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State Estimation-Based Event-Triggered H_∞ Control for Multi-Delay Stochastic Network Control System

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ABSTRACT In this paper, the stability and stabilization problems are investigated for a class of event-triggered multiple time delays network control systems. Under the conditions of the event-triggered scheme and the system state estimation, the delays in the controlled object and the network transmission delay are both taken into account in the hybrid stochastic network control system by the free weighting matrix method and the integral inequality method. This paper will construct a new Lyapunov-Krasovskii functional and linear matrix inequality to analyze the problems. Finally, two numerical examples are provided to prove the effective performance of the multi-delay stochastic network control system.

INDEX TERMS Event-triggered, H_∞ control, linear matrix inequalities (LMIs), multiple time delays, network control, state estimation, stochastic system.

I. INTRODUCTION

In recent years, the time delay system has attracted more and more investigations [1]–[3]. All kinds of the time delay dependent problems [4]–[6] influence the performance of the control system. Especially the state time delay often causes the instability of the system. References [7]–[9] investigated stability problem of the systems with the single time delay. Considering that there exist multiple time delays in the actual systems [10], [11] analyzed multiple time delays systems and got the less conservative upper bound of time delay. Correspondingly, the stability performance of single time delay systems such as the linear discrete time system [12], stochastic system [13] and singular system [14] could be improved by considering the multi-delay. Therefore, there is still a lot of research space for the multiple time delays.

With the advent of the era of big data, the analysis of network control system becomes critical. The non-fragile synchronisation control for complex networks was studied in [15]. In order to deal with the external disturbance, the fuzzy PID controller [16] was designed for the network control system. The [17] has researched the delay-dependent stability problem of network control system. Furthermore, in the network control system, the transmission signals can occupy a certain channel resources. Under the time-triggered scheme, there exist the computation resource waste and large transmission load in the limited bandwidth. Therefore, an event-triggered scheme was proposed to reduce the burden of communication [18]–[22]. Information will be sent when the event-trigger condition is satisfied. Now, the event-triggered scheme is applied extensively in various network control systems. For example, the paper [23] has studied the filter problem of positive continuous system with the event-triggered scheme. Under the event-triggered scheme, the first order stochastic system [24] has a better performance than the system with time-triggered scheme. The paper [25] has studied the network transmission problem of fuzzy control system. In this paper, we will apply the event-triggered scheme to replace the traditional time-triggered mechanism in stochastic network control systems.

In the event-triggered network control system, the time delay is a current hot spot of research. Many experts such as the Dong Yue, Yong He and James Lam have studied the delay problem in the control systems [26], [27]. The paper [28] has studied the stability and stabilisation of neural network delay system. The paper [29] has analyzed the distributed delay and disturbance phenomenon in the network control transmission process. However, few of them considered the time delay in the controlled object when they analyzed the network control system. In this paper, we focus on both the multiple time delays in the controlled object and network transmission delay, which can improve the stability of network control system. Furthermore, the external disturbance makes the output of the actual system different from what we expect [30]–[32]. During the network transmission processing, the external disturbance often causes noise and messy code [33], [34]. These phenomena will bring the instability of the system. Thus, the disturbance should be considered into the network control system.

Based on the Lyapunov-Krasovskii functional [35] and linear matrix inequality approaches, the stability criteria of stochastic network control system is constructed. The free weighting matrix method (FWM) [36] and integral inequality method [37] are applied to derive the linear matrix inequalities in the criteria. On the one hand, the free weighting matrix method is used to discuss the relationship between the any terms in the Newton-Leibniz formula [38]. On the other hand, the integral inequality is applied to eliminate the quadratic integral terms in Lyapunov-Krasovskii functional derivative. These techniques make the stability criteria be delay-dependent and lowly conservative. With the help of the LMI toolbox, we could solve out the upper bound of time delay and the controller gain K.

Generally, we assume the system states are fully available. But in practice, unfortunately, the obtained state information is usually partial due to the limited measurement [39], [40]. Therefore, in order to ensure the system stability and reliability, estimating the system states is very significant. Some estimator design approaches have been proposed in [41] and [42]. The estimator-based control for fuzzy linear systems has been analyzed in [43]. In addition, [44] has reported the result on the the state estimation of nonlinear system. However, the state estimate problem for event-triggered stochastic network control system has not been sufficiently investigated. Then, we will design an estimator to estimate the unmeasurable states. To the best knowledge of the authors, the state estimation-based event-triggered H_∞ control for the stochastic network control system with multi-delay has not been fully studied.

This paper is summarized as follows: This paper simultaneously employs the free weighting matrix (FWM) method and the integral inequality method to solve the estimation-based event-triggered H_∞ control of stochastic network control system with multi-delay, which is different from other papers and reduces the conservatism of stability criteria. In order to improve the transmission performance, we reduce the burden of the network communication in the limited bandwidth by employing the event-trigged mechanism. At the same time, we design an estimator to estimate system states. This paper considers both the delay in controlled object and the network transmission delay to improve the stability of system. An effective controller is designed and the controller gain K is solved by LMI toolbox. The numerical examples are provided to prove the effective performance of the multiple-delay stochastic network control system.

Notations: In this paper, unless otherwise specified, $\mathfrak{E}$ be the expectation operator, $\mathscr{L}$ be the weak infinitesimal generator. Let $\|\cdot\|$ be the Euclidean norm of a vector and its induced norm of a matrix. $L_2[0, \infty]$ be the space of square integrable vector functions over $[0,\infty]$ and its norm is denoted by $\|\cdot\|_2$.

If X is symmetric, then $X \geq 0$ means that the matrix X is positive semi-definite); if X is a square matrix, then $He(X)$ is defined as $He(X) = X+X^T$. Let I and 0 be the identity matrix and zero matrix with appropriate dimensions, respectively. In symmetric block matrices or long matrix expressions, we use an asterisk * to represent a term that is induced by symmetry. Moreover, matrices, if their dimensions are not explicitly stated, are assumed to have compatible dimensions for algebraic operations.

II. PROBLEM FORMULATION

Firstly, we consider a stochastic network control system described as:

$$\begin{aligned} dx(t) &= [Ax(t) + A_{\tau_1}x(t-\tau_1) + A_{\tau_2}x(t-\tau_2) \\ &\quad + B_1u(t) + D_1v(t)]dt + [Ex(t) \\ &\quad + E_{\tau_1}x(t-\tau_1) + E_{\tau_2}x(t-\tau_2)]d\omega(t), \\ y(t) &= C_1x(t) + C_{1\tau_1}x(t-\tau_1) + C_{1\tau_2}x(t-\tau_2) \\ &\quad + B_2u(t) + D_2v(t), \\ z(t) &= C_2x(t) + C_{2\tau_1}x(t-\tau_1) + C_{2\tau_2}x(t-\tau_2) \\ &\quad + D_3v(t), \quad t \geq 0, \end{aligned} \tag{1}$$

where the compatible dimensional coefficient matrices are A, A_{τ_1}, A_{τ_2}, B_1, B_2, C_1, $C_{1\tau_1}$, $C_{1\tau_2}$, C_2, $C_{2\tau_1}$, $C_{2\tau_2}$, D_1, D_2, D_3, E, E_{τ_1}, and E_{τ_2}; the τ_1, τ_2 are constant delays; the system state vector is $x(t)$; the measured output is $y(t)$; the controlled output is $z(t)$; the control input is $u(t)$; the external disturbance input is $v(t)$; and the stochastic Brownian motion is $\omega(t)$.

As is well known, the bandwidth of the channel is limited in network control systems. In order to save the channel resources, we will research event-triggered network control system [38] and give the control schematic diagram in Fig 1.

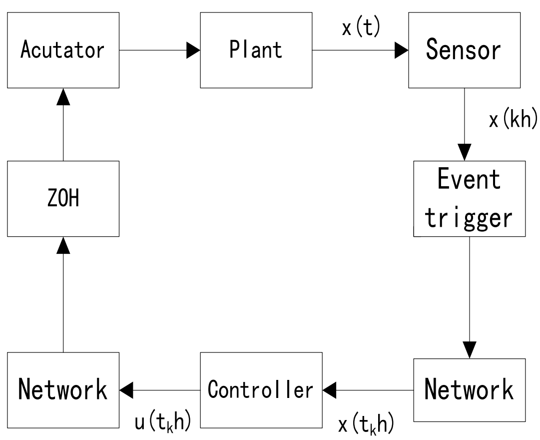

FIGURE 1. The schematic diagram of event-triggered network control system.

Then we consider the following event-triggered condition:

$$[x((k+j)h)-x(kh)]^T\Omega[x((k+j)h)-x(kh)] \le \sigma x^T(x(k+j)h)\Omega(x(k+j)h),$$

where Ω is symmetric positive definite matrix, $j=1,2,\ldots$, and scalar $\sigma \in [0,1)$, $x((k+j)h)$ is the sampled state. The $x((k+j)h)$ will be transmitted from sensor to controller when it exceeds the judgement condition threshold.

First of all, assume the release time is t_kh, $k=\{0,1,2,\cdots\}$, the scalar h is the sampling period. Thus, we can rewrite the stochastic network control system:

$$\begin{aligned} dx(t) &= [Ax(t)+A_{\tau_1}x(t-\tau_1)+A_{\tau_2}x(t-\tau_2) \\ &\quad +B_1u(t_kh)+D_1v(t)]dt+[Ex(t) \\ &\quad +E_{\tau_1}x(t-\tau_1)+E_{\tau_2}x(t-\tau_2)]d\omega(t), \\ y(t) &= C_1x(t)+C_{1\tau_1}x(t-\tau_1)+C_{1\tau_2}x(t-\tau_2) \\ &\quad +B_2u(t_kh)+D_2v(t), \\ z(t) &= C_2x(t)+C_{2\tau_1}x(t-\tau_1)+C_{2\tau_2}x(t-\tau_2) \\ &\quad +D_3v(t), \end{aligned} \tag{2}$$

Assume the sampling sequence as $\mathbb{S}_s=\{0,h,2h,\ldots,nh\}$. If after nh time, the next released time is $t_{k+1}h$, then the nh is the release interval of the transmit data under the event-triggered condition, thus, $t_{k+1}h=t_kh+nh$.

On the other hand, consider the network transmission delay $\hat{\tau}_k \in [0,\bar{\tau})$, where $k=\{0,1,2,\cdots\}$, $\bar{\tau}=max\{\hat{\tau}_k\}$, then the data will arrive at zero-order-holder (ZOH) at the time instant $t_kh+\hat{\tau}_k$.

