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A Failure Mechanism Cumulative Model for Reliability Evaluation of a k-Out-of-n System With Load Sharing Effect

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ABSTRACT In this paper, we propose a failure mechanism cumulative model that considers the loading history of the load sharing effect in a k-out-of-n system. Three types of failure mechanisms are considered, continuous degradation, compound point degradation, and sudden failure due to shock. By constructing a logic diagram with functional dependence gate, the load-sharing effect can be explained from the failure mechanism point of view using a mechanism-acceleration gate that shows that when one component fails, the failure mechanisms of the other surviving components will be accelerated. By deriving the total damage equation and a constructing failure behavior model, the system reliability of a k-out-of-n system with different types of failure mechanisms were evaluated. A voltage stabilizing system that contains a 1-out-of-2 subsystem, or a 2-out-of-3 subsystem was used to illustrate the practical applicability of the proposed approach. A combined Monte Carlo and binary decision diagram method was used in the numerical simulation process.

INDEX TERMS K-out-of-n system, load sharing effect, failure mechanism cumulative model, functional dependence, acceleration.

I. INTRODUCTION

In many real-world systems, the dependency of failure may increase joint-failure probabilities and reduce the overall system reliability [1], and increased attention is focused on improved models of failure prediction. Load sharing describes a functional dependency in which all components in a system share the working load with rules, so that when one component fails, the load will automatically be transmitted to the remaining surviving functional components [2]. There are many load-sharing system in the real world. For example the cables of a suspension bridge are used to share the total stress of the bridge, and if any of the cables break, the remaining cables share the stress [1]. Another example is the electric generators in a power plant, which can be arranged in parallel to share the electrical load if one or more of these generators fail [1]. Other common cases are servers in a distributed computer system and pumps in hydraulic systems [2]. With an increasing workload, the failure rate and reliability of each surviving component will increase and the overall system risk will increase [3], [4].

A. OVERVIEW OF LOAD SHARING RULES AND FAILURE TYPE AND DISTRIBUTION

A load-sharing rule dictates how stress or load is redistributed to the surviving components after a component fails or degrades within the system. This rule depends on the reliability application and how the components within the system interact through the structure function. Initially, research has concentrated on reliability estimation based on known load sharing rules. Durham *et al.* [5] discussed several sharing rules including equal load sharing, tapered load-sharing, local load-sharing, nearest-neighbor load-sharing, and hybrid local/nearest-neighbor load-sharing rules. For example, the equal load-share rule implies that a constant system load is distributed equally among the working components. Daniels [6] originally adopted this model to describe how the strain on yarn fibers increases as individual fibers within a bundle break. Lynch [7] characterized relationships between the failure rate and the load-share rule based on a monotonic load-sharing rule. Other studies of load share models characterized system reliability under unknown load

TABLE 1. Acronyms and abbreviations.

| Numbers | Acronyms | Details |
|---------|----------|----------------------------------|
| 1 | FMC | Failure Mechanism Cumulative |
| 2 | FDEP | Functional DEpendence |
| 3 | MACC | Mechanism Acceleration |
| 4 | BDD | Binary Decision Diagram |
| 5 | EM | Expectation-Maximization |
| 6 | PHM | Proportional Hazards Model |
| 7 | AFTM | Accelerated Failure-time Model |
| 8 | TFR | Tampered Failure Rate |
| 9 | CE | Cumulative Exposure |
| 10 | FM | Failure Mechanism |
| 11 | PPoF | Probabilistic Physics of Failure |
| 12 | FMG | Failure Mechanism Group |
| 13 | SM | Silver Migration |
| 14 | CD | Corona Discharge |
| 15 | TDDB | Time Dependent Dielectric Break |
| 16 | MD | Material Degradation |
| 17 | ELI | Electrical Shock |
| 18 | OVS | Over Voltage Stress |

sharing rules. Kvam and Pena [8] studied a statistical method to characterize systems with load sharing components by estimating unknown parameters of the equal load-share rule. Kim and Kvam [9] assumed that the load-sharing rule is unknown and derived methods for statistical inference on load-share parameters based on maximum likelihood.

As mentioned above, the key feature of load sharing for a k-out-of-n system is that workload has a significant influence on the failure rate of every component, and an exponential lifetime distribution for each component is assumed [9]–[11]. With this assumption, the entire system can be represented using a Markov transition diagram, simplifying system reliability evaluation. Park considered the case where the underlying lifetime distribution of the components is assumed to be Weibull [12] and lognormal or normal [13]. Maximum likelihood estimation and Expectation-Maximization (EM) algorithm were used to estimate the distribution parameters. Other studies instead described systems with components have arbitrary lifetime distributions [14], [15]. Singh et al [1] proposed a Bayesian treatment to estimate parameters of a k-components load-sharing parallel system in which some of the components follow a constant failure-rate and the remaining follow a linearly increasing failure-rate.

Other studies have analyzed component failure-type load sharing systems. Most early research on this dependent system assumed that component failure is sudden and catastrophic, causing the system to immediately cease functioning [2], [3]. Durham and Lynch [16] discussed the failure scenario

TABLE 2. Notations.

| Numbers | Symbols | Details |
|---------|---------------------|---|
| 1 | $C_{i'}$ | the i' th component |
| 2 | n | the total number of components in a k-out-of-n system |
| 3 | k | the minimum number of components during k-out-of-n system operation |
| 4 | x | the number of failed components |
| 5 | L | the total load of system |
| 6 | $L_i^{(i')}$ | the load shared by component $C_{i'}$, when i th load sharing occurs |
| 7 | $\sigma_i^{(i')}$ | the proportion of load shared by component $C_{i'}$, when i th load sharing occurs |
| 8 | FM_j | the j th mechanism |
| 9 | m | the total number of mechanism |
| 10 | τ_j | the lifetime of system suffering from the FM_j |
| 11 | Δd_j | the unit damage due to FM_j |
| 12 | t_p | the time of p th load change |
| 13 | $\Delta d_{p,j}$ | the unit damage due to FM_j at t_p |
| 14 | $d_{p,j}$ | the total damage due to FM_j at t_p |
| 15 | l | the number of sub-intervals |
| 16 | t_l | the time of the last load change |
| 17 | $d_j(t_l)$ | the total damage due to FM_j at t_l |
| 18 | l' | the time of load changes |
| 19 | θ_j | the damage ratio of the FM_j |
| 20 | $R_j(t)$ | reliability function of the system due to FM_j |
| 21 | $L_{th}^{(i')}$ | the load threshold for component $C_{i'}$ |
| 22 | b_p | the times a component bears the shock at $L_p^{(i)}$ |
| 23 | α_p | the ratio of the point load to the unit point load |
| 24 | t_i | the time of i th component fails |
| 25 | $b_{t-t_{n-k}}$ | the times a component bears the shock at time interval $(t_{n-k}, t]$ under $L_p^{(n-k)}$ |
| 26 | q | the q th random number |
| 27 | N | the total number of random numbers |
| 28 | $\tau_q^{(i')}$ | the value of the q th random number of component $C_{i'}$ |
| 29 | $\tau_q^{(i'*)}$ | the value of the q th random number of component $C_{i'}$, after a change |
| 30 | $T_{i,q}^{(n-k+1)}$ | the lifetime of C_{n-k+1} after i th load sharing |
| 31 | $T_{s,q}$ | the lifetime of the system |

of the loading sharing rule and defined phase I failures as ones that initiate a series of additional instantaneous failures (defined as phase II failures) as the load is transferred under the control of the load sharing rule. Another type of failure is degradation, which means that the condition of components degrades gradually until the system fails to meet the required performance threshold. In a load-sharing system

with degrading components, the workload shared on each surviving component will increase after a random component failure, resulting in a higher failure rate and increased performance degradation rate.

Ye *et al.* [2] estimated the probability of failure of a load sharing system under the assumptions that degradation is the dominant failure type and that the system will not be subject to sudden failure due to shock. As the primary cause of component degradation, the cumulative work load was adopted and exhibited an inverse Gaussian distribution. Yang *et al.* [17] proposed a method that combined a tampered failure rate model with a performance degradation model to analyze the reliability of a load-sharing k-out-of-n system with degrading components. Liu *et al.* [3] constructed reliability models for load sharing systems with degrading components, and showed that constant and varying load have a cumulative impact on the system.

