Received October 24, 2018, accepted November 12, 2018, date of publication November 16, 2018,
date of current version December 18, 2018.

Digital Object Identifier 10.1109/ACCESS.2018.2881684

A Structure-Aware DOA Estimation Method for Sources With Short Known Waveforms

YANG-YANG DONG[1], CHUNXI DONG[1], WEI LIU[2], (Senior Member, IEEE), MING-MING LIU[1], AND ZHENGZHAO TANG[1]
[1]School of Electronic Engineering, Xidian University, Xi'an 710071, China
[2]Department of Electronic and Electrical Engineering, University of Sheffield, Sheffield S1 4ET, U.K.

Corresponding author: Yang-Yang Dong (dongyangyang2104@126.com)

This work was supported in part by the Natural Science Basic Research Project of Shaanxi Province under Grant 2018JQ6046, in part by the China Postdoctoral Science Foundation under Grant 2017M623123, and in part by the Fundamental Research Funds for the Central Universities of China under Grant XJS18033.

ABSTRACT For direction of arrival (DOA) estimation with known waveform information, with a larger number of snapshots and a longer duration of the known waveforms, the required storage space for hardware implementation will increase. To save storage resources and also reduce the response time of the array system, the DOA estimation problem with short known waveforms in snapshot size is studied in this paper. We first establish the connection between DOA estimation for known waveform sources and the structured least squares (SLS) technique utilizing a potential rotation invariant property. Next, a constraint is formulated and then transformed into its real and imaginary parts to satisfy the requirements of SLS, and the resultant SLS optimization problem is solved iteratively. Simulation results show that the proposed method has a better performance than the existing DEML method for small numbers of snapshots.

INDEX TERMS Direction of arrival estimation, known waveform, short snapshot size, structured least squares.

I. INTRODUCTION

Estimating the direction of arrival (DOA) of multiple emitting sources is a fundamental problem in array signal processing, and has been applied to a variety of areas, such as wireless communications, radar, sonar, and electronic reconnaissance, etc [1], [2]. Many methods have been developed for DOA estimation, such as multiple signal classification (MUSIC) [3], estimation of signal parameters via rotational invariance technique (ESPRIT) [4], and the propagator method (PM) [5]. However, these methods only exploit statistical properties of the received array data, even if the signal waveform information can be acquired in some practical scenarios [2], [6], [7]. By incorporating the prior information of signal waveforms, a better performance could be achieved, and there has been an increasing interest in studying the DOA estimation problem for known waveform sources [8]–[19]. According to the accuracy in knowledge about the waveform, they can be classified into two major classes: the first one, which is also the main focus of research, can handle sources with exact waveform information, and this type can be further divided into two subclasses, one for uncorrelated sources, such as DEML [9], SB [10], and LR [13], and the other for coherent sources in the presence of multipath, such as CDEML [15], WCDEML [16], PADEC [17], and LP [18]; the second class, which is a recent development, is for sources with inaccurate waveform information. One example for the latter is the work in [19], which deals with the known waveforms in the presence of unknown Doppler shifts, suitable for applications with fast moving platforms.

Since the known waveforms need to be stored in the system, the more and longer the known waveforms, the larger the required storage space for hardware implementation. To save storage resources and also reduce the response time of the array system, in this work, we focus on the DOA estimation problem with short known waveforms in snapshot size. For the conventional DEML method [9], it is derived from the maximum likelihood (ML) method and designed for a large number of samples. When the available number of snapshots is limited and small, the magnitude of cross-correlation between the source signal vector and noise vector cannot be ignored any more in comparison with that of noise covariance, which results in errors in the estimates of array manifold matrix and noise covariance matrix, and in turn a large bias in its angle and complex amplitude estimation performance.

To solve the problem, inspired by the work of structured least squares (SLS) based ESPRIT in [20], we first treat the cross-correlation between the source and noise vectors as a variable to be determined and establish a link between DOA estimation incorporating known waveform information and SLS utilizing its potential rotation invariant property. Then, to make use of the structure information, an amplitude constraint on the vector elements, consisting of the rotation invariance factor, is developed. However, the constraint involves a conjugate operator, which is nonlinear and the SLS technique cannot handle it directly. As a solution, we divide the complex matrix equation into its real and imaginary parts and form more constrains to create an over-determined optimization problem, which is then solved via the iterative least squares method. Finally, the DOAs and complex amplitudes are obtained by examining the inherent relationship of the matrix estimated from the SLS optimization step. Simulation results show that the proposed method has a better estimation performance than the DEML method for a small number of snapshots.

The rest of the paper is organised as follows. In Section 2, the studied signal model is introduced. The proposed estimation method is derived and analyzed in Sections 3 and 4, respectively. In Section 5, simulation results are provided for performance comparison. Finally, conclusions are drawn in Section 6.

Notations: matrices and vectors are denoted by boldfaced capital letters and lower-case letters, respectively. The notations used in this paper are listed below:

$(\cdot)^*$	conjugate
$(\cdot)^T$	transpose
$(\cdot)^H$	conjugate transpose
$(\cdot)^{-1}$	inverse
$(\cdot)^\dagger$	Moore-Penrose inverse
$\|\cdot\|_F$	Frobenius norm
$\|\cdot\|_2$	ℓ_2 norm
$\circ$	Hadamard product
$\otimes$	Kronecker product
$\odot$	Khatri-Rao product
$\mathrm{diag}\{\cdot\}$	diagonalization
$\mathrm{vec}\{\cdot\}$	vectorization
$\mathrm{Re}\{\cdot\}$	real part of a complex number
$\mathrm{Im}\{\cdot\}$	imaginary part of a complex number
$\mathbf{I}_M$	identity matrix of size M
$\mathbf{0}_{MN}$	zero matrix of size $M \times N$.

II. SIGNAL MODEL

Consider a uniform linear array (ULA) consisting of M sensors with an inter-sensor spacing d. Q narrowband far-field uncorrelated sources with known waveforms $\{s_q(n)\}_{q=1}^{Q}$ ($n = 1, \cdots, N$, with N being the number of snapshots) of wavelength λ from unknown directions $\{\theta_q\}_{q=1}^{Q}$ impinge on the array. The received signal vector at the nth snapshot can be expressed as

$$\begin{aligned}\mathbf{x}(n) &= \mathbf{A}(\boldsymbol{\theta})\boldsymbol{\Gamma}(\boldsymbol{\gamma})\mathbf{s}(n) + \mathbf{w}(n)\\ &= \mathbf{B}(\boldsymbol{\theta}, \boldsymbol{\gamma})\mathbf{s}(n) + \mathbf{w}(n)\end{aligned} \tag{1}$$

where

$$\begin{aligned}\mathbf{x}(n) &= [x_1(n), x_2(n), \cdots, x_M(n)]^T,\\ \mathbf{A}(\boldsymbol{\theta}) &= [\mathbf{a}(\theta_1), \mathbf{a}(\theta_2), \cdots, \mathbf{a}(\theta_Q)],\\ \boldsymbol{\Gamma}(\boldsymbol{\gamma}) &= \mathrm{diag}\{\gamma_1, \gamma_2, \cdots, \gamma_Q\},\\ \mathbf{a}(\theta_q) &= [1, e^{-j2\pi d \sin\theta_q/\lambda}, \cdots, e^{-j2\pi(M-1)d \sin\theta_q/\lambda}]^T,\\ \boldsymbol{\theta} &= [\theta_1, \theta_2, \cdots, \theta_Q]^T,\\ \boldsymbol{\gamma} &= [\gamma_1, \gamma_2, \cdots, \gamma_Q]^T,\\ \mathbf{s}(n) &= [s_1(n), s_2(n), \cdots, s_Q(n)]^T,\\ \mathbf{w}(n) &= [w_1(n), w_2(n), \cdots, w_M(n)]^T.\end{aligned}$$

Similar to [9], it is assumed that the additive noise vector $\mathbf{w}(n)$ is temporally and spatially white and uncorrelated with the signal vector $\mathbf{s}(n)$, and $\mathbf{w}(n)$ follows the circularly-symmetric complex jointly-Gaussian distribution, i.e., $\mathbf{w}(n) \sim CN(\mathbf{0}, \sigma_w^2\mathbf{I}_M)$. Besides, without causing confusion and for simplicity, $\mathbf{B}$, $\mathbf{A}$, and $\boldsymbol{\Gamma}$ are used to denote $\mathbf{B}(\boldsymbol{\theta}, \boldsymbol{\gamma})$, $\mathbf{A}(\boldsymbol{\theta})$, and $\boldsymbol{\Gamma}(\boldsymbol{\gamma})$, respectively.

III. PROPOSED METHOD

With the aid of known waveform $\mathbf{s}(n)$, similar to [9], we first obtain the estimate of $\mathbf{B}$ as follows,

$$\mathbf{B}_0 = \mathbf{R}_{xs}\mathbf{R}_{ss}^{-1} = \mathbf{B} + \mathbf{R}_{ws}\mathbf{R}_{ss}^{-1} \tag{2}$$

where

$$\begin{aligned}\mathbf{R}_{xs} &= \sum\nolimits_{n=1}^{N} \mathbf{x}(n)\mathbf{s}^H(n)/N,\\ \mathbf{R}_{ss} &= \sum\nolimits_{n=1}^{N} \mathbf{s}(n)\mathbf{s}^H(n)/N,\\ \mathbf{R}_{ws} &= \sum\nolimits_{n=1}^{N} \mathbf{w}(n)\mathbf{s}^H(n)/N.\end{aligned}$$

Note that if N is very large, the second term $\mathbf{R}_{ws}\mathbf{R}_{ss}^{-1}$ tends to zero, and $\mathbf{B}_0$ can be a good estimate of $\mathbf{B}$. However, for a small N, $\mathbf{R}_{ws}\mathbf{R}_{ss}^{-1}$ may be too large and cannot be ignored [21]. A good choice is to make use of the potential structural information of $\mathbf{B}$. Fortunately, the structured least squares (SLS) technique has been proved to be very effective in handling similar problems [20], [22]–[28]. Following the core idea of SLS-ESPRIT [20], we formulate a similar optimization problem with the relationship

$$\mathbf{J}_1\mathbf{B} \cdot \mathrm{diag}\{\boldsymbol{\phi}\} = \mathbf{J}_2\mathbf{B} \tag{3}$$

where $\mathbf{J}_1 = [\mathbf{I}_{M-1}, \mathbf{0}_{(M-1)\times 1}]$, $\mathbf{J}_2 = [\mathbf{0}_{(M-1)\times 1}, \mathbf{I}_{M-1}]$, $\boldsymbol{\phi} = [\phi_1, \phi_2, \cdots, \phi_Q]^T$, and $\phi_q = e^{-j2\pi d \sin\theta_q/\lambda}$.