Next, based on [22] and [45], we will construct a network time delay model for the stochastic network control system. Assume that

$$\rho_k = min\{j|t_kh+\hat{\tau}_k+jh \ge t_{k+1}h+\hat{\tau}_{k+1}, j=0,1,2,\ldots\}.$$

The interval $[t_kh+\hat{\tau}_k, t_{k+1}h+\hat{\tau}_{k+1})$ can be rewritten as

$$[t_kh+\hat{\tau}_k, t_{k+1}h+\hat{\tau}_{k+1}) = \bigcup_{j=1}^{\rho_k} I_j,$$

where

$$\begin{aligned} I_j &= [t_kh+\hat{\tau}_k+(j-1)h, t_{k+1}+\hat{\tau}_k+jh), \\ &\qquad j=1,2,\ldots,\rho_k-1, \\ I_{\rho_k} &= [t_kh+(\rho_k-1)h+\hat{\tau}_k, t_{k+1}h+\hat{\tau}_{k+1}). \end{aligned} \tag{3}$$

$$\tau(t) = \begin{cases} t-t_kh, & t\in I_1 \\ t-t_kh-h, & t\in I_2 \\ \cdots & . \\ t-t_kh-(\rho_k-1)h, & t\in I_{\rho_k} \end{cases} \tag{4}$$

$$e_k(t) = \begin{cases} 0, & t\in I_1 \\ x(t_kh)-x(t_kh+h), & t\in I_2 \\ \cdots & . \\ x(t_kh)-x(t_kh+(\rho_k-1)h), & t\in I_{\rho_k} \end{cases} \tag{5}$$

where $0 \le \tau(t) \le \bar{\tau}+h$, we set the $\tau_M=\bar{\tau}+h$, then $0\le\tau(t)\le\tau_M$. For the $t\in[t_kh+\hat{\tau}_k, t_{k+1}h+\hat{\tau}_{k+1})$, scalar $\sigma\in[0,1]$, the event-triggered scheme is:

$$e_k^T(t)\Omega e_k(t) \le \sigma x^T(t-\tau(t))\Omega x(t-\tau(t)), \tag{6}$$

therefore, according to the formula (2), (4), (5), (6), we can obtain:

$$\begin{aligned} u(t_kh) &= Kx(t_kh) \\ &= Ke_k(t)+Kx(t-\tau(t)), \end{aligned} \tag{7}$$

where $t\in[t_kh+\hat{\tau}_k, t_{k+1}h+\hat{\tau}_{k+1})$, $K\in R^{m\times n}$ is the stochastic network controller gain, and we can rewrite the system as:

$$\begin{aligned} dx(t) &= [Ax(t)+A_{\tau_1}x(t-\tau_1)+B_1Kx(t-\tau(t)) \\ &\quad +B_1Ke_k(t)+A_{\tau_2}x(t-\tau_2)+D_1v(t)]dt \\ &\quad +[Ex(t)+E_{\tau_1}x(t-\tau_1) \\ &\quad +E_{\tau_2}x(t-\tau_2)]d\omega(t), \\ y(t) &= C_1x(t)+C_{1\tau_1}x(t-\tau_1)+C_{1\tau_2}x(t-\tau_2) \\ &\quad +D_2v(t), \\ z(t) &= C_2x(t)+C_{2\tau_1}x(t-\tau_1)+C_{2\tau_2}x(t-\tau_2) \\ &\quad +D_3v(t), \\ x(t) &= \widetilde{\Phi}(t) \quad \forall t\in[-\eta,0]. \end{aligned} \tag{8}$$

where $t\in[t_kh+\hat{\tau}_k, t_{k+1}h+\hat{\tau}_{k+1})$, the scalar $\eta = max\{\tau_M,\tau_1,\tau_2\}$, $\widetilde{\Phi}(t)$ is initial condition function.

Next, we will design a state estimator as the following forms:

$$\begin{aligned} \dot{\bar{x}}(t) &= A\bar{x}(t)+A_{\tau_1}\bar{x}(t-\tau_1)+A_{\tau_2}\bar{x}(t-\tau_2) \\ &\quad +(L+\Delta L)(y(t)-\bar{y}(t)), \\ \bar{y}(t) &= C_1\bar{x}(t)+C_{1\tau_1}\bar{x}(t-\tau_1) \\ &\quad +C_{1\tau_2}\bar{x}(t-\tau_2), \end{aligned} \tag{9}$$

where $\bar{x}(t)\in R^n$ is the estimate state, the $\bar{y}(t)\in R^n$ is the estimate output, $L\in R^{n\times p}$ is the state estimator gain, ΔL is unknown real matrix as

$$\Delta L = M_LF_LN_L, \tag{10}$$

where the F_L, M_L, N_L are uncertain constant matrices, and the nominal matrix L is affected by these parameters. F_L satisfies the following forms:

$$F_L^TF_L \le I. \tag{11}$$

By defining the state estimator error as $e(t)=x(t)-\bar{x}(t)$, we get the following augmented system:

$$\begin{aligned} d\xi(t) &= [\mathscr{A}\xi(t)+\mathscr{A}_{\tau_1}\xi(t-\tau_1)+\mathscr{A}_{\tau_2}\xi(t-\tau_2) \\ &\quad +\mathscr{A}_{\tau_3}\xi(t-\tau(t))+WE_k(t)+\mathscr{B}v(t)]dt \\ &\quad +[\epsilon\xi(t)+\epsilon_{\tau_1}\xi(t-\tau_1)+\epsilon_{\tau_2}\xi(t-\tau_2)]d\omega(t), \\ z(t) &= \mathscr{C}\xi(t)+\mathscr{C}_{\tau_1}\xi(t-\tau_1)+\mathscr{C}_{\tau_2}\xi(t-\tau_2) \\ &\quad +\mathscr{D}v(t), \end{aligned} \tag{12}$$

where

$$\xi(t)=[\bar{x}^T(t), e^T(t)]^T, \quad E_k(t)=[0, e_k^T(t)]^T,$$

and

$$
\begin{aligned}
\mathscr{A} &= \begin{bmatrix} A & \bar{L}C_1 \\ 0 & A-\bar{L}C_1 \end{bmatrix}, \quad \mathscr{A}_{\tau_1} = \begin{bmatrix} A_{\tau_1} & \bar{L}C_{1\tau_1} \\ 0 & A_{\tau_1}-\bar{L}C_{1\tau_1} \end{bmatrix}, \\
\mathscr{A}_{\tau_2} &= \begin{bmatrix} A_{\tau_2} & \bar{L}C_{1\tau_2} \\ 0 & A_{\tau_2}-\bar{L}C_{1\tau_2} \end{bmatrix}, \quad \mathscr{A}_{\tau_3} = \begin{bmatrix} 0 & 0 \\ B_1K & B_1K \end{bmatrix}, \\
W &= \begin{bmatrix} 0 & 0 \\ B_1K & B_1K \end{bmatrix}, \quad \epsilon = \begin{bmatrix} 0 & 0 \\ E & E \end{bmatrix}, \\
\epsilon_{\tau_1} &= \begin{bmatrix} 0 & 0 \\ E_{\tau_1} & E_{\tau_1} \end{bmatrix}, \quad \epsilon_{\tau_2} = \begin{bmatrix} 0 & 0 \\ E_{\tau_2} & E_{\tau_2} \end{bmatrix}, \\
\mathscr{B} &= \begin{bmatrix} D_2^T\bar{L}^T & D_1^T - D_2^T\bar{L}^T \end{bmatrix}^T, \\
\mathscr{C} &= \begin{bmatrix} C_2 & C_2 \end{bmatrix}, \quad \mathscr{C}_{\tau_1} = \begin{bmatrix} C_{2\tau_1} & C_{2\tau_1} \end{bmatrix}, \\
\mathscr{C}_{\tau_2} &= \begin{bmatrix} C_{2\tau_2} & C_{2\tau_2} \end{bmatrix}, \\
\mathscr{D} &= D_3, \\
K &= K, \quad \bar{L} = L + \Delta L. \qquad (13)
\end{aligned}
$$

In order to facilitate the stability analysis of the system, next, we introduce the following definitions and lemmas.

Definition 1: For a scalar $\gamma > 0$, if the system satisfies the following formula:

$$\|z(t)\|_{\mathfrak{E}_2} \leq \gamma \|v(t)\|_2, \qquad (14)$$

for any non-zero $v(t) \in L_2[0, \infty)$, where

$$\|z(t)\|_{\mathfrak{E}_2} = \mathfrak{E}\{\int_0^\infty |z(t)|^2 dt\}^{1/2},$$

then the system has a H_∞ performance γ under the zero initial condition.

Definition 2: Consider the system (2) without of input $u(t)$ and disturbance $v(t)$, for any $\varepsilon > 0$, there is a $\delta(\varepsilon) > 0$, if the following formula

$$\mathfrak{E}|x(t)|^2 < \varepsilon, t > 0,$$

when,

$$sup\mathfrak{E}|\widetilde{\Phi}(s)|^2 < \delta(\varepsilon),_{-\eta \leq s \leq 0},$$

then the system is mean square stable.

Lemma 1: For any appropriate dimensional matrix $R \in R^{n\times n} \geq 0$, and the function $\omega : [0, l]$, the following integral inequality holds:

$$(\int_0^l \omega(s)ds)^T R \int_0^l \omega(s)ds \leq l \int_0^l \omega(s)^T R\omega(s)ds.$$

Lemma 2: Consider compatible dimensional matrices $\Omega_1 < 0$, $\Omega_2 < 0$, $\Omega_3 < 0$, then

$$\begin{bmatrix} \Omega_1 & \Omega_2 \\ * & \Omega_3 \end{bmatrix} < 0$$

is equivalent to $\Omega_1 - \Omega_2\Omega_3^{-1}\Omega_2^T < 0$.

Lemma 3: Consider any matrices $\mathbb{R}$, $\mathfrak{S}$, $\mathbb{P} > 0$, then:

$$2\mathbb{R}^T\mathfrak{S} \leq \mathbb{R}^T\mathbb{P}^{-1}\mathbb{R} + \mathfrak{S}^T\mathbb{P}\mathfrak{S}.$$

Lemma 4: For the scalar $\rho > 0$, consider the parameters Λ, U_i and V_i and W_i(i=1,...,N), if we have:

$$\begin{bmatrix} \Lambda & U_1 + \rho V_1 \ldots U_N + \rho V_N \\ * & diag\{-\rho W_1 - \rho W_1^T \ldots - \rho W_N - \rho W_N^T\} \end{bmatrix} < 0, \qquad (15)$$

then

$$\Lambda + \sum_{i=1}^{N} He(U_i W_i^{-1} V_i^T) < 0.$$

Then, we will introduce the following proof.