B. OVERVIEW OF LOADING HISTORY AND LIFE RELATIONSHIP

In order to evaluate the reliability of load-sharing systems, the relationships between load and component lives or failure rate and loading history must be considered. The Proportional Hazards Model (PHM), accelerated failure-time model (AFTM), and Tampered Failure Rate (TFR) model are three load and life relationship models, and the Cumulative Exposure (CE) model describes the relationship of failure rate with loading history. The PHM model assumes that the failure rate is the product of the baseline hazard rate and accumulative factors [18], and the AFTM model emphasizes the time effect of loading on lifetime [19]. Liu [14] proposed a generalized AFTM model for reliability analysis of load sharing for a k-out-of-n system with an arbitrary load-dependent component lifetime distribution.

In the TFR model, the failure rate of a component completely depends on the current applied load and the age of the component, and is independent of the loading history [20]. Amari *et al.* [21] provided a closed-form analytical solution to assess reliability of a TFR load-sharing k-out-of-n system. This work was extended by the same authors to provide closed-form analytical solutions for the reliability of TFR load-sharing k-out-of-n: G systems with identical or non-identical components. Amari's work concentrated on non-repairable systems with arbitrary baseline distributions. Wang *et al.* [22] proposed a repairable TFR model with exponential baseline distributions.

The cumulative failure rate of the CE model is calculated using the effective age of each component, which is the sum of all loading durations multiplied by the corresponding acceleration factors. Amari and Bergman [20] presented an efficient procedure to compute the reliability and mean life of k-out-of-n load-sharing systems with identical or non-identical components following general failure distributions determined with the CE model. Based on the Miner cumulative damage theory and total probability formula, Hao *et al.* [23] analyzed the reliability of load sharing of

parallel systems and provided insight into the load-redistribution process. Huang and Xu [24] introduced the cumulative time concept to reflect the aging effect for each state with arbitrary failure distributions. The cumulative time in all states are the combined in a unified manner to express the reliability function of a single component.

C. THE MOTIVATION OF THE PROPOSED LOAD SHARING MODEL

The PHM, AFTM, TFR, and CE models described the failure rate change due to load variation, but did not examine the causes of the change. The data required (data) to evaluate these models can only be collected during different loading phases, making these models difficult to use in practice. In the previous work, two of the current authors have studied failure mechanisms (FM) and their relationships [25], [26], modeling of the system reliability with a failure mechanism tree and Binary Decision Diagram model, and data analysis with a PPOF (Probabilistic Physics of Failure) method.

In this paper, we propose a Failure Mechanism Cumulative (FMC) model to describe the complex loading history of a k-out-of-n system with load sharing effect. We introduce the concept of failure mechanism damage accumulation to explain the change of failure rate at each loading phase with arbitrary failure distributions. The damage accumulated in different phases can be summed and the reliability of the entire system can be assessed by the system modeling method.

There are several contributions of this work. First, the load sharing effect was logically modeled using a DEPENDENCE(FDEP) gate and an MACC gate. Second, a Failure Mechanism Cumulative (FMC) model was constructed to consider the loading history of the load sharing effect. Third, the load sharing effect of three main types of failure processes, the continuous degradation process, the compound point degradation process, and the shock process was studied with the FMC model. Finally the total damage amount, the damage function and reliability of the system are studied.

The remainder of this paper is organized as follows. Section 2 presents the modeling load sharing effect with FDEP and MACC gates. Section 3 describes the FMC model. The damage accumulation rule and reliability evaluation method are discussed in this section. The FMC model proposed in Section II is a general model, but it is not convenient to calculate the reliability, therefore the Section III explains about how the FMC model is used in analyzing the load sharing for different types of FMs. Section 4 presents a case study of a k-out-of-n voltage stabilizing system. The dynamic reliability of the system with load-sharing effect was evaluated. Finally, the conclusions of this study are summarized in section 5, and directions for future work are described.

II. THE PROPOSED FAILURE MECHANISM CUMULATIVE MODEL

When one component in k-out-of-n system fails, the load will be redistributed to the surviving components. As a direct

result, the development rate of the failure mechanisms in the surviving components will increase, i.e. these failure mechanisms will be accelerated. This will lead to two kinds of results. The component may not be able to bear the increased load and will fail within a very short of time. Alternatively, the degradation process of the component will accelerate, thus the lifetime will be shortened. The failure behavior of the load-sharing effect can thus be described with the following failure mechanism correlations.

As described above, the following assumptions are made.

- 1) The shared load will change instantaneously.
- 2) All active components in this system share the load according to load-sharing rule.
- 3) The objective system is non-repairable; i.e., none of the mechanisms, elements, or products can recover from a failure or an unusable condition.
- 4) The development speed of an FM under a constant stress level is invariable.
- 5) Only two kinds of FM dependence are considered here, the accumulation effect of the same FM in different phases and the competition different FMs.
- 6) For a continuous degradation failure mechanism, the total load of k/n system is invariant.
- 7) For shock-type loads, only one component fails or no component fails for one shock.

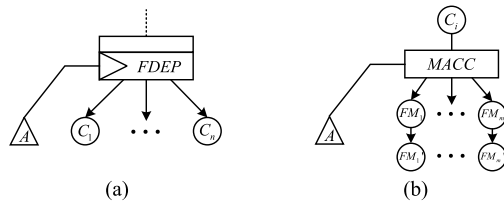


FIGURE 1. Logic diagram of the load sharing effect using a DEpendence (FDEP) gate and an MACC gate. (a) FDEP gate. (b) MACC gate.

In the fault tree analysis method, a special dynamic gate, Functional DEpendence (FDEP) gate [27] is used to model the functional dependence behavior, and the general structure is illustrated in Figure 1(a). The FDEP gate has a single trigger input and one or more dependent basic events. Each dependent component fails when it fails intrinsically, or when its trigger component fails.

FDEP can also express the load-sharing effect, where the trigger event A is defined as failure of one component of the k-out-of-n system, and C_i ($i = 1, 2, \dots, n$) are the components that share more loads and increase the developing rate of failure mechanisms in other components, until all of them fail. This process can be expressed with the MACC gate of failure mechanism tree shown in Figure 1(b), which shows that when one component fails, the failure mechanisms of the other components will be accelerated.

For a k-out-of-n system, each time a component fails, the load applied on the surviving components will change. According to this assumption, if the system total load is invariant, when the i th load sharing occurs and there x ($0 \leq x \leq n - k$) components have failed, the total load will

be distributed on the remaining $n - x$ components, which is,

$$L = L_i^{(1)} + L_i^{(2)} + \dots + L_i^{(n-x)} = \sigma_i^{(1)} \times L + \sigma_i^{(2)} \times L + \dots + \sigma_i^{(n-x)} \times L \quad (1)$$

Where L is the total load of the system, $L_i^{(i')}$ ($i' = 1, 2, \dots, n - x$) is the load shared by component $C_{i'}$, and $\sigma_i^{(1)} + \sigma_i^{(2)} + \dots + \sigma_i^{(n-x)} = 1$.

Assuming that only one component fails each time, then before the surviving components number reaches k , the system experiences $n - k + 1$ phases, and the final surviving component will have $n - k$ number of changes, i.e. there are $n - k$ times when load-sharing occurs in the system. For example, if component $C_{i'}$ failed when the i th trigger event occurred, this component experienced i times of load change.

To consider the loading history of the component, a Failure Mechanism Cumulative(FMC) model is presented here. According to [25], the accumulation effect includes damage accumulation and parameter union. Some failure mechanisms act on the same part of the component, and result in the change of same parameters, this is parameter combination. Some types of failure mechanisms have deterioration features that cannot be measured by parameters, such as thermal fatigue or vibration fatigue. For this type of FM, the damage accumulates until it makes the component fail. For the sake of simplicity, the destructive effects of damage accumulation and parameter union are (the destructive effects of these two types) all simply referred to as damage in the following section.