However, there are two distinct features in the structural information of $\mathbf{B}$: (i) $\mathbf{B}$ is different from the signal subspace matrix $\mathbf{U}_s$ in [20], which makes the rotation invariance matrix in (3) diagonal, i.e, $\mathrm{diag}\{\boldsymbol{\phi}\}$; (ii) the norm of each element in vector $\boldsymbol{\phi}$ should be one, i.e.,

$$\boldsymbol{\phi} \circ \boldsymbol{\phi}^* = \mathbf{1}_{Q\times 1}. \tag{4}$$

Since the conjugate operation is a non-linear operator, i.e., (4) cannot be applied to the SLS technique directly. A simple solution is to split the complex matrix into its real and imaginary parts as

$$\mathbf{J}_1(\mathbf{B}^{\Re} \cdot \mathrm{diag}\{\boldsymbol{\phi}^{\Re}\} - \mathbf{B}^{\Im} \cdot \mathrm{diag}\{\boldsymbol{\phi}^{\Im}\}) = \mathbf{J}_2\mathbf{B}^{\Re} \tag{5}$$

$$\mathbf{J}_1(\mathbf{B}^{\Re} \cdot \mathrm{diag}\{\boldsymbol{\phi}^{\Im}\} + \mathbf{B}^{\Im} \cdot \mathrm{diag}\{\boldsymbol{\phi}^{\Re}\}) = \mathbf{J}_2\mathbf{B}^{\Im} \tag{6}$$

$$\boldsymbol{\phi}^{\Re} \circ \boldsymbol{\phi}^{\Re} + \boldsymbol{\phi}^{\Im} \circ \boldsymbol{\phi}^{\Im} = \mathbf{1}_{Q\times 1} \tag{7}$$

where $\boldsymbol{\phi}^{\Re} = \mathrm{Re}\{\boldsymbol{\phi}\}$, $\boldsymbol{\phi}^{\Im} = \mathrm{Im}\{\boldsymbol{\phi}\}$, $\mathbf{B}^{\Re} = \mathrm{Re}\{\mathbf{B}\}$, and $\mathbf{B}^{\Im} = \mathrm{Im}\{\mathbf{B}\}$.

From (5)-(7), it can be seen that the number of unknown variables is larger than that of equations, which is underdetermined. Thanks to (3)-(4) and the non-linear property of conjugate operator, we also have the following expression

$$\mathbf{J}_2\mathbf{B} \cdot \mathrm{diag}\{\boldsymbol{\phi}^*\} = \mathbf{J}_1\mathbf{B} \tag{8}$$

Similar to (5)-(6), (8) can be split into two equations as follows,

$$\mathbf{J}_2(\mathbf{B}^{\Re} \cdot \mathrm{diag}\{\boldsymbol{\phi}^{\Re}\} + \mathbf{B}^{\Im} \cdot \mathrm{diag}\{\boldsymbol{\phi}^{\Im}\}) = \mathbf{J}_1\mathbf{B}^{\Re} \tag{9}$$

$$\mathbf{J}_2(\mathbf{B}^{\Im} \cdot \mathrm{diag}\{\boldsymbol{\phi}^{\Re}\} - \mathbf{B}^{\Re} \cdot \mathrm{diag}\{\boldsymbol{\phi}^{\Im}\}) = \mathbf{J}_1\mathbf{B}^{\Im} \tag{10}$$

Since $\mathbf{B}_0$ in (2) can only provide a rough estimate of $\mathbf{B}$, its accuracy needs to be improved. Besides, there are some other approaches which can give a similar rough estimate, and for simplicity, we denote the rough estimate of $\mathbf{B}$ as $\hat{\mathbf{B}}$ and assume that there exists a small perturbation $\Delta\hat{\mathbf{B}}$ between them; therefore, an improved estimate can be expressed as $\tilde{\mathbf{B}} = \hat{\mathbf{B}} + \Delta\hat{\mathbf{B}}$. Similarly, define the improved estimate of $\boldsymbol{\phi}$ as $\tilde{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}} + \Delta\hat{\boldsymbol{\phi}}$. Recalling (5)-(7) and (9)-(10), we have the following five residual matrices:

$$\begin{aligned}\mathbf{F}_1(\tilde{\mathbf{B}}^{\Re}, \tilde{\mathbf{B}}^{\Im}, \tilde{\boldsymbol{\phi}}^{\Re}, \tilde{\boldsymbol{\phi}}^{\Im}) = {} & \mathbf{J}_1\tilde{\mathbf{B}}^{\Re} \cdot \mathrm{diag}\{\tilde{\boldsymbol{\phi}}^{\Re}\} - \mathbf{J}_2\tilde{\mathbf{B}}^{\Re} \\ & - \mathbf{J}_1\tilde{\mathbf{B}}^{\Im} \cdot \mathrm{diag}\{\tilde{\boldsymbol{\phi}}^{\Im}\}\end{aligned} \tag{11}$$

$$\begin{aligned}\mathbf{F}_2(\tilde{\mathbf{B}}^{\Re}, \tilde{\mathbf{B}}^{\Im}, \tilde{\boldsymbol{\phi}}^{\Re}, \tilde{\boldsymbol{\phi}}^{\Im}) = {} & \mathbf{J}_1\tilde{\mathbf{B}}^{\Re} \cdot \mathrm{diag}\{\tilde{\boldsymbol{\phi}}^{\Im}\} - \mathbf{J}_2\tilde{\mathbf{B}}^{\Im} \\ & + \mathbf{J}_1\tilde{\mathbf{B}}^{\Im} \cdot \mathrm{diag}\{\tilde{\boldsymbol{\phi}}^{\Re}\}\end{aligned} \tag{12}$$

$$\begin{aligned}\mathbf{F}_3(\tilde{\mathbf{B}}^{\Re}, \tilde{\mathbf{B}}^{\Im}, \tilde{\boldsymbol{\phi}}^{\Re}, \tilde{\boldsymbol{\phi}}^{\Im}) = {} & \mathbf{J}_2\tilde{\mathbf{B}}^{\Re} \cdot \mathrm{diag}\{\tilde{\boldsymbol{\phi}}^{\Re}\} - \mathbf{J}_1\tilde{\mathbf{B}}^{\Re} \\ & + \mathbf{J}_2\tilde{\mathbf{B}}^{\Im} \cdot \mathrm{diag}\{\tilde{\boldsymbol{\phi}}^{\Im}\}\end{aligned} \tag{13}$$

$$\begin{aligned}\mathbf{F}_4(\tilde{\mathbf{B}}^{\Re}, \tilde{\mathbf{B}}^{\Im}, \tilde{\boldsymbol{\phi}}^{\Re}, \tilde{\boldsymbol{\phi}}^{\Im}) = {} & \mathbf{J}_2\tilde{\mathbf{B}}^{\Im} \cdot \mathrm{diag}\{\tilde{\boldsymbol{\phi}}^{\Re}\} - \mathbf{J}_1\tilde{\mathbf{B}}^{\Im} \\ & - \mathbf{J}_2\tilde{\mathbf{B}}^{\Re} \cdot \mathrm{diag}\{\tilde{\boldsymbol{\phi}}^{\Im}\}\end{aligned} \tag{14}$$

$$\mathbf{F}_5(\tilde{\boldsymbol{\phi}}^{\Re}, \tilde{\boldsymbol{\phi}}^{\Im}) = \tilde{\boldsymbol{\phi}}^{\Re} \circ \tilde{\boldsymbol{\phi}}^{\Re} + \tilde{\boldsymbol{\phi}}^{\Im} \circ \tilde{\boldsymbol{\phi}}^{\Im} - \mathbf{1}_{Q\times 1} \tag{15}$$

With the principle of SLS, we should minimize the Frobenius norm of the five residual matrices and keep the Frobenius norm of $\Delta\hat{\mathbf{B}}$ as small as possible. Therefore, we arrive at the following optimization problem

$$\min_{\substack{\Delta\hat{\mathbf{B}}^{\Re}, \Delta\hat{\mathbf{B}}^{\Im} \\ \tilde{\boldsymbol{\phi}}^{\Re}, \tilde{\boldsymbol{\phi}}^{\Im}}} \left\| \begin{bmatrix} \mathbf{F}_1(\tilde{\mathbf{B}}^{\Re}, \tilde{\mathbf{B}}^{\Im}, \tilde{\boldsymbol{\phi}}^{\Re}, \tilde{\boldsymbol{\phi}}^{\Im}) \\ \mathbf{F}_2(\tilde{\mathbf{B}}^{\Re}, \tilde{\mathbf{B}}^{\Im}, \tilde{\boldsymbol{\phi}}^{\Re}, \tilde{\boldsymbol{\phi}}^{\Im}) \\ \mathbf{F}_3(\tilde{\mathbf{B}}^{\Re}, \tilde{\mathbf{B}}^{\Im}, \tilde{\boldsymbol{\phi}}^{\Re}, \tilde{\boldsymbol{\phi}}^{\Im}) \\ \mathbf{F}_4(\tilde{\mathbf{B}}^{\Re}, \tilde{\mathbf{B}}^{\Im}, \tilde{\boldsymbol{\phi}}^{\Re}, \tilde{\boldsymbol{\phi}}^{\Im}) \\ \mathbf{F}_5(\tilde{\boldsymbol{\phi}}^{\Re}, \tilde{\boldsymbol{\phi}}^{\Im}) \\ \kappa \cdot \Delta\hat{\mathbf{B}}^{\Re} \\ \kappa \cdot \Delta\hat{\mathbf{B}}^{\Im} \end{bmatrix} \right\|_F \tag{16}$$

where $\Delta\hat{\mathbf{B}}^{\Re} = \mathrm{Re}\{\Delta\hat{\mathbf{B}}\}$, $\Delta\hat{\mathbf{B}}^{\Im} = \mathrm{Im}\{\Delta\hat{\mathbf{B}}\}$. $\kappa = \sqrt{(5M-3)/(\alpha M)}$ is a normalization factor to make the minimization of $\Delta\hat{\mathbf{B}}^{\Re}$ and $\Delta\hat{\mathbf{B}}^{\Im}$ independent of the block matrix size, and $\alpha > 1$[1].

