciseness and comparability. Actually, the main contribution of this paper is the comprehensive summary and systematic categorization of the representative wavelet based approaches, rather than the performance competition of them, for ECG analysis. Although these results can reflect which methods perform better or worse for the given tasks, we do not plan to overemphasize the advantages or disadvantages of them. In our opinion, every coin has two sides, and any method can be studied and developed. Moreover, all the inspiring results represent the past, and technologies progress everyday. Therefore, studying the good ideas contained in these methods ought to be much more meaningful than addressing the pure performance competition, which may perhaps be unfair due to different experimental conditions.

III. CONCLUSION AND OUTLOOK

A. CONCLUSION

For the issues of signal denoising, wave detection, and heartbeat classification in ECG analysis, plentiful wavelet based methods come forth every year and have trended diversification up to date. This overview and taxonomy has concentrated on comprehensive summary and systematic categorization of the methods based on wavelets for ECG analysis. By doing so, this paper has not only uncovered the inner mechanism why wavelet based methods are suitable for ECG analysis, but also revealed the designing principles these methods potentially follow.

All the methods have been categorized according to the summarized designing principles in this paper. Although these principles seems various, we can further conclude these principles as "3-C" patterns: choosing, cooperating, and creating. Choosing strategy means manually or optimally selecting the functions, parameters, or coefficients of wavelet relevant models; cooperating strategy means borrowing strengths from the techniques that supplement or complement the adopted wavelet based approaches; creating strategy means improving or innovating novel methods based on wavelet transform that may partially or fully break through the current solution stereotypes. Although the 3-C patterns are conceptually distinct from each other, they are usually unified in one scheme in practice. More broadly, technology cannot leave human knowledge and intelligence, and innovation has been deeply marked by the spirit of times. In this era, technology develops rapidly and world changes dramatically. The 3-C patterns also coincide with the humanistic implications: sometimes, choice is as important as efforts, cooperation is as important as competition, and creativity is as important as skills.

B. OUTLOOK

Although wavelet based methods have achieved encouraging performances for ECG analysis, there are still challenges haunting them. For signal denoising, useful information and noises are sometimes entangled together, and in such a situation, the useful information may perhaps be damaged if the noises are forcibly suppressed by the denoisers. For wave detection, signal distortions often occur in real scenarios, so that the waves are probably different from their standard morphologies more or less, which brings certain difficulty to the detectors. For heartbeat classification, it is usually hard to ensure the discriminative power and generalization ability of the classifiers especially when the signals undergo serious variations in the feature space.

Progress of artificial intelligence technologies will promote the development of ECG analysis methods. We tentatively look forward to the prospective trends of these methods in the following. For signal denoising, progress of the subspace learning technology may stimulate the reformation of wavelet based methods. Wavelet based denoisers usually map ECG signals to the wavelet space for better processing the noises. Subspace learning can help alter the structure of the wavelet space so that useful signal components are concentrated on a low-dimensional embedding while undesired noises are scattered in other dimensions. For wave detection, progress of the compressed sensing technology may motivate the renovation of wavelet based methods [117]. Compressed sensing has advantage in capturing the sparse and salient information of signals. Typical waves in ECG signals are sparse and salient in nature, so compressed sensing can be quite beneficial for wavelet based detectors to accurately locate these waves even they are distorted to a certain degree. For heartbeat classification, progress of the deep learning technology may inspire the innovation of wavelet based methods [118]. Deep learning simulates the process of human perception and cognition, and it has strong discrimination power and generalization ability for handling complicated ECG heartbeats. The collaboration between deep learning and wavelet based classifiers can be expected to effectively capture the useful local characteristics in signal details and accurately discover the class information in complicated data variations.

We further provide an outlook for the technological development trends for ECG analysis in the future at a more macroscopic level. From the methodology perspective, doing denoising, detection, and classification together in a seamless end-to-end system will become a new direction for ECG analysis technologies. Systemizing these three procedures together for the same final target will be not only beneficial for avoiding the information loss caused by the procedural gap, but also conducive to largely increasing the practicality of wavelet based or other kinds of methods. From the application perspective, development of wearable technologies has largely broadened the application scope of health and medical monitoring. Wearable ECG analysis plays a crucial role in the wearable ECG monitoring systems [119]. Wearable ECG data inevitably confronts much more severe challenges than the traditional Holter ECG data, and these challenges will promote the progress of the analysis techniques which also include the methods based on wavelets. From the data perspective, huge amounts of ECG data will be produced with the rapid development and promotion of heart monitoring systems. The requirement of analyzing big ECG data will necessarily lead to the revolution of relevant technologies, which will also have a profound influence on wavelet based methods [120].

The era of wavelet methodology for ECG analysis seems close to end, because currently no proposal only depends on wavelet techniques to do denoising, detection, or classification. However, end is another beginning. More and more wavelet techniques have been collaborating with or integrated into new approaches. Based on the performances of wavelets for ECG in history and today, we have enough confidence to believe that such methodology will be able to survive with the development of technologies and thrive in the new epoch of innovation revolution.

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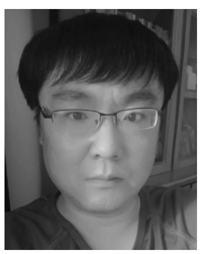

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