Under certain loads, the failure mechanism group(FMG) of component C_i is $\{FM_1, \dots, FM_j, \dots, FM_m\}$. The unit damage of C_i when FM_j acts alone is defined as Δd_j , and

$$\Delta d_j = \frac{1}{\tau_j} \quad (j = 1, \dots, m) \quad (2)$$

where, τ_j is the lifetime of C_i when FM_j acts alone. During a minimum time interval $(t_{p-1}, t_p]$, the damage amount of FM_j is $d_{p,j}$, and

$$d_{p,j} = \Delta d_{p,j} \cdot (t_p - t_{p-1}) \quad (3)$$

where $\Delta d_{p,j}$ is the unit damage of FM_j over the interval $(t_{p-1}, t_p]$. The total damage $d_j(t_l)$ at t_l is calculated by accumulation of each sub-interval damage in $(0, t_l]$,

$$d_j(t_l) = \sum_{p=1}^l \Delta d_{p,j} \cdot (t_p - t_{p-1}) \quad (4)$$

l is the number of sub-intervals, let $t_1 = 1$, then

$$d_j(t) = \sum_{p=1}^l \Delta d_{p,j} \cdot (t_p - t_{p-1}) \quad (5)$$

III. LOAD SHARING EFFECT OF DIFFERENT TYPE OF FMS

There are three types of failure mechanisms categorized by the triggered loading condition. They are continuous degradation FMs, compound-point degradation FMs, and sudden

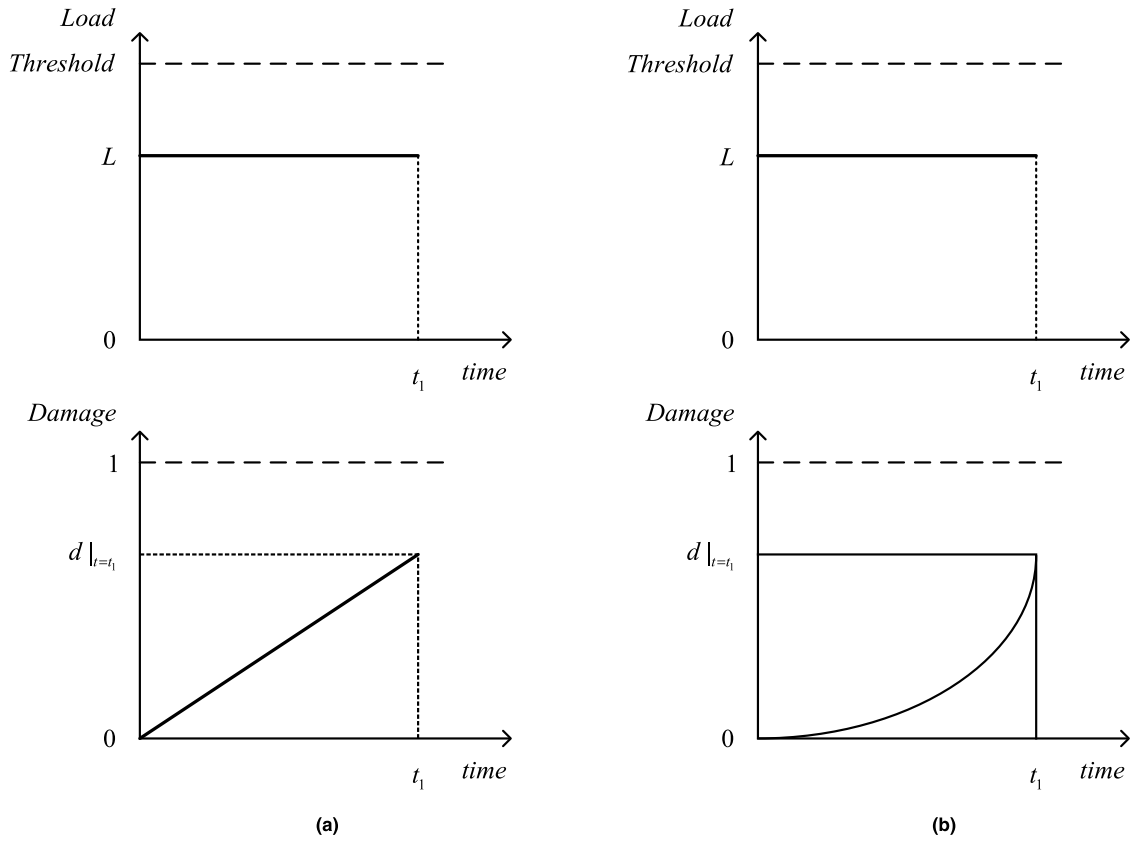


FIGURE 2. Load and damage of continuous degradation process. (a)Linear. (b)Non-linear.

shock FMs. The following explains how the FMC model performs load sharing for different types of FMs.

A. CONTINUOUS DEGRADATION FMs

For a continuous degradation failure mechanism, the load will remain constant during operation. For example, in Figure 2 when is applied, the FM induced by this load will degrade gradually, and some parameters will appear to be linearly or nonlinearly increasing (or decreasing), as shown in Figure 2(a)~(b).

According to (5), if the load changes l' times, then the time can be divided into l' sub-intervals. The accumulation equation of a continuous degradation FM_j is,

$$d_j(t) = \sum_{p=1}^{l'} \int_{t_{p-1}}^{t_p} \Delta d_{p,j} dt \tag{6}$$

If the degradation occurs linearly (as shown in Figure 2(a)), the total damage of FM_j to component $C_{i'}$ is,

$$d_j(t) = \sum_{p=1}^{l'} \frac{1}{\tau_{p,j}} \cdot (t_p - t_{p-1}) \tag{7}$$

If the degradation is nonlinear (as shown in Figure 2(b)), the total damage of FM_j to component $C_{i'}$ will be,

$$d_j(t) = \sum_{p=1}^{l'} \left[\left(\frac{t_p}{\tau_{p,j}} \right)^{\theta_j} - \left(\frac{t_{p-1}}{\tau_{p,j}} \right)^{\theta_j} \right] \tag{8}$$

where θ_j is the damage ratio of component $C_{i'}$ when FM_j acts alone.

As mentioned before, for a k-out-of-n system with $n - k + 1$ phases, the damage or parameter variation has the characteristic of stepwise increment. The damage or parameter variation should be accumulated to achieve the total effect of this kind of load. For example, for the nonlinear degradation condition (as shown in Figure 2(b)), assume that the load increases during each phase, and then damage accumulation of component $C_{i'}$ is as illustrated in Figure 3. Note that the order of loads is $L_0^{(i')} < L_1^{(i')} < \dots < L_i^{(i')}$ ($0 \leq i \leq n - k$). After the i_{th} load change, the time matrix of each load change is $[T]_{1 \times (i+1)} = [t_1 \dots t_p \dots t_i]$.

Given that component $C_{i'}$ is under the load shown in Figure 3 and FM_j acts alone, then the unit damage matrix is $[\Delta D]_{(i+1) \times 1}$,

$$[\Delta D]_{(i+1) \times 1} = [\Delta d_{0,j} \quad \dots \quad \Delta d_{p,j} \quad \dots \quad \Delta d_{i,j}]^T \tag{9}$$

$$[T]_{1 \times (i+1)} = [t_0 \quad \dots \quad t_p \quad \dots \quad t_i] \tag{10}$$

where $t_0 = 0$ is the initial time. Combining (2) and (8), the damage amount at time t_l ($t_l \geq t_i$) is,

$$d_j(t_l) = (t_1 \cdot \Delta d_{0,j})^{\theta_j} + (t_2 \cdot \Delta d_{1,j})^{\theta_j} - (t_1 \cdot \Delta d_{1,j})^{\theta_j} + \dots + (t_i \cdot \Delta d_{i-1,j})^{\theta_j} - (t_{i-1} \cdot \Delta d_{i-1,j})^{\theta_j} + (t_l \cdot \Delta d_{i,j})^{\theta_j} - (t_i \cdot \Delta d_{i,j})^{\theta_j}$$

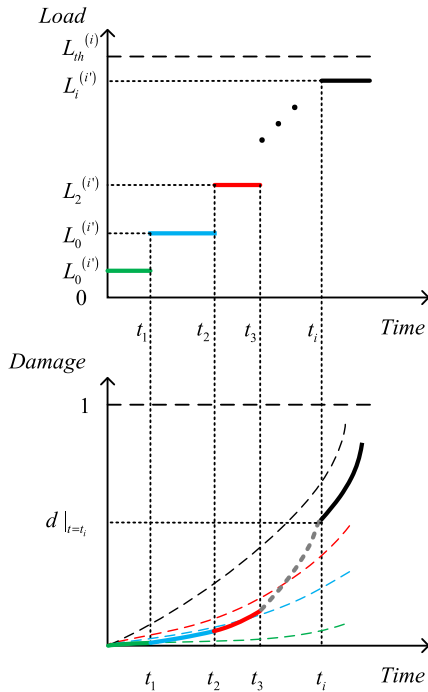


FIGURE 3. Damage accumulation of the continuous degradation process.