According to [20], the optimization in (16) can be solved efficiently in an iterative fashion using the first order linear approximation technique. Assume that the improved estimates of $\tilde{\mathbf{B}}$ and $\tilde{\boldsymbol{\phi}}$ between the $(k+1)$th and the kth iterations can be expressed as $\tilde{\mathbf{B}}^{\Re}_{k+1} = \tilde{\mathbf{B}}^{\Re}_k + \Delta\hat{\mathbf{B}}^{\Re}_k$, $\tilde{\mathbf{B}}^{\Im}_{k+1} = \tilde{\mathbf{B}}^{\Im}_k + \Delta\hat{\mathbf{B}}^{\Im}_k$, $\tilde{\boldsymbol{\phi}}^{\Re}_{k+1} = \tilde{\boldsymbol{\phi}}^{\Re}_k + \Delta\hat{\boldsymbol{\phi}}^{\Re}_k$, and $\tilde{\boldsymbol{\phi}}^{\Im}_{k+1} = \tilde{\boldsymbol{\phi}}^{\Im}_k + \Delta\hat{\boldsymbol{\phi}}^{\Im}_k$, and hence, we can derive the relationship of each residual matrix in (11)-(15), between its $(k+1)$th and kth iterations as follows,

$$\begin{aligned}&\mathbf{F}_1(\tilde{\mathbf{B}}^{\Re}_{k+1}, \tilde{\mathbf{B}}^{\Im}_{k+1}, \boldsymbol{\phi}^{\Re}_{k+1}, \boldsymbol{\phi}^{\Im}_{k+1}) \\ &\quad = \mathbf{F}_1(\tilde{\mathbf{B}}^{\Re}_k + \Delta\hat{\mathbf{B}}^{\Re}_k, \tilde{\mathbf{B}}^{\Im}_k + \Delta\hat{\mathbf{B}}^{\Im}_k, \tilde{\boldsymbol{\phi}}^{\Re}_k + \Delta\hat{\boldsymbol{\phi}}^{\Re}_k, \tilde{\boldsymbol{\phi}}^{\Im}_k + \Delta\hat{\boldsymbol{\phi}}^{\Im}_k) \\ &\quad = \mathbf{J}_1(\tilde{\mathbf{B}}^{\Re}_k + \Delta\hat{\mathbf{B}}^{\Re}_k) \cdot \mathrm{diag}\{\tilde{\boldsymbol{\phi}}^{\Re}_k + \Delta\hat{\boldsymbol{\phi}}^{\Re}_k\} - \mathbf{J}_1(\tilde{\mathbf{B}}^{\Im}_k \\ &\qquad + \Delta\hat{\mathbf{B}}^{\Im}_k) \cdot \mathrm{diag}\{\tilde{\boldsymbol{\phi}}^{\Im}_k + \Delta\hat{\boldsymbol{\phi}}^{\Im}_k\} - \mathbf{J}_2(\tilde{\mathbf{B}}^{\Re}_k + \Delta\hat{\mathbf{B}}^{\Re}_k) \\ &\quad \approx \mathbf{F}_1(\tilde{\mathbf{B}}^{\Re}_k, \tilde{\mathbf{B}}^{\Im}_k, \tilde{\boldsymbol{\phi}}^{\Re}_k, \tilde{\boldsymbol{\phi}}^{\Im}_k) + \mathbf{J}_1\tilde{\mathbf{B}}^{\Re}_k \cdot \mathrm{diag}\{\Delta\hat{\boldsymbol{\phi}}^{\Re}_k\} \\ &\qquad - \mathbf{J}_1\tilde{\mathbf{B}}^{\Im}_k \cdot \mathrm{diag}\{\Delta\hat{\boldsymbol{\phi}}^{\Im}_k\} + \mathbf{J}_1\Delta\hat{\mathbf{B}}^{\Re}_k \cdot \mathrm{diag}\{\tilde{\boldsymbol{\phi}}^{\Re}_k\} \\ &\qquad - \mathbf{J}_1\Delta\hat{\mathbf{B}}^{\Im}_k \cdot \mathrm{diag}\{\tilde{\boldsymbol{\phi}}^{\Im}_k\} - \mathbf{J}_2\Delta\hat{\mathbf{B}}^{\Re}_k\end{aligned} \tag{17}$$

$$\begin{aligned}&\mathbf{F}_2(\tilde{\mathbf{B}}^{\Re}_{k+1}, \tilde{\mathbf{B}}^{\Im}_{k+1}, \tilde{\boldsymbol{\phi}}^{\Re}_{k+1}, \tilde{\boldsymbol{\phi}}^{\Im}_{k+1}) \\ &\quad = \mathbf{F}_2(\tilde{\mathbf{B}}^{\Re}_k + \Delta\hat{\mathbf{B}}^{\Re}_k, \tilde{\mathbf{B}}^{\Im}_k + \Delta\hat{\mathbf{B}}^{\Im}_k, \tilde{\boldsymbol{\phi}}^{\Re}_k + \Delta\hat{\boldsymbol{\phi}}^{\Re}_k, \tilde{\boldsymbol{\phi}}^{\Im}_k + \Delta\hat{\boldsymbol{\phi}}^{\Im}_k) \\ &\quad = \mathbf{J}_1(\tilde{\mathbf{B}}^{\Re}_k + \Delta\hat{\mathbf{B}}^{\Re}_k) \cdot \mathrm{diag}\{\tilde{\boldsymbol{\phi}}^{\Im}_k + \Delta\hat{\boldsymbol{\phi}}^{\Im}_k\} + \mathbf{J}_1(\tilde{\mathbf{B}}^{\Im}_k \\ &\qquad + \Delta\hat{\mathbf{B}}^{\Im}_k) \cdot \mathrm{diag}\{\tilde{\boldsymbol{\phi}}^{\Re}_k + \Delta\hat{\boldsymbol{\phi}}^{\Re}_k\} - \mathbf{J}_2(\tilde{\mathbf{B}}^{\Im}_k + \Delta\hat{\mathbf{B}}^{\Im}_k) \\ &\quad \approx \mathbf{F}_2(\tilde{\mathbf{B}}^{\Re}_k, \tilde{\mathbf{B}}^{\Im}_k, \tilde{\boldsymbol{\phi}}^{\Re}_k, \tilde{\boldsymbol{\phi}}^{\Im}_k) + \mathbf{J}_1\tilde{\mathbf{B}}^{\Im}_k \cdot \mathrm{diag}\{\Delta\hat{\boldsymbol{\phi}}^{\Re}_k\} \\ &\qquad + \mathbf{J}_1\tilde{\mathbf{B}}^{\Re}_k \cdot \mathrm{diag}\{\Delta\hat{\boldsymbol{\phi}}^{\Im}_k\} + \mathbf{J}_1\Delta\hat{\mathbf{B}}^{\Re}_k \cdot \mathrm{diag}\{\tilde{\boldsymbol{\phi}}^{\Im}_k\} \\ &\qquad + \mathbf{J}_1\Delta\hat{\mathbf{B}}^{\Im}_k \cdot \mathrm{diag}\{\tilde{\boldsymbol{\phi}}^{\Re}_k\}\end{aligned} \tag{18}$$

$$\begin{aligned}&\mathbf{F}_3(\tilde{\mathbf{B}}^{\Re}_{k+1}, \tilde{\mathbf{B}}^{\Im}_{k+1}, \tilde{\boldsymbol{\phi}}^{\Re}_{k+1}, \tilde{\boldsymbol{\phi}}^{\Im}_{k+1}) \\ &\quad = \mathbf{F}_3(\tilde{\mathbf{B}}^{\Re}_k + \Delta\hat{\mathbf{B}}^{\Re}_k, \tilde{\mathbf{B}}^{\Im}_k + \Delta\hat{\mathbf{B}}^{\Im}_k, \tilde{\boldsymbol{\phi}}^{\Re}_k + \Delta\hat{\boldsymbol{\phi}}^{\Re}_k, \tilde{\boldsymbol{\phi}}^{\Im}_k + \Delta\hat{\boldsymbol{\phi}}^{\Im}_k) \\ &\quad = \mathbf{J}_2(\tilde{\mathbf{B}}^{\Re}_k + \Delta\hat{\mathbf{B}}^{\Re}_k) \cdot \mathrm{diag}\{\tilde{\boldsymbol{\phi}}^{\Re}_k + \Delta\hat{\boldsymbol{\phi}}^{\Re}_k\} + \mathbf{J}_2(\tilde{\mathbf{B}}^{\Im}_k \\ &\qquad + \Delta\hat{\mathbf{B}}^{\Im}_k) \cdot \mathrm{diag}\{\tilde{\boldsymbol{\phi}}^{\Im}_k + \Delta\hat{\boldsymbol{\phi}}^{\Im}_k\}) - \mathbf{J}_1(\tilde{\mathbf{B}}^{\Re}_k + \Delta\hat{\mathbf{B}}^{\Re}_k) \\ &\quad \approx \mathbf{F}_3(\tilde{\mathbf{B}}^{\Re}_k, \tilde{\mathbf{B}}^{\Im}_k, \tilde{\boldsymbol{\phi}}^{\Re}_k, \tilde{\boldsymbol{\phi}}^{\Im}_k) + \mathbf{J}_2\tilde{\mathbf{B}}^{\Re}_k \cdot \mathrm{diag}\{\Delta\hat{\boldsymbol{\phi}}^{\Re}_k\} \\ &\qquad + \mathbf{J}_2\tilde{\mathbf{B}}^{\Im}_k \cdot \mathrm{diag}\{\Delta\hat{\boldsymbol{\phi}}^{\Im}_k\} + \mathbf{J}_2\Delta\hat{\mathbf{B}}^{\Re}_k \cdot \mathrm{diag}\{\tilde{\boldsymbol{\phi}}^{\Re}_k\} \\ &\qquad + \mathbf{J}_2\Delta\hat{\mathbf{B}}^{\Im}_k \cdot \mathrm{diag}\{\tilde{\boldsymbol{\phi}}^{\Im}_k\} - \mathbf{J}_1\Delta\hat{\mathbf{B}}^{\Re}_k\end{aligned} \tag{19}$$