Proof: Pre-and post-multiplying equation (15) by

$$[I, \rho^{-1}U_1W_1^{-1} \ldots \rho^{-1}U_NW_N^{-1}],$$

and its transpose respectively yields. □

III. STABILITY ANALYSIS

This part analyzes the mean square stability of stochastic network control system. Define that

$$
\begin{aligned}
f(t) &= \mathscr{A}\xi(t) + \mathscr{A}_{\tau_1}\xi(t-\tau_1) + \mathscr{A}_{\tau_2}\xi(t-\tau_2) \\
&\quad + \mathscr{A}_{\tau_3}\xi(t-\tau(t)) + WE_k(t) + \mathscr{B}v(t), \qquad (16) \\
g(t) &= \epsilon\xi(t) + \epsilon_{\tau_1}\xi(t-\tau_1) + \epsilon_{\tau_2}\xi(t-\tau_2), \qquad (17)
\end{aligned}
$$

then

$$d\xi(t) = f(t)dt + g(t)d\omega(t). \qquad (18)$$

Assume $\tau_1 > \tau_2$, the stability criteria is shown in Theorem 1.

Theorem 1: For given the scalars $\tau_1 > 0$, $\tau_2 > 0$, $\tau_{12} = (\tau_1 - \tau_2) > 0$. If there exist matrices $P > 0$, $Q_1 > 0$, $Q_2 > 0$, $Q_3 > 0$, $Z_1 > 0$, $Z_2 > 0$, $Z_3 > 0$, $R > 0$, $M > 0$ and any appropriate matrices J_1, J_2, J_3, make:

$$
\Xi = \begin{bmatrix}
\Xi_{11} & \Xi_{12} & \Xi_{13} & \Xi_{14} & -N & \Xi_{16} & Z_1 & Z_2 & 0 \\
* & \Xi_{22} & \Xi_{23} & \Xi_{24} & 0 & \Xi_{26} & -Z_1 - J_1 & 0 & -Z_3 + J_3 \\
* & * & \Xi_{33} & \Xi_{34} & 0 & \Xi_{36} & 0 & -Z_2 - J_2 & Z_3 - J_3 \\
* & * & * & \Xi_{44} & 0 & \Xi_{46} & 0 & 0 & 0 \\
* & * & * & * & -Q_3 & \Xi_{56} & 0 & 0 & 0 \\
* & * & * & * & * & \Xi_{66} & 0 & 0 & 0 \\
* & * & * & * & * & * & -Z_1 - R & 0 & 0 \\
* & * & * & * & * & * & * & -Z_2 - R & 0 \\
* & * & * & * & * & * & * & * & -Z_3 - R
\end{bmatrix} < 0, \qquad (19)
$$

then, the system (12) is mean square stable, where

$$
\begin{aligned}
\Xi_{11} &= He(P\mathscr{A}) + Q_1 + Q_2 + Q_3 \\
&\quad + \tau_M NR^{-1}N^T - Z_1 - Z_2 \\
&\quad + \epsilon^T(P + \tau_1 Z_1 + \tau_2 Z_2 + \tau_{12} Z_3)\epsilon
\end{aligned}
$$

$$
\begin{aligned}
&\quad + \mathscr{A}^T[({\tau_1}^2 + {\tau_2}^2 + {\tau_{12}}^2 + \tau_M)R]\mathscr{A},\\
\Xi_{12} &= P\mathscr{A}_{\tau_1} + Z_1 + J_1^T\\
&\quad + \epsilon^T(P + \tau_1 Z_1 + \tau_2 Z_2 + \tau_{12} Z_3)\epsilon_{\tau_1}\\
&\quad + \mathscr{A}^T[({\tau_1}^2 + {\tau_2}^2 + {\tau_{12}}^2 + \tau_M)R]\mathscr{A}_{\tau_1},\\
\Xi_{22} &= -Q_1 - Z_1 - Z_3 - J_1 - J_1^T + J_3 + J_3^T\\
&\quad + \epsilon_{\tau_1}^T(P + \tau_1 Z_1 + \tau_2 Z_2 + \tau_{12} Z_3)\epsilon_{\tau_1}\\
&\quad + \mathscr{A}_{\tau_1}^T[({\tau_1}^2 + {\tau_2}^2 + {\tau_{12}}^2 + \tau_M)R]\mathscr{A}_{\tau_1},\\
\Xi_{13} &= P\mathscr{A}_{\tau_2} + Z_2 + J_2^T\\
&\quad + \mathscr{A}^T[({\tau_1}^2 + {\tau_2}^2 + {\tau_{12}}^2 + \tau_M)R]\mathscr{A}_{\tau_2}\\
&\quad + \epsilon^T(P + \tau_1 Z_1 + \tau_2 Z_2 + \tau_{12} Z_3)\epsilon_{\tau_2},\\
\Xi_{23} &= Z_3 - J_3 - J_3^T\\
&\quad + \mathscr{A}_{\tau_1}^T[({\tau_1}^2 + {\tau_2}^2 + {\tau_{12}}^2 + \tau_M)R]\mathscr{A}_{\tau_2}\\
&\quad + \epsilon_{\tau_1}(P + \tau_1 Z_1 + \tau_2 Z_2 + \tau_{12} Z_3)\epsilon_{\tau_2},\\
\Xi_{33} &= -Q_2 - Z_3 - Z_2 + J_3^T + J_3 - J_2 - J_2^T\\
&\quad + \mathscr{A}_{\tau_2}^T[({\tau_1}^2 + {\tau_2}^2 + {\tau_{12}}^2 + \tau_M)R]\mathscr{A}_{\tau_2}\\
&\quad + \epsilon_{\tau_2}^T(P + \tau_1 Z_1 + \tau_2 Z_2 + \tau_{12} Z_3)\epsilon_{\tau_2},\\
\Xi_{14} &= N + P\mathscr{A}_{\tau_3}\\
&\quad + \mathscr{A}^T[({\tau_1}^2 + {\tau_2}^2 + {\tau_{12}}^2 + \tau_M)R]\mathscr{A}_{\tau_3},\\
\Xi_{24} &= {\mathscr{A}_{\tau_1}}^T[({\tau_1}^2 + {\tau_2}^2 + {\tau_{12}}^2 + \tau_M)R]\mathscr{A}_{\tau_3},\\
\Xi_{34} &= {\mathscr{A}_{\tau_2}}^T[({\tau_1}^2 + {\tau_2}^2 + {\tau_{12}}^2 + \tau_M)R]\mathscr{A}_{\tau_3},\\
\Xi_{44} &= \sigma\Omega - M - M^T + \tau_M M R^{-1} M^T\\
&\quad + {\mathscr{A}_{\tau_3}}^T[({\tau_1}^2 + {\tau_2}^2 + {\tau_{12}}^2 + \tau_M)R]\mathscr{A}_{\tau_3},\\
\Xi_{16} &= PW + \mathscr{A}^T[({\tau_1}^2 + {\tau_2}^2 + {\tau_{12}}^2 + \tau_M)R]W,\\
\Xi_{26} &= {\mathscr{A}_{\tau_1}}^T[({\tau_1}^2 + {\tau_2}^2 + {\tau_{12}}^2 + \tau_M)R]W,\\
\Xi_{36} &= {\mathscr{A}_{\tau_2}}^T[({\tau_1}^2 + {\tau_2}^2 + {\tau_{12}}^2 + \tau_M)R]W,\\
\Xi_{46} &= {\mathscr{A}_{\tau_3}}^T[({\tau_1}^2 + {\tau_2}^2 + {\tau_{12}}^2 + \tau_M)R]W,\\
\Xi_{56} &= 0,\\
\Xi_{66} &= -\Omega + W^T[({\tau_1}^2 + {\tau_2}^2 + {\tau_{12}}^2 + \tau_M)R]W. \qquad (20)
\end{aligned}
$$

Proof: At first, in order to express the relationship of the time delays to reduce the conservatism of stability criteria, we give the equations (21)-(23). According to (18), for any $s \geq 0$ and time delay scalar τ, we have:

$$\xi(s) - \xi(s-\tau) = \int_{s-\tau}^{s} f(\alpha)d\alpha + \int_{s-\tau}^{s} g(\alpha)d\omega(\alpha). \quad (21)$$

Noting that $\mathfrak{E}\{\xi^T(s-\tau)J\int_{s-\tau}^{s} g(\alpha)d\omega(\alpha)\} = 0$ [31], setting any compatible dimensional matrix J, we have:

$$
\begin{aligned}
0 = 2\xi^T(s-\tau)J[\xi(s) - \xi(s-\tau) - \int_{s-\tau}^{s} f(\alpha)d\alpha\\
- \int_{s-\tau}^{s} g(\alpha)d\omega(\alpha)],
\end{aligned}
$$

therefore,

$$
\begin{aligned}
0 &= \mathfrak{E}[2\xi^T(s-\tau_1)]J_1[\xi(t) - \xi(t-\tau_1) - \int_{s-\tau_1}^{s} f(\alpha)d\alpha],\\
0 &= \mathfrak{E}[2\xi^T(s-\tau_2)]J_2[\xi(t) - \xi(t-\tau_2) - \int_{s-\tau_2}^{s} f(\alpha)d\alpha],\\
0 &= \mathfrak{E}\{2[\xi^T(s-\tau_2) - \xi^T(s-\tau_1)]\}J_3[\xi(t-\tau_2)\\
&\quad - \xi(t-\tau_1) - \int_{s-\tau_1}^{s-\tau_2} f(\alpha)d\alpha]. \qquad (22)
\end{aligned}
$$

In addition, we consider the compatible dimensional matrices M, N, then we have:

$$
\begin{aligned}
0 &= \mathfrak{E}[2\xi^T(t-\tau(t))]M[\xi(t) - \xi(t-\tau(t))\\
&\quad - \int_{t-\tau(t)}^{t} f(\alpha)d\alpha],\\
0 &= \mathfrak{E}[2\xi^T(t)]N[\xi(t-\tau(t)) - \xi(t-\tau_M)\\
&\quad - \int_{t-\tau_M}^{t-\tau(t)} f(\alpha)d\alpha]. \qquad (23)
\end{aligned}
$$

Next, we will deal with the terms in (23) to prepare for the derivation of the inequality in Theorem 1. According to the Lemma 3, we have:

$$
\begin{aligned}
&-2\xi^T(t)(\tau_M - \tau(t))N\int_{t-\tau_M}^{t-\tau(t)} f(\alpha)d\alpha\\
&\quad \leq (\tau_M - \tau(t))^2\xi^T(t)NR^{-1}N^T\xi(t)\\
&\quad + (\int_{t-\tau_M}^{t-\tau(t)} f(\alpha)d\alpha)R(\int_{t-\tau_M}^{t-\tau(t)} f(\alpha)d\alpha), \qquad (24)\\
&-2\xi^T(t-\tau(t))\tau(t)M\int_{t-\tau(t)}^{t} f(\alpha)d\alpha\\
&\quad \leq (\tau(t))^2\xi^T(t-\tau(t))MR^{-1}M^T\xi(t-\tau(t))\\
&\quad + (\int_{t-\tau(t)}^{t} f(\alpha)d\alpha)R(\int_{t-\tau(t)}^{t} f(\alpha)d\alpha). \qquad (25)
\end{aligned}
$$