$$\begin{aligned}
 &= \left(\frac{t_1}{\tau_{0,j}}\right)^{\theta_j} + \left(\frac{t_2}{\tau_{1,j}}\right)^{\theta_j} - \left(\frac{t_1}{\tau_{1,j}}\right)^{\theta_j} + \dots + \left(\frac{t_i}{\tau_{i-1,j}}\right)^{\theta_j} \\
 &\quad - \left(\frac{t_{i-1}}{\tau_{i-1,j}}\right)^{\theta_j} + \left(\frac{t_{i'}}{\tau_{i,j}}\right)^{\theta_j} - \left(\frac{t_j}{\tau_{i,j}}\right)^{\theta_j} \\
 &= \left(\frac{t_{i'}}{\tau_{i,j}}\right)^{\theta_j} - \left(\frac{t_i}{\tau_{i,j}}\right)^{\theta_j} + \sum_{p=1}^i \frac{(t_p)^{\theta_j} - (t_{p-1})^{\theta_j}}{\tau_{p-1,j}^{\theta_j}} \quad (11)
 \end{aligned}$$

If $i = n - k$, and there are $n - k$ components in the system that still survive, then the damage accumulation function of the next failure component $C_{i'}$ is,

$$\begin{aligned}
 d_j(t_{i'}) &= \left(\frac{t_{i'}}{\tau_{i,j}}\right)^{\theta_j} - \left(\frac{t_i}{\tau_{i,j}}\right)^{\theta_j} + \sum_{p=1}^i \frac{(t_p)^{\theta_j} - (t_{p-1})^{\theta_j}}{\tau_{j,p-1}} \\
 &= \left(\frac{t_{i'}}{\tau_{n-k,j}}\right)^{\theta_j} - \left(\frac{t_{n-k}}{\tau_{n-k,j}}\right)^{\theta_j} + \sum_{p=1}^{n-k} \frac{(t_p)^{\theta_j} - (t_{p-1})^{\theta_j}}{\tau_{j,p-1}^{\theta_j}} \quad (12)
 \end{aligned}$$

Let $t_{i'} = t$, and $0 < t_1 < t_2 < \dots < t_{n-k} \leq t$, from (12), the damage function of $C_{i'}$ under multi-phase continuous degradation process is,

$$\begin{aligned}
 d_j(t) &= \left(\frac{t_{i'}}{\tau_{n-k,j}}\right)^{\theta_j} - \left(\frac{t_{n-k}}{\tau_{n-k,j}}\right)^{\theta_j} + \sum_{p=1}^{n-k} \frac{(t_p)^{\theta_j} - (t_{p-1})^{\theta_j}}{\tau_{p-1,j}} \\
 &= \left(\frac{t}{\tau_{n-k,j}}\right)^{\theta_j} - \left(\frac{t_{n-k}}{\tau_{n-k,j}}\right)^{\theta_j} + \sum_{p=1}^{n-k} \frac{(t_p)^{\theta_j} - (t_{p-1})^{\theta_j}}{\tau_{p-1,j}^{\theta_j}} \quad (13)
 \end{aligned}$$

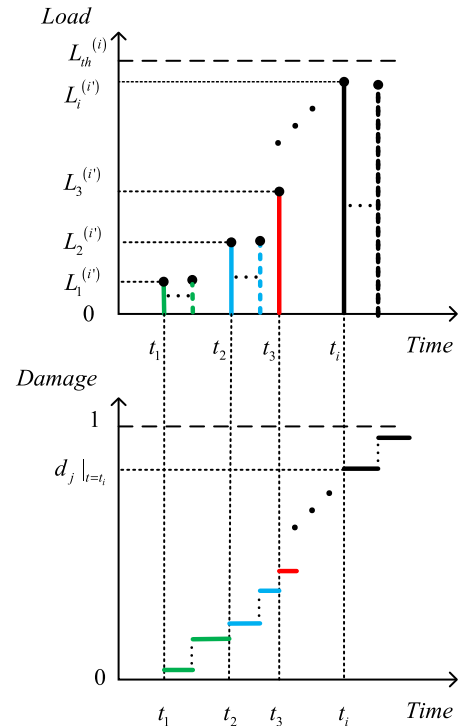


FIGURE 4. Damage accumulation of the compound point degradation process under condition I.

The reliability of component $C_{i'}$ under the condition that FM_j acts alone is,

$$\begin{aligned}
 R_j(t) &= P(d_j(t) < 1) \\
 &= P\left(\left(\frac{t}{\tau_{n-k,j}}\right)^{\theta_j} - \left(\frac{t_{n-k}}{\tau_{n-k,j}}\right)^{\theta_j} + \sum_{p=1}^{n-k} \frac{(t_p)^{\theta_j} - (t_{p-1})^{\theta_j}}{\tau_{p-1,j}^{\theta_j}} < 1\right) \\
 &= P\left(\tau_{n-k,j} \cdot \left(1 + \left(\frac{t_{n-k}}{\tau_{n-k,j}}\right)^{\theta_j} - \sum_{p=1}^{n-k} \frac{(t_p)^{\theta_j} - (t_{p-1})^{\theta_j}}{\tau_{p-1,j}^{\theta_j}}\right)^{\frac{1}{\theta_j}} > t\right) \quad (14)
 \end{aligned}$$

B. COMPOUND POINT DEGRADATION FMS

Degradation is not always induced by continuous load, and a series of impacts can also result in degradation, assuming that the system does not exceed failure threshold from a single shock. This kind of degradation is a compound point degradation process. $C_{i'}$ is under the compound point load as shown in Figure 4, When FM_j acts alone, where $[L]_{i \times 1}$ is the load of $C_{i'}$ when the point load is arriving, $L_{th}^{(i)}$ is the load threshold for component $C_{i'}$. Every time a component fails, the time matrix of failure time (Load sharing) is $[T]_{1 \times i}$, $[D]_{i \times 1}$ is the damage for each time of t_i , and the unit damage matrix is $[\Delta D]_{i \times 1}$.

$$\begin{aligned}
 [T]_{1 \times i} &= [t_1 \ \dots \ t_p \ \dots \ t_i] \\
 [D]_{i \times 1} &= [d_{1,j} \ \dots \ d_{p,j} \ \dots \ d_{i,j}]^T
 \end{aligned}$$

$$[L]_{i \times 1} = [L_i^{(i)} \quad \dots \quad L_p^{(i)} \quad \dots \quad L_i^{(i)}]^T$$

$$[\Delta D]_{i \times 1} = [\Delta d_{1,j} \quad \dots \quad \Delta d_{p,j} \quad \dots \quad \Delta d_{i,j}]^T \quad (15)$$

Each time a shock occurs, the component will fail or survive, so a component may bear many shocks. Assume that the load is $[L]_{i \times 1}$, and a component bears the shock $[B]_{i \times 1}$ times, then $[B] = [b_1 \dots b_p \dots b_i]$.

Assume the damage follows the linearity accumulation rule under the compound point degradation condition. Reference (5), the damage can be expressed as,

$$d_j(t) = \sum_{p=1}^i b_p \cdot \Delta d_{p,j} \cdot (t_p - t_{p-1})$$

$$= \sum_{p=1}^i b_p \cdot d_{p,j} \quad (16)$$

The coefficient is defined by the load strength and the development characteristic of failure mechanism. The amount of damage at a given time point is defined by the coefficient matrix and the damage at the previous time point.

For a k-out-of-n system, with a failure of the system component, the shared load for each surviving component will increase, and the point load may exceed the damage threshold. Then there are two cases.