$$\begin{aligned}&\mathbf{F}_4(\tilde{\mathbf{B}}^{\Re}_{k+1}, \tilde{\mathbf{B}}^{\Im}_{k+1}, \tilde{\boldsymbol{\phi}}^{\Re}_{k+1}, \tilde{\boldsymbol{\phi}}^{\Im}_{k+1}) \\ &\quad = \mathbf{F}_4(\tilde{\mathbf{B}}^{\Re}_k + \Delta\hat{\mathbf{B}}^{\Re}_k, \tilde{\mathbf{B}}^{\Im}_k + \Delta\hat{\mathbf{B}}^{\Im}_k, \tilde{\boldsymbol{\phi}}^{\Re}_k + \Delta\hat{\boldsymbol{\phi}}^{\Re}_k, \tilde{\boldsymbol{\phi}}^{\Im}_k + \Delta\hat{\boldsymbol{\phi}}^{\Im}_k) \\ &\quad = -\mathbf{J}_2(\tilde{\mathbf{B}}^{\Re}_k + \Delta\hat{\mathbf{B}}^{\Re}_k) \cdot \mathrm{diag}\{\tilde{\boldsymbol{\phi}}^{\Im}_k + \Delta\hat{\boldsymbol{\phi}}^{\Im}_k\} + \mathbf{J}_2(\tilde{\mathbf{B}}^{\Im}_k \\ &\qquad + \Delta\hat{\mathbf{B}}^{\Im}_k) \cdot \mathrm{diag}\{\tilde{\boldsymbol{\phi}}^{\Re}_k + \Delta\hat{\boldsymbol{\phi}}^{\Re}_k\} - \mathbf{J}_1(\tilde{\mathbf{B}}^{\Im}_k + \Delta\hat{\mathbf{B}}^{\Im}_k) \\ &\quad \approx \mathbf{F}_4(\tilde{\mathbf{B}}^{\Re}_k, \tilde{\mathbf{B}}^{\Im}_k, \tilde{\boldsymbol{\phi}}^{\Re}_k, \tilde{\boldsymbol{\phi}}^{\Im}_k) + \mathbf{J}_2\tilde{\mathbf{B}}^{\Im}_k \cdot \mathrm{diag}\{\Delta\hat{\boldsymbol{\phi}}^{\Re}_k\} \\ &\qquad - \mathbf{J}_2\tilde{\mathbf{B}}^{\Re}_k \cdot \mathrm{diag}\{\Delta\hat{\boldsymbol{\phi}}^{\Im}_k\} - \mathbf{J}_2\Delta\hat{\mathbf{B}}^{\Re}_k \cdot \mathrm{diag}\{\tilde{\boldsymbol{\phi}}^{\Im}_k\} \\ &\qquad + \mathbf{J}_2\Delta\hat{\mathbf{B}}^{\Im}_k \cdot \mathrm{diag}\{\tilde{\boldsymbol{\phi}}^{\Re}_k\} - \mathbf{J}_1\Delta\hat{\mathbf{B}}^{\Im}_k\end{aligned} \tag{20}$$

[1]Similar to [20], $\alpha > 1$ accounts for the fact that the entries of $\Delta\hat{\mathbf{B}}^{\Re}$ or $\Delta\hat{\mathbf{B}}^{\Im}$ should be larger than the other entries of the block matrix.

$$
\begin{aligned}
&\mathbf{F}_5(\tilde{\boldsymbol{\phi}}_{k+1}^{\Re}, \tilde{\boldsymbol{\phi}}_{k+1}^{\Im}) \\
&\quad = \mathbf{F}_5(\tilde{\boldsymbol{\phi}}_k^{\Re} + \Delta\hat{\boldsymbol{\phi}}_k^{\Re}, \tilde{\boldsymbol{\phi}}_k^{\Im} + \Delta\hat{\boldsymbol{\phi}}_k^{\Im}) \\
&\quad = (\tilde{\boldsymbol{\phi}}_k^{\Re} + \Delta\hat{\boldsymbol{\phi}}_k^{\Re}) \circ (\tilde{\boldsymbol{\phi}}_k^{\Re} + \Delta\hat{\boldsymbol{\phi}}_k^{\Re}) + (\tilde{\boldsymbol{\phi}}_k^{\Im} + \Delta\hat{\boldsymbol{\phi}}_k^{\Im}) \\
&\qquad \circ (\tilde{\boldsymbol{\phi}}_k^{\Im} + \Delta\hat{\boldsymbol{\phi}}_k^{\Im}) - \mathbf{1}_{Q\times 1} \\
&\quad \approx \mathbf{F}_5(\tilde{\boldsymbol{\phi}}_k^{\Re}, \tilde{\boldsymbol{\phi}}_k^{\Im}) + 2\cdot \operatorname{diag}\{\tilde{\boldsymbol{\phi}}_k^{\Re}\} \cdot \Delta\hat{\boldsymbol{\phi}}_k^{\Re} \\
&\qquad + 2\cdot \operatorname{diag}\{\tilde{\boldsymbol{\phi}}_k^{\Im}\} \cdot \Delta\hat{\boldsymbol{\phi}}_k^{\Im}
\end{aligned} \tag{21}
$$

where the second-order terms of $\Delta\hat{\mathbf{B}}_k^{\Re}$, $\Delta\hat{\mathbf{B}}_k^{\Im}$, $\Delta\hat{\boldsymbol{\phi}}_k^{\Re}$, and $\Delta\hat{\boldsymbol{\phi}}_k^{\Im}$ have been omitted for the last approximations of (17)-(21). Moreover, the approximation of (21) has utilized the property of Hadamard product, i.e,

$$
\mathbf{z}_1 \circ \mathbf{z}_2 = \operatorname{diag}\{\mathbf{z}_1\} \cdot \mathbf{z}_2, \tag{22}
$$

where $\mathbf{z}_1$ and $\mathbf{z}_2$ are two vectors with the same size.

Similar to the way done in SLS-ESPRIT [20], we vectorize (17)-(20) to generate the linear programming structure of $\Delta\hat{\mathbf{B}}_k^{\Re}$, $\Delta\hat{\mathbf{B}}_k^{\Im}$, $\Delta\hat{\boldsymbol{\phi}}_k^{\Re}$, and $\Delta\hat{\boldsymbol{\phi}}_k^{\Im}$, i.e.,

$$
\begin{aligned}
&\operatorname{vec}\{\mathbf{F}_1(\tilde{\mathbf{B}}_{k+1}^{\Re}, \tilde{\mathbf{B}}_{k+1}^{\Im}, \tilde{\boldsymbol{\phi}}_{k+1}^{\Re}, \tilde{\boldsymbol{\phi}}_{k+1}^{\Im})\} \\
&\quad \approx \operatorname{vec}\{\mathbf{F}_1(\tilde{\mathbf{B}}_k^{\Re}, \tilde{\mathbf{B}}_k^{\Im}, \tilde{\boldsymbol{\phi}}_k^{\Re}, \tilde{\boldsymbol{\phi}}_k^{\Im})\} + (\mathbf{I}_Q \odot (\mathbf{J}_1\tilde{\mathbf{B}}_k^{\Re}))\Delta\hat{\boldsymbol{\phi}}_k^{\Re} \\
&\qquad - (\mathbf{I}_Q \odot (\mathbf{J}_1\tilde{\mathbf{B}}_k^{\Im}))\Delta\hat{\boldsymbol{\phi}}_k^{\Im} + (\operatorname{diag}\{\tilde{\boldsymbol{\phi}}_k^{\Re}\} \otimes \mathbf{J}_1 - \mathbf{I}_Q \otimes \mathbf{J}_2) \\
&\qquad \cdot \operatorname{vec}\{\Delta\hat{\mathbf{B}}_k^{\Re}\} - (\operatorname{diag}\{\tilde{\boldsymbol{\phi}}_k^{\Im}\} \otimes \mathbf{J}_1)\operatorname{vec}\{\Delta\hat{\mathbf{B}}_k^{\Im}\}
\end{aligned} \tag{23}
$$

$$
\begin{aligned}
&\operatorname{vec}\{\mathbf{F}_2(\tilde{\mathbf{B}}_{k+1}^{\Re}, \tilde{\mathbf{B}}_{k+1}^{\Im}, \tilde{\boldsymbol{\phi}}_{k+1}^{\Re}, \tilde{\boldsymbol{\phi}}_{k+1}^{\Im})\} \\
&\quad \approx \operatorname{vec}\{\mathbf{F}_2(\tilde{\mathbf{B}}_k^{\Re}, \tilde{\mathbf{B}}_k^{\Im}, \tilde{\boldsymbol{\phi}}_k^{\Re}, \tilde{\boldsymbol{\phi}}_k^{\Im})\} + (\mathbf{I}_Q \odot (\mathbf{J}_1\tilde{\mathbf{B}}_k^{\Im}))\Delta\hat{\boldsymbol{\phi}}_k^{\Re} \\
&\qquad + (\mathbf{I}_Q \odot (\mathbf{J}_1\tilde{\mathbf{B}}_k^{\Re}))\Delta\hat{\boldsymbol{\phi}}_k^{\Im} + (\operatorname{diag}\{\tilde{\boldsymbol{\phi}}_k^{\Im}\} \otimes \mathbf{J}_1)\operatorname{vec}\{\Delta\hat{\mathbf{B}}_k^{\Re}\} \\
&\qquad + (\operatorname{diag}\{\tilde{\boldsymbol{\phi}}_k^{\Re}\} \otimes \mathbf{J}_1 - \mathbf{I}_Q \otimes \mathbf{J}_2)\operatorname{vec}\{\Delta\hat{\mathbf{B}}_k^{\Im}\}
\end{aligned} \tag{24}
$$