Applying the Lemma 1, we have:

$$
\begin{aligned}
&-2\xi^T(t)N\int_{t-\tau_M}^{t-\tau(t)} f(\alpha)d\alpha\\
&\quad \leq (\tau_M - \tau(t))\xi^T(t)NR^{-1}N^T\xi(t)\\
&\quad + \int_{t-\tau_M}^{t-\tau(t)} f(\alpha)Rf(\alpha)d\alpha, \qquad (26)\\
&-2\xi^T(t-\tau(t))M\int_{t-\tau(t)}^{t} f(\alpha)d\alpha\\
&\quad \leq (\tau(t))\xi^T(t-\tau(t))MR^{-1}M^T\xi(t-\tau(t))\\
&\quad + \int_{t-\tau(t)}^{t} f(\alpha)Rf(\alpha)d\alpha. \qquad (27)
\end{aligned}
$$

Due to the $\tau(t) \in [\tau_m, \tau_M]$, thus, we get:

$$
\begin{aligned}
&-2\xi^T(t)N\int_{t-\tau_M}^{t-\tau(t)} f(\alpha)d\alpha\\
&\quad \leq \tau_M\xi^T(t)NR^{-1}N^T\xi(t)\\
&\quad + \int_{t-\tau_M}^{t-\tau(t)} f(\alpha)Rf(\alpha)d\alpha, \qquad (28)\\
&-2\xi^T(t-\tau(t))M\int_{t-\tau(t)}^{t} f(\alpha)d\alpha\\
&\quad \leq (\tau_M)\xi^T(t-\tau(t))MR^{-1}M^T\xi(t-\tau(t))\\
&\quad + \int_{t-\tau(t)}^{t} f(\alpha)Rf(\alpha)d\alpha, \qquad (29)
\end{aligned}
$$

Then, choose the following Lyapunov-Krasovkii functional candidate to analyze the mean-square stability of system (12):

$$V(t) = V_1(t) + V_2(t) + V_3(t) + V_4(t) + V_5(t), \quad (30)$$

where

$$\begin{aligned}
V_1(t) &= \xi^T(t)P\xi(t),\\
V_2(t) &= \int_{t-\tau_1}^{t} \xi^T(\alpha)Q_1\xi(\alpha)d\alpha\\
&\quad + \int_{t-\tau_2}^{t} \xi^T(\alpha)Q_2\xi(\alpha)d\alpha\\
&\quad + \int_{t-\tau_M}^{t} \xi^T(\alpha)Q_3\xi(\alpha)d\alpha,\\
V_3(t) &= \tau_1\int_{-\tau_1}^{0}\int_{t+\beta}^{t} f^T(\alpha)Rf(\alpha)d\alpha d\beta\\
&\quad +\tau_2\int_{-\tau_2}^{0}\int_{t+\beta}^{t} f^T(\alpha)Rf(\alpha)d\alpha d\beta\\
&\quad +\tau_{12}\int_{-\tau_1}^{-\tau_2}\int_{t+\beta}^{t} f^T(\alpha)Rf(\alpha)d\alpha d\beta,\\
V_4(t) &= \int_{-\tau_1}^{0}\int_{t+\beta}^{t} g^T(\alpha)Z_1g(\alpha)d\alpha d\beta\\
&\quad + \int_{-\tau_2}^{0}\int_{t+\beta}^{t} g^T(\alpha)Z_2g(\alpha)d\alpha d\beta\\
&\quad + \int_{-\tau_1}^{-\tau_2}\int_{t+\beta}^{t} g^T(\alpha)Z_3g(\alpha)d\alpha d\beta,\\
V_5(t) &= \int_{-\tau_M}^{0}\int_{s+\beta}^{s} f^T(\alpha)(s)Rf(\alpha)d\alpha d\beta.
\end{aligned}$$

And by using the itô's formula, we obtain that:

$$\mathscr{L}V(s)=\mathscr{L}V_1(t)+\mathscr{L}V_2(t)+\mathscr{L}V_3(t) + \mathscr{L}V_4(t) + \mathscr{L}V_5(t), \quad (31)$$

where

$$\begin{aligned}
\mathscr{L}V_1(t) &= 2\xi^T(s)Pf(s) + g^T(s)Pg(s),\\
\mathscr{L}V_2(t) &= \xi^T(s)Q_1\xi(s) + \xi^T(s)Q_2\xi(s)\\
&\quad + \xi^T(s)Q_3\xi(s)\\
&\quad - \xi^T(s-\tau_1)Q_1\xi(s-\tau_1)\\
&\quad - \xi^T(s-\tau_2)Q_2\xi(s-\tau_2)\\
&\quad - \xi^T(s-\tau_M)Q_3\xi(s-\tau_M),\\
\mathscr{L}V_3(t) &= {\tau_1}^2f^T(s)Rf(s)\\
&\quad - \tau_1\int_{s-\tau_1}^{s} f^T(\alpha)Rf(\alpha)d\alpha\\
&\quad + {\tau_2}^2f^T(s)Rf(s) - \tau_2\int_{s-\tau_2}^{s} f^T(\alpha)Rf(\alpha)d\alpha\\
&\quad + {\tau_{12}}^2(f^T(\alpha)Rf(\alpha))\\
&\quad - \tau_{12}\int_{s-\tau_1}^{s-\tau_2} f^T(\alpha)Rf(\alpha)d\alpha,\\
\mathscr{L}V_4(t) &= \tau_1g(s)Z_1g(s)\\
&\quad - \int_{s-\tau_1}^{s} g^T(\alpha)Z_1g(\alpha)d\alpha\\
&\quad + \tau_2g(\alpha)Z_2g(\alpha)\\
&\quad - \int_{s-\tau_2}^{s} g(\alpha)Z_2g(\alpha)\\
&\quad + \tau_{12}(g^T(\alpha)Z_3g(\alpha))\\
&\quad - \int_{s-\tau_1}^{s-\tau_2} g^T(\alpha)Z_3g(\alpha)d\alpha,\\
\mathscr{L}V_5(t) &= \tau_Mf^T(\alpha)Rf(\alpha)\\
&\quad - \int_{t-\tau_M}^{t-\tau(t)} f^T(\alpha)Rf(\alpha)d\alpha\\
&\quad - \int_{t-\tau(t)}^{t} f^T(\alpha)Rf(\alpha)d\alpha.
\end{aligned}$$

According to lemma 1, the following inequations hold:

$$\begin{aligned}
&-\tau_1\int_{s-\tau_1}^{s} f^T(\alpha)Rf(\alpha)d\alpha\\
&\quad \le -(\int_{s-\tau_1}^{s} f(\alpha)d\alpha)^TR(\int_{s-\tau_1}^{s} f(\alpha)d\alpha), \quad (32)\\
&-\tau_2\int_{s-\tau_2}^{s} f^T(\alpha)Rf(\alpha)d\alpha\\
&\quad \le -(\int_{s-\tau_2}^{s} f(\alpha)d\alpha)^TR(\int_{s-\tau_2}^{s} f(\alpha)d\alpha), \quad (33)\\
&-\tau_{12}\int_{s-\tau_1}^{s-\tau_2} f^T(\alpha)Rf(\alpha)d\alpha\\
&\quad \le -(\int_{s-\tau_1}^{s-\tau_2} f(\alpha)d\alpha)^TR(\int_{s-\tau_1}^{s-\tau_2} f(\alpha)d\alpha), \quad (34)
\end{aligned}$$

In the view of the isometry property of the stochastic integral and (31), we have

$$\begin{aligned}
&\mathfrak{E}[\int_{s-\tau_1}^{s} g^T(\alpha)Z_1g(\alpha)d\alpha]\\
&\quad = \mathfrak{E}[(\int_{s-\tau_1}^{s} g(\alpha)d\omega(\alpha))^TZ_1(\int_{s-\tau_1}^{s} g(\alpha)d\omega(\alpha))],\\
&\quad = \mathfrak{E}[(\xi(s) - \xi(s-\tau_1) - \int_{s-\tau_1}^{s} f(\alpha)d\alpha)^TZ_1(\xi(s)\\
&\qquad - \xi(s-\tau_1) - \int_{s-\tau_1}^{s} f(\alpha)d\alpha)], \quad (35)\\
&\mathfrak{E}[\int_{s-\tau_2}^{s} g^T(\alpha)Z_2g(\alpha)d\alpha]\\
&\quad = \mathfrak{E}[(\int_{s-\tau_2}^{s} g(\alpha)d\omega(\alpha))^TZ_2(\int_{s-\tau_2}^{s} g(\alpha)d\omega(\alpha))],\\
&\quad = \mathfrak{E}[(\xi(s) - \xi(s-\tau_2) - \int_{s-\tau_2}^{s} f(\alpha)d\alpha)^TZ_2(\xi(s)\\
&\qquad - \xi(s-\tau_2) - \int_{s-\tau_2}^{s} f(\alpha)d\alpha)], \quad (36)\\
&\mathfrak{E}[\int_{s-\tau_1}^{s-\tau_2} g^T(\alpha)Z_3g(\alpha)d\alpha]\\
&\quad = \mathfrak{E}[(\int_{s-\tau_1}^{s-\tau_2} g(\alpha)d\omega(\alpha))^TZ_3(\int_{s-\tau_1}^{s-\tau_2} g(\alpha)d\omega(\alpha))],
\end{aligned}$$

$$
\begin{aligned}
&= \mathfrak{E}[(\xi(s-\tau_2)-\xi(s-\tau_1) \\
&\quad -\int_{s-\tau_1}^{s-\tau_2} f(\alpha)d\alpha)^T Z_3(\xi(s-\tau_2)-\xi(s-\tau_1) \\
&\quad -\int_{s-\tau_1}^{s-\tau_2} f(\alpha)d\alpha)]. \qquad (37)
\end{aligned}
$$

Next, add the event-triggered condition

$$
[x((k+j)h)-x(kh)]^T\Omega[x((k+j)h)-x(kh)] \leq \sigma x^T(x(k+j)h)\Omega(x(k+j)h)
$$

into the stochastic network control system. At the same time, we employ the (23) and (24) to the $\mathscr{L}V(s)$, then, when $v(s)=0$, it is easy to show that

$$
\mathfrak{E}\{\mathscr{L}V(s)\} \leq \mathfrak{E}\{\eta(s)^T\Xi\eta(s)\},
$$

where the

$$
\begin{aligned}
\eta(t) = [&\xi^T(s), \xi^T(s-\tau_1), \xi^T(s-\tau_2), \\
&\xi^T(s-\tau(s)), \xi^T(s-\tau_M), E_k^T(t), \\
&\int_{s-\tau_1}^{s} f^T(\alpha)d\alpha, \int_{s-\tau_2}^{s} f^T(\alpha)d\alpha, \\
&\int_{s-\tau_1}^{s-\tau_2} f^T(\alpha)d\alpha].
\end{aligned}
$$

According to the Definition 2 and Theorem 1, we can obtain the stochastic network control system is mean square stable. The proof is completed.