1) *Condition I:* If the point load does not exceed the damage or failure threshold after i times of load sharing, then the damage is the accumulation of the previous damage. Figure 4 shows the accumulation of component C_i ,

When the point load remains constant, the number of component that shares the load decrease, and then the load for each surviving component will increase. The strength coefficient matrix of the point load, which is defined as the ratio of the point load with the unit point load is $[A] = [\alpha_1 \dots \alpha_p \dots \alpha_i]$. According to (16)

$$d_j(t_i) = \alpha_1 b_1 \cdot \Delta d_{1,j} + \alpha_2 b_2 \cdot \Delta d_{2,j} + \dots + \alpha_i b_i \cdot \Delta d_{i,j}$$

$$= \sum_{p=1}^i \alpha_p b_p \cdot \Delta d_{p,j}$$

$$= \sum_{p=1}^i \alpha_p b_p \cdot \frac{1}{\tau_{p,j}} \quad (17)$$

If $i = n - k$, the accumulated damage value at time t_{n-k} for the next failure component C_i is,

$$d_j(t_{n-k}) = \sum_{p=1}^{n-k} \alpha_p b_p \cdot \frac{1}{\tau_{p,j}} \quad (18)$$

Assume that from time t_{n-k} to $t - t_{n-k}$, the number of shocks is a random number $b_{t-t_{n-k}}$, and after $n - k$ times of load-sharing, the load value and unit damage of component C_i are respectively α_{n-k+1} and $\frac{1}{\tau_{n-k+1,j}}$, then the damage accumulation of this component at time t is,

$$d_j(t) = \alpha_{n-k+1} b_{t-t_{n-k}} \cdot \frac{1}{\tau_{n-k+1,j}} + \sum_{p=1}^{n-k} \alpha_p b_p \cdot \frac{1}{\tau_{p,j}} \quad (19)$$

Given the load threshold for component C_i is $L_{th}^{(i)}$, combining (1) and (19), then the reliability at time t is,

$$R_j(t) = P(\text{The load} < \text{failure threshold at } t_{n-k})$$

$$\cdot P(\text{The accumulation damage} < 1 \text{ at } t)$$

$$= P(L_{n-k}^{(i)} < L_{th}^{(i)}) \cdot P(d_j(t) < 1)$$

$$= P(\sigma_{n-k}^{(i)} \times L < L_{th}^{(i)})$$

$$\cdot P\left(\alpha_{n-k+1} b_{t-t_{n-k}} \cdot \frac{1}{\tau_{n-k+1}} + \sum_{p=1}^{n-k} \alpha_p b_p \cdot \frac{1}{\tau_{p,j}} < 1\right) \quad (20)$$

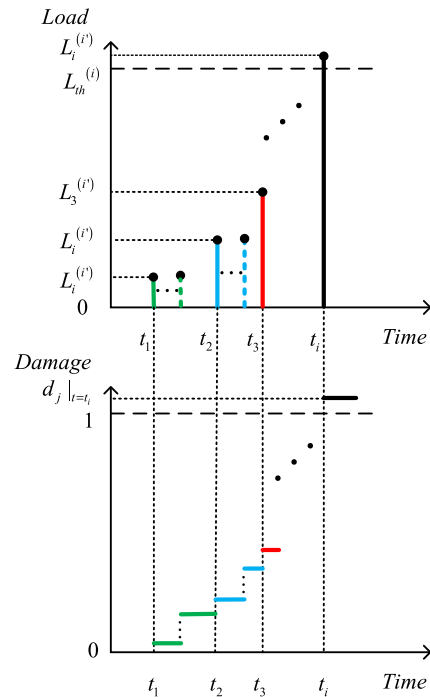


FIGURE 5. Damage accumulation of the compound point degradation process under condition II.

2) *Condition II:* If the point load exceeds the damage or failure threshold after i phases of load sharing, then the component will fail in response to shock. Figure 5 shows the accumulation of component C_i , at time t_i , the load $L_i^{(i)}$ will cause the component to fail.

The failure probability due to one point load is,

$$P(\text{Overstress failure happens at } t_i)$$

$$= P(\text{The accumulation damage} < 1 \text{ before } t_i)$$

$$\times P(\text{The load} > \text{failure threshold at } t_i) \quad (21)$$

According to the assumption that only one component will fail when a point load arrives, then there are $i - 1$ components that fail before the i th point load. Also, the load on the surviving components has been changed $i - 1$ times. According to (21), then the probability that the component will fail due

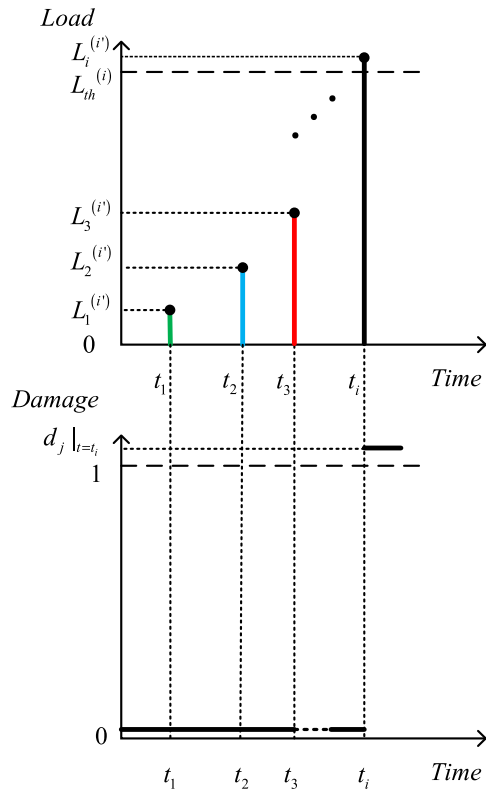


FIGURE 6. Load and damage of sudden shock type of FM.

to shock is,

$$P(\text{Overstress failure happen at } t_{i-1}) = P(d_j(t_{i-1}) < 1) \cdot P(L_i^{(i)} > L_{th}^{(i)}) \quad (22)$$

For this case, when $i = n - k + 1$, combining (18), (20), and (22), the probability that the component will fail due to shock is shown in equation (23)

$$P(\text{Overstress failure happen at } t_{n-k+1}) = P(d_j(t_{n-k}) < 1) \cdot P(L_{n-k+1}^{(i)} > L_{th}^{(i)}) = P\left(\sum_{p=1}^{n-k} a_p b_p \cdot \frac{1}{\tau_{p,j}} < 1\right) \cdot P(L_{n-k+1}^{(i)} > L_{th}^{(i)}) \quad (23)$$

Let $t_{n-k+1} = t$, the reliability of shock failure occur is,

$$R_j(t) = 1 - P\left(\sum_{p=1}^{n-k} a_p b_p \cdot \frac{1}{\tau_{j,p}} < 1\right) \cdot P(L_{n-k+1}^{(i)} < L_{th}^{(i)}) \quad (24)$$

C. SUDDEN SHOCK FMS

The load of sudden shock failure mechanisms is similar with compound point degradation, as both are sudden or catastrophic type of loads, as illustrated in Figure 6. The difference between these two types of loads is that a shock will not lead to damage of the component until it reaches a certain value

and exceeds the failure threshold of a component. However the compound point loads will result in damage or degradation, which can accumulate until the threshold is reached.

If the first shock load occurs at t_i , the damage of component C_i under this kind of load can be expressed as,

$$d_j(t) = \begin{cases} 0, & t < t_i \\ 1^+, & t \geq t_i \end{cases} \quad (i = 1, \dots, n) \quad (25)$$

For each impact or shock, the load will not exceed the failure threshold of all the components in a system, and we assume that it is only larger than the threshold of the weakest component. That is to say, when a shock or impact occurs, only the weakest component will fail, and the other components will survive. We also assume that there is no damage caused by a single shock on other components. Due to the decrease of the component, when the next shock occurs, the load on the surviving components will increase, which will make the next weakest component fail.

When the i th shock occurs, if the load that is shared by component C_i exceeds the failure threshold, the probability of C_i to fail under this condition is,

$$P(C_i \text{ Failure}) = P(L_i^{(i)} > L_{th}^{(i)}) \quad (26)$$

Combining (1) and (26),

$$P(C_i \text{ Failure}) = P(L_i^{(i)} < L_{th}^{(i)}) = P(\sigma_i^{(i)} \times L < L_{th}^{(i)}) \quad (27)$$

When $i = n - k + 1$, let $t_{n-k+1} = t$, the reliability probability of component C_i is,

$$R_j(t) = P(\sigma_{n-k+1}^{(i)} \times L < L_{n-k+1}^{(i)}) \quad (28)$$

IV. CASE STUDY

A. DESCRIPTION

A voltage-stabilizing system includes three protective resistances in series (R_1 , R_2 , and R_3 , metal film resistors), two regulators (F_1 and F_2), and a load (R_L), as illustrated in Figure 7. The input is voltage signal U_i . The three serially connected resistors may protect the voltage regulator from over-voltage, which can more easily occur when there is only one resistor, potentially resulting in failure of the regulator due to the instantaneous high voltage. The three serially connected resistors constitute a 2-out-of-3 subsystem. Similarly, two voltage regulators together form another 1-out-of-2 subsystem, so that if at least one regulator works, the load R_L will work normally.