$$
\begin{aligned}
&\operatorname{vec}\{\mathbf{F}_3(\tilde{\mathbf{B}}_{k+1}^{\Re}, \tilde{\mathbf{B}}_{k+1}^{\Im}, \tilde{\boldsymbol{\phi}}_{k+1}^{\Re}, \tilde{\boldsymbol{\phi}}_{k+1}^{\Im})\} \\
&\quad \approx \operatorname{vec}\{\mathbf{F}_3(\tilde{\mathbf{B}}_k^{\Re}, \tilde{\mathbf{B}}_k^{\Im}, \tilde{\boldsymbol{\phi}}_k^{\Re}, \tilde{\boldsymbol{\phi}}_k^{\Im})\} + (\mathbf{I}_Q \odot (\mathbf{J}_2\tilde{\mathbf{B}}_k^{\Re}))\Delta\hat{\boldsymbol{\phi}}_k^{\Re} \\
&\qquad + (\mathbf{I}_Q \odot (\mathbf{J}_2\tilde{\mathbf{B}}_k^{\Im}))\Delta\hat{\boldsymbol{\phi}}_k^{\Im} + (\operatorname{diag}\{\tilde{\boldsymbol{\phi}}_k^{\Re}\} \otimes \mathbf{J}_2 - \mathbf{I}_Q \otimes \mathbf{J}_1) \\
&\qquad \cdot \operatorname{vec}\{\Delta\hat{\mathbf{B}}_k^{\Re}\} + (\operatorname{diag}\{\tilde{\boldsymbol{\phi}}_k^{\Im}\} \otimes \mathbf{J}_2)\operatorname{vec}\{\Delta\hat{\mathbf{B}}_k^{\Im}\}
\end{aligned} \tag{25}
$$

$$
\begin{aligned}
&\operatorname{vec}\{\mathbf{F}_4(\tilde{\mathbf{B}}_{k+1}^{\Re}, \tilde{\mathbf{B}}_{k+1}^{\Im}, \tilde{\boldsymbol{\phi}}_{k+1}^{\Re}, \tilde{\boldsymbol{\phi}}_{k+1}^{\Im})\} \\
&\quad \approx \operatorname{vec}\{\mathbf{F}_4(\tilde{\mathbf{B}}_k^{\Re}, \tilde{\mathbf{B}}_k^{\Im}, \tilde{\boldsymbol{\phi}}_k^{\Re}, \tilde{\boldsymbol{\phi}}_k^{\Im})\} + (\mathbf{I}_Q \odot (\mathbf{J}_2\tilde{\mathbf{B}}_k^{\Im}))\Delta\hat{\boldsymbol{\phi}}_k^{\Re} \\
&\qquad - (\mathbf{I}_Q \odot (\mathbf{J}_2\tilde{\mathbf{B}}_k^{\Re}))\Delta\hat{\boldsymbol{\phi}}_k^{\Im} - (\operatorname{diag}\{\tilde{\boldsymbol{\phi}}_k^{\Im}\} \otimes \mathbf{J}_2)\operatorname{vec}\{\Delta\hat{\mathbf{B}}_k^{\Re}\} \\
&\qquad + (\operatorname{diag}\{\tilde{\boldsymbol{\phi}}_k^{\Re}\} \otimes \mathbf{J}_2 - \mathbf{I}_Q \otimes \mathbf{J}_1)\operatorname{vec}\{\Delta\hat{\mathbf{B}}_k^{\Im}\}
\end{aligned} \tag{26}
$$

where the derivations of the four expressions have used the following two equations,

$$
\operatorname{vec}\{\mathbf{Y}_1\mathbf{Y}_2\mathbf{Y}_3\} = (\mathbf{Y}_3^T \otimes \mathbf{Y}_1)\operatorname{vec}\{\mathbf{Y}_2\} \tag{27}
$$

$$
\operatorname{vec}\{\mathbf{Y}_1 \cdot \operatorname{diag}\{\mathbf{z}\} \cdot \mathbf{Y}_3\} = (\mathbf{Y}_3^T \odot \mathbf{Y}_1)\mathbf{z} \tag{28}
$$

with $\mathbf{Y}_1$, $\mathbf{Y}_2$, $\mathbf{Y}_3$, and $\mathbf{z}$ having appropriate dimensions.

Meanwhile, let $\Delta\hat{\mathbf{B}}_{0,k}^{\Re} = \sum_{p=1}^{k-1} \Delta\hat{\mathbf{B}}_p^{\Re}$ and $\Delta\hat{\mathbf{B}}_{0,k}^{\Im} = \sum_{p=1}^{k-1} \Delta\hat{\mathbf{B}}_p^{\Im}$ be the kth estimation change of $\mathbf{B}^{\Re}$ and $\mathbf{B}^{\Im}$compared with the initial estimates $\tilde{\mathbf{B}}_0^{\Re}$ and $\tilde{\mathbf{B}}_0^{\Im}$. Then, the improved estimates $\tilde{\mathbf{B}}_k^{\Re}$ and $\tilde{\mathbf{B}}_k^{\Im}$ can be expressed as,

$$
\tilde{\mathbf{B}}_k^{\Re} = \tilde{\mathbf{B}}_0^{\Re} + \Delta\hat{\mathbf{B}}_{0,k}^{\Re} = \tilde{\mathbf{B}}_0^{\Re} + \sum_{p=0}^{k-1} \Delta\hat{\mathbf{B}}_p^{\Re} \tag{29}
$$

$$
\tilde{\mathbf{B}}_k^{\Im} = \tilde{\mathbf{B}}_0^{\Im} + \Delta\hat{\mathbf{B}}_{0,k}^{\Im} = \tilde{\mathbf{B}}_0^{\Im} + \sum_{p=0}^{k-1} \Delta\hat{\mathbf{B}}_p^{\Im} \tag{30}
$$

Therefore, with (21) and (23)-(26), the SLS problem in (16) can be solved iteratively via the following optimization,

$$
\min_{\Delta\hat{\mathbf{B}}_k^{\Re}, \Delta\hat{\mathbf{B}}_k^{\Im}, \Delta\hat{\boldsymbol{\phi}}_k^{\Re}, \Delta\hat{\boldsymbol{\phi}}_k^{\Im}} \left\| \mathbf{H}_k \cdot \begin{bmatrix} \Delta\hat{\boldsymbol{\phi}}_k^{\Re} \\ \Delta\hat{\boldsymbol{\phi}}_k^{\Im} \\ \operatorname{vec}\{\Delta\hat{\mathbf{B}}_k^{\Re}\} \\ \operatorname{vec}\{\Delta\hat{\mathbf{B}}_k^{\Im}\} \end{bmatrix} + \begin{bmatrix} \operatorname{vec}\{\mathbf{F}_1(\tilde{\mathbf{B}}_k^{\Re}, \tilde{\mathbf{B}}_k^{\Im}, \tilde{\boldsymbol{\phi}}_k^{\Re}, \tilde{\boldsymbol{\phi}}_k^{\Im})\} \\ \operatorname{vec}\{\mathbf{F}_2(\tilde{\mathbf{B}}_k^{\Re}, \tilde{\mathbf{B}}_k^{\Im}, \tilde{\boldsymbol{\phi}}_k^{\Re}, \tilde{\boldsymbol{\phi}}_k^{\Im})\} \\ \operatorname{vec}\{\mathbf{F}_3(\tilde{\mathbf{B}}_k^{\Re}, \tilde{\mathbf{B}}_k^{\Im}, \tilde{\boldsymbol{\phi}}_k^{\Re}, \tilde{\boldsymbol{\phi}}_k^{\Im})\} \\ \operatorname{vec}\{\mathbf{F}_4(\tilde{\mathbf{B}}_k^{\Re}, \tilde{\mathbf{B}}_k^{\Im}, \tilde{\boldsymbol{\phi}}_k^{\Re}, \tilde{\boldsymbol{\phi}}_k^{\Im})\} \\ \operatorname{vec}\{\mathbf{F}_5(\tilde{\boldsymbol{\phi}}_k^{\Re}, \tilde{\boldsymbol{\phi}}_k^{\Im})\} \\ \kappa \cdot \Delta\hat{\mathbf{B}}_{0,k}^{\Re} \\ \kappa \cdot \Delta\hat{\mathbf{B}}_{0,k}^{\Im} \end{bmatrix} \right\|_2 \tag{31}
$$

where the expression of $\mathbf{H}_k$ is given in (32), as shown at the bottom of this page. As discussed in [20], (31) is a least squares problem and can be solved efficiently using the real-valued QR decomposition (see Appendix VI for more details). $\mathbf{B}_0$ in (2) is chosen as the initial estimate of $\mathbf{B}$, i.e., $\tilde{\mathbf{B}}_0^{\Re} = \mathbf{B}_0^{\Re}$ and $\tilde{\mathbf{B}}_0^{\Im} = \mathbf{B}_0^{\Im}$, while the initial value for $\boldsymbol{\phi}$ is obtained via a simple least squares solution using (2)-(3) as follows,

$$
\tilde{\boldsymbol{\phi}}_0 = \operatorname{diag}^{-1}\{(\mathbf{J}_1\mathbf{B}_0)^{\dagger}(\mathbf{J}_2\mathbf{B}_0)\} \tag{33}
$$

where $\operatorname{diag}^{-1}\{\cdot\}$ is to construct a vector using the diagonal elements of a matrix.