On the other hand, if in the case: $\tau_1 \leq \tau_2$, there will be the similar proof procedure as above. The following theorems just discuss the case: $\tau_1 > \tau_2$. □

IV. H_∞ STABILITY ANALYSIS

According to the Theorem 1, under the event-triggered condition and the state estimation, we have proved the stability of stochastic network control system.

In this section, we further study H_∞ performance of the stochastic network control system and obtain the following theorem.

Theorem 2: Consider the scalars $\gamma > 0$, $\tau_1 > 0$, $\tau_2 > 0$, $\tau_{12} = \tau_1 - \tau_2 > 0$, if there exist matrices $P > 0$, $Z_1 > 0$, $Z_2 > 0$, $Z_3 > 0$, $Q_1 > 0$, $Q_2 > 0$, $Q_3 > 0$, $R > 0$, $M > 0$, and any appropriate matrices J_1, J_2, J_3, such that the following LMI holds:

$$
\Pi = \begin{bmatrix}
\Pi_{11} & \Pi_{12} & \Pi_{13} & \Pi_{14} & -N & \Pi_{16} & \Pi_{17} & Z_1 & Z_2 & 0 \\
* & \Pi_{22} & \Pi_{23} & \Pi_{24} & 0 & \Pi_{26} & \Pi_{27} & -Z_1-J_1 & 0 & -Z_3+J_3 \\
* & * & \Pi_{33} & \Pi_{34} & 0 & \Pi_{36} & \Pi_{37} & 0 & -Z_2-J_2 & Z_3-J_3 \\
* & * & * & \Pi_{44} & 0 & \Pi_{46} & \Pi_{47} & 0 & 0 & 0 \\
* & * & * & * & -Q_3 & \Pi_{56} & 0 & 0 & 0 & 0 \\
* & * & * & * & * & \Pi_{66} & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & \Pi_{77} & 0 & 0 & 0 \\
* & * & * & * & * & * & * & -Z_1-R & 0 & 0 \\
* & * & * & * & * & * & * & * & -Z_2-R & 0 \\
* & * & * & * & * & * & * & * & * & -Z_3-R
\end{bmatrix} < 0, \qquad (38)
$$

then, system (12) is mean square stable with H_∞ performance γ, where

$$
\begin{aligned}
\Pi_{11} &= He(P\mathscr{A}) + Q_1 + Q_2 + Q_3 \\
&\quad - Z_1 - Z_2 + \tau_M NR^{-1}N^T \\
&\quad + \epsilon^T(P + \tau_1 Z_1 + \tau_2 Z_2 + \tau_{12} Z_3)\epsilon \\
&\quad + \mathscr{A}^T[(\tau_1^2 + \tau_2^2 + \tau_{12}^2 + \tau_M)R]\mathscr{A} \\
&\quad + \mathscr{C}^T\mathscr{C}, \\
\Pi_{12} &= P\mathscr{A}_{\tau_1} + Z_1 + J_1^T + \mathscr{C}^T\mathscr{C}_{\tau_1} \\
&\quad + \epsilon^T(P + \tau_1 Z_1 + \tau_2 Z_2 + \tau_{12} Z_3)\epsilon_{\tau_1} \\
&\quad + \mathscr{A}^T[(\tau_1^2 + \tau_2^2 + \tau_{12}^2 + \tau_M)R]\mathscr{A}_{\tau_1}, \\
\Pi_{13} &= P\mathscr{A}_{\tau_2} + Z_2 + J_2^T + \mathscr{C}^T\mathscr{C}_{\tau_2} \\
&\quad + \mathscr{A}^T[(\tau_1^2 + \tau_2^2 + \tau_{12}^2 + \tau_M)R]\mathscr{A}_{\tau_2} \\
&\quad + \epsilon^T(P + \tau_1 Z_1 + \tau_2 Z_2 + \tau_{12} Z_3)\epsilon_{\tau_2}, \\
\Pi_{23} &= Z_3 - J_3^T - J_3 + \mathscr{C}_{\tau_1}^T\mathscr{C}_{\tau_2} \\
&\quad + \mathscr{A}_{\tau_1}^T[(\tau_1^2 + \tau_2^2 + \tau_{12}^2 + \tau_M)R]\mathscr{A}_{\tau_2} \\
&\quad + \epsilon_{\tau_1}^T(P + \tau_1 Z_1 + \tau_2 Z_2 + \tau_{12} Z_3)\epsilon_{\tau_2}, \\
\Pi_{22} &= -Q_1 - Z_1 - Z_3 - J_1 - J_1^T + J_3 + J_3^T \\
&\quad + \epsilon_{\tau_1}^T(P + \tau_1 Z_1 + \tau_2 Z_2 + \tau_{12} Z_3)\epsilon_{\tau_1} \\
&\quad + \mathscr{A}_{\tau_1}^T[(\tau_1^2 + \tau_2^2 + \tau_{12}^2 + \tau_M)R^2]\mathscr{A}_{\tau_1} + \mathscr{C}_{\tau_1}^T\mathscr{C}_{\tau_1}, \\
\Pi_{33} &= -Q_2 - Z_2 - Z_3 + J_3 + J_3^T - J_2^T - J_2 \\
&\quad + \epsilon_{\tau_2}^T(P + \tau_1 Z_1 + \tau_2 Z_2 + \tau_{12} Z_3)\epsilon_{\tau_2} \\
&\quad + \mathscr{A}_{\tau_2}^T[(\tau_1^2 + \tau_2^2 + \tau_{12}^2 + \tau_M)R]\mathscr{A}_{\tau_2} + \mathscr{C}_{\tau_2}^T\mathscr{C}_{\tau_2}, \\
\Pi_{14} &= P\mathscr{A}_{\tau_3} + N + \mathscr{A}^T[(\tau_1^2 + \tau_2^2 + \tau_{12}^2 + \tau_M)R]\mathscr{A}_{\tau_3}, \\
\Pi_{24} &= \mathscr{A}_{\tau_1}{}^T[(\tau_1^2 + \tau_2^2 + \tau_{12}^2 + \tau_M)R]\mathscr{A}_{\tau_3}, \\
\Pi_{34} &= \mathscr{A}_{\tau_2}{}^T[(\tau_1^2 + \tau_2^2 + \tau_{12}^2 + \tau_M)R]\mathscr{A}_{\tau_3}, \\
\Pi_{44} &= \sigma\Omega - M - M^T + \tau_M MR^{-1}M^T \\
&\quad + \mathscr{A}_{\tau_3}{}^T[({\tau_1}^2 + {\tau_2}^2 + {\tau_{12}}^2 + \tau_M)R]\mathscr{A}_{\tau_3}, \\
\Pi_{16} &= PW + \mathscr{A}^T[({\tau_1}^2 + {\tau_2}^2 + {\tau_{12}}^2 + \tau_M)R]W, \\
\Pi_{26} &= \mathscr{A}_{\tau_1}{}^T[({\tau_1}^2 + {\tau_2}^2 + {\tau_{12}}^2 + \tau_M)R]W, \\
\Pi_{36} &= \mathscr{A}_{\tau_2}{}^T[({\tau_1}^2 + {\tau_2}^2 + {\tau_{12}}^2 + \tau_M)R]W, \\
\Pi_{46} &= \mathscr{A}_{\tau_3}{}^T[({\tau_1}^2 + {\tau_2}^2 + {\tau_{12}}^2 + \tau_M)R]W, \\
\Pi_{56} &= 0, \\
\Pi_{66} &= -\Omega + W^T[({\tau_1}^2 + {\tau_2}^2 + {\tau_{12}}^2 + \tau_M)R]W, \\
\Pi_{17} &= P\mathscr{B} + \mathscr{A}^T[(\tau_1^2 + \tau_2^2 + \tau_{12}^2 + \tau_M)R]\mathscr{B} \\
&\quad + \mathscr{C}^T\mathscr{D},
\end{aligned}
$$

$$\begin{aligned}\Pi_{27} &= \mathscr{A}_{\tau_1}^T[(\tau_1^2+\tau_2^2+\tau_{12}^2+\tau_M)R]\mathscr{B}\\ &\quad +\mathscr{C}_{\tau_1}^T\mathscr{D},\\ \Pi_{37} &= \mathscr{A}_{\tau_2}^T[(\tau_1^2+\tau_2^2+\tau_{12}^2+\tau_M)R]\mathscr{B}\\ &\quad +\mathscr{C}_{\tau_2}^T\mathscr{D},\\ \Pi_{47} &= \mathscr{A}_{\tau_3}^T[(\tau_1^2+\tau_2^2+\tau_{12}^2+\tau_M)R]\mathscr{B},\\ \Pi_{77} &= \mathscr{B}^T[(\tau_1^2+\tau_2^2+\tau_{12}^2+\tau_M)R]\mathscr{B}\\ &\quad +\mathscr{D}^T\mathscr{D}-\gamma^2 I. \end{aligned}\tag{39}$$