B. MODELING

After analyzing the material, working environment, and load condition of this system, the failure mode, mechanisms, and their correlation were determined and are listed in Table 3. Each component has one or more failure mechanisms, and will suffer degradation or catastrophic failure. Several failures

TABLE 3. Failure mechanism and their correlations.

| Component | FM symbol | FM distribution | FM type | Mechanism feature | Effect | Effect symbol |
|-------------------------------------|-----------|-------------------------------------|----------------------------|---------------------|-----------------|------------------|
| Resistance (R ₁) | SM | $R_1 f_1 \sim Weibull(12652, 3.26)$ | Continuous degradation | Damage accumulation | Short circuit | MR ₁ |
| | CD | $R_1 f_2 \sim Exp(12542)$ | Continuous degradation | Damage accumulation | Short circuit | |
| Resistance (R ₂) | SM | $R_2 f_1 \sim Weibull(12673, 3.28)$ | Continuous degradation | Damage accumulation | Short circuit | MR ₂ |
| | CD | $R_2 f_2 \sim Exp(12562)$ | Continuous degradation | Damage accumulation | Short circuit | |
| Resistance (R ₃) | SM | $R_3 f_1 \sim Weibull(12663, 3.27)$ | Continuous degradation | Damage accumulation | Short circuit | MR ₃ |
| | CD | $R_3 f_2 \sim Exp(12552)$ | Continuous degradation | Damage accumulation | Short circuit | |
| Voltage regulator (F ₁) | ELI | $F_1 f_1 \sim Log-N(9.92, 0.27)$ | Compound point degradation | Damage accumulation | Open circuit | MF ₁₁ |
| | TDDDB | $F_1 f_2 \sim Log-N(8.67, 0.3)$ | Continuous degradation | Damage accumulation | Parameter drift | MF ₁₂ |
| Voltage regulator (F ₂) | ELI | $F_2 f_1 \sim Log-N(9.82, 0.26)$ | Compound point degradation | Damage accumulation | Open circuit | MF ₂₁ |
| | TDDDB | $F_2 f_2 \sim Log-N(8.69, 0.31)$ | Continuous degradation | Damage accumulation | Parameter drift | MF ₂₂ |
| Load(R _L) | MD | $R_L f_2 \sim Log-N(8.72, 0.27)$ | Continuous degradation | Damage accumulation | Degradation | MR _{L1} |
| | OVS | $R_L f_2 \sim Log-N(8.69, 0.31)$ | Sudden shock | Over damage | Short circuit | MR _{L2} |

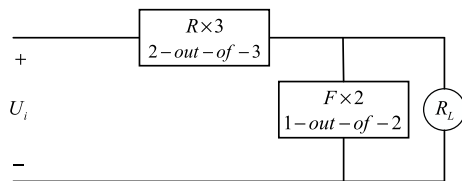


FIGURE 7. The example voltage stabilizing system.

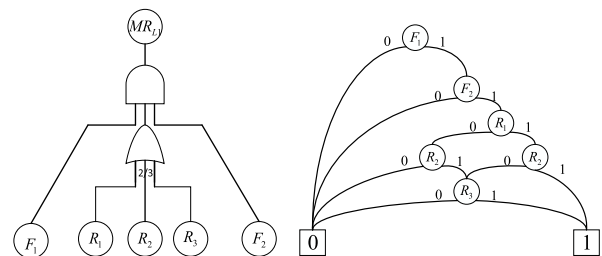


FIGURE 8. Failure mechanism tree and BDD of R_L.

are listed in the table: silver migration(SM), corona discharge(CD), time-dependent dielectric break(TDDDB), material degradation(MD), intermittent electrical shock(ELI), and over voltage stress(OVS). Of these, SM, TDDM, and MD are continuous degradation failure mechanisms, ELI is a compound point degradation failure mechanism and OVS is a sudden shock type of failure mechanism. ELI will occur when there are voltage or load current fluctuations in AC power. Each voltage fluctuation is a point shock, and will result in instability and damage of the voltage regulator. The damage will accumulate until failure.

There are two failure mechanisms for the load resistor R_L, degradation and over-voltage, and the relationship is shown in Figure 8. Because there is constant voltage because of the regulators, the R_L will fail only when both of the regulators and at least two protective resistors fail.

Figure 9 illustrates the load sharing effect with the FDEP gate, where a) shows that when one protective resistor failures (trigger event C_R), the surviving resistor and the voltage regulator will share the total voltage. If any of the two regulators occurs open circuit (trigger event C_F), the other one will share

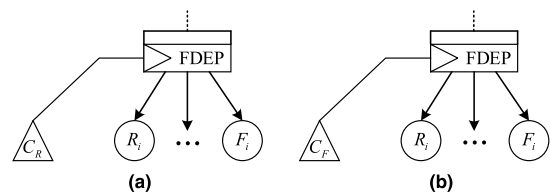


FIGURE 9. FDEP gate of the load sharing effect. (a) Protective resistor fails first. (b) Voltage regulator fails first.

more voltage because they are parallelly connected, as shown in Figure 9(b).

When one of the protective resistors short, the voltage on R_L will also increase, which increases the MD and OVS mechanisms in the regulator. Figure 10 shows the failure mechanisms and their correlation.

If the voltage regulators share more voltage, TDDDB and ELI will also be accelerated. The failure mechanism tree is similar to that of R_L in Figure 10.

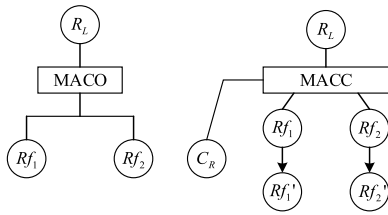


FIGURE 10. Failure mechanism tree of R_L .

When any of the three protective resistors is shorted, the total resistance will decrease and the load shared by the surviving two resistors will increase. If the regulator is open, the total resistance will increase, and the current in the circuit the voltage on the resistor will decrease. The developing rate of the failure mechanism is directly dependent on the voltage. That is to say, the trigger event, a “short of the protective resistor,” will accelerate the FMs in the resistors, whereas the trigger event of an open regulator will inhibit the FMs in the resistors.

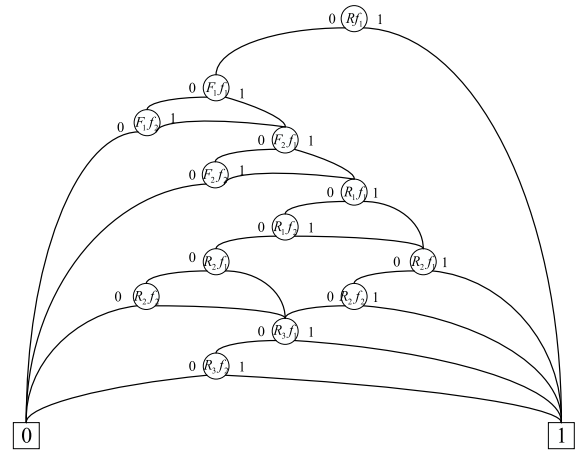


FIGURE 12. The BDD of the example system.

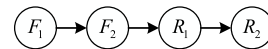


FIGURE 13. One failure scenario of the system.

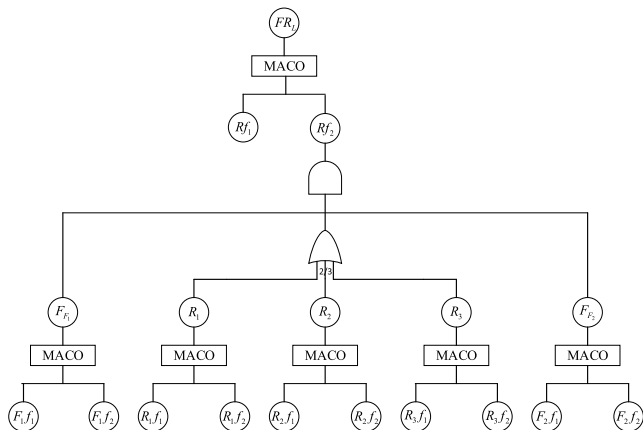


FIGURE 11. The failure mechanism tree of the example system.

When both regulators fail and at least one protective resistor is functioning, over-voltage failure will not occur on the load resistance. Figure 11 and Figure 12 show the fault tree and the BDD of the example system.

The failure times of the three protective resistors may be different, as well as the failure times of the two regulators. Thus, there may be many failure scenarios for the example system. For example, one failure scenario is shown in Figure 13, where the failure sequence is regulator 1, regulator 2, resistor1, and resistor 2.