$$
\mathbf{H}_k = \begin{bmatrix}
\mathbf{I}_Q \odot (\mathbf{J}_1\tilde{\mathbf{B}}_k^{\Re}) & -\mathbf{I}_Q \odot (\mathbf{J}_1\tilde{\mathbf{B}}_k^{\Im}) & \operatorname{diag}\{\tilde{\boldsymbol{\phi}}_k^{\Re}\} \otimes \mathbf{J}_1 - \mathbf{I}_Q \otimes \mathbf{J}_2 & -\operatorname{diag}\{\tilde{\boldsymbol{\phi}}_k^{\Im}\} \otimes \mathbf{J}_1 \\
\mathbf{I}_Q \odot (\mathbf{J}_1\tilde{\mathbf{B}}_k^{\Im}) & \mathbf{I}_Q \odot (\mathbf{J}_1\tilde{\mathbf{B}}_k^{\Re}) & \operatorname{diag}\{\tilde{\boldsymbol{\phi}}_k^{\Im}\} \otimes \mathbf{J}_1 & \operatorname{diag}\{\tilde{\boldsymbol{\phi}}_k^{\Re}\} \otimes \mathbf{J}_1 - \mathbf{I}_Q \otimes \mathbf{J}_2 \\
\mathbf{I}_Q \odot (\mathbf{J}_2\tilde{\mathbf{B}}_k^{\Re}) & \mathbf{I}_Q \odot (\mathbf{J}_2\tilde{\mathbf{B}}_k^{\Im}) & \operatorname{diag}\{\tilde{\boldsymbol{\phi}}_k^{\Re}\} \otimes \mathbf{J}_2 - \mathbf{I}_Q \otimes \mathbf{J}_1 & \operatorname{diag}\{\tilde{\boldsymbol{\phi}}_k^{\Im}\} \otimes \mathbf{J}_2 \\
\mathbf{I}_Q \odot (\mathbf{J}_2\tilde{\mathbf{B}}_k^{\Im}) & -\mathbf{I}_Q \odot (\mathbf{J}_2\tilde{\mathbf{B}}_k^{\Re}) & -\operatorname{diag}\{\tilde{\boldsymbol{\phi}}_k^{\Im}\} \otimes \mathbf{J}_2 & \operatorname{diag}\{\tilde{\boldsymbol{\phi}}_k^{\Re}\} \otimes \mathbf{J}_2 - \mathbf{I}_Q \otimes \mathbf{J}_1 \\
2\cdot\operatorname{diag}\{\tilde{\boldsymbol{\phi}}_k^{\Re}\} & 2\cdot\operatorname{diag}\{\tilde{\boldsymbol{\phi}}_k^{\Im}\} & \mathbf{0}_{Q\times MQ} & \mathbf{0}_{Q\times MQ} \\
\mathbf{0}_{MQ\times Q} & \mathbf{0}_{MQ\times Q} & \kappa\cdot\mathbf{I}_{MQ} & \mathbf{0}_{MQ\times MQ} \\
\mathbf{0}_{MQ\times Q} & \mathbf{0}_{MQ\times Q} & \mathbf{0}_{MQ\times MQ} & \kappa\cdot\mathbf{I}_{MQ}
\end{bmatrix} \tag{32}
$$

TABLE 1. **Summary of the proposed method.**

Input: $\mathbf{x}(n)$, $\mathbf{s}(n)$, α, ε, maxIter, c
Ouput: $\{\hat{\theta}_q\}_{q=1}^{Q}$, $\{\hat{\gamma}_q\}_{q=1}^{Q}$
1. Calculate the initial values $\tilde{\mathbf{B}}_0$ and $\tilde{\boldsymbol{\phi}}_0$ of $\mathbf{B}$ and $\boldsymbol{\phi}$ with (2) and (33);
2. Set initial value c such that $c > \varepsilon$, $\kappa = \sqrt{(5M-3)/(\alpha M)}$, $k = 0$;
WHILE ($c > \varepsilon$ AND $k <$maxIter)
3. Solve (31), and obtain $\Delta\hat{\mathbf{B}}_k^{\Re}$, $\Delta\hat{\mathbf{B}}_k^{\Im}$, $\Delta\hat{\boldsymbol{\phi}}_k^{\Re}$, and $\Delta\hat{\boldsymbol{\phi}}_k^{\Im}$;
4. Update: $\Delta\hat{\mathbf{B}}_{0,k+1}^{\Re} = \sum_{p=0}^{k} \Delta\hat{\mathbf{B}}_p^{\Re}$, $\Delta\hat{\mathbf{B}}_{0,k+1}^{\Im} = \sum_{p=0}^{k} \Delta\hat{\mathbf{B}}_p^{\Im}$,
$\tilde{\mathbf{B}}_{k+1}^{\Re} = \tilde{\mathbf{B}}_0^{\Re} + \Delta\hat{\mathbf{B}}_{0,k+1}^{\Re}$, $\tilde{\mathbf{B}}_{k+1}^{\Im} = \tilde{\mathbf{B}}_0^{\Im} + \Delta\hat{\mathbf{B}}_{0,k+1}^{\Im}$,
$\tilde{\boldsymbol{\phi}}_{k+1}^{\Re} = \tilde{\boldsymbol{\phi}}_k^{\Re} + \Delta\hat{\boldsymbol{\phi}}_k^{\Re}$, $\tilde{\boldsymbol{\phi}}_{k+1}^{\Im} = \tilde{\boldsymbol{\phi}}_k^{\Im} + \Delta\hat{\boldsymbol{\phi}}_k^{\Im}$;
5. $c = \min\{1/Q\left\Vert\Delta\hat{\boldsymbol{\phi}}_k^{\Re}\right\Vert_2, 1/Q\left\Vert\Delta\hat{\boldsymbol{\phi}}_k^{\Im}\right\Vert_2, 1/\sqrt{MQ}\left\Vert\Delta\hat{\mathbf{B}}_k^{\Re}\right\Vert_F, 1/\sqrt{MQ}\left\Vert\Delta\hat{\mathbf{B}}_k^{\Im}\right\Vert_F\}$;
6. $k = k + 1$;
END WHILE
7. Obtain the final estimates of $\{\hat{\theta}_k\}_{k=1}^{2}$ and $\{\hat{\gamma}_q\}_{q=1}^{Q}$ with (34)-(35).

Let $\hat{\mathbf{B}}^{\Re}$, $\hat{\mathbf{B}}^{\Im}$, $\hat{\boldsymbol{\phi}}^{\Re}$, and $\hat{\boldsymbol{\phi}}^{\Im}$ represent the final estimates obtained by (31). Then, the estimates of DOA and complex amplitudes can be expressed as

$$\hat{\theta}_q = \arcsin\{-\text{angle}\{\hat{\phi}_q^{\Re} + j\hat{\phi}_q^{\Im}\}\lambda/(2\pi d)\} \tag{34}$$

$$\hat{\gamma}_q = \frac{1}{M}\sum_{m=1}^{M}[\hat{b}_{mq}^{\Re} + j\hat{b}_{mq}^{\Im}]/(\hat{\phi}_q^{\Re} + j\hat{\phi}_q^{\Im})^{m-1} \tag{35}$$

where $\hat{b}_{mq}^{\Re}$, $\hat{b}_{mq}^{\Im}$, $\hat{\phi}_q^{\Re}$, and $\hat{\phi}_q^{\Im}$ denote the qth column of $\hat{\boldsymbol{\phi}}^{\Re}$ and $\hat{\boldsymbol{\phi}}^{\Im}$, and the mth row and qth column of $\hat{\mathbf{B}}^{\Re}$ and $\hat{\mathbf{B}}^{\Im}$, respectively.

The procedure of the proposed solution is summarized in Table 1. As shown, termination of the iterative process is set when a fixed number of iterations is reached or one of the norms of $\Delta\hat{\mathbf{B}}_k^{\Re}$, $\Delta\hat{\mathbf{B}}_k^{\Im}$, $\Delta\hat{\boldsymbol{\phi}}_k^{\Re}$, and $\Delta\hat{\boldsymbol{\phi}}_k^{\Im}$ is close to the fixed threshold value ε. In practice, the iterative process is often terminated after less than 10 and sometimes even 1 iterations.

IV. ALGORITHM ANALYSIS

A. LIMITING FACTORS

For the proposed method, the number of snapshots should be no less than the number of signals, i.e., $N \geq Q$, to guarantee the existence of $\mathbf{R}_{ss}^{-1}$. While for the DEML method [9], both $\mathbf{R}_{ss}$ and $\mathbf{R}_{ww}$ should be invertible, i.e., $N \geq \max\{M, Q\}$. Apparently, if $M > Q$, which is a common case for DOA estimation applications, the proposed method requires less number of snapshots than the DEML method.

B. COMPUTATIONAL COMPLEXITY

Now we analyse the computational complexity of the proposed method compared with the DEML method [9], [29] in terms of the number of complex-valued multiplications. Since the proposed method involves the real-valued QR decomposition, we transform the number of real-valued multiplications into complex-valued ones by a factor 1/4. Since the proposed method is designed for ULA, the DEML method with polynomial rooting is used for comparison.

For the proposed method,

(i) Initial estimations of $\mathbf{B}$ and $\boldsymbol{\phi}$ with (2) and (33): $O\{MQN + Q^2N + Q^3 + 2(M-1)Q^2\}$;

(ii) Estimation of $\hat{\mathbf{B}}^{\Re}$, $\hat{\mathbf{B}}^{\Im}$, $\hat{\boldsymbol{\phi}}^{\Re}$, and $\hat{\boldsymbol{\phi}}^{\Im}$ by solving (31) with real-valued QR decomposition: $O\{6K(M+1)^2(2M-1)Q^3\}$, where K is the total number of iterations;

(iv) DOA and complex amplitude estimation using (34)-(35): $O\{2Q + 2QM\}$.

Then, with $N > M > Q$ and K being less than 10 in practice, the overall computational complexity of the proposed method is approximately $O\{MQN + 24KM^3Q^3\}$.

Similarly, for the DEML method,

(i) Estimation of $\hat{\mathbf{B}}$ and $\hat{\mathbf{Q}}$ using [9, eqs. (17) and (18)]: $O\{M^2N + MQN + Q^2(N + 3M) + 2Q^3\}$;

(ii) DOA and complex amplitude estimation using [29, (34) and (51)–(56)]: $O\{2(M-1)^2 + 4M + 2(M-1)M^2 + (M-1)^3 + 4M^2\}$.

For the case discussed above, its overall computational complexity is about $O\{M^2N + MQN + 3M^3\}$.

It can be seen that the proposed method has a larger computational complexity than the DEML method owing to the large matrix $\mathbf{H}_k$ involved in the SLS optimization step.