Proof: It is easy to see if (38) holds then (19) holds; therefore, the system (12) is mean square stable. Next, we shall show that system (12) has a H_∞ performance γ. To this end, we introduce the following index:

$$J(t)=\mathfrak{E}\{\int_0^t(|z(s)|^2-\gamma^2|v(s)|^2)ds\}.$$

Under zero initial condition, by $it\hat{o}'s$ formula we have

$$\begin{aligned}J(t) &= \mathfrak{E}\{\int_0^t(|z(s)|^2-\gamma^2|v(s)|^2+\mathscr{L}V(s))ds\}-\mathfrak{E}\{V(t)\}\\ &\le \mathfrak{E}\{\int_0^t(|z(s)|^2-\gamma^2|v(s)|^2+\mathscr{L}V(s))ds\},\end{aligned}$$

where $V(t)$ is defined in (30). Combined with (12), (16), (17), and (30)-(37), then:

$$\begin{aligned}J(t)\le \int_0^t &\mathfrak{E}\{[\xi^T(s),\xi^T(s-\tau_1),\\ &\xi^T(s-\tau_2),\xi^T(s-\tau(t)),\\ &\xi^T(s-\tau_M),E_k^T(t),v^T(s),\\ &\int_{s-\tau_1}^s f^T(\alpha)d\alpha,\int_{s-\tau_2}^s f^T(\alpha)d\alpha,\\ &\int_{s-\tau_1}^{s-\tau_2} f^T(\alpha)d\alpha]\\ &\Pi[\xi^T(s),\xi^T(s-\tau_1),\xi^T(s-\tau_2),\\ &\xi^T(s-\tau(t)),\xi^T(s-\tau_M),\\ &E_k^T(t),v^T(s),\int_{s-\tau_1}^s f^T(\alpha)d\alpha,\\ &\int_{s-\tau_2}^s f^T(\alpha)d\alpha,\int_{s-\tau_1}^{s-\tau_2} f^T(\alpha)d\alpha]^T\}ds,\end{aligned}\tag{40}$$

where the Π is defined in (38). According to the (38) and (40), we have $J(t)\le 0$, for $t>0$. The proof is completed. □

V. H_∞ CONTROLLER DESIGN

Now we start to discuss the state estimation-based event-triggered H_∞ controller of the system (12). Firstly, we detach the gain L from matrix A and set

$$\mathcal{A}=\begin{bmatrix}A & 0\\ 0 & A\end{bmatrix},\quad \mathcal{A}_{\tau_1}=\begin{bmatrix}A_{\tau_1} & 0\\ 0 & A_{\tau_1}\end{bmatrix},$$
$$\mathcal{A}_{\tau_2}=\begin{bmatrix}A_{\tau_2} & 0\\ 0 & A_{\tau_2}\end{bmatrix},$$
$$\mathcal{B}_1=\begin{bmatrix}0 & 0\\ B_1 & B_1\end{bmatrix},\quad \mathcal{B}_1=\begin{bmatrix}0 & 0\\ B_1 & B_1\end{bmatrix},$$
$$\begin{aligned}\mathcal{C}_1 &= \begin{bmatrix}0 & C_1\end{bmatrix},\\ \mathcal{C}_{1\tau_1} &= \begin{bmatrix}0 & C_{1\tau_1}\end{bmatrix},\\ \mathcal{C}_2 &= \begin{bmatrix}C_2 & C_2\end{bmatrix},\\ \mathcal{C}_{2\tau_1} &= \begin{bmatrix}C_{2\tau_1}, C_{2\tau_1}\end{bmatrix},\\ \mathcal{C}_{2\tau_2} &= \begin{bmatrix}C_{2\tau_2}, C_{2\tau_2}\end{bmatrix},\\ \mathcal{D}_1 &= \begin{bmatrix}0 & D_1,\end{bmatrix},\\ \mathcal{D}_2 &= D_2,\quad \mathcal{D}_3=D_3,\\ \epsilon &= \begin{bmatrix}0 & 0\\ E & E\end{bmatrix},\\ \epsilon_{\tau_1} &= \begin{bmatrix}0 & 0\\ E_{\tau_1} & E_{\tau_1}\end{bmatrix},\\ \epsilon_{\tau_2} &= \begin{bmatrix}0 & 0\\ E_{\tau_2} & E_{\tau_2}\end{bmatrix},\\ \mathcal{I}_1 &= \begin{bmatrix}I & 0\end{bmatrix},\quad \mathcal{I}_2=\begin{bmatrix}I\\ -I\end{bmatrix},\end{aligned}$$

and rewrite the coefficient matrices of system (12) as:

$$\begin{aligned}\mathscr{A} &= \mathcal{A}+\mathcal{I}_2(L+\Delta L)\mathcal{C}_1,\\ \mathscr{A}_{\tau_1} &= \mathcal{A}_{\tau_1}+\mathcal{I}_2(L+\Delta L)\mathcal{C}_{1\tau_1},\\ \mathscr{A}_{\tau_2} &= \mathcal{A}_{\tau_2}+\mathcal{I}_2(L+\Delta L)\mathcal{C}_{1\tau_2},\\ \mathscr{A}_{\tau_3} &= \mathcal{B}_1K,\ W=\mathcal{B}_1K,\\ \mathscr{B} &= \mathcal{D}_1+\mathcal{I}_2(L+\Delta L)\mathcal{D}_2,\ \mathscr{C}=\mathcal{C}_2,\\ \mathscr{C}_{\tau_1} &= \mathcal{C}_{2\tau_1},\ \mathscr{C}_{\tau_2}=\mathcal{C}_{2\tau_2},\\ \mathscr{D} &= \mathcal{D}_3,\quad \epsilon=\epsilon,\ \epsilon_{\tau_1}=\epsilon_{\tau_1},\ \epsilon_{\tau_2}=\epsilon_{\tau_2}.\end{aligned}\tag{41}$$

Then, we propose a new approach for designing an H_∞ controller for the stochastic network control system with multiple time delays by the following theorem.

Theorem 3: For given scalars $\gamma>0$, $\tau_1>0$, $\tau_2>0$, $\tau_{12}>0$, and $\rho>0$, set $k_1=\tau_1^2+\tau_2^2+\tau_{12}^2+\tau_M$. If exist scalar $\epsilon_1>0$, and matrices $P>0$, $Q_1>0$, $Q_2>0$, $Q_3>0$, $Z_1>0$, $Z_2>0$, $Z_3>0$, $R>0$, $M>0$, any appropriate matrices J_1, J_2, J_3, X_L, Y_L and such that the following LMI holds:

$$\Phi=\begin{bmatrix}\Phi_1 & \Phi_2\\ * & \Phi_3\end{bmatrix}<0,$$

then the event-triggered stochastic network control system is solvable, where the matrices Φ_1, Φ_2, Φ_3, Φ_4 are in the following formulas, $\Phi_1=$

$$\begin{bmatrix}\Phi_{11} & \Phi_{12} & \Phi_{13} & \Phi_{14} & -N & \Phi_{16} & Z_1\\ * & \Phi_{22} & \Phi_{23} & 0 & 0 & 0 & -Z_1-J_1\\ * & * & \Phi_{33} & 0 & 0 & 0 & 0\\ * & * & * & \Phi_{44} & 0 & 0 & 0\\ * & * & * & * & -Q_3 & 0 & 0\\ * & * & * & * & * & \Phi_{66} & 0\\ * & * & * & * & * & * & -Z_1-\mu_1 P\\ * & * & * & * & * & * & *\\ * & * & * & * & * & * & *\\ * & * & * & * & * & * & *\\ * & * & * & * & * & * & *\\ * & * & * & * & * & * & *\end{bmatrix}$$

$$
\begin{matrix}
Z_2 & 0 & \Phi_{1t} & \mathcal{C}_2^T & k_1\mathcal{A}^T\mu_1 P \\
0 & -Z_3+J_3 & \Phi_{2t} & \mathcal{C}_{2\tau_1}^T & k_1\mathcal{A}_{\tau_1}^T\mu_1 P \\
-Z_2-J_2 & Z_3-J_3 & \Phi_{3t} & \mathcal{C}_{2\tau_2}^T & k_1\mathcal{A}_{\tau_2}^T\mu_1 P \\
0 & 0 & 0 & 0 & k_1\mu_1 Y^T \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & k_1\mu_1 Y^T \\
0 & 0 & 0 & 0 & 0 \\
-Z_2-\mu_1 P & 0 & 0 & 0 & 0 \\
* & -Z_3-\mu_1 P & 0 & 0 & 0 \\
* & * & \Phi_{tt} & \mathcal{D}_3^T & 0 \\
* & * & * & -I & k_1\mathcal{D}_1^T\mu_1 P \\
* & * & * & * & -k_1\mu_1 P
\end{matrix}
\Bigg] , \tag{42}
$$

$$
\Phi_2 = \begin{bmatrix}
-\mathcal{I}_2X_L + P\mathcal{I}_2 + \rho\mathcal{C}_1^T Y_L^T & P\mathcal{I}_2 M_L \\
\rho\mathcal{C}_{1\tau_1}^T Y_L^T & 0 \\
\rho\mathcal{C}_{1\tau_2}^T Y_L^T & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
\rho\mathcal{D}_2^T Y_L^T & 0 \\
0 & 0 \\
k_1\mu_1 P\mathcal{I}_2 & k_1\mathcal{I}_2 M_L
\end{bmatrix},
$$

$$
\Phi_3 = diag\{-\rho X_L - \rho {X_L}^T, -\epsilon_1 I\}, \tag{43}
$$

with

$$
\begin{aligned}
\Phi_{11} &= He(P\mathcal{A} + \mathcal{I}_2 Y_L \mathcal{C}_1) + Q_1 + Q_2 + Q_3 \\
&\quad - Z_1 - Z_2 + \mu_1\tau_M N P^{-1} N^T \\
&\quad + \epsilon_1 \mathcal{C}_1^T {N_L}^T N_L \mathcal{C}_1 \\
&\quad + \epsilon^T (P + \tau_1 Z_1 + \tau_2 Z_2 + \tau_{12} Z_3)\epsilon, \\
\Phi_{12} &= P\mathcal{A}_{\tau_1} + Z_1 + J_1^T + \mathcal{I}_2 Y_L \mathcal{C}_{1\tau_1} \\
&\quad + \epsilon_1 \mathcal{C}_1^T {N_L}^T N_L \mathcal{C}_{1\tau_1} \\
&\quad + \epsilon^T (P + \tau_1 Z_1 + \tau_2 Z_2 + \tau_{12} Z_3)\epsilon_{\tau_1}, \\
\Phi_{13} &= P\mathcal{A}_{\tau_2} + Z_2 + J_2^T + \mathcal{I}_2 Y_L \mathcal{C}_{1\tau_2} \\
&\quad + \epsilon_1 \mathcal{C}_1^T {N_L}^T N_L \mathcal{C}_{1\tau_2} \\
&\quad + \epsilon^T (P + \tau_1 Z_1 + \tau_2 Z_2 + \tau_{12} Z_3)\epsilon_{\tau_2}, \\
\Phi_{22} &= -Q_1 - Z_1 - Z_3 - J_1 - J_1^T + J_3 + J_3^T \\
&\quad + \epsilon_1 \mathcal{C}_{1\tau_1}^T {N_L}^T N_L \mathcal{C}_{1\tau_1} \\
&\quad + \epsilon_{\tau_1}^T (P + \tau_1 Z_1 + \tau_2 Z_2 + \tau_{12} Z_3)\epsilon_{\tau_1}, \\
\Phi_{23} &= Z_3 - J_3 - J_3^T + \mathcal{I}_2 Y_L \mathcal{C}_{1\tau_2} \\
&\quad + \epsilon_1 \mathcal{C}_{1\tau_1}^T {N_L}^T N_L \mathcal{C}_{1\tau_2} \\
&\quad + \epsilon_{\tau_1}^T (P + \tau_1 Z_1 + \tau_2 Z_2 + \tau_{12} Z_3)\epsilon_{\tau_2}, \\
\Phi_{33} &= -Q_2 - Z_2 - Z_3 - J_1 - J_1^T + J_3 + J_3^T \\
&\quad + \epsilon_1 \mathcal{C}_{1\tau_2}^T {N_L}^T N_L \mathcal{C}_{1\tau_2} \\
&\quad + \epsilon_{\tau_2}^T (P + \tau_1 Z_1 + \tau_2 Z_2 + \tau_{12} Z_3)\epsilon_{\tau_2}, \\
\Phi_{14} &= Y + N, \\
\Phi_{44} &= \sigma\Omega + \mu_1\tau_M M P^{-1} M^T - M - M^T, \\
\Phi_{16} &= Y, \\
\Phi_{66} &= -\Omega, \\
\Phi_{1t} &= P\mathcal{D}_1 + \mathcal{I}_2 Y_L \mathcal{D}_2 + \epsilon_1 \mathcal{C}_1^T N_L^T N_L \mathcal{D}_2, \\
\Phi_{2t} &= \epsilon_1 \mathcal{C}_{1\tau_1}^T N_L^T N_L \mathcal{D}_2, \\
\Phi_{3t} &= \epsilon_1 \mathcal{C}_{1\tau_2}^T N_L^T N_L \mathcal{D}_2, \\
\Phi_{tt} &= -\gamma^2 I + \epsilon_1 \mathcal{D}_2^T N_L^T N_L \mathcal{D}_2.
\end{aligned} \tag{44}
$$