When regulator 1 fails, the load shared by regulator 2 will increase, and thus the developing rate of the FMs related to voltage will increase. At the same time, the voltage applied on the three protective resistors will decrease, causing a decrease in the developing rate of FMs. After regulator 2 fails, the voltage on the protective resistor will increase. After one of the protective resistor fails, the voltage shared by the surviving two resistor will also increase. Every time, when the voltage is rearranged, the developing rate of the failure mechanism

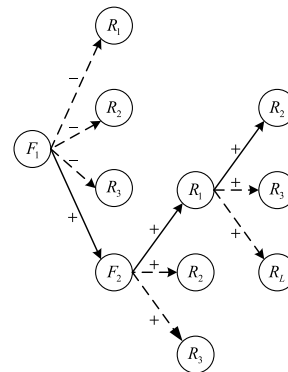


FIGURE 14. The failure sequence of the example voltage stabilizing circuit.

in the load resistance will be accelerated or inhibited until the system fails. Figure 14 illustrates the scenario in another way, in which the “+” and “-” symbols indicate FM acceleration and inhibition. The solid line in Figure 14 represents the sequence of the failure components, and the dashed line represents the failure of a component then altering the load change on other components. For example, the failure of F_1 will cause the load re-distribution of $R_1 \sim R_3$.

There are 120 scenarios in this example, and all of them are considered in the simulation. To solve the load-sharing problem in the example system, the following procedures are carried out. First, samples are selected according to the distribution to obtain a sample group of each failure mechanism and discretize the time. Second, the load sharing effect is calculated according to the FMC model. For a certain sample, there is a corresponding failure scenarios. With the load-sharing rule and the FMC model, the failure accumulation and acceleration effect can be taken into consideration. Third, the failure probability of component, subsystem, and system

can be calculated for each discrete time point. This allows calculation of the failure probability and reliability over the full dynamic curve. Procedure 1 summarizes the proposed reliability simulation strategy.

C. SIMULATION AND DISCUSSION

To evaluate the system reliability, all scenarios are considered. Figure 15 shows the component damage accumulation as calculated with Procedure 1.

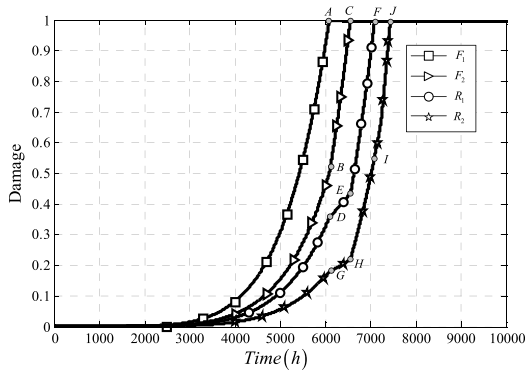


FIGURE 15. Degradation and accumulation of components.

Although R_1 and R_2 , F_1 and F_2 occupy the similar position in the system shown in the Fig. 7, however, due to the fact that the devices are not identical, the damage rates of the same type of mechanism in different components are different, which results in the difference in the four curves in Figure 15.

The three time points 6116 (point “A” in Figure15), 6599 (point “C” in Figure 15) and 7110 (point “F” in Figure15) are the failure time of F_1 , F_2 , and R_1 , respectively. The accumulation figure changed at these points.

Component F_1 failed at 6116h (point “A” in Figure 15), which caused the voltage on F_2 to increase, and the voltage on $R_1 \sim R_3$ decrease. After point “B”, the damage curve of F_2 became sharp, which showed that the damage accumulation rates for ELI and TDDDB became higher. In contrast, the damage curve of R_1 and R_2 become flatter, such as the part between points “D” and “E” or between points “G” and “H”. Component F_2 failed at time 6559h (point “C” in Figure 15), which resulted in a voltage increase in $R_1 \sim R_3$. After point “E”, the damage curve of R_1 became sharp, until failure at time 7110h (point “F” in Figure 15). The voltage on R_2 then increased, and the curve sharpened from point “I”. R_2 failed at time 7469h (point “J” in Figure 15), which damaged the system further, and caused R_L to fail due to over-voltage.

The voltage regulation components F_1 and F_2 constitute a 1-out-of-2 subsystem. In Figure 16, the dashed line shows the reliability when the load-sharing effect was not considered and the solid line indicates the behavior with load-sharing. It is obvious that the reliability would be decreased if load-sharing effect is considered. Although the difference should appear after this effect, these two curves showed an initial difference because the component failure time is distributed.

Procedure 1 System Evaluation Strategy With Monte-Carlo Method

Input: FM lifetime distribution obtained by PPoF

1: Component lifetime distribution considering FM correlation

The first layer: different failure scenarios

2: **for** $q = 1, 2, \dots, N$ **do**

3: Obtain a group of lifetimes by sampling each component lifetime

$$[\tau]_{n \times 1}^q = (\tau_q^{(1)}, \dots, \tau_q^{(i)}, \tau_q^{(n)})$$

4: Sort the value in $[\tau]_{n \times 1}^q$ from big to small, obtain a new lifetime group, which is

$$[\tau']_{n \times 1}^q = (\tau_q^{(1')}, \tau_q^{(i')}, \dots, \tau_q^{(n')})$$

5: Calculate the component damage $\Delta d_q^{(i')} = \frac{1}{\tau_q^{(i'')}}$

The second layer: different failure component

6: **for** $i = 1, 2, \dots, n - k$ **do**

The third layer: survived component

7: **for** $i' = i + 1 = 2, 3, \dots, n - k + 1$ **do**

8: Calculate $d_{i,q}^{(i')}$

end

9: Calculate the lifetime of C_{n-k+1} after load sharing

$$T_{i,q}^{(n-k+1)} = \tau_q^{(n-k+1)} (1 - d_{i,q}^{(n-k+1)})$$

end

10: Evaluate system lifetime $T = T_{s,q}^{(n-k+1)}$

End

11: After calculation of $[T]_{N \times 1}$, fit reliability with all the data

Output: System reliability

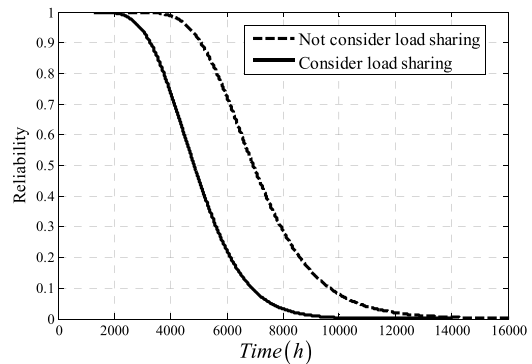


FIGURE 16. Reliability of the voltage regulation subsystem.

The simulation results showed that when the load sharing effect is not considered, the average lifetime of the voltage regulation subsystem is 7207 hours, but the value is 4994 if this effect is considered. Thus, the load-sharing effect had a strong influence on this subsystem.

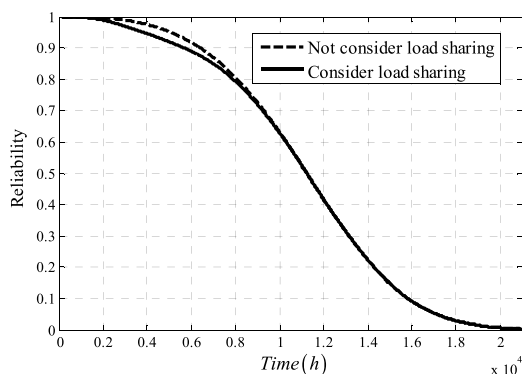


FIGURE 17. Reliability of the protective resistance subsystem.

Similarly, Figure 17 shows the reliability of the protective resistance subsystem with or without consideration of the load sharing effect. The average lifetime of this subsystem is 11242 hours without consideration of this effect and 11082 when this effect is considered. From Figure 16 and Figure 17, it can be seen that the load sharing effect has a relatively larger impact on the voltage regulation subsystem compared to the effect on the protective resistance subsystem. This is determined by the system function. The premise of the system over-stress failure is the failure of the voltage regulator subsystem, but there is no such requirement for the protective resistor subsystem. More often than not, when the system fails, the protective resistance still functions, which means that the system may fail before all three resistors fail.