V. SIMULATION RESULTS

In this section, performance of the proposed method is investigated in comparison with that of DEML [9], [29], and the Cramer-Rao bound (CRB) for known waveforms [9], [19] and unknown waveforms [30], respectively. It is assumed that $d = \lambda/2$, and the waveforms of all sources are known with unit power. For the proposed method, we set $\alpha = 10$, $\varepsilon = 10^{-12}$, and maxIter $= 100$.[2]

Example 1: In the first example, we investigate the performance of the proposed method against SNR. The DOAs, and complex amplitudes of two sources are set to $-15°$, $0°$,

[2]For different simulation conditions, various simulations have been conducted and shown that a smaller α can reduce the number of iterations, but may not give the highest DOA estimation accuracy, and the DOA estimation accuracy is not sensitive to the value of α. Therefore, to achieve a balance between angle estimation performance and running time, $\alpha = 10$ is chosen here.

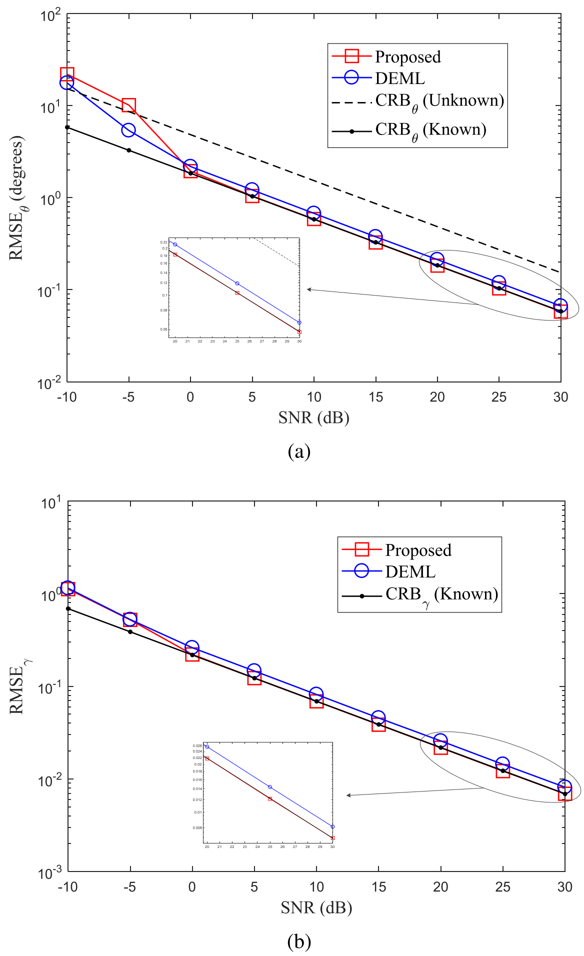


FIGURE 1. RMSE versus SNR, $K = 2$, $M = 4$, $N = 10$. (a) DOA. (b) Complex amplitude.

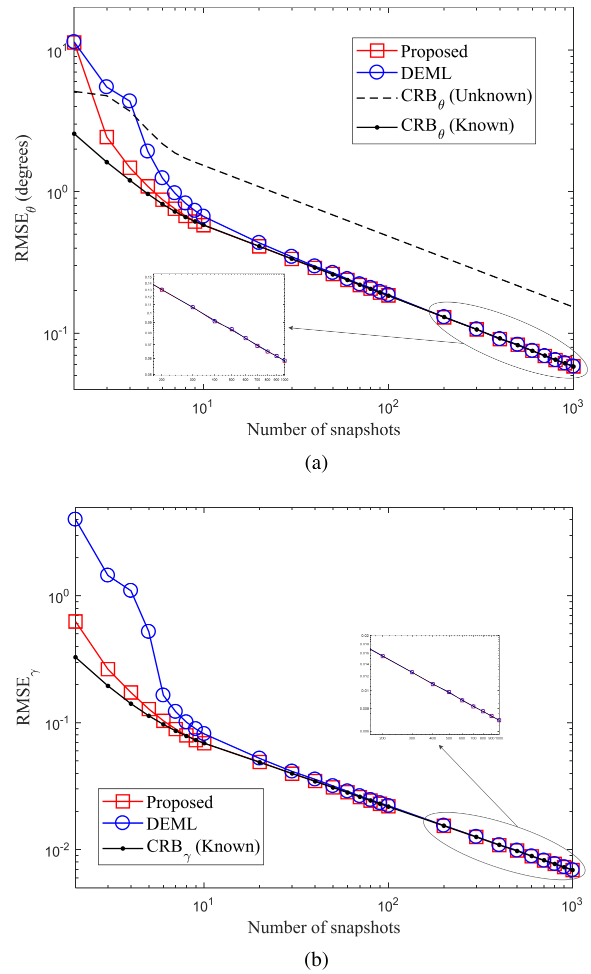


FIGURE 2. RMSE versus number of snapshots, $K = 2$, $M = 4$, SNR = 10 dB. (a) DOA. (b) Complex amplitude.

$e^{j0.3\pi}$, and $e^{-j0.4\pi}$, respectively. With $M = 4$, and $N = 10$, the input SNR varies from -15 dB to 30 dB with an interval of 5 dB. 10000 independent Monte Carlo trials are conducted for each SNR and the root mean square error (RMSE) results are shown in Fig. 1.

Example 2: In this example, the performance of the proposed method with respect to the number of snapshots is examined. The settings are the same as Example 1 except that SNR = 10 dB and N ranges from 2 to 1000. The estimation results are provided in Fig. 2.

As shown in Figs. 1a-2b, the proposed method has provided a much better performance than the DEML method for small numbers of snapshots ($N \leq 10$) even when SNR is very high, since the proposed method has exploited more structure information and treated the bias between the true $\mathbf{B}$ and its estimate as a parameter to estimate. Besides, when N becomes large i.e., $N \geq 100$, performance of the two methods becomes almost the same and agrees with the CRB. However, from Fig. 1a, the DOA estimation performance of the proposed method is worse than that of DEML for low SNR values, and a possible reason is that the DEML method exploits more statistical information about noise while the proposed method does not. Moreover, according to Section IV-B and Figs. 1a-2b, the proposed method has a higher computational complexity than the DEML method, but does not provide a significantly better performance. However, the proposed method is applicable to any array geometry with a rotation invariance structure, while, for low computational complexity, the DEML method can only be applied to ULA using the polynomial rooting technique, and requires time consuming spectrum search for other array geometries.

VI. CONCLUSIONS

A DOA estimation method for sources with short known waveforms in snapshot size has been introduced. The connection between the known waveform source DOA estimation and the structured least squares (SLS) method was first established; then, with the special structural information of rotation invariance factor, a constraint on the amplitudes of the rotation invariance factor vector was proposed. Since it involves the conjugate operator, which is nonlinear and cannot be applied directly to the SLS optimization problem, we have to split it into a real part and an imaginary part. Finally, the problem was solved via the iterative least squares method.

As demonstrated by simulation results, when the number of snapshots of known waveforms is small, the proposed method has provided a better DOA estimation performance than the existing DEML method.

APPENDIX
SOLUTION TO (31)

For simplicity, define

$$\hat{\mathbf{g}}_k = [(\Delta\hat{\boldsymbol{\phi}}_k^{\Re})^T, (\Delta\hat{\boldsymbol{\phi}}_k^{\Im})^T,\ \mathrm{vec}^T\{\Delta\hat{\mathbf{B}}_k^{\Re}\}, \mathrm{vec}^T\{\Delta\hat{\mathbf{B}}_k^{\Im}\}]^T \tag{36}$$

$$\begin{aligned}\hat{\mathbf{b}}_k = -[&\mathrm{vec}^T\{\mathbf{F}_{1k}\}, \mathrm{vec}^T\{\mathbf{F}_{2k}\}, \mathrm{vec}^T\{\mathbf{F}_{3k}\}, \mathrm{vec}^T\{\mathbf{F}_{4k}\},\\ &\mathrm{vec}^T\{\mathbf{F}_{5k}\}, (\kappa\cdot\Delta\hat{\mathbf{B}}_{0,k}^{\Re})^T, (\kappa\cdot\Delta\hat{\mathbf{B}}_{0,k}^{\Im})^T]^T\end{aligned} \tag{37}$$

where $\mathbf{F}_{tk}$ denotes $\mathbf{F}_t(\tilde{\mathbf{B}}_k^{\Re}, \tilde{\mathbf{B}}_k^{\Im}, \tilde{\boldsymbol{\phi}}_k^{\Re}, \tilde{\boldsymbol{\phi}}_k^{\Im})$, $t = 1, 2, 3, 4$, and $\mathbf{F}_{5k}$ represents $\mathbf{F}_5(\tilde{\boldsymbol{\phi}}_k^{\Re}, \tilde{\boldsymbol{\phi}}_k^{\Im})$.

With (36)-(37), the optimization problem in (31) can be transformed into the following linear programming problem,

$$\mathbf{H}_k\hat{\mathbf{g}}_k = \hat{\mathbf{b}}_k \tag{38}$$

Applying QR decomposition to $\mathbf{H}_k$, we have

$$\mathbf{H}_k = \mathbf{Q}_k\mathbf{R}_k \tag{39}$$

where $\mathbf{Q}_k$ is orthogonal with size of $3Q(2M-1)\times 3Q(2M-1)$ and $\mathbf{R}_k$ has the size of $3Q(2M-1)\times 2Q(M+1)$. And, it is easy to prove that

$$\mathbf{Q}_k^T\mathbf{H}_k\mathbf{R}_k = \begin{bmatrix}\mathbf{U}_k\\ \mathbf{0}_{Q(4M-5)\times 2Q(M+1)}\end{bmatrix}. \tag{40}$$

where $\mathbf{U}_k$ is a square and triangular matrix with size of $2Q(M+1)\times 2Q(M+1)$.

Then, from (40), (38) can be simplified further as follows,

$$\mathbf{U}_k\hat{\mathbf{g}}_k = \mathbf{Q}_k^T\hat{\mathbf{b}}_k \tag{41}$$

It is noticed that $\mathbf{U}_k$ is triangular, and we can use the back substitution technique to obtain $\hat{\mathbf{g}}_k$ [31].

REFERENCES

[1] H. L. Van Trees, *Detection, Estimation, and Modulation Theory, Optimum Array Processing*. Hoboken, NJ, USA: Wiley, 2004.

[2] G.-Q. Zhao, *Principle of Radar Countermeasure*, 2nd ed. Xi'an, China: Xidian University, 2012.