In this case, the desired gains were given as

$$
K = B_1^{-1} P^{-1} Y, \quad L = X_L^{-1} Y_L.
$$

Proof: According to Theorem 1 and its derivation process, it is easy to see that if there exist matrices $P > 0$, $R > 0$, $M > 0$, $Q_1 > 0$, $Q_2 > 0$, $Q_3 > 0$, $Z_1 > 0$, $Z_2 > 0$, $Z_3 > 0$, and J_1, J_2, J_3 satisfying (38), at the same time, applying the Schur's complement, we have:

$$
\Xi = \Bigg[\begin{matrix}
\Xi_{11} & \Xi_{12} & \Xi_{13} & \Xi_{14} & -N & \Xi_{16} & Z_1 \\
* & \Xi_{22} & \Xi_{23} & 0 & 0 & 0 & -Z_1-J_1 \\
* & * & \Xi_{33} & 0 & 0 & 0 & 0 \\
* & * & * & \Xi_{44} & 0 & 0 & 0 \\
* & * & * & * & -Q_3 & 0 & 0 \\
* & * & * & * & * & \Xi_{66} & 0 \\
* & * & * & * & * & * & -Z_1-R \\
* & * & * & * & * & * & * \\
* & * & * & * & * & * & * \\
* & * & * & * & * & * & * \\
* & * & * & * & * & * & * \\
* & * & * & * & * & * & *
\end{matrix}
$$

$$
\begin{matrix}
Z_2 & 0 & P\mathscr{B} & \mathscr{C}^T & k_1\mathscr{A}^T R \\
0 & Z_3-J_3 & 0 & \mathscr{C}_{\tau_1}^T & k_1\mathscr{A}_{\tau_1}^T R \\
-Z_2-J_2 & -Z_3+J_3 & 0 & \mathscr{C}_{\tau_2}^T & k_1\mathscr{A}_{\tau_2}^T R \\
0 & 0 & 0 & 0 & k_1\mathscr{A}_{\tau_3}^T R \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & k_1 W^T R \\
0 & 0 & 0 & 0 & 0 \\
-Z_2-R & 0 & 0 & 0 & 0 \\
* & -Z_3-R & 0 & 0 & 0 \\
* & * & -\gamma^2 I & \mathscr{D}^T & 0 \\
* & * & * & -I & k_1\mathscr{D}_1^T R \\
* & * & * & * & -k_1 R
\end{matrix}
\Bigg] < 0, \tag{45}
$$

where

$$
\begin{aligned}
\Xi_{11} &= He(P\mathscr{A}) + Q_1 + Q_2 + Q_3 - Z_1 - Z_2 \\
&\quad + \tau_M N R^{-1} N^T \\
&\quad + \epsilon^T (P + \tau_1 Z_1 + \tau_2 Z_2 + \tau_3 Z_3)\epsilon, \\
\Xi_{12} &= P\mathscr{A}_{\tau_1} + Z_1 + J_1^T \\
&\quad + \epsilon^T (P + \tau_1 Z_1 + \tau_2 Z_2 + \tau_2 Z_3)\epsilon_{\tau_1}, \\
\Xi_{13} &= P\mathscr{A}_{\tau_2} + Z_2 + J_2 \\
&\quad + \epsilon^T (P + \tau_1 Z_1 + \tau_2 Z_2 + \tau_2 Z_3)\epsilon_{\tau_2}, \\
\Xi_{14} &= P\mathscr{A}_{\tau_3} + N, \\
\Xi_{22} &= -Q_1 - Z_1 - Z_3 - J_1 - J_1^T - J_3 - J_3^T
\end{aligned}
$$

$$
\begin{aligned}
&\quad +\epsilon_{\tau_1}^T(P+\tau_1 Z_1+\tau_2 Z_2+\tau_2 Z_3)\epsilon_{\tau_1},\\
\Xi_{23} &= Z_3+J_3+J_3^T\\
&\quad +\epsilon_{\tau_1}^T(P+\tau_1 Z_1+\tau_2 Z_2+\tau_2 Z_3)\epsilon_{\tau_2},\\
\Xi_{33} &= -Q_2-Z_2-Z_3-J_2-J_2^T-J_3-J_3{}^T\\
&\quad +\epsilon_{\tau_2}^T(P+\tau_1 Z_1+\tau_2 Z_2+\tau_2 Z_3)\epsilon_{\tau_2},\\
\Xi_{44} &= \sigma\Omega-M-M^T+\tau_M MR^{-1}M^T,\\
\Xi_{16} &= PW,\\
\Xi_{66} &= -\Omega, \qquad (46)
\end{aligned}
$$

and

$$P\mathcal{I}_2 LW_1=\mathcal{I}_2 Y_L W_1+(-\mathcal{I}_2 X_L+P\mathcal{I}_2)X_L{}^{-1}Y_L W_1, \qquad (47)$$

where

$$W_1=[\mathcal{C}_1,\mathcal{C}_{1\tau_1},\mathcal{C}_{1\tau_2},0,0,0,0,0,0,\mathcal{D}_2,0,0].$$

Then, consider the equation (10), (47), we can rewrite (45) as

$$\Xi+He(U_1X_L^{-1}V_1^T+U_2F_LV_2^T)<0, \qquad (48)$$

and

$$
\Xi=\begin{bmatrix}
\Xi_{11} & \Xi_{12} & \Xi_{13} & \Xi_{14} & -N & \Xi_{16} & Z_1 & Z_2 & 0 & \Xi_{1t} & \mathcal{C}^T & k_1\mathcal{A}^TR\\
* & \Xi_{22} & \Xi_{23} & 0 & 0 & 0 & -Z_1-J_1 & 0 & Z_3-J_3 & 0 & \mathcal{C}_{\tau_1}^T & k_1\mathcal{A}_{\tau_1}^TR\\
* & * & \Xi_{33} & 0 & 0 & 0 & 0 & -Z_2-J_2 & -Z_3+J_3 & 0 & \mathcal{C}_{\tau_2}^T & k_1\mathcal{A}_{\tau_2}^TR\\
* & * & * & \Xi_{44} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & k_1K^T\mathcal{B}_1^TR\\
* & * & * & * & -Q_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
* & * & * & * & * & \Xi_{66} & 0 & 0 & 0 & 0 & 0 & k_1K^T\mathcal{B}_1^TR\\
* & * & * & * & * & * & -Z_1-R & 0 & 0 & 0 & 0 & 0\\
* & * & * & * & * & * & * & -Z_2-R & 0 & 0 & 0 & 0\\
* & * & * & * & * & * & * & * & -Z_3-R & 0 & 0 & 0\\
* & * & * & * & * & * & * & * & * & -\gamma^2I & \mathcal{D}_3^T & 0\\
* & * & * & * & * & * & * & * & * & * & -I & k_1\mathcal{D}_1^TR\\
* & * & * & * & * & * & * & * & * & * & * & -k_1R
\end{bmatrix}, \qquad (49)
$$

where

$$
\begin{aligned}
\Xi_{11} &= He(P\mathcal{A}+\mathcal{I}_2Y_L\mathcal{C}_1)+Q_1+Q_2+Q_3\\
&\quad -Z_1-Z_2+\tau_M NR^{-1}N^T\\
&\quad +\epsilon^T(P+\tau_1 Z_1+\tau_2 Z_2+\tau_{12} Z_3)\epsilon,\\
\Xi_{12} &= P\mathcal{A}_{\tau_1}+\mathcal{I}_2Y_L\mathcal{C}_{1\tau_1}+Z_1+J_1^T\\
&\quad +\epsilon^T(P+\tau_1 Z_1+\tau_2 Z_2+\tau_{12} Z_3)\epsilon_{\tau_1},\\
\Xi_{13} &= P\mathcal{A}_{\tau_2}+\mathcal{I}_2Y_L\mathcal{C}_{1\tau_2}+Z_2+J_2\\
&\quad +\epsilon^T(P+\tau_1 Z_1+\tau_2 Z_2+\tau_{12} Z_3)\epsilon_{\tau_2},\\
\Xi_{22} &= -Q_1-Z_1-Z_3-J_1-J_1^T+J_3+J_3^T\\
&\quad +\epsilon_{\tau_1}^T(P+\tau_1 Z_1+\tau_1 Z_2+\tau_{12} Z_3)\epsilon_{\tau_1},\\
\Xi_{23} &= Z_3-J_3-J_3^T\\
&\quad +\epsilon_{\tau_1}^T(P+\tau_1 Z_1+\tau_2 Z_2+\tau_{12} Z_3)\epsilon_{\tau_2},\\
\Xi_{33} &= -Q_2-Z_2-Z_3-J_2-J_2^T-J_3-J_3^T\\
&\quad +\epsilon_{\tau_2}^T(P+\tau_1 Z_1+\tau_2 Z_2+\tau_{12} Z_3)\epsilon_{\tau_2},\\
\Xi_{14} &= P\mathcal{B}_1K+N,\\
\Xi_{44} &= \sigma\Omega-M-M^T+\tau_M MR^{-1}M^T,\\
\Xi_{16} &= P\mathcal{B}_1K,\\
\Xi_{66} &= -\Omega,\\
\Xi_{1t} &= P\mathcal{D}_1+\mathcal{I}_2Y_L\mathcal{D}_2,\\
U_1 &= [-X_L^T\mathcal{I}_2^T+\mathcal{I}_2^TP,0,0,0,0,0,0,0,0,0,0,k_1\mathcal{I}_2^TR^T],\\
U_2 &= [M_L^T\mathcal{I}_2^TP,0,0,0,0,0,0,0,0,0,0,k_1M_L^T\mathcal{I}_2^TR],\\
V_1 &= W_1^TY_L^T,\\
V_2 &= W_1^TN_L^T. \qquad (50)
\end{aligned}
$$