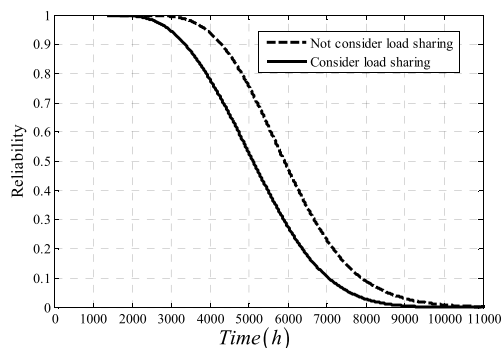


FIGURE 18. Reliability of the example system.

Figure 18 shows the system reliability of the overall example system. When the load sharing effect is not considered, the average lifetime of the whole system is 6027 hours and the value is only 5167 when this effect is considered. Although the premise of the system over-stress failure is the failure of the voltage regulator subsystem, R_L is still subject to another failure mechanism, material degradation, which is also affected by the load-sharing effect. This lowers the system reliability more than the voltage regulation subsystem.

V. CONCLUSIONS AND FUTURE WORK

This paper proposed a Failure Mechanism Cumulative model to consider the loading history of the load sharing effect in

a k-out-of-n system. With FDEP and MACC gates, the dynamic behavior of the load sharing effect is described, and the functional dependence can also be explained by the failure mechanism acceleration at different loading phases. Compared with the aforementioned PHM, AFTM, TFR and CE models that illustrate the change of failure rate with each time of load distribution, the FMC model explains the change of failure rate is due to the change of the failure mechanism developing rate, and the failure accumulation model is the sum of the damage at different load stages.

With the damage accumulation rule, the FMC model can integrate the effect of the loading history into the load sharing system. This model can adapt to arbitrary failure distribution, including Exponential, Weibull and Lognormal. It can also apply to different failure mechanism types, including continuous degradation, compound point degradation, and sudden failure due to shock.

We illustrate the advantages of the approach through a detailed analysis of an example voltage-stabilizing system with a 1-out-of-2 subsystem and a 1-out-of-3 subsystem. The results show that the reliability of the k-out-of-n system varies significantly when considering the load sharing effect with the FM-DC model. The lifetime of the system will be lower when the load-sharing effect is considered in the simulation.

Future studies will concentrate on the efficiency of solving more complicated load sharing reliability problems. In addition, more FM correlations should be considered to illustrate the failure behavior of the system with more complexity and accuracy.

REFERENCES

- [1] B. Singh, K. K. Sharma, and A. Kumar, "A classical and Bayesian estimation of a k-components load-sharing parallel system," *Comput. Statist. Data Anal.*, vol. 52, pp. 5175–5518, Aug. 2008.
- [2] Z. Ye, M. Revie, and L. Walls, "A load sharing system reliability model with managed component degradation," *IEEE Trans. Rel.*, vol. 63, no. 3, pp. 721–730, Sep. 2014.
- [3] B. Liu, M. Xie, and W. Kuo, "Reliability modeling and preventive maintenance of load-sharing systems with degrading components," *IIE Trans.*, vol. 48, no. 8, pp. 699–709, 2016.
- [4] R. K. Iyer and D. J. Rossetti, "A measurement-based model for workload dependence of CPU errors," *IEEE Trans. Comput.*, vol. C-35, no. 6, pp. 511–519, Jun. 1986.
- [5] S. D. Durham, J. D. Lynch, W. J. Padgett, T. J. Horan, W. J. Owen, and J. Surles, "Localized load-sharing rules and Markov-Weibull fibers: A comparison of microcomposite failure data with Monte Carlo simulations," *J. Compos. Mater.*, vol. 31, no. 18, pp. 1856–1882, 1997.
- [6] H. E. Daniels, "The statistical theory of the strength of bundles of threads. I," *Proc. Roy. Soc. A, Math. Phys., Eng. Sci.*, vol. 183, pp. 405–435, Jun. 1945.
- [7] J. D. Lynch, "On the joint distribution of component failures for monotone load-sharing systems," *J. Stat. Planning Inference*, vol. 78, pp. 13–21, May 1999.
- [8] P. H. Kvam and E. A. Peña, "Estimating load-sharing properties in a dynamic reliability system," *J. Amer. Stat. Assoc.*, vol. 100, pp. 262–272, Dec. 2005.
- [9] H. Kim and P. H. Kvam, "Reliability estimation based on system data with an unknown load share rule," *Lifetime Data Anal.*, vol. 10, pp. 83–94, Mar. 2004.
- [10] J. Shao and L. R. Lamberson, "Modeling a shared-load k-out-of-n: G system," *IEEE Trans. Rel.*, vol. 40, no. 2, pp. 205–209, Jun. 1991.
- [11] H.-H. Lin, K.-H. Chen, and R.-T. Wang, "A multivariate exponential shared-load model," *IEEE Trans. Rel.*, vol. 42, no. 1, pp. 165–171, Mar. 1993.

- [12] C. Park, "Parameter estimation for the reliability of load-sharing systems," *IIE Trans.*, vol. 42, no. 10, pp. 753–765, 2010.
- [13] C. Park, "Parameter estimation from load-sharing system data using the expectation–maximization algorithm," *IIE Trans.*, vol. 45, no. 2, pp. 147–163, 2013.
- [14] H. Liu, "Reliability of a load-sharing k-out-of-n: G system: Non-iid components with arbitrary distributions," *IEEE Trans. Rel.*, vol. 47, no. 3, pp. 279–284, Sep. 1998.
- [15] W. Y. Yun and J. H. Cha, "A stochastic model for a general load-sharing system under overload condition," *Appl. Stochastic Models Bus. Ind.*, vol. 26, pp. 624–638, Sep./Oct. 2009.
- [16] S. D. Durham and J. D. Lynch, "A threshold representation for the strength distribution of a complex load sharing system," *J. Stat. Planning Inference*, vol. 83, pp. 25–46, Jan. 2000.
- [17] C. Yang, S. Zeng, and J. Guo, "Reliability analysis of load-sharing K -out-of- N system considering component degradation," *Math. Problems Eng.*, vol. 2015, Apr. 2015, Art. no. 726853.
- [18] W. B. Nelson, "Accelerated life testing—Step-stress models and data analyses," *IEEE Trans. Rel.*, vol. R-29, no. 2, pp. 103–108, Jun. 1980.
- [19] W. B. Nelson, *Accelerated Testing: Statistical Models, Test Plans, and Data Analysis*. Hoboken, NJ, USA: Wiley, 1990.
- [20] S. V. Amari and R. Bergman, "Reliability analysis of k-out-of-n load-sharing systems," in *Proc. 54th Annu. Rel. Maintainability Symp.*, Jan. 2008, pp. 440–445.
- [21] S. V. Amari, K. B. Misra, and H. Pham, "Reliability analysis of tampered failure rate load-sharing k -out-of- n : G systems," in *Proc. 12th ISSAT Int. Conf. Rel. Quality Design*, Aug. 2006, pp. 30–35.
- [22] L. Wang, J. Zhang, W. Chen, and X. Jia, "Reliability evaluation of a load-sharing parallel system with failure dependence," *Commun. Statist. Simul. Comput.*, vol. 45, no. 9, pp. 3094–3113, 2016.
- [23] G. Hao, L. Xie, and X. He, "The reliability mathematical model of load-sharing parallel systems based on fatigue cumulative damage," *Int. J. Performance Eng.*, vol. 5, no. 4, pp. 397–399, 2009.
- [24] L. Huang and Q. Xu, "Lifetime reliability for load-sharing redundant systems with arbitrary failure distributions," *IEEE Trans. Rel.*, vol. 59, no. 2, pp. 319–329, Jun. 2010.
- [25] Y. Chen, L. Yang, C. Ye, and R. Kang, "Failure mechanism dependence and reliability evaluation of non-repairable system," *Rel. Eng. Syst. Saf.*, vol. 138, pp. 273–283, Jun. 2015.
- [26] Y. Y. Li, Y. Chen, Z. H. Yuan, N. Tang, and R. Kang, "Reliability analysis of multi-state systems subject to failure mechanism dependence based on a combination method," *Rel. Eng. Syst. Saf.*, vol. 166, pp. 109–123, Oct. 2017.
- [27] L. Xing, B. A. Morrisette, and J. B. Dugan, "Combinatorial reliability analysis of imperfect coverage systems subject to functional dependence," *IEEE Trans. Rel.*, vol. 63, no. 1, pp. 367–382, Mar. 2014.



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