[3] R. O. Schmidt, "Multiple emitter location and signal parameter estimation," *IEEE Trans. Antennas Propag.*, vol. AP-34, no. 3, pp. 276–280, Mar. 1986.

[4] R. Roy and T. Kailath, "Esprit-estimation of signal parameters via rotational invariance techniques," *IEEE Trans. Acoust., Speech, Signal Process.*, vol. 37, no. 7, pp. 984–995, Jul. 1989.

[5] S. Marcos, A. Marsal, and M. Benidir, "The propagator method for source bearing estimation," *Signal Process.*, vol. 42, no. 2, pp. 121–138, 1995.

[6] J. Ward and R. T. Compton, Jr., "Improving the performance of a slotted ALOHA packet radio network with an adaptive array," *IEEE Trans. Commun.*, vol. 40, no. 2, pp. 292–300, Feb. 1992.

[7] A. Jakobsson, A. L. Swindlehurst, and P. Stoica, "Subspace-based estimation of time delays and Doppler shifts," *IEEE Trans. Signal Process.*, vol. 46, no. 9, pp. 2472–2483, Sep. 1998.

[8] J. Li and R. T. Compton, Jr., "Maximum likelihood angle estimation for signals with known waveforms," *IEEE Trans. Signal Process.*, vol. 41, no. 9, pp. 2850–2862, Sep. 1993.

[9] J. Li, B. Halder, P. Stoica, and M. Viberg, "Computationally efficient angle estimation for signals with known waveforms," *IEEE Trans. Signal Process.*, vol. 43, no. 9, pp. 2154–2163, Sep. 1995.

[10] N. Wang, P. Agathoklis, and A. Antoniou, "A new DOA estimation technique based on subarray beamforming," *IEEE Trans. Signal Process.*, vol. 54, no. 9, pp. 3279–3290, Sep. 2006.

[11] J.-F. Gu, S. C. Chan, W.-P. Zhu, and M. N. S. Swamy, "DOA estimation and tracking for signals with known waveform via symmetric sparse subarrays," in *Proc. IEEE 55th Int. Midwest Symp. Circuits Syst. (MWSCAS)*, Aug. 2012, pp. 952–955.

[12] J.-F. Gu, W.-P. Zhu, and M. N. S. Swamy, "Sparse linear arrays for estimating and tracking DOAs of signals with known waveforms," in *Proc. IEEE Int. Symp. Circuits Syst. (ISCAS)*, May 2013, pp. 2187–2190.

[13] J.-F. Gu, W.-P. Zhu, and M. N. S. Swamy, "Fast and efficient DOA estimation method for signals with known waveforms using nonuniform linear arrays," *Signal Process.*, vol. 114, pp. 265–276, Sep. 2015.

[14] J.-F. Gu, W.-P. Zhu, and M. N. S. Swamy, "Direction of arrival tracking for signals with known waveforms based on block least squares techniques," *J. Franklin Inst.*, vol. 354, no. 11, pp. 4573–4594, 2017.

[15] M. Cedervall and R. L. Moses, "Efficient maximum likelihood DOA estimation for signals with known waveforms in the presence of multipath," *IEEE Trans. Signal Process.*, vol. 45, no. 3, pp. 808–811, Mar. 1997.

[16] H. Li, G. Liu, and J. Li, "Angle estimator for signals with known waveforms," *Electron. Lett.*, vol. 35, no. 23, pp. 1992–1994, Nov. 1999.

[17] L. N. Atallah and S. Marcos, "DOA estimation and association of coherent multipaths by using reference signals," *Signal Process.*, vol. 84, no. 6, pp. 981–996, 2004.

[18] J.-F. Gu, P. Wei, and H.-M. Tai, "Fast direction-of-arrival estimation with known waveforms and linear operators," *IET Signal Process.*, vol. 2, no. 1, pp. 27–36, Mar. 2008.

[19] Y.-Y. Dong, C.-X. Dong, W. Liu, M.-M. Liu, and Z.-Z. Tang, "Robust DOA estimation for sources with known waveforms against Doppler shifts via oblique projection," *IEEE Sensors J.*, vol. 18, no. 16, pp. 6735–6742, Aug. 2018.

[20] M. Haardt, "Structured least squares to improve the performance of ESPRIT-type algorithms," *IEEE Trans. Signal Process.*, vol. 45, no. 3, pp. 792–799, Mar. 1997.

[21] M. Shaghaghi and S. A. Vorobyov, "Subspace leakage analysis and improved DOA estimation with small sample size," *IEEE Trans. Signal Process.*, vol. 63, no. 12, pp. 3251–3265, Jun. 2015.

[22] H. Park, J. Ben Rosen, and J. Glick, "Structured total least norm method for Toeplitz problems," in *Proc. IEEE Int. Conf. Acoust., Speech, Signal Process. (ICASSP)*, vol. 2, May 1995, pp. 1141–1144.

[23] J. Ben Rosen, H. Park, and J. Glick, "Total least norm formulation and solution for structured problems," *SIAM J. Matrix Anal. Appl.*, vol. 17, no. 1, pp. 110–126, 1996.

[24] M. Haardt and J. A. Nossek, "Structured least squares to improve the performance of ESPRIT-type high-resolution techniques," in *Proc. IEEE Int. Conf. Acoust., Speech, Signal Process. (ICASSP)*, vol. 5, May 1996, pp. 2805–2808.

[25] J. Ben Rosen, H. Park, and J. Glick, "Structured total least norm for nonlinear problems," *SIAM J. Matrix Anal. Appl.*, vol. 20, no. 1, pp. 14–30, 1998.

[26] M. R. J. A. E. Kwakkernaat, Y. L. C. Jong, J. C. Bultitude, and M. H. A. J. Herben, "Improved structured least squares for the application of unitary ESPRIT to cross arrays," *IEEE Signal Process. Lett.*, vol. 13, no. 6, pp. 349–352, Jun. 2006.

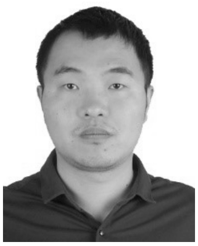
[27] J. Steinwandt, F. Roemer, and M. Haardt, "Performance analysis of ESPRIT-type algorithms for strictly non-circular sources using structured least squares," in *Proc. 5th IEEE Int. Workshop Comput. Adv. Multi-Sens. Adapt. Process. (CAMSAP)*, Dec. 2013, pp. 316–319.

[28] Y. Shi, L. Huang, C. Qian, and H.-C. So, "Direction-of-arrival estimation for noncircular sources via structured least squares–based esprit using three-axis crossed array," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 51, no. 2, pp. 1267–1278, Apr. 2015.

[29] J. Li, B. Halder, P. Stoica, M. Viberg, and T. Kailath, "Decoupled maximum likelihood angle estimation for signals with known waveforms," Chalmers Univ. Technol., Gothenburg, Sweden, Tech. Rep. CTH-TE-8, 1994.

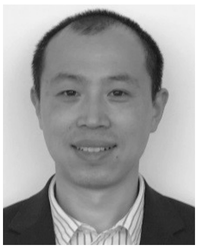
[30] S. M. Kay, *Fundamentals of Statistical Signal Processing*. Upper Saddle River, NJ, USA: Prentice-Hall, 1993.

[31] G. H. Golub and C. F. Van Loan, *Matrix Computations*. Baltimore, MD, USA: The Johns Hopkins Univ. Press, 1996.



YANG-YANG DONG was born in Bengbu, Anhui, China. He received the B.Eng. and Ph.D. degrees in electronic science and technology from Xidian University, Xi'an, China, in 2012 and 2017, respectively. Since 2017, he has been a Lecturer with the School of Electronic Engineering, Xidian University, where he also held a post-doctoral position. His current research interests include array signal processing, multilinear algebra, and information geometry.

CHUNXI DONG was born in Sanmenxia, Henan, China. He received the Ph.D. degree from Xidian University in 2004. He is currently an Associate Professor and the Director of the Information Technology Department, School of Electronic Engineering, Xidian University. His current research interests include high speed signal processing and system simulation.



WEI LIU (S'01–M'04–SM'10) received the B.Sc. degree in space physics and the LL.B. degree in intellectual property law from Peking University, China, in 1996 and 1997, respectively, the M.Phil. degree from the University of Hong Kong in 2001, and the Ph.D. degree from the School of Electronics and Computer Science, University of Southampton, U.K., in 2003. He held a post-doctoral position first at Southampton and later at the Department of Electrical and Electronic Engineering, Imperial College London. Since 2005, he has been with the Department of Electronic and Electrical Engineering, University of Sheffield, U.K., first as a Lecturer and then a Senior Lecturer. He has published about 260 journal and conference papers, three book chapters, and a research monograph about wideband beamforming *Wideband Beamforming: Concepts and Techniques* (John Wiley, 2010). Another book about low-cost smart antennas will appear in 2019 *Low-cost Smart Antennas* (Wiley-IEEE Press). His area of research covers a wide range of topics in signal processing, with particular focus on sensor array signal processing and its various applications in wireless communications, radar, sonar, satellite navigation, human–computer interface, and big data analytics.

He is an elected member of the Digital Signal Processing Technical Committee of the IEEE Circuits and Systems Society and the Sensor Array and Multichannel Signal Processing Technical Committee of the IEEE Signal Processing Society (elected Vice-Chair from 2019). He was the General Co-Chair for the 2018 IEEE Sensor Array and Multichannel Signal Processing Workshop, Sheffield, U.K. He is currently an Associate Editor for the IEEE Transactions on Signal Processing and the IEEE Access, and an Editorial Board Member of the *Journal Frontiers of Information Technology and Electronic Engineering*.

MING-MING LIU was born in Ningxia, China. She received the M.Eng. degree in electronic science and technology from Xidian University, Xi'an, China, in 2016, where she is currently pursuing the Ph.D. degree in electronic science and technology. Her research interests include compressed sensing, and array and radar signal processing.

ZHENGZHAO TANG was born in Baoji, Shaanxi, China. He received the B.Eng. degree in electronic science and technology from Xidian University, Xi'an, China, in 2014, where he is currently pursuing the Ph.D. degree with the School of Electronic Engineering. His current research interests include inverse synthetic aperture radar imaging and its countermeasure technology.