According to the Lemma 2, for any real scalar $\epsilon > 0$ and any real matrices U and V with appropriate dimensions, we have $UV^T+VU^T \le \epsilon^{-1}UU^T+\epsilon VV^T$. At the same time, for scalar $\rho > 0$, applying Lemma 4, Φ can be rewritten as the following forms:

$$\begin{bmatrix}\Xi+\epsilon_1^{-1}V_1V_1^T+\epsilon_1V_2V_2^T & U_1+\rho V_1\\ * & -\rho X_L-\rho X_L^T\end{bmatrix}<0.$$

By Lemma 3,

$$\Xi+He(U_1X_L^{-1}V_1^T)+\epsilon_1^{-1}V_1V_1^T+\epsilon_1V_2V_2^T<0, \qquad (51)$$

$$He(U_1F_L^{-1}V_1^T)\le\epsilon_1^{-1}U_1U_1^T+\epsilon_1V_1V_1^T. \qquad (52)$$

By (51) and (52), we obtain the (48). In addition, due to the $\mathscr{A}_{\tau_3} = \mathcal{B}_1K$, we set $K = \mathcal{B}_1^{-1}P^{-1}Y$, $R = \mu_1P$. Then we have $P\mathscr{A}_{\tau_3} = P\mathcal{B}_1\mathcal{B}_1^{-1}P^{-1}Y$. Next, we replace the relevant parameters of Ξ with them, and then we can obtain (42), (43), (44). At last, we can get the Theorem 3. Then the proof is completed. □

VI. NUMERICAL EXAMPLES

Example 1: According to the Theorem 1, under the event-triggered scheme, we obtained a new delay-dependent stability criteria for the stochastic network control system with multiple time delays. To demonstrate this point, we consider a system of the form (9) *with parameters:*

$$\mathscr{A}=\begin{bmatrix}-5 & 0\\ 0 & -5\end{bmatrix},\quad \mathscr{A}_{\tau_1}=\mathscr{A}_{\tau_2}=\mathscr{A}_{\tau_3}=\begin{bmatrix}-1 & 0\\ -0.5 & -1\end{bmatrix},$$

$$\epsilon=\begin{bmatrix}-0.5 & 0.2\\ 0 & 0.3\end{bmatrix},\quad \epsilon_{\tau_1}=\epsilon_{\tau_2}=\begin{bmatrix}-0.5 & 0.3\\ 0 & -0.5\end{bmatrix}.$$

Now, we assume the system state time delay $\tau_1>\tau_2$, set $\sigma = 0.3$, at the same time, fixed the time delay $\tau_1 = 1.1018$, $\tau_2 = 0.2155$, the upper bound of network transmission delay τ_M is given in Table 1: Paper [22] and [38] only considered the network transmission delay without considering the delay

TABLE 1. The upper bound of τ_M.

Method	upper bound τ_M
Theorem 1 setτ_1=1.1018, τ_2=0.2155,$\sigma = 0.3$	0.7045
Theorem 1 setτ_1=0, τ_2=0,$\sigma = 0.3$	0.5021
Theorem 1 of [22]	0.08
Collary 2 of [38]	0.22

in controlled object. From the Table 1, under the $\sigma = 0.3$, we can see that the $\tau_M = 0.08$ in paper [22] and the $\tau_M = 0.22$ in paper [38]. By comparison, the τ_M obtained in this paper is bigger than them and $\tau_M = 0.7045$.

Example 2: This example concerns the state estimation-based event-triggered H_∞ control for multi-delay stochastic network control system with the following parameters:

$$A_1 = \begin{bmatrix} 0 & 0 \\ 0 & -0.5 \end{bmatrix}, \quad A_{\tau_1} = A_{\tau_2} = \begin{bmatrix} -1 & -0.2 \\ 0 & -1 \end{bmatrix},$$
$$B_1 = \begin{bmatrix} 0 & -0.5 \end{bmatrix}^T, \quad B_2 = 0.5,$$
$$C_1 = \begin{bmatrix} 0.5 & 0.5 \end{bmatrix},$$
$$C_{1\tau_1} = C_{1\tau_2} = \begin{bmatrix} -0.1 & -0.3 \end{bmatrix},$$
$$C_2 = \begin{bmatrix} 0 & -0.7 \end{bmatrix},$$
$$C_{2\tau_1} = \begin{bmatrix} -0.2 & -0.2 \end{bmatrix}, \quad C_{2\tau_2} = \begin{bmatrix} -0.7 & -0.7 \end{bmatrix},$$
$$D_1 = \begin{bmatrix} -0.2 & 1.2 \end{bmatrix},$$
$$D_2 = 0.9, \quad D_3 = 1, \; E = E_{\tau_1} = E_{\tau_2} = \begin{bmatrix} -0.2 & 0 \\ 0 & -0.2 \end{bmatrix},$$
$$M_K = 0.2, N_K = \begin{bmatrix} 0.1 & 0.6 \end{bmatrix},$$
$$M_L = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}, \quad N_L = 0.3,$$

when we assume the scalar $\tau_M = 0.7$, $\tau_1 = 0.3$, $\tau_2 = 0.2$, $\rho = 0.2$, $\gamma = 1.8640$, and $v(t) = \frac{1}{1+t}e^{-t}$, by the way, set state estimator perturbation $\Delta L = 0.5 M_L N_L$. According to the Theorem 3, the state feedback controller gain is obtained:

$$K = \begin{bmatrix} 0.5194 & -0.2006 \\ 4.3198 & 7.4127 \end{bmatrix}.$$

The state response of the closed-loop stochastic network control is given in Fig 2. It is easy to see that the system will achieve a steady state in 8 seconds.

From Example 1, we get $\tau_M = 0.7$, $\tau_M = h + \bar{\tau}$. Assume $\bar{\tau} = 0$, the maximum sampling period is $h = 0.7$. Choose the $h = 0.26$. The event-based release instants and release interval of the system are shown in Fig 3. From the Fig 3, we find that only 46 samples need to be sent to the controller. This only accounts for 59.7% of all sampling points. This percentage is larger than the one in [22]. In addition, we fix the $\sigma = 0.3$, just change sampling period h, we find the number of triggers increases as h increases. Due to $\tau_M = h + \bar{\tau}$ and we assume $\bar{\tau} = 0$ is unchanged, thus, the upper bound of h is depended on the τ_M. Combined with Example 1, our τ_M is bigger than other papers, which means that this paper is more effective in reducing the burden of communication bandwidth.

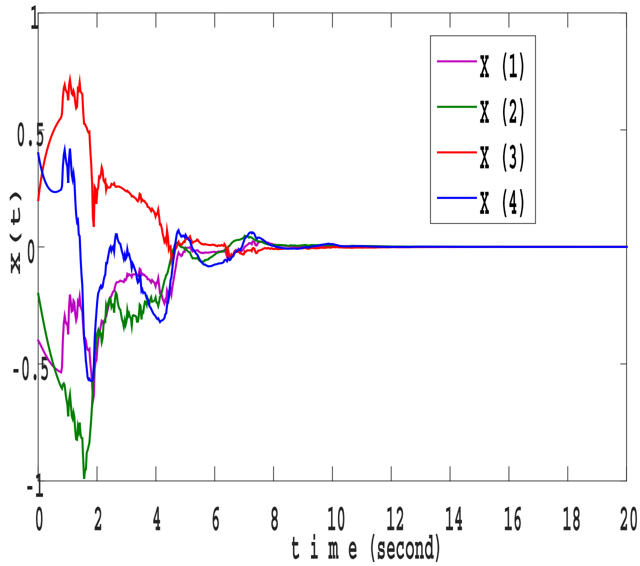

FIGURE 2. State response of the closed-loop stochastic network control system.

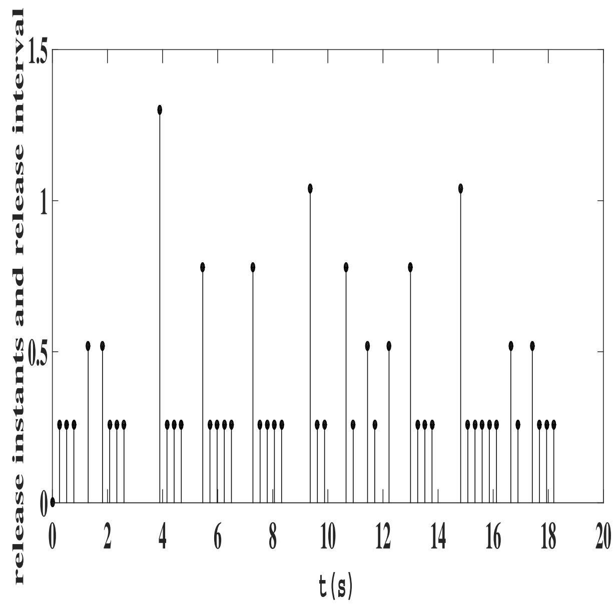

FIGURE 3. release instants and release interval for h=0.26.

VII. CONCLUSION

In this note, we handled with the H_∞ stability and the stabilization of the stochastic network control system with multiple time delays. The delays in this paper not only include the network transmission delay. We also took the delay in the controlled object which is multiple time delays into account. We employed the event-triggered mechanism to reduce the burden of network channel and decrease the occupation of sensor, controller and actuator. In addition, we designed an estimator to better control the system. The less conservative H_∞ stability criteria has been received by using both FWM method and integral inequality method. We designed a controller and solved the controller gain K with the help of LMI toolbox. Finally, the numerical examples were shown to demonstrate the advantages of this paper. Further research topics can be focused on communication protocol issues for the stochastic network control system.

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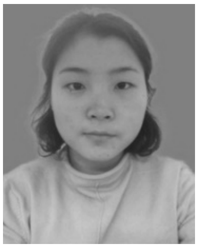

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